Power System Generation, Transmission and Distribution

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Lecture No # 14

Power Flow through a Line

Good morning ladies and gentleman, today we starting lecture 14 on Power Flow through a Line. Now, let us first do what was that gamma, which we used in earlier derivations, if we recall this propagation constant is a complex number.

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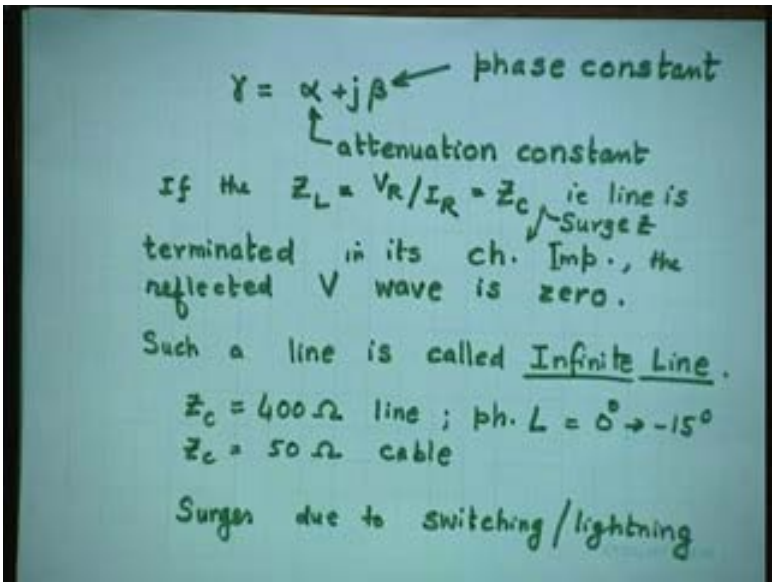

So, it is real thing alpha is called attenuation constant and the machinery part detail is called phase constant. Now, if the load impedance is exactly equal to characteristic impedance or surge impedance, then, if line is terminated in that load impedance, which is equal to its characteristic impedance, the reflected wave, voltage wave is zero this is a very important property, that if you terminate the line in its characteristic impedance or it is also called surge impedance.

Then what happens then such a line is called infinite line, why this called infinite line? The receiving and chinks as if the line is continuing, the no reflection absolutely no reflection and if there is no reflection is it has not terminated. Now the characteristic

typical values of characteristic impedance is 400 ohm line value, for line the value for the transmission line and same value is roughly 8 times less for cable, can you tell me why? Why should there be characteristic impedance, so high for a line and so, low for a cable because, the definition of characteristic impedance is under root l by c or under root z by y c is very high in case of cables, because it is right, there in the ground and the effect of ground is very predominant there and is very close also and hence, this z c is always low for cable.

Of course, you are assuming 50 hertz frequency of phase angle varies 0 degree to minus 15 degree for z c, because it is also a complex quantity. Suggest to switching a lighting is very important this chapter might have done, this topic might have done in your under graduate otherwise, you can revised in the brown book there is full fledged chapter on power system tangents, right and you can revised it, reviewed it, read it, as and when you get time.

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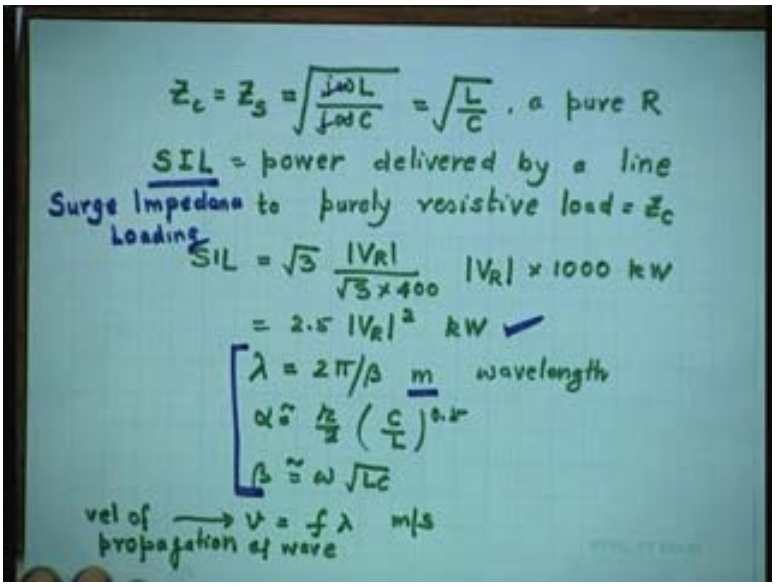

Now I was just repeating z c is equal to z s is equal to j omega L over j omega C, j omega j omega gets cancel, so we get under root L by C, what is under root L by C it is a pure resistance if we carry out a dimensional analysis which a way you might done in your earlier classes you will get a unique as ohm which is a pure resistance, what is S I L? It is called Surge Impedance Loading, say very important parameter power delivered by a line to purely resistive load, which is equal to z C, S I L can be computate by

finding out root 3 V R by root 3400 V R into 1000 kilo watt, this is the power delivered now which on simplification you get 2.5 times V R square kilo watt, this is the formula for calculating S I L, once you know the receiving a voltage.

Now, lambda is the wavelength is defined by 2 pi by beta naturally units of wavelength of meters, so alpha is roughly r by 2 C by L under root and the beta is omega L C, so these three field you can used in numerical, the detailed derivation is given in the book you can read but these three quantities are very important. What is the velocity of propagation of wave? V is equal to f lambda f is frequency and lambda is wavelength computed like this, the natural units of velocities meters per second, if we recall did talk about Ferranti effect sometime back.

What is Ferranti effect and why this is important? It is very rare that receiving end voltage becomes more than sending end normally, most of the time 99 percent of time receiving end voltage is always lower than sending a voltage, at best it can be equal if you have compensation, but nobody will like to compensate so much, that it becomes higher than the sending end, that is never done, at the most we have flat voltage start. By flat voltage start I mean the nodes all node voltages all bus bus voltages are roughly same which is equal to 1per unit time as per as possible, but here is in vocation, here is in instance in which, the receiving end no load voltage becomes more than sending end voltage and this effect is called Ferranti effect.

Normally, this is asked in interviews in viva vice examination, even in examination if there are small questions, one mark questions are asked some examinations like gate is now purely objective type, so you must have all appeared in gate those of you got fellowship, effect of line capacitance is this is because, of line capacitance and that is why short line there is no Ferranti effect, so, whenever line capacitance is important that is the medium line, onwards especially for long line where capacitance is really quite predominant.

In that case, it becomes more visible and as the line length increases the effect becomes more pronounced, say 1000 kilo meter the phenomenon is called Ferranti effect; let us prove this, here I written without proof.

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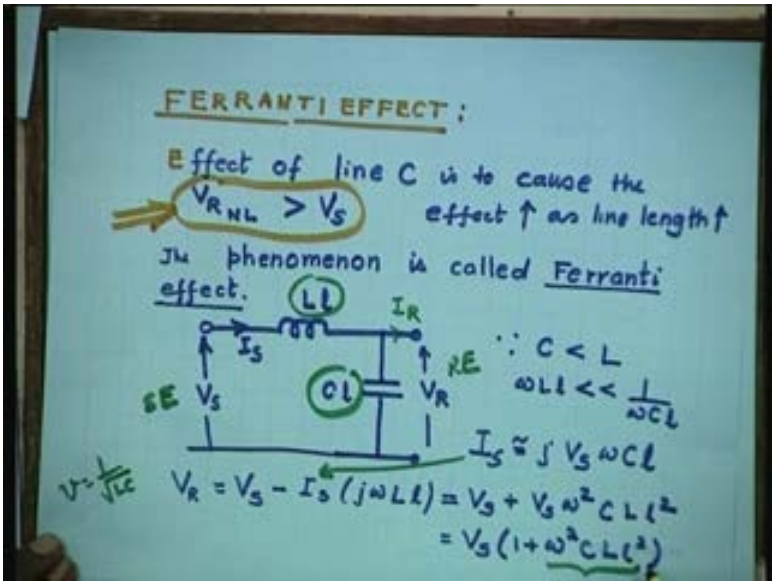

I mean you like a nice students may agree whatever I say, but some of you may privately questioned, by how do you say, this can you prove it. So, here is a very small derivation given, to show that the indeed the receiving end voltage is more than sending end, this because this phenomenon is primarily associated with light load conditions or no load conditions. Now this a very simple model, there is a capacitance c very first model, which we used before t and pi write the L model, so here there is the capacitance at the end of the line and there is an inductances and l is the length of the line, so total inductance is l into l, total capacitance is c into l, this is sending end and this is receiving end, this is V r, this is L l, this is I S and this is I R.

Now, because c is less than L, L is the most important parameter in transmission line, that why L never gets neglected, never even in if it is a one foot line there will be L smallest possible line, so omega L l will be much, much less than one over omega C l alright what is I S? j V S in to omega C l alright. So, V R is V S minus the drop in the inductances, drop is I S into the impedances, which in this case is purely inductive reactance, value omega L into l. Now V S plus substitute the value of I S from here, you get V S omega S square C L l square, take V S common 1 plus omega square c L l square write; now, still is not clear, whether this is a positive quantity, unless until you show that, this is more than one sorry more than 0 how do you say that V R is more than V S in order to do that, all of we know that is c l is nothing but, not always positive velocity what is velocity v is equal to 1 upon under root L C.

So, this will be one upon V squared, so all I squared that and no square quantity in the world can be negative, even if the quantity itself is negative, so some some fellow can say well am not convinced, now is convinced because, we know that V is equal to…

So now, this is positive, is no height of doubt left, so we have proved in a very small brief concise way that, Ferranti effect indeed exist and under that condition V S becomes less than V R that all it is Ferranti effect, so we small questions comes, explain Ferranti effect show that, we show that V R is more than V S, so this is what we do, there is a complicated proof also given in the book the exits proof, if we want you can see that also, I am not sure whether you studied tuned power lines, did you? Yeah, so in case all of you studied tuned power lines.

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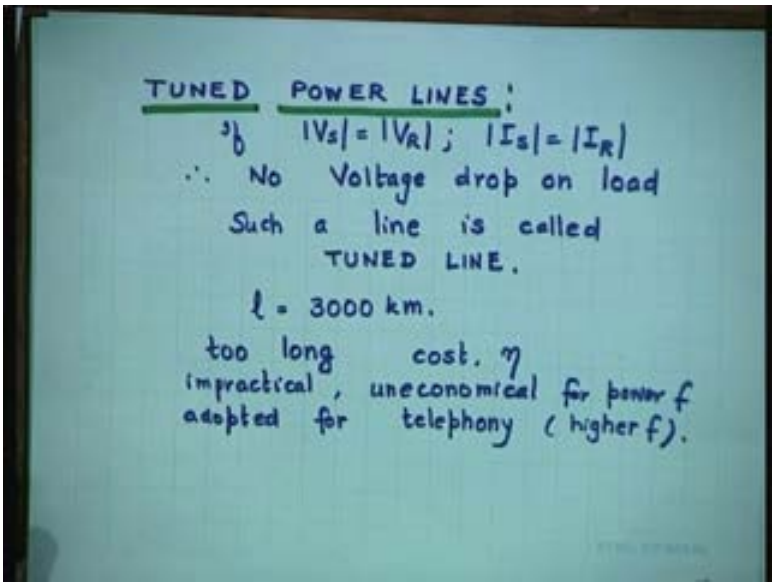

Then you will all recall that, these are important in electronics communication and electronics, which is the number one field today, even ahead of computer, though computer and I T are now again looking back and their again coming sort of one top, but there are fluctuating wave, you know after four years whether they will remain, but electronics will always remaining for front; so unfortunately this tuned power lines are not used in power systems, but there used in electronics, why? Let us examine that, what is definition of tune power line? Anybody?

Student: Magnitude

Magnitude word is very see, each keyword is very important if you missed that word, you will get zero and do not say sir I meant magnitude only, so it will be very careful in viva because, their nobody give an another chance, in fact they nobody will let you know whether you are giving correct answer or wrong answer, they may just very good, very good you feel wow, I am getting appreciated. In fact, he just laughing on your no sense, is not only voltage magnitude of current as well should be equal, because magnitude is equal normal case anyway it is suppose to be equal, that is what voltage start is all about you know the all voltages throughout this system is suppose to be nearly same, because all equipments are rated at a given voltage, but current also same that makes it special case and a such a line is called tuned power lines; that means no voltage drop on load, such a line is called tuned line.

Unfortunately, if you derive such a condition, you get the value of length as 3000 kilometer, not every where it is possible to have transmission line of this length there are very few countries in the world having their length or breadth more than 3000 kilometers, our leave China and India and formal soviet union or even today Russia is a big country. So, too long, too costly proposition, even efficiency is a problem, so it is impractical and economical for power frequency, at power frequency, it is better not to have such tuned power line, adopted for telephony, that what is I said electronics and telecommunication because, they have higher frequency Megahertz, Gigahertz at that point it is ok.

Just was a passing reference, let me tell you is sea on ships and airport airplanes, we have 400 hertz, even that, not sufficient they have, why have 400 hertz because, weight becomes a weighty problem, an important problem, you do not want higher weight on plane, so the all equipments, all devices, all pure, even power plant if you have known that will be 400 hertz, so that, the the size is small, the weight is small. Equivalent circuit of long line we had derived last time, if you recall, we did equivalent exact analysis of long line and we proved a b c d constants in terms of hyperbolic sine and cosine, then we have given two alternate ways of computing a b c d constants and those formula can be used; however, some people still want that, I want equivalent T circuit, I want equivalent pi circuit even for a long line.

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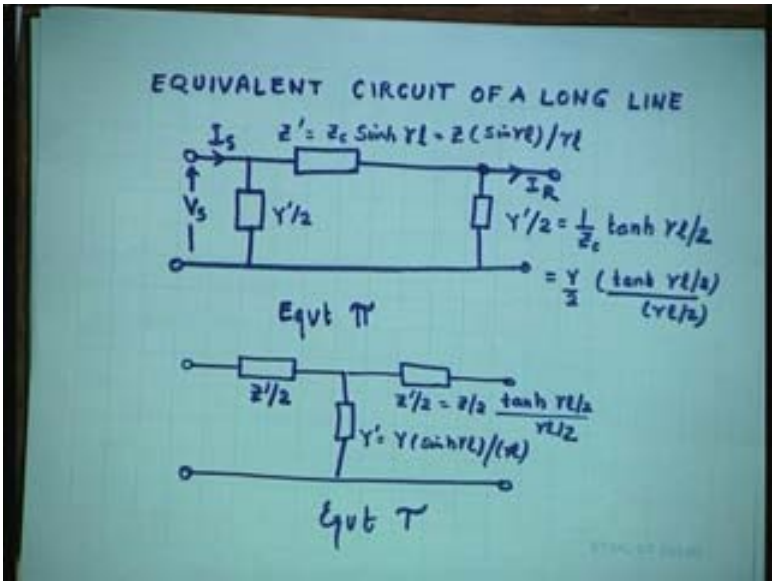

So, this can be derived, but I thought since all of you have done it, so I just give the final answer for equivalent pi as well as equivalent T and here it is no longer z and Y by 2, Y by 2, but here z dash and which is equal to z c hyperbolic sine gamma l or z sine gamma l upon gamma l both are same. Similarly, the stunt range is equal to 1 over z c hyperbolic tangent gamma l by 2 on Y by 2 tangent hyperbolic of course, gamma l by 2 gamma l by 2, right you can verify this and see it is alright, I feel this should be alright.

Similarly in t z by 2 z dash by 2 z dash by 2 is equal to z 2 hyperbolic time gamma l by 2 gamma l by 2 and then Y dash is Y hyperbolic sine gamma l upon gamma l, so this is equivalent t, this is equivalent pi, there is a problem given in the book, where I have asked you to show it in tabular form, the values of sending in quantities, power factors, currents, voltages for all the models small line, medium line T, pi, long line, exact long line T, long line equivalent pi and you compare how the values vary.

So, that problem you solve attempt at home, if you have any problem please let me know, may be will have one tutorial class next week and in that class will solve some problems, so please do bring calculators.

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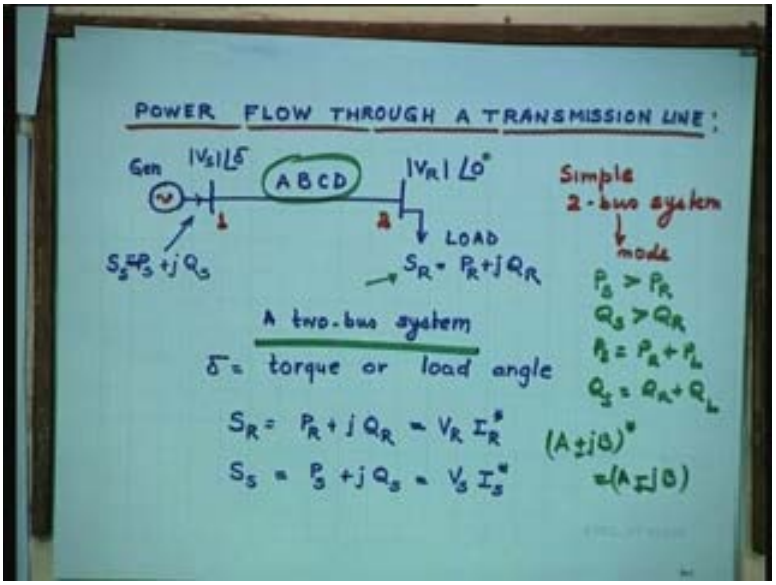

Now, we come to the very important topic of power flow through a transmission line, because the whole this exercise is for what, you are generating somewhere and fortunately and unfortunately load is somewhere else, for all 1000 of kilometer survey take a example of Uttar Pradesh U P, most of the generation in historic U P Singroli

So, super thermal power station, but most of the load the industries are invest in U P for various reasons, you can see Hardwar though of course, it is more in U P, but it say there is a industry there is B H E L there, then Ghaziabad, Noida all these things are in the Western side and the generation is in the Eastern side, so you have to flow power through a transmission line from Singroli to Hardwar, Ghaziabad and Noida and perhaps neighboring states, if any who is the power shortage state. Just you study this we consider a simple two bus system, this is called a simple two bus system, bus is what node, bus is nothing but node this is number 1, this is number 2, you cannot have a simpler system than this you one bus is no system that means; you generate and use it then and there.

So, there no question of power flow, generated power and consume them right, so the minimum power flow only comes if there is a two bus system, that is minimum, maximum then be 1000 bus system, the Indian power system is 1000 bus system, if you at all imagine the national grid which is yet not in place if there only thinking. Generated some people show this sign, some people do not show, assuming that now there is hardly

any D C generation, this is assume whenever you write generation it is A C generation, only transmission now there is a possibility you may have h v t c, but generation as well as distribution remains largely A C, this no chance of decision relation is no chance of decision distribution; however, in Calcutta we still have D C lines in the houses, every house has some part connection A C part D C. So, something works all the time. Now this is the line which is implied by A B C D, this is V R 0 degree is 0 degree is the reference, this is the load, this is receiving end power, this is received, P R plus j Q R real power reactive power, this is the sending in power P S plus j Q S, all of we know that P S will not be equal to P R, it losses on the way there are losses.

So, P S in all probability there is no exception here like Ferranti effect is going to be greater than P R, if we ignore losses, then you will be have equal to, but that will be a theoretical case, you cannot ignore losses; however, small they may be. Similarly Q S is also going to be greater than Q R and if you want equality, then you will have P S is equal to P R equal to P losses, similarly Q S is equal to Q R equal to plus Q L reactive power loss, active power loss P L and Q L denote real and reactive power losses respectively.

No, not transmitted, that is because of load, that is because of compensation not inherently because of transmission line, there is bound to be losses in the line, so that rate to power which is generated here and pumped to this point by the time it reaches here has to be less unless not, here compensation along the root, then you are disturbing a system, system is not left on its own, then you will have, you can have anything depends on how much power you have pumping in midstream or one third of the point or two third of the point, that is a separate issue, otherwise I think so, that Q S is always gone to be more than here, do you agree.

So, this is the two bus system, delta is a very important parameter is called torque angle or load angle; we already talk about delta earlier on, in fact you drawn at spatial diagram also with, if you recall chapter four the sell in pole synchronize machine and non selling pole that is cylindrical rotary machine and so on, S R is P R plus j Q R is equal to V R I R star S s is P S plus j Q S is equal to I s star star is conjugate, any any complex variable will have a conjugate. Suppose the complex variable is A plus j B and if you take a conjugate of this it will be a minus j B, but if we have minus here, it will become plus

sign, this you must have studied in your mathematics courses, if you recall your old equation which is very important equation.

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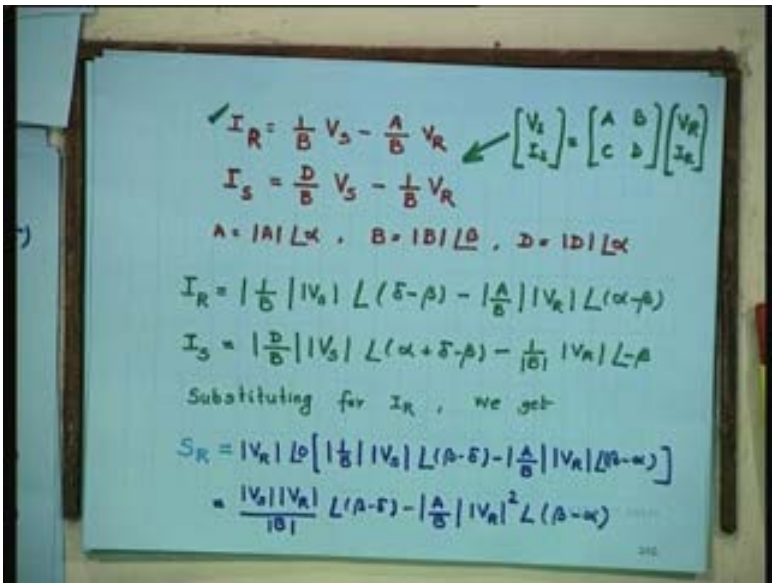

I will rewrite here V S I S, A B C D, V R I R find out I R? What will be I R V S is equal to A V R plus B I R, now take the other R in the other side, divide by b you get I r is this equation clear to all of you, is there anybody sitting here who is not clear about how this equation is come just come from here. Similarly I s, I s is, one minute this has to be I R ah it is D B here B here minus, one minute I think we are doing some jugglery, then only it will come, it won't be that easy, so it will come, but you have do certain mathematical manipulation.

Anyway A B C D are also complex variables as I told you earlier, several times A is roughly one the magnitude, alpha is roughly close to 0 degree or five degree, B is an impedance beta is close to 90 degree and D in a r are roughly same alright, so this I R and I S, now you substitute magnitude and angles, so you will get 1 by B magnitude V S magnitude delta minus beta, because this is delta and this is beta, so where, when beta goes upstairs becomes minus similarly, minus A by B, V R, A is alpha V is beta; so alpha minus beta I S D by V, V S alpha plus delta minus beta minus 1 by V, V R beta is 1 by minus beta, so 0 minus beta is minus beta, substituting for I R we get in S R.

Yes. So, you get this big equation this I just leave it here for two minutes, so that you can go through various steps, there anything is not clear please relate me now is it. Similarly

in parallelly or similarly you find out S S that is sending a complex part, these two equation gives three phase m V A if V S and V R are expressed in cable line alright, that can be very fine very easily, but there very complicated, who will remain search or who will derived it in the exam, very complicated why not make it bit simple, so what we do we separate real and reactive power, imaginary real and reactive powers, so that we get our old friends cosine and sine back. So, P R is V S V R by B cosine beta minus delta minus A by B V R square cosine beta minus alpha in Q R is V S V R 1 b sine beta minus delta minus A by B V R square sine.

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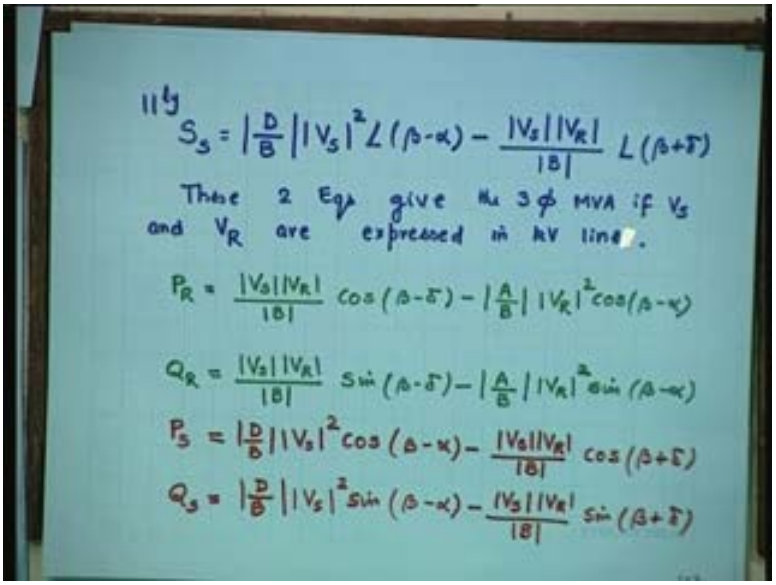

So, will be P S and Q S I will leave for just two minutes to verify, so that anyone of you wants to verify, wishes to verify these equations can do, so in two minutes time we are not anything just separated real and imaginary parts, they are still very complicated I mean is very difficult to remember them. So, why not do something and make them for this simply, before we do that I want to ask you one question, our main emphasis is on receiving end quantities, we are not much bother what is happening in Singroli that is why it is in because the policy of the government is to have power plants near the pitted coal mine, so that you do not have to transmit or transfer or transport coal, which is not easy, then you to request railways, you to request coaler minister and so on, so, it is whether to have power plant right there, so at least railways are not involve just purchase the coal and start power generation.

So, I am not much worried about sending an power and sending an reactive power, I am worried, but not much worried like if you get a headache you are not much worried, but it is something happens more serious immediately you go to the doctor, headache you can tolerate you can take rest putting whatever. So, I want to know what the maximum receiving end power is; my aim is to evacuate as much power from Singroli as I can. So, that I can take it to Ghaziabad, Hyderabad, Noida, Hardwar or any load anybody who is paying me money, now it is a market driven economy, the transmission line may be won by somebody, but you can book like you used to book computer center once upon a time when p c is where not their, everyone of us who had doing p hd is whatever the students there to go and write even, now I think same thing must be happening those who go and work in the computer center booking. I will come from 10 to 11 is it happening that way or a it is nobody their, all field is wide open anybody can go and work, anywhere there was a time when you are to wait and you are to book timing on mainframe computer.

Similarly, I am now interested in what maximum receiving end real power I can be more more emphasis on real power, so I am interested in finding out the condition for p r max and our teachers are told us in earlier classes, that how do you maximize, by maximizing or very simple the first order derivative with respect to the control variable should be 0. So, what is the control variable here, all are fixed, line is fix, B is fix, beta is fixed, once line is already late V S V R; normally we fixed at 1 per unit, so then at very well then what is the variable here. Delta, delta is the only fellow, whom you can change rest of them are fixed more or less, so, delta is the only variable, if I carry out this exercise the result is note P R is maximum at delta is equal to not 90 beta, there is no 90 here and we will come after some time at 10, 11 55, now it is 11 43, 90 will come after sometime.

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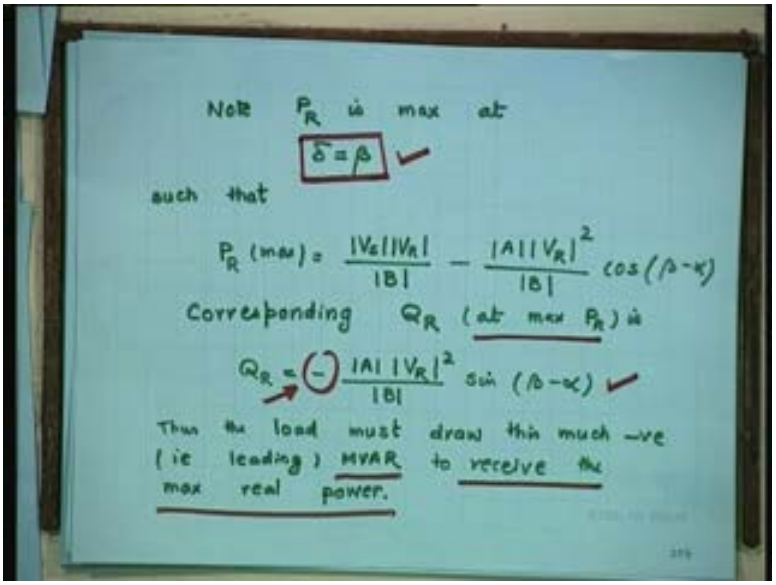

So, today I just now it is delta is equal to beta, so if I ask you question, derived the condition for receiving maximum real power at the receiving end, this is the condition and you have to come at this point, such that the value of P R max is this, because this initial cosine 0 is.

1, if you put this value here what is this, is it not this this is next this is given here you come here and this beta is equal to delta will make it 0, I do not write it 0 here to be wrong and then this becomes 1, this is what is happening here V S V R by D minus A V R square by D cosine beta minus alpha. Corresponding q r at maximum p r is this you can use same condition here because if real power changes rate to power also changes, they are core travelers is sitting on the pillion, so, he cannot going upon opposite direction, he has to be with the graver.

So, Q R is this value and notice this sign, thus the load must draw this much negative, that is leading M V A R to receive the maximum real power, so the question can further continue, also find the reactive power received at this time precision, so you continue not also now we are to respect what this little girl said about ninety degrees. So, let us go towards that slowly will surely.

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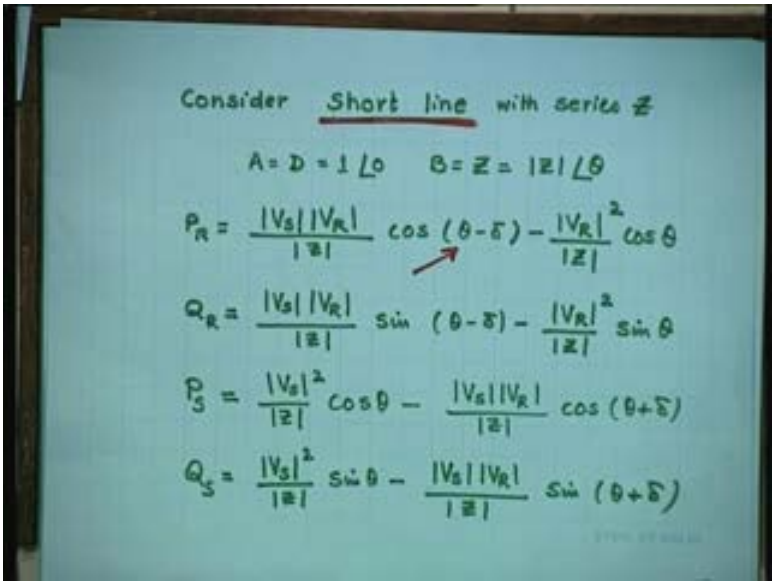

Consider a short line, because most of the countries are small most of the states now we are going towards distributed power or disperse power, which is given in this book in a big way, the market, the private diversion, the reforms., so now, we are no more getting power from their, so d v b, now we getting for b s n l, tata, tata power, so that is all given here, the whole thing is change, now, so I am not interested in long lines anymore shorter line shortest system smaller generators right. So, short line has assumed more important than merely academic importance, the real life lines are also now shorter. So, must so, we want the get rid of transmission as much as possible you generate and use it then and there, that is the called distributor power or disperse power, which I did it earlier in the class.

Naturally A and D will be highly simplified 1 0 degree B and z will be impedance with angle theta and now, this P R Q R P S Q S are change to theta and no more beta's now, z and I will give two minutes to see whether to arrive at this point. Naturally if you maximize this, the condition will be, delta should be equal to beta, not yet 90 this is second step where beta is taken given way to theta it may also happen for a short line, derived the conditions then for god sake do not derived that condition and then say beta, now will gets replaced by theta, same thing as I told you earlier. So, then you have to derived from those equations with short line and I was talking to you; now the condition for maximum receiving end power is delta is equal to theta.

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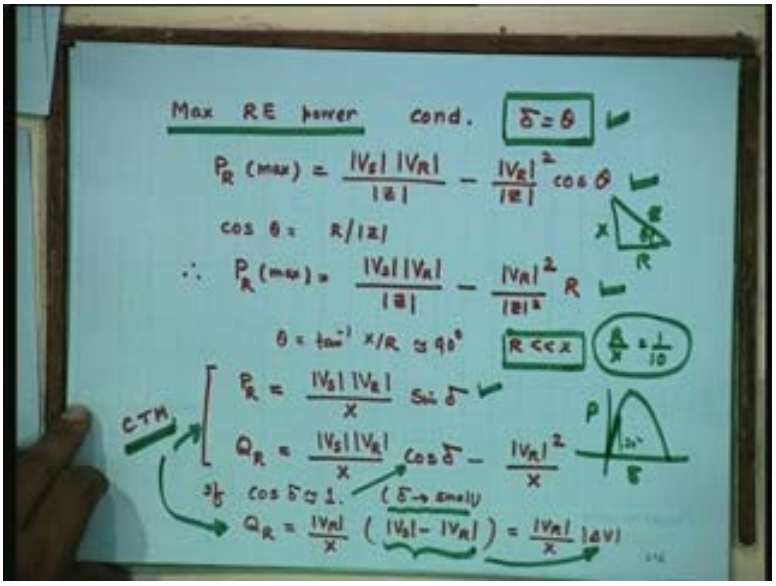

And so, P R max is this where cosine theta is R by z this is the triangle, transmission on angle, this triangle is not to this size, not to this scale, normally r is very, very small to x. So, this will be this and goes out of the paper, so I have drawn like this and theta is therefore, 90 degrees the theta as you can see from this triangle may look like 60 or 70, but indeed it is roughly 90 or 89 or 81 or something like that, So, the value of p r max is this where theta is 10 inverse x by r same same theta drawn in this triangle roughly 90 degrees as I said, why? Because r is very very small as compared to X. In fact, R by x is roughly 1 by 10 and if R becomes close to X or more than X what do you call such power systems.

There are instances where r is as good as x or more than x you know no no not. See if you do not know the answer, do not give the wrong answer that gives still lower marks in interview, such systems are called ill condition power systems, like a bad bad behavior, the boy you gets spoil, you get pampered, goes out in the night time, when is suppose to go in the day time. Similarly this power system behaves in a erratic manner to which it is not suppose to be and such if you somebody asked you what is ill condition power systems, this is one property of ill condition power system that resistance becomes more phenomenon, is it realistic system do not think it is only friction, when it happens see the index ill condition system read the book, I want you to cultivate the habit of reading various books.

So, that you gain knowledge, classes you cannot get all the things because time is limited 60 minutes, but you have to supplement at least in a graduate level and must have to read journals not only books need to read journals the papers is to generate engineers IEEE, IEE etcetera, etcetera. If theta is 90 degree then what happens, then P R becomes our good old friend V S V R upon X sine delta and now, you maximize P R delta becomes 90 for this, that already was same we reached 90 finally and for this Q R is V S V R X cosine delta minus V R square by X some phonetics some people will not still be satisfied, they want everything very simplify. So, if you want to further simplify this equations, at least Q R you cannot to simplify P R anymore, P R cannot be further simplified indivisible, but q r can be how, delta is very small in power system if you recall thus power angle curve, we normally operate on this region, linear region that is up to 30 degree, so 30 degree delta is not ok

And hence cosine delta is roughly 1, cosine 0 is 1 and slight deviation from 0 will continue to be nearly 1, so if you put one here what we will get V R by X V S minus V R and what is what is this quantity V S minus V R? Difference drop, voltage drop, so I can call delta v can you reduce it further no if you reduce it further, you go back home leave your m tech, so this is the minimum simplest formula anybody in the world can achieve, so for nothing further.

So, Q R is equal to V R upon X into delta V this is the simplest formula for Q R and simplest formula for P R is this and this should be C T M come into matter along with this of course, this three formulas, what are the conclusions, what ever we have done today, what are the conclusions for resistances nearly 0.

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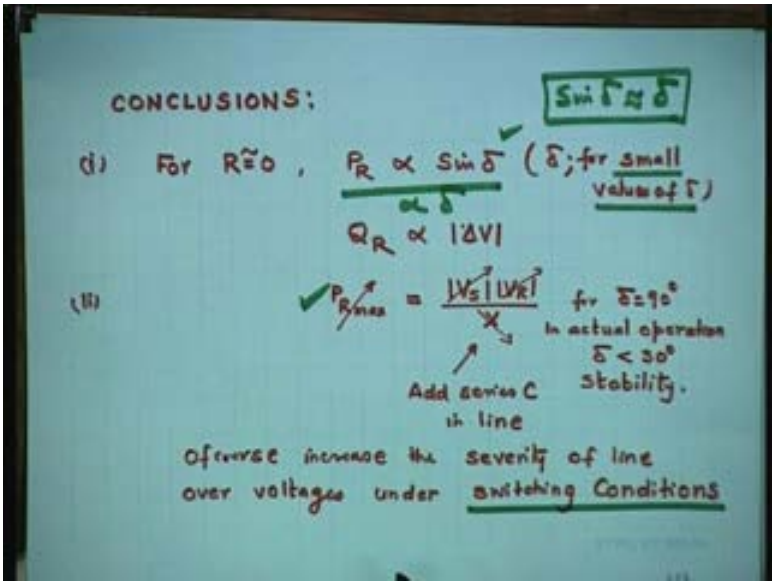

P R is proportional to sine delta V S V R by X is anyway constant given this quantity is known, this quantity is known, this is known, once line is built x is known and v r and v s are roughly same fixed. So, only variable is sine delta, that is what I am writing here, that when R is low or 0 P R is directly proportional to sine delta and if some more fellow whose interested in getting further simplification, this becomes proportional to delta itself for small delta, because delta is 0 sine delta is also 0 right.

So, you can this sine delta is roughly equal to delta for small delta like for cos delta it was one for del sine delta it is delta itself and Q R becomes directly proportional to the deviation, because other things are constant they are also, other thing is what v r and x v r is also known constant that is what your job is to keep it constant, if it is not, that is why you spend lot of money u p s and was this what it called voltage stabilizer, for anything you purchase in India you need voltage stabilizer, to stabilize the voltage to a constant quantity, but I am more interested in P R as I told you continue to told continue to tell you see is the very first lecture whole idea is to reach power should reach the consumer the customer the industry. So, I want to maximize that further of course, delta is 90 degree agreed, but whatever is left I want to still maximize, like a good business man. I want to maximize my profits how do I achieve that increase V S and V R as much as you can, but there are physical limitations you cannot make it 100000 volts or 10000000 volts; so there is a limit of 800 k v 7 65 k v or whatever in the world 30 100 k v.

So, after exhausting that avenue, the next is to reduce X, how do you reduce x in the transmission line is already built, before building X bundle conductor these aluminum, copper, s c s r whatever you can do, but having built having purchased car, auto or whatever then you can only play with that car only, you cannot say I need Mercedes, you stuck with that car. So, once line is built then only manipulation possible is with X, that is all built, you can only you can add of course, but there is one time solution after adding 1 line, 2 line, 3 line then what next, next is used to reduce x and that is what is the topic of compensation, add series capacitance in line, how much, where what quantity, is the fresh chapter added compensation.

So, you to read, that the time permits will read otherwise will be happy with whatever we are able to do it of course, at a price whatever you achieved in life is at a price, as we say the higher will go you feel more and more tension here may, they may come sitting in the chair, taking fat or whatever some diabetic, blood pressure, something, so within at a price, what is the price here increase the seniority of line over voltages under switching conditions because you have to bring in an out capacitance switching and switching tangents against the same chapter 13, whatever it is in, that brown book power system tangents; so you have to read, that switching tangents are more serious than lightening tangents now a days.

So, this are resonances, sub resonances these are the few problems associated with series capacitances, so I think will finishing today, any questions please, whatever we are done today. See loading depends on two many things, there is now voltage stability, there is a angle stability, there is a thermal load equilibrium, then there is serge impedance loading, so it depends on what is your design objective depending on, that you will decide the, the bottom line is I want as much as power transmitted as possible subject to all these constraints. That is all that is your explaining the Ferranti effect. No that, you have to read and then come back and tell me, you cannot explain Ferranti effect using this.

No Ferranti effect is the capacitances, is this is nothing to which loading is not a normal thing, nobody will terminate the load, load can vary load is the real fellow is the random variable you cannot make it load to, show its theoretical concept, impedance loading is the theoretical concept well the thermal loading is the real thing that should become the temperature rise to be within a limit you cannot gentle warming is in the heat. yes

Student: Sir

Yeah.

Yeah its right.

No it varies means, dependence on the line, it can be 0 or it is very small, the value of alpha is very small it varies along 0 degree or less than 0, that is all. It becomes solve the numerical you will get the exact values and you will have a feel of it when you solve those numerical right, then you have the feel of those values alright then next.