

## Power System Generation, Transmission and Distribution

Prof. D. P. Kothari

Department of Electrical Engineering

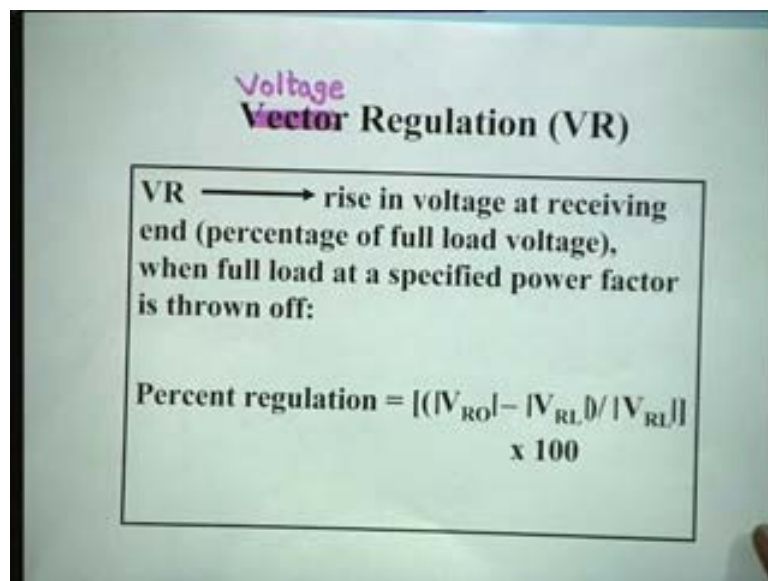
Indian Institute of Technology, Delhi

Lecture No. # 13

Voltage Regulation

Good morning ladies and gentlemen, last time we started the chapter 5, we have done up to pi network, that is the medium transmission line, of course we have to do now voltage regulation.

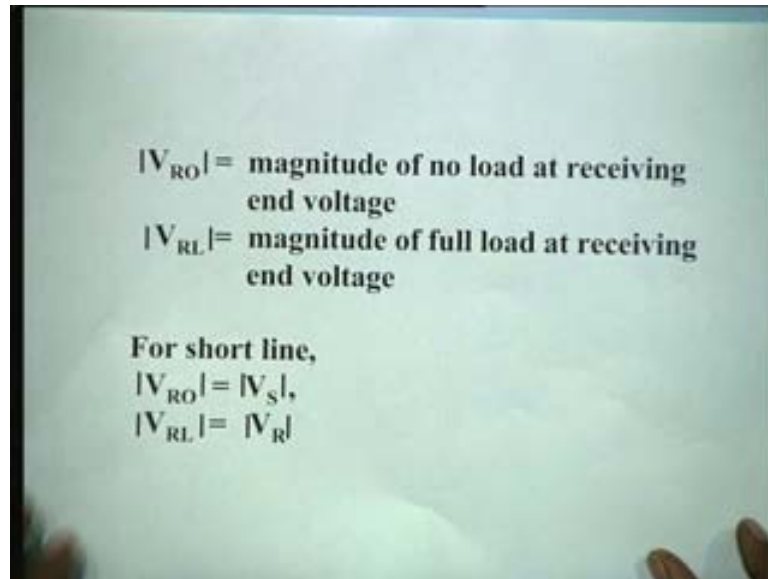
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What is voltage regulation, why do we need to regulate voltage, that is a million dollar question, because as you all know we have to keep voltage constant, why? Because all equipments, all devices, all machines, all motors they are suppose to operate at given voltage. And the voltage variation is injurious to their health because, voltage varies, their performance will vary, vary in the sense, it would not be what it is meant to be.

So, we need to regulate the voltage in a very very narrow range, it is defined as a rise in voltage at receiving end, as a percentage of full load voltage, when full load at a specified power factor is thrown off:

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So, what is the rise in voltage? That is no load minus full load upon the full load voltage into 100, just for percentage.  $V_{RO}$  is nothing, but magnitude of no load at receiving end voltage,  $V_{RL}$  is a gain magnitude, and vertical lines denote the magnitude or amplitude of full load at receiving end voltage. What is important is receiving end, why it is important, because the loads are there.

The whole electric engineering is there, because there are loads, if there is no load, no need to generate any power. As you know for a short line, anything less than 100 kilometer is a short line, lines have already be categorized into short, medium and long, the  $V_{RO}$  is nothing but,  $V_S$  for short line and  $V_{RL}$  is nothing but  $V_R$ .

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The image shows two handwritten equations on a whiteboard. The first equation is for a lagging load, with a plus sign between the two terms in the numerator. The second equation is for a leading load, with a minus sign between the two terms in the numerator. Both equations are multiplied by 100. A hand is visible at the bottom left of the whiteboard.

$$\text{Percent regulation} = \frac{(V_S - V_R)/V_R \times 100}{= [(I R \cos \Phi_R + I X \sin \Phi_R)/V_R] \times 100}$$

for lagging load

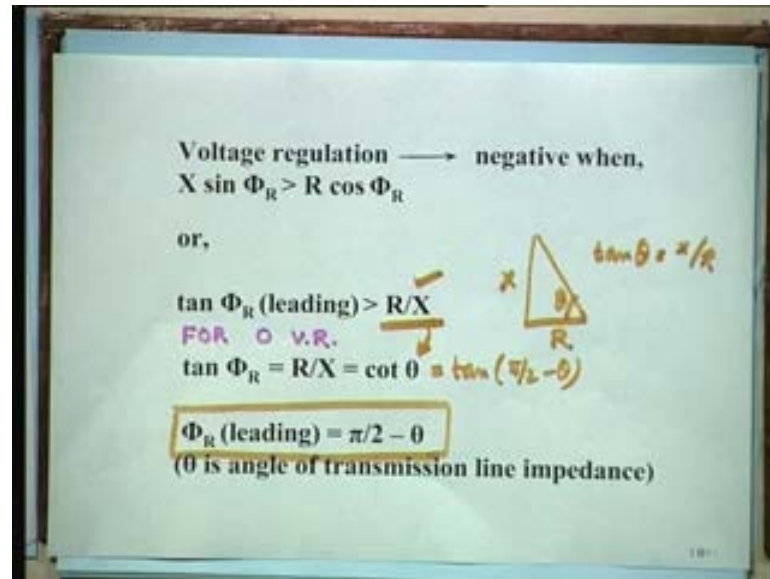
$$\text{Percent regulation} = \frac{(V_S - V_R)/V_R \times 100}{= [(I R \cos \Phi_R - I X \sin \Phi_R)/V_R] \times 100}$$

for leading load

So, it will be practically  $V_S$  minus  $V_R$  upon  $V_R$  into 100 that will be the definition of percentage regulation, and that is what coming in the next slide. This is what it is percentage regulation, which I have just defined; of course you can always multiply by 100. We have just derived an expression last time if you recall, using that phasor diagram and I talked to you about Pythagoras theorem, all those things,  $I R \cos \phi_R$  plus  $I X \sin \phi_R$  is the difference between  $V_S$  and  $V_R$ , that is a drop upon  $V_R$  is the percentage regulation for a lagging load.

I have also told you other day, if it is leading, then the plus sign get change to minus, here it is plus for a lagging load and it becomes minus for a leading load. I have also told you to recall that, if you if I ask you to derive for leading, do not derive for lagging, and then thus change the sign, if you recall I told you that also.

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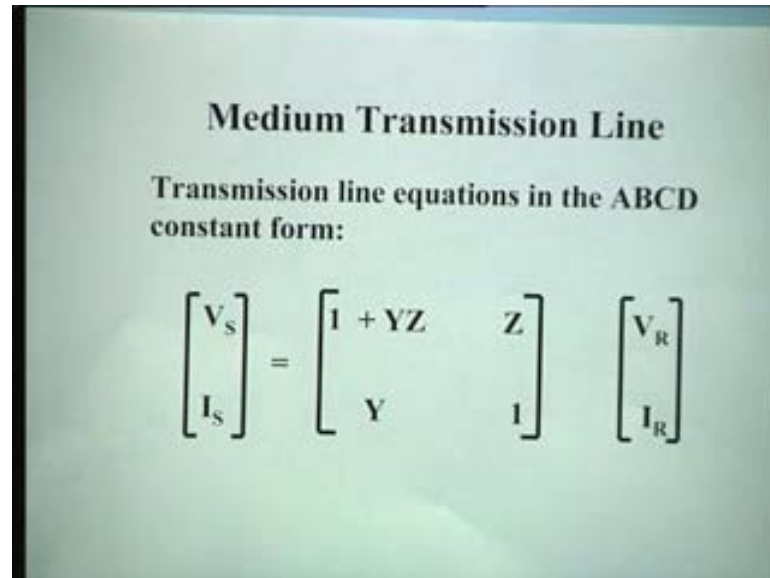
For negative voltage regulation as you can see the formula, you will notice that  $X \sin \phi$  should be more than  $R \cos \phi$  or tangent  $\phi$  it is leading, because negative can be there only when it is leading, it is greater than  $R$  by  $X$ . Now, you may ask me why not zero voltage regulation, which is an ideal condition, zero voltage regulation means no change in the voltage; that means, the drop should be equal to 0 and you put drop is equal to 0, what does it mean? It means the tangent of  $\phi$  should be equal to  $R$  by  $X$ , that means  $R$  by  $X$  is nothing but cotangent  $\theta$ . I hope you know this triangle  $R$ ,  $X$ , this is  $\theta$ ,  $X$  by  $R$  is  $\tan \theta$ , I think this much you know from your trigonometry knowledge. So, cotangent  $\theta$  is reciprocal of tangent  $\theta$  which is base upon hypothesis.

So,  $R$  by  $X$  is given here and this nothing but  $\cot \theta$ , so tangent of something is equal to cotangent of something means, if I do  $\pi/2$  minus  $\theta$  it becomes a tangent of, this is nothing but, tangent of so again cancel  $\tan$  and get  $\pi/2$  are leading should be this is a condition this derivation is very important, what is  $\theta$ ,  $\theta$  is angle of transmission line impedance; what is  $\phi$ ,  $\phi$  is a power factor angle.

So, what will be the question like, derive the condition for zero voltage regulation. Now, this is up to you from where you should start, you have to derive the condition for voltage regulation and from there you have to say, for 0 that drop should be equal to 0

for zero voltage regulation. If there is no drop, there is no zero voltage regulation. So, this derivation is important.

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**Medium Transmission Line**

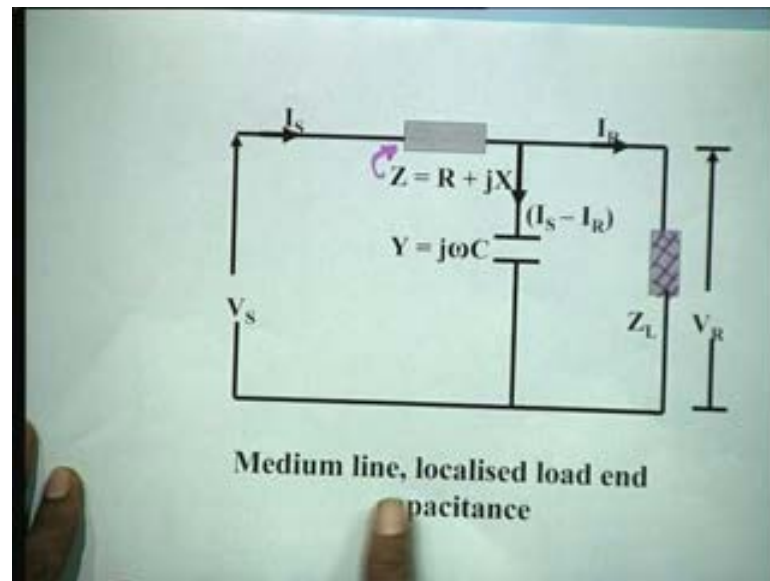
Transmission line equations in the ABCD constant form:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + YZ & Z \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Now, after short line that is the last article for the short line, now we come to the medium transmission line, and in many countries in the world, most of the transmission lines are medium because, their length and breadth are very small. For example, Srilanka cannot have a long transmission line by its nature, Nepal cannot have long transmission line, it can have medium, Srilanka can even have only short, Bhutan is hardly any length and breadth, well Bangladesh can have perhaps one or two long lines, but most of the lines should be adheres short or medium.

India will have all the three varieties: there will be long lines, medium lines and short lines. Obviously, when you say medium, you cannot ignore inductions and capacitance as we did for short, medium is roughly up to 250 kilometers. Now, the transmission line equations in the ABCD constant form are  $V_S, I_S, 1 + YZ, Z, Y, V_R, I_R$ , that is the normal equations used for medium transmission line.

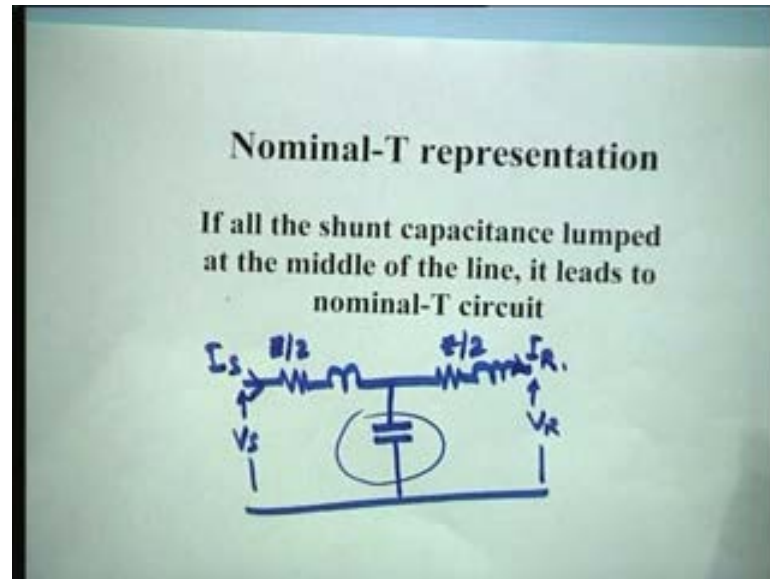
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Suppose, load capacitor is put across the load medium line, localized load end capacitance, what does it mean? It means the capacitance is not in the middle, it is put at the end of transmission line across the load, parallel to the load, in shunt to the load. Naturally, this current and etcetera you can find out by repetitive application of Kirchhoff's current law at this node, so  $I_s$  minus  $I_R$  will be this, this will be  $I_R$ , this arrow is for this, and this impedance is this, the transmission line, this is the transmission line impedance, which is  $R$  plus  $jX$ , this is sending an sending an voltage sending an current sending an voltage sorry receiving an voltage, receiving an current, sending an current, sending an voltage (Refer Slide Time: 10:30).

And this which I Showed you corresponds to this figure, ABCD constants you can always write Kirchhoff's voltage law and current law and show that, this equation will be correct, that you can derive from this figure (Refer Slide Time: 11:17).

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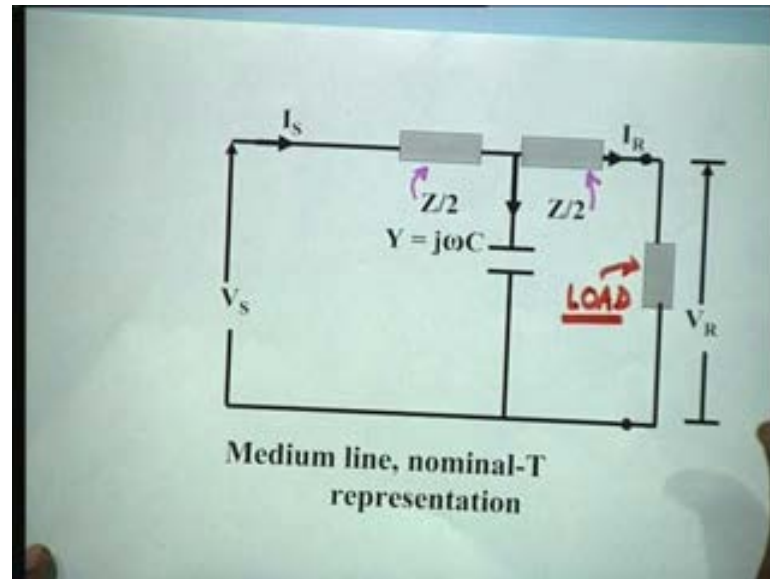


What is nominal T, I am sure you heard about nominal T, T junctions not only in power system, on roads also there are T junction, where there is no fourth road if you had like pi shape, if you had to the T junction either you go to the right that is IIT or left that is Nehru place, if you are coming from that south extension or Asian village or nasal plaza, where you must be visiting some time.

If all the shunt capacitance is lumped at the middle of the line, it leads to T network; T network simply means, that you have the shunt portion is lumped at the middle, whereas the series is divided by divide equally and this is  $V_s$ , this is  $V_r$ , this is  $I_s$ , this is  $I_r$ , it is a symmetrical (Refer Slide Time: 12:53).

Of course, same thing I have drawn here, which are just in a slightly better fashion, though it is does not look to be symmetrical, this should be shifted here.  $I_s$ ,  $I_r$  same thing I drew earlier,  $Y$  is equal to  $j\omega C$ , this is  $V_r$  and this is load, this is load, this is medium line, nominal T representation. Let us derive ABCD constants for T; earlier one I have left that to you, because it is so simple.

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Now, this one I want I wish to derive for the benefit of all of you. So, keep this T figure in mind when you write this equation.

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For the nominal-T circuit, the following circuit equations can be written:

$$V_C = V_R + I_R (Z/2)$$
$$I_S = I_R + V_C Y$$
$$= I_R + Y V_R + I_R (Z/2) Y$$
$$V_S = V_C + I_S (Z/2)$$

What is  $V_C$ ;  $V_C$  is nothing but, the voltage across the capacitors. So, naturally that will be  $V_R$  plus  $I_R$  into  $Z$  by 2, if you look from this end, load end,  $V_R$  is the voltage at the load end, at the drop, drop is nothing but  $I_R Z$  by 2, you get this  $V_C$ . What is  $I_S$ , Kirchhoff's current law apply at the middle of the circuit that is at the T point, you get  $I_R$  plus  $V_C Y$ , across capacitance multiply with the voltage you get the current, this is



what it is, substitute the value of V C, V C is the intermediate variable, nobody wants V C everybody want V S, V R, I S, I R, so V C can be eliminated by substituting from equation number one. Hence, rearrange it then you get, it should be V R, this V S is the again V C plus drop, and drop is I S into Z by 2 this equation is over here. This is this equation is coming from this side, V C plus I S into Z by 2. Now, you can see here, this V S we are just written, substitute the value of V C and I S which you have just obtain.

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The image shows a whiteboard with the following handwritten equations:

$$V_S = V_R + I_R(Z/2) + (Z/2) [I_R(1 + ZY/2) + YV_R]$$

$$= V_R(1 + ZY/2) + I_R Z (1 + YZ/4)$$

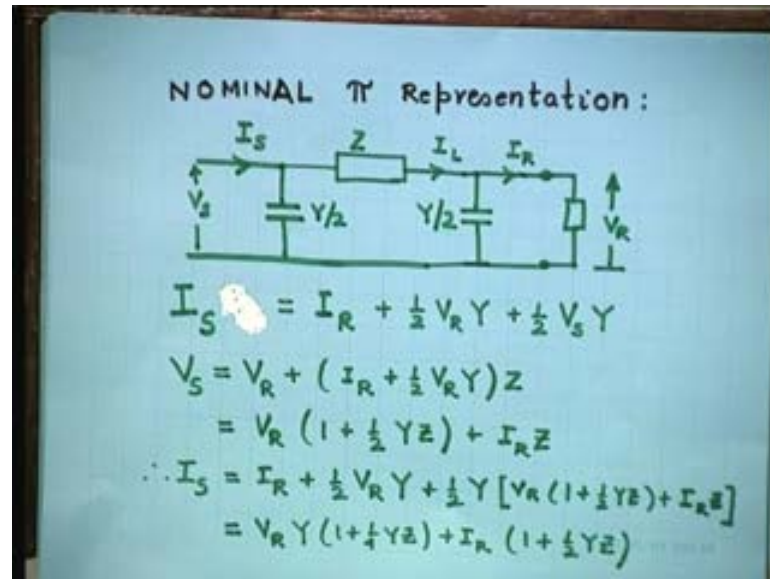
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}ZY & Z(1 + \frac{1}{4}YZ) \\ Y & 1 + \frac{1}{2}YZ \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The matrix elements are labeled with green letters: A is above  $1 + \frac{1}{2}ZY$ , B is above  $Z(1 + \frac{1}{4}YZ)$ , C is below  $Y$ , and D is below  $1 + \frac{1}{2}YZ$ . A green checkmark is visible at the bottom right of the whiteboard.

So, what you will get, we will get V R and I R, because we do not want in terms of I S also, I S should be on left hand side, and not on right hand side. So finally, we have got this equation, this is A, this is B, this is C, this is D, so you have got ABCD constants (Refer Slide Time: 16:40).

I am sure, you will be able to derive if asked ABCD constant for L network, that was the beginning the load in parallel with capacitance, T network, that we have just finished here, this is the T network you should be able to derive ABCD. I just ask you if anybody asks you a question, derive ABCD constant for nominal T network, you should know, where to start and where to end, those who do not know this, they cannot progress in life, they must know the starting point, they must know the end point, then only athlete can win. If you are away from the starting point you are disqualified, if you do not touch the end point, again you are disqualified, so same rule is applicable here, in any derivation.

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Now, we come to the last of the T model, which we have allocated for the medium transmission line L, T and the pi, now pi is just the reverse of T, what will be do in T we divided the series impedance into two parts and put the shunt admittance located in the middle part of the line. Here, the series impedance or series admittance rather is divided equally, what is the two ends? Sending end and receiving end, whereas the series impedance is lumped, shown in the middle of the line, just the reverse Z, Y by 2, Y by 2, I S, here I L is the intermediate variable which should be eliminated, I R, V R, V S. Now, you are all friendly to these various nominal clarion various symbol, they are very familiar, find out I S which is nothing but, I R plus I V R Y by 2 plus I V S Y this is this drop, this drop plus this drop.

So, you have to this I S, I S is current going to this branch, current going to this branch plus current going here, that is what I have done three terms, I R plus this term, plus this term. So, Y by 2 into V S here, the voltage across this branch is V S, the voltage across this branch is V R similar should, so this is I S. What is V S? V S is again nothing but, V R plus drop in the series, series the current is I R plus current going here, this two is equal to I L, this is nothing but I L.

I hope you understand everybody, combining the terms which must you have done from your 4th class or 6th class I do not know, which class. So, taking collecting all coefficients V R we get this 1 from here, and half Y Z from here, and taking this I R Z

outside, so we can get this term. Coming back to I S again, why I want to get read out the V S, I do not want to see any sending end quantities in right hand side, so right hand side is reserved for receiving end, left hand side only sending end.

So, substituting V S just obtain fresh, we get the value of I S, and I hope similarly you can take 2 minutes to see, whether what we have got here, it is otherwise you can always verify at home and these are given any book, you can nominal pi is very well known model, now we have obtained final two equation; V S in terms of V R and I R, and I S in terms of V R and I R. So, collect the coefficient of I R and V R, you will get ABCD and finally, I can write ABCD constants.

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Finally we have

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} (1 + \frac{1}{2} YZ) & Z \\ Y(1 + \frac{1}{2} YZ) & (1 + \frac{1}{2} YZ) \end{bmatrix}$$

Nominal T  $\equiv$  Nominal  $\pi$   
 Y- $\Delta$  Transf. better?

Ex. 5.4.  
 LONG TRANSMISSION LINE - Rigorous Sol.  
 length > 250 km  
 parameters not lumped  
 but distributed throughout its length

This is A, this is B, this is C and this is D, notice A and D are equal (Refer Slide Time: 22:22). Most of the time for bilateral network, their balance network is always be equal, and it is close to 1, that also I told you and it is dimension less, that also I told you, B is having a dimension of impedance here it is a impedance itself.

So, the angle will be close to 90 degree, if you get angle for B as 2 degree or 3 degree, you should immediately understand you are gone wrong, drastically somewhere, in calculation to make a note, if you have no time left to correct your calculation, the examiner will be happy to note that, at least you know the Z cannot have the angle of 5 degree or 10 degree, Z angle will always be the 80 degree, 89 degree or 78 degree something like that; C is of course, having a units (( )) whatever, I want to ask you one

question, do you think the nominal T and nominal pi are same, they are not, if you want to conform it, you can do star delta transformation and you will see that, you cannot you cannot obtain one from another, just change the formula, use the formula for star delta and you will see that, there are not same though, both of them represent same transmission line, but there are not quit same.

Naturally any one will ask this question, if there are not same, which is better, which one is better why? Any reason, ok. Any other reason, well we are considering the lump impedance that is one of the truth that account, you cannot say, that is pi is better than T, there is one lump in one case another lump in another case, there is one divided into two parts in one case, and other parameter is divided into two parts in another case on that account, there exactly same, yes, any other reason.

(( ))

Any other thing

(( ))

No, what you are saying T is better than pi.

(( ))

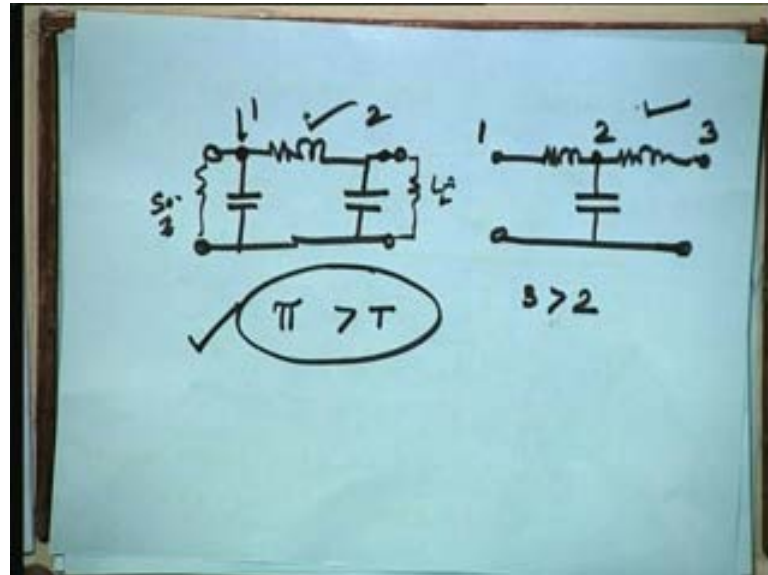
No, that will be a wrong answer and that is not correct, pi is decidedly to better than T for following reasons; reason number one, in pi you are not increasing the number of nodes, reduction of order is always welcome. In fact, there is a full control branch in control systems, those of you like control system course, they should know reduced order modeling is one of the very important topics in control system, where people do PhD's, how do you reduced model of the system, so here you are increasing a node instead of you are increasing, you are creating a middle node, that node is not their in pi network, that is an extra node, the middle node where there is a connection; secondly in pi, shunt branches close to the load and source impedance, so it can combine with them.

And hence, you are left with practically, the series impedance because load is shunt and the pi or leg is also shunt. So, you can add the two admittances and it will help you load flow studies, when you derive your Y bus or Z bus or whatever. So, thus pi network is.

(())

No, no, that means, I have to draw both here and I explain you, see see all of you have confusion 1 minute see there is a confusion I must remove it that is my job.

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These both are two port network four terminal networks you must have done it in electronics and in circuit theory, here the total number of nodes are two; this is one, this is two. Here, the total numbers of nodes are three, so here the three is more than two, so this is higher order model than this. Secondly, this capacitance cannot be combining with anything, it is middle, whereas this capacitance can be combined with load here, this capacitance can be combined with source impedance here, this is load impedance and hence, you left with only this, so you get tremendously simplified.

So, these two are the main reasons for preferring pi network to T network. Normally no book gives this explanation, but every teacher expect you to know the reason. I hope it is clear to you, that is why I left it here, just to asking a question which is better, you can again think and reads some other book to find out, if any books says, T is better than pi, so I will immediately bring fourth edition.

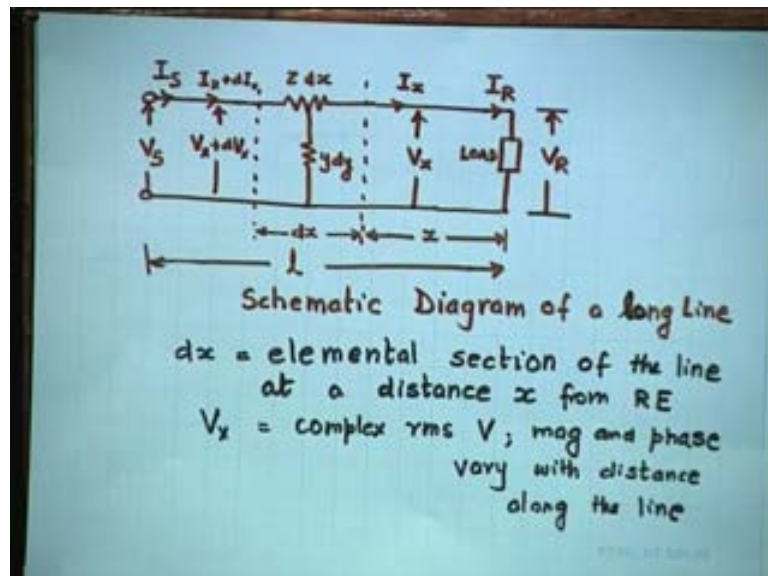
That will fetch you some award, now solve all the examples, we have solved yesterday examples 5.1, 5.2, 5.3 and 5.4, 5.4 is on pi network that is why I am written examples

5.4. There is also a examples on T network, so you can see the book, you can read other books also, and solve the problems, solve, as well as answer.

Now, we come to the real thing, the long transmission line from Jammu to Kanyakumari or Junagadh to maybe Guwahati or whatever, a line you want to take the big line, the length is more than 250 kilometers, we are having thousands of kilometer for transmission line even in UP itself, why (( )) country from Ghaziabad to maybe Banaras will be 500 kilometers or from Delhi to Kolkata will be more than 1000 kilometers, you cannot in such a long transmission line, lump the parameters, parameters cannot be lumped if you lump it, that means you doing approximation which is by no means can be justify, the parameter have got to be distributed.

So, you have to distribute the parameter throughout its length, and hence the solution has got to be rigorous, the word is written here is you have to have a rigorous solution. It would not be that simple as we have doing is so far, by writing few Kirchhoff's law equation's and loop loop equation and nodal equation and solving them, collecting co efficient V S, I S, V R, I R it would not be that easy, because parameters are not lump but there are distributed.

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Let us considering, systematic diagram of a long line obviously, there has to be sending end, there has to be a receiving end, there has to be a sending end current, there has to be a receiving end current, the total l is the line of length or length of the line or line length

whichever, you want to say. Now, we are considering a section element of the line at a distance  $x$  from the receiving end, we are calculating  $x$  is equal to 0 at this point,  $x$  is equal to  $l$  at this point. Now, when you are considering the element let us say, is of length  $dx$  and we are considering the T network, so  $z dx$  and  $y dx$ . In fact, it is not T it is elemental because,  $z$  is not divided into two parts, so it is not T even, it is just an element.

$z dx$  is series impedance and  $y dx$  is the shunt admittance. Naturally, the  $V_x$  is voltage here,  $I_x$  is the current here, here the voltage will be  $V_x$  plus  $dV_x$ , and the current here will be  $I_x$  plus  $dI_x$ , current and voltage are increasing as we move away from receiving end, that is a well known fact,  $V_S$  is higher than  $V_R$  (( )) which will be doing today, if time permits. So,  $dx$  is a elemental section of the line at a distance  $x$  from the receiving end,  $V_x$  is the complex RMS voltage, it has a magnitude, it has a angle, so it is a complex, magnitude and phase vary with distance along the line, if you go move along the line, the magnitude also varies, and phase also varies this should be very clear.

Now, you remember this figure we are going to do further analysis, based on this figure. I do not know whether we can continue to shows the figure some some how. So,  $dV_x$  is nothing but  $I_x z dx$  ok, the impedance drop into current is a voltage drop.

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$$dV_x = I_x z dx \quad \text{or} \quad \frac{dV_x}{dx} = z I_x$$

$$\text{||} \frac{dI_x}{dx} = y V_x$$

$$\therefore \frac{d^2V_x}{dx^2} = \frac{dI_x}{dx} z = y z V_x$$
 Diff eq<sup>n</sup> → Linear Gen Sol<sup>n</sup> :
 
$$V_x = C_1 e^{\gamma x} + C_2 e^{-\gamma x} \quad \gamma = \sqrt{yz}$$
 arbitrary Constants

So,  $dV_x$  by  $dx$  is equal to  $z I_x$ , the theory of duality or principle of duality tells us, if I replace  $V$  by  $I$  and  $I$  by  $V$ ,  $z$  by  $y$  the equation is valid, that is the duality.  $V$  is equal to  $I$

R, I is equal to  $V_g$ , R than g, z than y proper dualition is done ok. So, these two equation are, this is similarly, this symbol I have written is parallel lines, ly similarly. If I again differentiate once more with respect to x, what do I get, z d I x by dx and I can immediately substitute d I x by dx from here, so I get y z V x, this is a differential equation.

And in your under graduate or 11th class, 12th class you might have read differential equation, I do not know (( )) which book you have read differential equation. The general solution of the second order differential equation is given by this equation, C 1 e has to power gamma x plus C 2 e has to power minus gamma x, where gamma is under root y z, C 1 and C 2 are nothing but, arbitrary constants which can be evaluated using the initial conditions or end conditions or some conditions, which are given to you by the knowledge of the system so for, there is no problem.

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The image shows a handwritten derivation on a screen. The equations are as follows:

$$\frac{dV_x}{dx} = C_1 \gamma e^{\gamma x} - C_2 \gamma e^{-\gamma x} = z I_x$$

$$\therefore I_x = \frac{C_1}{z_c} e^{\gamma x} - \frac{C_2}{z_c} e^{-\gamma x}; z_c = \sqrt{\frac{z}{y}}$$

End Conditions:

$$x=0 \quad V_x = V_R \quad I_x = I_R$$

$$V_R = C_1 + C_2$$

$$I_R = \frac{1}{z_c} (C_1 - C_2)$$

Solving

$$C_1 = \frac{1}{2} (V_R + z_c I_R)$$

$$C_2 = \frac{1}{2} (V_R - z_c I_R)$$

Now, we differentiate the general solution by x again, so I get d V by x dx C 1 gamma e has to power gamma x minus C 2 gamma e has to power minus gamma x is equal to z I x, because this is this is what we know, we know that d V x by dx is z I x this what I am writing here, d V x by dx is equal to z I x therefore, what will be the I x just divide by z, C 1 z C e gamma x minus C 2 z C e is to power minus gamma x, and this z C is nothing but under root z y how did I get z C, the gamma I know under root y z or z y, divide by z means top half z gone, so remaining is y by z. So, if it is a denominator it becomes z by



y, and this  $z_c$  is called characteristics impedance of line or shunt impedance of line, as we will see later, shortly.

What are the end condition, all of us known for a beginning of the line, the  $V_x$  is  $V_R$ , I have started with that and then  $I_x$  is  $I_R$  substitute that, we get  $V_R$  is  $C_1 + C_2$ ,  $I_R$  is  $C_1 - C_2$  into  $C_1$  minus  $C_2$ , two equations and two unknowns,  $C_1$  and  $C_2$  are unknown arbitrary constants. So, I get the values of  $C_1$  and  $C_2$  like this, no problem, any problem any where, any difficulty, should be crystal clear everybody. Once we know  $C_1$  and  $C_2$  the natural next step is, to substitute them and get rid of  $C_1$  and  $C_2$ , once I do that, I get this equations  $V_x$  and  $I_x$ .

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The image shows a whiteboard with the following handwritten equations and definitions:

$$V_x = \left( \frac{V_R + z_c I_R}{2} \right) e^{\gamma x} + \left( \frac{V_R - z_c I_R}{2} \right) e^{-\gamma x}$$

$$I_x = \left( \frac{V_R/z_c + I_R}{2} \right) e^{\gamma x} - \left( \frac{V_R/z_c - I_R}{2} \right) e^{-\gamma x}$$

$z_c =$  characteristic impedance of the line

$\gamma =$  propagation constant

$$V_x = V_R \left( \frac{e^{\gamma x} + e^{-\gamma x}}{2} \right) + I_R z_c \left( \frac{e^{\gamma x} - e^{-\gamma x}}{2} \right)$$

$$I_x = V_R \frac{1}{z_c} \left( \frac{e^{\gamma x} - e^{-\gamma x}}{2} \right) + I_R \left( \frac{e^{\gamma x} + e^{-\gamma x}}{2} \right)$$

Now again, I am defining  $z_c$  is the characteristics impedance of line and gamma is a propagation constants, do not write gamma like a r, if you want to write gamma properly, you have enough time you can write like this, otherwise, it looks like a r. Again what we are doing our familiar game we are playing, we are the collecting the coefficient of  $V_R$  and  $I_R$  and putting them together, why? we are looking for a ABCD constant ok, and we know the equations  $V_x$  is equal to  $A V_R$  plus  $B I_R$ , so coefficient of  $V_R$  is automatically A.

So that is what I have done, I am regrouping it and forming the coefficient here, coefficient here, coefficient here. So, the coefficients are obvious this is A, this is B, this is C, this is D, of course with one changed, x changed to l which we are

going to do next (Refer Slide Time: 40:33), if you have done trigonometry which I am sure you have done in 11th class, 12th class or first year of your under graduate degree course.

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$$V_x = V_R \cosh \gamma x + I_R Z_0 \sinh \gamma x$$

$$I_x = I_R \cosh \gamma x + V_R \frac{1}{Z_0} \sinh \gamma x$$
 when  $x=l$ ,  $V_x = V_S$ ,  $I_x = I_S$ 

$$\therefore \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A \cosh \gamma l & Z_0 B \sinh \gamma l \\ \frac{1}{Z_0} \sinh \gamma l & C \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\gamma = \alpha + j\beta$$

Those bracketed things are nothing but, hyperbolic cosine, hyperbolic sine, hyperbolic cosine and hyperbolic sine. So, it is nice to write smaller things, rather than big thing. And when  $x$  becomes  $l$ , just now told you  $V_x$  is  $V_S$  and  $I_x$  is  $I_S$ . So, if you do this two things, we will get finally  $V_S$ ,  $I_S$ ,  $V_R$ ,  $I_R$ ,  $ABCD$  and this is very important equation, this equation is CTM, CTM means commit to memory, because any numerical you will need this equations, if you forget it, you will derive it, it will take a lot of time, you will loose time in exam; obviously, sometime  $V_S$  and  $I_S$  is given to you.

So, how do you find out  $V_R$  and  $I_R$ ,  $V_R$  and  $I_R$  will be the inverse of this matrix into  $V_S$  by  $I_S$ , how do you invert 2 by 2 matrix, exchange the diagonal terms and change the sign of half diagonal terms and divide by determinant, what is the determinant here a  $B$   $A D$  minus  $B C$ , what is the value of  $A D$  minus  $B C$ , is 1. This identity I have already told you sometime back that,  $A D$  minus  $B C$  equal to 1, use this identity and hence this equation is very clear which follows very straight forward without any proof, without any detailed competition.

So, these needs not to be remember, once you remember this, this easily follows. So, why did you remember the rules of inversion of a 2 by 2 matrix, no cofactors, etcetera

are required, it is just a mechanical rule, change the diagonal terms just swap them and change the sign of the half diagonal term and divide by determinant, and determinant is I hope you could, you know how to find out determinant in this case, it is A D minus B C, which is nothing but 1.

So, you do not have to be divided by anything, please remember gamma is again complex quantity, so there is a real part and there is an imaginary part, real part is called alpha and imaginary part is called beta; alpha is the attenuation constant, beta is the phase constant (Refer Slide Time: 43:36). Attenuation constant is one by which, the way is attenuated, way does not remain same throughout sinusoidal or whatever it gets affected. And similarly, beta is a phase constant all of all these things you might have already done we are just revise them, how do you evaluate the ABCD constant, that is our next topic because you have to evaluate them.

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Evaluation of ABCD constants:  
Method 1  
 $\cosh(\alpha l + j\beta l) = \cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l$   
 $\sinh(\alpha l + j\beta l) = \sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l$  {Std Table Calculators  
A=D=Z  
(1+YZ/6)}  
 2.  $\cosh \gamma l = 1 + \frac{\gamma^2 l^2}{2!} + \frac{\gamma^4 l^4}{4!} + \dots \approx (1 + \frac{\gamma^2 l^2}{6})$   
 $\sinh \gamma l = \gamma l + \frac{\gamma^3 l^3}{3!} + \dots \approx \sqrt{\gamma Z} (1 + \frac{\gamma Z}{6})$   
 $B \approx Z (1 + \frac{\gamma Z}{6}); C \approx \gamma (1 + \frac{\gamma Z}{6})$

So, our next topic is what, evaluation of ABCD constants, there are two methods: method one these identity you must have read in your some mathematics course, hyperbolic cosine alpha l plus j beta l is hyperbolic cosine alpha l into cosine, ordinary cosine look there is no h here, is ordinary cosine of beta l plus j times hyperbolic sine alpha l into ordinary sine of beta l. I hope you will remember this equation, which you have must done in your trigonometry course whenever you are or wherever you are studied it. Similarly hyperbolic sine, sine means sinh, you do not need to say hyperbolic

twice or something, hyperbolic sine of something,  $\alpha l + j \beta l$  is nothing but, hyperbolic sine of  $\alpha l \cos$  of  $\beta l + j$  times hyperbolic cosine of  $\alpha l$  into sine  $\beta l$ .

To find out the values of hyperbolic sine and cosine, we used to struggling in nowadays, they used to be standard tables or calculators, you now use exclusively calculators, without any difficulty you can find out sine and cosine hyperbolic. Of course, you can use shift button, if you do not then, there is a problem then, you may get the values of sine and the marks will be result 0 because, you cannot say it is only a numerical mistake; that means, you do not understand the difference between the sine and sine or cosine and cosine though, it is a matter of operation of calculators, it is not a power system knowledge which is at fault, it is the use of calculators you do not know.

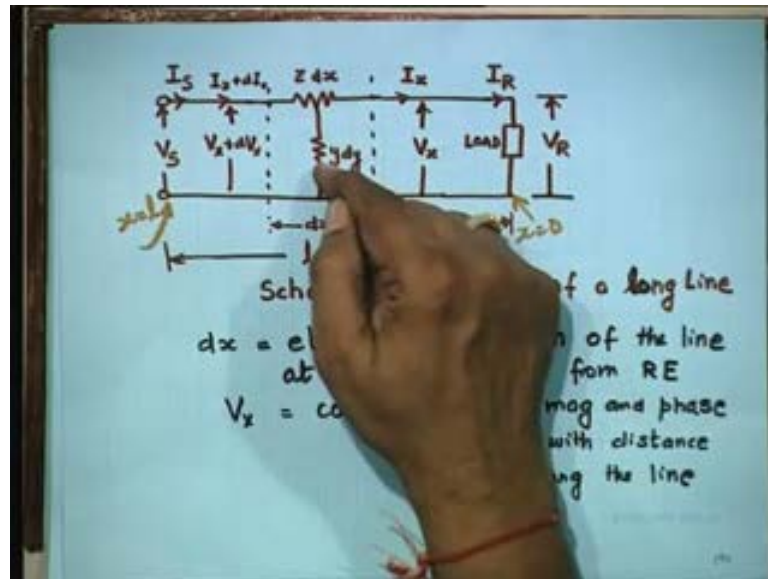
Of course, you can always expand, these are all the infinite series, sine and cosine. That is why whenever they are present, they introduce non-linearity understand that, so as you can expand sine and cosine, cosine is  $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$ , and sine is of course,  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ , similarly cosine is  $1 + \frac{\gamma^2}{2!} + \frac{\gamma^4}{4!} + \dots$  and so on, and sine is  $\gamma - \frac{\gamma^3}{3!} + \frac{\gamma^5}{5!} - \dots$ .

So the approximate value is  $1 + \frac{yz}{2}$ , under root  $\gamma yz$  into  $1 + \frac{yz}{6}$ . So, B is this, C is this, A and D is equal to this (Refer Slide Time: 47:27). So, finally we are able to obtain the approximate values for ABCD for long transmission line in terms of  $y$  and  $z$ , which we are familiar with T network and pi network, and all those things are obtained. So, this is how the second method, which is approximate the first method is the exact method, there is no approximation in first method, second is approximate method, I think we will stop here today, any questions you have.

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$z dx, y dy$ , it should be  $y dx$  not  $dy$ , it should be  $dx$ .

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No, there is no need, the only variable is  $x$  that is length, but there is no second dimension, there is no second dimension there any other question.

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Because, the reference point is always low voltage, the receiving end is in your hand and the sending end may be Sapura or jungle we do not know, may be singuroli we do not know, what it is, but the load end is in your hand, where you are, where the customer is, where the industry is. So, you are more bothered about the viewer place, not from where it is coming. In fact, in the latest power reforms, the power quality will not be guaranteed by the power companies but that is up to you, they will give raw power make it, as beautiful as you can.

So, you can establish various improvements, equipments, fax, SVS or whatever, you have to spent money and make it as beautiful you can. So, that is what the latest thinking is, rather than blaming the power company for saying the frequency is not good, voltage is not good, well, we give you a power, whatever it is, pay as less we do not mind and then, you ask for your requirement. So, if you want to make a tea I have given you milk, full milk tea, half milk tea or one spoon or without milk, it is up to you, customer

depends on customer rather than, just making the readymade tea and giving that is why, they give you part tea, water separate, sugar separate and milk separate.

Same thing is happening in the power sector, have power of your quality, of your test, your requirement, your needs anything else, that is coming next time, that will be the next topic is only that, what is alpha, what is beta, what do you do with that, all those things, anyway that is the part of travelling waves. I do not think I have to I will cover that, I am sure you must have read Tran giants in your under graduate and travelling waves in your under graduate, well it is given in the book, you can see the book, no, that gives you characteristic of the transmission lines that is why it is called characteristic, it significance is if you terminate the line in characteristic impedance as if there is no reflection, it is the infinite line, that line is called infinite line; all these things will form the part of next lecture.