Control Engineering
Prof. Madan Gopal
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture - 9
Dynamic Systems and Dynamic Response (Contd….)

Well friends, let us discuss today the last topic I will say, the last lecture on dynamic systems and dynamic response and in this lecture I am going to introduce to you another important plant for process control applications, the liquid level control. Let me put it this way in many process control applications you are required to control the height of the liquid in the tank. This is let us say a tank I have taken and let me assume that the height of the liquid in the tank is H bar plus h. Recall the nomenclature; H bar represents to the equilibrium position and h is a disturbance or perturbation with respect to the equilibrium position because of any reason the control manipulated variable or the disturbance variable. So, let us say that the pressure at this particular point is P bar plus p again the same meaning P bar being the equilibrium position pressure and the outflow here I am taking Q bar plus q. The variable to be manipulated is the input flow rate input flow rate and let us say that this is Q i bar plus q i that is all. These are the variables. Look at these variables carefully and then see the requirements.

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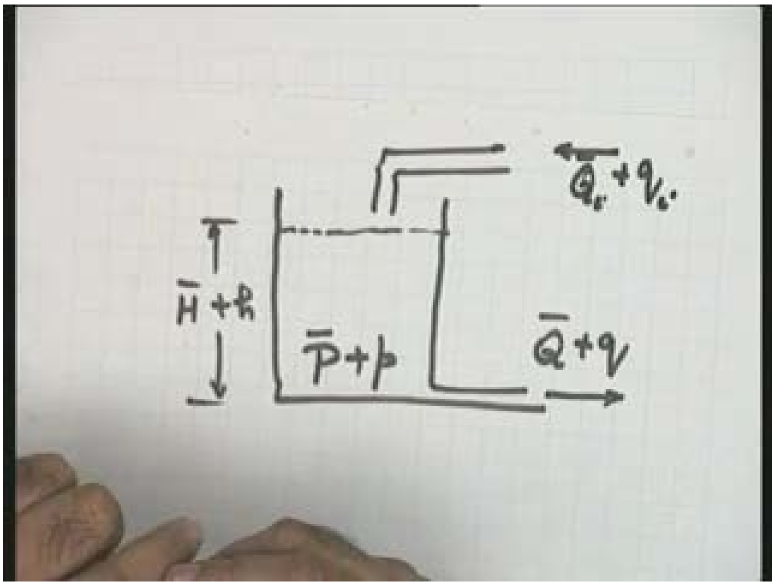

I am taking a single tank but there may be a cascade of tanks in any process control application and you are required to control the outflow rate or the pressure of the liquid or the height of the liquid in the tank and you will find that all the three are equivalent; it is visible I think from this application from this schematic also that if you control the height you are indirectly controlling the pressure here or the flow through this point and this will become more evident when we write equations for the system. So you see that the manipulated variable in this particular case is q I, the controlled variable in this particular case is h, p or q depending upon the mathematical model, in what way we write our mathematical model.

To get a transfer function model between the two variables let us say h is the controlled variable and q i is the manipulated variable. In that particular case I want to write the transfer function between the two and the step is exactly identical to what we have been doing earlier. First I will write the differential equations describing the dynamics of the system and then transform those differential equations to a suitable transfer function model. And if you require a state variable model naturally you will define suitable state variables and accordingly get a state variable model.

Come on let us take the dynamical equations for this particular system. In this case I will put it this way, I will write the volume balance equations assuming that density throughout remains the same. So I can take up volume balance equations for this system and therefore the rate of change of the volume of the liquid in the tank is equal to the rate of change of volume entering the tank minus the rate of change of volume leaving the tank that becomes my basic equation.

So let me say that this q i (Refer Slide Time: 4:33) is meter cube per second; it is the rate of change of volume entering the tank. q is in meter cube per second it is the rate of change of volume leaving the tank. So the volume balance will be the rate of change of volume within the tank, I think is obvious, will be given as A the cross-sectional area in meter scale dh by dt equal to q i minus q. See that this becomes my basic mathematical equation for the system. The cross-sectional area is A which is not changing because of the perturbation. A is a constant that is why that is why it has been taken outside the derivative, h is changing and therefore the rate of change of volume within the tank in meter cube per second is given by A dh by dt this is equal to q i minus q. So this is the basic equation. I will simply manipulate this equation to write my mathematical model in a suitable form convenient to me.

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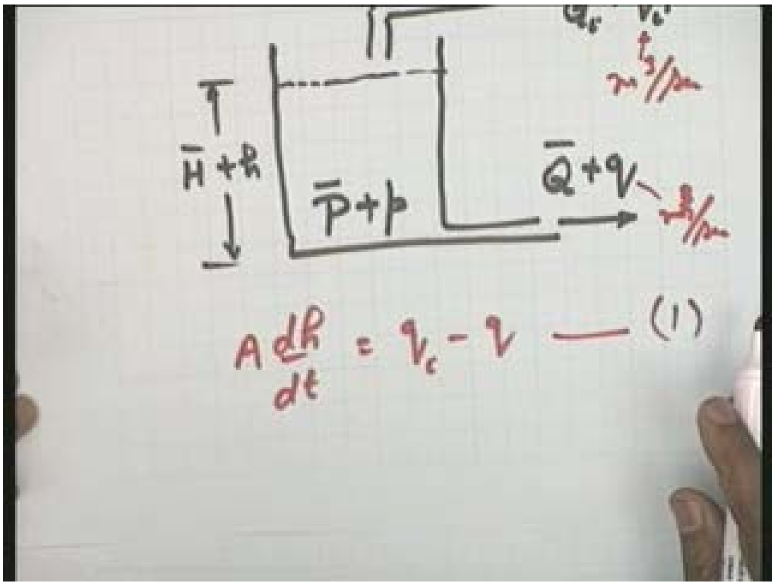

Now, to make it suitable to make it more convenient first of all I will like to define in this particular case also the concept of resistance and capacitance. Instead of going to the basic parameters of the flow system again and again, when we come to control systems application, I will be concentrating on the two parameters: the hydraulic resistance and hydraulic capacitance; the terms equivalent to electrical resistance and capacitance. So, for any situation the time constant will be given by R into C where R is the hydraulic resistance and C is the hydraulic capacitance.

Let me look at the hydraulic resistance and capacitance parameters. First I Let me use this slide itself, I refer to the hydraulic resistance first. You will please note that in this particular case this liquid (Refer Slide Time: 6:35) which is flowing through this particular opening this change in the cross-sectional area of the pipe is going to offer resistance to the flow. In general, let me say that the resistance to the flow may be because of any reason the sudden change in the cross-sectional area of the pipe or in the pipe you have a resistance like this, you are changing the cross-sectional are like this or you have a controlled valve, let me symbolically put it this way, this is the symbol to open the opening to control the opening of this particular valve so this also changes the cross-sectional area to flow. So whenever you change the area to flow there will be a resistance to flow.

I will use the terminology that this is your orifice flow and all this represented by an orifice as we say in hydraulic systems. So the orifice flow resistance now I am going to define and this should be very clear that orifice flow resistance may be because of this nature, well, there may be a bend in the liquid flow pipe, this is also going to offer a resistance to flow. So, instead of going to the particular structure the structure the mechanism in which the resistance is being offered I will symbolically represent the resistance by this symbol. This is my input this is output. This may be offered in anyway and I can consider this as orifice flow resistance.

Please recall, I am going to explain to you two parameters: the resistance and the capacitance. First I am taking up resistance and the resistance as is obvious is the resistance to flow because of the cross-sectional area change.

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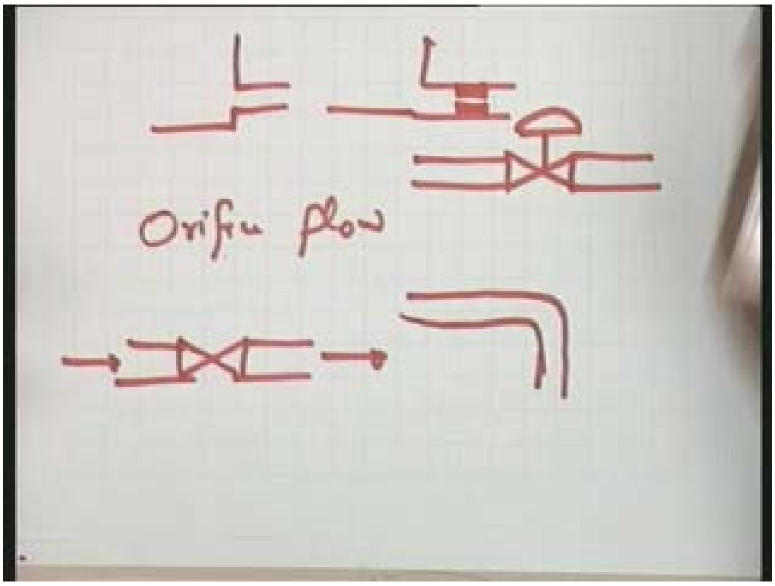

So in this particular case this is the symbol in which the controller is not put and if I use this symbol (Refer Slide Time: 8:34) it means it is controlled resistance like a potentiometer. This is a pure resistor and here is a potentiometer because you can change the area and hence you can change the resistance to flow. Fine.

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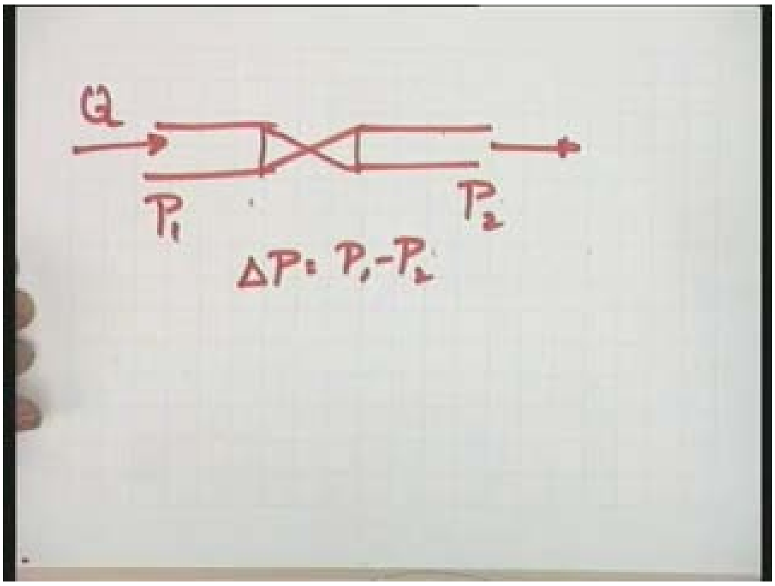

So let me take now this situation and let me say on one side of the pipe the pressure is P 1 and on the other side of the pipe the pressure is P 2. So the net differential in pressure across the pipe is P 1 minus P 2 and let me call this as del P is equal to P 1 minus P 2 is the del net differential across the pipe pressure differential. And let me say that Q is the flow rate through the pipe. Please see that the relationship between pressure difference and flow Q is highly non-linear; it cannot be captured by a linear mathematical model. So what normally is done in practice, this non-linear relationship is experimentally obtained and then a suitable model is derived from that experimental data.

Let me put it this way; that let us say that for a particular orifice for a typical orifice the experimentally obtained curve is like this (Refer Slide Time: 10:05). On this side I have del P and on this side I have flow Q and this is a non-linear relationship I have got. So in any system wherever you come across restriction to flow which I am naming as an orifice, any type of restriction, the parameter for the restriction can be obtained by conducting this type of experiment getting a curve between Q and del P.

Now let me say that at steady state the situation is like this: It is del P bar bar I have been using for the steady state situation and at steady state the flow is Q bar. This is the steady state and at steady state you can get it by a linear type of relation. You see that if you are operating at this particular point you can say Q bar is equal to some constant into del P bar. But naturally you cannot operate at a point because there will be disturbance. And let me assume that disturbance forces this operating point to deviate and let me limit my deviation to a small distance meaning thereby I can model the behaviour by a linear mathematical model only if the disturbance or the perturbation as assumed to be limited to a small distance.

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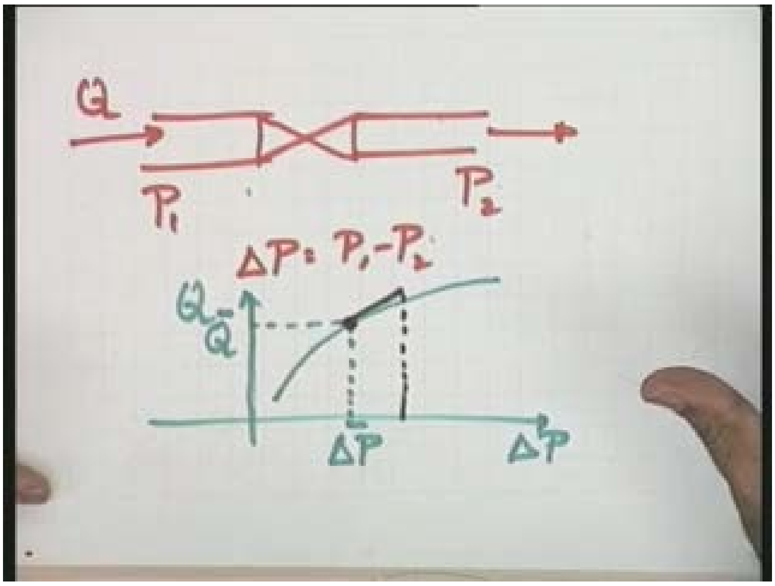

You will please note that in a controlled system situation this assumption is valid because after all if your controlled system is effective as soon as the deviation comes it has to curve the deviation; it has to reduce the deviation. So this deviation cannot increase if it is a controlled system because after all in any controlled situation the purpose of the controlled system is to minimize the deviation to reduce the deviation to zero as soon as it occurs. So it means this approximation or this assumption which I am making that this (Refer Slide Time: 12:09) is a small deviation around the equilibrium point though it is a highly non-linear curve you may please note that it is not a big assumption if this model is being derived for a feedback controlled system particularly. So, if it is an open general model your mathematical model is in doubt. But if it is a planned model which is going to be embedded in a feedback controlled situation naturally this assumption is quite valid because we hope we will be able to make a good controlled system which will not let the deviations grow indefinitely and hence this assumption will be valid.

So, in that particular case I assume it is a small perturbation and this small perturbation I say del p small p stands for a perturbation variable. So del p is a small perturbation and on this side let me say that this gives rise to a perturbation inflow and let me call this as q small q meaning thereby it is a perturbation variable. And it is obvious from this equation that from this particular system that your q is going to be equal to some constant into del p where the constant is the slope of this line (Refer Slide Time: 13:34). This constant is the slope of this line.

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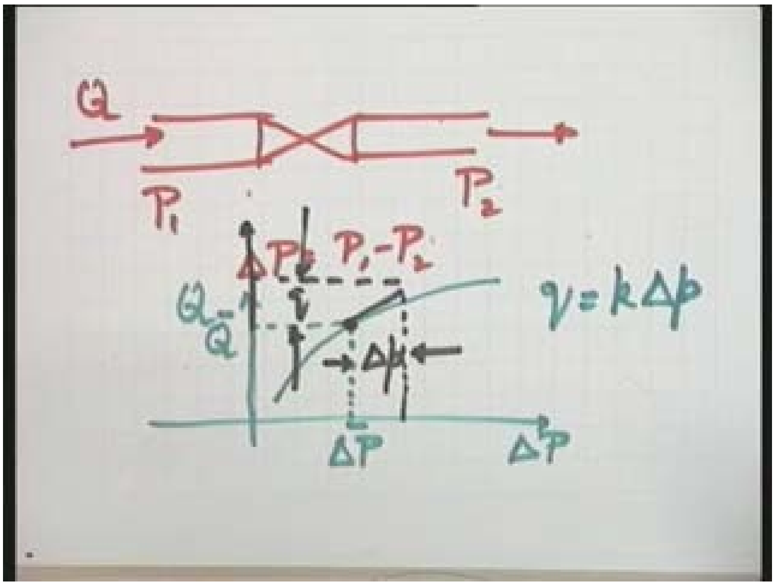

I hope this is okay that for given any situation where restriction to flow takes place I will tell you that actually this particular curve is supplied by the manufacturer of the equipment, the user need not go for this, this is the data the manufacturer of the equipment normally conducts these experiments and gives you not the value of k but rather gives you entire curve because the value of k will depend upon your operating point it is not a constant. So the manufacturer gives you the entire curve and from that particular curve you can obtain the value of small k and hence this mathematical model q is equal to k into del p is available for restriction to flow.

Well, in some situations the manufacturer instead of giving you this type of curve gives you a mathematical equation which captures the behaviour of the restriction to flow or the orifice. And a standard mathematical model for this type of behaviour is Q the flow is equal to C not sorry here it is A 0 (P 1 minus P 2) divided by rho under root.

This is the flow equation: C 0 A 0 (P 1 minus P 2) divided by rho where rho is the density of the liquid, A 0 is the area to flow the orifice area and C 0 is the orifice coefficient. You may write down please, orifice is coefficient and this data is supplied by manufacturer.

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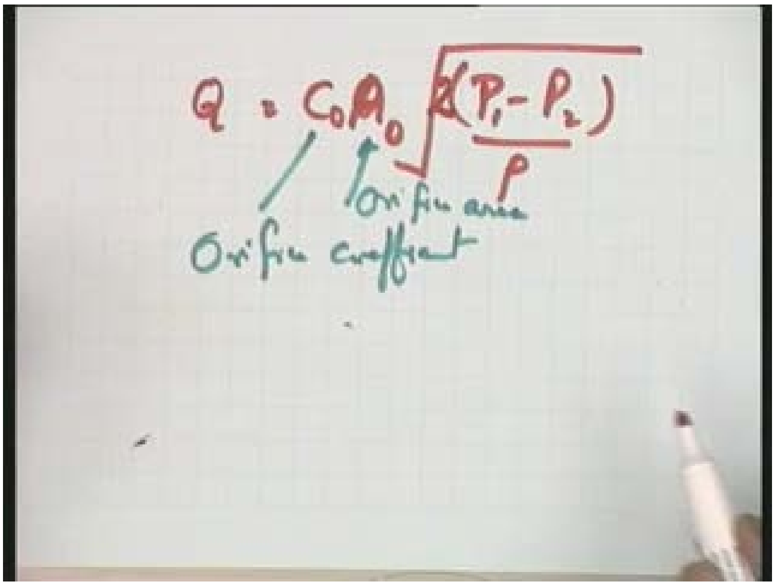

Now if I recall this could be checked. Probably in mathematical model it is 2 into (P 1 minus P 2) divided by rho this could be examined, there is no problem there. Now you see that this is a highly non-linear relationship between Q and pressure. So, if all these numerical values are given you can linearize this using Taylor series expansion the operating point and hence get a mathematical model q is equal to k into del p. So I will leave to the type of data available to us whether the type of manufacturers' data is available in terms of an experimental curve or the data is available in terms of this type of non-linear relationship. The point is this that, by linearizing this or by writing the slope on that particular curve we are able to write the equation q is equal to k into del p.

Now look at analogy. If you look at the analogy you will find that this equation is actually equal to i equal to e by R; i is the current, e is the potential difference and R is the resistance. So if that is the case I can say the hydraulic resistance, now I am defining the parameter please. For me now onwards k probably will not appear it is R which will appear again and again once you understand my point. It is hydraulic resistance R is equal to 1 by k. Whatever way k is determined it is determined from the data supplied by the manufacturer. Once we come to analysis in design I assume that this are the hydraulic resistance is available to us and therefore the equation becomes q is equal to del p divided by R this becomes the equation for me. Okay? I hope this is okay.

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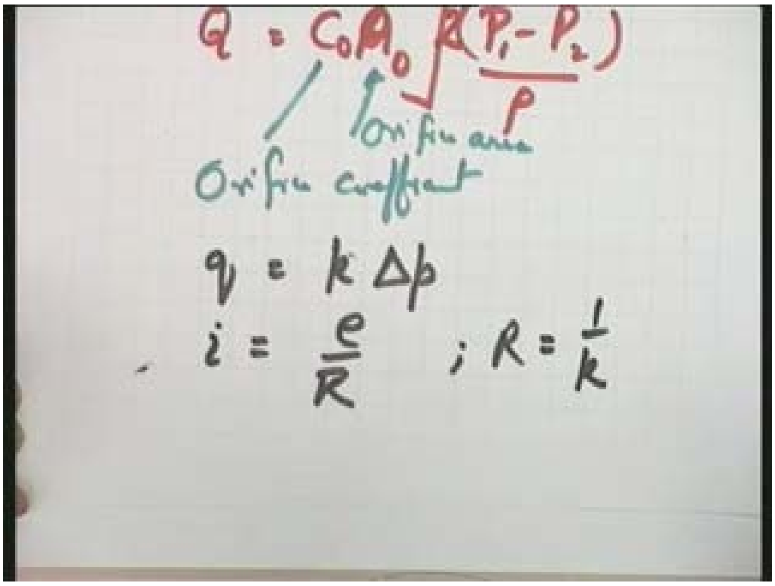

Now I go back to the original problem. This was your tank. Let me redraw that situation and now here symbolically let me put this particular restriction (Refer Slide Time: 17:34) and let me say that the R is the parameter of this restriction. So now this is the flow Q bar plus q and here I have H bar plus h and input variable as given earlier is Q i bar plus q i. Now since now we understand the restriction I can put a restricted flow at the input side also. After all as you have said your q i is a manipulated variable; it is the variable you will control; how will you exert your control? You are going to exert your control by controlling of an opening of a valve. So I think it will be appropriate if I put a suitable valve over here but with a control available in your hand. So in this particular case this becomes the situation. So there it is a controlled valve and through this controlled valve you are going to have the flow Q i bar plus q i.

So, with respect to steady state you can say what will happen?
At steady, please help me, what will happen? I think the situation is very clear. At steady state your H bar is equal to constant and Q i bar is equal to Q i equal to Q. This becomes a steady state situation. If there is no disturbance you are happy, the flow output is equal to the flow rate into the system and the height of the liquid and hence the pressure at this particular point P bar remains constant. And at this point (Refer Slide Time: 19:17) your flow control valve is at some particular situation and that particular point let me call as X bar. Capital X bar is the steady state opening of the flow control valve. So with respect to the steady state opening Q i bar is the inlet flow, P bar is the pressure, H bar is the height and Q bar is the outlet flow following this particular equation. I hope this situation is clear.

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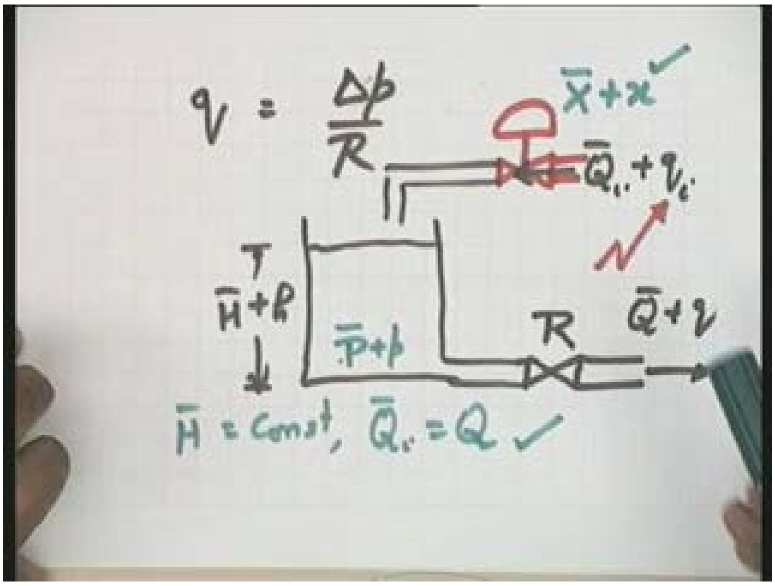

Now let us say some disturbance occurs or let me assume that I myself disturb the system by changing the inlet flow and that change is captured by this variable x (Refer Slide Time: 19:59). So, if I change this x naturally q i will change. So it means in this particular case, please see, your q i variable is definitely has to be definitely a function of x, has to be a function of x.

So in this particular case I hope you will agree with me that if I have a valve of this nature where the flow opening is X bar plus x, inlet is Q, pressure is del P your net equation becomes Q is a function of the flow opening x let me call it as small q; small q is a function of small flow opening x and del p the differential pressure across this. Or if you want to write the total equation the total equation could also be written. In that particular case it will be: Q is equal to f the total flow is equal to x and the total flow rate the total pressure is del P.

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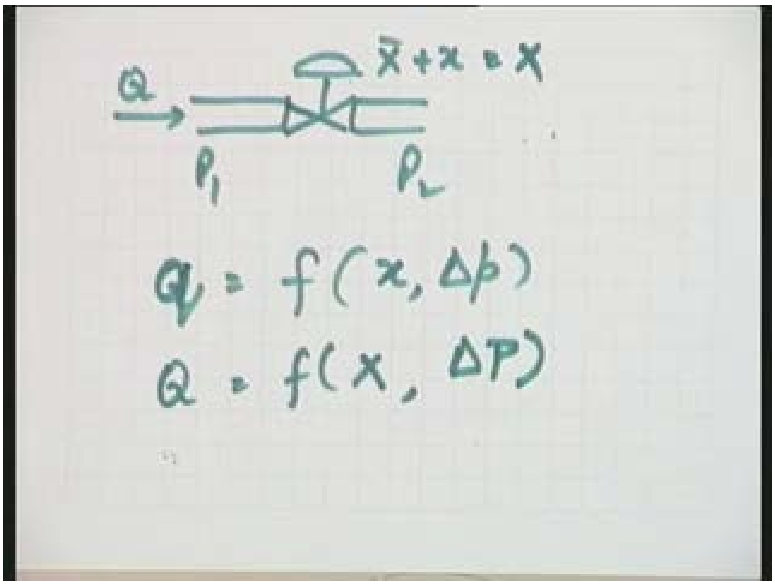

So I want to put it this way. If it is a controlled valve then the outlet flow is a function of the controlled opening which is in your hand which is in controller's hand and the differential pressure across the valve. This type of equation I will be using when I write the complete mathematical model for my system. This point may please be understood very carefully. If there is no control available in that particular case naturally Q is a function of del P only which we have linearized it with the help of a hydraulic resistance.

Well, the same situation again. R here, the inlet here, Q i bar plus q i. now you have to help me please. As far as the equation is concerned it is Q bar plus q H bar plus h and it is P bar plus p. What is the rate of change of valve? In this particular case let me write the complete equation. You recall the equation you had written: A dh by dt equal to q i minus q. I need your attention now. To transform this equation to a suitable form the electrical analogue should become clear. And surely you can help me. Keep the electrical analogues in mind. I have taken pressure as analogues to voltage e; I have taken flow as analogues to current i this is what I have taken; pressure as analogous to voltage e and flow as analogues to current i that is why the equation using hydraulic resistance as one of the parameters was written.

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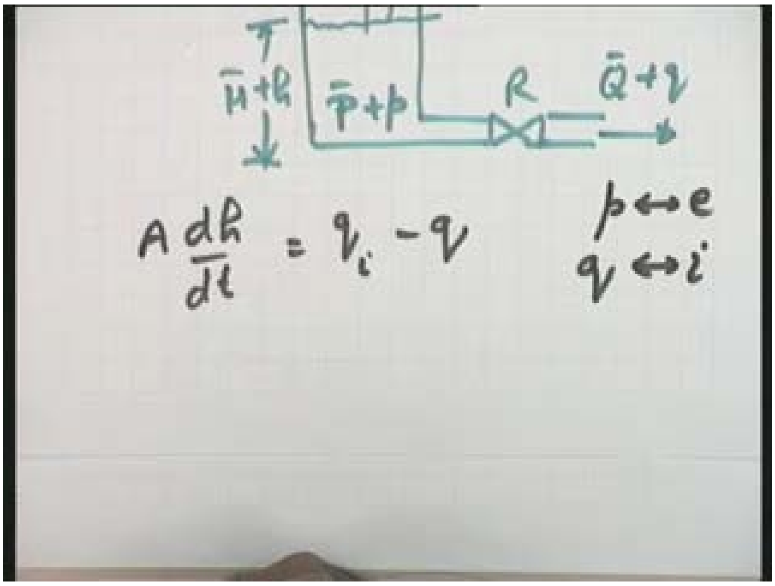

Now, in this particular case I find the equation is not in that form. That is instead of pressure I have the variable h here. Can you help me? Can I change this h variable into pressure variable so that I have the equations in terms of these two variables p and q?

[Conversation between Student and Professor – Not audible ((00:23:23 min))]

Yeah that is fine. So I can please see, write this equation, this particular pressure as is obvious in this particular case; it is rho g into h. the pressure and height are related by rho g into h and hence your equation becomes A by rho g dp by dt equal to q i minus q q i minus q, q i is the input variable.

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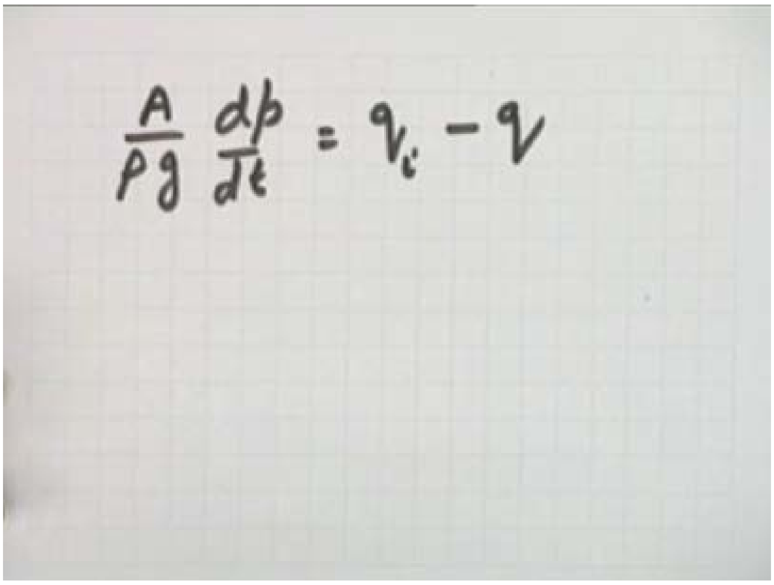

Well, as you helped me in this particular situation I need one more help please.

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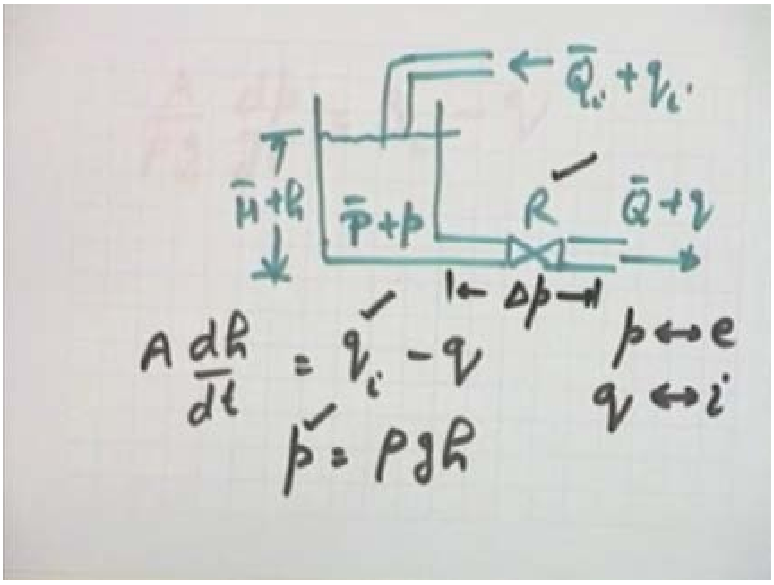

Please see that the pressure and q these are not independent variables. So you see actually they are controlled variables. If I am controlling pressure p I am controlling the flow q as well. So I really want to relate p and q so that in my mathematical model I have only the input variable q i and the output variable p. So it means this q the output flow or the outflow should also be eliminated. I need your help here. The flow should be eliminated. And you know the equation, the equation as you know in this particular case is this: You consider this restriction; the flow in this particular restriction is given by the equation that flow is equal to pressure difference across the restriction divided by the resistance offered by the restriction.

I assume that the manufacturer of the equipment has given you this parameter R. So it means this q (Refer Slide Time: 25:06) is equal to pressure difference across this let me call it del p divided by the resistance R.

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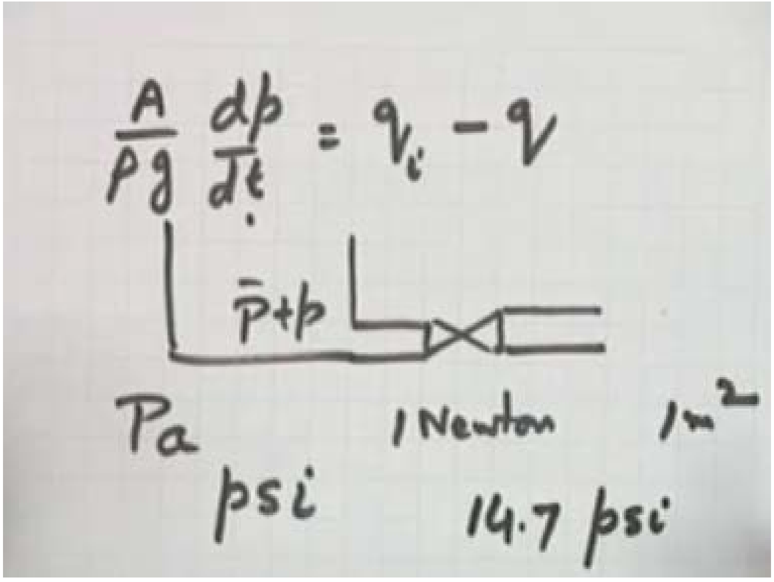

Now you have to help me for del p, what is del p?
Del p in this particular situation, this is your tank, del p in this particular……. this is P bar plus p will be affected by this pressure and the atmospheric pressure (Refer Slide Time: 25:36). Please see, the pressure across this orifice. On this side there is no tank here, it is the atmospheric pressure as far as the present situation is concerned. So what is the atmospheric pressure and how do I take the variables.

Please here, again I will put it set it alright once for all because these are the units I will be taking throughout. I know that the pressure unit a standard unit is Pascal which means 1 Newton of force applied on 1 metered squared area 1 meter per meter square is 1 Pascal. But you see that in process control industry let us go closer to the industry the units to the best of my knowledge even today are being used as psi pounds per square inch. Pounds per square inch are the units used in the process industry. I mean they should switch over, we should quickly switch over but still the situation is this that the manufacturers are giving the data in terms of psi mostly. 14.7 psi is your atmospheric pressure.

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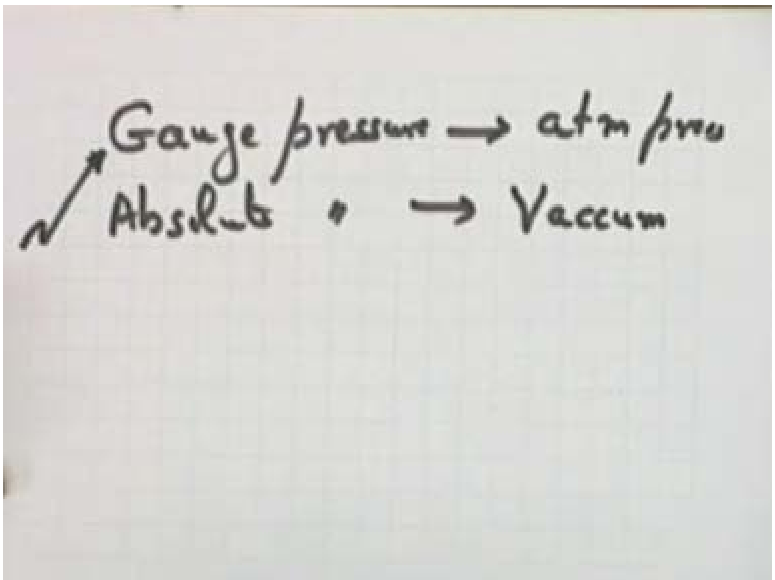

I will be taking mostly the gauge pressure. I need your attention here. The gauge pressure is a reference to atmospheric pressure and absolute pressure is a reference to perfect vacuum. If I take gauge pressure as the units throughout in that particular case please see the pressure the pressure here (Refer Slide Time: 27:24) in terms of gauge units could be taken as 0 because to convert it to atmospheric pressure the gauge pressure units will have to be added by 14.7. So this atmospheric pressure, absolute pressure is equal to 14.7 psi is equivalent to saying that the gauge pressures at this particular point is 0. So, if all the pressures in the system are in terms of gauges pressure in that particular case the reference is atmospheric pressure and hence I can write as far as this particular unit is concerned if R is the resistance and Q is the flow Q bar plus q, please see my equation q is equal to p dived by R. You have to agree to this equation where p this has to be very clear has to be gauge pressure that is why the reference has been taken as 0.

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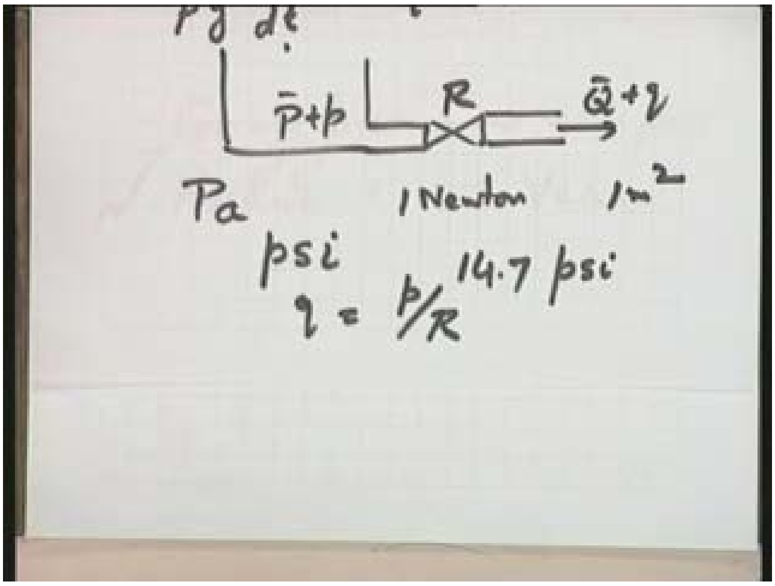

In this particular case of course if you take q as absolute pressure this atmospheric pressure as absolute pressure 14.7 view will cancel away and you will get the same value again. But to avoid this confusion all through my discussion I will be taking the gauge units and therefore the flow q is equal to p divided by R where R is the resistance offered by the orifice. So, in that particular case let me go to the basic equation for this system; the equation for this system now will become A by rho g dp by dt plus I am writing it directly in the form it is p divided by R equal to q i. please see, this is the equation now i get for this particular tank.

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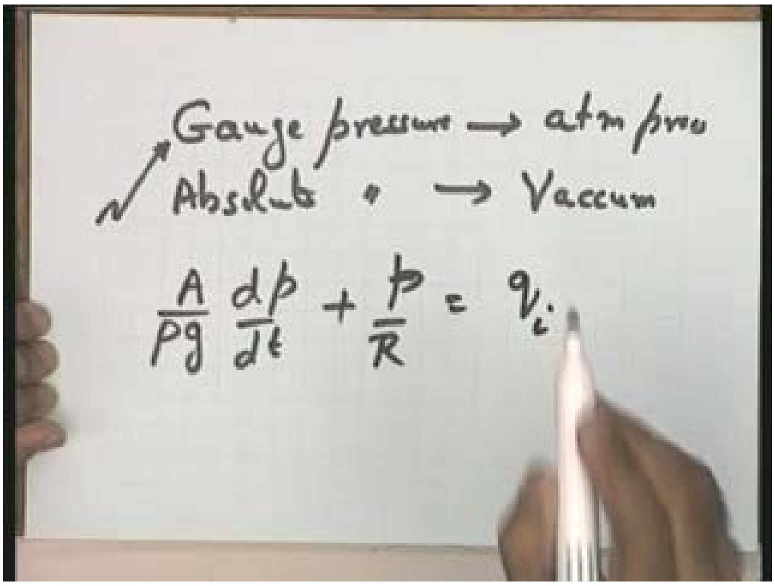

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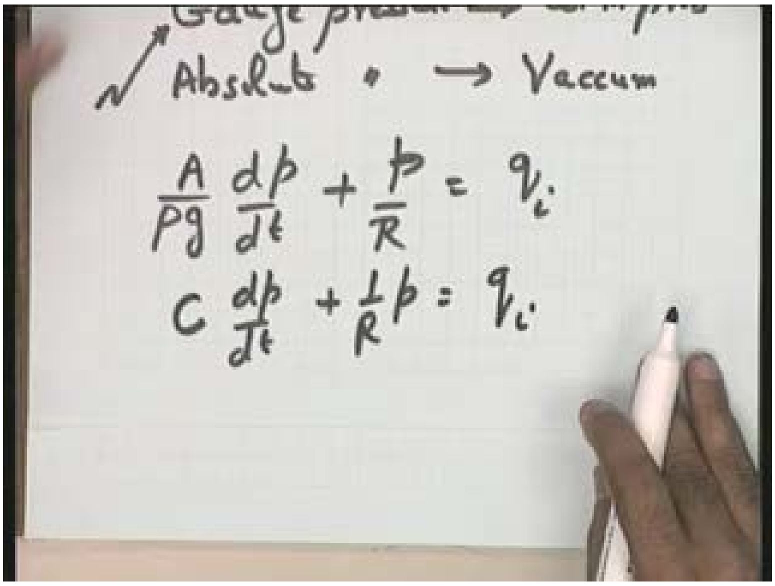

Recall, just compare it with the electrical analogue. I call this as hydraulic capacitance C dp by dt plus 1 by R p equal to q i.

Equivalent electrical circuit will appear as C de by dt plus e by R equal to i. So now, for me you see the hydraulic tank the liquid level control problem is equivalent to an electrical circuit you say in terms of at least developing the model and therefore now onwards as I made a statement in the case of thermal systems I will be mostly referring to C and R as the two parameters of the liquid level system. Assuming that in terms of manufacturers data or in terms of your own experimental experiments conducted on the system you will be able to determine these parameters R and C. So in terms of these parameters I can write this as RCs plus this equation RCs plus 1 P(s) equal to…. help me please, what should I write here? R Q i(s) I hope you are getting it, I just project the equation, okay I can write the equation here C dp by dt plus p by R equal to q i. This equation has been written in the transformed domain this way. This is the transformed domain equation.

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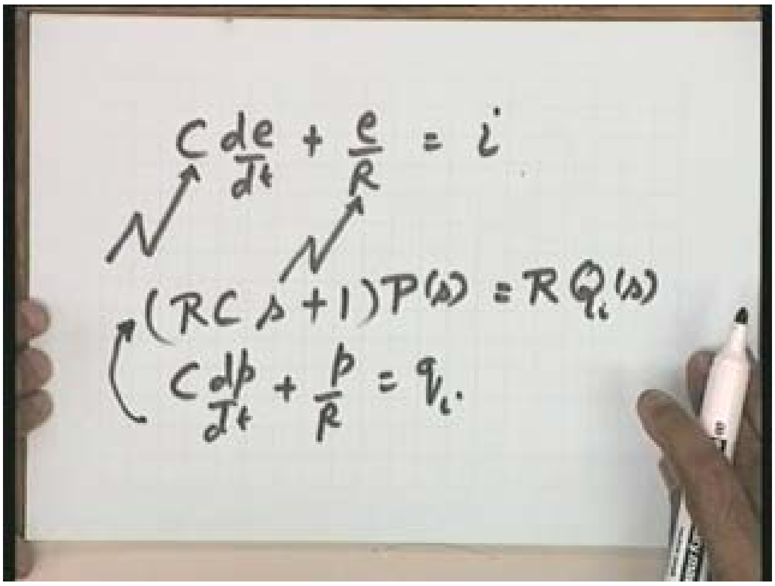

So now you see in this case, if I consider RC as tau parameter of the tank the time constant it is a simple exercise for you, if you take all consistent units in the system the units of R into C will turn out to be time units in seconds. Let me not go through this exercise, this you can do. This is tau the time constant of the system so in this case (tau s plus 1) P(s) equal to RQ i(s) becomes the mathematical model. In terms of the block diagram I can write this as Q i(s) as the input variable P(s) is the output variable R over tau s plus 1 is the function of the system which is a first-order system. So the liquid level tank turned out to be a first-order system.

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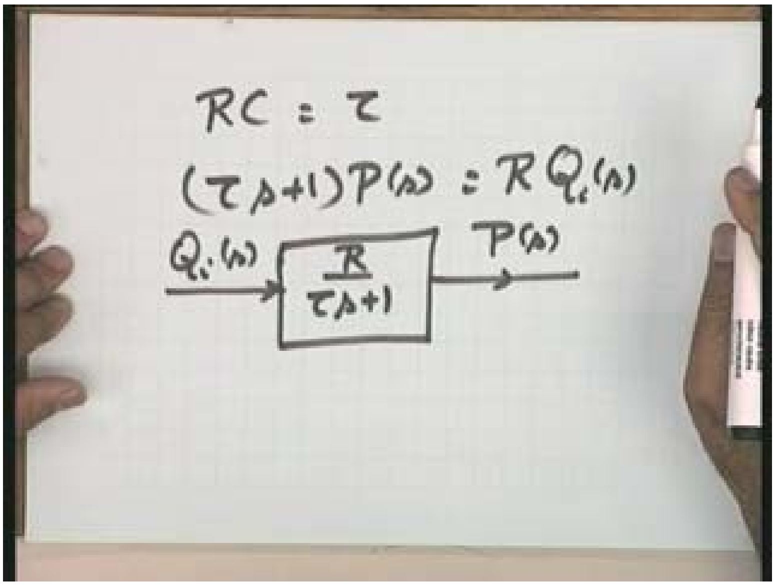

This in general let me make a statement. Since the parameter equivalent to inductance has not been taken in our thermal models as well as liquid level models. Can you tell me what is the implication of that on the total modelling? The implication is this that the models will turn out to be suitable cascades of first-order models. The inductance parameters in mechanical systems you had it. In the mechanical systems it was the spring which was giving that effect. So, whether it is a mechanical translational system or a rotational system a second-order s model or quadratic lag is possible because of that inductance parameter. But in our modelling we have made assumptions and those assumptions have led to the situation that there is no parameter equivalent to inductance and hence always it will be a suitable cascade of first-order models when I talk of thermal and liquid level systems.

So in this case you see, as I said that, the control variable is mostly the height of the liquid tank and not pressure. Pressure we have taken as the variable to develop a suitable analogue with electrical systems or to write the parameters the resistance and the capacitance parameters of the system. Come on, make a quick change, give me a change in this particular block diagram when I want the height H(s) as the output variable. Well p Whether you are controlling p or whether you are controlling H is equivalent. If I consider height H(s) as the input variable please see whether it is okay, R by rho g over tau s plus 1 becomes the equation. This becomes the equation.

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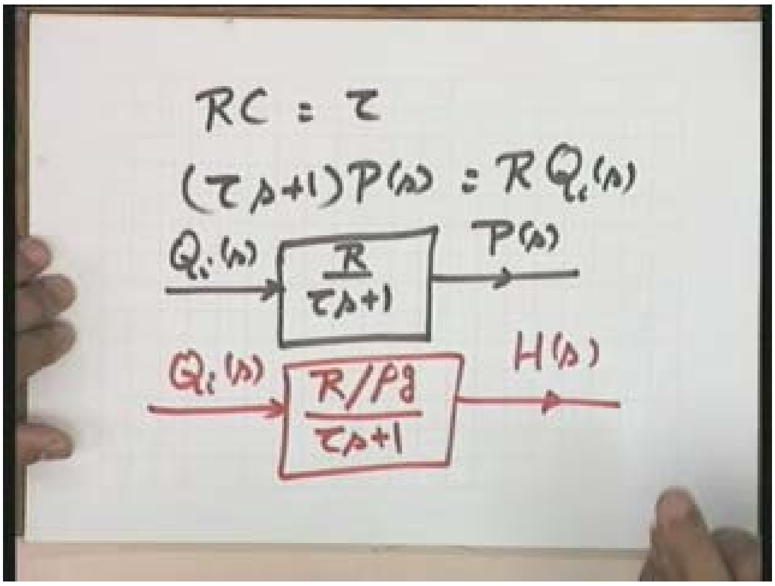
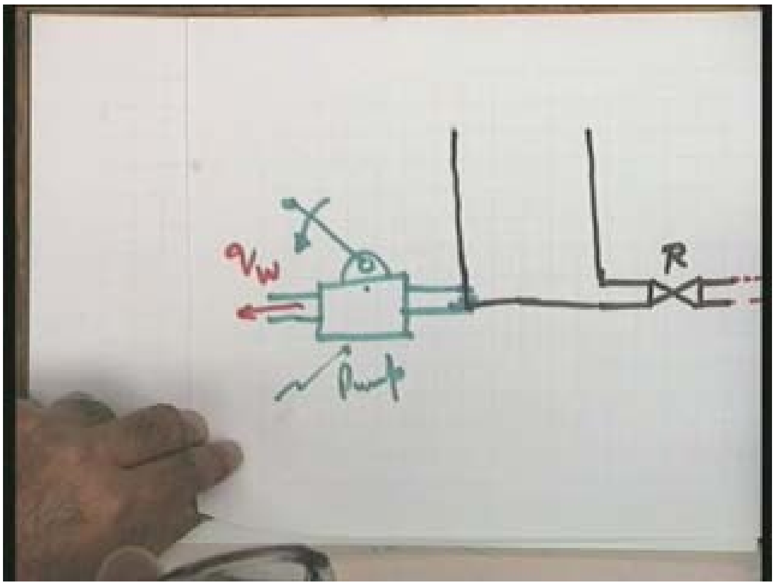


So P and Q i are the variables we will be taking with respect to electrical analogue and here it is the variable which normally is the controlled variable of the system. Well, I have to rush to complete this problem looking at the available time. So now let me take…., this was the mathematical model, I now take a practical situation, a practical situation in process industry is the following: a tank A tank now not an isolated tank, you will make the changes now in your model. This particular tank has a resistance to flow I represent it by R.

Now this tank (Refer Slide Time: 34:47) may be connected to other tanks in the system. So it means, the pressure on this side need not be atmospheric pressure. This is one change I make. Other change which again mostly is there in the process control applications is that there is an opening here an exit and this is not in uncontrolled exit this is a controlled exit. I have a pump here and this flow from the pump is controllable, you can control it.

Why do I show this situation?
This situation you see, there may be some processing going on in this particular tank and the output of that process is going the output of that processing is going to the other tank through this particular orifice. Now, may be in some situations you may require a bulk of this material at intermittent times for some other processing and that I have shown over here (Refer Slide Time: 35:54) and let me say this is q w and naturally as far as the modelling is concerned I will call this q w as a disturbance because it is not a regular flow. I have given a provision in this system that any time if I require certain flow for other process I should be able to operate this pump and I should be able to withdraw the liquid from the tank as far as from this particular opening and I call that as q w; naturally I will term this as a disturbance flow to the system.

(Refer Slide Time: 36:23)



Now what is the flow which you are going to control, or the manipulated flow? The manipulated flow is going to be through this particular valve and this particular valve I will put it this way is a controlled valve. So it means it has got x the opening which you can control and let me say that the flow is Q i bar plus q i. Now please develop the equations, yes please.

[Conversation between Student and Professor – Not audible ((00:36:52 min))]

Yes, provided you are using it, it depends you see, when you you have a are given you are given a particular situation it depends on your control configuration the designer's control configuration. If you are using this as the controlled variable then that flexibility which the user may require that sometimes you require a bulk of liquid, certain processing, let us say some composition change is going on in the process, intermittently you require that so that quantity will not be available to you if you are using this as controlled variable to control the height of the liquid in the tank. So that is why I am using taking a different provision over here that to control the variable I am controlling this flow opening (Refer Slide Time: 37:42) there is absolutely no problem.

If this type of requirement would not have been there then you could have used the manipulation at this particular point to control the liquid in the tank. In that particular case you could have given a constant flow rate here. This is one configuration because control configuration is not unique, I hope all of you are getting it; the control configuration for a system is not unique. I have given this configuration to give this provision as well that here is a pump here and here is an outlet available over here. If you do not require this flow then the disturbance from this outlet is zero and hence you are giving the processed liquid to the other tanks only. But if you require it the provision is available. It is only with respect to this requirement. I hope you get the point.

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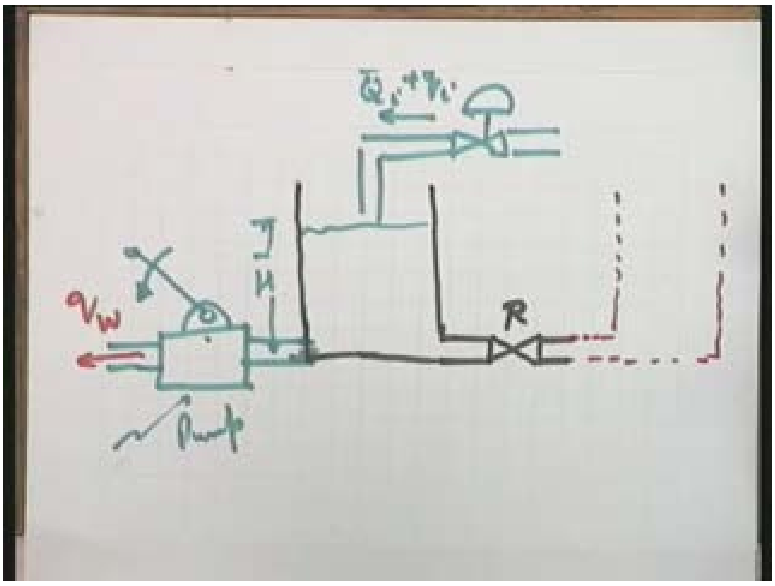

[Conversation between Student and Professor – Not audible ((00:38:33 min))]

Disturbance, I am calling it from the point of view of modelling because this. What is disturbance? Disturbance word you just recall the bathroom toilet tank example we had taken. the flow which is your useful flow we had called it a disturbance because this fitted very well into our mathematical model because we modelled that particular situation as a regulator system, the idea was to regulate the system to a constant head level, to a constant tank level. So, if the idea is to regulate the system to a controlled H to controlled height H, now it may be your useful variable, you are making it q w but if your requirement is this (Refer Slide Time: 39:22) you see that there is a dual requirement, you require this as well as you require the liquid in this particular tank at a constant height or as a constant pressure. So it means whenever a flow is demanded the control should take action so that this height becomes constant. So it means it is a regulator problem; it is a regulator control problem.

In setting it into a regulator control problem I am calling it a disturbance so control action is to reduce the effect of this disturbance to zero. It means, as soon as you have demanded q w the control action should set the height H to the preset location immediately. So you see that your purpose is this is a flow which you have demanded but as far as the mathematical modelling is concerned or as far as the setting of the controlled system is concerned I think there is no problem in that because I am making a setting a regulator setting in which the control action is to bring the height of the liquid to the preset height because this particular tank requires that. I hope you have got this point.

So, taking this q w as a disturbance now, you have to help me please, give me the mathematical model for this now. The tank the situation is same, the volume balance.

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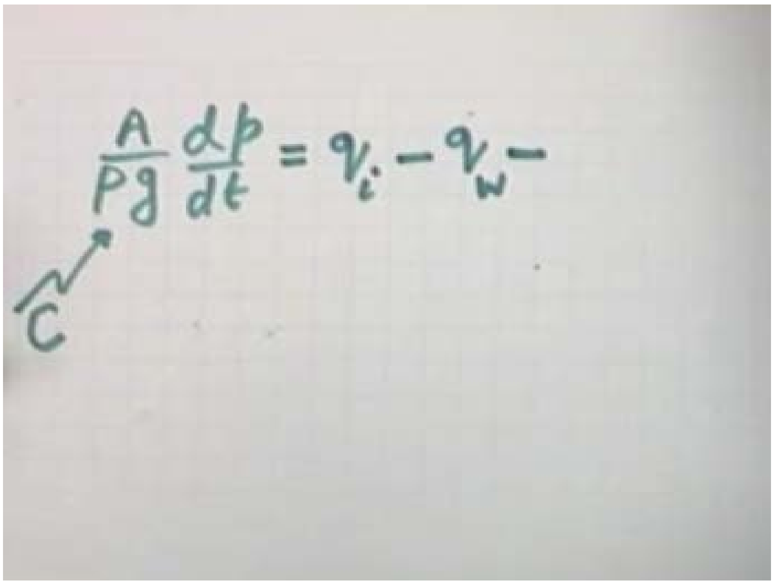
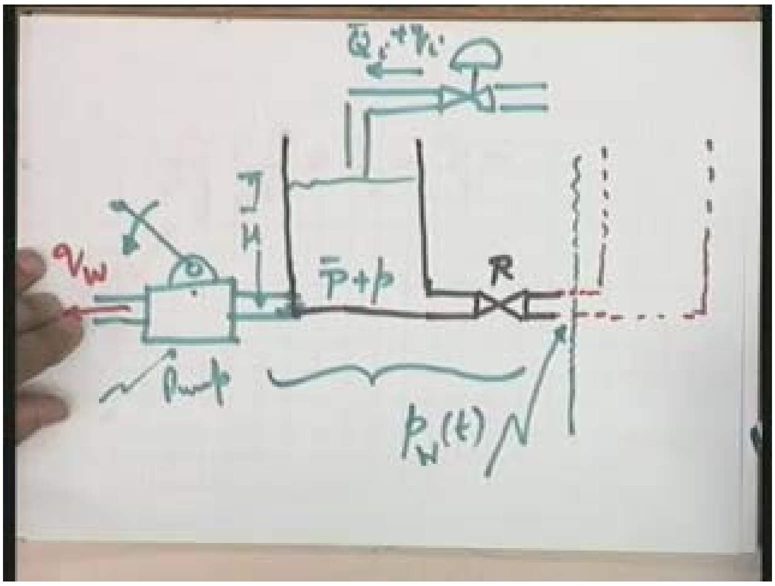


To write the volume balance I go for the rate of change of liquid in the tank which I write as A by rho g dp by dt. I am writing my equation in terms of p I will quickly change in H there is no problem there A by rho g dp by dt is the volume balance, this just to give you a confidence I am writing this; I think probably for the last time in these parameters and then we will start writing directly in terms of hydraulic capacitance C. So it is C into dp by dt help me please what is the flow rate situation. The inflow rate is q i the inflow rate is q i, how about the outflow rate, the outflow rate, if now, you do not mind the disturbance I say, q w is the disturbance which is the outflow rate which may be zero if you do not demand that and how about the other outflow rate? Yes, you have to help me here, see this variable P bar plus p.

What is the situation here?
In the earlier case it was taken as zero gauge pressure, but now, you see, what is happening in this part of the system is not in your control (Refer Slide Time: 42:00) you have been assigned the control of this particular tank and how the other tanks are operating is not in your control but if there are disturbances in this particular in these tanks in cascades of tanks there is a possibility that the pressure at this particular point will not remain constant will not remain equal to atmospheric pressure, it will not remain see at a constant value; forget about atmospheric pressure it may not be constant and let me say that variation is p w(t) the change in the pressure. I am taking this (Refer Slide Time: 42:36) as change with respect to the steady state value p w. This I think should be clear because it is not in my control; it depends upon the cascade of tanks to which the tank I am designing is interfaced.

(Refer Slide Time: 42:49)



If that is the situation…….. [Conversation between Student and Professor – Not audible ((00:42:51 min))] yes, [Conversation between Student and Professor – Not audible ((00:42:54 min))] no, you have to control everything in a practical situation but you see that if you have to control everything that is what we are doing. In this particular case also I am modelling the disturbances in other tanks by p w(t) and those disturbances again are not in your control. So it means, as far as this tank is concerned, if I am enforcing this so it means my controller should minimise or should reduce the effect of these p w also to zero you are very right. the control for I have to control everything that is why in my mathematical model I am taking into account all the possible disturbances which this can this tank has to has to tackle. This particular tank has disturbance from this side as well as disturbance from this side and his point must be kept in mind that when we go to the controller's design we will control everything including this disturbance as well as this disturbance.

So, in my mathematical model if I am not including p w I will not be able to use a suitable controller that is why a complete mathematical model should account for all the types of disturbances which can appear which can come on the system. Well, even in addition to you see, do not be very happy that well, once we have got q w and p w we have got all the disturbances which can act on the system, no. Even now there may be disturbances which have not been captured by our mathematical model. In that particular case you can only be happy with the situation; after all, your controller is going to be a feedback controller. Feedback controller inherently has got the property of curbing of filtering the disturbances affects.

Let us hope without modelling that any other disturbances which will occur on the system will automatically be taken care of by the feedback controller you design for the system. But the important thing is that important disturbances having dominating effect on the performance of the system should be accounted for and we hope that q w and p w are dominating disturbances acting on the system. This point I have made it very clear that mathematical model we have derived in every situation you model, it is always an approximate model. You have always neglected many physical phenomena acting in the system which you could capture or which you did not want to capture because otherwise your mathematical model will become too complex, purposely you have neglected them. However, the good point is this that we have a feedback controller whose inherent properties will be encashed when such disturbances act on the system.

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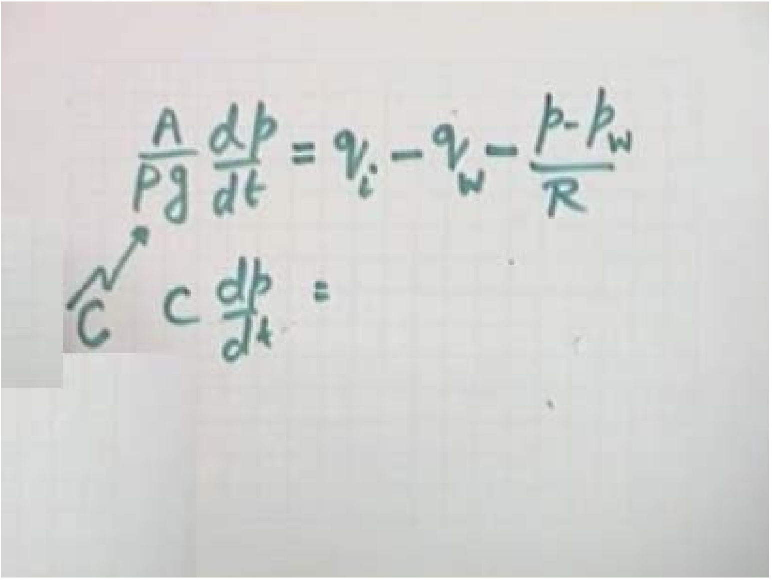

So with this background I can now go to the mathematical model: A by rho g dp by dt is equal to q i minus q w here it is going to be, yes, what did we plan? It is p minus p w divided by R permit me to write R directly or let me write the equation: C dp by dt equal to q i can you help me here.

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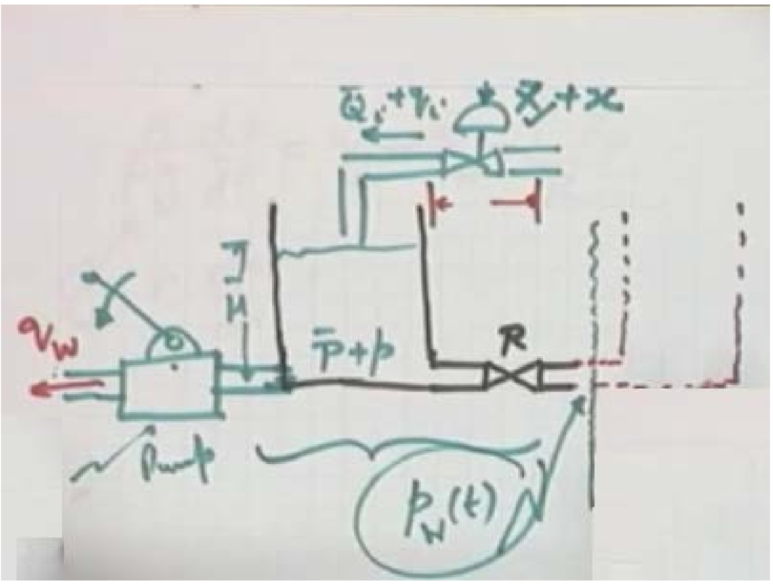

Suppose the controlled variable I keep is x you will recall X bar plus x small x, help me here please. What into what expression do you give me between q i and x approximations are bound to be there. In this particular case please see, the pressure differential across this particular control valve is going to be negligible it is not going to change the pressure differential across this wall and therefore in this situation I can write q i as a linear function of x. Again the assumptions involved may be noted very carefully, the assumptions involved may be noted that in this particular case how the pressure differential is getting established, please see.

What is the pressure differential across this?
On this side it is the atmospheric pressure, on this side (Refer Slide Time: 46: 58) this is the over head tank or the pump or any source which is supplying liquid. If I assume that input pressure which may be the over head tank or the pump and the atmospheric pressure on this side remain constant so under the perturbed situation the pressure differential across the control valve will not change. I need your attention here. Not that there is no pressure differential, at steady state there is a pressure differential; I am simply taking the perturbation and pressure to be zero because the model I am writing is a perturbation model and this is based on the assumption that the over head tank I have here or the pump I have here and the atmospheric pressure on this side they remain constant and hence the mathematical model for this particular control valve will be q i is equal to….. well, let me write it directly here K v into x hoping that you have agreed to K v is the valve gain but keeping in mind it is a perturbation model K v into x minus q w is disturbance I cannot touch it plus p minus sorry p by R and plus pw by R, help me please.

Write the equation now, the complete equation could be written as: C dp by dt plus p by R equal to K v x minus q w plus p w by R this becomes your equation and writing in terms of time constant I can write this as (tau s plus 1) you have to help me if I make an error, (tau s plus 1) P(s) on this side is equal to R K v X(s) is your manipulated variable minus yes RQ w(s) okay this is also alright I hope plus P w(s) please check whether this is okay. R has been taken on the right hand side and I get this equation. Therefore I can now get the mathematical model directly.

Input variable please, help me, the input variable in this particular case is X(s). Let me take the system gain here. Recall, the personality of a first-order system is described by the system gain and the time constant. Let me put a system gain here which is R K v. I am just writing this mathematical equation this way: this is plus and (Refer Slide Time: 49:56) this is a disturbance acting on the system all others are disturbances acting on the system, this is here in this block I have 1 over (tau s plus 1) and here I have P(s) the output variable P(s) here the output variable. Well, to create space if you do not mind I will remove this and put it on this side here. Here I have a space on this slide.

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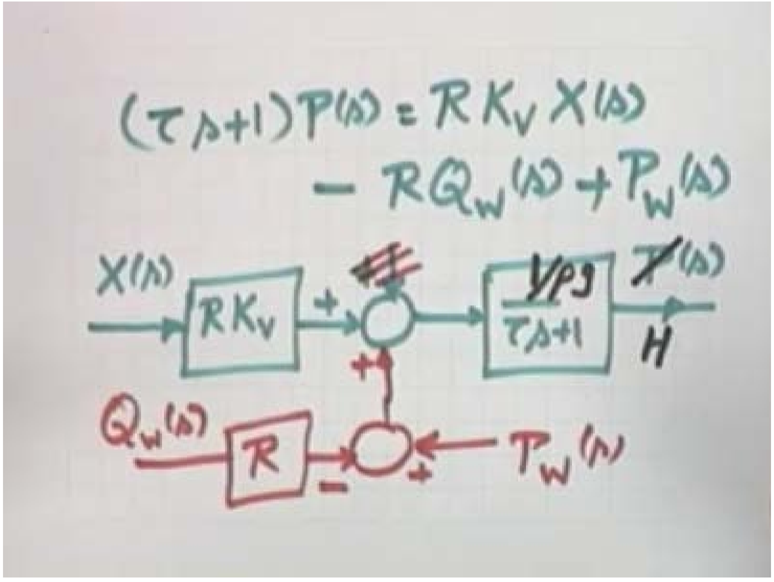

Now what are the other two things?
You see that the transfer function between P w and P if you find gets completed if I write over here P w. Please see the channel. The transfer function between P w and P is given by 1 over (tau s plus 1) only and the transfer function between Q w and P. Well, I put this as the system gain when your variables are Q w and P and I put a negative sign here it is a Q w(s). Please see very carefully whether this block diagram is okay. It shows three input variables: The manipulated variables the variable to be manipulated X, Q w the disturbance, P w the disturbance and you have the transfer function between P and P w, P and Q w and P and X.

Well, here itself I can make a change. If I make this P to the variable H the net effect will be this one will become 1 over rho g. It is possible to write this as 1 over rho g if your controlled variable is H so this becomes the total mathematical model of the system which we will be taking in our controlled system applications where the plant models for the liquid level systems will be required. I conclude with the statement that the plant models will most of the times turn out to be first-order models. Thank you.