Control Engineering
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Lecture - 8
Dynamic Systems and Dynamic Response (Contd….)

Well friends, let us resume our discussion on thermal system modeling. Last time I gave you basic equations and it is an electrical analogue [1:07.. o….] was also given of a typical thermal system. Well, I think the revision of those equations will come now through a systematic example in which the resistance, capacitance elements will be visible.

I take the example of a tank, let us say I take this as the tank, this is the input and here is the output, the liquid flowing in is at the rate Q and the liquid flowing out is also assume to be at rate Q so that there is no change in the hold up of the tank; the volume of the tank remains the same. In this particular case, as you know that the entire volume of the tank will not be at a uniform temperature so I use a stirrer so that using appropriate stirring (Refer Slide Time: 2:16) I assume that the temperature of the liquid in the tank is uniform and can be represented by a single temperature.

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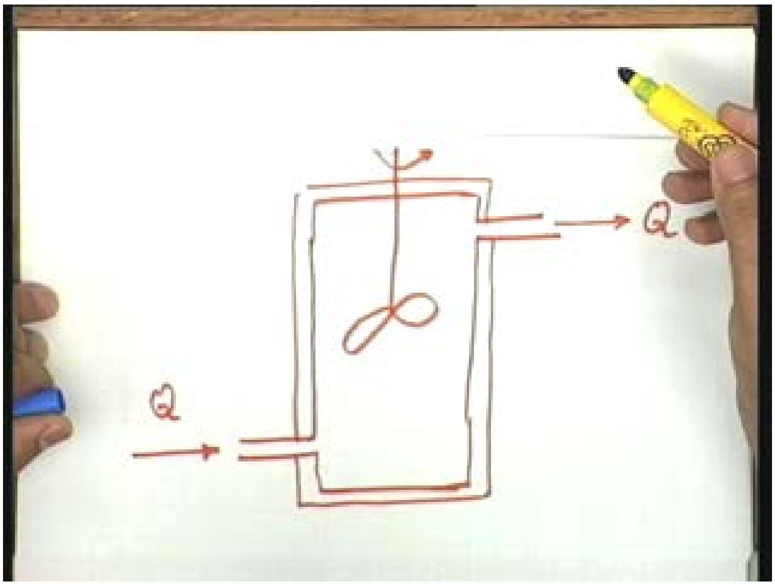

You will please note this point that the stirring is giving us the sufficient reasoning to assume the temperature of the tank to be uniform. Of course it is an assumption otherwise you would have got a distributed parameter system model and we are heading towards a lumped parameter system model. This particular type of system in process industry in the short form is referred to as CSTR Continuously Stirred Tank Reactor is the standard name used there, a CSTR. I repeat, Continuously Stirred Tank Reactor.

So in this case we have taken Q, let me take the units Q is in meter cube per second the rate of flow of the liquid in the tank. Let me assume that this liquid is entering at a temperature theta i bar plus theta i. You will note that the two theta i bar and theta i I have taken, theta i bar is the steady state temperature that is the temperature corresponding to nominal operation of the system and theta i is a deviation with respect to steady state temperature; it is a perturbation in the system may be in uncontrolled perturbation and you want to study the effect this perturbation on the dynamics of the system. This will become my standard symbol you see. That is I will be representing a steady state and a perturbation, steady state corresponds to nominal operation of the system. So I assume that the input conditions correspond to Q a constant flow rate and a temperature theta i bar plus theta i the units being degree centigrade.

Now let us see what the process is. In this particular case it is a heating process and I take up the situation where heating is going on through a heater. This is the heater element (Refer Slide Time: 4:35) and through this heater the heating is going on so naturally this you will say is the heat given to the liquid. And again as for the terminology used earlier I will be using H bar plus h, you have to very careful and note this point; H bar is the steady state heat flow rate the units of flow being Joules per seconds and h is the perturbation in the heat flow rate which in this case may be a controlled perturbation. May be the objective of the system is this that when theta i which is uncontrolled takes place you will like to control small h so that the effect of theta i is reduced to 0 or reduced to a negligible value. So you may see that h is also a perturbation but it could be or it will be a controlled perturbation to filter out the effects of disturbances acting on the system.

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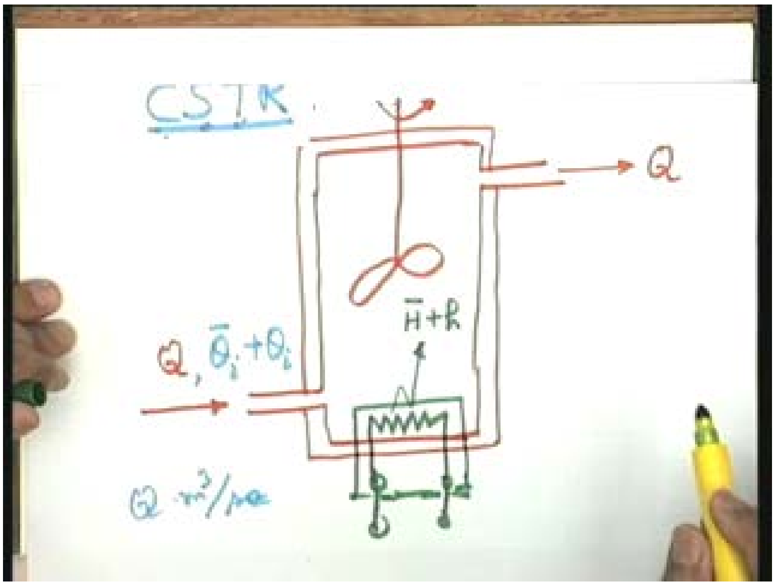

Joules per second here please note, so it means this particular liquid which is being heated using a heater, now this exit has got the flow Q and I assume that the heater temperature here is theta 1. Let me say again theta 1 bar plus theta 1. Theta 1 bar is the steady state value and theta 1 is the variation from the with respect to the steady state value. This is your heater mass. Now, as far as the liquid in the tank is concerned I assume that its temperature is theta 2 bar plus theta 2. So it means exit temperature is theta 2 bar plus theta 2 assuming that there is no change and uniformity of the temperature has been assumed. This is the exit condition (Refer Slide Time: 6:26), this is the condition within the tank, this is the inlet condition and this is the heater position. And the purpose is to write the dynamical model for this particular system.

In addition to this, let me take one more variable here and that variable is that this particular tank wall is definitely radiating energy is definitely giving energy thermal energy to the environment and let me assume that the environment temperature is theta a bar plus theta a. Again theta a bar will be the nominal environmental temperature at the time when the total system operation was at steady state and theta a is a variation in the environmental temperature itself which is beyond your control. So this is the total situation and you will help me now we will apply the basic equations I gave you last time on this particular model on this particular system and write systematically the dynamical equations of the system. You will keep in mind that theta i is a disturbance for me, theta a is a disturbance for me and small h is the controlled variable when I design a feedback control system for this particular situation I will really control this h in such a way that the effects of theta i and theta a are reduced to 0. This is the objective of the system, the control system is to come later; at this particular juncture the objective is to study the dynamical model of this thermal plant or the temperature process you call it.

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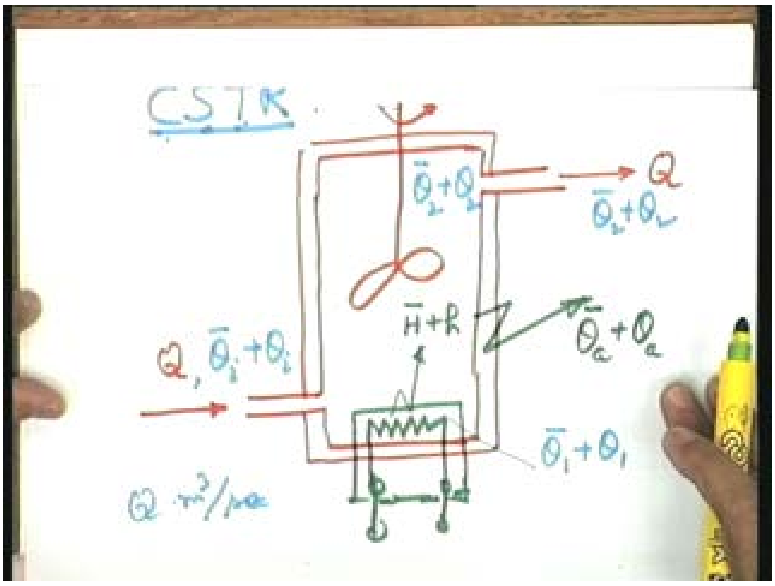

Equations: I will start with the heater equations. Heat balance for the heater; help me please, the input storage and the output conditions you have to tell me. Let me first see what is the input condition. The heat input, no, heat input, yes, you can say as far as the heater is concerned H bar plus h is the total heat which electrical energy is creating. Now this particular heat, well, electrical units I will convert to heat units later but let us say H bar plus h h is the heat produced because of the electrical energy supplied to the heater H bar plus h is the heat produced. Now where is this heat going? This heat number one is part of it is getting stored in the heater mass itself and part of it is going to the liquid in the tank. Is it alright please? The part of it which is stored in the heater is given by, let us say M is the mass of the heater and let me take it the units as kilogram. Here it is a more or less revision of basic laws also through this example: M is the mass of the heater and C I take as the specific heat of the heater substance. Let me take the units of C as Joules per kilogram degree centigrade.

Now you tell me what is rate of heat storage in the heater?
The rate should come in terms of Joules per second. You will please note that the rate of heat storage in the heater will be given by Mc d theta 1 by dt where theta 1 is the heater

temperature. You can look at the units: Joules, this is kilogram into Joules per kilogram degree centigrade into degree centigrade per second is equal to Joules per second. It is the rate of heat storage in the heater. I hope this is okay.

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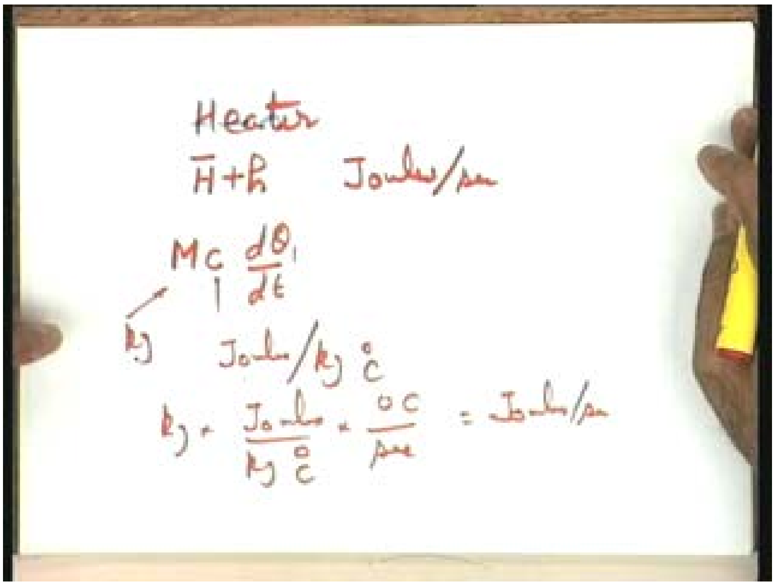

Now, how about the other component that is the rate at which the heat is being given to the liquid? And as you will see, in this particular case the heat is going to the liquid through a convective process. This (Refer Slide Time: 10:34) is a solid liquid interface over here solid liquid interface and through the convective process the heat is being given to this particular liquid. Now, again at this particular point the equation can be written as the theta 1 is the temperature of the heater, theta 2 is the temperature of the liquid mass and therefore the rate at which the heat is being given to the liquid is UA into theta 1 minus theta 2. Recall the units: A in meter squared and U is the film coefficient, the film coefficient at the liquid solid interface. Yes what should be the units please? Joules per meter squared degree centigrade seconds. You can see that this is the units this is for this particular U I have to written this the units as Joules per meter squared degree centigrade second. Now you please see that UA(theta 1 minus theta 2) will turn out to be in Joules per second. This is the rate at which the heat through convection process is going to the liquid in the tank.

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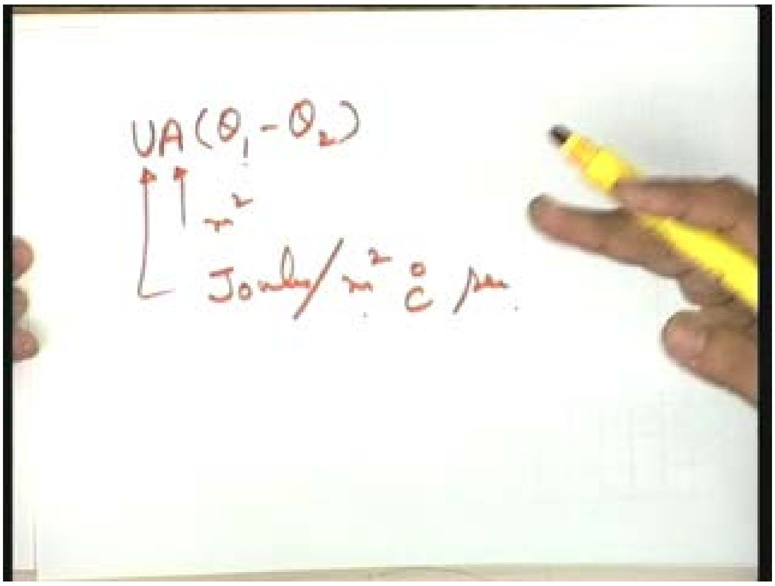

Come on now, with this information, if there is any question you are welcome to raise it here otherwise with this information I write the heat balance equation. The heat balance equation will be Mc d theta by dt the rate at which the heat is being stored into the heater mass is equal to the rate at which the heat is coming into the heater mass. it is, yes what is the value, h is the rate coming the rate at which heat is coming into the heater mass minus the rate at which it is going out it will become (theta 1 minus theta 2) into UA this becomes your heat balance equation.

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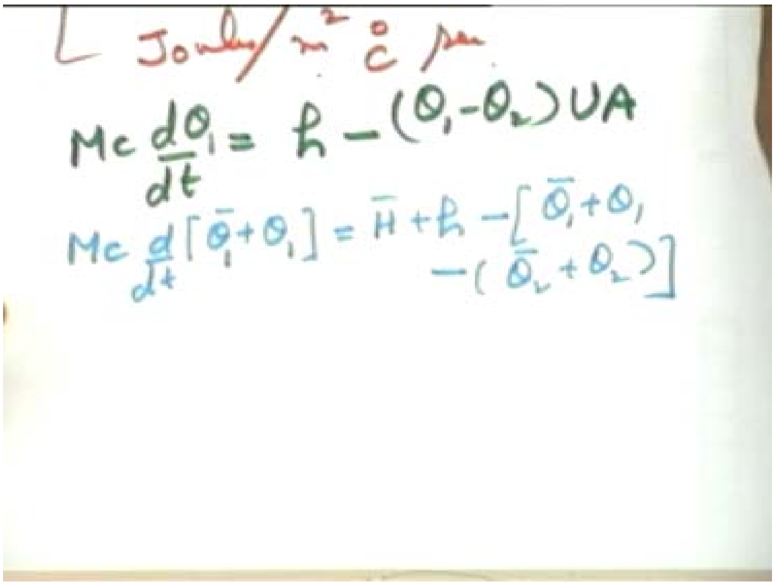

You will note one point, let me explain it here that I have not taken the theta bar terms the total equation if I want to write will look like this: Mc d by dt of theta bar plus theta sorry theta 1 here [theta 1 bar plus theta 1] is equal to H bar plus h minus [theta 1 bar plus theta 1 minus (theta 2 bar plus theta 2)] this is the total equation you may please note. I have directly written the perturbation model but let me explain how the perturbation model has come. this is the total equation: theta 1 bar plus theta 1 is the total temperature at any time t as far the heater mass is concerned, H bar plus h is the total heat generated in the heater, theta 1 bar plus theta 1 the heater temperature, theta 2 bar plus theta 2 is the liquid temperature.

Now, if I break this equation in two parts at the steady state you find d by dt theta 1 bar is equal to 0. So help me please, what is the equation at steady state? [Conversation between Student and Professor – Not audible ((00:13:46 min))] I can write the equation at steady state as: H bar equal to…. yes into UA your right please (Refer Slide Time: 13:54) UA into theta 1 bar minus theta 2 bar is it okay please? This is my steady state equation, all of you please note. Because the derivative will become 0 theta 1 bar being a constant the derivative will become 0 and at steady the left hand side is equal to 0 and from the right hand side I write H bar is equal to UA theta 1 bar minus theta 2 bar. It means if there is no disturbance acting on a system, if there is no variation in the various temperatures which are there as far as the system is thermal system is concerned then at steady state the heater mass balance is given by H bar the heat produced is equal to UA[theta 1 bar minus theta 2 bar].

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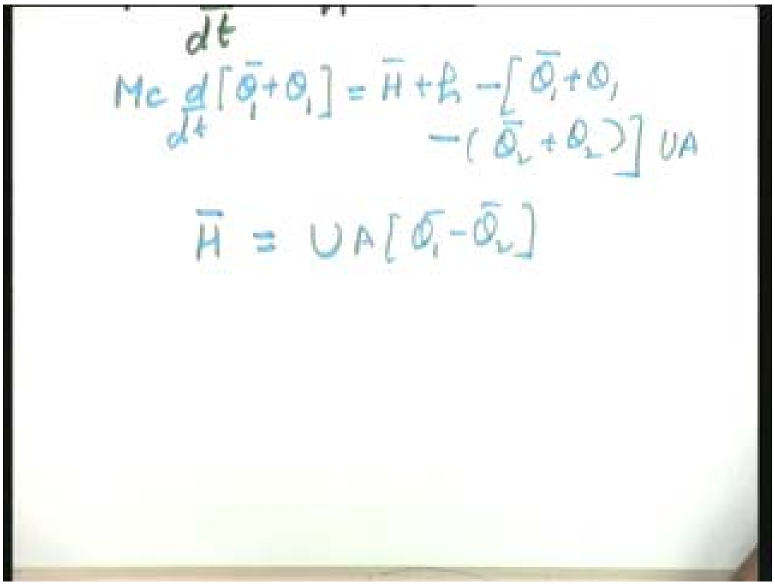

Now, coming to the perturbation model, I am using the steady state balance and the perturbation model separately. Coming to the perturbation model from this equation I write Mc d theta 1 by dt equal to h minus [theta 1 minus theta 2] into UA this becomes my equation. And if you understand this situation please now onwards I may not write the steady state balance again and again assuming that with respect to steady state the input the energy conditions will be satisfied my specific objective is to take the perturbation because if there is no perturbation there is no need of a control system. The need of a control system arises if and only if disturbances act on the system and these disturbances will deviate the various variables from the nominal point and that is why the need for controlling these disturbances arises. I hope this is okay.

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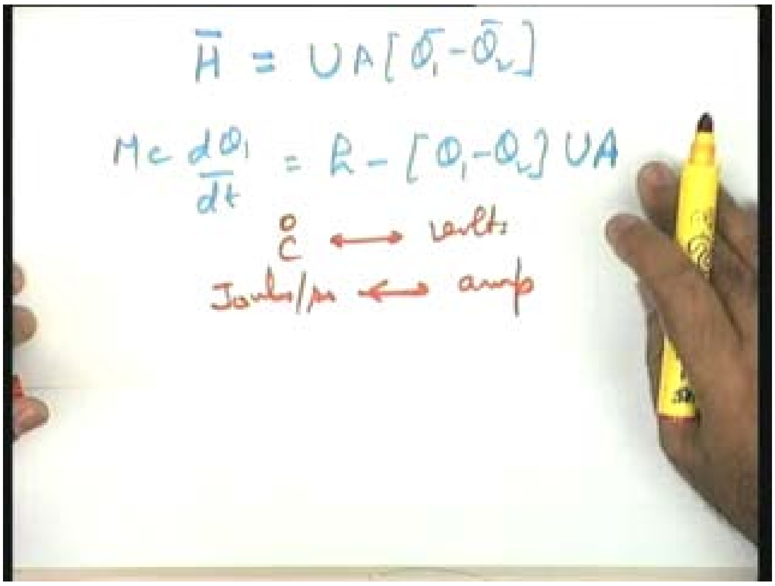

Under steady state open-loop system will work and closed-loop system and feedback system is not needed and therefore the steady state model need not be written again and again. We will directly write the perturbation model henceforth. Now this particular equation, if you say, if I assume that temperature degree centigrade is analogous to voltage volts, heat Joules per second is analogous to current amps can you help me please what is the equivalent electrical equation for this system? I hope it will come easily, come on please. In this particular case if degree centigrade, volts, Joules per second, amps relationship is taken I will write it directly and wait for questions from you if necessary.

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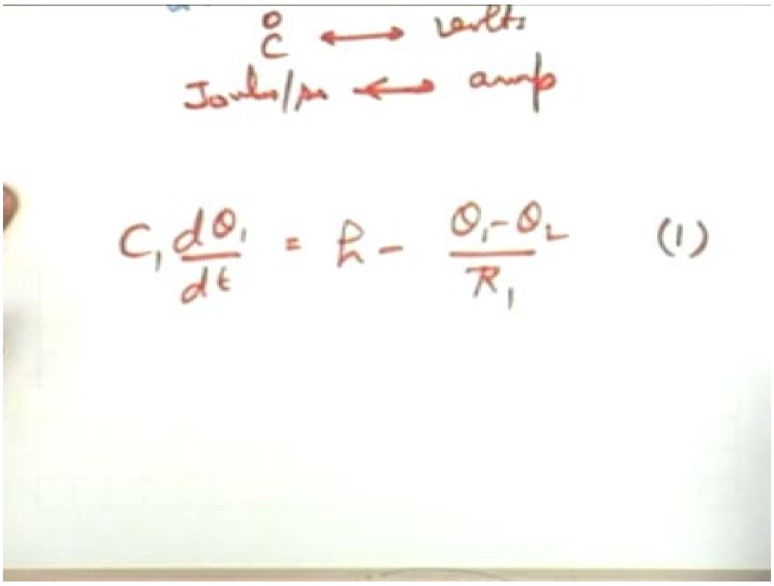

You see that C d theta 1 by dt let me call it C 1 dt is equal to h minus theta 1 minus theta 2 divided by R 1 where let me call C 1 as the thermal capacitance of the system and R 1 as thermal resistance of the system and now onwards if you understand this point I need not take up these variables in terms of mass, specific heat, film coefficient, area and all that. I assume that given a particular situation I will be able to get the values of thermal capacitance and thermal resistance and therefore the dynamical equations now onwards could be written directly in terms of Cs and Rs if this point is very well taken here.

You will please note that the identical equation is C de by dt is equal to current source minus potential difference divided by resistance. This is your equivalent electrical analogue. And in terms of electrical analogue for the thermal systems I will be writing the thermal capacitance and thermal resistance as the parameters of the system instead of going to the parameters M C U A and other parameters again and again. This (Refer Slide Time: 17:59) becomes the first dynamical equation as far as this thermal system is concerned. I hope this is okay up to this stage.

If this is okay I like to go the second equation. What is second equation? The second equation is given by the fluid or the liquid in the tank itself because the same situation as you have seen in the case of a heater applies to be liquid; it stores energy, there is an input of energy and there is an output of energy and I am going to write the heat balance for the liquid in the tank. Now, here also you have to help me please. First, help me for the storage of thermal energy. This, last time also you had given me.

Storage of the thermal energy the liquid now I am taking in the tank, rate of energy storage will be given by……. Yes, what is the total mass? V the volume, the volume as you see will remain constant because the inflow rate has been assumed to be equal to the outflow rate so the volume of the liquid in the tank is a constant parameter. So V the volume, rho the density, units you can take, meter cube, kilograms per meter cube and of course if I multiply it by specific heat V rho C becomes equivalent to Mc and V rho C d theta 2 by dt the units will turn out to be Joules per second and it is the rate of heat storage in the tank.

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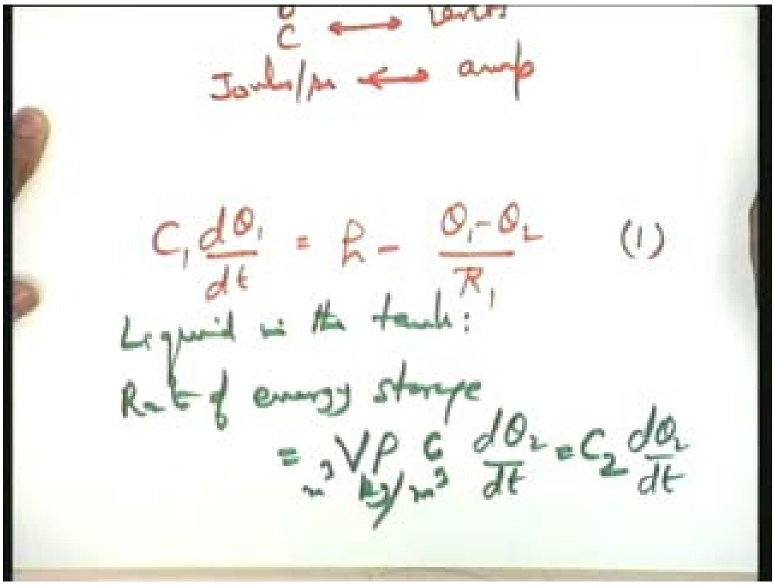

And now I hope you won't mind if I write this as directly C 2 d theta 2 by dt where C 2 is the thermal capacitance of the tank liquid, thermal capacitance of the tank liquid. The equation being i is equal to C de by dt e is the voltage, c is the capacitance and i is the current. You see that Joules per second or the rate of heat flow has been taken as equivalent to current. I hope the C 2 variable is alright for you now and it is coming in terms of the parameters: the volume, the density and the specific heat. Now help me for the heat input and output. Yes, what is the heat input please? Look at the tank situation and let us see what are the sources of heat input.

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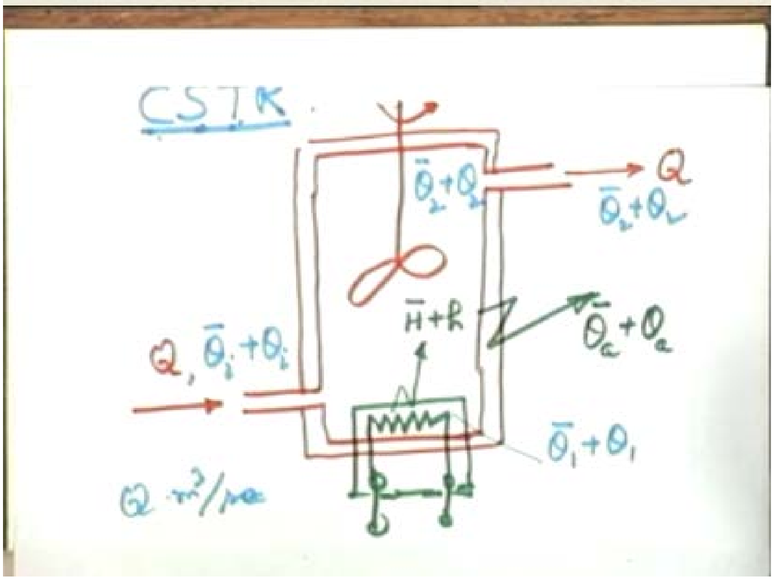

Yes, heat input in this particular case as you see is through this particular heater (Refer Slide Time: 20:42), any other source please? The inlet or the inflowing liquid is also a source of heat because any variation in the tank……. you see, at steady state a certain amount of heat is going on along with this liquid but if there is a variation in the temperature naturally there will be a variation in the heat carried by this liquid and therefore there is going to be a variation in the heat input to the tank through this particular source. So I will say, though it is a disturbance for the system with respect to the nominal point I will say that there is a heat inflow from this source, there is a heat inflow from this source and where is the heat outflow? The heat outflow will be here as well as here (Refer Slide Time: 21:30) that is going to be environment.

So help me please, now what is the heat inflow?

Look at this now. At this point Q is the Q meter squared per second Q meter squared per second, rho is the density becomes kilograms per second into specific heat C into theta i. Note that I am now directly writing the perturbation variable; I will save my effort and I will avoid writing theta i bar again and again. Otherwise theta i bar equations I will simply take out from the total equation and get the perturbation model. So here (Refer Slide Time: 22:11) I am writing theta i so it means it is the variation or perturbation in the heat flow rate and units will turn out to be Joules per second. This is the heat flow rate from this particular inlet point and how about the heat flow rate from here? The heat flow rate from here is going to be, yes, (theta 1 minus theta 2) into UA. This is the source. This we have already seen. This is the heat given to the liquid. It is the input for the liquid. It was output for the heater mass becomes input for the liquid naturally. This is as far as the heat flow rate of this is concerned, from the heater to this source is concerned.

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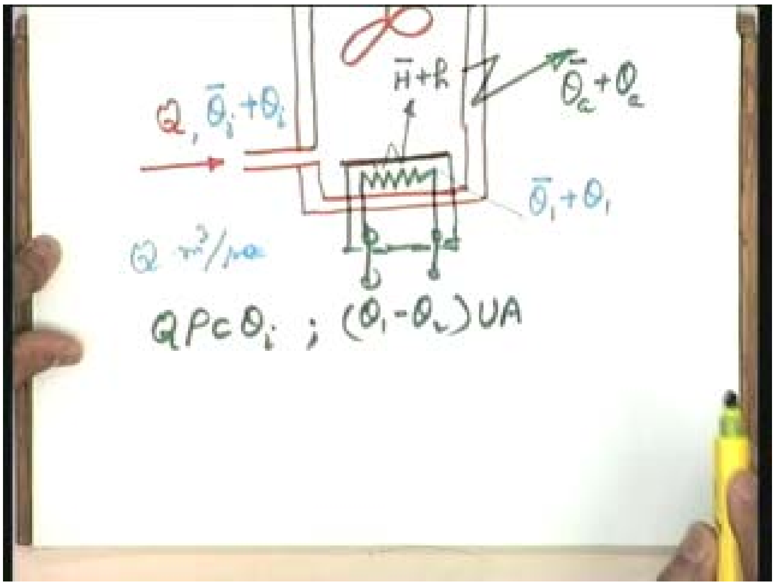

Coming to this now, what is the heat carried away by the out flowing liquid, is it not identical? Q rho C theta 2 Joules per second. It is the heat carried away by the out flowing liquid. And lastly let us see what is the heat lost to the environment. Here you help me, what is the lee heat lost to the environment. Well, there could be some storage of heat in the tank walls. Well, certain approximations are to be made and let me make an approximation that the heat storage of the tank walls is negligible and I assume the tank walls to be at the temperature theta 2 itself. If that is the case then you please see, because of this particular interface between the solid mass of the tank wall and the fluid which is the air again I may say that through the convective process the heat flow is taking place from the tank wall to the environment. It is the convective process because the tank wall gives you the fluid solid interface and U the film coefficient for that will come, A the area of cross-section will come and therefore between theta 2 and theta A there will be a flow of heat.

Now, if you see that Q rho C theta i and Q rho C theta 2 these are the terms which give the heat carried away through the liquid and therefore now I can write the heat balance equation.

What is the heat balance equation?
V rho C d theta 2 by dt the rate of heat storage the perturbation I am again repeating, it is the perturbation in the heat storage, it will be Joules per second, this is equal to the source. The sources are going to be Q rho C theta i plus (theta 1 minus theta 2) UA minus Q rho C theta 2 minus it is going to be theta 2 minus theta A into let me say just to make it different from UA let me make it U 1 A 1 or UA or U 1 A 1 is alright there is no problem. U 1 is the film coefficient at the wall air interface and A 1 is the area of the contact, the surface area. Is it okay please? This could be written as, as you see: Q rho C (theta i minus theta 2) plus (theta 1 minus theta 2) as UA minus (theta 2 minus theta a) as U 1 A 1 into U 1 A 1. Is it okay?

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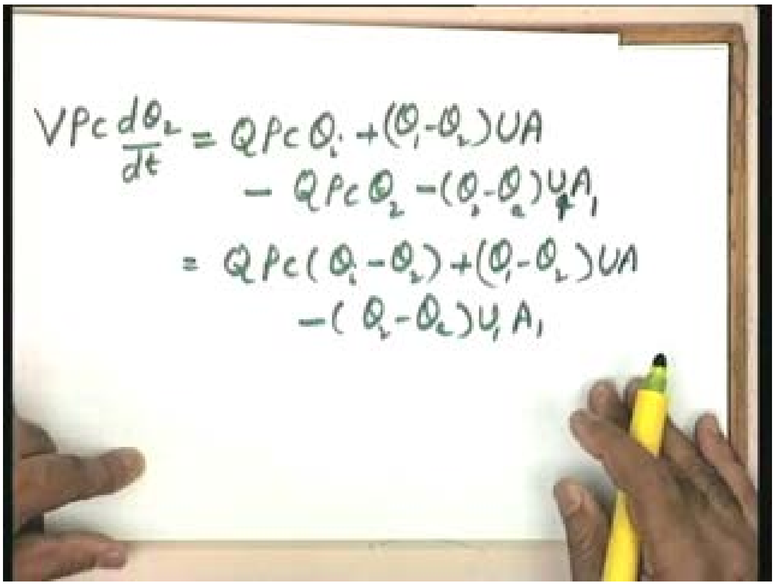

Now let us go to an equivalent electrical parameters. If I take equivalent electrical parameters I will write this equation as C 2 the thermal capacitance of the liquid in the tank, d theta 2 by dt equal to theta 1 minus theta 2 by R 1 is the thermal resistance of the heater mass plus theta i minus theta 2 into R 2 thermal resistance of the inflowing of the liquid in the tank. that is R 2 is equal to how much 1 over Q rho C is I think obvious; R 2 the thermal resistance is equal to 1 over Q rho C minus let me put it theta 2 minus theta a divided by R 3 it is the convective thermal resistance at the air wall interface. So, if we forget about the basic parameters of the system and concentrate on R 1 R 2 R 3 and C 2 I think well it should not create any confusion rather it should give a simplified version in terms of time constants as you will see in terms of dynamical equations which you are going to write. So from here, this point onwards I will take these parameters only.

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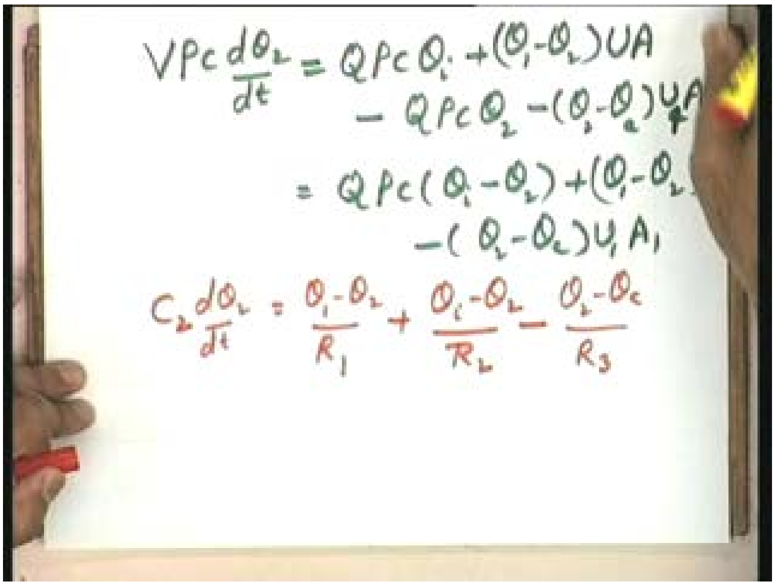

Let me summarize my equations: C 1 d theta 1 by dt equal to h minus theta 1 minus theta 2 by R 1. C 2 d theta 2 by dt equal to theta 1 minus theta 2 by R 1 plus theta i minus theta 2 by R 2 minus theta 2 minus theta a by R 3.

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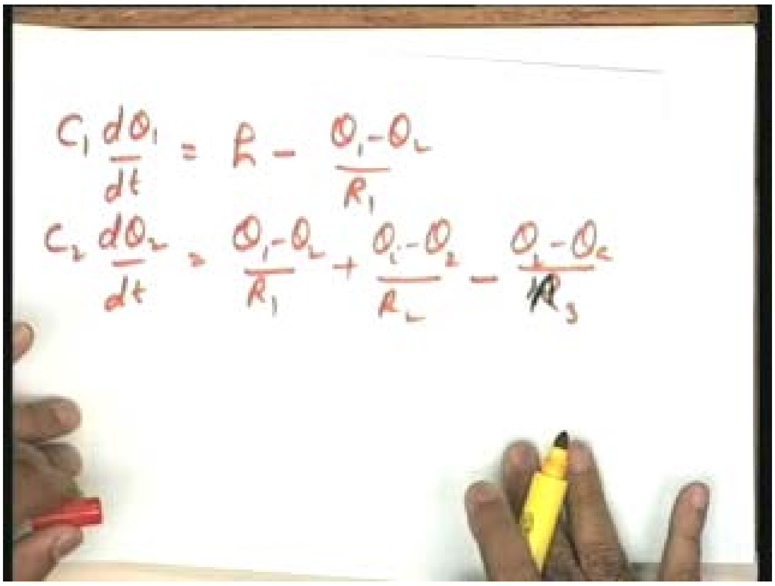

These are the two equations and hence the dynamical model. I will say the effort I wanted to put in to obtain the two differential equations has been obtained. It is a second-order system because there are two differential equations involved.

Now you please see that, as far as this system is concerned, if want to write the state variable model the equations are more or less in the state variable format because you could take x 1 is equal to theta 1 and x 2 is equal to theta 2 as the two state variables and you have got the equations in the form you want for the state variable formulation; it is x 1 dot equal to in this side you will write, you will please note, what are the input variables in this particular case h is an input variable which is a controlled input variable, theta i is an input variable, it is a disturbance input variable, theta a is also an input variable. So in this particular case there are three input variables: h, theta i and theta a and two state variables theta 1 and theta 2 (Refer Slide Time: 29:06). This structure should be seen.

Now, if I ask you to give me a transfer function model what will you say, you will have to first ask a question as to transfer function for the transfer function what attribute of the system I am interested in. May be I am interested in theta 2 as the output variable and h as the input variable in that particular case I will get the transfer function between theta 2 and h assuming theta i and theta a to be 0 this point is to be noted please. As far as the state variable model is concerned x 1 is equal to theta 1 x 2 is equal to theta 2, input variable r is equal to h it is a vector now h theta i and theta a these are the input variables. If you are interested in a transfer function model take one input at a time. Say, I am taking h so I will let theta i and theta a go to 0 because I am interested in the study of the dynamics of the system through transfer function transfer function being input output formulation you will have to take one output and one input taking theta 2 as the output h as the input, theta i and theta a equal to 0 I will eliminate theta 1 and get a transfer function model between theta 2 and h. identically if you are interested to study from the input output model the effect of the disturbance variables in that case the reference input or the controlled input will be assumed to be equal to 0 and you will set up a mathematical model between theta i and theta 2 or theta a and theta 2 as the study demands.

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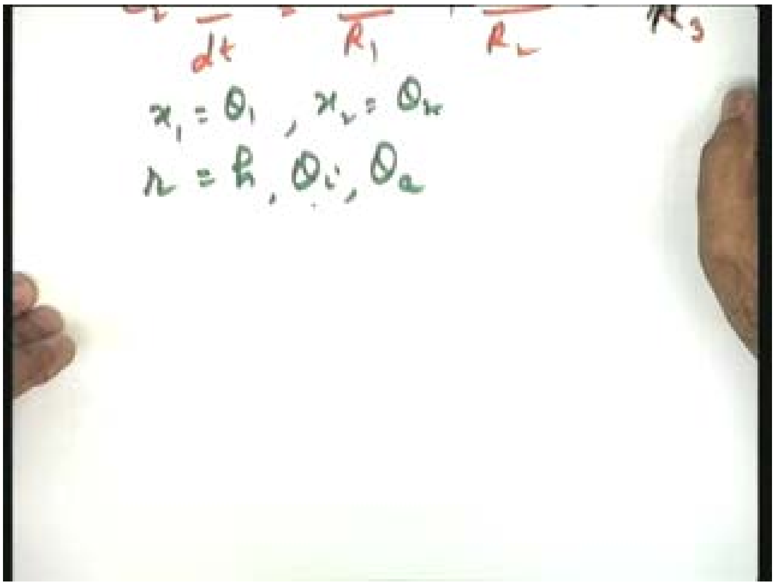

I can leave this exercise to you that this type of manipulation you should be able to do to get the required formulation either in the transfer function model or in the state variable format. Now let me take a particular case which I will be using later as a case study for a control system. It is a simplified version of this. I assume that the heat storage….. please note this point and modify your equations….. heat storage in heater mass is negligible compared to the heat storage of the liquid, heat storage in heater mass I am neglecting. Based on this assumption I want to set up a mathematical model here, a plant model and I will like you to store that model in your memory because I will recall that from your memory when I take a control system and for that I will like you to have a look at this particular model itself.

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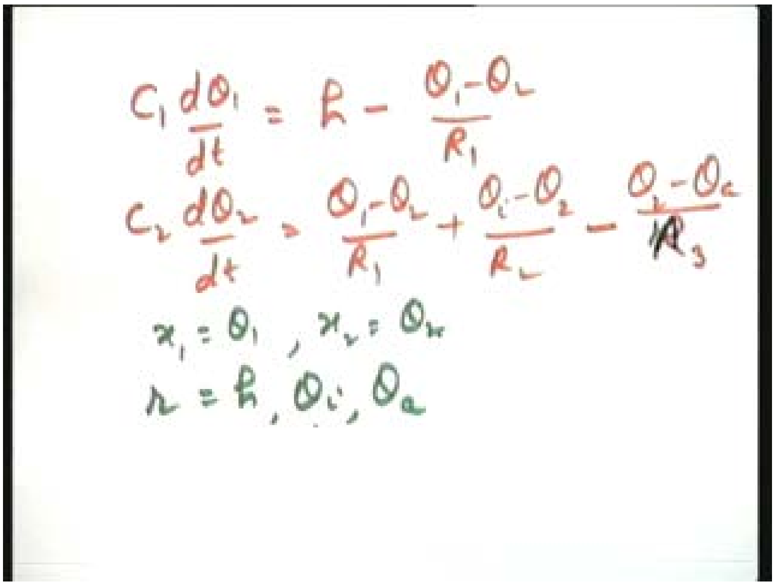

If you take the heat storage is equal to 0 in that particular case please see the equation 1 gets modified to…….. the left hand side becomes equal to 0…. it gets modified to h the controlled heat is equal to theta 1 minus theta 2 by R 1. Under this assumption this becomes your equation h is equal to theta 1 minus theta 2 by R 1. How about the second equation please? I want you to give me the second equation. the second equation under this assumption…….first equation let me write it: h is equal to theta 1 minus theta 2 by R 1 and the second equation will turn out to be C 2 d theta 2 by dt equal to I can replace this by h the input variable theta 1 minus theta 2 by R 1 plus theta i minus theta 2 by R 2 minus theta 2 minus theta a by R 3. This becomes our equation.

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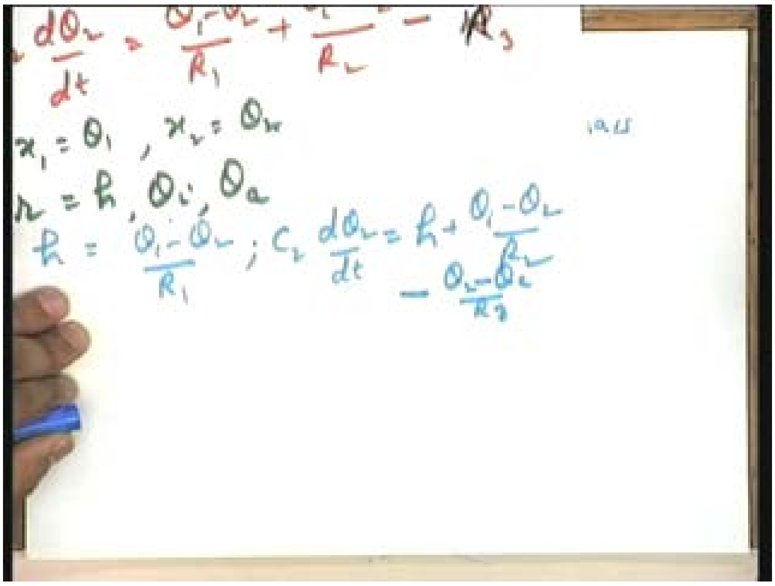

I like to rewrite this equation neatly. this very equation now becomes: C 2 d theta 2 by dt equal to h plus theta 1 minus theta 2 theta i sorry minus theta 2 by R 2 minus theta 2 minus theta a by R 3. I hope this is okay. This becomes a single equation. And in the process, please note, I have reduced the second-order system to a first-order system. Naturally the order of the system depends upon the number of energy storage elements. In the earlier situation there were two energy storage elements: The heater mass and the liquid in the tank. If I assume that the heater mass energy storage is negligible then by physical reasoning also you can say that the system now is a first-order system and it is amply demonstrated by the corresponding mathematical model which turns out to be first-order differential equation and hence a first-order straight variable model.

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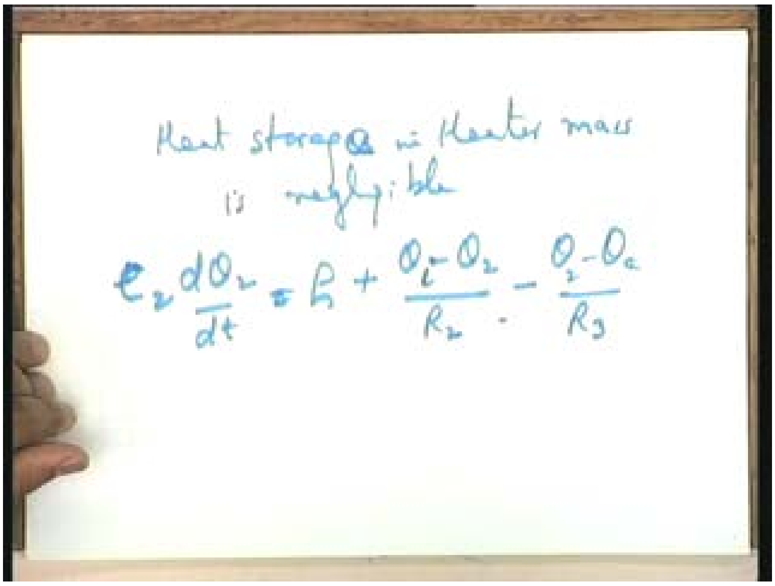

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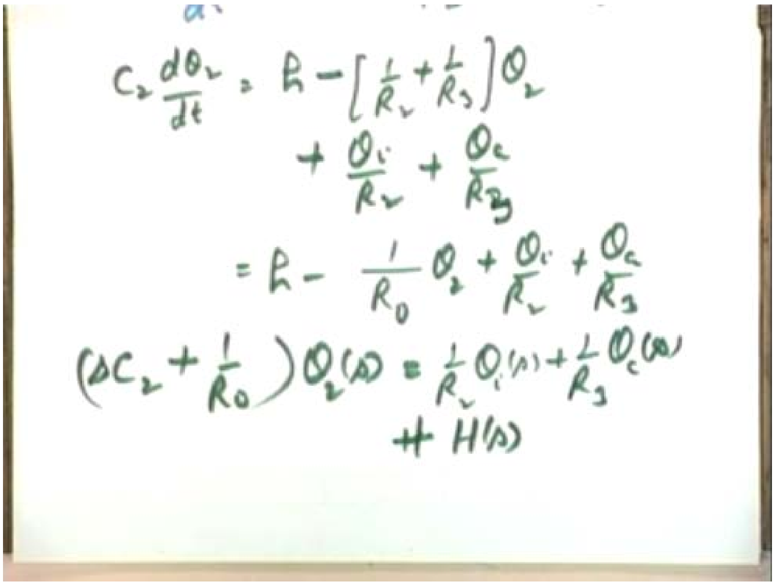

So in this particular case now this is the model, this is the equation and this equation can be rewritten in the form: C 2 d theta 2 dt equal to h minus please see little manipulation here 1 by R 2 plus 1 by R 3 into theta 2. I need your attention here because I am going to put it in a format which I will be extensively using later. You recall, what is the personality of a first-order system? The personality of a first-order system we had said depends on the system gain and the time constant. This being a first-order system I will like to bring this in the standard form that is somehow I want to relate the system gain and the time constant in terms of the system parameters. This is what I am going to do in terms of little manipulation on this equation. So, for that what I am doing is, this h I have written here, theta 2 I have taken and all these the terms the coefficients of theta 2 have been assembled (Refer Slide Time: 35:35).

Look at the other terms. The other terms are minus sorry plus theta i by R 2 and plus theta a by R 2. Is it okay please? This I could write as: equal to h minus, could I write this as 1 over

R some new variable R 0 now new parameter R 0 is a parallel combination of R 2 and R 3 so minus 1 over R 0 theta 2 plus theta i over R 2 plus theta a over R 3 this let me modify R 3. You see that in electrical circuit resistance into capacitance is the time constant and its units always are seconds.

Now, in thermal systems identically, a little exercise for you, look at the units of thermal capacitance and the units or thermal resistance, you will find that the units of capacitance into resistance will turn out to be seconds and therefore equivalent to electrical systems C into R will turn out to be the units in seconds and I will call it as the time constant of the thermal system. So what I do in this case I take this term on this side C 2 or can I write it directly in Laplace domain that will be helpful (sC 2 plus 1 over R 0) theta 2(s) equal to 1 over R 2 theta i(s) plus 1 over R 3 theta a(s) plus H(s), H(s) is the Laplace transform of small hp that is the control variable. This is in the Laplace domain.

Now putting this equation in this format I write: (s R 0 C 2 plus 1) theta 2(s) but let me take it divide it by R 0, yes, equal to equal to 1 over R 2 theta i(s) plus 1 over R 3 theta a(s) and plus H(s) this becomes the equation. And now let us define the variables which define the which describe your first-order system. I take tau is equal to R 0 C 2. It is the time constant of the thermal tank. And the units as I said you will you should really verify the units of tau will turn out to be seconds. So in this particular case what I am doing is I am writing this equation in this form output theta 2(s) is equal to some constant K into tau s plus 1 H(s) plus some constant K D 1 over tau s plus 1 theta i(s) plus some constant K D 2 over tau s plus 1 theta a(s). I hope this will be alright, make an attempt please. some suitable constants K, K D 1 and K D 2 I define and in terms of these constants this becomes my equation: theta 2 is the output variable H(s) is the controlled variable, theta i(s) and theta a(s) are the disturbance variables.

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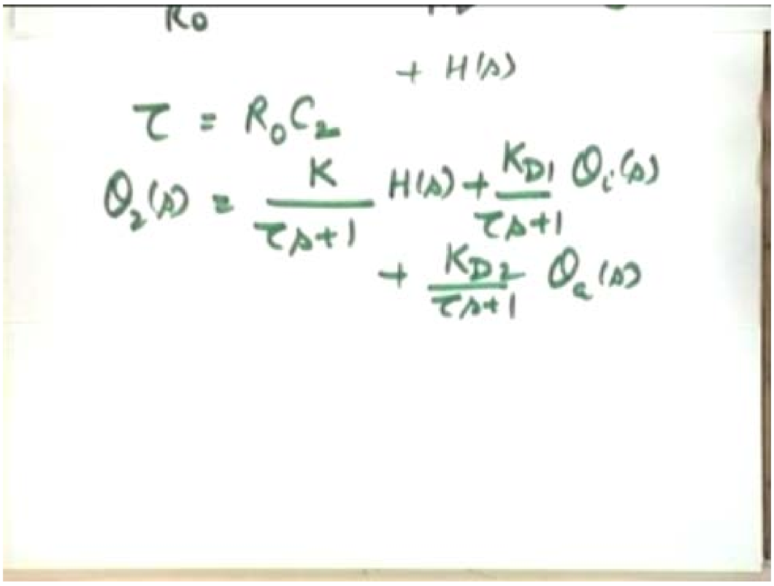

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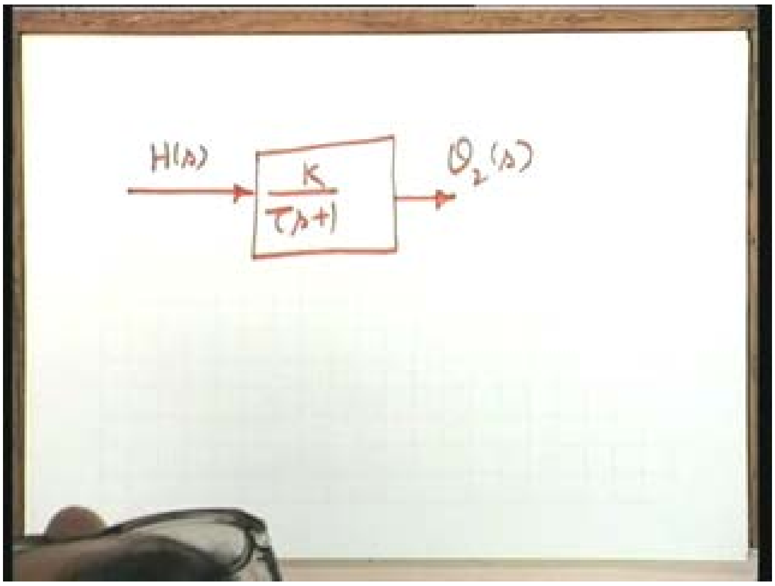

Well, suitably in the form of a block diagram if I take, and this is the block diagram I will like you to recall when I take a control system around this plant. What is the input to the system it is H(s), what is the plant model it is K over tau s plus 1, what are the disturbances acting on the system the output of the system is theta 2(s) this is your plant system as far as input output variables are concerned without disturbances. You can see, this your plant model if you take your disturbances to be 0 this becomes your plant model. And if you are writing a model between theta i(s) and theta 2(s) K D 1 yes it is K D 1 tau s plus 1 it is theta 2(s) variable here theta 2(s) variable here K D 1 tau s plus 1 and theta a(s) variable if I take because all of them are inputs K D 2 over tau s plus 1 and this is my theta 2(s) variable here.

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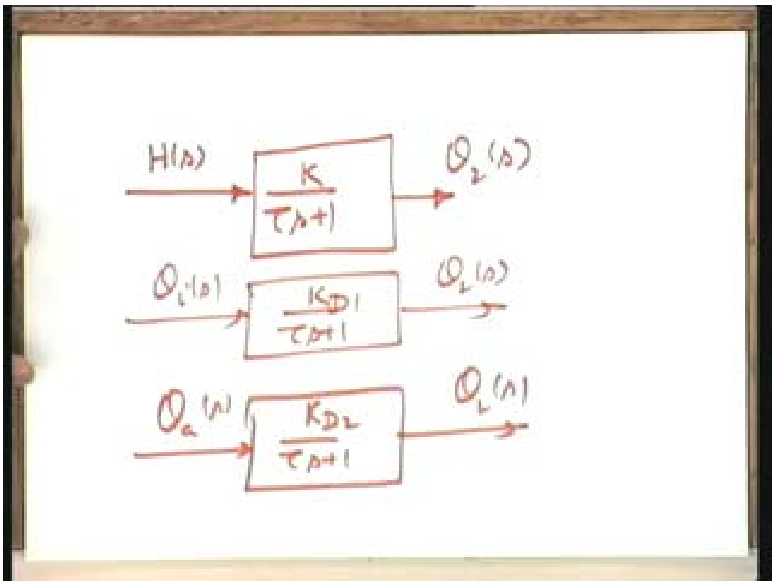

So look, you see, these are the three models input output models in terms of the output variable of interest and the three input variables of interest to me because I will like to study the effects of disturbances on the process. Now, instead of putting the models this way I will like to club all the three into a single transfer function model and for that I can make a diagram like this: help me if I make an error H(s) is the input, let me put the system gain K here, a summing point here and let me take a summing point here as well.

Let me take here as K D 2, it is for you to see that the three block diagrams I have given are being now put in a single block diagram, this I put as K D 1, with K D 1 I have got the variable theta i, with K D 2 I have got the variable theta a and here I put 1 over tau s plus 1 block and this becomes my theta 2(s). Please see whether this is okay; this is the input output relationship.

Now, from this you get between theta 2 and H setting theta i and theta a equal to 0 naturally whatever we have written we are getting. Similarly, setting these two variables 0 you have got the transfer function between theta 2 and theta i and identically the third transfer function. So this is my summing block. It is a symbolic representation of the additive effect which is going on in the process and these three input variables in this particular block diagram (Refer Slide Time: 42:48) have been expressively shown and this is the output variable theta 2. This becomes the total block diagram of the system.

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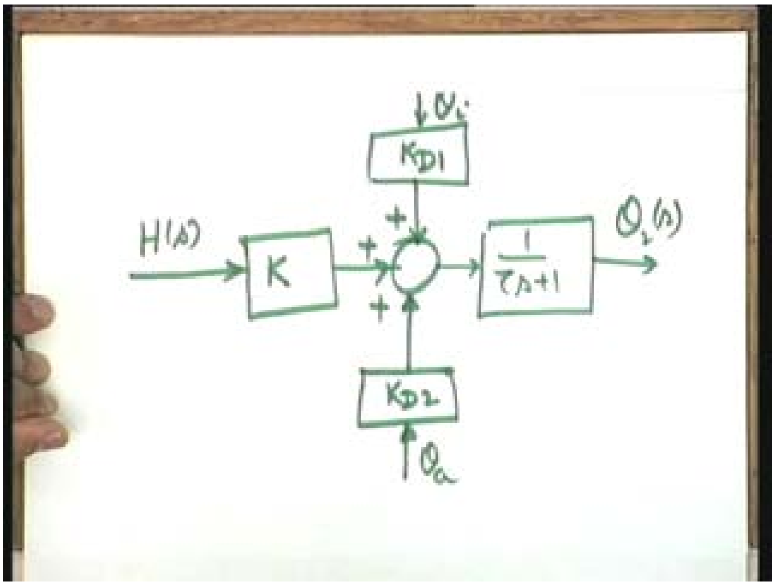

And as I said that, this becomes the plant model of the system where the input, any plant model I will say is complete for the study of control systems; if you give me the controlled variable that is manipulated variable which finally your controller will activate and the disturbance is vary disturbance variables as the input and the particular attribute of interest as the output and it is well connected by suitable dynamical blocks, the transfer function blocks that is the plant model. Well, I do not think the liquid level systems I had envisaged for today's discussion we can take today. But one point I will like to take here today itself because it is in continuation and that point is regarding the sensing of the variables.

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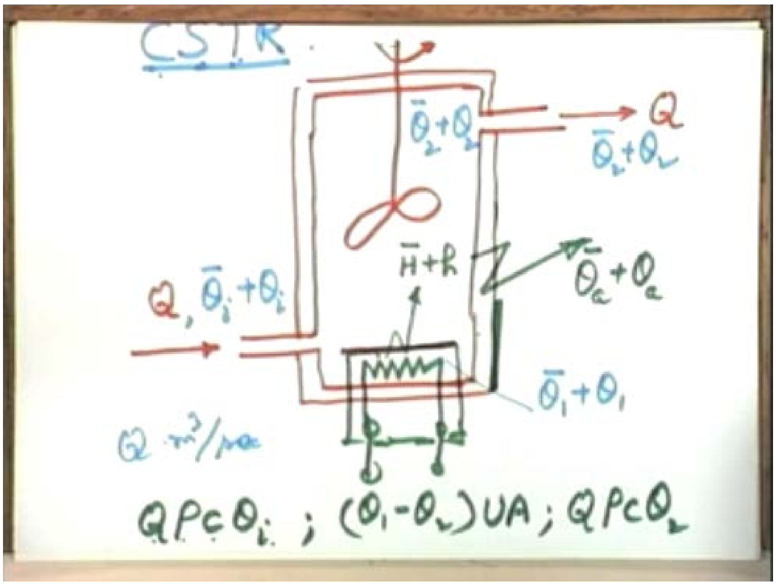

I need your attention please. A very important point from the point of view of process applications; extremely important I will say. Sensing: now you say that if I want to control this h (Refer Slide Time: 44:09) the controlled variable to nullify the effects of disturbances I will require a feedback loop and the feedback loop will require the sensing of the controlled variable or the output variable.

What is the controlled variable?
The controlled variable is of course theta 2. So, if I put my sensor here, I need your attention please, in that particular case what will happen, my sensor let us say thermocouple, the thermocouple output you know is in millivolts very small voltage. Now if the stirring here, rigorous mechanical motion of the liquid is going on because of the vibrational effects the noise signals will be produced and these noise signals will be high frequency signals. Note my point please. And therefore if a sensor is placed over here in that particular case, in addition to the useful information that is a voltage proportional to the controlled temperature the noise signal will also be going in the feedback loop through the feedback loop into the plant and the noise may even dominate the useful signal and herefore therefore the control action will not be the way you want it to happen.

So what are the alternatives?
One alternative, one way is this that, instead of installing sensor over here, you install this particular sensor little away, let us say at this particular point so that the effect of thermal agitation, so that the effect of vibrations is reduced if not eliminated completely. Electrical filters of course can be used but why to first create noise and then filter it. It is better first to avoid noise signals right in the beginning and to avoid noise signal the sensor, instead of this particular point, may be put at this particular and this is what is normally happening in the process industry.

Now you will note that if you put a sensor here your mathematical model which you have derived the plant model which I have stored in your memory will change. The reason being, now in this particular case the controlled variable is not theta 2 but the controlled variable is this quantity because controlled variable is actually that variable which you are sensing;

anything is happening to the system is unknown to you, the variable which you are sensing and feeding back in the control loop. As far as the controller is concerned the controller does not have any information at this particular point, the controller has information of the temperature at this point only. So the controller is intelligent with respect to this information (Refer Slide Time: 47:10) and not with respect to this information. So it means in this particular case for the purpose of control system design if you are setting up a control system model a plant model in that particular case the output variable should be this temperature and not this temperature.

Let me say that this temperature is theta only, let us say theta, let me use this variable for this, this temperature is theta. So little change in your model but very important change. You have to now tell me, what is the relationship between theta and theta 2? And, from that relationship, using that relationship I will modify my earlier model and will get the overall system model. Yes can you help me please?

What is the relationship between theta and theta 2?
Assuming that this particular length L this particular length is L and the liquid is moving with the velocity v how much time will it take for the liquid to move from this point to this point. L in meters, v in meters per second, L in meters and v in meters per second please, how much time will it take? L by v seconds. And what is v? v let me write here is equal to Q is the meter cube per second Q divided by area of cross-section of this point meter squared. If I take v is equal to Q by A meter cube divided by meter squared per second, meter cube per second divided meter squared I have got v in meter per second. This becomes the velocity. If this velocity (Refer Slide Time: 48:47) the liquid is moving and it has to move a distance L, in that particular case the time taken is L by v. And let me use the symbol tau D. I am reserving it for throughout the course, the tau D the delay time or in the literature it is also referred to as the transportation lag.

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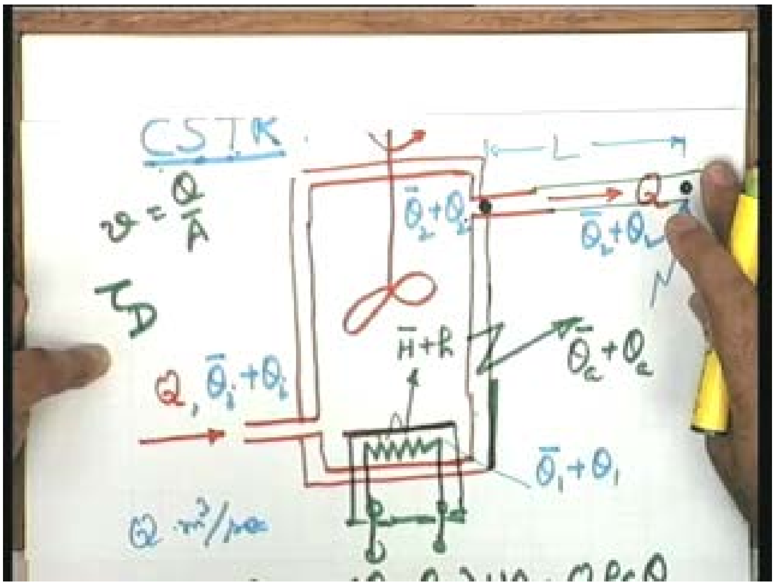

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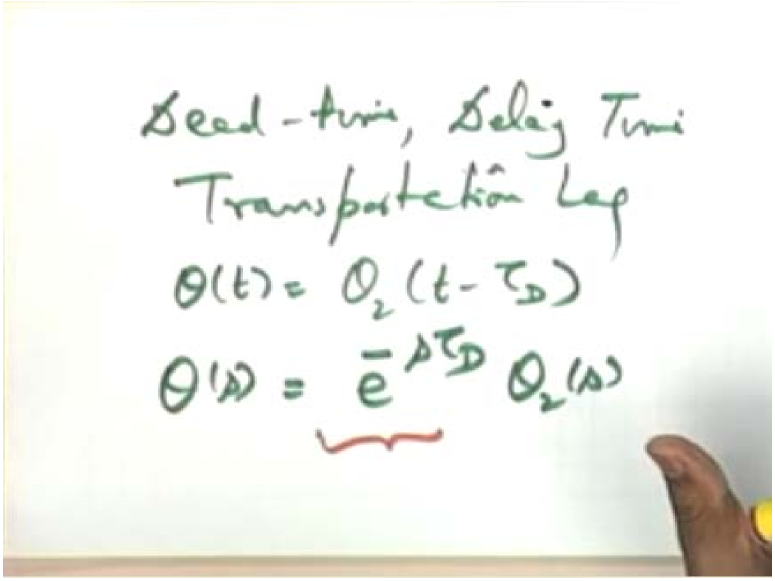

I will like you to really get these terms very clearly, dead time because this is the dead time the name the nomenclature is obvious. This is the time which the system is not using. Delay time, transportation lag, ideally you would like that such a thing should not happen transportation lag. In that particular case your model turns out to be theta t is equal to please see theta 2(t minus tau D) this becomes your model. A minute more please; theta(t) is equal to theta 2(t minus tau D) becomes your transportation model. And in Laplace domain theta(s) help me please what is the Laplace transform of this; e to the power of minus s tau D theta 2(s) this is your standard Laplace operator let me not explain it.

This is time shift operator (Refer Slide Time: 50:11) theta(s) is equal to e to the power of minus s tau D theta 2(s). So it means e to the power of minus s tau d becomes the transfer function of the delay element if you call it. It is not a physical separate element, it is a phenomena taking place. But in the in the form of a block diagram a can take it as a dead time element in the system. This is the transfer function of the dead time element in the system.

Now please see, what will become your transfer function model. If I take this as theta 2 (Refer Slide Time: 50:46) yes I know time, theta 2 let me put e to the power of minus s tau D here this will become theta please see. theta 2 Between theta 2 and theta I am putting another transfer function e to the power of minus s tau D and therefore the total model from this point to this point as you see is given by e to the power of minus s tau D over tau s plus 1.

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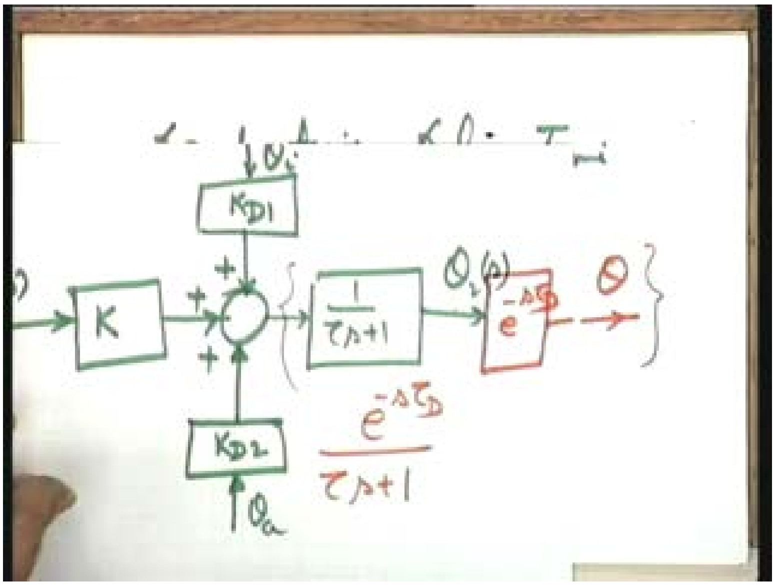

This, the concluding statement for today's problem I am giving that this transportation lag or dead time is a problem for the control designer, it creates instability problems and the control designer has to account for the problem introduced by transportation lag in its control system design. We will come to these problems but in the modeling exercise whenever there is a transportation lag I will like to model that situation by a block of this form e to the power of minus s tau D where tau D is the dead time. Thank you very much.