Control Engineering
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Lecture - 7
Dynamic Systems and Dynamic Response (Contd….)

Well friends, let us continue with our discussion on modeling of physical systems. If you recall, during the last lecture we had considered the model of models of mechanical systems: the translational as well as rotational. Basic equations of the mechanical systems were given to you. The analogue with the electrical circuits was also presented with the comment that, for mechanical systems, may be considering the analogue it may be convenient for us to write the differential equations. I think before I go to the thermal and other systems it will be worthwhile considering a couple of examples.

My first example is that of one wheeled model of an automobile. You will see that how the physical behavior, the physical attributes of a system or modeled into a mass, spring, damper system. Consider this one wheeled model in which I consider this M 1 as the mass of the vehicle, K 1 is the model of the springs of the vehicle and let me take B as the model for the dampers. Consider another mass M 2 this is the mass of the wheels and I will consider one more spring over here representing the elastance of the tyres.

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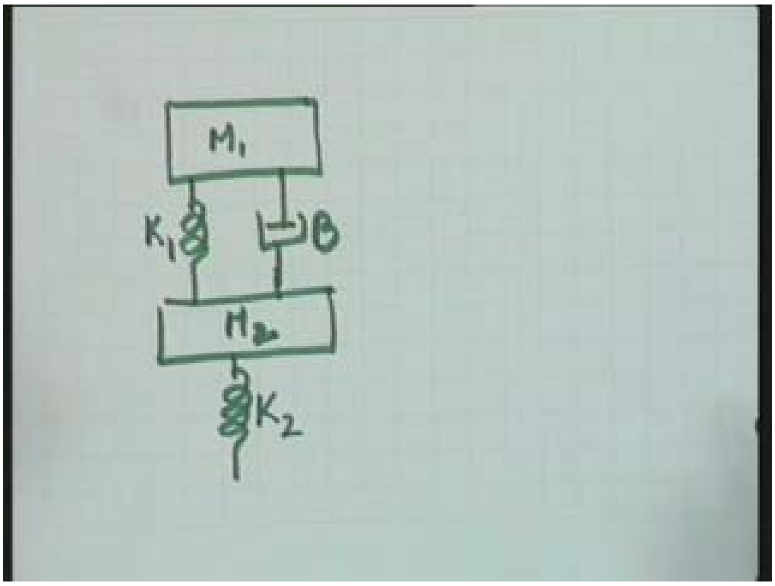

So you see that this becomes the one wheeled model of an automobile. Under consideration is the study of effect of the road surface variations. You see that, it means I am considering I am interested in this study of vertical motion of the vehicle; the vertical motion of the vehicle will come as and when the bumps come on the road or there are other disturbances, the roughness and other things. so it means, as far as this attribute of study of the vehicle is concerned this input is a vertical input and which you can definitely declare to be the disturbance input. So this is the disturbance input to the system.

What is your interest?

Your interest is that your body should not vibrate. So it means let me say that this y 2 is the output of interest the vertical motion of the automobile. Let me consider this mass M 2 and this y 1 as I consider another variable in the system this is the variable y 1 displacement of the mass the vertical direction of course. Now you see that the input variable is of w which is the disturbance in the system and output variable is y 2 the attribute of interest to us and y 1 and y 2 and their derivatives as you will see will turn out to be the state variables of the system.

Let us look at the way we will write the differential equation model for this system. You will recall my statement that we will apply the physical laws to systems. Obtain the differential equations and there from we can get a state variable model or transfer function model as the need be. So, in this particular case I will obtain the differential equations considering the electrical analogue. One point I like to mention before I take the analogue of this particular system is that in this particular system I may not take the gravitational force explicitly and that is possible only if you consider your reference point properly.

Let me say that in this particular case the zero reference for position corresponds to the level surface but with gravitation acting on the system. So, in that particular case under that equilibrium position or zero reference position you will note your springs and dampers are not in the relaxed position they are a getting the gravitational force on them, however, the corresponding position is declared as the zero position of the automobile and the and the wheels. So, with respect to this position naturally the model does not take care of the explicitly, does not use explicitly the gravitational effect because it has been accounted for in our definition of zero reference. Yes please… [Conversation between Student and Professor – Not audible ((00:05:38 min))]

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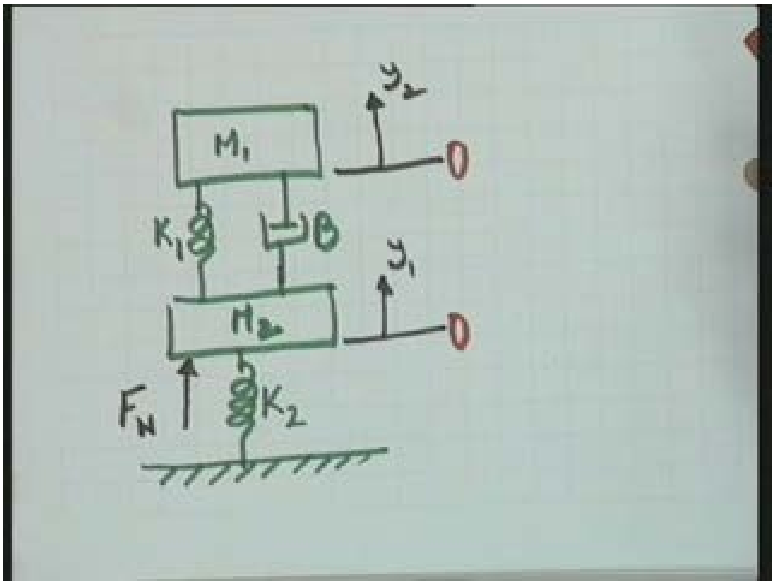

Yes initial rest position. At the level surface when the disturbance is 0 the initial rest position is the zero reference position and if I measure my variations with respect to that position the gravitation need not be taken into account. Look at this particular case now. I will like to right the electrical model for this. Well, there are two nodes. Let me say that the two nodes for me are the velocity v 1 of the mass M 1 and velocity v two of the mass M 2. Between these two velocities there are two elements this spring and the damper. Between these two velocities spring and the damper and the values are K 1 and B.

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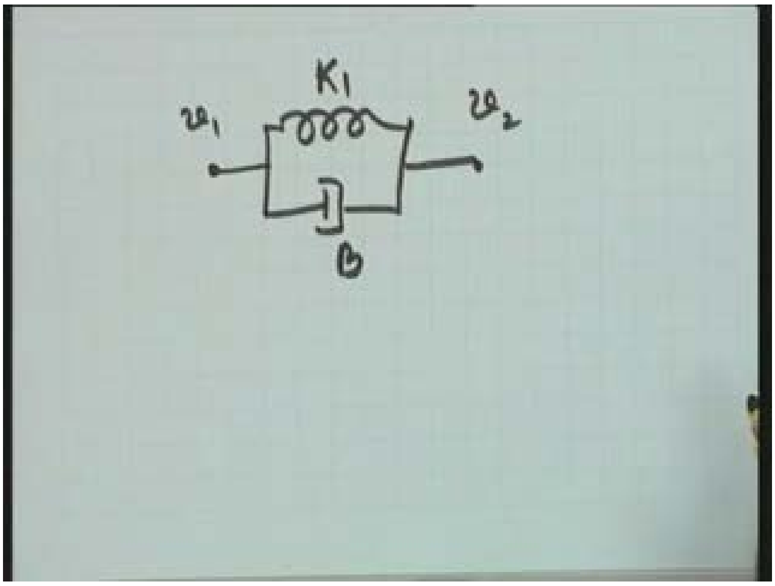

Now, between v 2 and the ground reference between v 2 and the ground reference you have, let me say the mass M 1.

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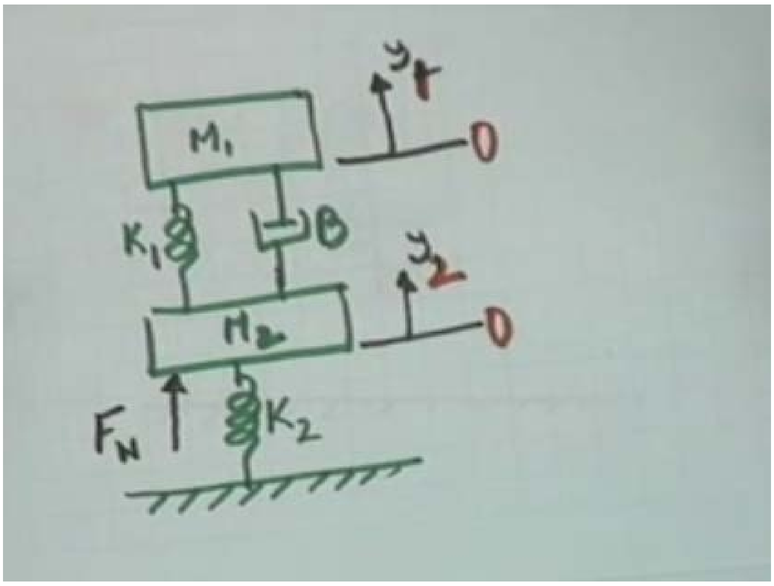

Well, I think, looking at this, if you don't mind, if there is no confusion, looking at this, if this is mass M 1 let me declare this to be y 1 and this as y 2 (Refer Slide Time: 7:05) there will be compatibility, there is no absolutely no problem, there will be compatibility here. Now, between v 1 and the ground you can say that there is a mass M 1 and let me represent it this way (Refer Slide Time: 7:22) as a two terminal element where one terminal is fixed that is the velocity is measured with respect to this terminal which is the reference terminal.

Between v 2 and ground there will be this spring k 2 and the mass M 2, and how about the force corresponding to the current? This is your force let me call it F W please see.

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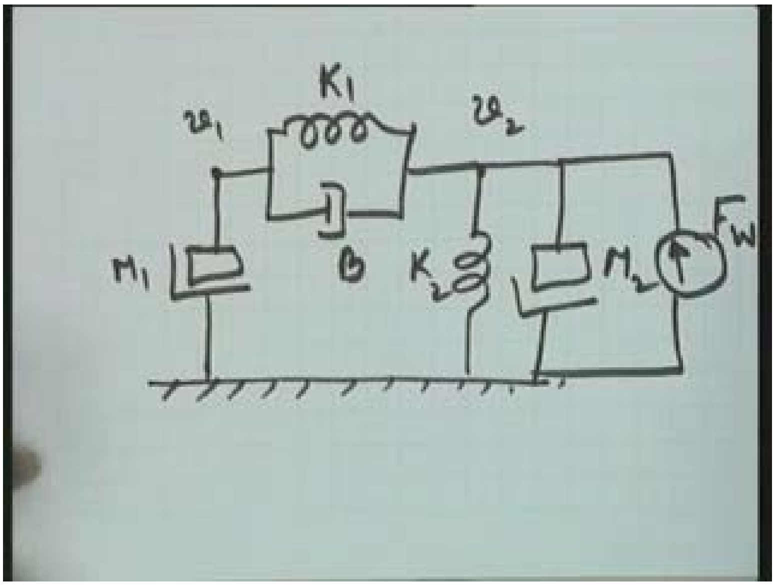

I give you the diagram again. Look at this diagram (Refer Slide Time: 7:52) M 1 is a node and M 2 is another node these are the two variables of the node, electrical voltages are analogous to velocities. So I am considering the two velocities as these two node variables. Between these two nodes I have got these two quantities, between this node and the ground I have k 2 and mass M 2; of course M 1 is with respect to ground and the node v 1. So, that is why I get this as the equivalent network and let me call this as a mechanical network. And if you appreciate this chain, in that particular case equations can directly be written in terms of mechanical network rather than converting them into resistance, capacitance, inductance and current elements we can directly write the equations. Instead of the current balance now you will have the force balance, current being analogous to force.

In that particular case help me; write the equation at node 1. I will write the equation and just you compare whether it tallies with your equation. At node 1 the equation is this the current through mass M 1 or the force M 1 dv 1 by dt plus k 1 v 1 minus sorry it will be B (v 1 minus v 2) plus k 1 (y 1 minus y 2) equal to 0. I hope you are getting it. The way you write your nodal equations I have written the force balance equation keeping in mind that force is analogous to current.

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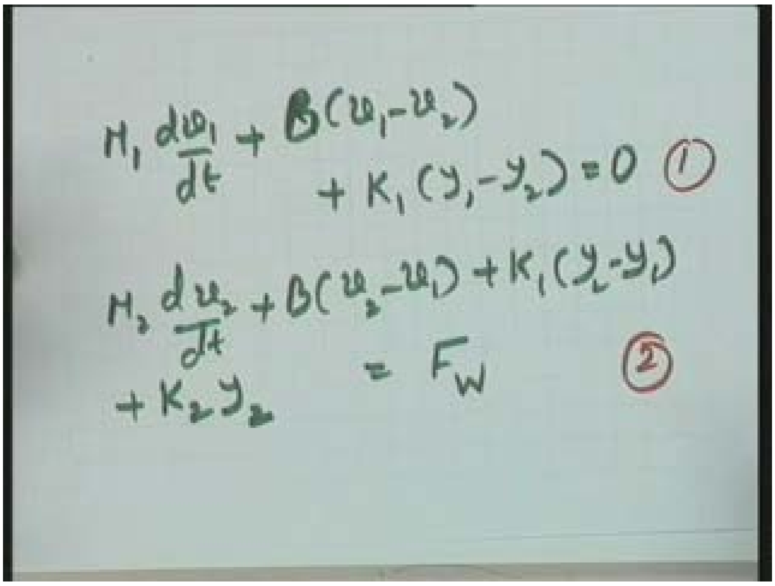

Look at the second equation now. The second equation will be, well, M 2 dv 2 by dt plus B (v 2 minus v 1) plus (k 1 y 2 minus y 1) what else equal to F W. This becomes….. did I miss anything? Yes I have missed, and the thing I have missed is another force here (Refer Slide Time: 9:54) plus k 2 y 2 equal to F W. y 2 is the displacement of the mass M 2. So this become the second equation which is the equation at node 2 the force balance equation. Keeping that the force is analogous to current I have taken the currents entering into the node is equal to currents leaving the node this is what I have done. I hope this is okay. Equivalently you can write the free body diagrams corresponding to the two masses and from the free body diagrams you can definitely write these force balance equations, they will definitely tally.

If there is any question on this I will like to welcome that at this stage. I hope this is okay. [Conversation between Student and Professor – Not audible ((00:10:49 min))] Fine, fine reason…… please repeat your question please? Little loudly….. Sir, what is the reason for the presentation of M 1 and M 2 as used by you? Yes,

I have taken M 1 as a two terminal element. This is the free terminal, it means this is the node which is the motional variable and the motion of the mass that is this is with respect to certain reference which is fixed. So, if you consider this (Refer Slide Time: 11:13) the velocity of the mass M 1 is with respect to the ground terminal that is the fixed reference. Similarly, the velocity of the mass M 2 is with respect to the fixed reference and therefore M 1 and M 2 though in this diagram is not clearly shown to be a two terminal component can easily be taken as a two terminal component one being the ground terminal or any reference terminal with respect to which the two velocities are measured. That is why it is in my diagram taken as a two terminal component. I hope this is okay. Fine.

If the first step is over, that is I have written the two differential equations, equation 1 and equation 2. Now this constitutes the complete dynamical model of the system. The only thing is that this model may not be convenient for use; I may be looking for as a state variable model. If that is the case, help me, what are the state variables you propose? In this particular case two masses are there and recall the statement, normally the displacements and velocities associated with the masses constitutes an independent set of state variables for a spring, mass, damper system. If that is the case then x 1 is equal to y 1; x 2 equal to y 2; x 3 equal to y 1 dot equal to v 1 and x 4 equal to y 2 dot equal to v two constitutes a set of state variables for the system.

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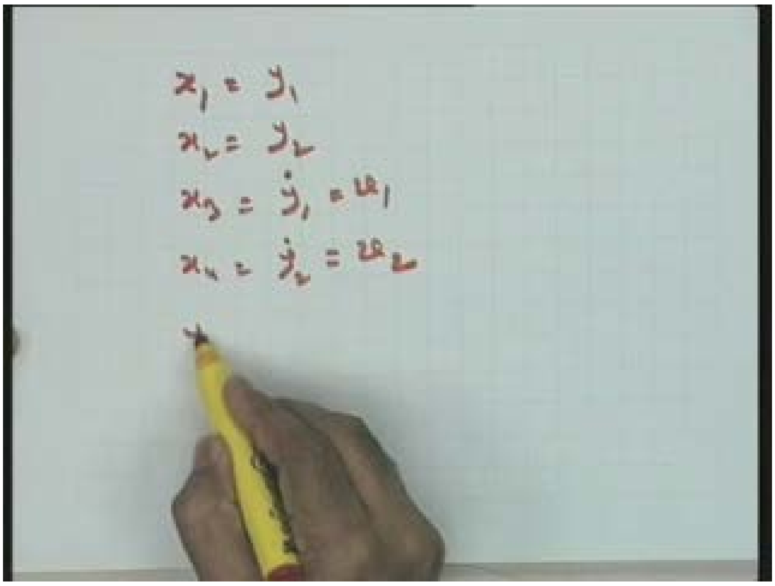

Interest……. the output variable y of course in this particular case is x 1 itself because I am interested in the motion y 1 of the automobile as an important attribute of the system so that becomes the output variable. Now you see that it is just the rearrangement of the two differential equations we have obtained in terms of this, that is I should write d x 1 by dt equal to….. one equation; the other equation will turn out to be dx 2 by dt equal to this dx 3 by dt equal to this dx 4 by dt equal to this these are the four equations and you can easily manipulate the given set of equation the given equations that is the two differential equations to obtain these four equations.

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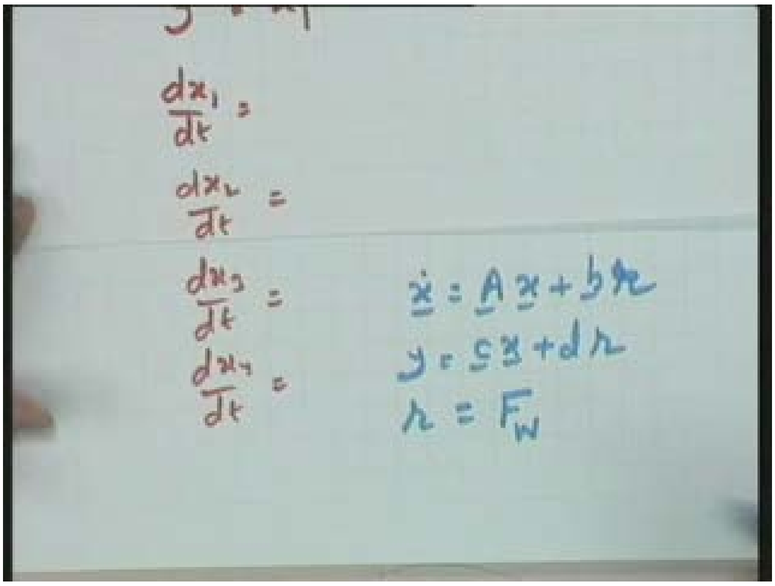

And then these four equations can suitably be put in the standard format of x dot equal to A x plus b u b r; y is equal to c x c plus d r where r as you know is nothing but the F W the input to this system which is the disturbance input. Well, I have rearranged these equations. I project it here and I leave it to you for verifications.

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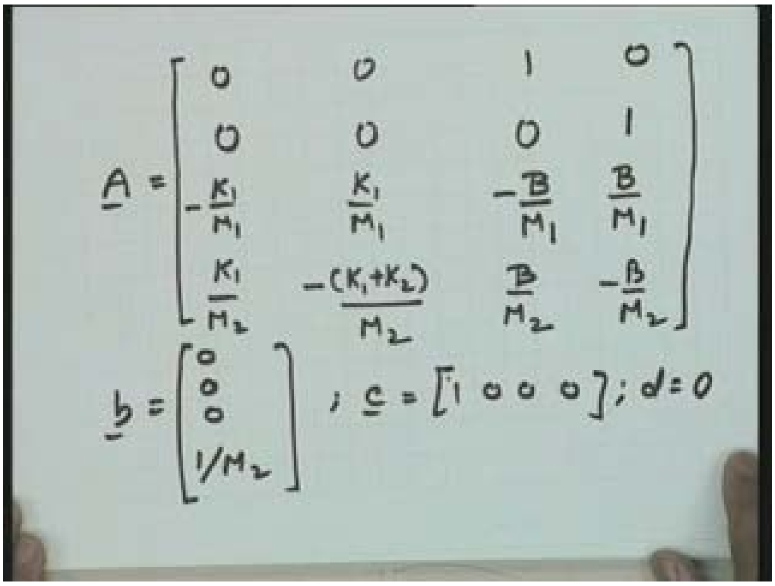

These are the a b and c A B C and D matrices I get just by rearrangement with x 1 x 2 x 3 and x 4 defined the way we have done that is the two displacements and the two velocities. This is the state variable model I get; a 4 into 4 model for a fourth order system. You will clearly note that the C in the particular case turns out to be only one element because we had seen your y is equal to x 1 itself which will naturally give you this value of C. So this can easily be verified and I hope I can leave this as a simple exercise for you.

Now look at the transfer function model. If I am interested in a transform function model what should I do; I should eliminate the variables in which I am not interested. I am interested in y 1 the output of the system and F W the input to the system. So, naturally y 2 the displacement of the mass displacement of the wheels should be eliminated. So write the two equations in Laplace domain so that elimination becomes an algebraic manipulation.

You see that if I write it two equations in the Laplace domain the equations will turn out to be this way. I can take out take the same slide here (Refer Slide Time: 15:50). Just see, this equation in Laplace domain will be s M 1 the other s squared M 1 Y 1(s) I hope you are getting it. Directly in terms of y 1 I have written: s squared M 1 Y 1(s) plus it is now B I write here as [s Y 1(s) minus s Y 2(s)] plus K 1 [Y 1(s) minus Y 2(s) equal to 0. This is one equation in Laplace domain which I get.

Look at the second equation, similarly, will get s squared M 2 Y 2(s) plus s B[Y 2(s) minus Y 1(s) plus K 1[Y 1(s) sorry Y 2(s) [Conversation between Student and Professor – Not audible ((00:16:42 min))] fine fine ((I will do it)) ((00:16:46 min)) plus K 2 Y 2(s) equal to F W(s).

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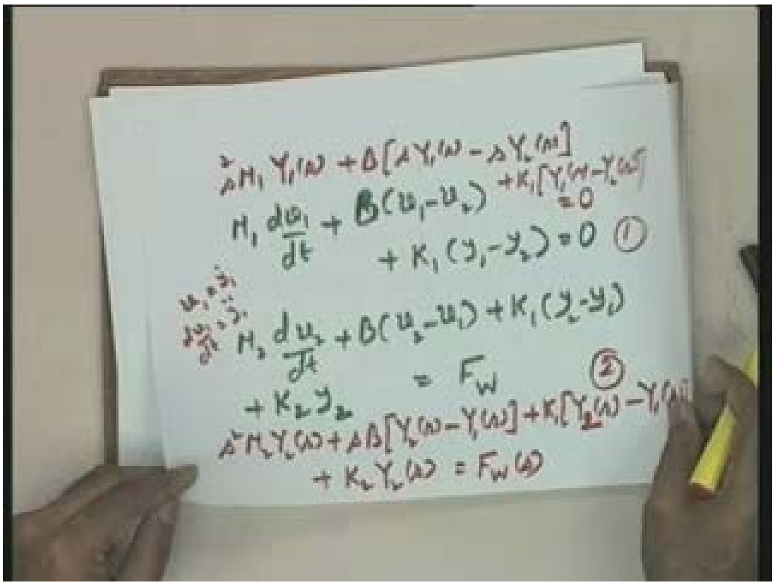

I repeat, look at this equation: Y 1 is the velocity so it means v 1 is the velocity so it means v 1 is equal to y 1 dot or d y v 1 by dt equal to y 1 double dot so this is M 1 Y 1 double dot hence I have written s squared M 1 Y 1(s) I hope this is okay plus what is v 1? v 1 is y 1 dot and therefore B[s Y 1(s) minus s Y 2(s) and this is simple (Refer Slide Time: 17:30) plus K 1 into these variables in Laplace domain, I hope this okay now.

Similarly, look at this equation. This component is Y 2 double dot gives me s square M 2 Y 2(s) plus s I have taken out s B[Y 2(s) minus Y 1(s) plus this is directly coming there is no derivative involved, this term corresponds to this and this is the Laplace transform of the disturbance variable. So now you will see in this particular case there are three variables Y 1(s), Y 2(s) and F W(s). You are interested in Y 1(s) and F W(s) so you have two equations you eliminate Y 2(s) and this gives you the differential the transfer function model which I have written for you and I will leave it for you to verify.

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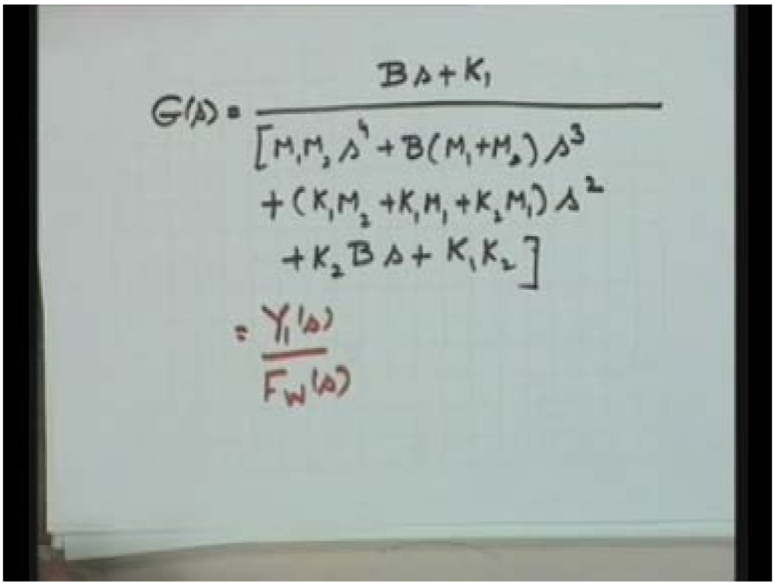

This is the transfer function model which is equal to Y 1(s) divided by F W(s) it is a fourth order system as you see the state variable model we had seen. From the state variable model I observed that it is a fourth order system so the transfer function model should give me a fourth order characteristic equation. So I find that the characteristic equation in this particular case turns out to be a fourth order characteristic equation.

Even if you have not noted any specific expression I think you can derive these expressions, you have to concentrate only on the basics. That is, this example I have taken only to bring home this point; that given any physical situation I will apply the basic physical laws. In the mechanical system I may use the free body diagrams or I may use an equivalent mechanical network. I get the differential equation model and then transform that model into appropriate form; the two forms useful to me of the state variable model and the transfer function model. This is what I wanted to convey.

Next example as you said, let me take a little slowly so that you can appreciate and check my calculations. The next example I take is that of a rotational system will run quite parallel to it a rotational system I take this as the load J 2 is the inertia of the load. This is driven by a motor and let me say the rotor of the motor and other rotating parts have got inertia J 1. I am making this system a little complex compared to the system handled earlier. Now I consider that it is a high power servo and in under this condition that system is a high power servo and this shaft (Refer Slide Time: 20:40) is a long shaft the flexibility of the shaft the torsional effect of the shaft cannot be neglected. So I assume that the shaft can be twisted and K is the corresponding constant which models this behavior of the shaft.

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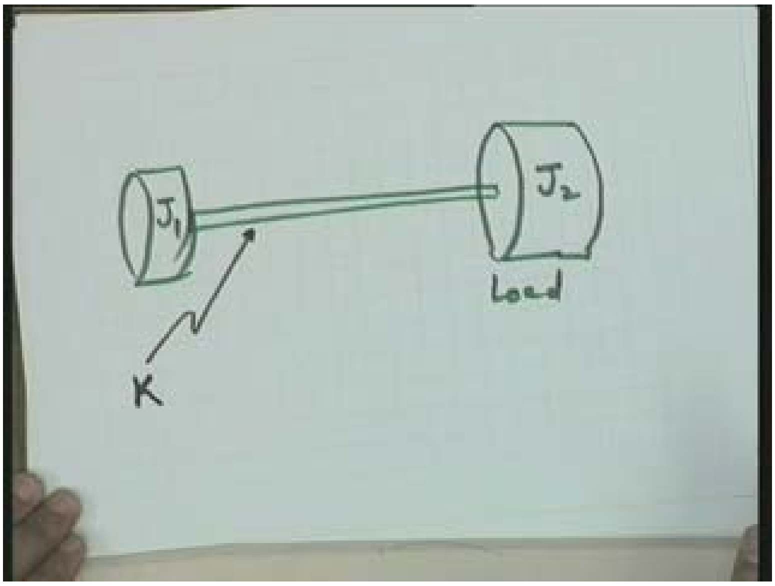

Earlier I had mentioned that normally servo systems will have two parameters J and b but such situations also we do come across where the third parameter K is also important. Let me assume that the frictional effect in the system is modeled by this damper and (Refer Slide Time: 21:19) this damper has got this B as the frictional coefficient viscous frictional coefficient. So in this case now I take the input here as the applied torque by the motor and therefore this give rise to the motor rotor the motion of the motor rotor is given by theta 1 or omega 1 theta 1 being the angle of displacement and omega 1 is the angular velocity. The two things at the load end let me take as theta 2 and omega 2.

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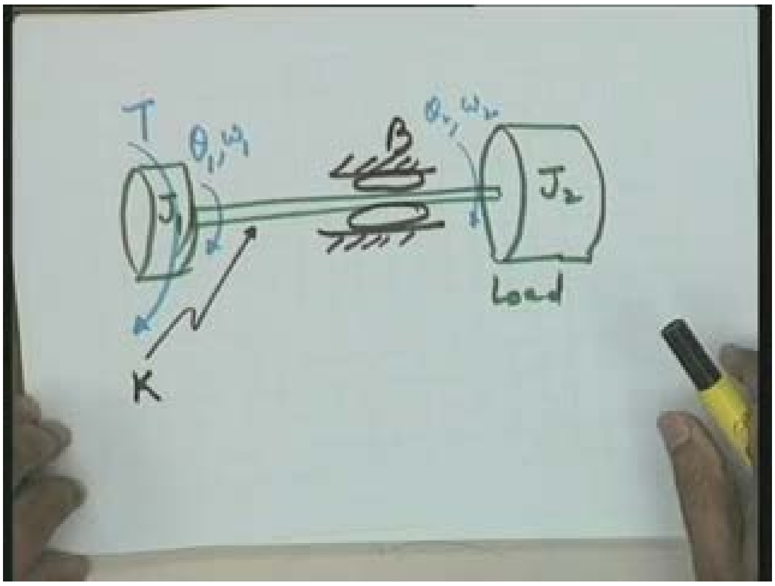

Now you help me please and all your queries regarding whether I can take these moment of inertia or mass as a two terminal component or not should be clear. Make an attempt yourself and compare with my result.

These are the two basic nodes corresponding to moment of inertia J 1 and moment of inertia J 2. So let me say let me make this diagram these are the two basic nodes and let me associate the corresponding variables the node variables as omega 1 and omega 2 analogous to two voltages. Now, between these two nodes you see that there is damper and a spring. So let me put is spring over here and a damper K and B this is between these two nodes. Now let us see what is happening at node W 1 that is this particular node. You see that the moment of inertia J 1 you take the dated node as the inertial reference with respect to which the displacement or the velocity of J 1 is measured.

If you consider this in that particular case (Refer Slide Time: 23:17) this could be taken as a two terminal component and let me use the same symbol as I used for mass there is no problem there as far as symbolic representation is concerned. So J 1 is this terminal now. Look at the torque t. So, naturally in this particular case torque is analogous to current so this torque becomes the force variable or the torque becomes analogous to current variable in an electrical network. Go to the other side J 2 is the second terminal corresponding to omega 2 the angular velocity. So obviously your mechanical network becomes complete if I write J 2 here this is your mechanical network.

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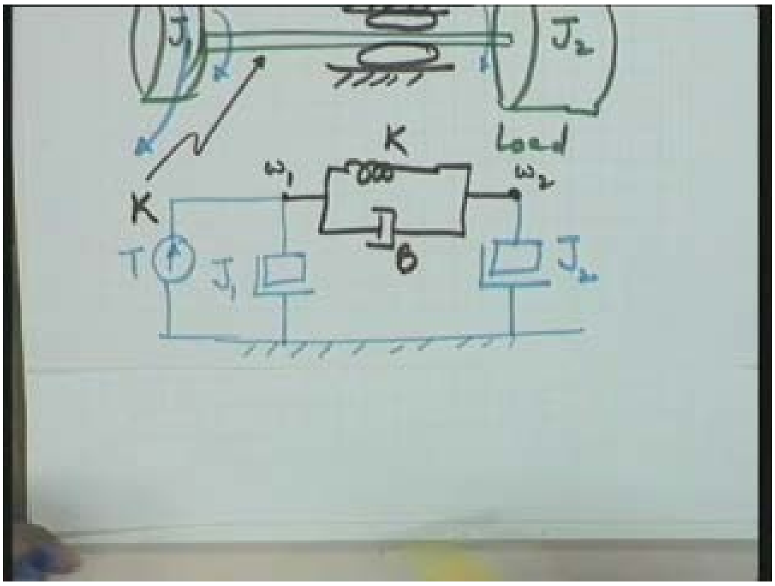

I am giving these networks because I presume that we will be more comfortable with this type of network as far as writing of differential equations are concerned otherwise you please write the free body diagram for J 1, the free body diagram J 2, take the force balance equations you get the corresponding equation uh differential equation model. So in this particular case the two equations turn out to be…….. Just look at this J 1 theta 1 double dot… now I am directly writing in terms of theta 1 so that when I transform it to transfer function model I am more comfortable J 1 theta 1 double dot yes, what is next please? plus B theta 1 dot minus theta 2 dot plus k theta 1 minus theta 2 equal to t. fine in Laplace domain right here let me write the equation s squared J 1 theta 1(s) plus s B(theta 1(s) minus theta 2(s)) plus K theta 1(s) minus theta 2(s) equal to T(s).

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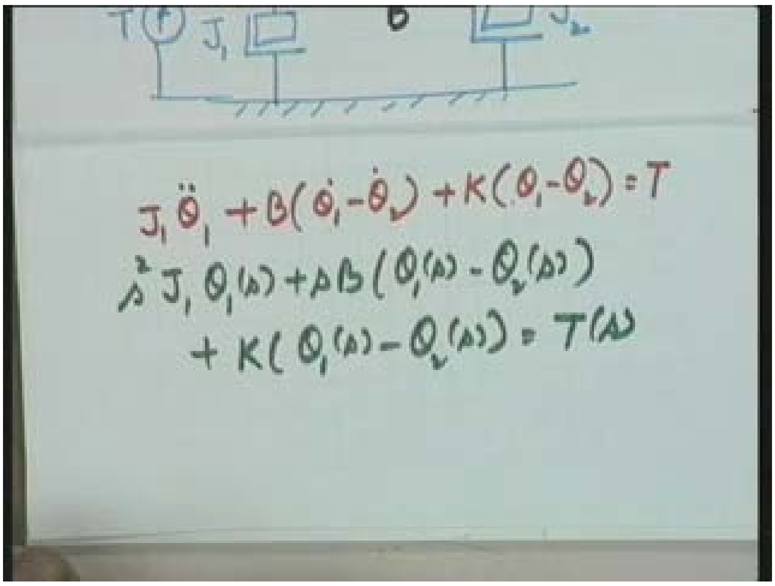

Now let me get the second equation. The second equation is J 2 theta 2 double dot plus B(theta 2 dot minus theta 1 dot) plus K(theta 2 minus theta 1) equal to 0 and the corresponding Laplace domain equation will turn out to be s squared J 2 theta 2(s) plus sB(theta 2(s) minus theta 1(s)) plus K(theta 2(s) minus theta 1(s) equal to 0. I hope your speed is alright now. This is the situation which is identical to the earlier situation. So, in the earlier case if there was any confusion, please, you can bridge the gap by looking at this example. There are three variables, now the input variable t and the two variables are theta 1 theta 2. If I am interested in theta 2 the moment the the displacement of the load, so, naturally theta 1 variable has to be eliminated. And I hope this I can leave as an exercise for you. You can just look at the result elimination of the variable theta 1 and rearrangement in this standard form of quadratic lag.

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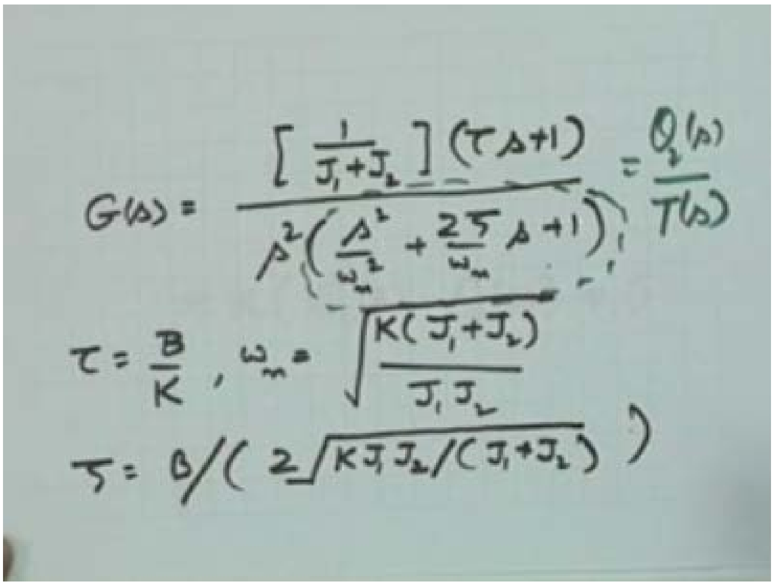

You will recall this is a standard quadratic lap this particular term is a second order term is a standard quadratic lag. So rearranging this equation in the quadratic lag gives me this result. So do not worry, even if you do not note it you just keep this in mind that G(s) equal to theta 2(s) over T(s) should be rearranged in a form where the time constants and quadratic lags do appear because these are the terms which represent the personality of various factors in a transfer function model as we have seen earlier. I think, with this I will like to conclude that the modeling of mechanical systems is over. I will take more examples in the tutorial classes only.

Let me go to another phase of modeling. As far as the overall system is concerned we know that here is a plant the so-called mechanical systems means which we have considered means actually we have considered the models of the plant, various plants we have considered, more of them will appear later. Now if this is the situation after that I have an actuator for a typical mechanical system say motor is an actuator because it will drive your load which is the plant and after this you see, if motor is an actuator so you are controlling the power to this particular actuator so that your output follows the command so it means here is what I call as a controller.

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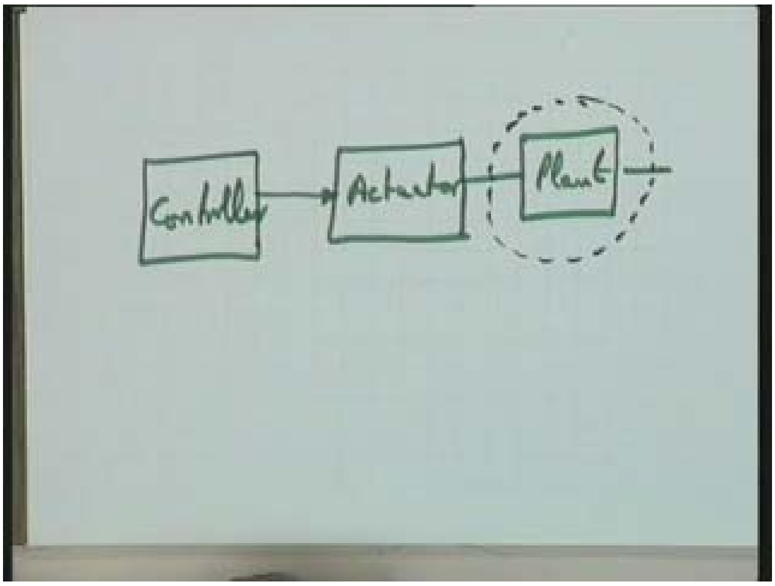

The controller should get information this is what we have done already; should get information about the command signal and it should also get information about the actual controlled variable. On the basis of this information it gets it controls the power supply to the actuator. The actual information about the controlled variable will normally come through a suitable sensor. I need your specific attention here because the role of electrical engineer I am going to define as far as total control system is concerned. You see that we have seen our plant to be of mechanical nature; the other plants which will come across will be mostly from the thermal field or for the control of liquid level and other variables in which we normally encounter in industry. So where is the electrical engineer's role as far as this system is concerned.

You will please note that a sensor, we have, I have already said this point told this point to you, a sensor normally gives rise to high frequency noise and therefore to filter this noise you require suitable filters otherwise thee noise will also go in the loop and the performance of the control system of the feedback control systems will be deteriorated. One point, you require noise filters that you will design as an electrical engineer. The sensors output normally will be in millivolt milliampere, you require amplification of that to make it compatible with the standard input signal or to make it usable by the controller hardware. So it means suitable amplifications or suitable conditioning of the sensor output will be required and that also you will do. Lastly and the most important the controller over here which is the brain, all these muscles are being controlled by this brain and the brain is electrical in nature. This controller as far as the today's trend is concerned is either an Op-Amp circuit or a digital computer. Well, in the process industry particularly pneumatic controllers, hydraulic controllers were used earlier. But these either have been replaced or are being replaced gradually. The trend is towards Op-Amp circuits and digital computers giving you in this particular case the brain of the total control system.

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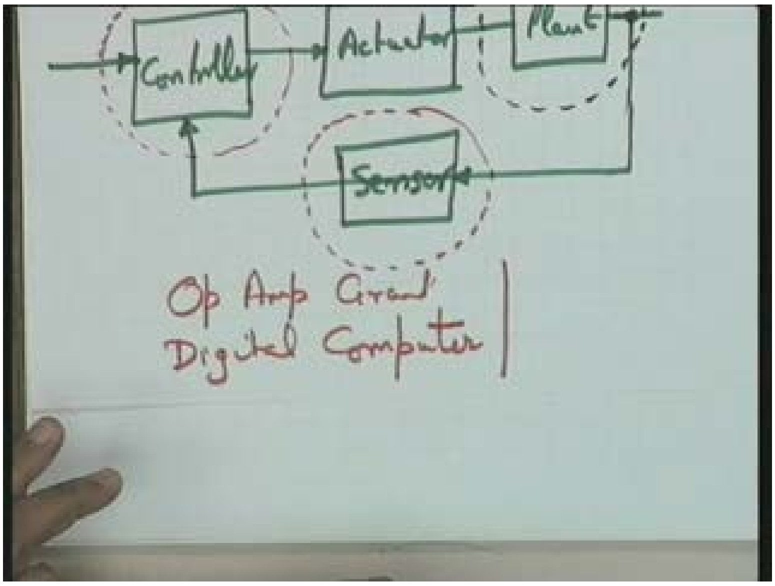

Well, I do not think I need take any examples of filters or amplifiers or Op-Amp circuits because there are very well know to you. I simply wanted to bring home this point that the electrical circuit will be embedded in the total control configuration at appropriate locations and they are crucial part of the total control system; the controller being the most important part which is the brain of the system. So let me quickly glance at the type or Op-Amp circuit which will sit in this particular block, the controller. It will it is only a quick glance and not the details of the circuits will come. Look at this basic Op-Amp circuit; what is the circuit doing? Look at this please. I can draw the write the transfer function of this circuit immediately: E 0(s) over E i(s) equal to please help me minus R 2 by R 1. So it is acting as an amplifier and I require an amplifier or a proportional controller in my control configuration.

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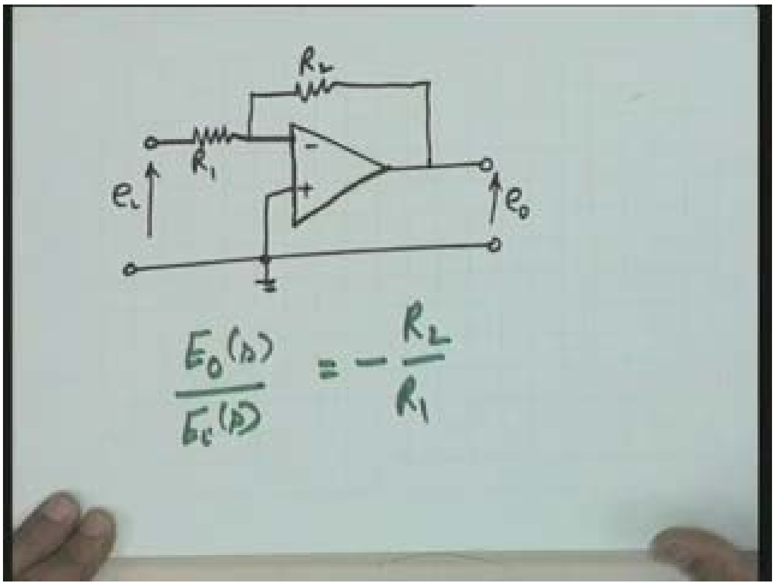

Note the minus sign; I may like to get rid of the minus sign, well, put an inverting amplifier in series in cascade. Let me say R R and now if I take this as E 0 so E 0(s) in that particular case E 0(s) over E i(s) becomes equal to R 2 by R 1. It is an amplifier and the gain of the amplifier can suitably be can be controlled by selection of appropriate register values.

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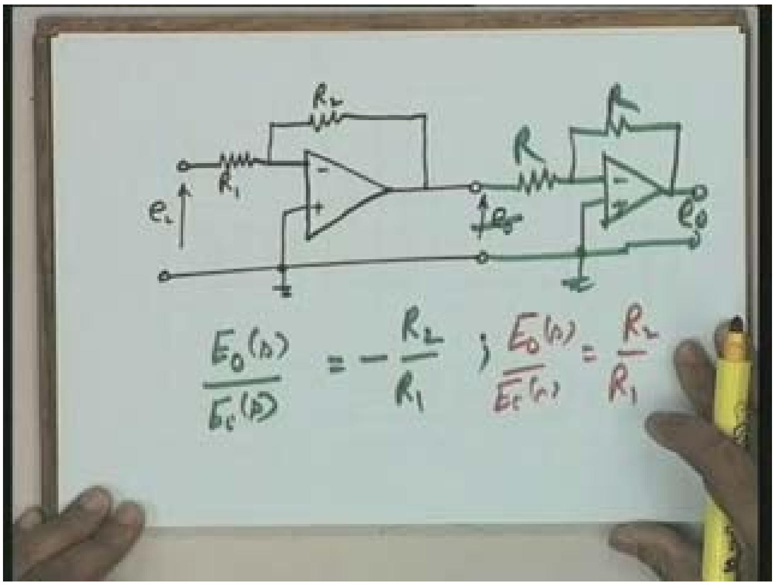

This, as I mentioned, well, I will be using the proportional type of control logic or proportional control and wherever such a situation comes this type of circuit will be utilized there. Well, take up another circuit, a simple circuit again and let us see what is the behavior of this particular circuit.

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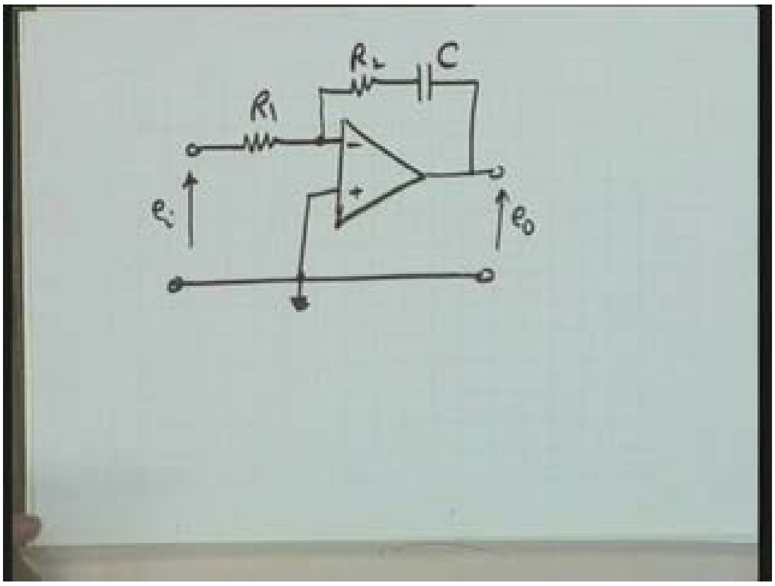

I have now a resistor and capacitor and parallel in series over here. So let us write this model for this particular circuit: E 0(s) over E i(s) equal to this impedance divided this by this impedance; what is this impedance? This is, as you see R 2 plus 1 over sc divided by R 1 this becomes the gain of the transfer function of this particular circuit. Now help me please, what is this equation? This equation as you see is equal to minus R 2 by R 1 minus 1 over R 1C 1 over s. I will like to look at this equation in a different way. For me, as far as the control system is concerned, it is a PI controller. Let us see how. Can I write this equation as e 0(t) equal to minus R 2 by R 1 e i(t) minus 1 by R 1C e(tau) d tau? It is a PI circuit for me. And you will see that in the controller block many a times PI circuits as used. Rather, if you do not mind I can make a statement here, almost 75 percent of the industries in the process field use PI controllers. So it means naturally this type configuration is going to be important as far as the process control applications are concerned.

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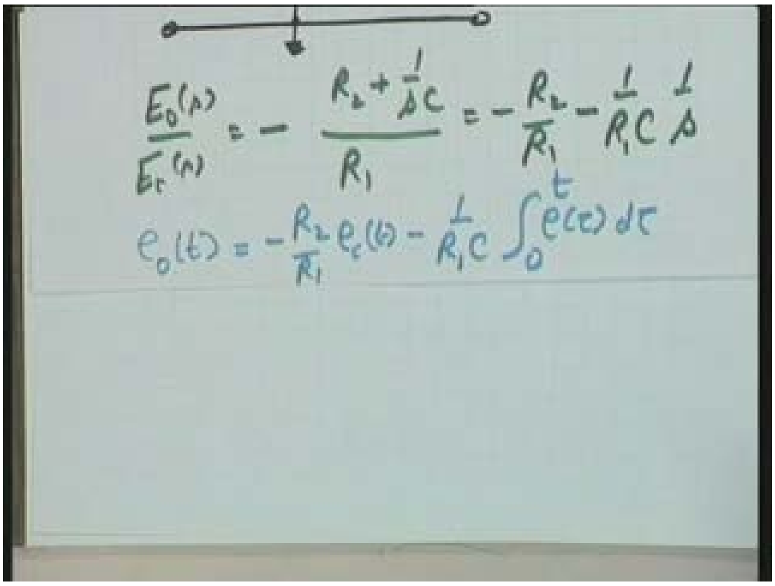

[Conversation between Student and Professor – Not audible ((00:35:56 min)) PI controller as you see, though the details are yet to come, I will like to take the complete block or let me redraw it does not matter. This is your command circle command signal. Let me call this as r reference signal or command signal, this is an error detector plus minus here (Refer Slide Time: 36:20) let me see this is an error detector and the error is e rather e cap we have used the actuating error, here is a controller and let me say it is the plant, I am merging the actuator also into the controlled system and calling it a plant to simple get a simplified situation and here is a sensor.

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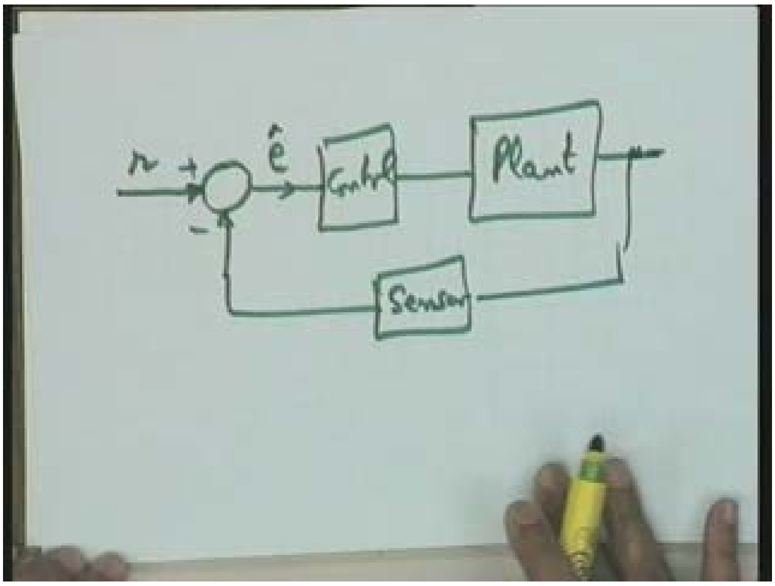

Now you see that this controller has got information about the error between the command signal and the actual signal and this brain or this particular block is going to control the power being supplied to the actuator of the plant. Now you will find that there are different ways this brain will act. One of the ways is this that it controls the power proportional to the error signal, I call it a proportional controller and the first Op-Amp circuit could be used over here in this particular situation. The other situation is which is commonly used in the industry that this controller generates a signal which is proportional to the error as well as proportional to the integral of the error. So it means that the signal at this particular point is a sum of two components one component being proportional to the error itself and the other component being proportional to the integral of the error. So the concept as to why do I need it, I think we can leave it for discussion later.

But at this juncture you can see that such a situation may be required or will be required by a controller where a signal which is a proportional signal to the error and a signal which is proportional to the integral of that particular error is required and hence a PI circuit will be in use and the term PI controller I am referring to corresponding to this situation.

Similarly, since the block diagram is in front of me I can refer to other situations as well. if this signal at this particular point is proportional (Refer Slide Time: 38:23) that is, let me say let me call it u, u is equal to e K c ecap(t) plus K D d ecap(t) by dt which is the derivative of the error the corresponding controller is a PD controller proportional derivative controller. In general, a controller block may consist of a circuit which generates all the three components the proportional component, the derivative component and the integral component and suitably [ad…39:02] these components to generate a signal which then in turn is going to control the power supply to the actuator, power to be generated by the actuator. So in that particular case that will become PID controller and PID controller is a term which will be very frequently coming in our discussion. The logic behind its use I will like to postpone for a later discussion.

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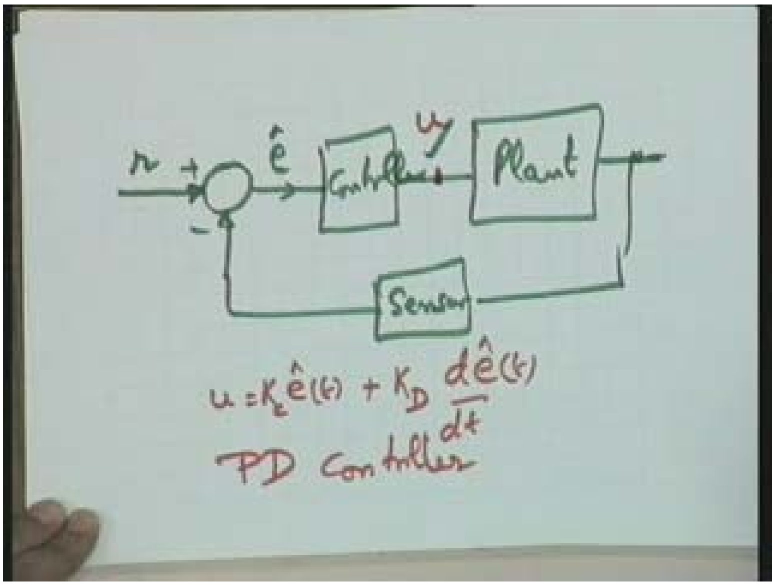

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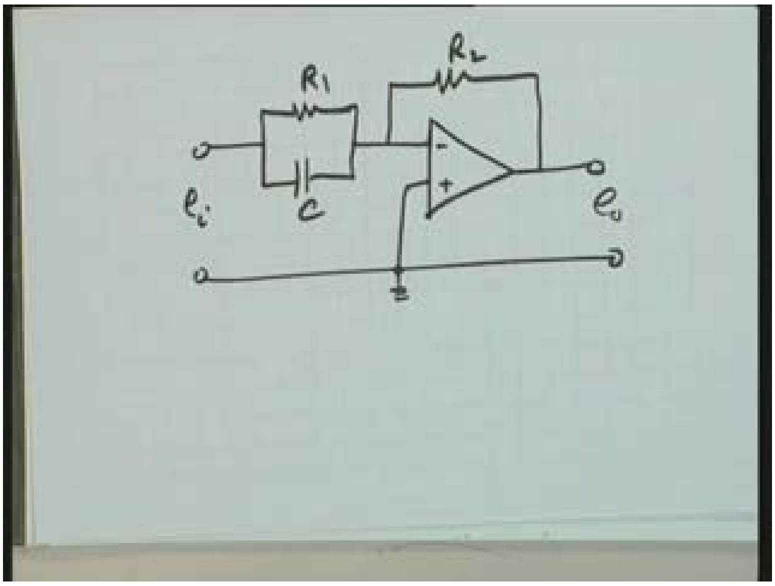

Quick look at this particular circuit. just look at this circuit now. The impedance here (Refer Slide Time: 39:38) is a parallel combination of R 1 and C and this impedance here is a pure resistor. therefore E 0(s) over E i(s) turns out be equal to……. please help me 1 over in this particular case it is R 1 into 1 by sc sorry so first let me take this as R 2 divided by this impedance R 1 into (1 by sc) divided by R 1 plus (1 by sc) I want you to manipulate and get convinced that it is a PD circuit proportional derivative circuit. Just the manipulation of this equation will give you proportional derivative circuit where the two constants will turn out to be in terms of the circuit parameters which are R 1 R 2 and C. I hope this manipulation will be easily done by you.

So let me look at this particular circuit. This circuit is a PID circuit. Again in this case, manipulation I like to leave it to you. The impedance Z 1 here the impends Z 2 here let me say E 0(s) over E i(s) equal to minus Z 2(s) over Z 2 Z 1(s) Z 2 here and a Z 1 here. now well, again a little exercise for you; get the values of Z 1 and Z 2 and get the transfer function model and from this particular model you will like to you will see by manipulation that E 0 by E transfer function is the transfer function of a PID circuit that is the output here is an appropriate combination of a component proportional to E i proportional to the derivative signal of e i and the integral signal of E i.

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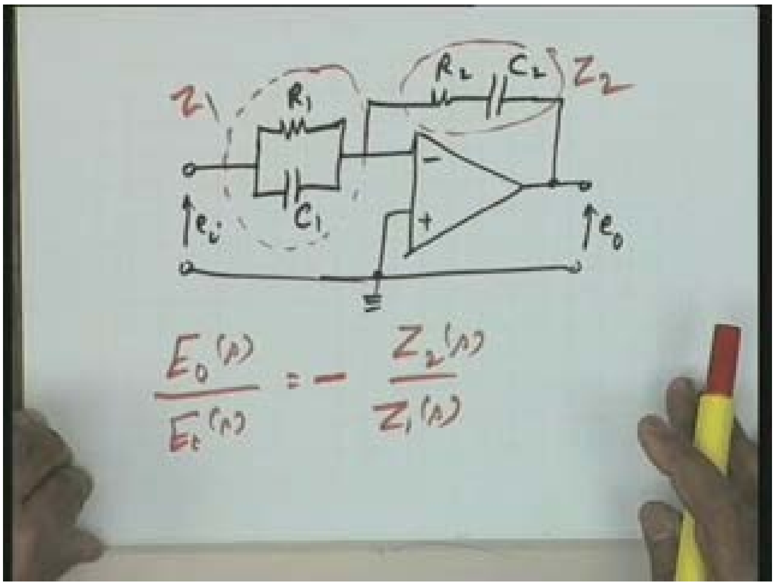

This I leave it to you please; I hope this is okay. So this is the electrical component in the overall control system; the electrical components being in coming in terms of the signal conditioning at various places. The controller circuits, Op-Amp circuits, digital computer circuits we have not taken and I do not think we will be able to accommodate the digital computer discussion in this course. We will primarily concentrate on the analogous controller and therefore Op-Amp circuits will come in our discussion. It does not mean that digital controller controlled circuits are not important, they are rather more important rather they are gaining importance, however, we have left this discussion to the second course on control engineering and the third component I said is the filter in the total system diagram.

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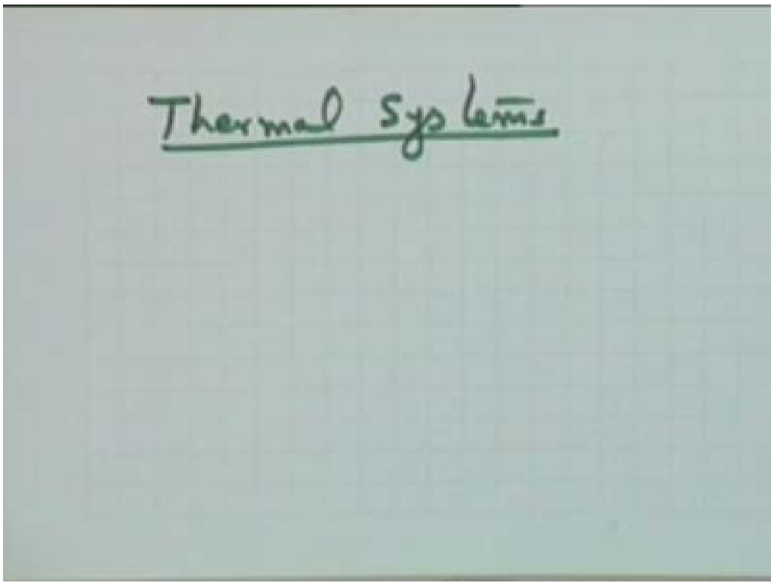

After this the electrical component is over. Now let me say that what are the equations from the thermal systems we will be requiring in our discussion. two more physical systems please: Thermal system and liquid level systems and after that I think we are ready to take on to the overall control system models and therefore one more lecture only on the dynamic models and dynamic responsible will be there. Thermal systems may be……. a system example may come next time looking at the available time but the basic equations can be given right away so that you can revise those equations before coming from the next lecture.

The basic equations are the following:
Let me consider a wall of any material with surface area A please see revision for you surface area A and the thickness let me take ℓ may be this type of diagram (Refer Slide Time: 43:43). Theta 1 is the temperature on this side, theta 2 is the temperature on the other surface; in that particular case the heat transfer through this wall is given by h equal to sigma A (theta 1 minus theta 2) divided by ℓ. This is the heat transfer equation. But again please note, all the time I will not be worried about the basic parameters of the thermal system. I will just see that, well, an analogous electrical system will help me very much in visualizing the dynamic behavior of this particular system.

Look at the units: h is in Joules per second, A in meter square, here I have meters and theta in degree centigrade and the units of thermal conductivity of the material then can appropriately come from this equation sigma the thermal conductivity of the material. But you see that this heat transfer equation for me becomes equal to is given by h equal to theta 1 minus theta 2 divided by R where R is equal to sigma A by ℓ ℓ by sigma A, R is equal to ℓ by sigma A. please see that this R is the conduction resistance the conductive resistance of the material and wherever I need this heat transfer equation I will be using this parameter (Refer Slide Time: 45:37) rather than going to the basic parameters of the system; h the heat transfer, Joules per second, the heat flow in Joules per second is equal to theta 1 minus theta 1 the temperatures in degree centigrade divided by R where the units of R can easily follow from the basic units given over here. So this is one basic equation which I will be using in my discussion.

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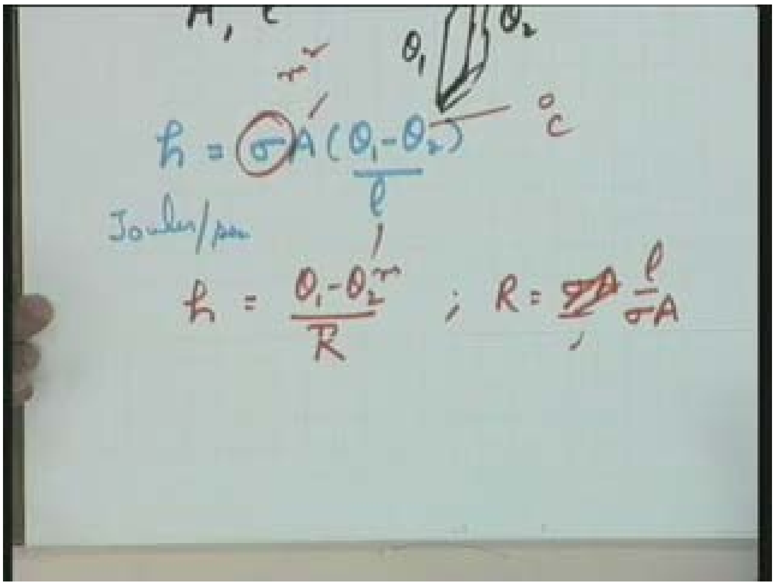

Now, the other way the heat transfer takes place is at the liquid solid interface. At the liquid solid interface my heat transfer equation will turn out to be of this nature: h equal to UA into (theta 1 minus theta 2). Again please see, I will view this equation as theta 1 minus theta 2 divided by R.

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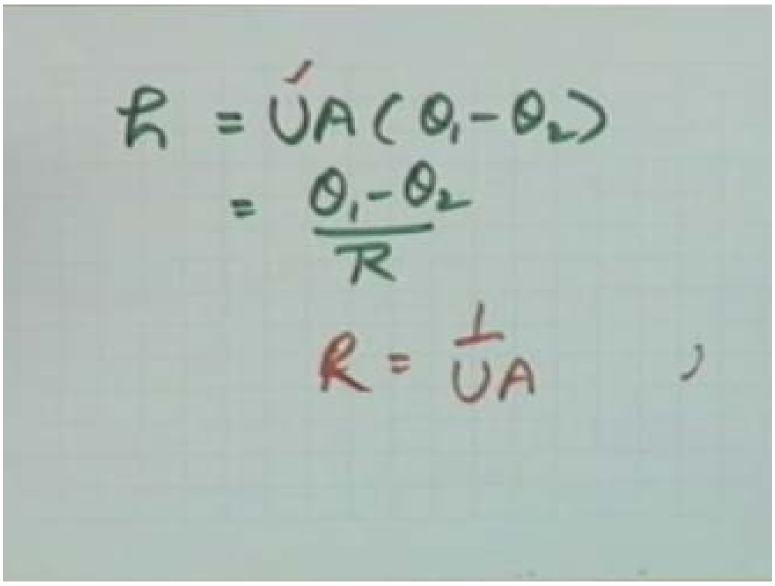

Look at the basic parameters of the system. U is the film coefficient at the solid liquid interface, A is the surface area of contact and I am not taking R equal to 1 by UA is the convective heat transfer resistance. So, for me the basic equation now becomes h equal to theta 1 minus theta 2 divided by R whenever the convective process is going on. You will see that the parameters of interest to me will be the resistance parameter as far as this dynamical behavior of the thermal system is concerned.

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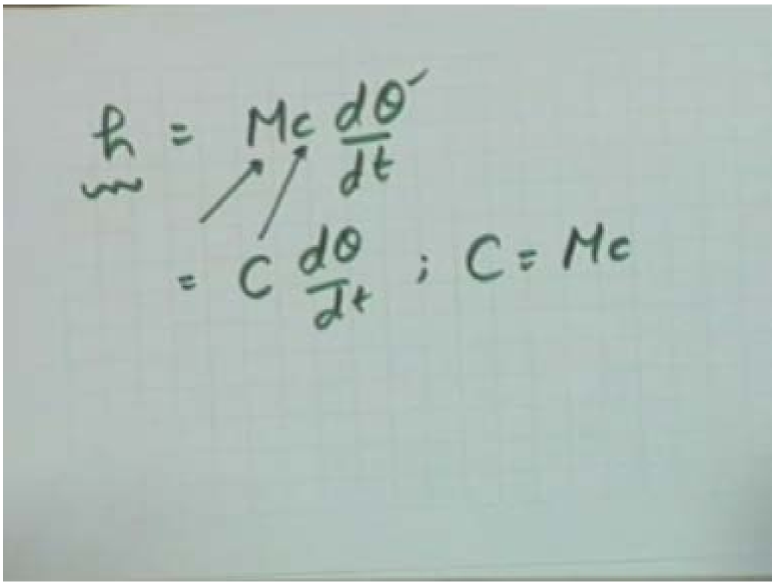

The capacity of parameter comes from the heat storage. Look at the heat storage the same wall example which I had taken. The heat storage will be given by Mc, you see that h equal to Mc d theta by dt is the basic equation you can see. this is heat flow rate in Joules per second, mass is the M is the mass of the material whose heat storage capacity is being considered, c is the specific heat of the material I am not writing here hoping that you already know it and you can quickly write and d theta by dt is the rate of change of temperature. So the basic equation the heat storage equation becomes h equal to Mc d theta by dt where theta is the temperature of the material under consideration; in that particular case this can be written as C d theta by dt where C equal to Mc is the thermal capacitance of the material. So, for me now as far as a solid material is concerned if…….[Conversation between Student and Professor – Not audible ((00:48:31 min))]

Well, this I can believe that the thermal systems have not been done. But anyhow I can repeat this that is no problem. How about the first equation please? The conduction and the conduction processes is it okay or does it need any repetition? [Conversation between Student and Professor – Not audible ((00:48:51 min))] Convection okay, if you see that the convection heat transfer should be should be explained, you see, consider a fluid solid interface. Let us say that the heat transfer is taking place at the fluid solid interface and the area of the contact is A and U is a parameter which is the film coefficient and which regulates the heat transfer. The U parameter is the film coefficient. So the basic thermal equation is this that the H per the heat flow rate is equal to as I have over given here, as I have given earlier UA into theta 1 the temperature of the solid minus theta 2 temperature of the liquid if the heat is flowing from the solid to the liquid the reverse will occur if the heat is flowing from the liquid to the solid.

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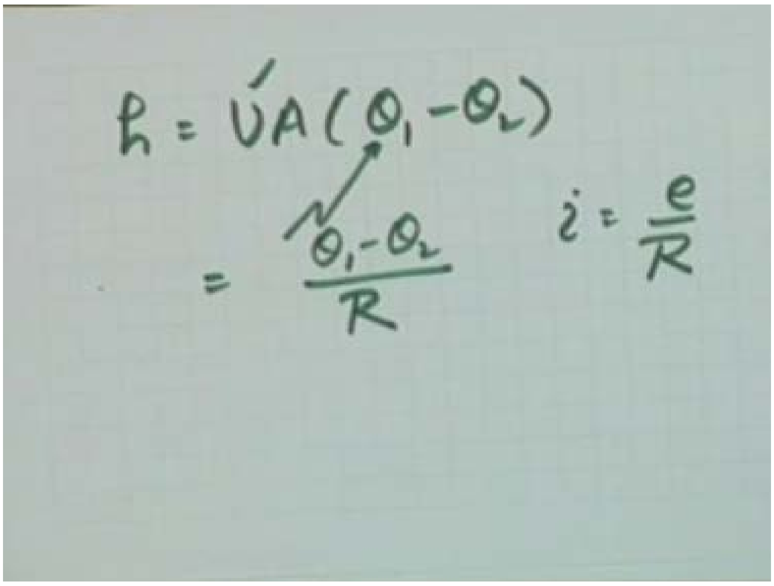

Let us assume that theta 1 is the temperature of the solid which is at higher temperature, theta 2 is the temperature of the liquid which is at lower temperature so theta 1 minus theta 2) is the net gradient across the film and this gradient the (theta 1 minus theta 2) into UA U is the film coefficient becomes your heat transfer rate.

Now what I have done is that I have recognized this equation considering h identical to current. Please see, i equal to e by R is my basic equation in an electrical circuit. So considering h analogous to current, temperatures analogous to voltage I recognize a resistant parameter and this I have written as theta 1 minus theta 2 the temperature difference as the potential difference, temperature difference across the film is equivalent to potential difference divided by R which is the resistance offered by the conductive process and the two parameters are the film coefficient and the area of cross-section. I hope this is okay. [Conversation between Student and Professor – Not audible ((00:50:49 min))]

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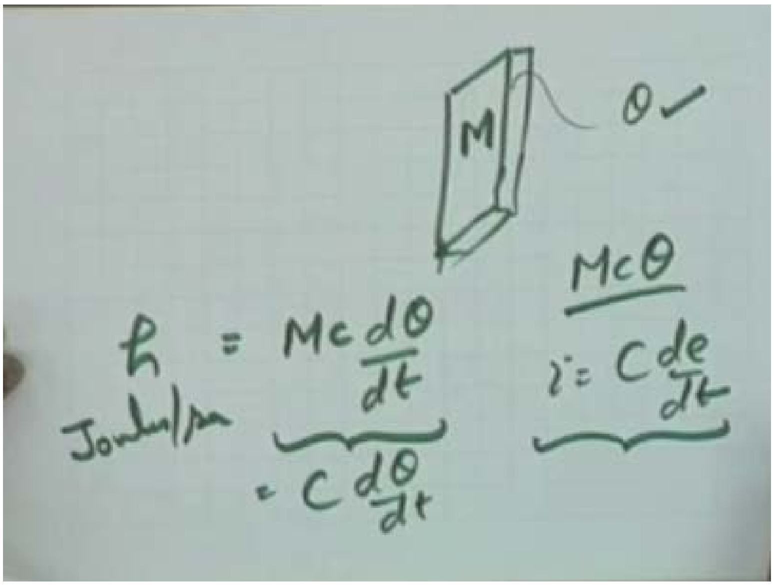

Fine, similarly if you want me to repeat I will take this material again. Let me say M is the mass of this material and let me say that theta is the temperature. Of course one thing is obvious say I am not repeating all those things. The system is actually a distributed parameter system, it is a distributed parameters system but you are approximating it by a lumped parameter system that is why the total mass is being approximated to be modeled at a point mass of temperature at temperature theta. So this is a point mass at temperature theta (Refer Slide Time: 51:37).

The total heat storage in this particular case is Mc and the basic equation which I have written is this h and the consistency in the units can be taken, c is the specific heat of the system. You will find that Mc represents the heat storage and the rate of this is the rate of heat storage is given by Mc d theta by dt, Mc theta is the heat storage you will find that please see, Mc temperature theta is the heat storage so I am considering it as rate of change of heat stored because this is h joules per second. Since it is joules per second I am taking here as to make it compatible Mc d theta by dt and corresponding to the equation i equal to C de by dt my equation becomes this is equal to C d theta by dt where c is the thermal capacitance. Since it is analogous to this particular circuit equation i is equal to C de by dt.

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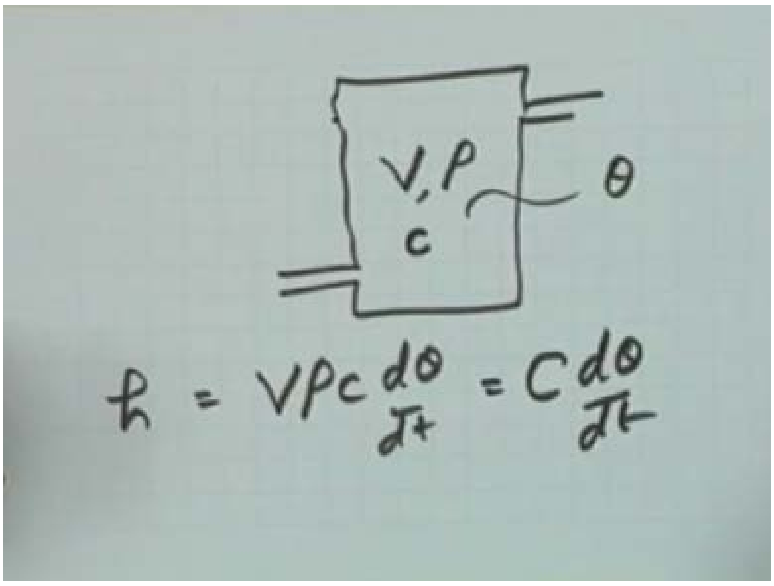

Consider just a couple of minutes are available with me and therefore I can write the tank also. This is your tank this tank has got volume V. please give me the equation now. If you have got my point you can give the equation to me. What is the thermal capacitance of the liquid in the tank? If I get it from you it will be it will confirm that you have understood my point. I want from you the thermal capacitance of the liquid in the tank. V is the volume in meter cube, rho is the density kilo gram per meter cube and c is the specific heat. [Conversation between Student and Professor – Not audible ((00:53:23 min))]

rho V see becomes obviously in this particular case, please see your equation h is equal to V rho c d theta by dt where theta is the temperature of the liquid in the tank which again I am considering to be a single temperature that is lumped parameter approximation is there. So in this particular case this can be modeled as C d theta by dt where C is the thermal capacitance of the liquid in the tank. With these two and you will please note, in thermal systems there is no parameter identical to inductance or equivalent to inductance so we will have resistance capacitance circuits only whenever I write dynamical equations for a thermal system and the example of the thermal system I will take in the next class, thank you.