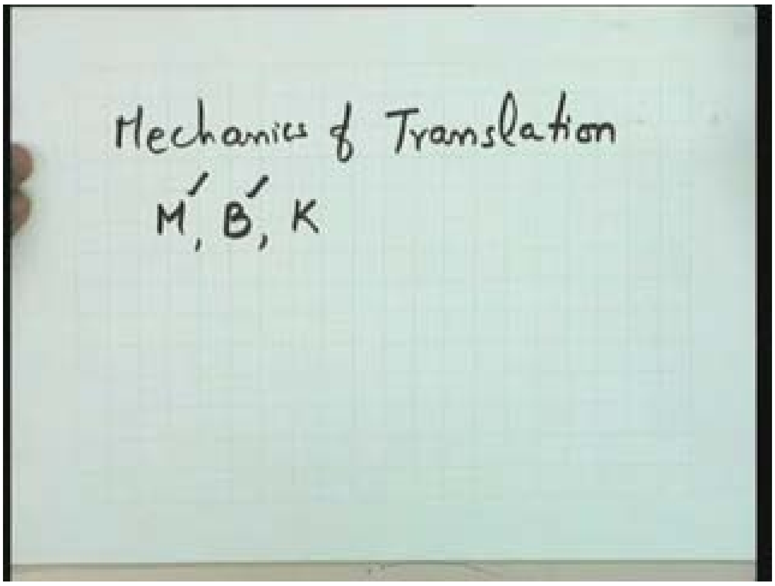
Control Engineering
Prof. Madan Gopal
Electrical Engineering Department
Indian Institute of Technology, Delhi
Lecture - 6
Dynamic Systems and Dynamic Response (Contd.)

Well friends, we have been talking about the dynamic systems and dynamic response for quite sometime. Well, I think I like to devote three more lectures on this subject and those three lectures are primarily concerned with the modeling of the dynamic systems. The dynamic systems will constitute of the plants in our systems, the controllers, the sensors and actuators and any other device which comes across so we will get a feel as how to establish equations for the dynamic systems.

As I told earlier that our major emphasis will be on application of the physical laws and there from transforming those differential equations to a more convenient form and the convenient forms we are going to use are the state variable models and transfer function models.

Let me get started with mechanics of translation. It is a sort of review no doubt but an important review because we have to use the models of the physical systems used in the controlled systems. Mechanics of translation; the three basic elements are: The ideal mass M, the friction coefficient B and the spring constant K. These are the three basic elements of a system coming under this topic mechanics of translation. You will recall that we have come across these parameters earlier; the M component the ideal mass; it is a particle of mass M is actually a lumped parameter idealization of a distributed mass concentrated at its centre of gravity. In a physical situation you will not come across a particle of mass M. So you see the approximations involved. It is the lumped parameter approximation of a distributed mass. But we will assume that the total mass is concentrated at its centre of gravity and therefore it can be approximated by a particle of mass M.

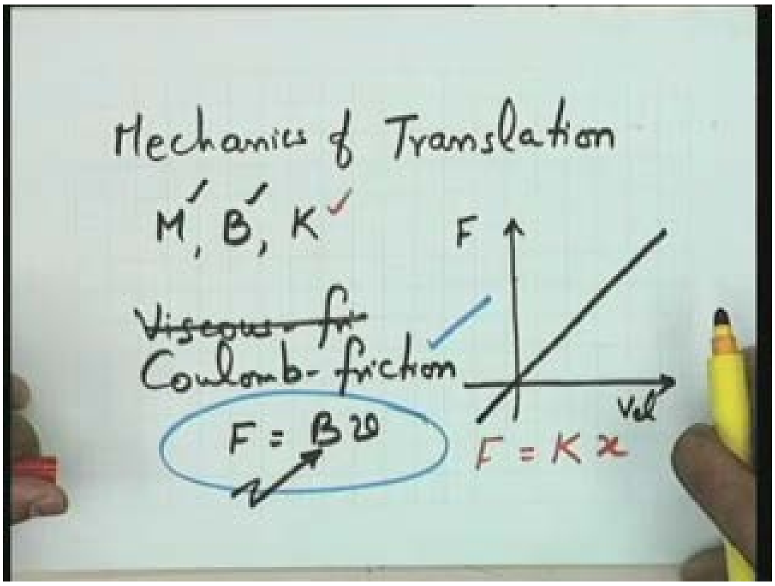
(Refer Slide Time: 3:17)



Come to B the frictional coefficient B. you see that the friction in a system can arise because of various phenomena and therefore their models are also different. For example, there may be a constant friction, constant drive friction existing between two surfaces. In that particular case that modeling of that will be a constant frictional force exists for all velocities and this is as you know known as the viscous friction in a system sorry Coulomb-friction. Now, if there is a mechanical that is, two sliding surfaces are there and there is a viscous medium in that particular case the frictional force is normally proportional to the velocity. Let us say this is velocity variable (Refer Slide Time: 4:16) this is frictional force, in that particular case if there is a viscous medium between the two surfaces the model is given by this linear relation and we get the equation F is equal to Bv where this B now becomes the coefficient of viscous friction.

This type of equation or this type of model is valid when a mechanical or a solid body is in contact with a fluidic medium. So we will mostly come across this particular situation as far as linear models are concerned. It does not mean that the coulomb friction does not exist. Well, coulomb friction for example, the in take the case of a dc motor; the friction due to the brush contact is an example of a Coulomb-friction. But since such a friction is modeled by a non-linear model in that particular case at the initial phase of our discussion where linear models are under consideration we will assume that all the non-linear frictions are disturbances acting on the system. instead of taking those frictions explicitly into the model I will like to take those frictions as disturbances acting on the system and our friction will be modeled as F is equal to BE where BE is the coefficient of the viscous friction, this becomes the second parameter.

(Refer Slide Time: 6:39)

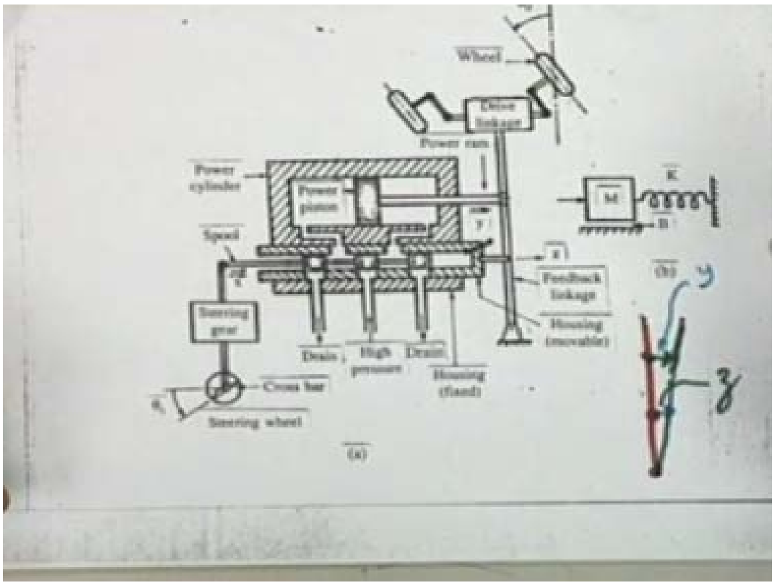


I will like you to understand very well the approximations involved. The M, B, K type of system the mass, spring, damper system when we take up for modeling it does not mean that there is a physical situation which exists which can model the system as a spring, mass, damper system. It is actually an idealization; it is a gross approximation but fortunately these approximations are working, they do give us useful result when our control system design is based on these approximations.

The third example, the third parameter I wanted to take is the spring constant K. Whenever there is elastic effect elastic deformation in a system you will like to modulate as a spring constant K and this as you know the force due to the springy effect is equal to K into x where x is the displacement and K you will call it the spring constant, F is the force due to the spring.

Now I will like you recall, you need not take up this diagram it is already there in your notes. Let us see that this is a physical situation.

(Refer Slide Time: 00:06:57 min)



In this particular physical situation now the first step will be to modulate to get the mathematical model for this if you want to analyze this particular system or you want to go for a design. Now let us visualize what are what are the mass, spring, damper equivalents in this particular case. Take for example the load on the system, the load is going to be just….. look at this the power piston mass, the mass of the drive linkage and the mass of the wheels and you can see its distributed nature: The power piston, the drive linkage and the wheels (Refer Slide Time: 7:31) but when you go for an approximate model you will like to model this as a particle of mass M and in this model this becomes the model of this physical situation.

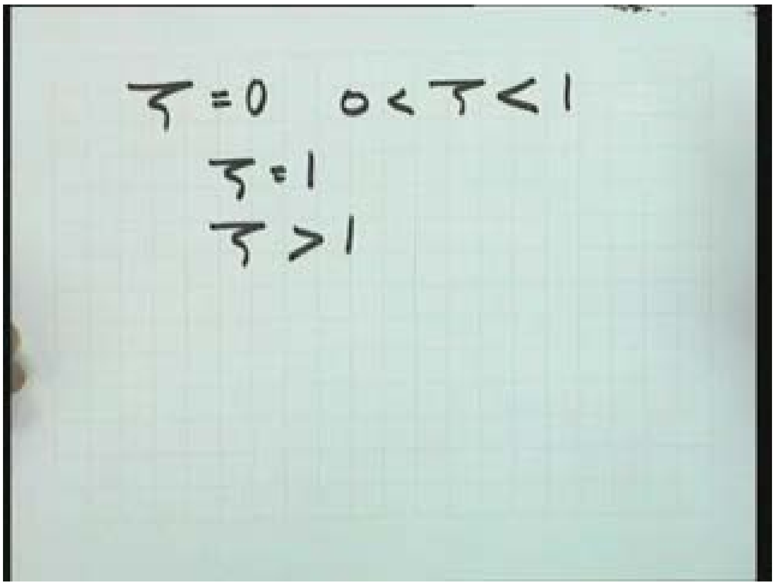
See the effect of the viscous friction. Where is the viscous friction in this particular case? Viscous friction, you can just see, this piston is moving in this power cylinder and there is a viscous medium. So the friction exerted on the piston, the friction which is opposing the motion of the piston is naturally a viscous friction situation. Similarly, there is a viscous medium in the drive linkage; the friction in this mechanical part is also viscous friction.

How about the Coulomb-friction or the constant friction?
Well, you can see that the tires on road, well, could give you a situation in which a Coulomb-friction type of situation exists. But let us take that as a disturbance acting on our system which is going to be modeled as M, B and K where K is the elastance of the tires. So these three parameters capture this particular situation and therefore now onwards whenever I give you a mass, spring, damper model keep in mind that this actually is an approximated situation

of a physical system wherein there may be different types of physical phenomena taking place. However, this approximation will work.

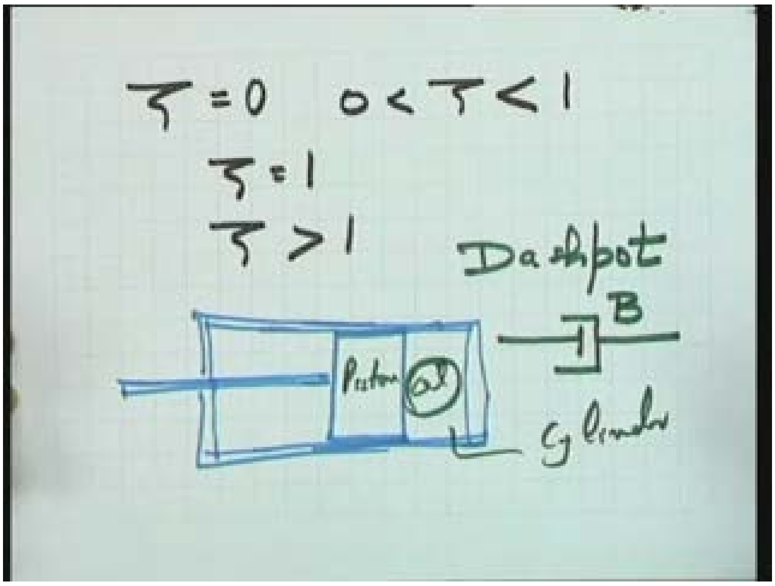
Well, there are situations where you will like to introduce friction yourself. You have seen that in considering the dynamic response we have taken up different situations. Zeta is equal to 0 is your un-damped system, zeta greater than 0 less than 1 is an under-damped system and zeta equal to 1 we call it a critically damped system and zeta greater that 1 is an over-damped system. And depending upon the situation, depending upon the demand on the control system you may like to go for one of this and therefore you will like to control the damping on the system.

(Refer Slide Time: 9:34)



If you want to control the damping it means you may like to introduce a damping parameter: One is unintentional damping, unintentional friction so there may be a situation wherein intentional friction is introduced in the system so that damping in the system can be properly controlled. And the physical system required to control damping is a dash pot. Say, consider this situation a dash pot. In this particular case here is an oil medium and this is your piston (Refer Slide Time: 10:30). Now you can very easily see that the motion between this cylinder and the piston is going to be resisted by the oil medium. So you can properly design this dash pot so that yes, an appropriate B parameter in the system is introduced.

(Refer Slide Time: 10:54)

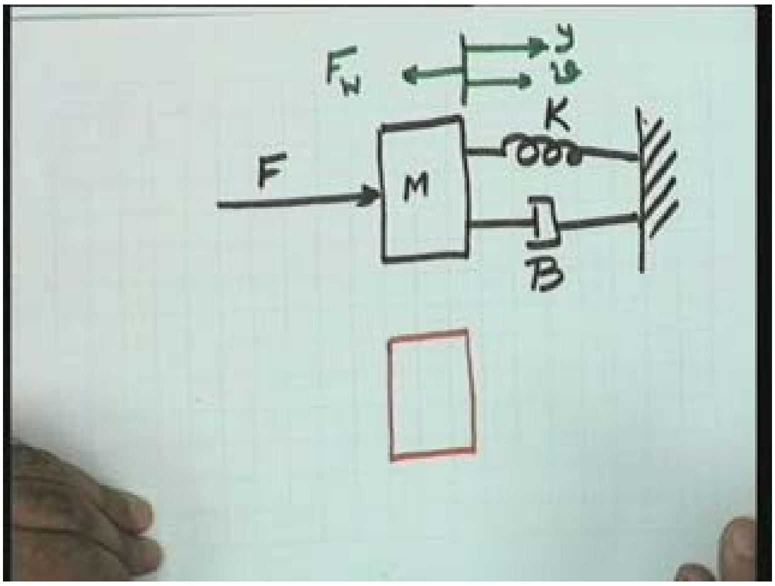


In the schematic diagrams which I take for mass, spring, damper systems please note that I will like to take this as the representation of the element B. Please see, this does not this is similar to the dash pot; you see I have copied this arrangement here in this schematic but keep in mind that whenever I give this schematic it does not mean that it always corresponds to intentional friction. This schematic I may be using for an intentional friction that is a dash pot or may be non-intentional friction which exists in the system which is beyond your control. This will become the symbol representing the B parameter.

If this is okay then in that particular case please see. Any physical situation existing in nature, any physical system in the industry which is a mechanical system in translational mode will suitably be modeled as this particular physical system (Refer Slide Time: 12:01). This will become my model. And from here onwards I will write the differential equations and transform those equations into a transfer function model or a state variable model. But up to this point also there are so many things involved, so many approximations involved and you have to keep those approximations in mind.

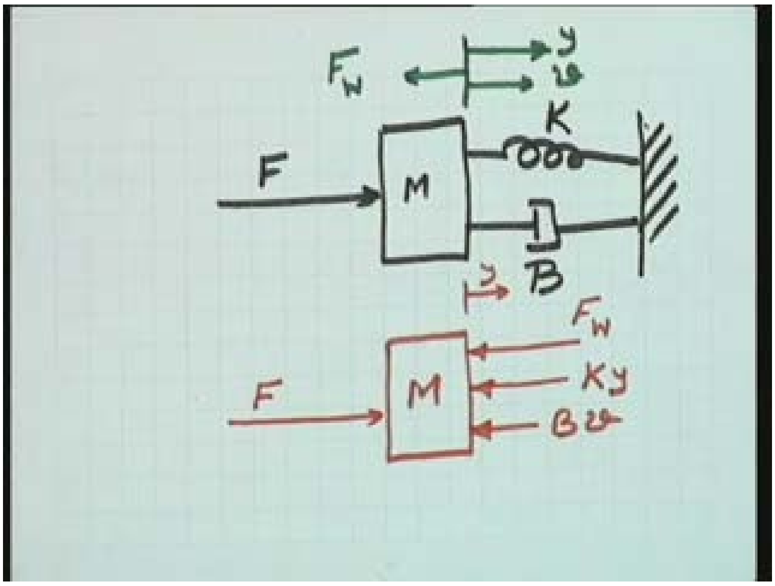
So let me say that M is the parameter here, K is the spring constant and B is the dash pot constant or the viscous the friction constant. In this particular case let me say that… this is F w I am taking, F w is a disturbance force acting on the system. the disturbance force in the power steering mechanism I told you could be an uncontrolled friction, the disturbance force may be coming from the environment so this disturbance force is represented by F w because w symbol we have kept for representing disturbance. And let us say that y is the displacement and v is the velocity; these two y and v are the variables in the system so this becomes a physical model. Now this physical model, well, if I want to write in the form of a differential equation model I will like will you can straightaway write the equation byjust summarizing the total procedure which may be useful when we come to complex diagrams. The procedure could be to write a free body diagram for the system.

(Refer Slide Time: 13:42)

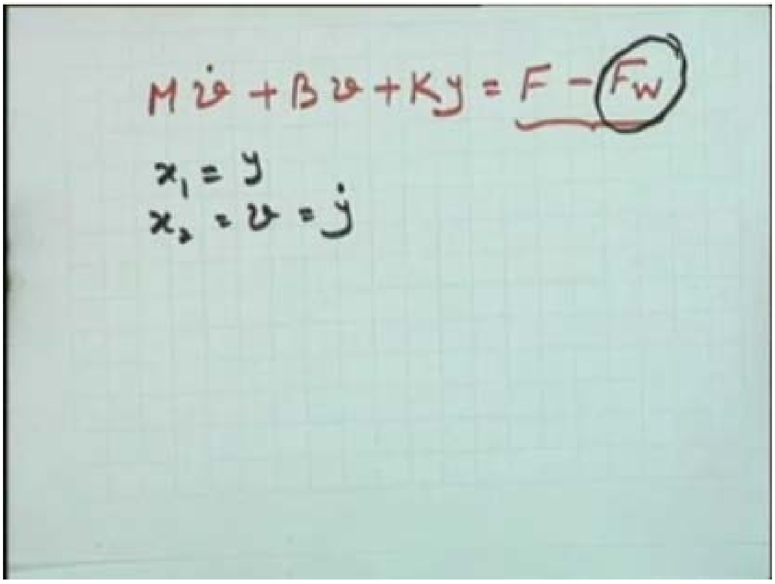


A free body diagram will show the masses as the nodes and all the forces acting on the masses will be represented by suitable arrows. So in this case let me see that this there is only one mass, this is the mass M shown over here in a free body and (Refer Slide Time: 14:11) here is a force F the applied force acting on it. There will be a force F w the disturbance force acting in the opposite direction. The motion is y in this direction, the spring force Ky will oppose the motion and hence in the opposite direction, the frictional force Bv will oppose the motion and therefore in this direction, this becomes the free body diagram. Keep in mind the inertial force due to mass M itself will also oppose the motion.

(Refer Slide Time: 14:34)



(Refer Slide Time: 00:14:51 min)

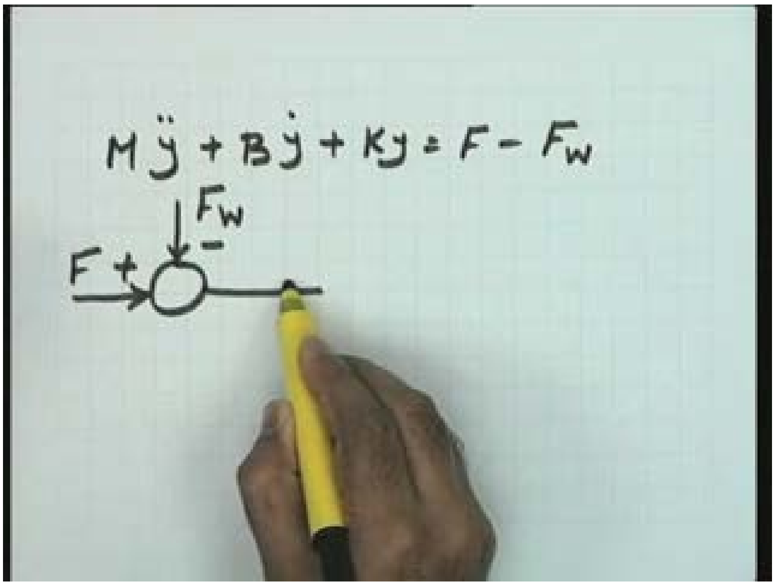


So what is the force balance equation?
The forces helping the motion, the forces helping the motion is equal to the forces opposing the motion and hence the equation which can immediately be written will look like this: Mv dot it is the inertial force due to mass plus Bv plus Ky equal to F minus F w. this is the net force acting on the system, please. F minus F w the applied and the disturbance force acting on net force acting on the system. One point may please be noted here that this is a disturbance which can even help the applied force because we are taking minus sign here, however, it is an algebraic quantity; a wind force may be helping the motion or a wind force or a mechanical mass may be opposing the motion. So F w is an algebraic quantity. In a mathematical model we will show F minus F w but keep in mind it can oppose the motion or it can even help the motion depends upon the nature of disturbance. But even helping the motion is a disturbance because you have not designed your system for that. So the word disturbance is still valid, the system has not been designed taking F w into consideration.

Now this is your model. Now you know it. Now if I want to convert it into a state variable model I will define x 1 is equal to y as one state variable, x 2 is equal to v is equal to y dot as the other state variable. Since we have already written a model of this nature let me not repeat that. Instead let me write the difference the transfer function model for this system. The transfer function model when you want to write even please note the transfer function is defined between an input and an output. So this is a system which is a two input one output system; the two inputs been the applied force F and the disturbance force F w.

So you will note that in this case, well I like to take another slide and write it in this form: My double dot, please see the change now, I am writing my equation in terms of the output variable y only. v By dot plus Ky equal to F minus F w, two input one output system and the transfer function is a single input single output description and therefore in a symbolic diagram the system looks like this: a plus symbol here a minus symbol here and F w here.

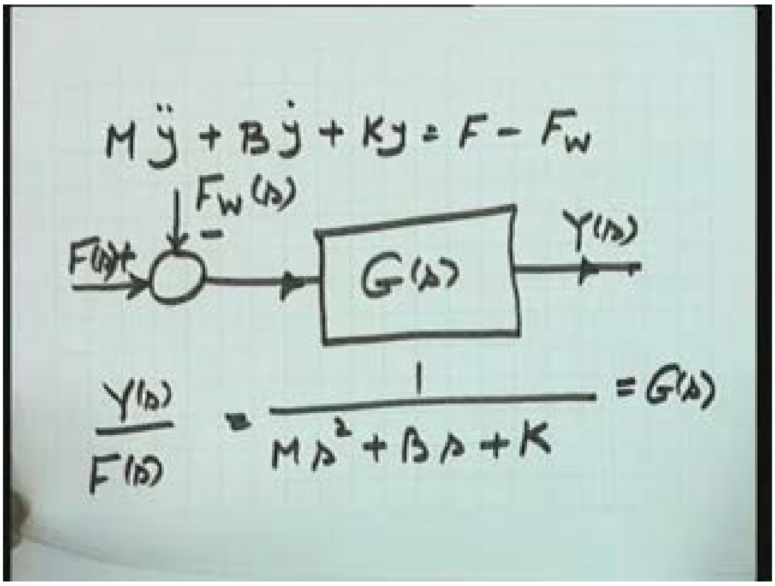
(Refer Slide Time: 17:30)



Now you see that this is the net input applied to the physical system and you can say that this is the transfer function G(s) and here is the output variable y. In terms of Laplace domain, in terms of Laplace variables this becomes the block diagram description. So you see that the transfer function you can write only either between y and F or between y and F w. So just to give you a transfer function example in this particular case, assuming F w is equal to 0 then between y and F your transfer function is Y(s) over F(s) equal to, which I think now, with your experience of modeling I can straightaway write the expression Ms squared plus Bs plus K.

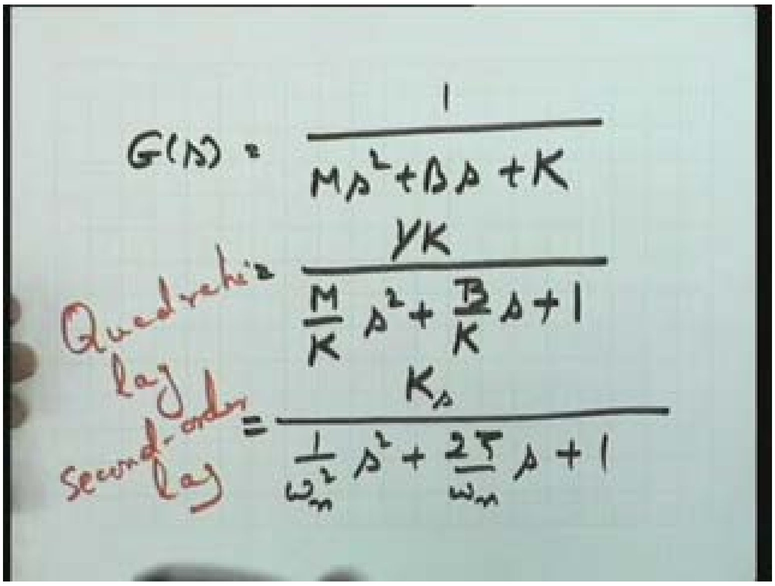
Now you see that this G(s); now in this particular case you see, since this very transfer function is the transfer function between Y(s) and F w (s) as well you can express this as G(s) there is no difficulty there because you can see the link. This is the transfer function (Refer Slide Time: 18:48) between Y and F w and therefore G s is represented by this.

(Refer Slide Time: 00:18:52 min)



This is in a spring mass damper system where G(s) has come in the form: 1 over Ms square plus Bs plus K. recall the standard form, the personality of a second-order system is represented by the parameters the constant system gain K, the damping zeta and the natural frequency omega N. In terms of that I will like to write this as: 1 by K M by K s squared plus B by K s plus 1 which is equal to….. well, the K symbol has….. now in this case this K has been used over here so K system gain K s let me use over here just to differentiate it from this K over 1 by omega n squared s squared plus 2 zeta omega n s plus 1. This becomes your standard transfer function and this transfer function I may call as the quadratic lag in contrast to the simple lag we had considered here the simple lag was a first-order lag and a quadratic lag is a second-order lag. And I had mentioned this point to you earlier that the first-order and second-order lags in our mathematical models will come too often.

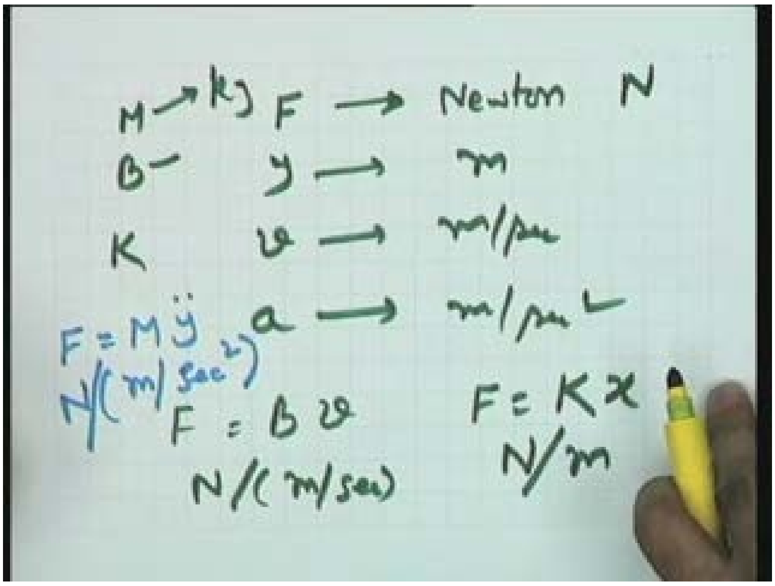
(Refer Slide Time: 20:18)



Could we just review the units also? I think it will be helpful.

The parameters are M B K; the variables are force, displacement y, velocity v. Quick review of this force Newton’s, y displacement and let me say displacement meters, velocity meters per second, may be acceleration meters per second square, mass in this particular case let us take it as kilogram, B, now B the force is B into velocity so can I take the units of B as Newton’s per meter per second. You can see this point please. Newton’s per meter per second will become the units of B, the units of K F is equal to K into x. so the units of this K parameter will become Newton’s per meter.

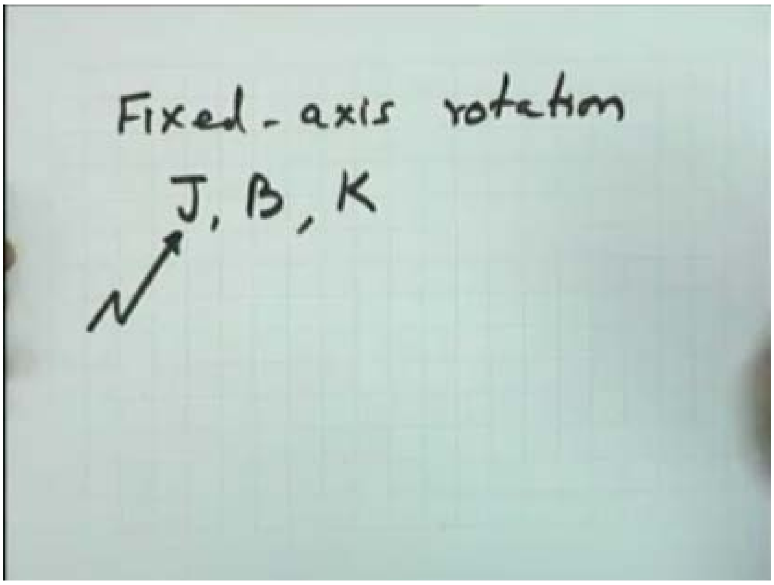
(Refer Slide Time: 21:37)



And how about the units of M the mass? I think I can use this space. We have taken this as kilogram. this could be taken as F is equal to M into y double dot acceleration so Newton's per meters per second square is also a unit of M the parameter, this is an M K S system of units though it is known to us but a review will be helpful because we will be coming across these units quite often.

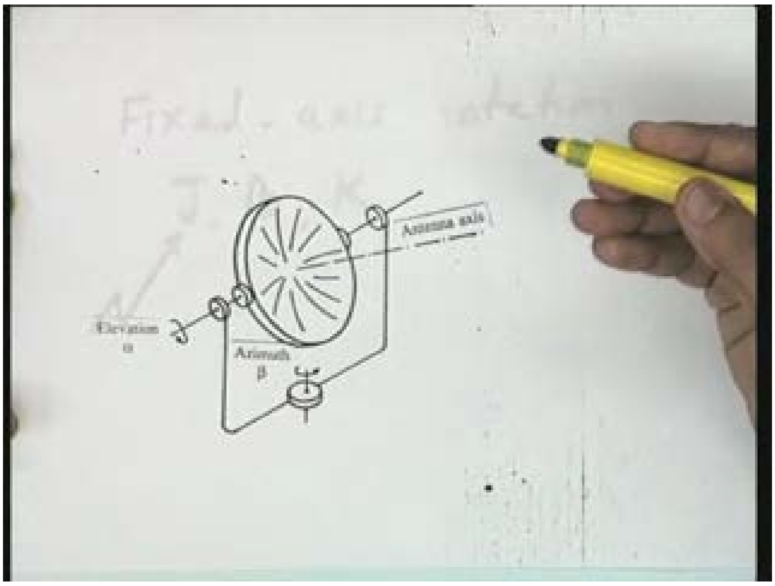
Now, after this, the review I like to take is that of a system fixed axis rotation; the rotational systems I like to consider now. The parameters are J the movement of inertia. It will run parallel to our earlier discussion so we have to be quicker now. J, B and K these are the three parameters. Look at the physical situations: J is the movement of inertia. Recall the example of steering of an antenna we had taken earlier.

(Refer Slide Time: 00:22:16 min)

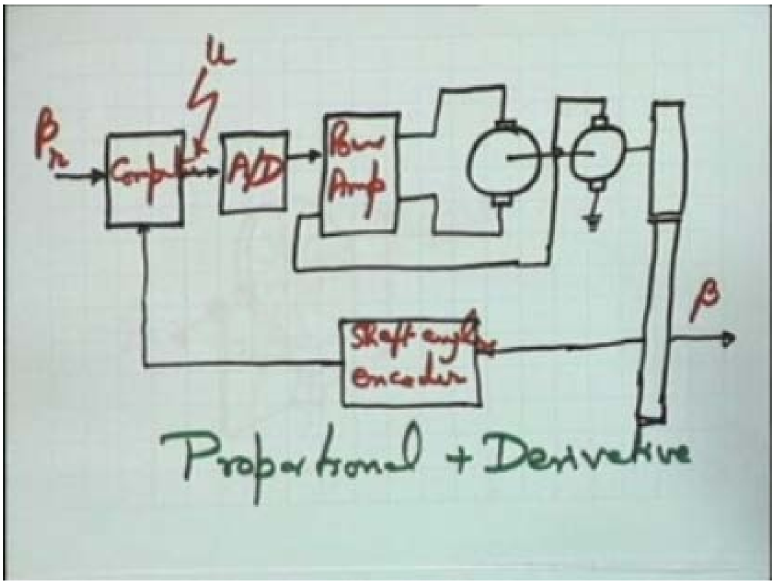


So the moment of inertia of the antenna could be this J parameter. I need your attention here. The moment of inertia of the antenna I said it is the load on the system it will be taken as the inertial parameter J. But if I gave you the block diagram of the system we had taken for steering an antenna this was the situation, this we have already discussed.

(Refer Slide Time: 22:59)



(Refer Slide Time: 00:23:20 min)



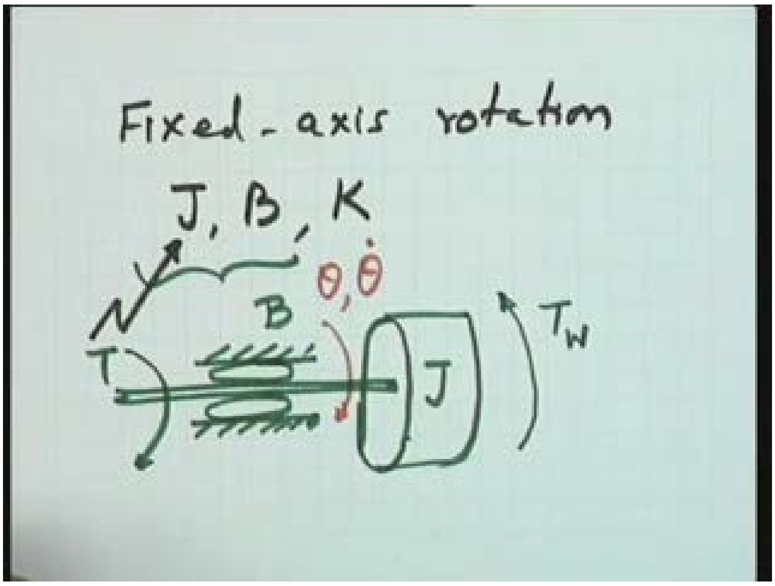
Now please see what is our modeling exercise. Our modeling will be like this: The actuator here is the motor; the motor shaft also has its own inertia, the tachometer installed on the motor shaft has an inertia because naturally though this inertial parameter (Refer Slide Time: 23:37) or this inertia affect may negligibly small compared to that of the antenna but in any situation where this is also appreciable you please note that what is the total parameter J the inertia due to the motor rotor and shaft, the inertia due to tachogenerator, the inertia contributed by the gears and the inertia of the antenna the total lumped into a single parameter J when it comes to modeling, when it comes to modeling of the system.

Now all these are moving in certain media so the viscous friction component is the total friction given to these particular rotors and shafts by the viscous medium. So B parameters correspond to the friction which may be due to bearings or other effects in the system. So you see that the effects are quite distributed but you are lumping them into the parameters J and B for the purpose of modeling.

Let me see the spring effect K. You see that in this particular case practically you see these shafts are not of very long length and if this servo or if the motor over here is a low power motor total torques generated in the system are not very high. What normally is done, the torsional effect on this spring is neglected and the K parameter is normally not taken. But it does not mean that it cannot exist. The robot controlled system, for example, when we take such a situation, well, the rigidness of the shaft may be ignored and you may have to go for including the flexibility in the system in that particular case the K parameter will also exist. So you can see that the K parameter is due to the torsional effect of the shaft which in many industrial applications you will like to neglect and therefore in many of the situations the parameters which we will come across in the mechanical rotational system will be these two components J and B (Refer Slide Time: 25:45). So, if you agree with this then let us take a physical model of the system.

The physical model of the system will apply this in that case. Well, I will represent the inertial element like this coming to a physical situation physical model of a of a situation which you have to handle which you have to control. so this is the shaft (Refer Slide Time: 26:10), here is the applied torque T the frictional effect, this you can say is a sort of damper for rotational systems, schematic of a damper or a dash pot for rotational systems, B is the parameter for this. Let me say that in this particular case T w I take as the disturbance torque on the system, the disturbance torque.

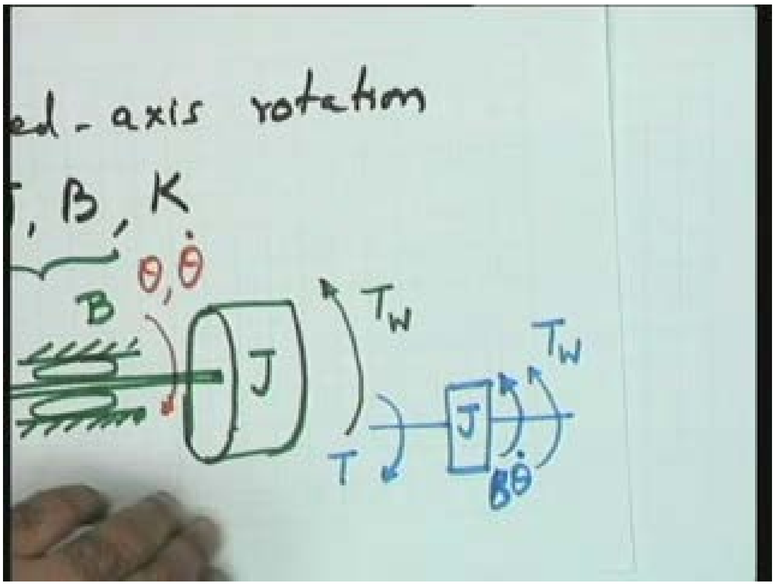
(Refer Slide Time: 26:44)



The rotation, well, will be theta and velocity is theta dot. Now, here itself I think let me make it in this space the free-body diagram. The free-body diagram will look like this:

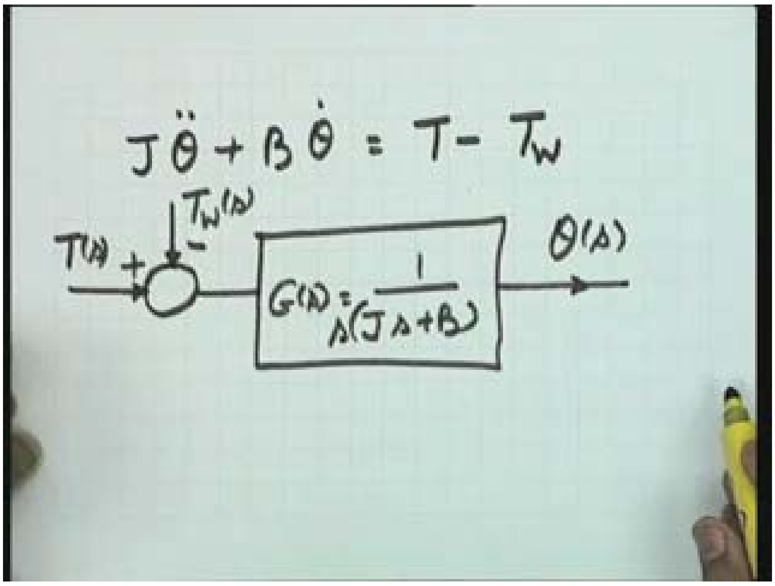
This is your free-body J, keep in mind that the inertial torque due to J will oppose the motion. That may not be shown express it as a as a force vector here, the applied torque T on the free-body, the disturbance torque T w, what are the other forces please? The other forces which I have to show because the K parameter has been ignored. The other force I have to show here which will oppose the motion is K theta K sorry I am sorry B theta dot.

(Refer Slide Time: 27:44)



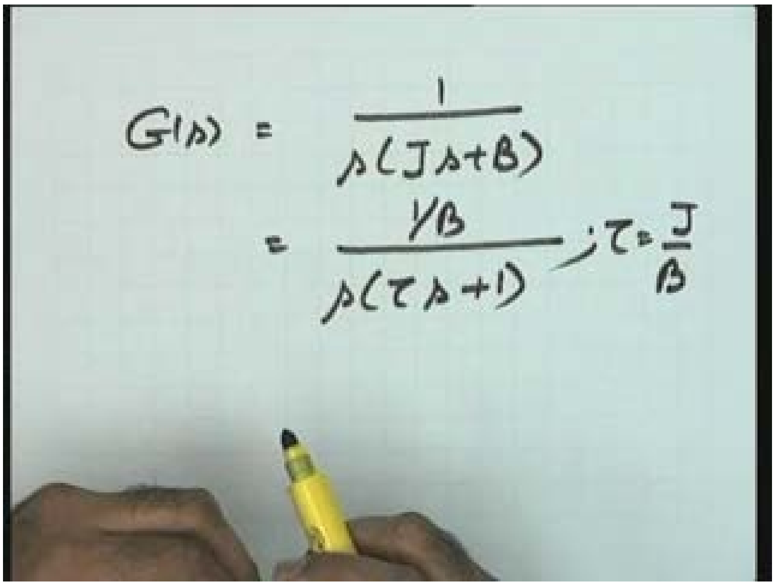
K has been ignored, it is B theta dot which is acting on the system and therefore the mathematical model a differential equation model for the system becomes J theta double dot plus B theta dot equal to T minus T w. This is the net torque acting on the system. Taking it in the transfer function form I can now directly draw this diagram and write the transfer function. I hope you will agree with the with me. G(s) I am writing as 1 over (Js plus B) into s this is the transfer function with theta(s) as the output variable.

(Refer Slide Time: 28:34)



This is your T w (s) here with a negative sign we have taken as a standard convention, this is your T(s) here (Refer Slide Time: 28:45) this becomes the model of this particular load which is a plant for a servo mechanism, a plant model for a servo mechanism.

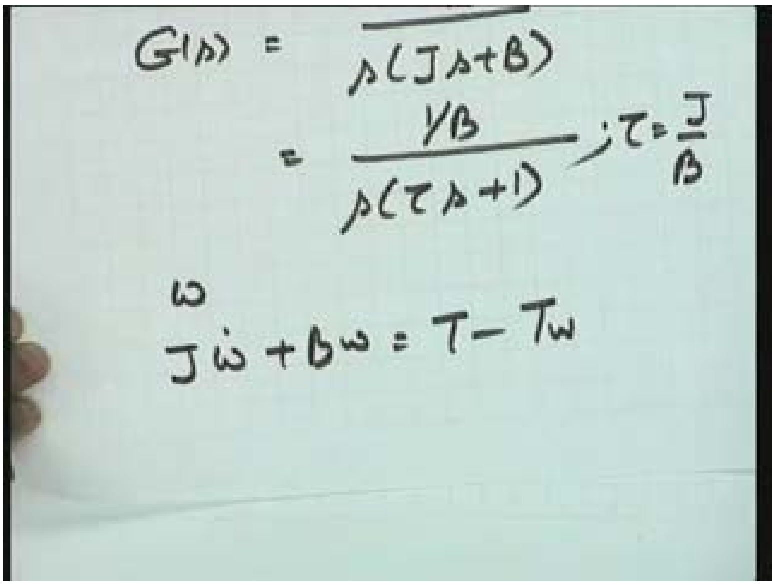
(Refer Slide Time: 29:55)



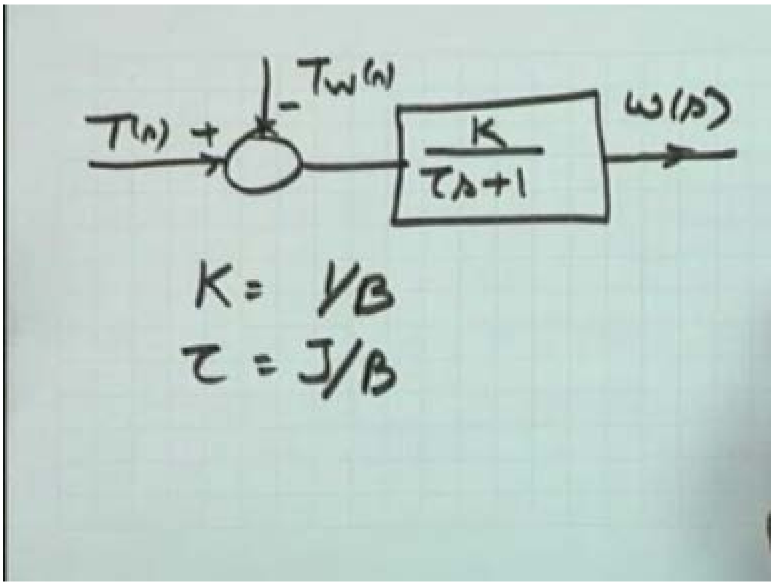
Look at this G(s). The G(s) we have taken is in this form: 1 over s into (Js plus B). You will please note, in the standard form it will be represented as 1 by B over s into (tau s plus 1) where tau parameter is equal to J by B. You will see, you will look at the units of J and B you will find that this tau parameter takes the units of time. It is a time constant of the system which naturally has to take the units of time if the units are consistent. This becomes the tau parameter and you can see, this is consisting of an integrator, look at the denominator 1 by s is an integrator and 1 by (tau s plus 1) is a simple lag or a first-order lag. So your mathematical model consists of an integrator and a simple lag cascade it together.

The same system you may take the same system but if I take omega as the output variable because output variable is an attribute in which I am interested in. If your servo system is a speed control system instead of position this speed will become the variable of interest. So, if omega is the variable of interest, please help me, your equation becomes J omega dot plus B omega equal to T minus T w. Well, equations you can write. Really I want you to give some space in your memory to the forms of mathematical models we are going to have. because then later on instead of bringing in these equations again and again I will be directly using these forms the first-order lag, the second-order lag, the integrator and so on. This you must store this information in your memory this is going to be an important information for us when we go to the important subject of analysis and design.

(Refer Slide Time: 30:40)



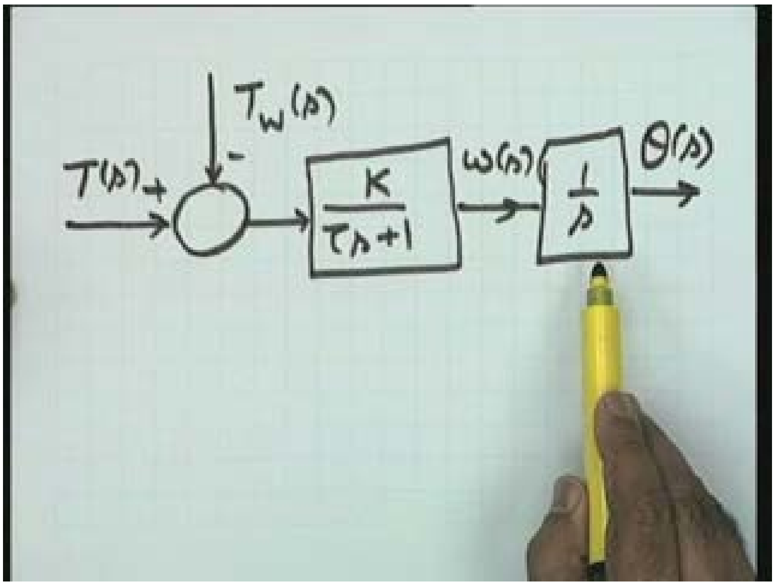
(Refer Slide Time: 00:31:04 min)



So look at this equation. If I make a block diagram for this T is the applied torque, T w (s) please see, in this particular case if you allow me I can directly write here as K tau s plus 1 with omega(s) as the output variable, omega(s) as the output variable this becomes K over tau s plus 1 where the variable K yes, K you will find is equal to 1 by B is it okay please? You could check, yes, I think it is 1 by B the variable K. Now it is not the spring, do not get confused with the nomenclature please, this K as per the standard first order term is the system gain, is the system gain, B and J parameters have already been defined, your tau becomes equal to J by B.

Can I write a block diagram here which actually will come to often later? And at this stage since your now equip to visualize this block diagram it will be in order if I put this block of the servo system here: K over tau s plus 1 omega(s) 1 over s theta(s). You will please see that a motor in a position controlled system has an integrating effect. There is no physical integrator as such present as an electrical circuit. But look at the effect, look at the total dynamics. In this particular case you find that T minus T w is the applied torque and if it is a motor is a na is an actuator for position controlled system it consists of these two blocks (Refer Slide Time: 33:02) a first-order lag as an and an integrator.

(Refer Slide Time: 33:09)

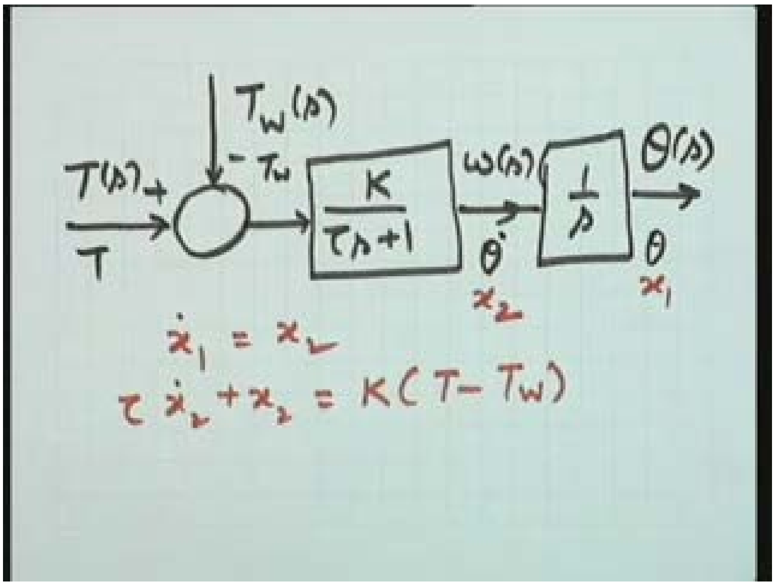


See the integrating effect of the motor. And in this particular case this if I put the time variables over here the variables in time domain theta, theta dot and T and T w become the time domain variables as far as this total system is concerned. In a Laplace domain block diagram a passing comment for you please, if I ask you, you see, transformation from the so-called state variable model to the transfer function model we have already taken and it was so simple because you had taken the Laplace transform of the differential equations assuming initial conditions as 0 manipulations of those equations gave you the transfer function.

Please see, if I give you a transfer function model in the block diagram form you will see that the outputs of the integrator are suitable state variables. In this particular block diagram if I ask you to write the term the state variable model for the system, please see, immediately you can take x 1 is the output of the integrator, this is a state variable one state variable (Refer Slide Time: 34:17) theta dot or omega is the output of this first order lag which is also an integrator or a filter along with the filter so this you can take as x 2 the state variable and your state variable model becomes, please see is directly from……..you need not you see go to the differential equation model for that. Given a transfer function model in the block diagram form or you arrange it in the block diagram form you can immediately write the state variable equations from that model.

In this particular case I get x 1 dot equal to x 2 you can see from this block and from this block I get x 2 dot tau x 2 dot plus x 2 is equal to K into T minus T w, these are the two equations. And now it is for you to identify the a b c and d matrices to put it in a standard form x dot is equal to a x plus b u y is equal to c x plus d u.

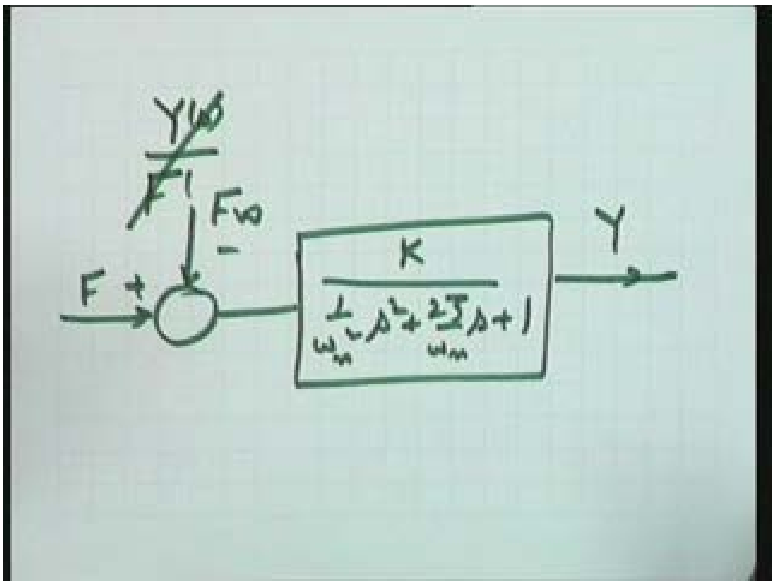
(Refer Slide Time: 35:19)



So what you have to do is just define the outputs of the integrators or the first-order lags as the state variables and straightaway right the corresponding differential equations and rearrange them into the suitable standard form x dot is equal to a x plus b u and the output equation y is equal to c x plus d u. So a review of the units will be in order here.

[Conversation between Student and Professor – Not audible ((00:35:51 min))]
Second-order lag directly, that is in that particular case the second-order lag will consist will be transformed to two state variables. So you take for example, since you have raised this point for example, let me take the mechanical system we have come across. The position Y(s) and the force, or I think I can take in response to his question the complete block diagram F w plus minus this was a second-order lag which I put it in the form K over 1 over omega n squared s squared plus 2 zeta omega n s plus 1 this is your output y.

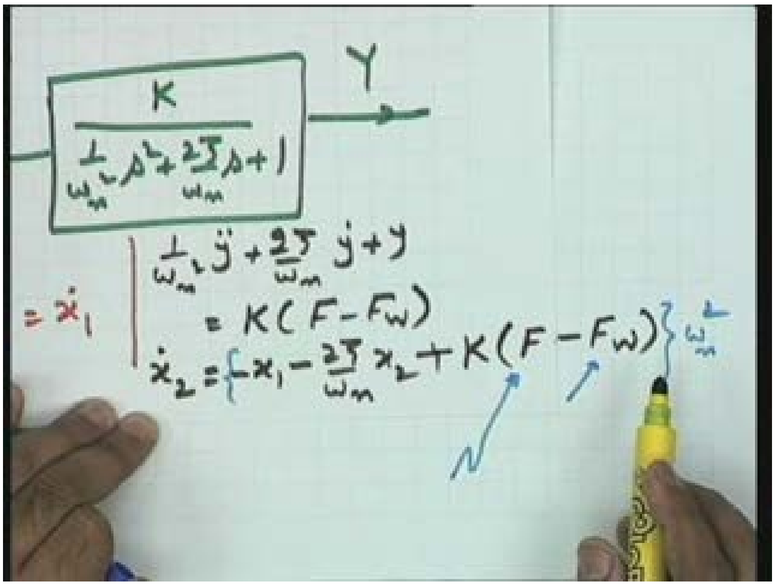
(Refer Slide Time: 36:40)



Please see, in this particular case the state variables are not available as the output of the integrator and first-order lags; it is a second-order system so naturally two state variables. The state variable selection is not unique as you know. But one of the selections which will work is take output as one of the state variables x 1 is equal to y and derivative of the output as the other state variable x 2 is equal to y dot is equal to x 1 dot which gives you this equation itself.

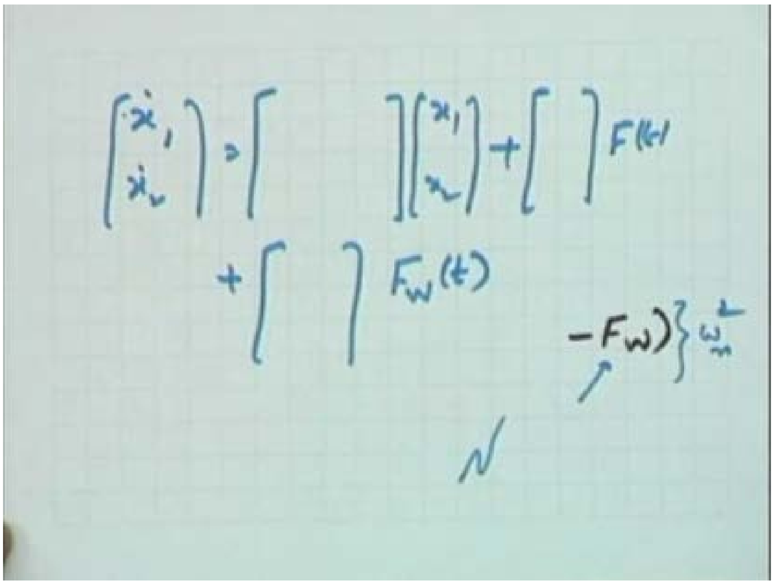
How about the second equation? Let me write here itself.
The second equation will be the following: 1 over omega n squared y double dot plus 2 zeta omega n y dot plus y equal to K(F minus F w). Now please see, it is just manipulation of this equation in terms of the variables x 1 and x 2. One equation you already have here: x 1 dot equal to x 2 in the standard format; from here you write the equation x 2 dot equal to what…. you see it is available, in what form? This is y so minus x 1 is clearly available minus 2 zeta by omega n squared x 2 is available and what else plus K into F minus F w these are the input variables and this total thing multiplied by omega N squared. Is it all right please? This is the equation which I have written for x 2 dot. Now you have to identify the suitable parameters. This is written (Refer Slide Time: 38:38) in terms of x 1 x 2 F and F w.

(Refer Slide Time: 38:50)



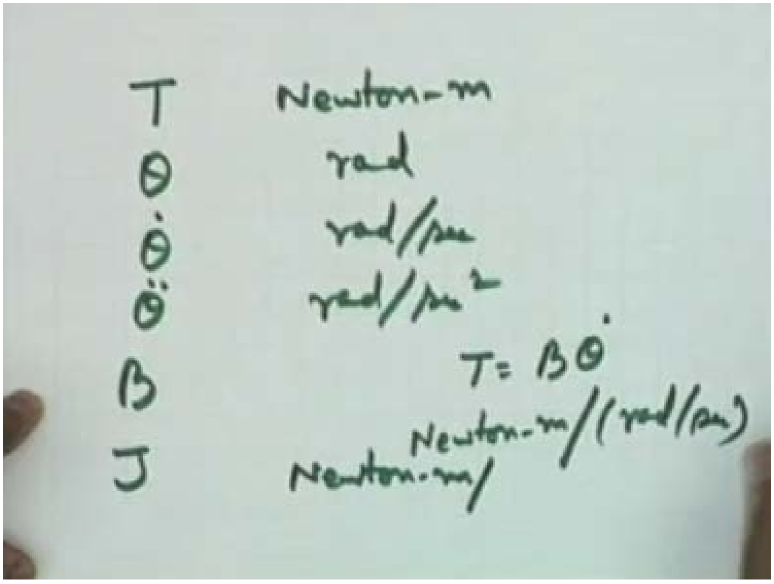
You will please note here, I mean, yes, it is good that I realize this point that there are two inputs (F and F w) and therefore your state variable model will come in the form: x 1 dot x 2 dot is equal to a matrix whose parameters you have to identify x 1 x 2 plus a b vector into F(t) plus another vector into F w because it is a two input system. It is a two input system there will be two vectors the b 1 vector and the b 2 vector; b 1 with F(t) as the input variable and b 2 with F w (t) as the disturbance. But as far as a state variable model is concerned this is also an input variable. For a control system it is a disturbance but for the x dot is equal to a x plus b u type of equation it is an input variable for you. Is this point okay or it needs further elaboration please? I hope this is okay. Fine.

(Refer Slide Time: 00:39:36 min)



Coming to the equations I said we will look at the equations as well. Variables are the torque Newton meter what else, you can take displacement. Let us set these though alternative units are also possible as you know but we will set these units for our course throughout we will be using these units. Theta as in radians, theta dot radians per second, theta double dot as required the angular acceleration, radians per second squared. Go to parameters the B parameter, let us look at the units T equal to B theta dot so naturally you can write the units appropriately Newton meter per radian per second will become the unit of B. Accordingly, if the flexible shaft is there and the K parameter is involved the units of K can be written.

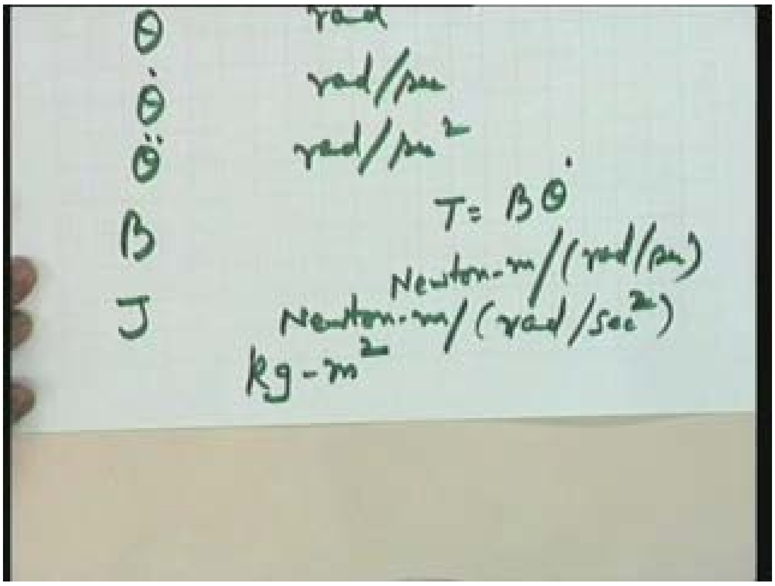
(Refer Slide Time: 00:40:02 min)



How about the units of J?
The units of J will become in this particular case Newton meter per radian per seconds squared. Please see, the equivalent units of J are equivalent to this kilogram meter squared.

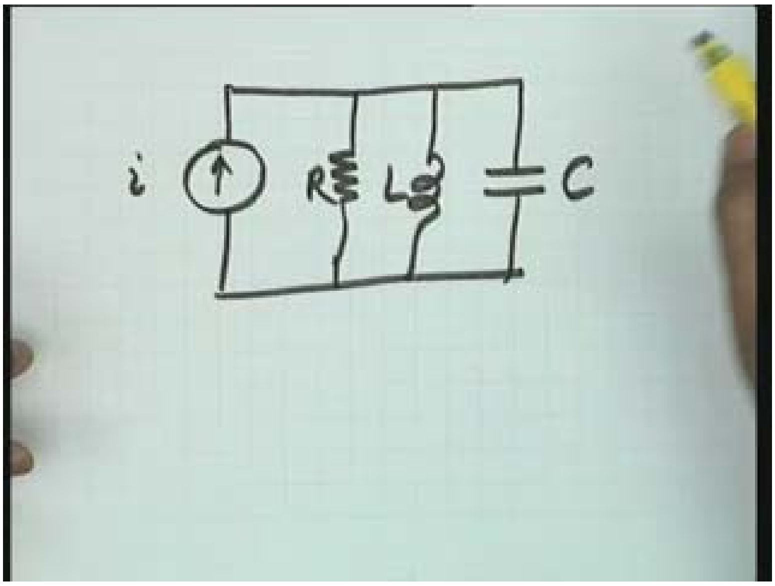
So I will be using sometimes Newton meter per radian per squared or kilogram meter squared.

(Refer Slide Time: 41:32)



Now I think I like to take up today because I have to take couple of examples, modeling examples. But looking at the time instead of taking a modeling example I like to set analogous variables into your mind. What is the principle of analogous variables and that is suitably illustrated to an electrical circuit.

(Refer Slide Time: 00:42:00 min)

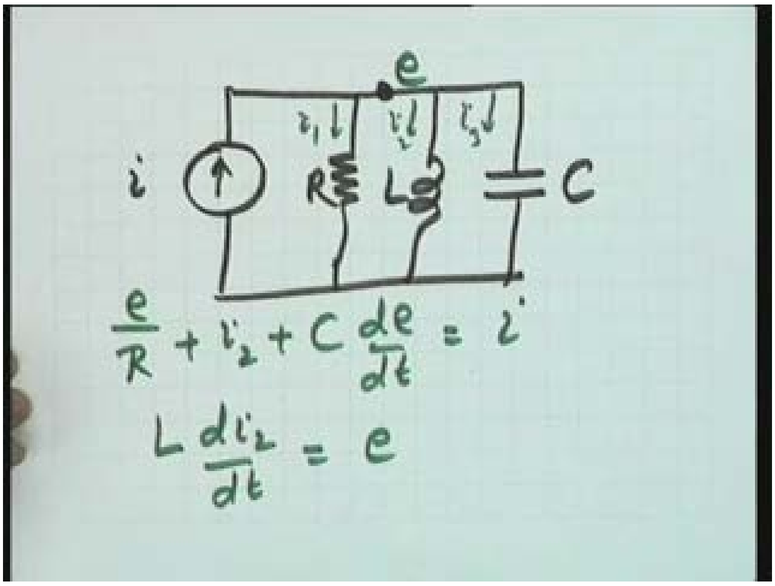


So I take an electrical circuit here with a current source I, the resistance R, the inductor parameter L and the capacitor parameter C. And if you understand this point the analogous variables you will find that any mechanical system the mass, spring, damper system for we the electrical engineers becomes an equivalent electrical circuit and all the equations of modeling, the force balance equation can be written in terms of, as you will see, the nodal equations. We will be able to visualize those equations in the form of nodal equations.

Look at this point (Refer Slide Time: 42:48) this is our node. This node is represented by let us say the voltage e. So the current I is the source current, let me take I 1 here, I 2 here and I 3 the three currents. As you know, the mathematical model for this system become e by R plus I 2 plus C de by dt is equal to I this becomes the mathematical model and if I want to write I 2 L d I 2 by dt becomes equal to e is the second equation.

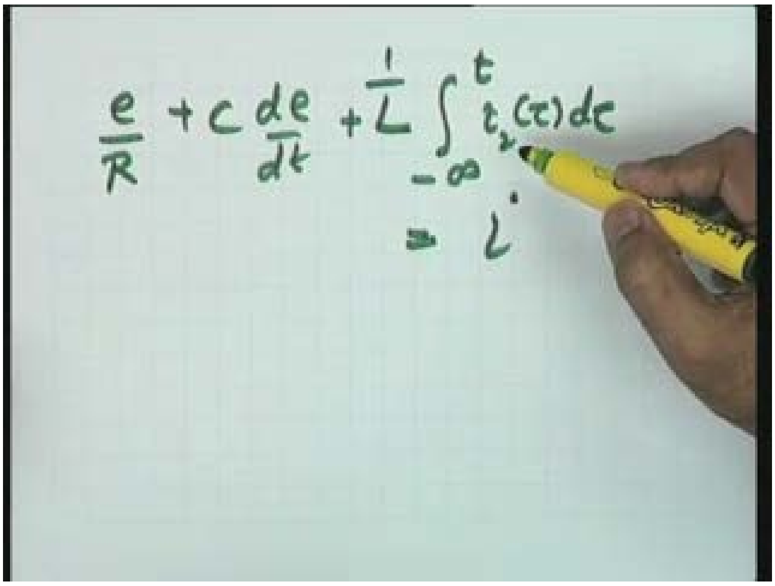
You know, if you want to transform these equations into a state variable model it is a simple situation; you will take current distributes current through the inductor and voltage across the capacitor as the state variables and the so-called x dot is equal to a x plus B u type of formulation will be available to you.

(Refer Slide Time: 43:47)



But at this point let me say that I am interested in getting an analogous mechanical system for that. This equation for a later use I am writing it in the form e by R plus C de by dt plus L integral minus infinity to t I 2 (tau) d tau equal to i.

(Refer Slide Time: 44:18)

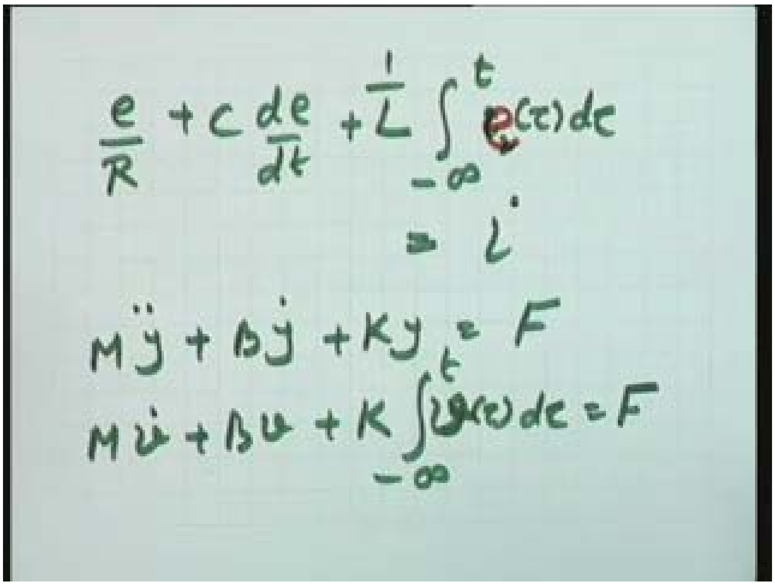


I hope you will realize this equation (Refer Slide Time: 44:22) just see please. I have substituted I 2 from this equation, it should be 1 over L is it not? 1 over L minus infinity to t I 2 (tau) d tau I. This is the differential equation model for this system. We have not been writing our model in this form: the integral differential equation from. I have specifically written for one objective of giving an analogous variable set.

Take the mechanical system. The mechanical system equation was in the form: My double dot plus By dot plus Ky is equal to force F. If you take only F, if you consider F w is equal to 0, if you consider only applied force this becomes your mechanical equation of the mechanical system. y was the displacement. Now this equation I rearrange in the form: Mv double dot plus Bv plus K please see minus infinity to t y(tau) d tau sorry I made a mistake again v(tau) d tau is equal to F.

v variable, yes please……..
[Conversation between Student and Professor – Not audible ((00:45:41 min))]
Oh yes, oh yes, okay.

(Refer Slide Time: 45:54)

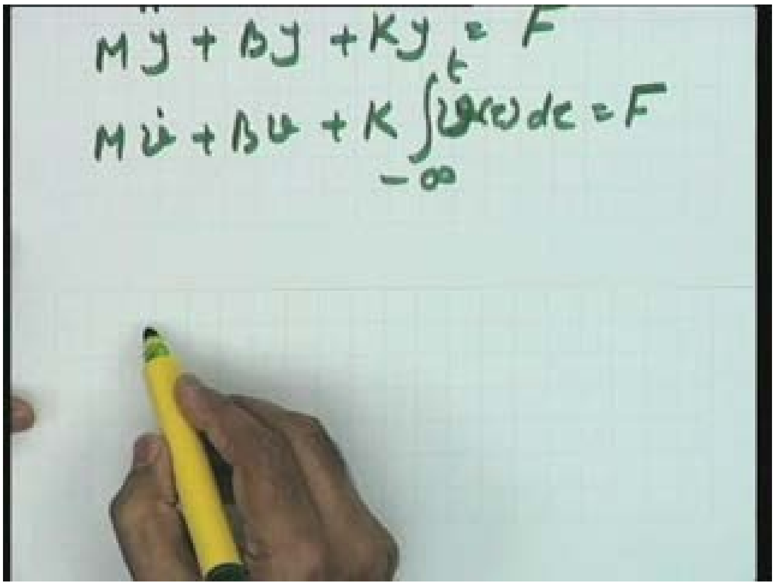


Now please look at these two equations. You see if these two differential equations are identical in that particular case whether you give a force step force input over here and measure the response v or you give a step current here and measure the response e the two will behave identically as far as dynamic response is concerned. Actually this is the basic principle of your analogue computer; the two equations are analogues meaning there by the two systems are analogous. If you want to study this particular system you can study this system and then translate the conclusions of this particular system into an appropriate form as far as the mechanical system is concerned.

Since electrical systems particularly electrical circuits can easily be manipulated can easily be worked with experimentally normally when we take up analogous systems we take the analogous to electrical form. A mechanical system it is an electrical analogue, a thermal system it is an electrical analogue, liquid system it is electrical analogue because the electrical circuit can easily be experimented with. So in this particular case also when we are using an analogous situation we will write an electrical analog of the mechanical system.

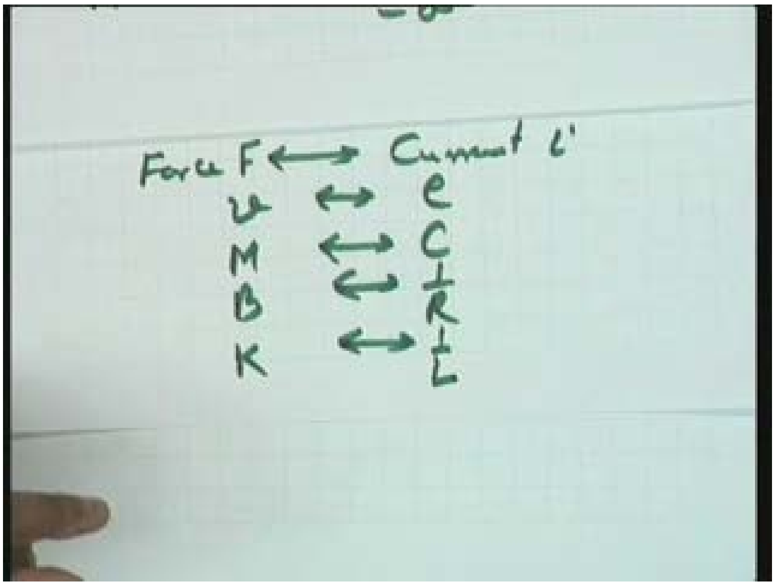
So look at this, let me retain these to equations over here so that we can compare the two.

(Refer Slide Time: 00:47:18 min)



You can find that the force F is analogous to current i. The velocity v is analogous v to the voltage e. Let us look at the parameters: M you find C B what is B? B if you see it is 1 by R on the electrical site and K you find is 1 by L as for us electrical circuit is concerned. These are analogous variables and since different analogies are possible let us give the name to this a force current analogy and we will be using wherever required will be using this type of analogy, a force current analogy; other could be a force voltage analogy where the equations are written in a different way, so that is why I said that.

(Refer Slide Time: 47:54)



If you visualize this then writing equations for mechanical system will become very simple for you because we will immediately write the nodal equations for the mechanical systems as well.

Now look at a mechanical translational system. There is no problem over here, you can write the equivalent variable here (Refer Slide Time: 48:37) force becomes the torque T so it is analogous to current, v the angular velocity omega is analogous to voltage, M replace it by a moment of inertia J, B is B itself and K it will exist for flexible shaft so these becomes the analogous variable in what is called the torque current analogy.

I think that is enough today for today and I will take up a couple of examples on modeling in the next lecture. Thank you.