Control Engineering
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Lecture - 5
Dynamic Systems and Dynamic Response (Contd…..)

Well friends, having introduced the the transfer function of a plant that is modeling of a plant and models of disturbances and the test signals; probably today onwards we can take up the dynamic response of the system. So I will like to take the block like this. This is the plant this is modeled by the transfer function G(s); to this plant we have the input R(s) and the output is given by Y(s) and I know the general nature of G(s) and the type of inputs we are going to handle.

Recall the step, RAM parabolic inputs are the models of the disturbances as well as they are the models of test signals for the purpose of analysis and design. So, as such the output becomes Y(s) equal to G(s) R(s) that is in the Laplace domain the convolution is transformed to an algebraic relationship. Now the dynamic response that is the output Y(t) if I am interested in I will get first the value of G(s) then the value of R(s) there from the value of Y(s), inversion of Y(s) will give me the value of Y(t).

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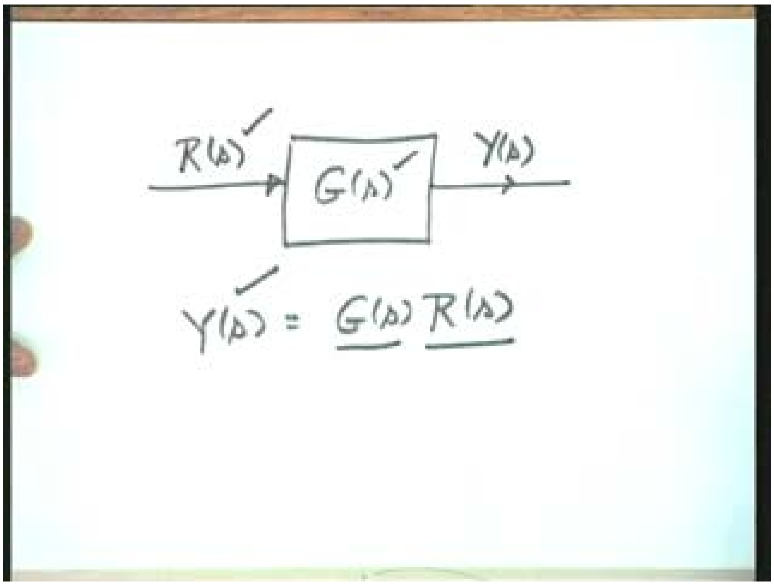

Since the Laplace transform and its inverse….., this is tabulated in various text books we will not go to the details of how these tables or how this inversion formulae are obtained; we will rather use these tables for the purpose of inversion. And I think it will be appropriate if I quickly give you couple of examples though nothing new in this particular case but still, as I said, in having an overall setting of the control systems analysis and design these quick review examples will be helpful.

The example I take is the following: G(s) the plant model is equal to 1 over s squared plus 3 s plus 2. You can just see, numerator polynomial is 1 and denominator polynomial is a second-order, numerator polynomial is a zero order polynomial let me put it this way and the denominator polynomial is a second-order polynomial. The system under consideration is a second-order system because the highest power of s determines the order of the system.

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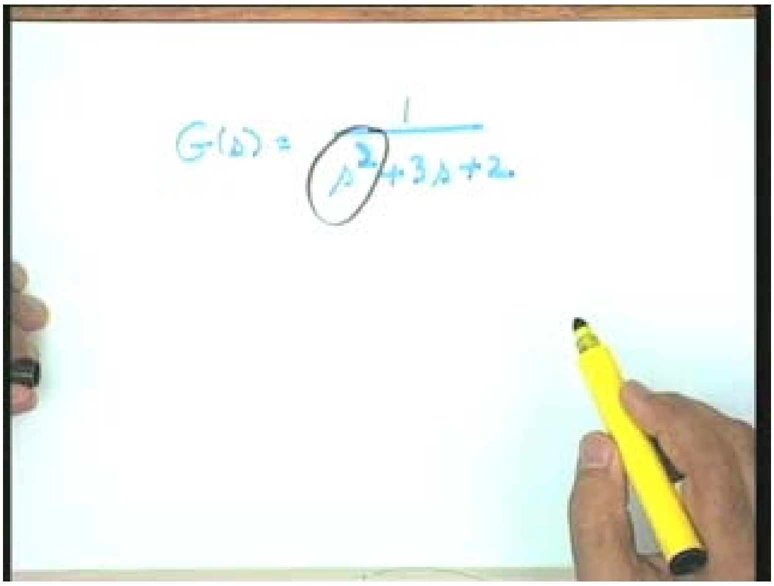

Let me put it in the pole zero form, there is no zero in this particular case and there are two poles at s is equal to minus 1 and s is equal to minus 2 these are the two poles of this particulars system. How about the input? Consider an input r(t) equal to 5 mu(t) a step input of magnitude 5. Therefore, as you know in this particular case R(s) will become 5 over s.

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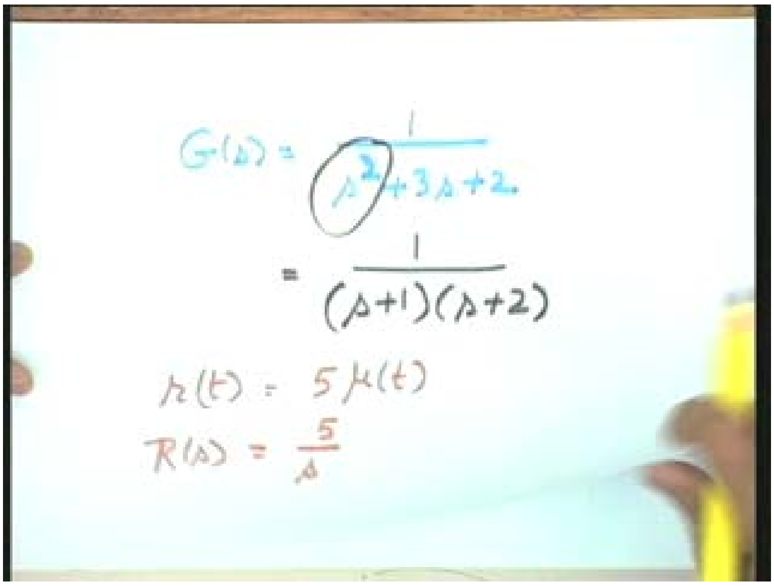

So the algebraic relationship Y(s) is equal to G(s) R(s) gives me the response transform as 5 over s into (s plus 2) into (s plus 2). Some of the terms which I will be defining you should please note because these will be refer to again and again later on. So, in this particular case there are three poles now as far as the response transform is concerned; the pole at s is equal to 0, the pole at s is equal to minus 1 and the pole at s is equal to minus 2. If I am interested in Y(t) so naturally the partial fraction expansion and inversion of Y(s) will give me the value of Y(t). So help me please, what is the partial fraction expansion for this? At s is equal to 0 I get the residue as 5 by 2 so one term is 5 by 2 by s. At s is equal to minus 1 what is the residue please? It is minus 5 so the other term becomes minus 5 over s plus 1, at s is equal to minus 2 it will become plus 5 by 2 I hope so it is plus 5 by 2 over s plus 2. Please see if there is any error here.

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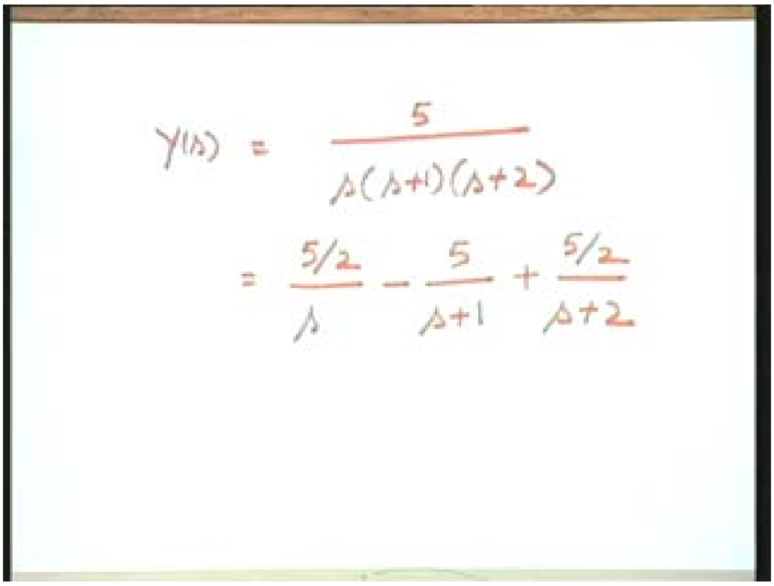

The point I want you to keep in mind is the following: these are the terms because of the system poles please. You recall the transfer function at the poles at s is equal to minus 1 and at s is equal to minus 2. This is the term which is because of the excitation pole. The step input gives rise to this particular pole.

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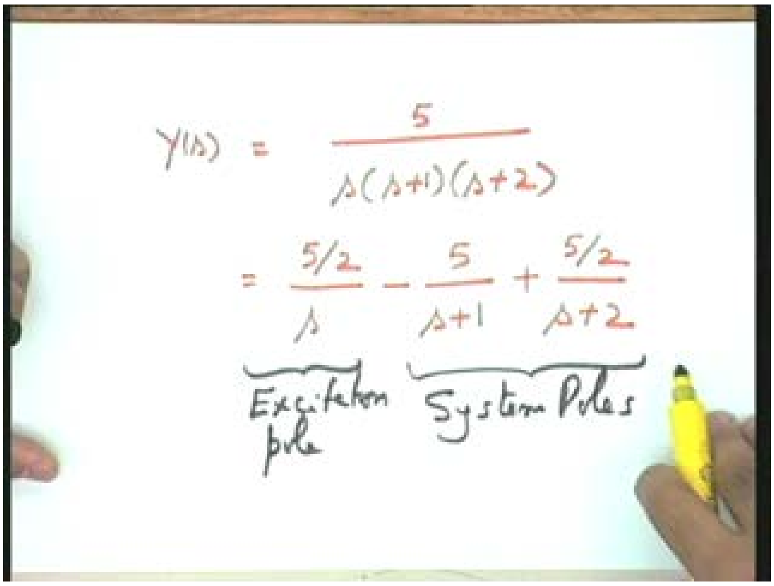

Now if I invert this Y(s) I get Y(t), very simple, this is equal to 5 by 2 t minus 5 e to the power of minus t plus 5 by 2 e to the power of minus 2t. Now just see, let us just classify the terms over here. What are these terms? These terms are contributed by these system poles when excited by the step input. Please see, the poles are excited by the step input and the resultant the result in the response is given by these two terms (Refer Slide Time: 7:05) and as you see these two terms die away with time; as t tends to 0 these terms die out and I refer to these terms as the transient response of the system.

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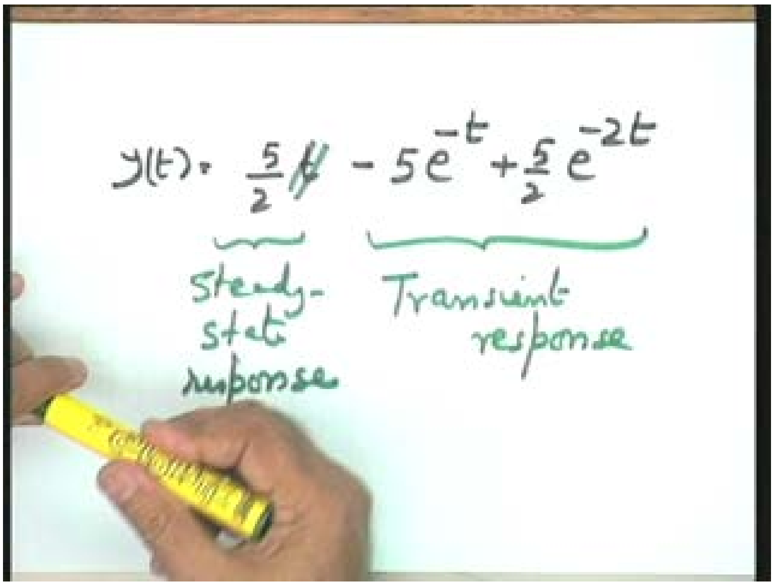

Yes please, any question?
[Conversation between Student and Professor – Not audible ((00:07:22 min))]
I am sorry, I am sorry it is 5 by 2 it is the inverse of 5 by 2 by s so it is 5 by 2. Now look at this term (Refer Slide Time: 7:33) this is a contribution due to the excitation pole and you will please note that the nature of this contribution 5 by 3 is same as the nature of the input itself; the only thing is that the magnitude has got modified and this modification in the magnitude is dependent on the reaction of the system to the input.

Since the input persists for all time this particular response will persist for all time and is refer to as the steady state response. Now, the steady state response as you know, you can also get without resorting to getting the total time response of the system you can apply the final value theorem to obtain the steady state response. Let me put this theorem also for your benefit, revise the theorem for you: limit t tends to infinity y(t). I am interested in the final value of the variable Y(t), this is equal to as you know limit s tends to 0 sY(s) were Y(s) is the transform of Y(t) provided this point may please be noted provided sY(s) has no poles on j omega axis right half plane. This point the applicability of the final value theorem please be noted. Many a times you make an error here. So this final value theorem will give the correct result. By using this particular relationship provided sY(s) has no poles on the j omega axis and are in the right half plane.

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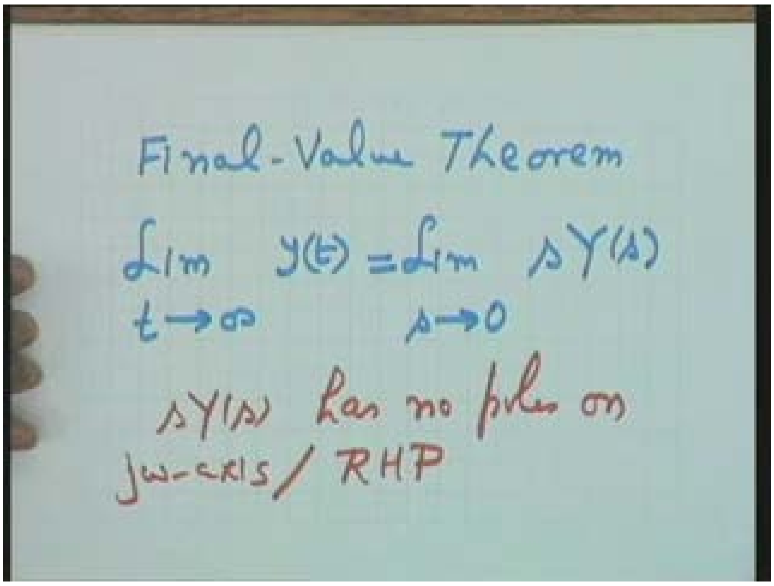

So if I apply this theorem. You recall in this particular example under consideration. The steady state value is 5 by 2. So if I apply the final value theorem sY(s) as you know is equal to 5 s over s into (s plus 1) into (s plus 2) and therefore Y double s naturally by the final value theorem turns out to be 5 by 2. So it means if we are interested in the steady state behavior of this system in that particular case directly this theorem can provide us with the results.

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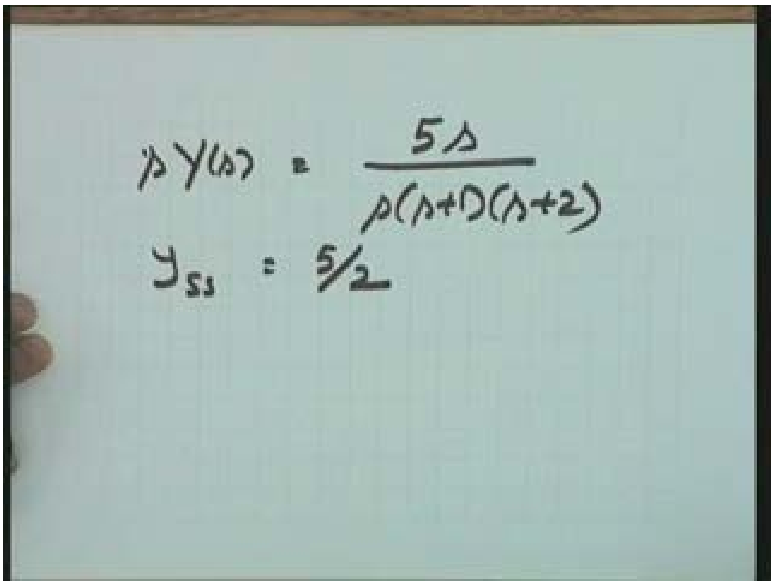

For this particular numerical example under consideration the response looks like this: (Refer Slide Time: 10:27) this is your t, this is y, this was the input the value is 5 and the output is an exponential rise to a value which is different than 5. This is the exponential rise as you see and this particular value, the final value; the steady state value is 5 by 2.

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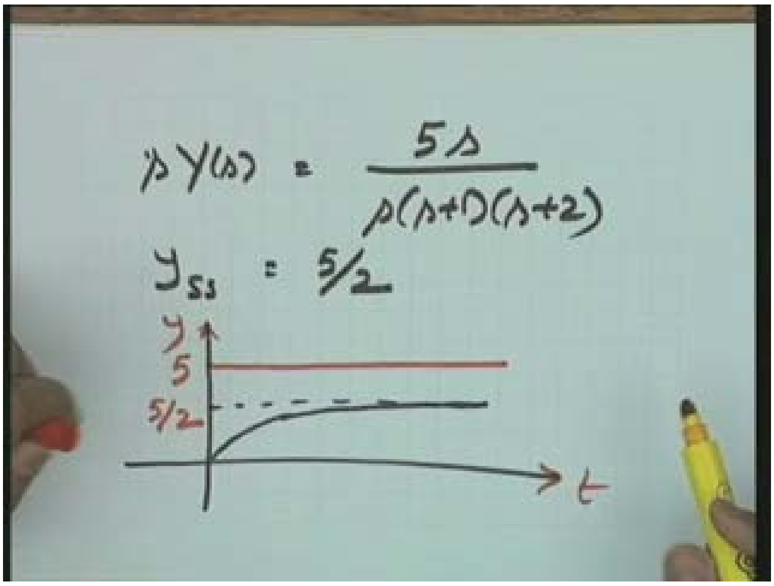

Nothing new, this example I know, but still some of the terms which we are using over here they will be used later on and hence the acquaintance with these terms will be quite useful. Another example, one more example if you can bear with me.

The other example I will like to take is the same plant: G(s) over (s plus 1) into (s plus 2) that is the two system poles at minus 1 and minus 2. But now I take the input as r(t) equal to 5 t mu(t). It is a ramp input as you see. So, for this ramp input 5 t mu(t) the transformed domain expression is as you know 5 by s squared and therefore the response transform Y is now becomes equal to 5 over s squared (s plus 1) (s plus 2) this is the response transform.

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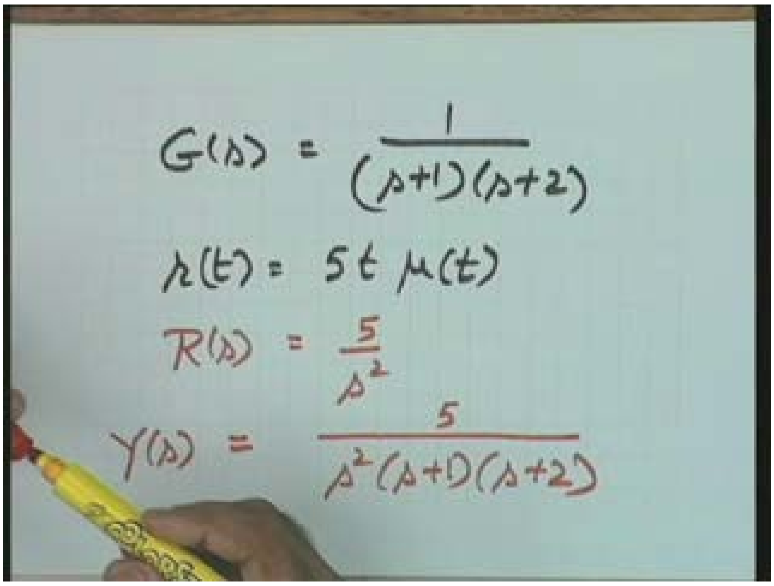

Now I want to invert this. The same procedure. Now let me retain this here. And I want to get the partial fraction expansion for this. This Y(s) (Refer Slide Time: 12:10) is equal to….. I need your help please what is the term corresponding to s squared; what is the residue? As you know in this particular case the residue will be substitute s is equal to 0 it is going to be 5 by 2. The other terms are going to be plus let me take some constant k over s, this k I am going to determine but let me proceed further the next I take is that s plus 1 so let me substitute s is equal to minus 1 s is equal to minus 1 gives me 1 here, the s is equal to minus 1 gives me 1 so it is going to be plus 5 over s plus 1. And the last term please; s is equal to minus 2 4 and a minus sign here so it is minus 5 by 4 over s plus 2.

Now look at this residue k; I hope you will recall this. What is k? k will be equal to d by ds of 5 over s plus 1 into s plus 2 at s equal to 0. This is because of the repeated pole. The pole at s is equal to 0 is repeated, the multiplicity is 2 and therefore it will give rise to two terms in the partial fraction expansion: one; s squared term coming in the denominator, other s term coming in the denominator and s squared is directly given by putting s is equal to 0 in the term reminder term when s squared is removed while s is given by this expression.

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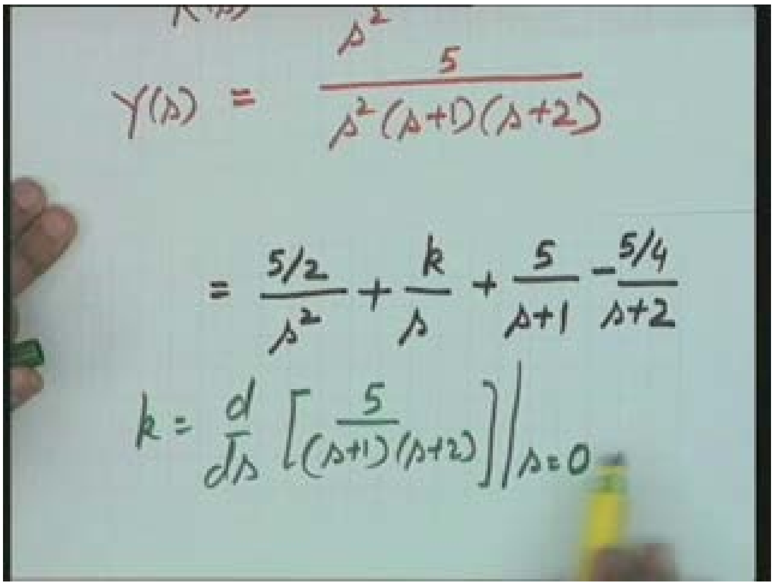

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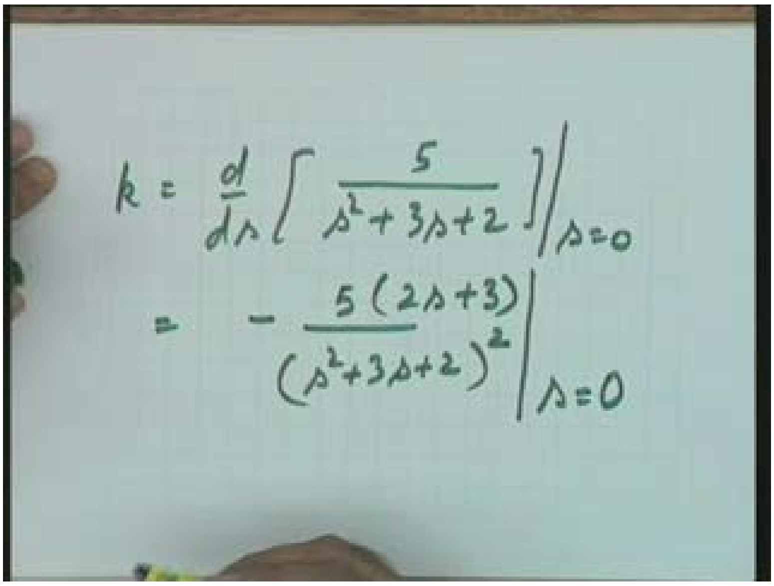

Now look at the value of k. In this case now k equal to d by ds of 5 divided by s squared plus 3s plus 2 where s is equal to 0. This is equal to yes please minus take the derivative of this 5 over s squared plus 3s plus 2 squared over here; what else? The term here will be 2s plus 3 (Refer Slide Time: 14:40). I hope this is okay. Please do help me if there is an error.

Now if I substitute s is equal to 0 here I get this s equal to minus 15 by 4 minus 15 by 4. So this term (Refer Slide Time: 15:08) now is replaced by minus 15 by 4 and therefore I can rewrite the total expression; Y(s) equal to 5 by 2 s squared minus 15 by 4 by s plus 5 by s plus 1 minus 5 by 4 divided by s plus 2. This becomes the partial fraction expansion for this case when the system is excited by a ramp input.

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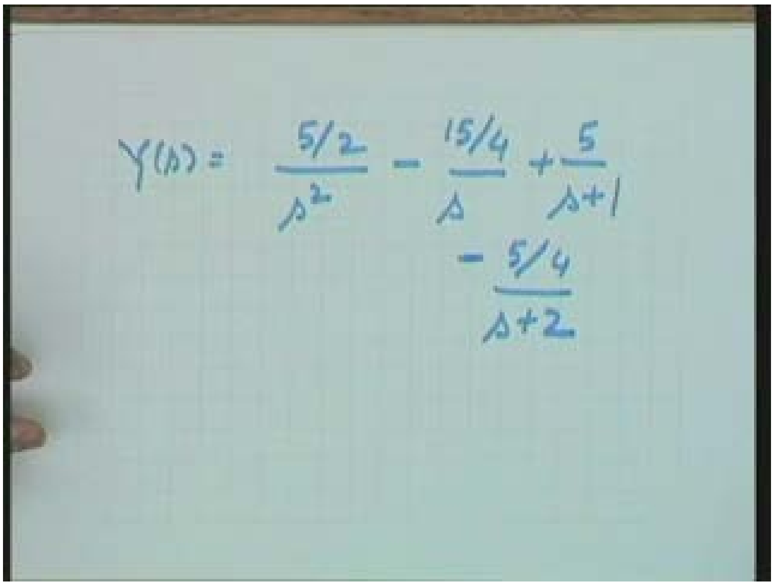

Now let us see the behavior of the plant when the plant has been excited by a ramp input; and the behavior will come out easily when I take the inverse transform of this (Refer Slide Time: 15:53). Inverse transform will be given by 5 by 2 t minus 15 by 4 plus 5 e to the power of minus t minus 5 by 4 e to the power of minus 2t. Look at these expressions now. You will find that these are the terms which have been contributed by the system poles and these terms reduce to 0 as t tends to infinity. This is the transient component of the total response. Look at the other component (Refer Slide Time: 16:34) this component is due to the reaction of the system to the input the input in this particular case being ramp. So this also is of the similar nature as you see and therefore I can write over here; y steady state the steady state is equal to 5 by 2 t minus 15 by 4 this is the steady state response of the system.

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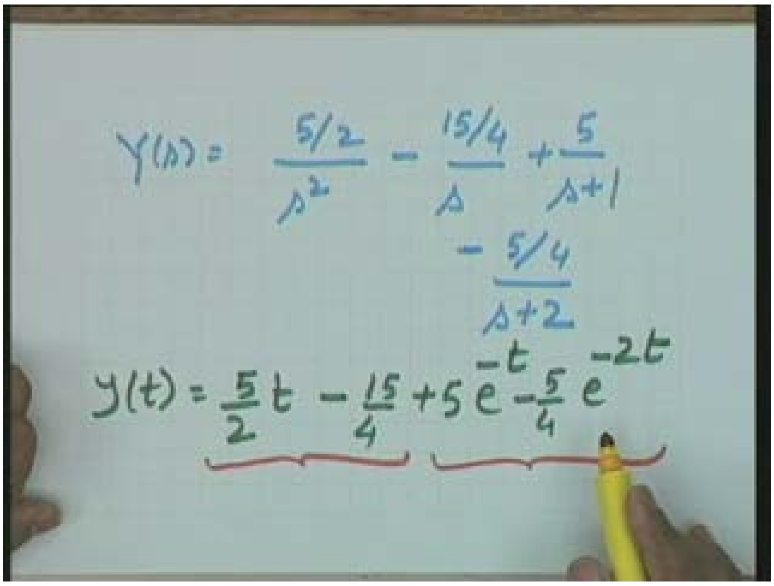

Look at the nature of the response; it looks like this: this is your value 5 t (Refer Slide Time: 17:10) that is the step the ramp input and the response will be something of this nature so I can put it this way that this is 5t and this is 5 by 2 t minus 15 by 4.

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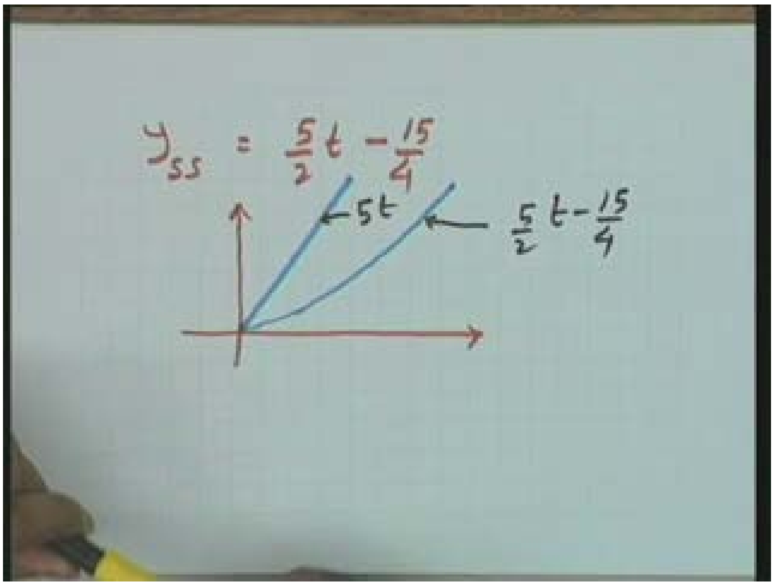

I told you to be a little cautious when applying the steady the final value theorem. Look at this particular case: what is y ss if I apply the final value theorem to this, limit s tends to 0 sY(s) equal to limit s tends to 0 it is s over s 5s over s squared (s plus 1) into (s plus 2).

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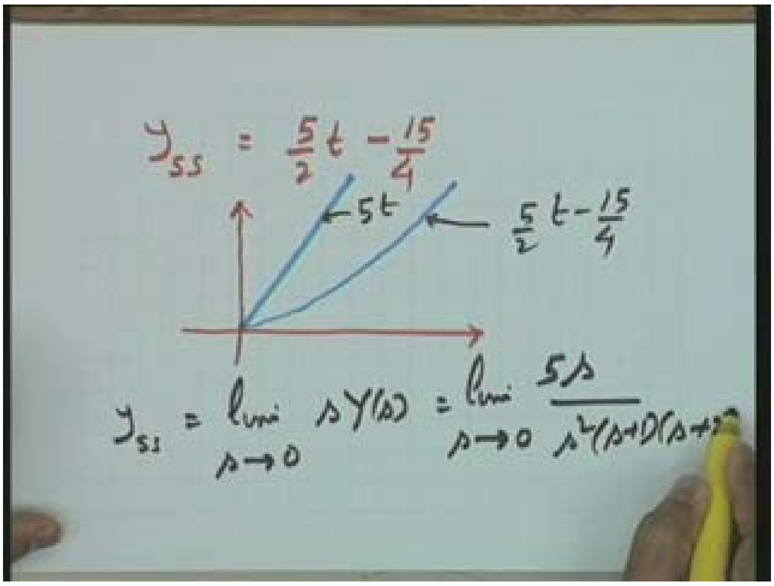

Sir, sir…..
[Conversation between Student and Professor – Not audible ((00:18:04 min))]
This has to be the starting…….
[Conversation between Student and Professor – Not audible ((00:18:10 min))]
Sorry, I am sorry, you are very right. Actually I have to plot the total transient. This graph is right, I'll I like to rectify this please, you are very right.
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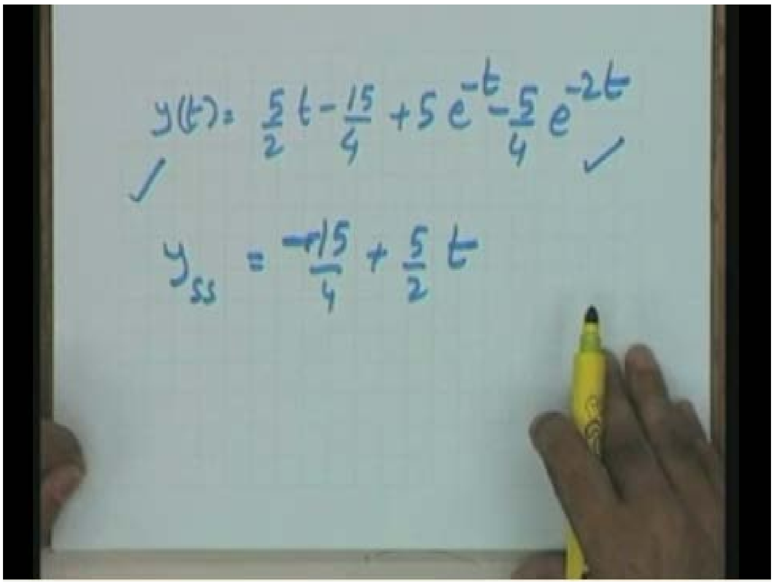

Y(t) is equal to 5 by 2 t minus 15 by 4, yes, let me look at the total response please plus 5 e to the power of minus t minus 5 by 4 e to the power of minus 2t. Please see, this is the graph, when t is equal to 0 you your what is the value; the value in this particular case at t is equal to zero please help me will turn out to be 0, check this. So it means the graph which I have plotted is the graph of this Y(t). y ss component is equal to minus 15 by 4 plus 5 by 2 t please.

This is your y ss component the steady state component as t tends to infinity. The graph I plotted is that of the total response Y(t) which is equal to this particular expression. This can be checked please.

Now I was referring to the final value theorem. So, coming to this expression, please see, your steady state given by this. Coming to this expression if I apply the final value theorem y ss is equal to this (Refer Slide Time: 19:37) now please see if you don't take care of the condition on the poles of sY(s) the final value turns out to be infinity. But however, what is sY(s)? sY(s) has a pole at the origin and therefore in this particular case final value theorem is not applicable; you have to be very careful about this. The final value theorem gives the result as infinity while the basic time response analysis shows you that the steady state response of the system is given by this expression: minus 15 by 4 plus 5 by 2 5 by 2 t.

The steady state the steady state on the time axis starts as soon as the transients die out. So this is the expression and this expression naturally, as t tends to infinity goes to infinity, so the two values will match at t tends to infinity but the total steady state expression is not given out by the final value theorem in this particular case because sY(s) has a pole at the origin.

Now, after giving you the dynamic response review, I mean more examples need not be given taking the Laplace transform of input on the plant, inverting that to get the response was nothing new to you. But I hope the review of the dynamic response analysis which has been given will be useful and I am going to use this particular dynamic response analysis review in setting up or in giving you the characteristics of first-order and second-order models.

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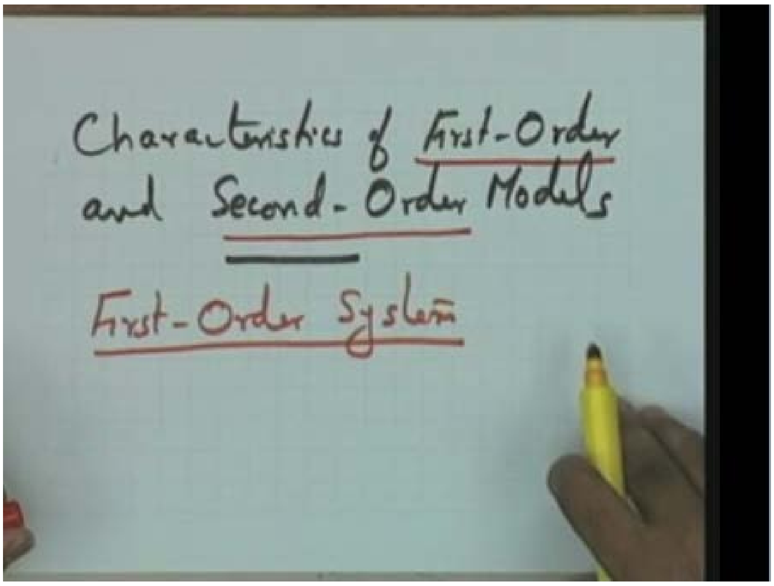

You may say that why out of nth-order model where n can vary from zero value to any number first and second-order models have been taken out for a specific study. The reason being, the systems we are going to come across, you will find, can appropriately be modeled in terms of first-order and second-order terms. They play a very important role in the modeling and in the design exercise of control systems. And your specific attention is needed to certain parameters which define the basic characteristics of first-order and second-order models.

I take first a simple first-order system. Well, an inertial load with moment of inertia J this I take as a rigid shaft. by rigid shaft I mean to say is that the spring constant for this particular shaft is zero it is not flexible and the friction environment of the system let me schematically represent it like this: (Refer Slide Time: 23:12) the friction environment and the viscous friction coefficient I take as B. Let me say the attribute of interest to me is omega the speed and the input is the torque T applied to this particular shaft. This is the system environment and as you will see this can be appropriately modeled by a first-order transfer function model. I hope the situation is very clear.

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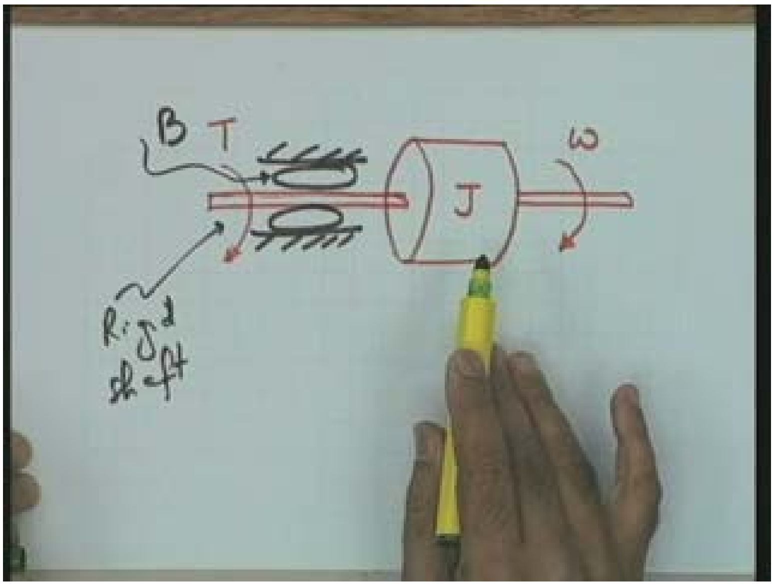

The two parameters of the of the system are J and B, the input variable is t the torque applied and the output variable of interest to me is omega the speed, omega(t), T(t) these are the functions of time, J and B are constant parameters of the system. I hope you don't need any explanation to the to this particular equation. I can write the equation as J omega dot J into acceleration plus B omega B into velocity is equal to T the torque applied. This is the basic differential equation model for this particular system. The Laplace transformation gives me Js omega(s) plus B omega(s) equal to T(s).

You will note, without specifically mentioning it again and again the system under consideration will be considered as a relaxed system that is why the initial conditions do not appear in this transformed equation. The transfer function model of this system becomes G(s) is equal to the output omega(s) divided by the input T(s) equal to, yes, 1 over Js plus B which obviously is a first-order model.

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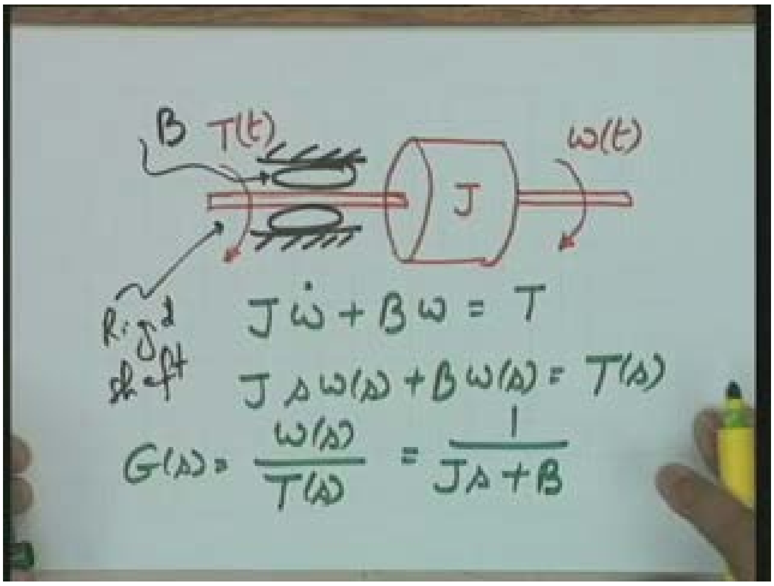

Let me repeat this expression; G(s) equal to omega(s) over T(s) equal to 1 over Js plus B. Now, in this particular case the parameters of the system are J and B. The characteristics of a first-order system, any first-order system be it mechanical, thermal, electrical, liquid level system or any other system you see they come out in this particular form very nicely that is a general model I am going to take as K over (tau s plus 1) this is also a first-order expression but written in a different way where the parameters now are K and tau. The K is referred to as, it will become to clear as to why this name is given, this K is referred to the system gain and the tau parameter is the time constant of the system time constant of the system.

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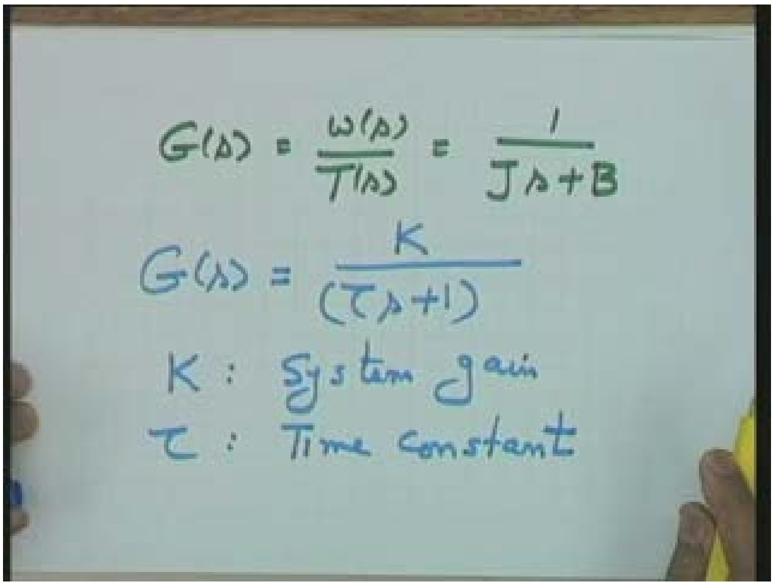

Now, see this particular situation, the input T(s) is equal to 1 by s. I give a unit step input to this system under consideration. In that particular case the response omega(s) is equal to K over s into (tau s plus 1) and the inversion of this gives me omega(t) equal to, I think now I can directly write; this could easily be verified that this is going to be the expression as for the time response is concerned.

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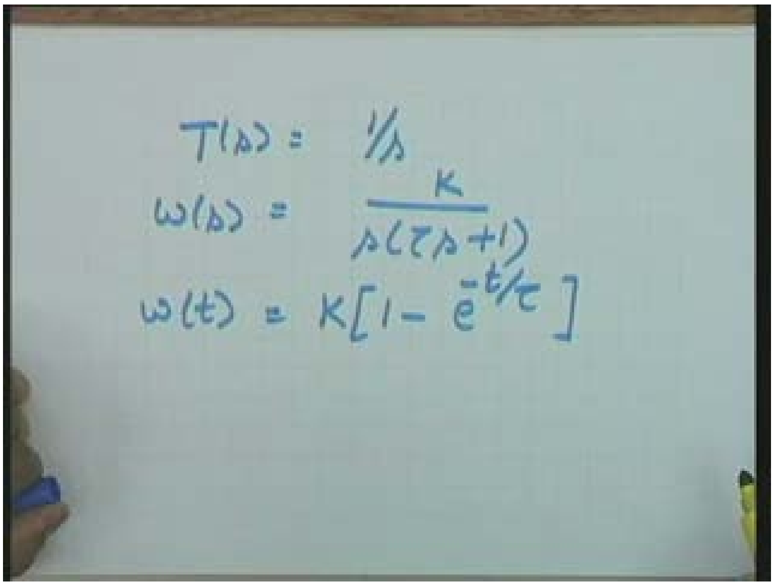

Look at the system gain please. K is a parameter, as t tends to infinity the speed omega becomes K. So it means the speed of the system changes to a value K in response to a unit input unit step input and that is why the name system gain is appropriate for this particular parameter K in the general first order model. It changes the output of the system by K. This is the reason probably, is the most appropriate reason to say that why the word system gain is used for this particular parameter.

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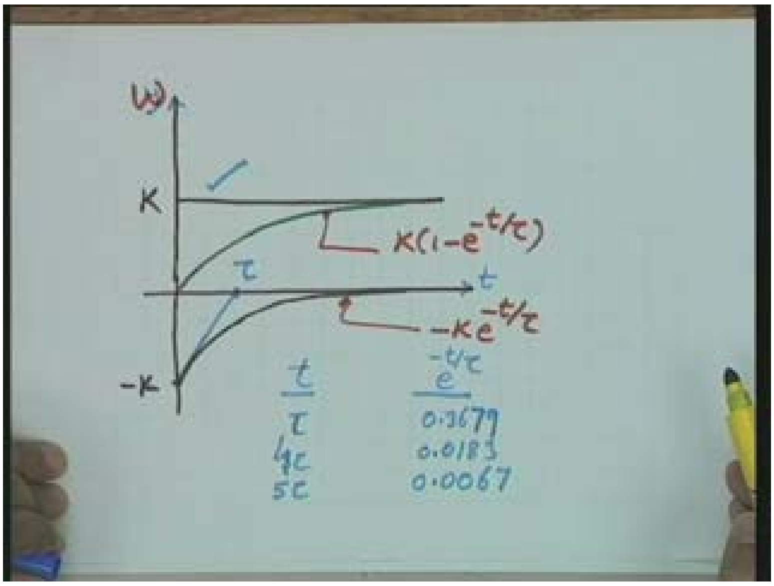

Now let us look at the time tau that is the time constant tau. For that I like to make a sketch of this particular response. If I make a sketch of this response, this is omega and not y please (Refer Slide Time: 28:17) t versus omega in that particular case you see that the final value is k given over here and this is the transient component minus ke to the power of minus t by tau which goes 0 as t tends to infinity and the total response is given by the this green curve k(1 minus e to the power of minus t by tau).

Now look at the decay of this transient term, look at the decay please. This particular transient term minus k e to the power of minus t by tau; if I take the derivative of this at t is equal to 0 you can easily ascertain, please see that this k by tau. So it means this is the slope of this particular transient term at t is equal to 0 initial slope. So it means this particular term starts decaying with an initial slope of k by tau if the slope would have been constant if the decayed slope at t is equal to 0 would have continued this particular term would have come to 0 at time t is equal to tau and is referred to as the time constant of this system. This is a very important parameter as I am going to explain to you. The time constant of the system represents the time for the transient to decay if the decay were dictated by the initial slope k by tau.

Now look at the actual value. Now the slope is not constant it is changing. Look at the actual value. At t is equal to tau the actual value is 0.3679.

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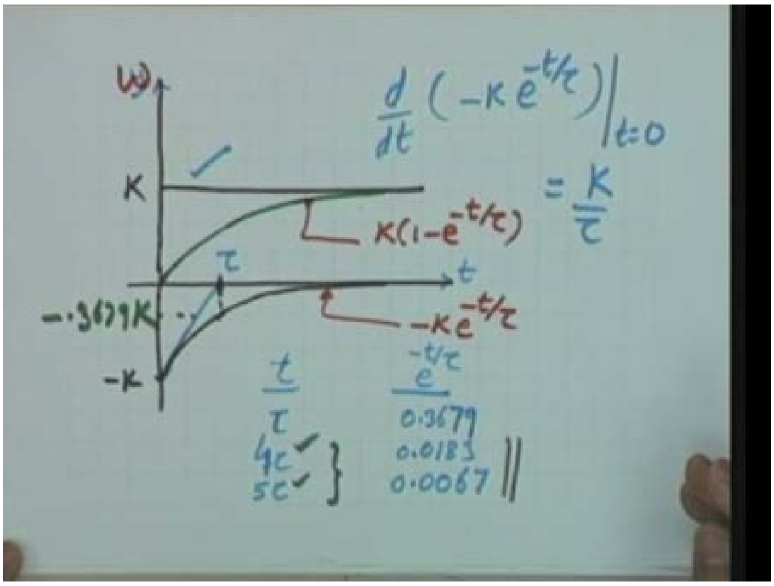

In that particular case I can say that the actual value at this particular point is minus 0.3679 k this is the actual value. Similarly, if I proceed and calculate at 4tau it is 0.0183, at 5tau it is 0.0067. So you can say that in this particular situation in a time roughly between 4 to 5tau where tau is a characteristic parameter of the system the transient of the system has almost decayed to 0, because you see, mathematically if you see the decay will come as t tends to infinity but we have to take up to the practical value. The practical value in this particular case 0.0067 can be taken as approximately equal to 0 and therefore in a time roughly equal to 4 to 5 time constants the transient of the system has decayed or equivalently I can make a statement this system has settled down to the final steady state. After all this is the total time response, this is the total curve of the system (Refer Slide time: 31:31). The system has settled down to the steady state in 4 to 5 time constants.

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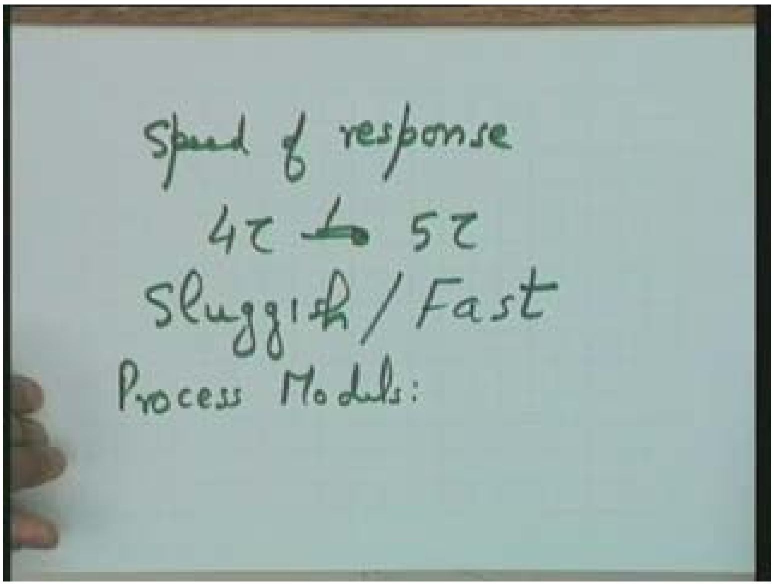

What is the time constant parameter?
Can I say that, in terms of control system terminology, the speed of response is a very important characteristic of the system, speed of response? When an input comes on a plant we will like that the plant immediately responds. Theoretically speaking I want the plant to respond instantaneously but it is not possible; the instantaneous response is not possible because of the lag components in the plant. Now you see that this particular response should be as fast as possible. Now, if I interpret this in terms of time constant of the system since the response decays in 4 to 5tau so it means the faster response will be obtained when the time constant of the system is low. So it means, larger the time constant of the system I can say sluggish is the response. These terms I will be using quiet often because these first-order factors will be appearing quiet often in our exercise on analysis and design.

So what is a sluggish system? A sluggish system will have a larger time constant. The process models let me make a mention over here, the process models, by process I have defined these terms: control of temperature, liquid level, pressure and the like the chemical composition these are the process control applications. These are typically characterized by large time constants that are these are sluggish systems. Take for example, the examples we have taken of speed control or radar tracking system these are characterized by smaller time constants and hence are fast systems.

Please see that the time constant of a typical system may vary from milliseconds to a few minutes. The example of milliseconds being that of the speed control system or a tracking system, the radar tracking system, servo mechanism for steering of an antenna and that of minutes could be the temperature control and the like. You see that this is an important characteristic parameter of this system the time constant and 4 to 5 time constants gives you the total time for the system to settle down to the steady state environment.

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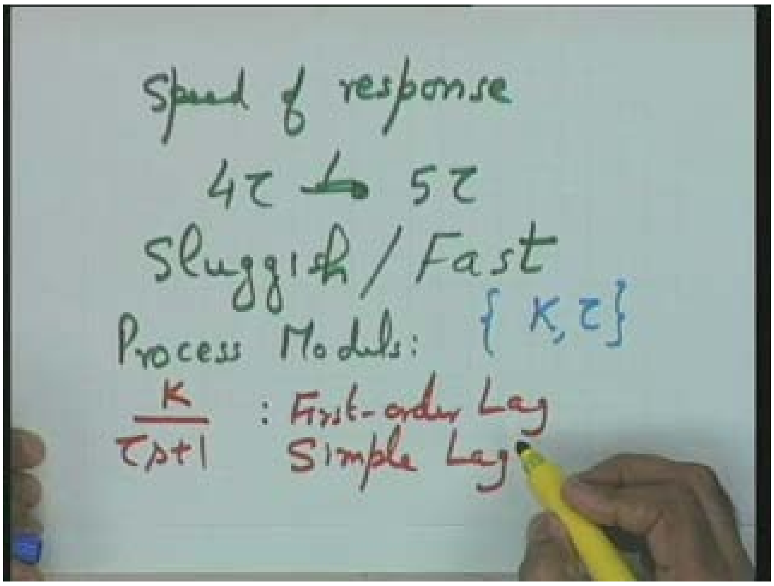

One more definition I like to give over here which I will be using the terms of the type k over taus plus 1. I will be using the terms for this first-order lag. Now, the word lag becomes clear why the word lag is coming, because it is not aligned for allowing the system to respond instantaneously. This is a first order lag or sometimes in the control system jargon simple lag is also used for terms of the form k over taus plus 1. So can I use the word this first order lag or a simple lag is completely characterized by these two parameters k and tau and therefore the personality of a first order system is completely characterized by the two parameters k and tau; k being the system gain and tau being the time constant of the system.

Similarly, I will like to take up a second order system and find out the minimum information which describes or which characterizes the total personality of a second-order system. For you to focus attention on the discussion on second-order system I like to make a mention that all the higher order systems may be fourth order, fifth order or even tenth order, as you will see later in our total design exercise, are appropriately approximated by a second-order system.

I need you a tension on second-order system because the second-order system is going to play crucial role in our total design exercise. What we are going to do later, I will I am going to convince you, not that just we are going to take a second order system as the approximation for any higher order system. I am going to convince you that in most of the situations we come across in practical situations, in practical systems the higher order system of any order even up to tenth order can be appropriately approximated by a second-order and hence our design can be carried out using a second order approximation and therefore we should concentrate on the parameters which describe the personality of a second-order system.

And I like to give an example of a second-order system as well and the example I give you is that again of a load with moment of inertia j. But in this particular case I make the shaft flexible. In this particular case the shaft is flexible. The torque T is the input, let me make it a function time and let me write it here specifically that it is a flexible shaft and let me take the spring constant for this, the parameters which describes this attribute of the shaft is k. As I did earlier the frictional environment is schematically represented like this and let me say that B is the viscous friction parameter.

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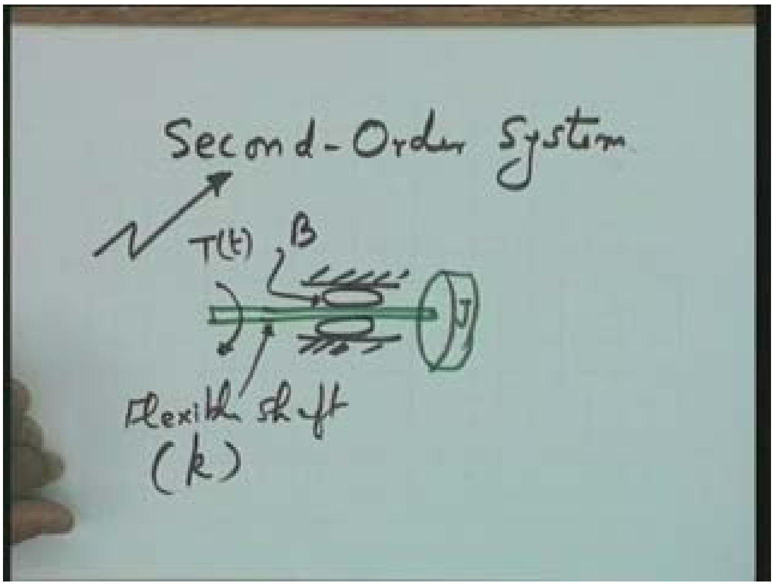

And in this example, now, I take the position theta as the attribute I am interested in, theta that is the position, the twist of the particular shaft is the attribute I am interested in so this becomes my output variable. So, input variable here, output variable and the system parameters which are the constant parameter are J, B and (k). You will note a change here please. I have specifically taken a small k here and not a capital K because I have reserved capital K for the system gain parameter. So this point may please be noted in your notes as well. This is a small (k) I have taken to distinguish it from capital K which I have reserved for use, as a system parameter as a system gain parameter in the general transfer function model.

So let me write the differential equation model for this particular expression. Help me if this is okay?
J theta double dot, double dot is your acceleration plus B theta dot velocity here plus k theta the displacement is equal to T the input. This becomes a differential equation model for this particular system. This is a second-order differential equation so it is a second-order system.

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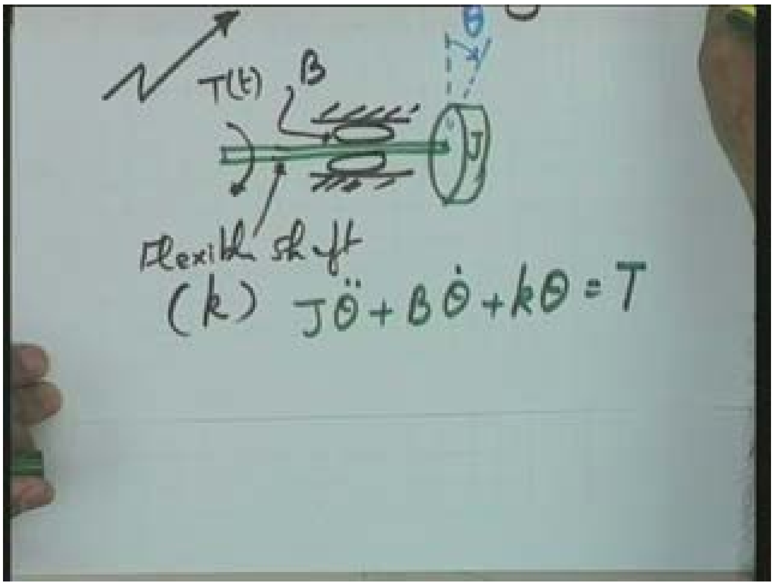

In the transfer function form as you will see a second-order characteristic polynomial will appear. G(s) is the transfer function of the system is now given by the output variable theta(s) divided by the input variable T(s) is equal to one over Js square plus Bs plus k this becomes the transfer function model of the system.

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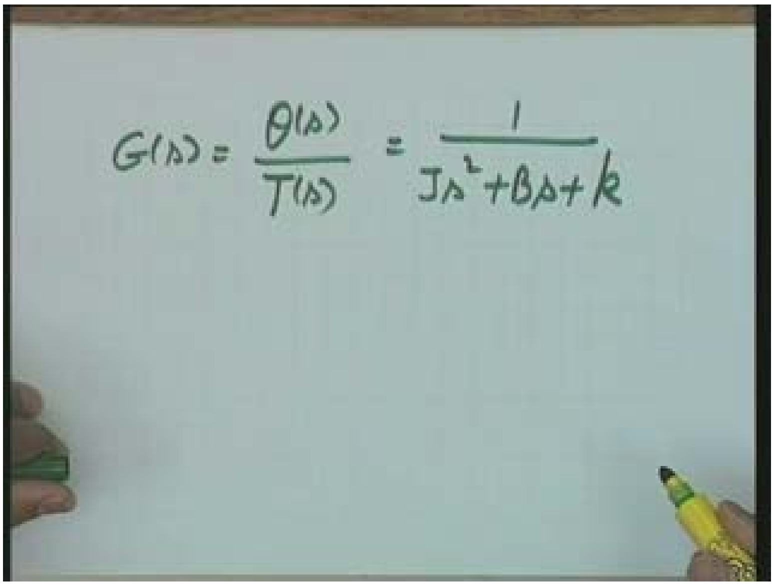

The output variable is displacement and the input variable is T and this of course as you is a second-order model with J, B and k as the three parameters of the systems. Again J, B and k are the specific parameters of the system under consideration. I am interested in general parameters so that those parameters have physical meaning and they become applicable to any system we come across, and the general parameters. You already have the information about those parameters as I said it is a review sort of thing to put it in the control system setting; the general parameters as you know are omega n the natural frequency, zeta the damping ratio and k the system gain. So it means this particular expression (Refer Slide

Time: 40:52) can be written in this form now and this is a second-order standard model. With these as the parameters specifically in terms of the parameters of the system under consideration the values of k the system gain, the value of omega n the un-damped natural frequency, the value of zeta the damping ratio can easily be obtained as given over here. You can easily check this. These are the three parameters.

Now I want to look at the physical meaning in terms of a control system. What is the physical meaning of the terms k, omega n and zeta. And to get the physical meaning I have processed the same way that is this second-order model will be excited by a step input. I will the study the response of this particular system to a step input and I will see how the system behaves or how the response behaves for different values of k, omega n and zeta.

The k has the same meaning. this you can check by the final value theorem as well. You see that for a step input please help me you can immediately check, for a step unit step input the final value of response is given by k, just immediately apply the final value theorem and check please. So it means this is the gain of the system. So I have to concentrate on the parameters zeta and omega n because my claim is this that k, zeta and omega n these three parameters describe the personality of a second-order system in general.

[Conversation between Student and Professor – Not audible ((00:42:45 min))]
Your question please, can you speak it again?
This is the final value theorem always applicable to this form. No, that it is for you to examine. I thought you have examined it straightaway whether it is applicable are not. Let us see.

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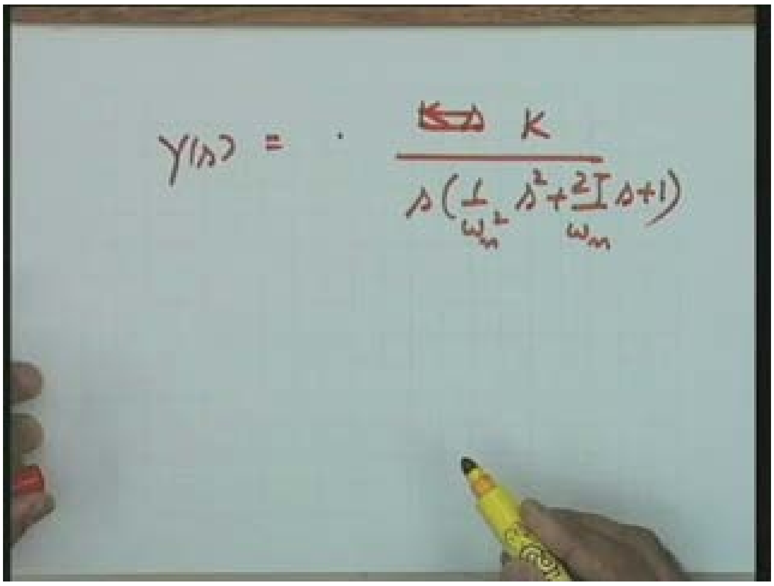

So in this particular case Y(s) for a unit step input is 1 over or k into sorry, I am sorry, this is k over s into 1 over omega n squared s squared plus 2 zeta omega n s plus 1. It is for you to test, I have given you the general condition of applicability sY(s) becomes equal to……. as you see in this case, k over 1 by omega n squared s squared plus 2 zeta by omega n s plus 1. So in this particular case as you see, as s tends to zero no sorry the poles of this should be there in the right half plane this is the only condition of getting the value of k as the final value or steady state value. So if I assume that the values of omega n and zeta are such that the roots of this equation are in the left half of the s plane the final value theorem is applicable and limit s tends to 0 sY(s) becomes equal to k that system gain under or the system under consideration.

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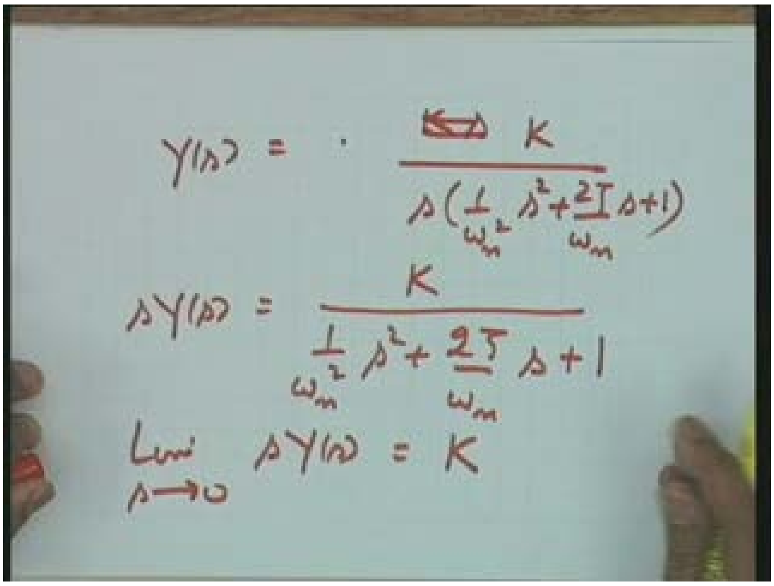

So, as I said that, to study the parameters zeta and omega n I like to take the response of the system. And the response now, having given you a couple of examples, really I will like to take the complete response of the system and I have too much of repetition. Since you already know I like to put it in the final expressions only: theta(s) over T(s) is equal to k omega n squared over s squared plus 2 zeta omega n s plus omega n squared is the expression. Remove T(s) from here and have the s value here. This becomes your transform response transform theta(s). What you have to do is you have to take theta(t) is equal to Laplace inverse of this theta(s).

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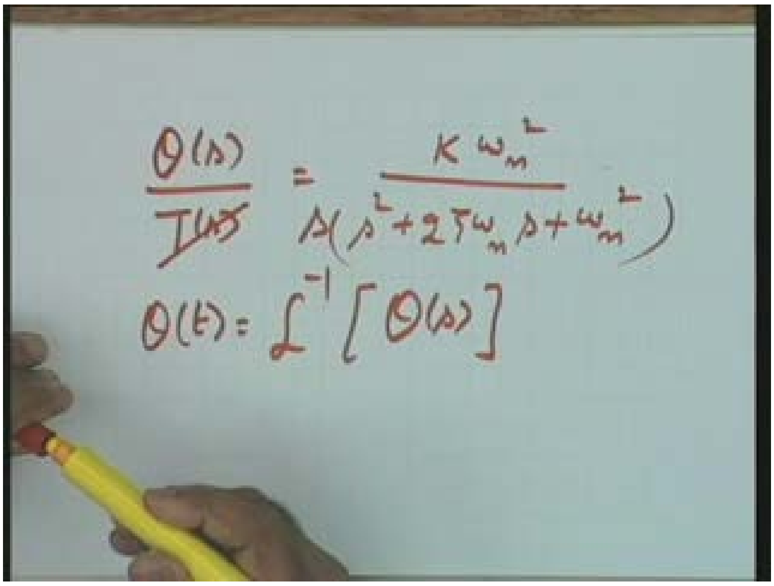

I will be writing the expression directly. I have taken it my notes but you can defiantly for with different values of omega n and zeta you can take the inverse Laplace transform and verify the expressions of theta(t) which I am going to write. I will take up four cases: Case 1 will correspond to zeta equal to 0. You see that zeta equal to 0 is a case corresponding to B is equal to 0 that is no damping under damped case. The second case I will take zeta is greater than 0 less than 1, the third case I am going to take zeta is equal to 1 and the fourth case will be zeta greater than 1 and all these cases correspond to damped case that is B is not equal to 0.

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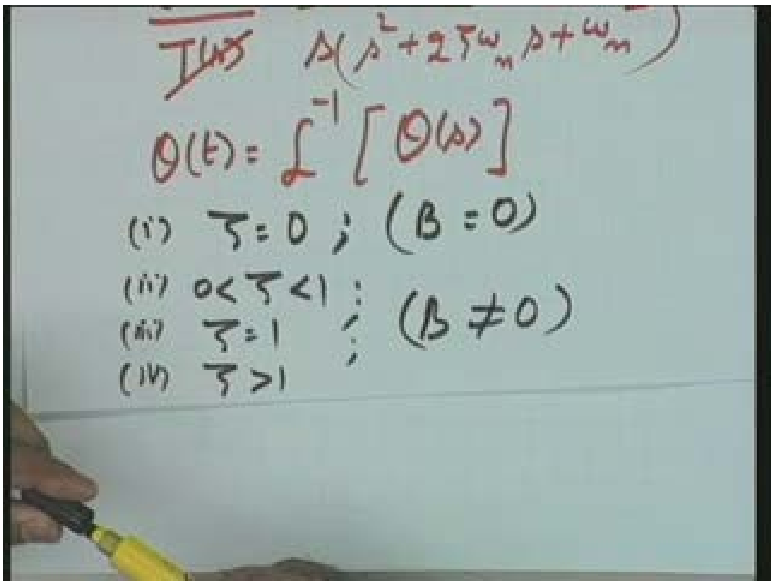

These are the four cases I will take and we will see as to how the response behaves and how it characterizes the behavior of a control system or a system under consideration.

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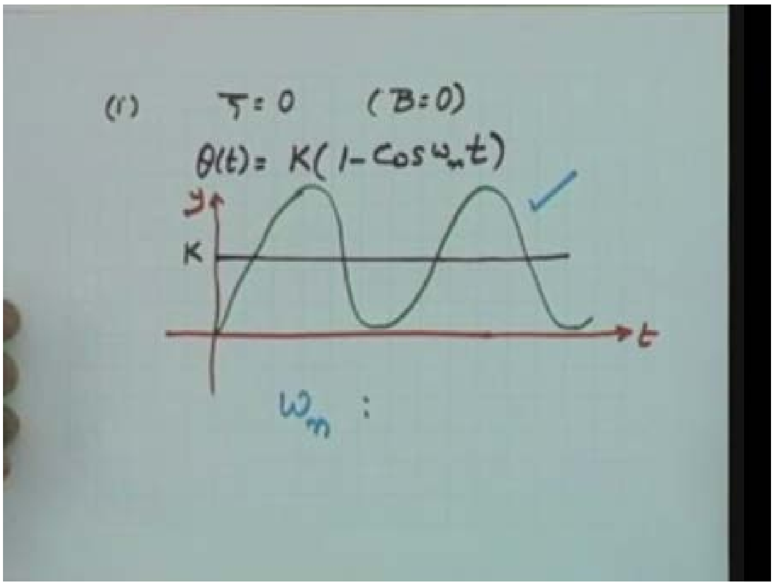

Case 1 I am taking and I have given you the expression for theta t directly here which can be obtained by inverting theta(s) corresponding to the response transform given to you earlier. So zeta is equal to 1 is the case and theta(t) is equal to K(1 minus cos omega n t). And as you see it is an oscillatory response oscillating around the value k. Since these are oscillations when damping is equal to 0 you can say that these are un-damped oscillations and the name omega n is given as the un-damped natural frequency because this is the oscillating frequency when damping B is equal to 0 or zeta is equal to 0. So this is the behavior of the system an oscillatory behavior when the damping is the 0 and naturally in a control system normally you will not like to go for this behavior. When you take up all the four cases, please keep in mind, you should relate to the acceptable behavior in an industrial environment that is how a system a control system should behave. Naturally in most of the cases this type of oscillating behavior will not be acceptable.

Can you recall an example in which I made a mention that this type of oscillating behavior may be acceptable?
If you recall, that is the example of residential heating. With the on/off control system in that residential heating system you see, the output of the system was oscillating output, oscillating within certain temperature band. So the acceptance of this oscillating behavior is dependent on your requirement of the system. But in the most of the situations industrial situations such a behavior is not acceptable.

Let me now take up the under-damped case. The under-damped case of zeta is greater than 0 less than 1. So this case corresponds to this restriction on the system parameters where can easily be verified and this is the corresponding response, this also you can easily obtain.

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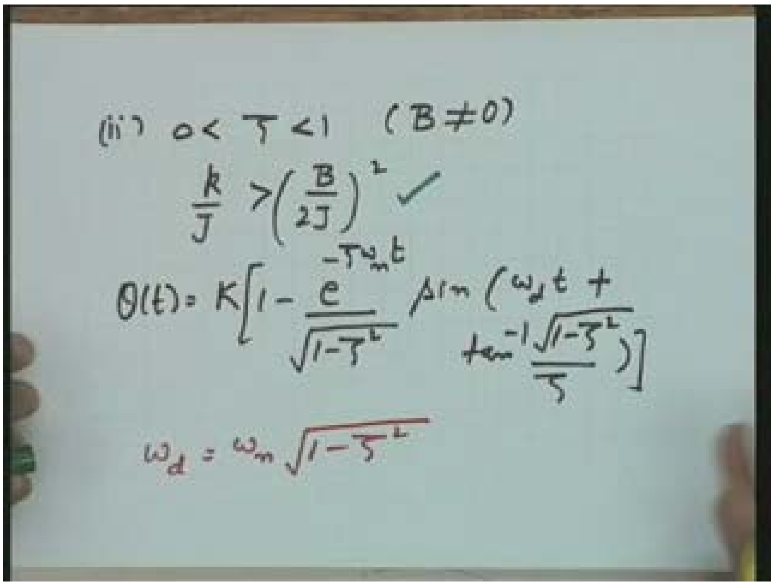

Now, in this omega d, is the term used over here and this omega d in terms of omega n and zeta is given by this expression (Refer Slide Time: 49:06) this omega d is referred to as damped natural frequency. The name will become, the reason for the this name will become obvious when you look at the response curves; the damped natural frequency is omega d.

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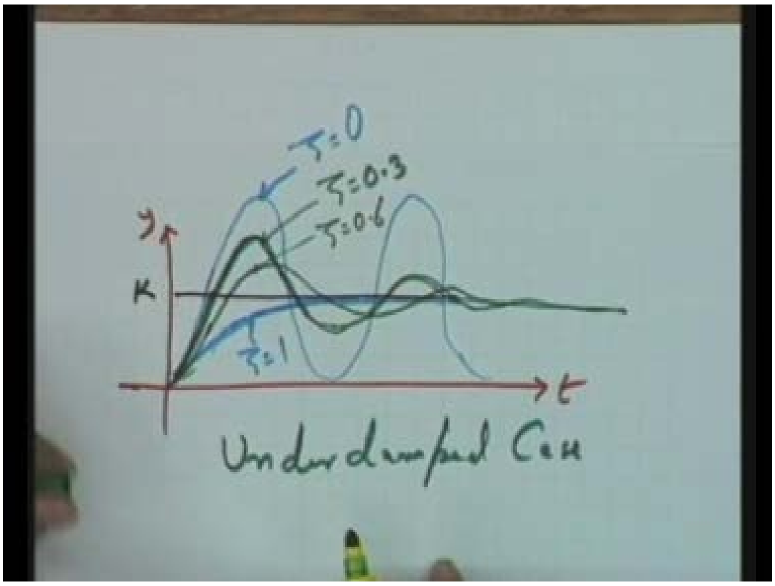

Look at the response curves; the green curves in this expression please. The green curves correspond to zeta greater than 0 less than 1. As you find over here that, this particular expression now, is again an oscillating behavior but the oscillations are damped. The oscillations in this particular case are damped, please see (Refer Slide Time: 49:53). If you increase the value of zeta let me take a typical value of zeta over here; typical value of zeta is equal to 0.3 let us say. So, if you increase the value of zeta equal to let us say 0.6 what will happen? You find that the change in the damped oscillations and the change in the peak overshoot which is appearing over here. If you further increase the value of zeta you find you get this particular curve (Refer Slide Time: 50:23) which is the curve corresponding to zeta equal to 1. So zeta is equal to 1 corresponds to the limiting situation where the damp where the oscillations have just died out. It is limiting situation. So what will happen, just for a value zeta little less than 1, for a value zeta little less than 1 there will be oscillation. So this is a limiting case.

On the other hand, the limiting case is given by zeta is equal to 0 where the oscillations are un-damped. So you find that the green curves correspond to the situation zeta greater than to the 0 less than 1 this so called under-damped case; under-damped I like to put, under-damped case. And most of the situations in control systems belong to this particular sub-class. That is, this is the requirement normally you impose on this control system. You can see at the juncture as to why you impose this requirement on the control system. You will find that as your green curve approaches this zee and blue curve and or further if you increase the value of the zeta the settling time or the speed of response of the system becomes poor. That is the system becomes sluggish as the value of zeta increases. But if the value of zeta decreases, that is, if you take the value zeta close to 0 what will happen, the system becomes too oscillatory.

So you please see that the situation the sandwich situation between zeta is equal to 0 and zeta is equal to 1 taking the response closer to zeta is equal to 0 gives you an oscillatory response too oscillatory too much oscillatory which may not be expectable to you. Taking zeta towards 1 gives you a sluggish response which again may not be acceptable to you. So, in most of the practical situations you impose a requirement on the system the zeta requirement to be between 0 and 1, well, specific situations could be zeta is equal to 1 as well.

Take for example a robot control system please. If a robot has picked up certain payload and it has to place the payload at some other location, probably you will not like that the system should oscillate. Take for example, the Maruthi car example, a robot really does the welding there. It is doing the welding; now you will not like the welding torch to oscillate. So it means, in that particular case for that particular situation, even at the cost of speed of response you may like to go for zeta is equal to 1 because the oscillations cannot be tolerated in this type of situation. So you see for a specific situations larger value of zeta could also be accepted but normally it is between zeta greater than 0 less than 1 that is mostly your industrial controlled systems are under-damped systems.

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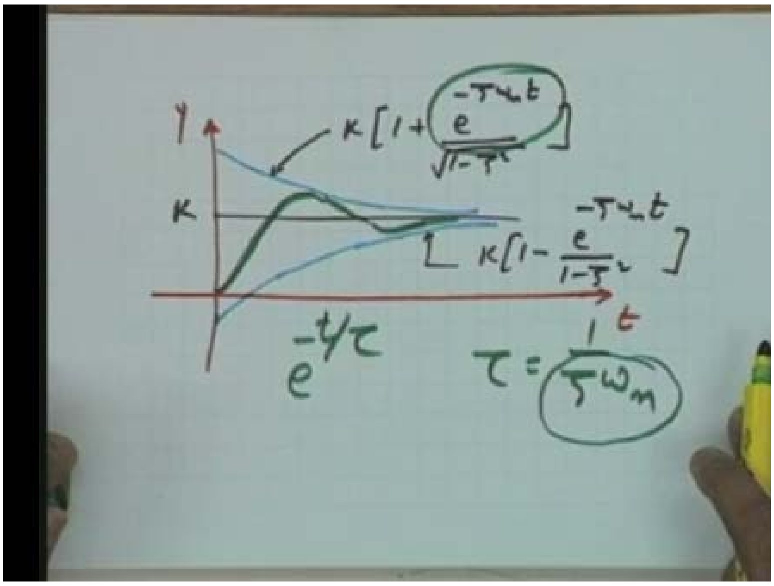

Now look at this place. This (Refer Slide Time: 53:55) is a typical under-damped to response I have taken over here and this is the envelop of this particular response the blue curve is the envelop of this response. So you see that this can easily be ascertained, this can easily be examined these are the two envelop expressions. So it means the decay of this response depends upon the decay of the envelop and this particular envelop decay is guided by this particular exponential factor. The time constant of this factor can you tell me please? What is the time constant of this factor? The time constant is given by the expressions of the type: e to the power of minus t by tau so you find that tau is equal to 1 over zeta omega n. So the time constant of the envelop is 1 over zeta omega n and therefore the decay of the envelop is guided by this particular product zeta into omega n. the larger the value of the product zeta into omega n smaller is the value of the time constant and hence faster is the is the response. So, one more slide I will quickly like to give you over here.

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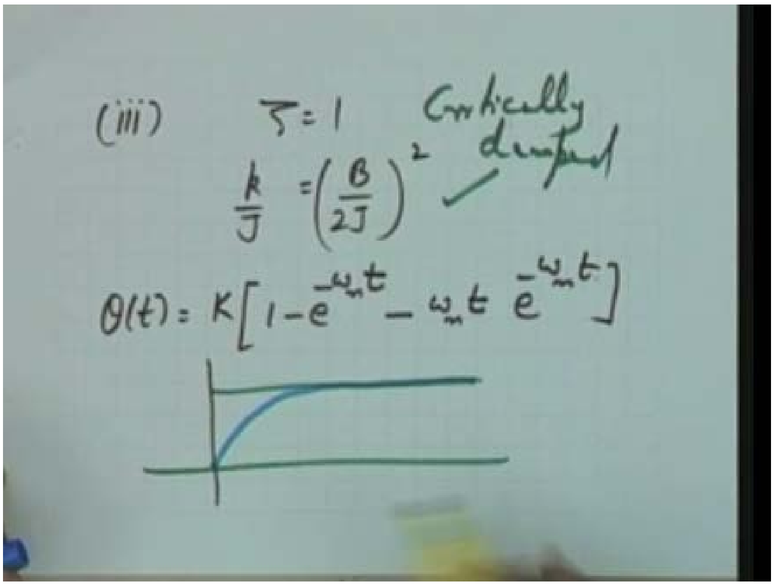

The third-case is zeta is equal to 1 so this corresponds to this k by J expression, this is the theta(t) expression as you find that the under-damped, the this is a critically damped case, critically damped, and in this particular case your response is just oscillatory, just the oscillations I have have been killed you see, over-damped case is not accepted. So it means our control system's performance specifications mostly demands the under-damped case zeta greater than less than 1 greater than 0 on the border line situations zeta equal to 0 or zeta equal to 1 may be accepted. Thank you very much.