Control Engineering
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Lecture - 4
Dynamic Systems and Dynamic Response

Under discussion is the topic of modeling. Various types of models which are used in control system analysis and design are under discussion. As I told you many of these models are already known to you through your earlier knowledge. However, a quick review is necessary or is useful in terms of setting the symbols and the terminology for the course. Last time I introduced the state variable model to you a quick review will be in order here. I said that the dynamical variable or the energy variables of the system can be defined in terms of state variables which give you the energy state of the system.

If, for a typical system I have state variables as x 1 x 2 to x n then the relationship which can be obtained from the differential equation model of the system can be rearranged in the following form: x 1 dot the time rate of change of the first variable x 1 is equal to a 11 x 1 plus a 12 x 2 plus a 1n x n plus b 1 r where x 1 x 2 x n are the state variables, r is the input variable and all others are the constants of the system the coefficients of a and b matrices. Similarly, x 2 dot is equal to a 21 x 1 plus a 22 x 2 plus a 2n x n plus b 2 r is second equation in the set of n equations. And I get x n dot identically a n1 x 1 plus a n2 x 2 plus a nn x n plus b n r and the output equation as we have discussed last time output is an attribute of the system you are interested in and it is in an algebraic read out function, it can be obtained directly from the state variable set of the system.

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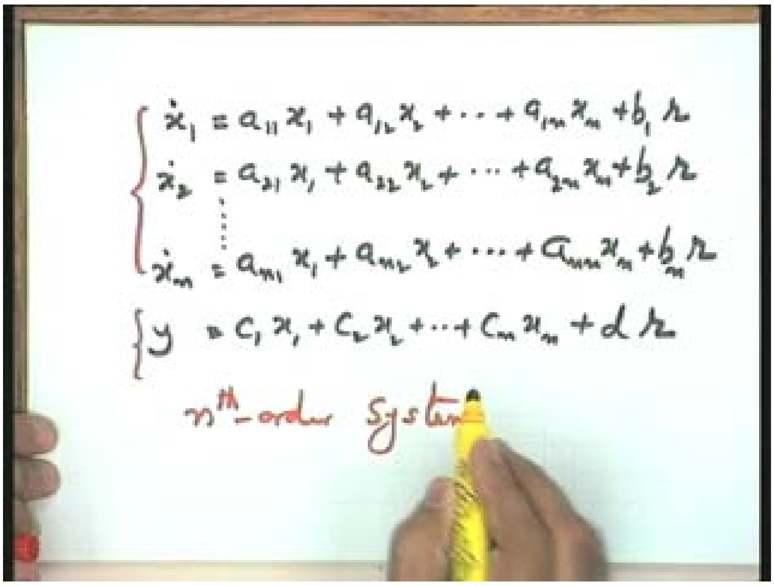

So output equation can be written as; y is equal to c 1 x 1 plus c 2 x 2 plus c n x n plus let me say d into r the output may be directly affected by the input. So these are the state equations here and (Refer Slide Time: 3:33) this is an output equation. Let me put it this way. A set of n state equations and a single output equation if single input single output systems are under consideration. So these n state equations represent an nth order system. So it means order of a system is directly linked to the number of state variables which represent the energy state of the system. So, in this particular case, under consideration, is an nth order system nth order single input single output system.

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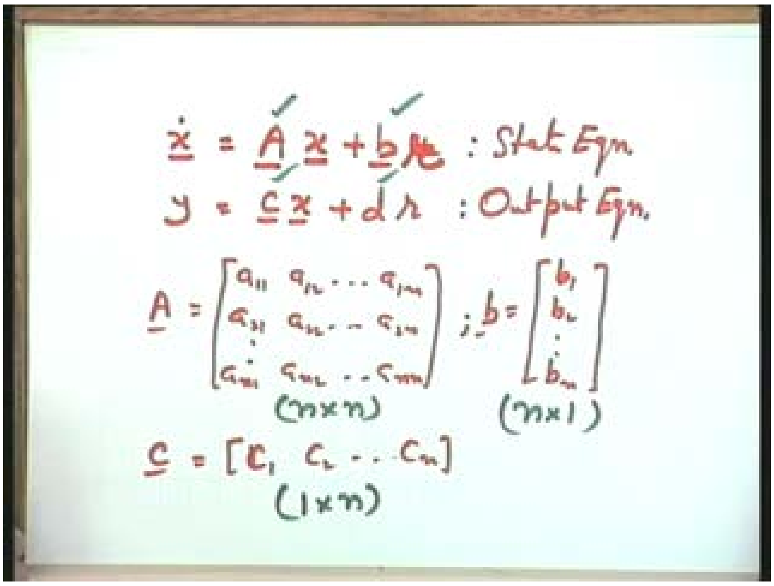

In terms of the matrix notation I will like to write this which I have written last time: x dot is equal to Ax plus bu. Please note that lower case letters with an underline represent vectors, upper case letters with an underline represent matrices and u sorry we have been taking r as the input r is the scalar variable and this is your state equation. And output equation will become y equal to cx plus dr the output equation where A matrix….. I think a quick revision will be helpful though we have written it already this your A matrix, b vector is b 1 b 2 b n an n into 1 vector, c is 1 sorry c 1 c 2 c n a 1 into n vector. so I can say that for an nth order system the system description is given by A b c and d. the A matrix is n into n, the b vector is n into 1, the c vector is n into 1 and d is a scalar constant.

A couple of examples were given last time. More of this will come when we go to detailed modeling when we take complex plants and total control systems around those plants.

Let me take up another mode another way of modeling a system and that I am going take as the impulse response model. You will see that it is an effective method of modeling a linear time invariant system. Impulse response model. I am going to define this model under the two conditions; what are the conditions please? One, please note these terms also which will be very frequently used in this sequel. One is that the system is relaxed. the word relaxed system I am using which simply means that initial energy storage in the system is zero which equivalently in terms of mathematical model means that the initial conditions that is the system state at t is equal to 0 is equal to 0. All initial conditions if taken zero the system is in the unenergized state the system is termed out to be relaxed and the impulse response model will be defined for a relaxed system.

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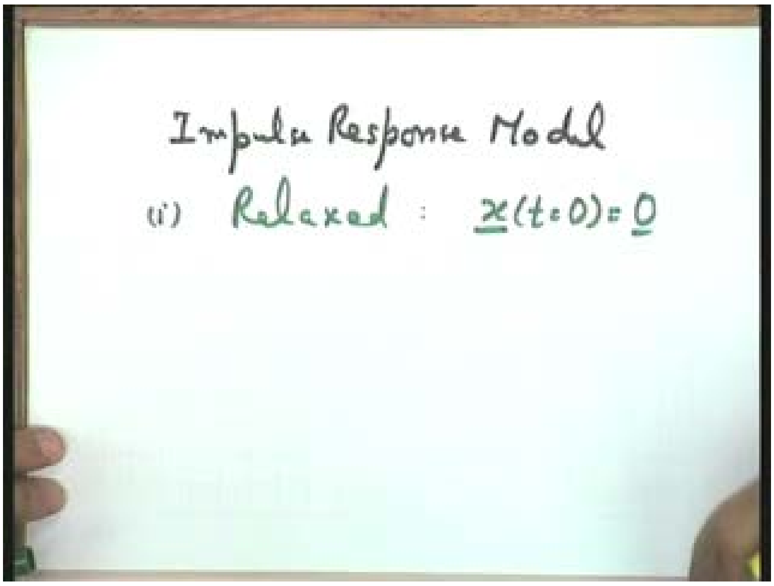

Second point I want to make out is that the impulse response model is an input output relationship. That is the impulse response model will not give you, as you know already this model will not give you an information about the system state, given an input it will be able to give you the information about the output of the system. So, if I put it in the block diagram form I will say that this r is the input, y is the output (Refer Slide Time: 7:50) x 1 x 2 x n are the state variables x(0) is the initial state or initial energy of the system, this particular block diagram represents the state variable formulation of a single input single output system.

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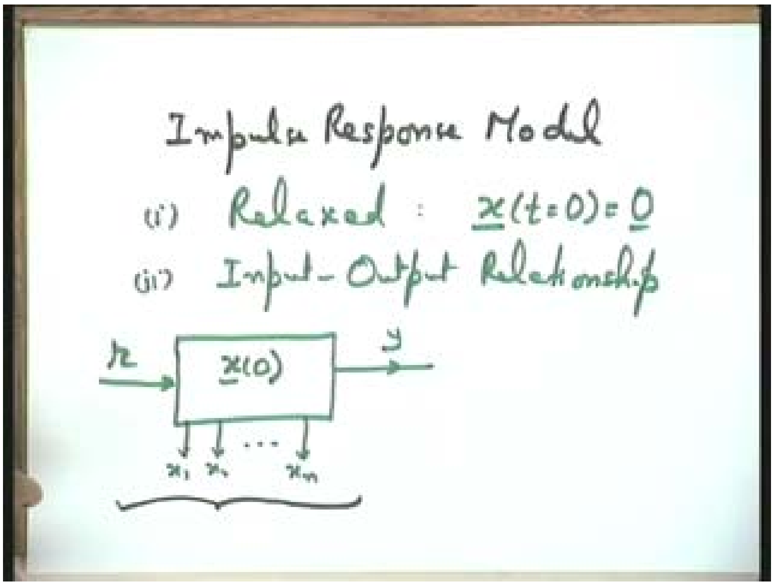

Coming to the impulse response formulation, so naturally impulse response model is silent about the state variables so you have only r and a y over here and here is a relaxed system (Refer Slide Time: 8:27). This I can say is the block diagrammatic view of representing a system in the impulse response format. This point may please be noted over here. Though this is a formulation in which the state variables are not coming explicitly none the less impulse response models as we will see are extremely useful in control system analysis and design.

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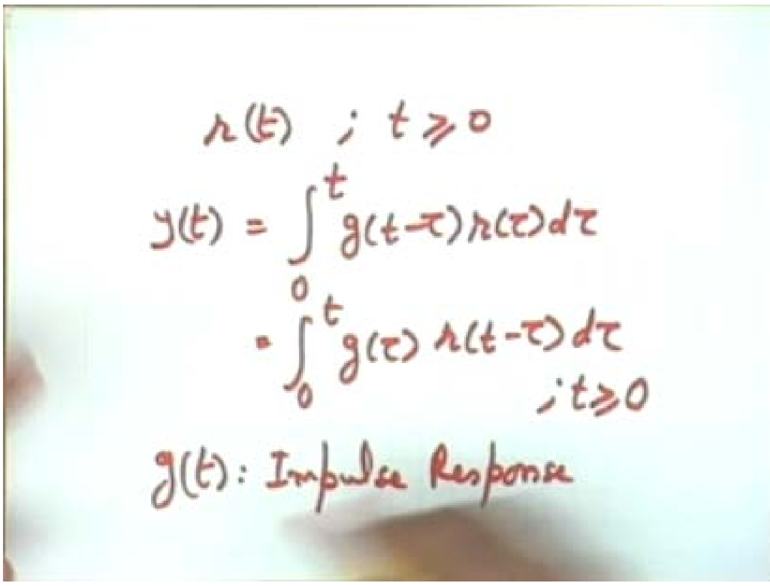

So naturally I am sure there is no need of going to the to the details of impulse response formulation, I can directly give you the result the result being if r(t) is the input defined for t greater then equal to 0 then the output y(t) for t greater then equal to 0 is given by this convolution integral or equivalently this is defined for t greater than equal to 0 what is after all a state variable model. It gives you the value of the system state x(t) for all time for given a input and initial conditions. This convolution integral gives you the value of the output for all t for given input under the assumption that initial conditions are zero. And therefore I can say that g(t) the impulse response of the system characterizes the system relaxed system completely. Impulse response of the system is a complete characterization of the system because given input and the impulse response using the convolution integral you can determine the value of the output for all time. And hence I can say that this is a system block impulse response g(t) is contained in this block which is a complete characterization of the system, external input r(t) is coming onto this particular system and y(t) is the response which is available using the information on r(t) and g(t).

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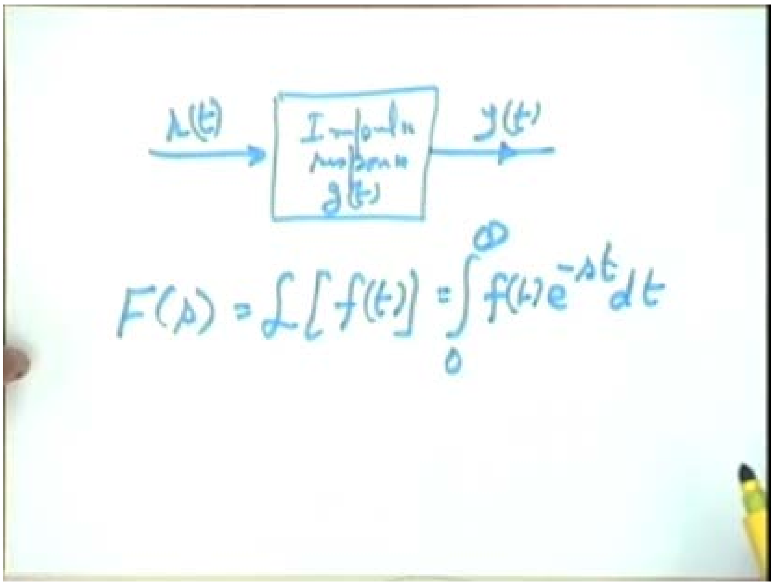

So this is another important model, the impulse response model which we will be using. However, as you will see the impulse response or the convolution difficult to compute compared to handling this in the equivalent form and that the equivalent form is in Laplace domain. So instead of handling this particular equation the convolution integral in time domain if I take the Laplace transform of this it becomes more convenient. The integration actually becomes an algebraic manipulation as you know and from there the concept of transfer function will evolve.

So I know that rather you know it very well that the Laplace transform F(s) s is a Laplace variable of a function f(t) where f(t) is a time function is equal to 0 to infinity ft e to the power of minus st dt. I am going to use this basic relationship onto the convulsion and setup the relationship between input and output in Laplace domain and that relationship I hope will turn out to be convenient to use. So, for that what I have to do is I have to take the Laplace transform of y(t) where y(t) is given by the convolution integral. So let us make an attempt.

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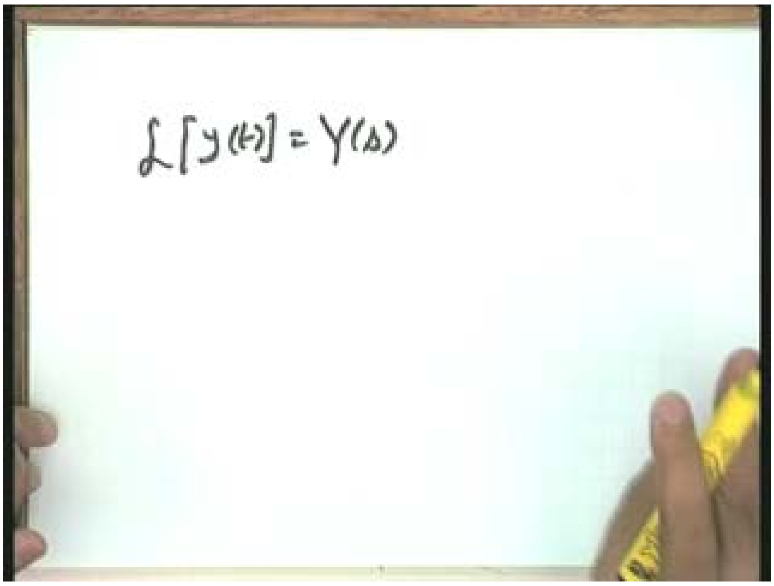

The Laplace transform of y(t) is equal to Y(s). Again please see the terminology; I will be mostly using a capital letter to represent the Laplace variable and a smaller one the lower case one to represent the time variable. As far as possible we will really try to keep this particular terminology all through the course so that there is no confusion. So in this particular case Laplace transform of y(t) is equal to Y(s) is equal to as per the definition given 0 to infinity. The y(t) will come over here e to the power of minus st dt. This is the Laplace variable.

Now what is y(t)?
y(t) is equal to 0 to t g(t minus tau) r (tau) d tau. Please see the manipulation and help me. G(t minus tau) is equal to 0 for t less than tau. Is it okay please because it is a causal system. The input t less than tau the input appears at t is equal to tau the response cannot appear before the input comes. So this response g(t minus tau) is equal to 0 for t less than tau and hence this expression……. because I am making this change because I need this manipulation when I take the Laplace transform. This expression (Refer Slide Time: 13:59) can be written has 0 to infinity g(t minus tau) r(tau) d tau because any how this is 0 for(tau) greater than t tau being the integration variable.

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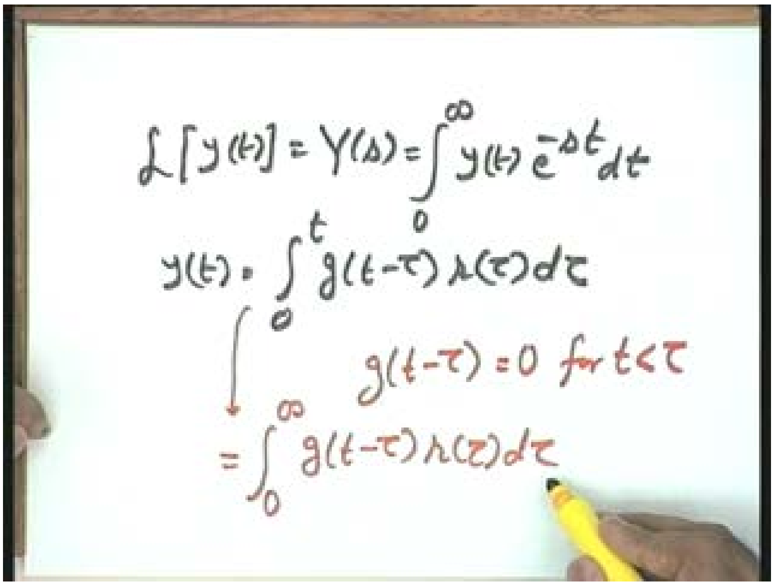

You will please note that t has been replaced by infinity without any change because this particular signal this particular response g(t minus tau) is equal to 0 for(tau) greater than t and hence t can be replaced by infinity without any change. As you will see this I am going to utilize when I take the Laplace transform. I can now write this as 0 to infinity 0 to infinity g(t minus tau) r(tau) d tau is y(t) variable into I have now e to the power of minus st dt.

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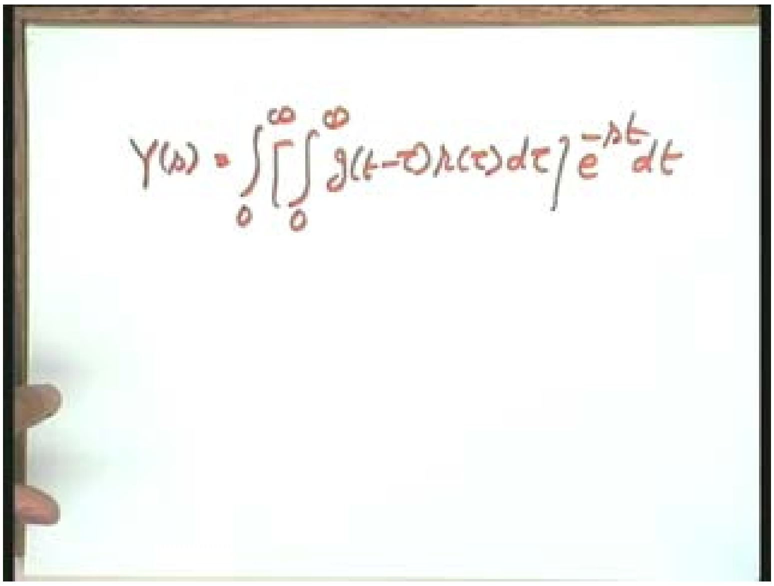

Since from here a very important definition is going to follow it is worth while taking up this particular manipulation, I could have given the result directly. So in this case now you see that just I interchange the order of integration 0 to infinity 0 to infinity here g(t minus tau) yes, let me put e to the power of minus st dt here and then r(tau) d tau here. I hope this is okay. Integrating first with respect to t and then with respect to tau. Please help me here: 0 to infinity, is it all right if I put this as 0 to infinity g theta e to the power of minus s theta d theta e to the power of minus s tau r(tau) d tau. Please see whether this is okay; I hope this will turn out to be okay.

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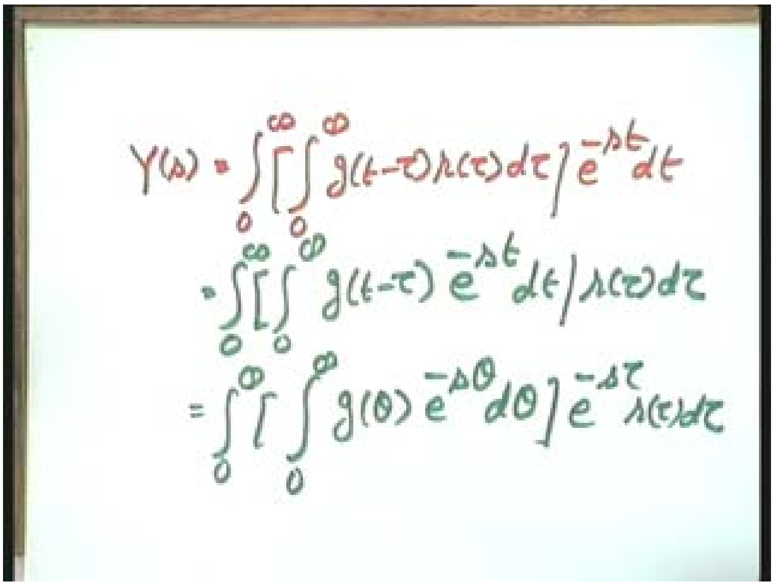

What I have done over here is the following: This particular expression; e to the power of minus st has been replaced by e to the power of minus st by tau with e to power of minus s tau appearing here. Just see, this has been changed by this (Refer Slide Time: 16:13) with e to the power of minus s tau appearing here. Now what I do is the following: t minus tau variable has been taken as equal to theta, change of variables give me this expression. Please see whether this is okay.

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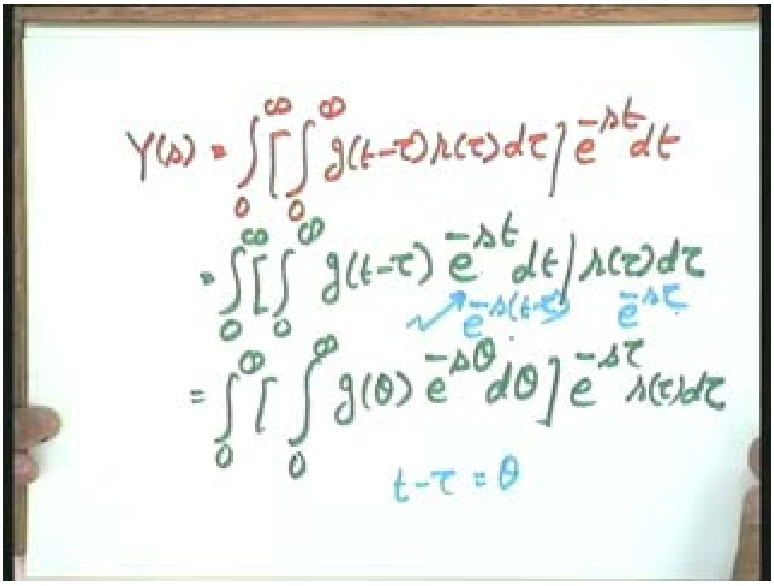

In the lower limit when t is equal to 0 you have theta is equal to minus tau. However, since g theta is equal to 0 for theta less than 0 so that minus tau has been replaced by 0 and therefore the expression……. I hope this is okay that this expression has been transformed to this particular expression wherein the theta variable is corresponding to this integration and tau variable is corresponding to the outer integration. Is it okay or it needs further elaboration please? I hope this is okay. So in that case this can be written as; 0 to infinity g(theta) e to the power of minus s theta d theta into 0 to infinity r(tau) e to the power of minus s tau d tau; rearrangement of the earlier expression please gives me this.

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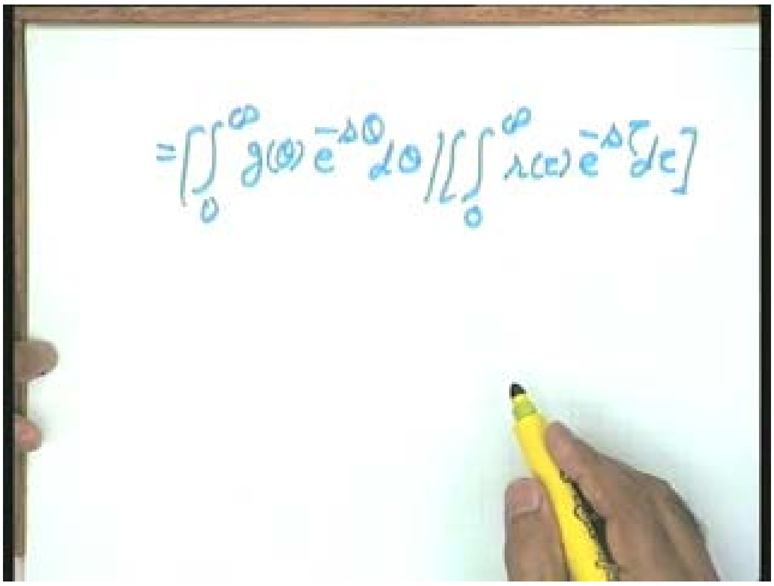

And now we have the result. This as you find, this particular expression, by definition is nothing but the Laplace transform of the impulse response and this (Refer Slide Time: 17:43) is nothing but the Laplace transform of the input signal. So, defining G(s) as the Laplace transform of the impulse response and R(s) as the Laplace transform of the input signal that is G(s) by definition is equal to Laplace transform of g(t) and R(s) by definition is equal to Laplace transform of r(t) the convolution relationship has been replaced by this simple algebraic relationship Y(s) is equal to G(s) R(s). This relationship I am sure is already known to you.

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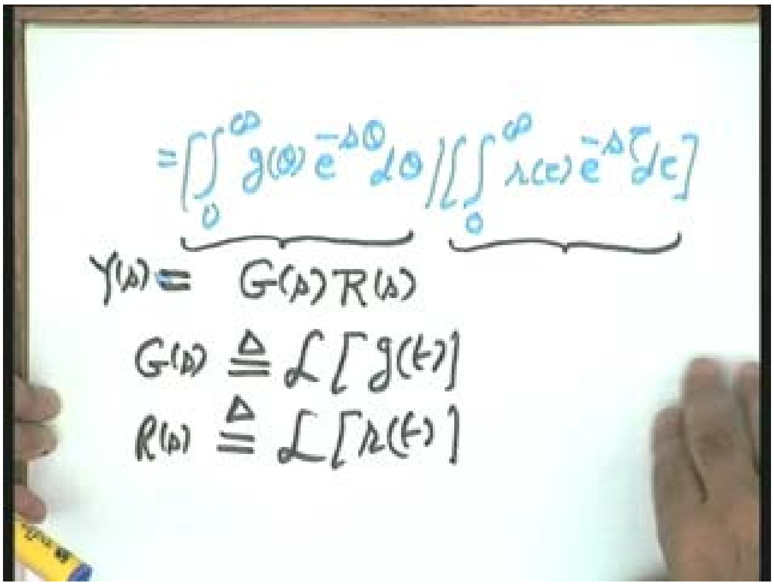

Now this G(s) is nothing but the transfer function of the system; transfer function of the system. And the transfer function as we will see is more convenient for analysis and design purposes compared to the impulse response. So mostly we will come across the transfer function models and the state variable models in our discussion.

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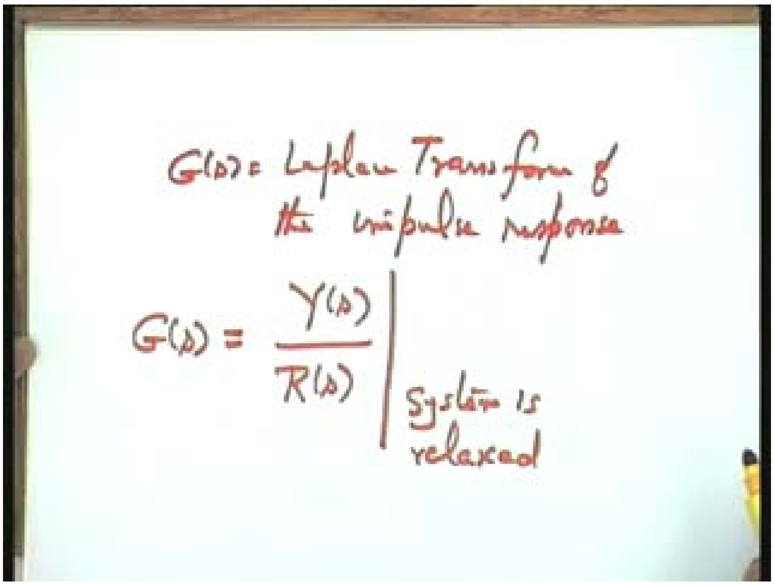

So this transfer function G(s) now I say is equal to Laplace transform. This becomes my basic definition of the impulse response. However, it may not be convenient to use this definition because the impulse response strictly will not be available to me from the basic laws of physics. I will get the differential equation model of the system and I will like to get the transfer function model directly from the differential equation model. And for that purposes I get the following definition for the transfer function: G(s) is equal to Y(s) the Laplace transform of the output variable of the system divided by R(s) the Laplace transform of the input variable under the condition that the system is relaxed. This point may please be noted. This is very important in the definition of a transfer function.

The transfer function is defined for relaxed system. Transfer function is the Laplace transform of the impulse response, which by definition, is the response of a relaxed system to an impulse input. So this becomes, as you will find that this definition is more convenient compared to this definition that is the Laplace transform of impulse response of the system when we come when we take up actual modeling of the system.

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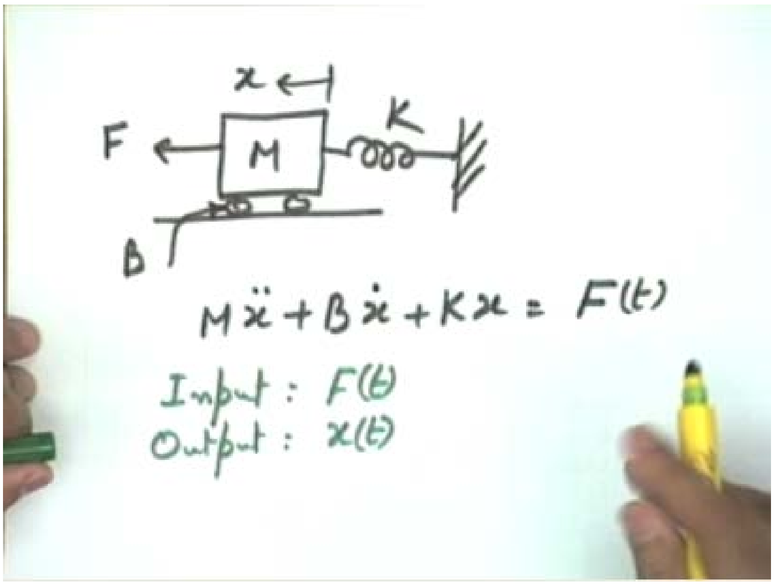

Take for example a very simple case we have been referring to earlier also; a mass and a spring and friction, this is a frictional coefficient B frictional coefficient B. I say that application of basic laws will give me the differential equation model. For this particular case the differential equation model can be written as Mx double dot double dot is second derivative I am directly writing the equation plus Bx dot that is the velocity plus Kx equal to the applied force F(t) this is the applied force here. So this is mass into acceleration, (Refer Slide Time: 21:26) viscous friction coefficient into velocity, spring constant into the displacement is equal to the applied force. Now this is the second order differential equation.

Now depending upon my requirement, if my requirement is, go for a state variable model I define the two state variables as the displacement and the velocity and get this equation in the form x dot is equal to ax plus bu y is equal to cx plus du. Instead, if the analysis and design requirements I should know beforehand, if the requirement is that of a transfer function model in that particular case I can directly get the transfer function from here applying the basic definition that the transfer function is the ratio of the Laplace transform of the output variable and the input variable under the assumption that the system is relaxed. The input variable in this particular case is F(t), the output variable let me say though it is for us to define let me say x(t) the displacement of the mass is the output variable. So now let me take the Laplace transforms.

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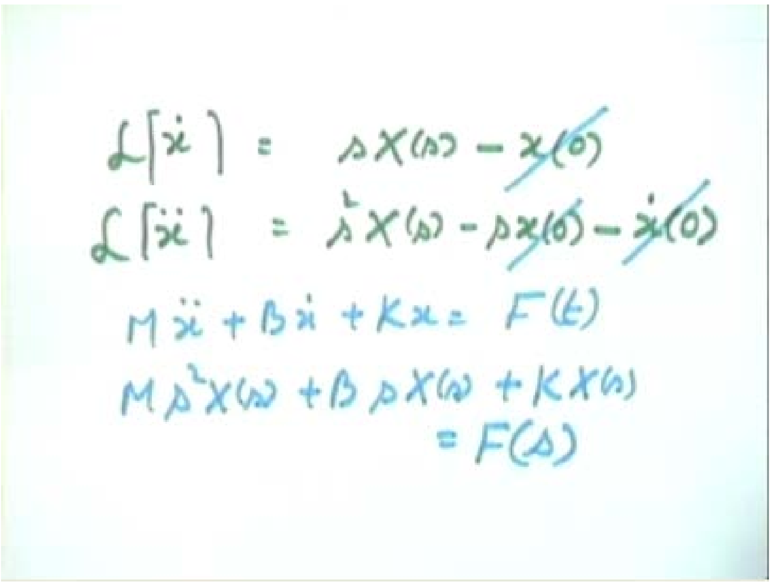

You will see that x dot Laplace transform as you know is X minus the initial condition x(0) and if I take the Laplace transform of x double dot the second derivative it is s square X(s) minus sx(0) minus x dot (0) that is the initial velocity. Now, if under consideration is a relaxed system so naturally x(0) is 0 and x dot 0 is equal to 0 and therefore these become the expression of Laplace transformation under the assumption of zero initial conditions. If that is the case Mx double dot plus Bx dot plus Kx equal to F(t) when transformed give me Ms squared X(s) plus Bs X(s) plus KX(s) equal to F(s) so t variable has been replaced by the Laplace variable s.

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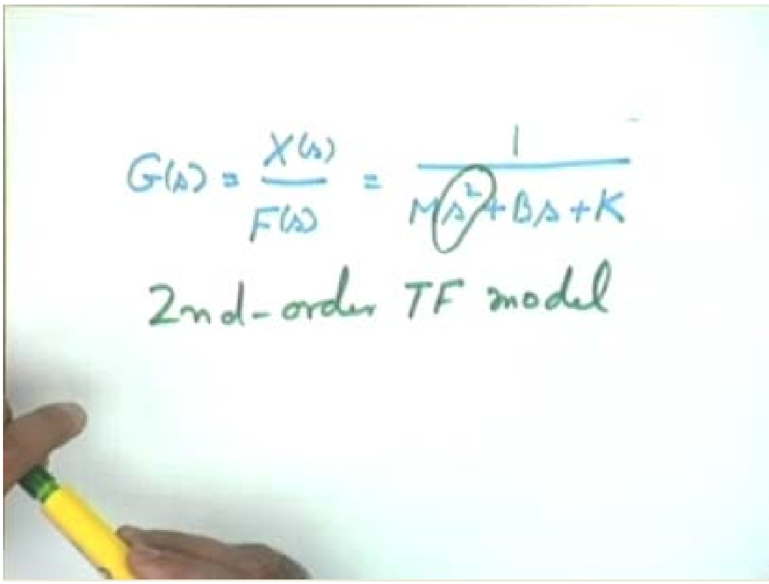

Rearrangement of this equation in terms of a transfer function: G(s) is the transfer function of the mass spring damper system. The output is X(s) the input is F(s) this obviously becomes equal to one over Ms squared plus Bs plus K. This becomes the transfer function model of the system. Note a point over here that this particular model the spring mass damper system was handled earlier in the last lecture and we found that it has two state variables. We really transformed that particular model to a second order state variable model meaning thereby the order of the system is 2 it is a second order system. You will please note that, in the transfer function model the order of the system is defined by the highest power of s in the denominator of the transfer function. In this particular case the denominator of the transfer function is Ms squared plus Bs plus K so the order of highest power of s is 2 so it means it is a second order transfer function transfer function model. Let me take the other example also for the sake of completeness though I will leave it in between for completion.

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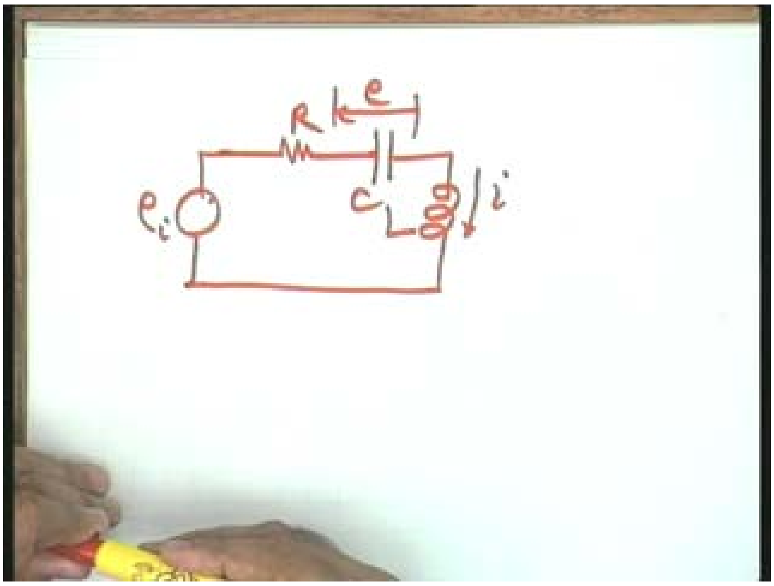

The other example I had taken was that of a simple electrical circuit R C and L and this is your input variable e i. This is L here, this is C here and here I had taken e the voltage across the capacitor and I the current through the inductor. Again please see, there are two energy storing elements in this particular case; the capacitor and the inductor and hence this particular system will be represented or has been represented by us by a second order model, the second order state model. So in this particular case we expect that this will be transformed to a second order transfer function model, that is, the highest power of s in the denominator will be 2.

So let me write, the basic rules are the same. Let us apply the basic laws of physics. Get the differential equations and those differential equations can then be transformed and rearranged to get it into the form of a transfer function G(s). The basic differential equation we had already written Ri plus e plus Ldi by dt is equal to e i. This is the first equation. Please see, if this equation is written in the Laplace domain it is RI(s) plus E(s) plus sLI(s) equal to E i(s). The second equation let me write. The second equation we had written was; C de by dt equal to I. Equivalently that is applying the transform operator on this equation I have sCE(s) is equal to I(s). Now you see, you have these two one and two algebraic equations. These algebraic equations can easily be manipulated to arrange them in the transfer function format.

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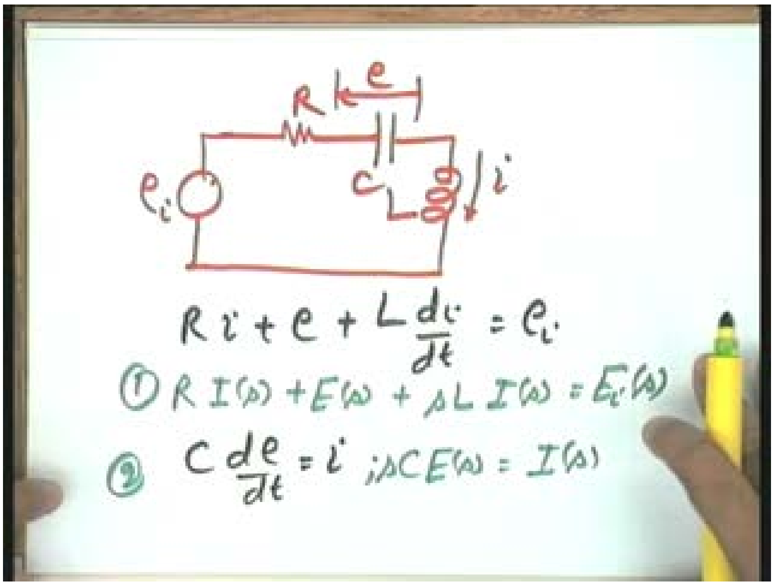

The manipulation in this particular case gives me G(s). Please help me, what is the output variable in this particular case? Output variable is Y(s) which you have to define divided by the input variable E i(s). Y(s) or y(t) is an attribute you are interested in. And in the last example y(t) was taken to be voltage across the inductor.

Let me assume; y(t) is the voltage across the inductor in that particular case as you see y(t) is equal to di by dt. So, if I write this in the Laplace transform Y(s) is equal to sL I(s). Rather now instead of two I have three equations to be manipulated to get the transfer function model for the system because the Y(s) has resulted in one equation in terms of the state variables.

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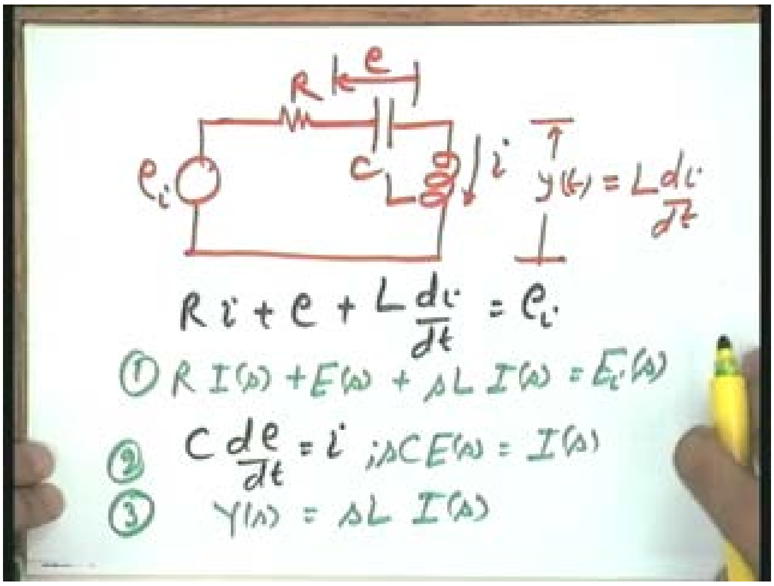

So in this case, well, I remember the result that is why I am giving you directly. It is s squared over s square plus R by L s plus 1 by LC the rearrangement of this equation. So naturally this also is a second order model as you see because the highest power of s in the denominator is 2. So now, I think with these two simple examples, though complex examples of modeling will follow, here the idea is to just give you the nomenclature and the definitions. From these simple examples I can say that G(s) could be written as b 0 s to the power of m plus b 1 s to the power of m minus 1 b m minus 1 s plus b m divided by s n plus a 1 n minus 1 plus a n minus s plus a n.

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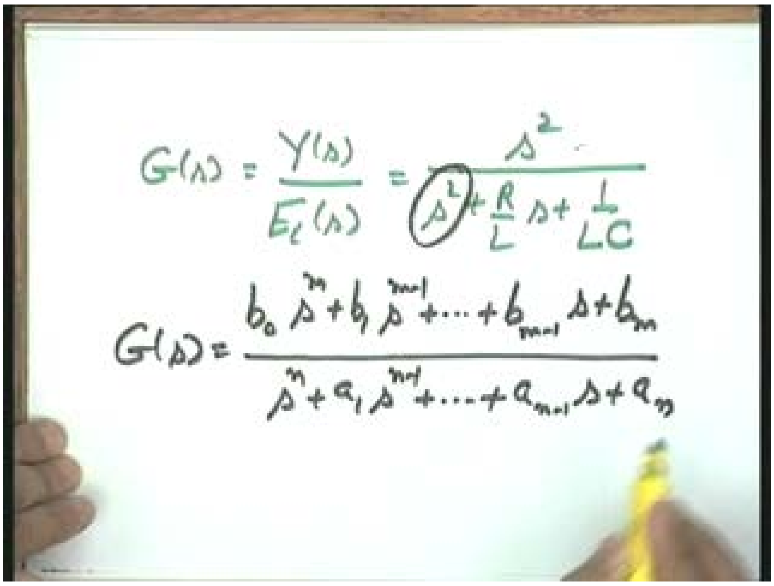

You will please note that all through the course whenever I write a general differential equation, general transfer function I will definitely write it in this form only. That is the form and the symbols get fixed for the course. In this particular case the order of the numerator polynomial m is less than are equal to the order of the denominator polynomial n. So see the basic recall the basic definitions; if m is equal to n the transfer function is a proper transfer function, if m equal to n, and if m is less then n as you know it is a strictly proper fraction or strictly proper transfer function.

[30:22] Student asking a question: can it be equal?
Yes, well, in this particular case let me say that we will mostly come across the situations in control system where m is less than n. mostly the physical systems belong to this type of models. But now coming to the general case whether m can be equal to n or not the answer is yes because it satisfies the realizability conditions. The transfer function is realizable if and only if the order of the numerator polynomial is less than or equal to n. it cannot be greater than n then it becomes an improper transfer function which is not realizable. But this point may please be noted: the physical plant you will come across will mostly be of the types which come under the category strictly proper transfer functions.

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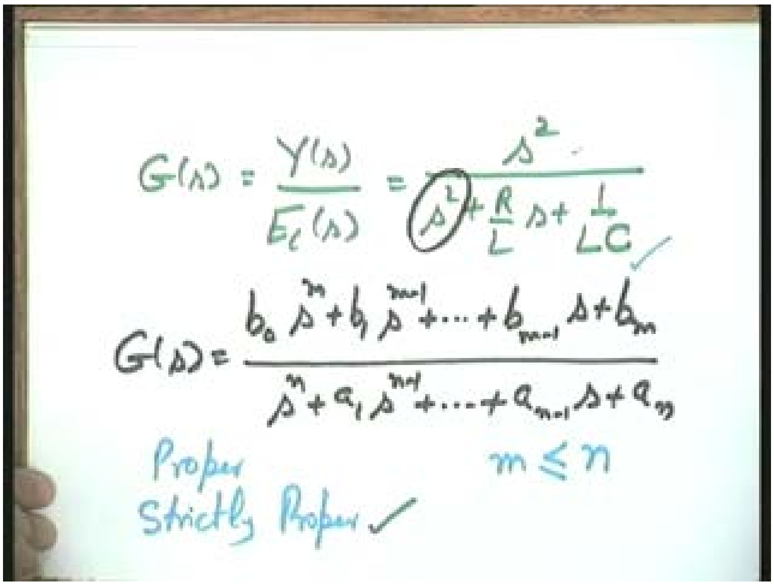

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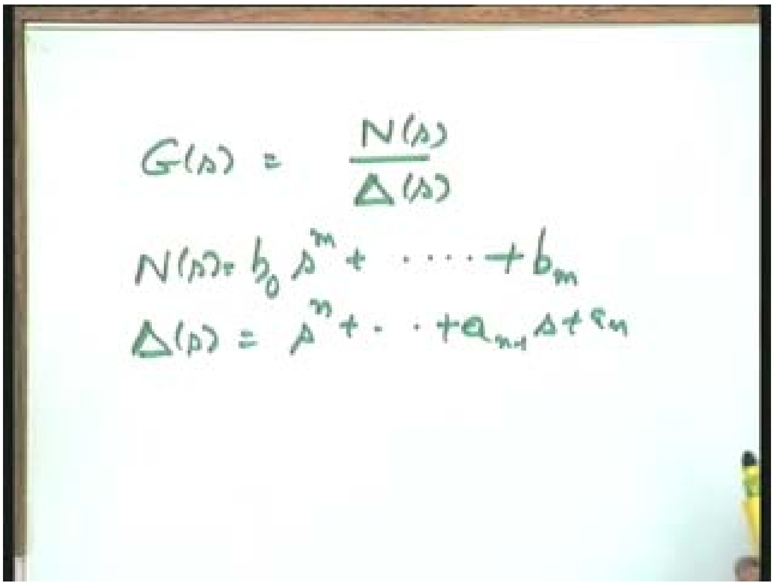

Now, let me put it in this form also: N(s) I write is equal to Ns over delta s. Again the symbols are to be fixed for the entire course. N(s) is equal to b 0 s to the power of m plus let me quickly write the last term which is b m. and delta s is equal to s n plus the last okay in this case one more term let me write; a n minus 1 s plus a n this is your denominator polynomial.

You will note one point here that, in the numerator polynomial the highest power of s has a coefficient b 0 (Refer Slide Time: 32:00). Well, the way I have written the denominator polynomial has the highest power of s has got the coefficient 1. You will please note that there is no loss of generality here because if the denominator polynomial has a coefficient other than 1 in that particular case it can be transformed to this type of transfer function wherein the coefficient is 1 by dividing all these coefficients by this constant. So there is absolutely no loss of generality. If the denominator polynomial is always written as a polynomial where in the highest power of s has a coefficient 1, this is just for convenience. At a later stage you will see that, well, this gives us some convenience that is why it is being written in this form. And this polynomial or this form of polynomial is referred is known as Monic polynomial.

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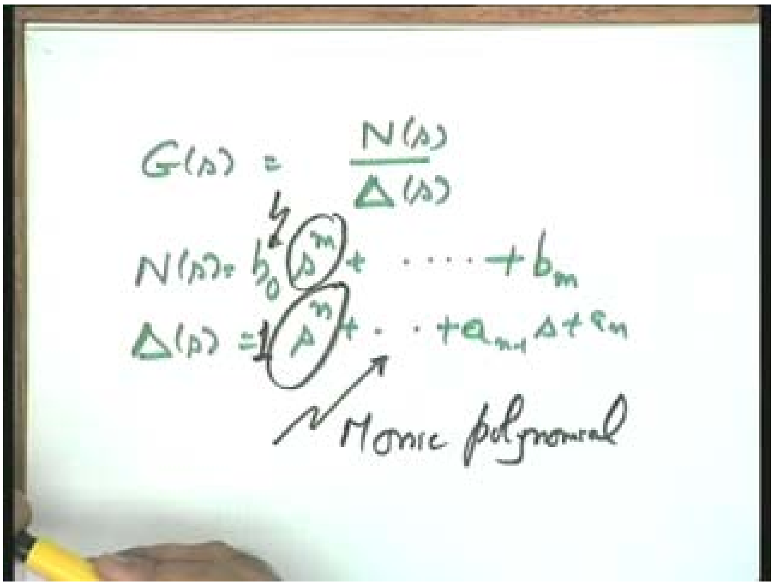

There is no absolutely no problem in rearranging a transfer function wherein the denominator polynomial is a Monic polynomial. Couple of more definitions please. The roots of this equation which is the numerator polynomial are referred to as the zeros of the transfer function and the roots of the denominator polynomial are referred to as the poles of the transfer function. Zeros of the transfer function and the poles of the transfer function please. And this polynomial whose roots are the poles of the transfer function plays an important role as you will see or as you know already, important role in the dynamical evolution of the system. The dynamics of the system, as you will see, is largely governed by the poles of the transfer function or the roots of the denominator polynomial and it is because of this that this polynomial is referred to as the characteristic polynomial of the system and the roots of the equation del s is equal to 0 or the characteristic roots of the system.

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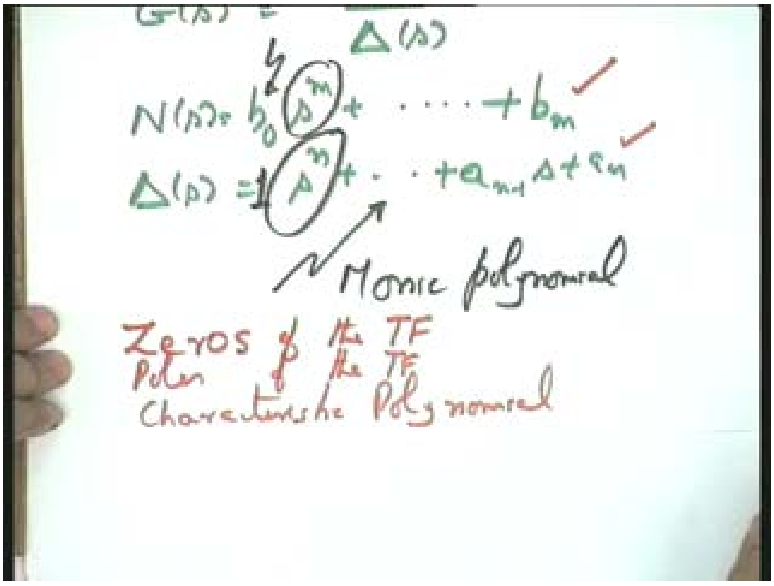

The specific name is being given because the dynamics is largely governed by this. The numerator polynomial that is the zeros of the transfer function will play with the amplitudes of the responses but the quality of the responses the nature of the responses will be given by the poles of the transfer function.

Writing it in the equivalent forms where the roots of the numerator and denominator polynomials appear explicitly the G(s) can be written as a constant K (s plus z 1) (s plus z 2)………..(s plus z m) divided by (s plus p 1) (s plus p 2) (s plus p n) where z i represent the zeros of the system and p i the poles and K the gain constant. You can easily see that this K is nothing but your b 0. The gain K is b 0 the gain constant of the system.

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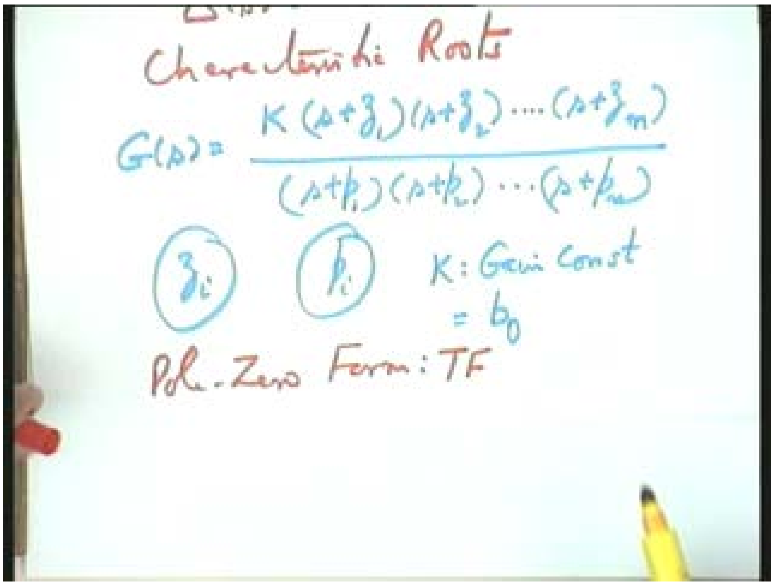

And this form of representation of the transfer function is referred to as the pole zero form. The earlier referred to as the the earlier form is referred to as the polynomial form, pole zero form transfer function. Fine, I think with this a review of various forms of models has been given. The state variable models and the transfer function models will have importance in our discussion in both analysis and design.

Now, once the model has been given, let us say if I look at the transfer function model I have this as G(s) the transfer function of the system, the input is R(s) here and the output is Y(s) here, this becomes the input output block diagram using the transfer function model. Taking it to the new topic now please a new point this is the input output block of the transfer function model.

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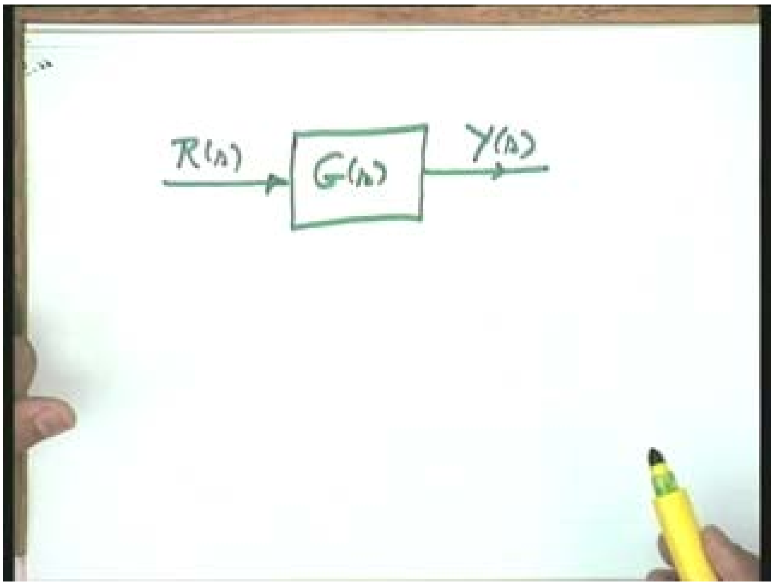

However, you will note that probably the input output relationship by this particular block diagram has not been fully defined. The reason being that this particular input R(s) is composed of the external input which you feed or which you control that is the manipulated input. Please see what are the different types of inputs. The manipulated inputs controlled by the controller in the feedback system so that the output is able to follow the command.

What is the other input?
The other input on a system is the disturbance input. So, it means if my interest is to find the response y(t) of this particular system (Refer Slide Time: 37:53) I should first define or I should first model the manipulated input and the disturbances disturbance input which act on the system. So it means know to get you Y(s); I know that Y(s) is available as an expression; Y(s) is equal to G(s) R(s); to get you the value of Y(s) I should concentrate on the input variable R(s) which I know could be manipulated input or a disturbance input. Or rather it will be better if I redraw the block diagram where the two inputs are explicitly shown.

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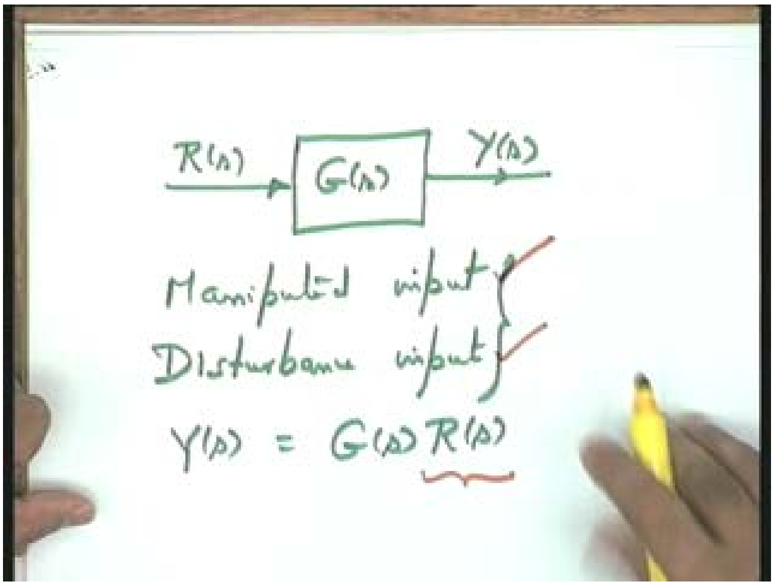

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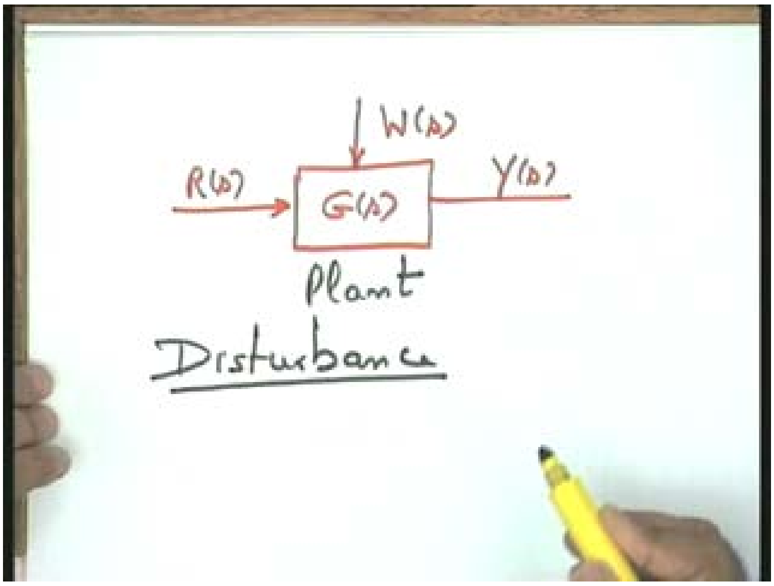

G(s) this is the input R(s), let me call this as the input you manipulate and this is the input here W(s), W letter you recall was used for disturbance so capital W(s) will be the disturbance in Laplace domain and this is your Y(s). This in fact should be the input output configuration where G(s) is a plant model because we know that these are the two types of inputs the plant will be subjected to.

So now let us talk about disturbance. Please see, again a very important concept is coming now. This may be coming to you for the first time but it is an extremely important concept because the purpose of the control system is to filter out this disturbance or to filter the effects of the disturbances. So let us see what are disturbance and how do we model them. By very definition the disturbance is something which is not under your control. So it means the nature of the disturbances also not known to you. If the disturbance signal was known to you probably the control would not have been difficult. You would have controlled, you would have installed a suitable controller so as to reduce or nullify the effect of the disturbance. But unfortunately disturbance is a signal which is unknown to you, it is erratic, its wave form is unknown.

So I will like to classify disturbances in two ways: One; erratic signals with unknown wave forms. Well, I think the term noise in that particular case is better suited for these signals. I need your attention here. These are normally high frequency disturbances acting on the system and the wave forms of these disturbances are unknown. And, deterministic methods of analysis and design are not applicable to such situations and you actual go for what is called stochastic modeling for such disturbances and this is an area which is excluded as far as our course is concerned. We will not come to this area the stochastic control systems wherein the disturbances are modeled in the form of stochastic models. And we will be referring to or we will be coming to the situation where the disturbances are slow-varying signals slow-varying signals. The wave forms if not exactly known the general nature of the wave forms are known to us for these slow-varying signals.

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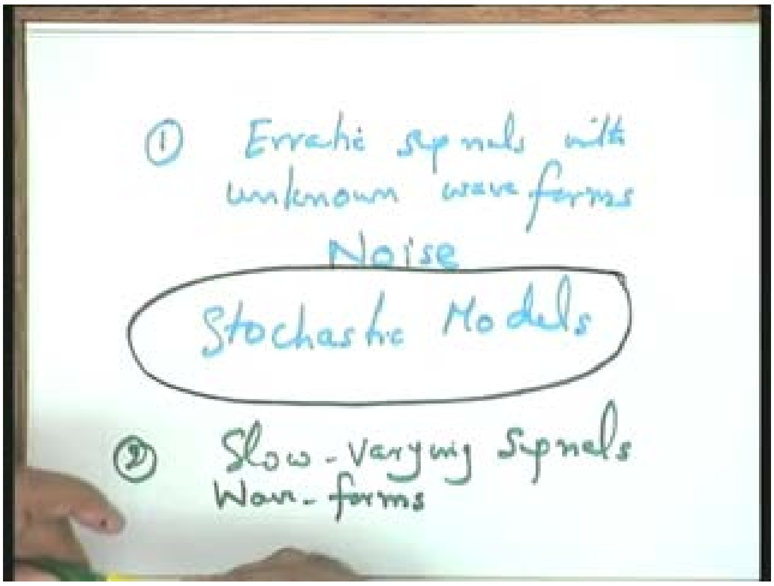

Come on it will be interesting if you generate some examples. The examples could be an electrical system, for example, a power system. Electrical load on the system is a disturbance. But the electrical load variations are not that random you see, you know its behavior you know that the disturbance will be more in the evening, the disturbance will taper off as the as the as you see that you can see the effect that during the time at about 7 pm the disturbances will be high and if you see the disturbance at about 12 12 noon 12 midnight or 1 am the disturbances will be low. So the general nature of the disturbances is known to us.

Take a robotic robotic system. A robot picks up a payload; payload, well, may not be fixed but the general nature of the payload as to how much variation in a payload can come is known to us and therefore the payload variation in a robotic system is a disturbance whose characteristics in some form or the other are known to you. Any other example please?

Let us take a thermal system. Residential heating example we have taken. What is the major disturbance in this system? The major disturbance is the environmental temperature. Now the environmental temperature behavior is not as much random you see; its behavior can be captured in a suitable model because it is its behavior is more or less is known to us over a period of time and therefore all such disturbances can be handled by deterministic methods and this is the range we are going to take up in our discussion.

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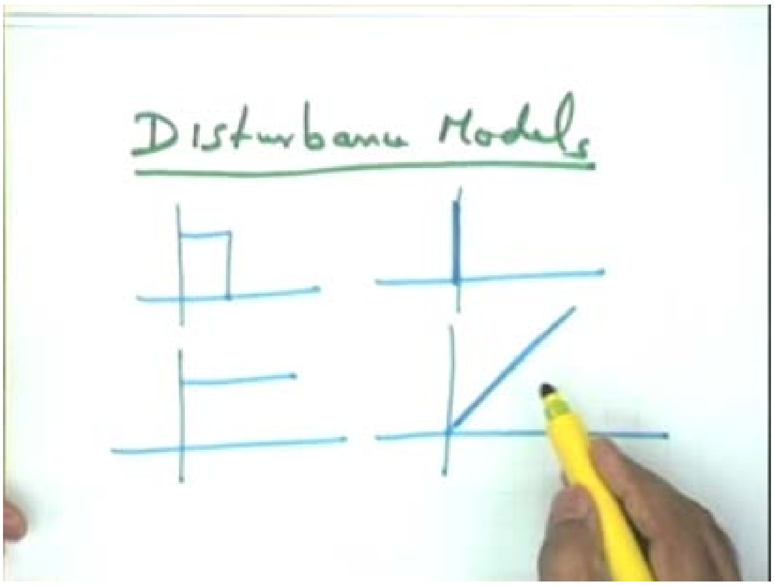

So, if this is the range of disturbances we are going to handle so we have to make now suitable disturbance models which we will use in our analysis and design. Disturbance models you see. Any situation cannot be captured mathematically. But if we use the models we are going to discuss this gives good amount of information about the system, how will it behave when it is subjected to actual disturbance conditions.

Let me take a pulse as a disturbance. Whenever a disturbance of short duration comes, a disturbance of short duration, a constant magnitude and short duration, well, it may not be an exact pulse but you can approximate it by a pulse. You see that if you go to the situation where disturbance is a sudden shock, in that particular case probably an impulse is a suitable model, a sudden shock of very high magnitude, may not be in finite magnitude but a shock of high magnitude it is a model, impulse model could be used to capture that particular situation.

Other signals please. Let us say a disturbance which is a constant step. A constant step signal, electrical load variation for example, in the evening hours there is a constant change in the disturbance signal. So this particular type of disturbance may be captured by a constant step signal. Now disturbance may be captured by a ramp signal which represents a situation where the disturbance variable drifts away; it does not remain constant it drifts away and keeps on increasing. So, ramp signal is a suitable representation or model of that particular situation.

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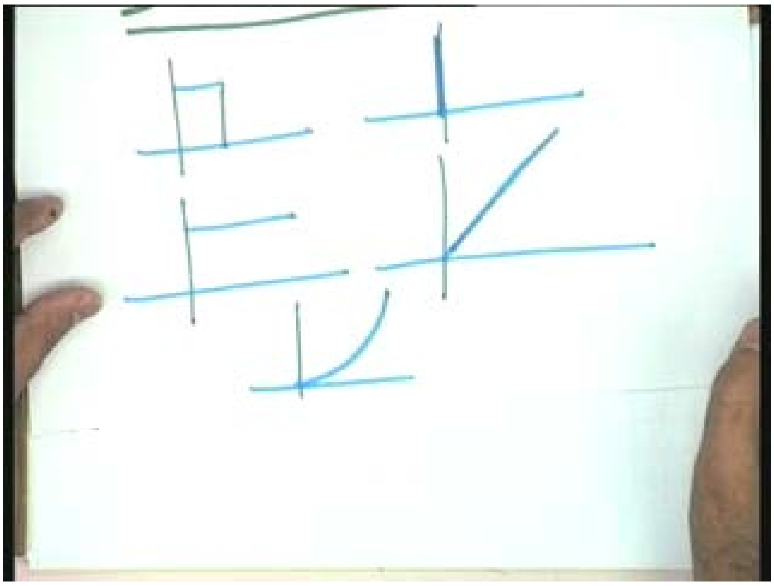

If you go further you see that this particular ramp signal (Refer Slide Time: 45:25) a faster mode is a parabolic signal, the variation of the disturbance or the drifting of the disturbance is faster. This is the situation where suitable which which represents the situation where the fast drift of the disturbance occurs. You see further, faster than these drifts are also possible but normally in control system analysis and design these disturbances are good enough and we get appropriate results by analyzing the system under the conditions of these disturbances.

Now, if I define these disturbances in Laplace domain, take for example delta(t) the impulse response, the impulse response, the Laplace variable of the impulse response as you know is 1. Take a step signal mu(t), unit step I am taking the magnitude is 1, the Laplace transform of mu(t) as you know is equal to 1 over s this is your step signal. You will note one point, an interesting point, again the way we are going to use it. In the literature as you would have come across the step signal is normally represented as u and not as mu. But in our course throughout we will be representing the step signal by mu the reason being, at a later stage we will use u signal as a control signal and this is universally used in control literature as a control signal. Therefore it will be better if we deviate from this standard convention of using u as a step and we have I have changed this to mu which is closer closer to you and our mu will represent this unit step signal for us.

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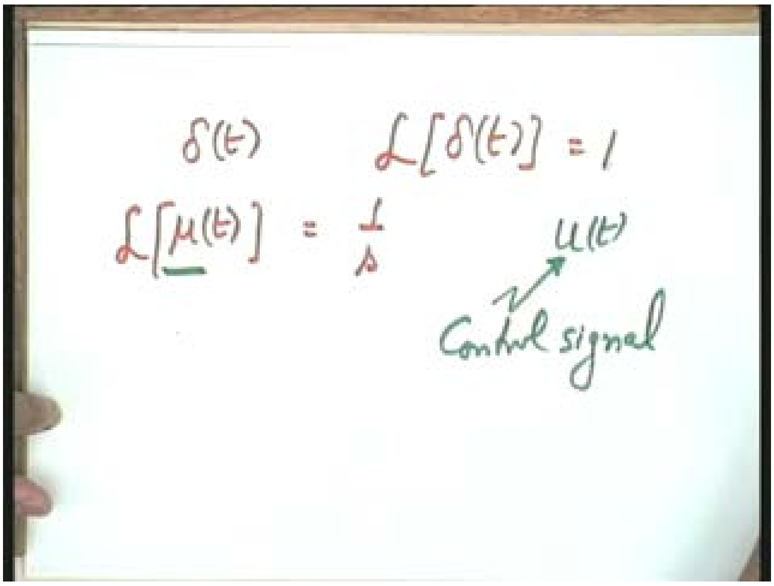

Now take the ramp, you see that the ramp f(t) is equal to t or this could be represented as t is greater than equal to 0; this could also be represented as, you know, t mu(t). What is the Laplace transform of this please? The Laplace transform of the ramp signal f(t) is equal to 1 over s square. These are standard signals you see we will come across.

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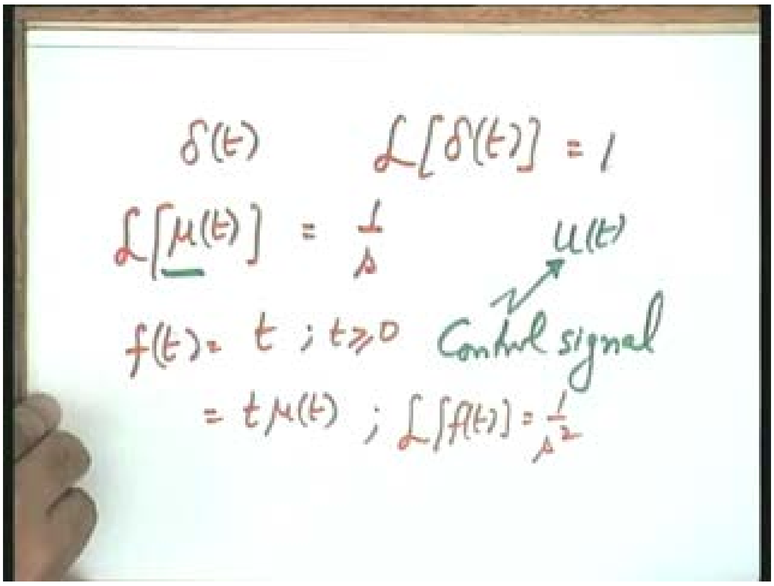

Take….. Now let me take the parabolic signal; let me define this as t squared by 2 for t greater than 0 is equal to 0 for t equal to 0. This I can define as f(t) is equal to t squared by 2 mu(t) and the Laplace transform of a f(t) is equal to 1 by s cube, this can easily be examined.

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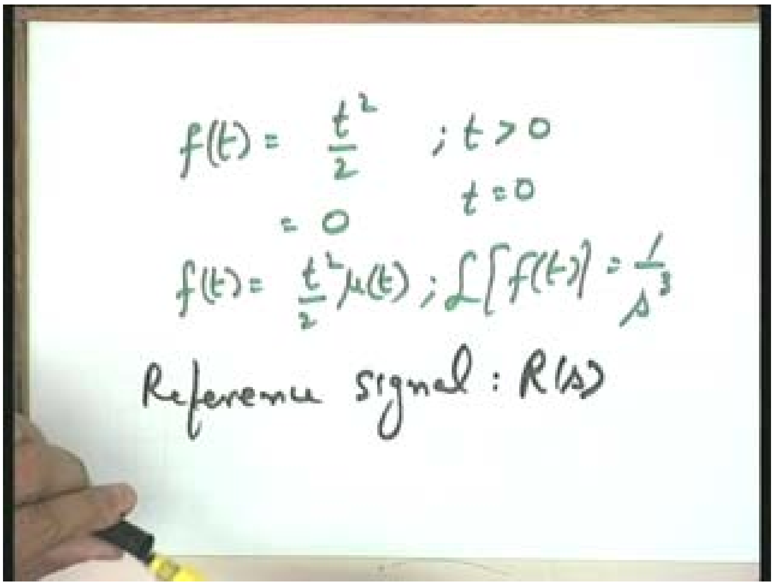

So 1 by s 1 by s squared 1 by s cube and 1 are the Laplace variables which will be taken as the disturbance signals. Now, once I have defined the disturbance signals let me come to the other signal the reference signal or the command signal. What are these signals? Please see then in many situations the reference signal is known to you. Take for example the situation of….. which example should we take? We can take the situation of residential heating. In that particular case what is reference signal; the reference signal is a temperature set point. So you know that reference signal, you might fix that to be equal to 20 degree centigrade depending upon your requirement so the reference signal in this particular case becomes a step signal.

On the other hand, you take the radar tracking problem. What is the reference signal in that particular case? The reference signal or command signal in this particular case is the target plane's position and target plane is the enemy's plane and the position of the target plane is something which is beyond your control which is not known to you. So please note that the command or the reference signal may also be an unknown signal. And you are going to design a system for the situation where the system will be subjected to these unknown signals. So it means, what should be the design strategy? The design strategy should be to design the system under the most strenuous situations. that is your system should be subjected to strains and if it is able to take the strains during the analysis and design phase hopefully it will be able to work satisfactorily under the actual conditions where the actual command or the disturbance signal come on this comes on this system.

So you see that, you have seen the disturbance signals. The disturbance signals we have seen are the impulse, the step, the ramp, the parabola. These signals you see represent various situations wherein the actual system under design is strained. And it is found experimentally that if a system performs satisfactorily for these inputs it is going to perform satisfactorily under the actual commanded conditions. And therefore the standard, I will use the word standard test signals now for me, because these standard test signals may not be the actual command signals; these are the test signals while designing a system.

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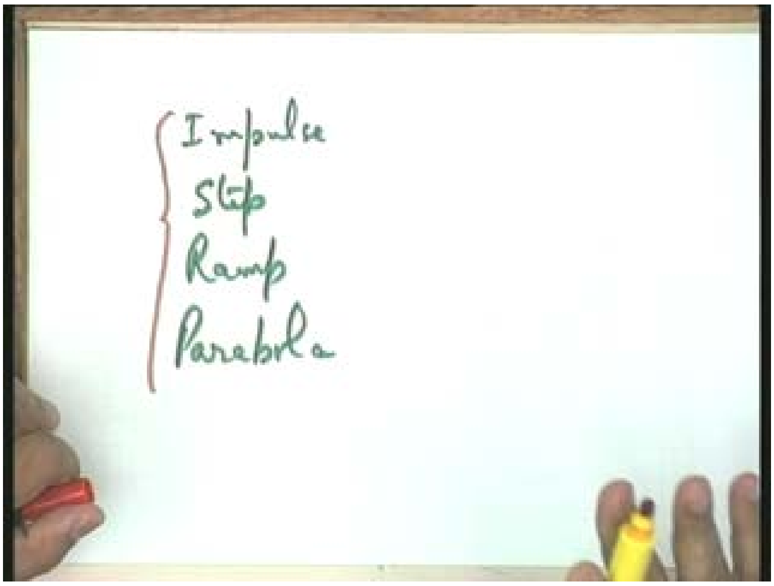

Under the assumption, under the experimental evidence that if the system behaves satisfactorily for standard test signals it will behave properly under the actual commanded situations and therefore these inputs; impulse, step, ramp, parabola for our study they become the models of the disturbances the slow-varying disturbances of course which can be captured but deterministic models, these becomes disturbance models and these are the models for the standard test signals as well. And our analysis which is going to start from the next lecture will be utilizing these input signals for the purpose of dynamic analysis of the system. Thank you.