Control Engineering
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Lecture - 39
Feedback System Performance Based on the Frequency Response

Well friends, the last phase of our discussion is on frequency domain design. Today's lecture and the remaining two lectures will be devoted to the design aspect in frequency domain. So, to give you the design algorithms the very first point will be to see as to how do we represent a system in frequency domain, how do we characterize a system in frequency domain. Today let us discuss the frequency domain specifications which completely characterize the steady state and transient behaviour of a system.

I will consider a feedback system let me say that (Refer Slide Time: 00:01:35 min) this is the open-loop transfer function forward path transfer function and here I have G the sensor transfer function, the feedback path transfer function. So the closed-loop transfer function let me represented by M so M(s) equal to G(s) over 1 plus G(s) H(s). This is what I have as the closed-loop transfer function of the system. If I view it in the frequency domain M(j omega) becomes equal to G(j omega) 1 plus G(j omega) H(j omega). If frequency varies between 0 to infinity I can see the behaviour of the closed-loop system if I am able to evaluate the magnitude and phase angle of omega.

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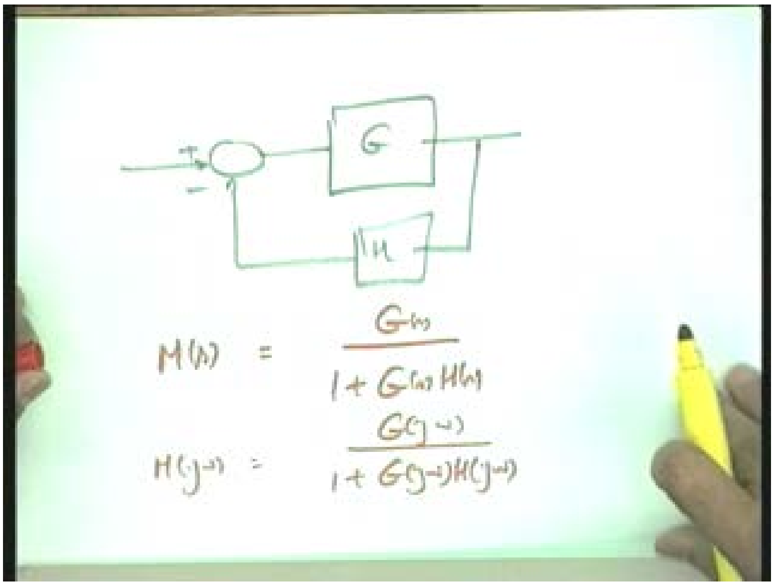

I put this function here I want you to store this information in your mind this question in your mind this is G(j omega) 1 plus G(j omega) H(j omega). What I said is the following that the closed-loop behaviour of the system in frequency domain is known to us if we know M(j omega) magnitude and angle M(j omega). Please note that what you have is the open-loop frequency response, the information on GH is known to you because the open-loop frequency response corresponds to the sensor corresponds to the plant, you have the model corresponding to the sensor or the plant or you have determined by experimentation the frequency response of the open-loop system. So, when I say that the closed-loop behaviour will be known to me it means really this is still a pending question that is given an open-loop frequency response I have to evaluate the closed-loop frequency response to get the values of the magnitude and phase angle of M this is a point you registered. Please see that when I come back to the closed-loop frequency response I like to pick it up again.

Now, before that I will like to take the standard characteristics of an ideal low-pass filter. You know that if I take this as the magnitude and this is omega this is my cut-off frequency omega c this is the characteristics of an ideal low-pass filter (Refer Slide Time: 4:04) which gives you the gain up to this point as unity let us say I take as unity and then there is a sharp cut-off at the frequency omega c so that the gain becomes 0.

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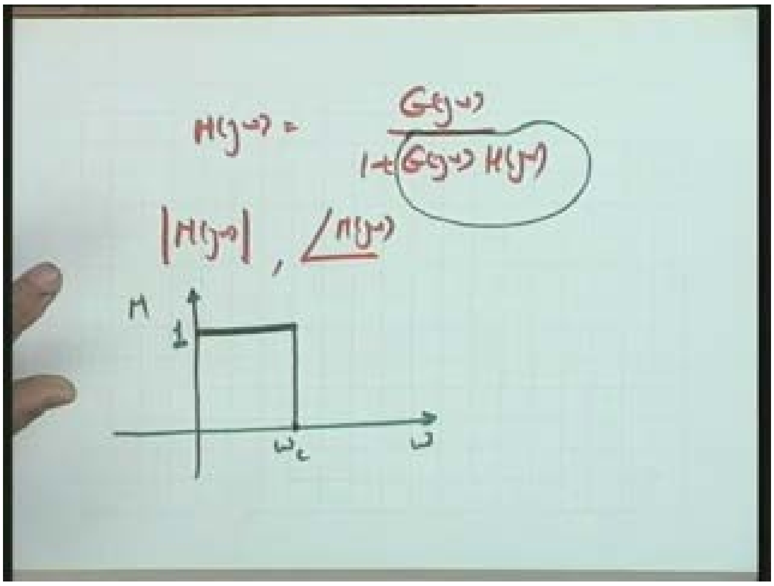

Hence you see that this type of characteristic for control system is quite ideal because in the frequency range of interest to you you can have the unity gain and whenever the frequencies you are not interested and you want to filter out the noise frequencies you can sharply cut-off. So it means as far as the low-pass characteristics are concerned its ideal characteristic is very much suited to control system. Therefore, you see that if you view it from the point of view of communication engineering what a low-pass filter is is very clearly known to us. I will like to say that all control systems are more or less low-pass filters as you will see because the signals are normally of low frequency range and the noise is of high frequency and you want the noise to be filtered off and therefore the characteristics of the control system which you are going to design in frequency domain is more or less a low-pass filter. But that also you know that an ideal low-pass filter characteristic is not realizable so in that particular case naturally no control system will approximate this because this is a function which is not realizable.

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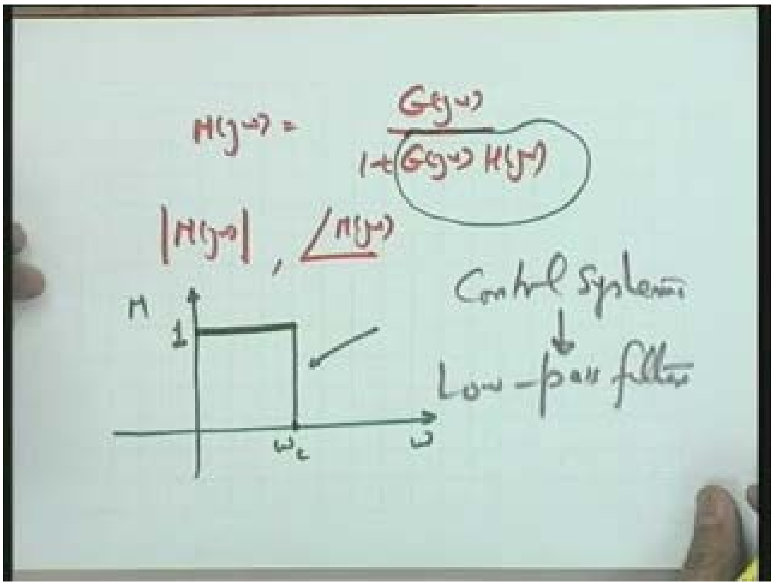

Hence a typical control system when I say control system I am talking of a feedback system; typical characteristics of a feedback system are of the following type. This is the typical characteristics let me say it is 1 here, this is omega and here it is magnitude M (Refer Slide Time: 6:00) because the ideal characteristics are non-realizable. In this particular case you will find that after 1 there is an overshoot also in frequency domain and you can say that this particular point where this is maximum this particular frequency I may refer to as the resonant frequency and this particular overshoot I may referred to as the resonant peak and then it falls off and you will note that there is no sharp cut-off here this is a typical control system characteristic. So, as far as the cut-off frequency is concerned where I put my cut-off frequency is not clear to me so the characteristics as you already know the low frequency range and high frequency range characteristics of a system are defined with respect to the bandwidth. So what I do is I consider that particular frequency as omega b the bandwidth where the magnitude has dropped to 1 by under root 2 value of unity not the peak overshoot please (Refer Slide Time: 7:07) this we take with respect to unity because actually you wanted a flat gain.

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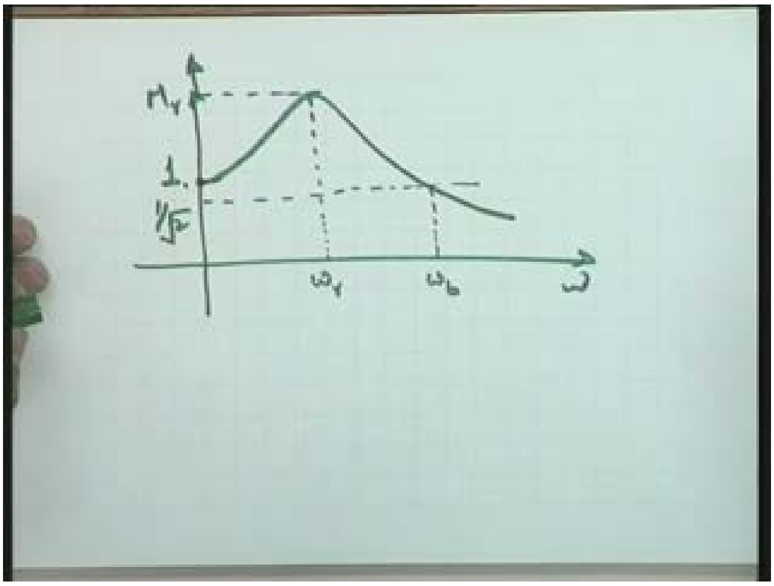

You see that if your magnitude gives a unity gain at all frequencies that are the ideal condition because that will correspond to perfect tracking. The magnitude for all frequencies is equal to 1 it means output will follow the input very correctly and smoothly. So this one is the ideal characteristics you want and there may be a peak overshoot so in that particular case the bandwidth for the control systems are low-pass filter characteristics systems with low-pass filtering characteristics is defined with respect to this flat gain of unity therefore 1 over under root 2 is the magnitude I take and this I take as the frequency omega b which gives me the characteristics of the system in the low frequency and high frequency region.

You will please note that for a typical control system what is omega b is very difficult to quantize and describe. As you will see when I give you the quantitative relationships omega b as large as possible is a very useful characteristic the reason being omega b is as large as possible it means your gain is flat for as large a range of frequencies as possible so naturally the tracking characteristics of such a system is going to be good. The rise time of such a system is going to be low and therefore a large omega b is a characteristic which is wanted by control designers. This is just a quantitative guideline; qualitative guideline; quantitatively I cannot say that what should be the value because the value will depend upon the system and the noise filtering requirements.

Now if you take a very large values what will happen the noise also enters the loop. So it means if you know the sensor noise if you know how the system is going to behave to the noise characteristics you know the cut-off point. So even if you are compromising tracking you will have to take a finite value of bandwidth so that the noise characteristics are taken care of so that high frequency noise is filtered this is one point.

You see that I am telling you that omega b is equal to infinity or very large of omega b are welcome from the point of view of trajectory tracking; from the point of view of control system performance but there are limiting factors the limiting factors are the noise characteristics and second factor as you will see to realize a large bandwidth you will require large gains. So the large gains….. you can see it from here, please see, help me please, if you want (Refer Slide Time: 00:09:53) the flat characteristics the flat characteristics means M is equal to 1 for most of the total frequency range how can you get M is equal to 1 can you help me from here? M is equal to 1 will come only if G(j omega) tends to infinity then and only then you can get M is equal to 1. You see G(j omega) tending to infinity only will approximate this function to 1 and hence this one means unity gain for a large range of frequencies which equivalently means their large bandwidth in the system.

Therefore, you find that G(j omega) tending to infinity will come only if large magnitudes are involved and large magnitudes will lead to saturation. So the limiting point may please be understood very well; though I need omega b to be a very large value but the noise filtering requirements and the requirements of preventing the system instead of going to large values of gain to make the system saturated you will like to prevent those values which lead to saturation so these two factors you can say lead to the limitation on the bandwidth of the system. The quantitative guidelines cannot be given they are system dependent.

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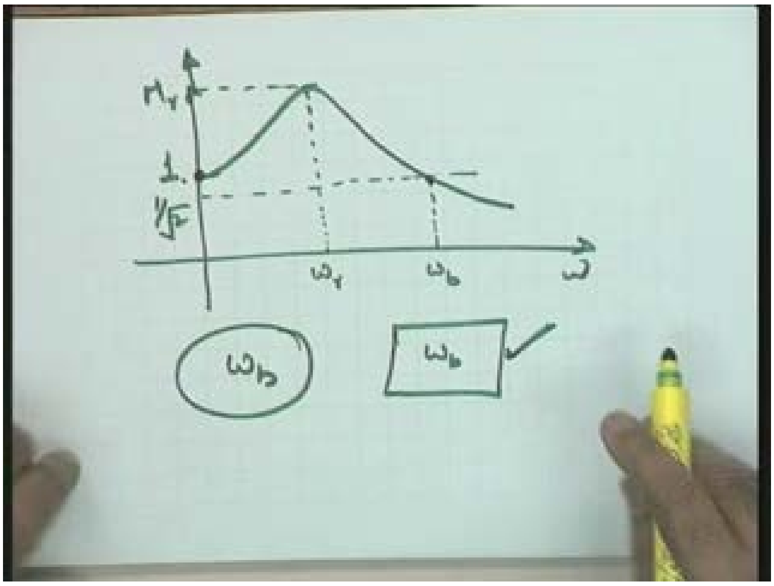

So you see that I now know that control system I am going to design is more or less a low-pass filter. Now let me say that how do I quantify the total behaviour of a system. I take this as a typical characteristics of a control system (Refer Slide Time: 11:30); this is 1 here and this now is the total frequency response; I am taking this as magnitude M, this I am taking omega; please note that this magnitude M I am referring to closed-loop frequency response though the data available with you is that of open-loop frequency. So it means you will have to obtain the closed-loop frequency response to make this sketch and from this sketch we will see what are the characterizing parameters.

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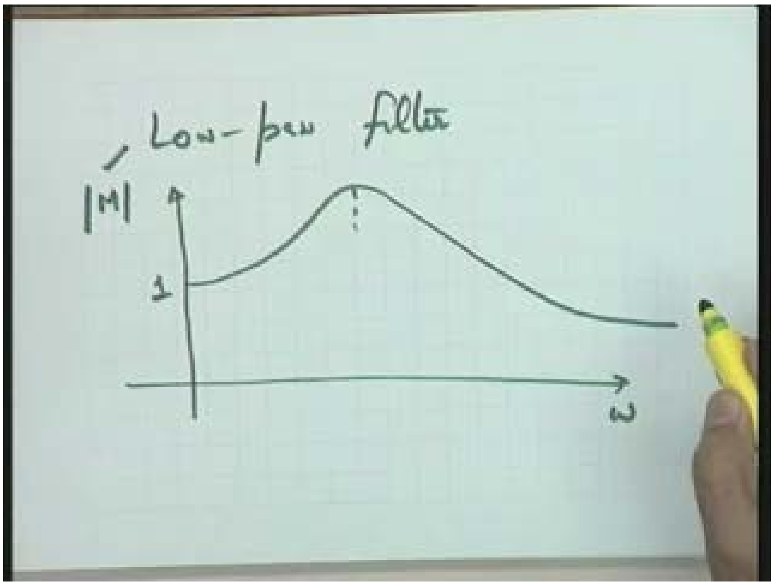

Like in the time domain I can say that the characterizing parameters for this particular system is the complete frequency response. But again if I take the complete frequency response to characterize the system the design algorithms becomes very difficult and therefore some indices I take and these three indices I was referring to are the optimum indices to characterize the closed-loop frequency response completely. The three indices are: the resonant frequency omega r the frequency at which the maximum occurs; the resonant peak let me call it M r the magnitude at the resonance frequency and the frequency omega b the frequency at which the magnitude is 0.707.

I say now these three are the characterizing parameters; qualitatively I know these values, qualitatively I say that M r should not be too high; what is this too high we have yet to quantify but M r should not be too high because a high value will mean that the flat gain requirement is not being met. The values of omega r and omega b as you will see are related to rise time and a shorter rise time requires large values of omega r and omega b. This let us see how do I compare this in the time domain but before I compare this in the time domain I like to say that there are two more indices and those indices are the phase margin and the gain margin.

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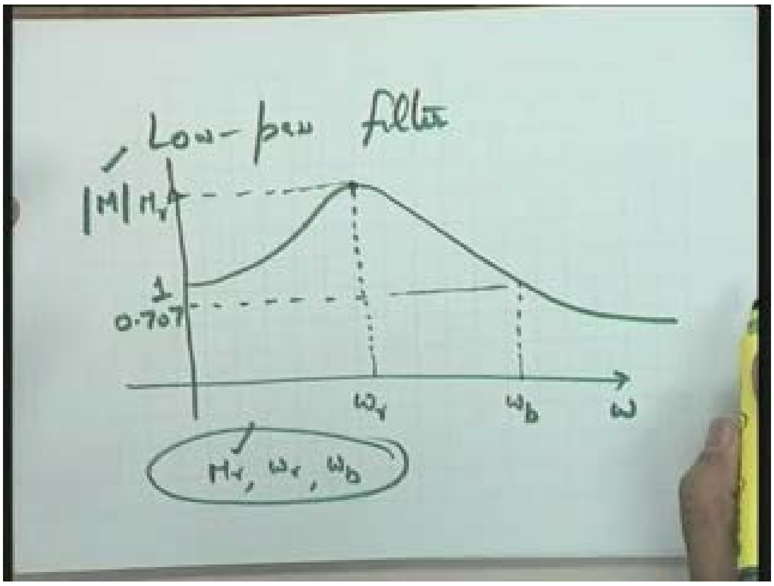

Now here we have; (Refer Slide Time: 13:48) this is minus 1 this is a, gain margin equal to 1 over a this is unity and phase margin is equal to phi phase margin is equal to phi measured positive with respect to negative real axis. I know that these two though they are obtained from the open-loop frequency response date they give you the degree of relative stability of the closed-loop system; obtained from the open-loop frequency response data this particular plot is an open-loop frequency response plot the values of gain margin and phase margin give you the indication of the closed-loop behaviour of the system. If these values are positive a large positive value will indicate the system is sufficiently stable and the small positive value will indicate that the system is close to instability. So it means the closed-loop behaviour of the system can be characterized in terms of gain margin and phase margin as well.

In the time domain if you recall there were more than one or two parameters to characterize a system, here also if I assemble everything I can say now the resonant peak M r, the resonant frequency omega r, the bandwidth omega b, the gain margin GM the phase margin PM the steady state error e ss or the constants K p K v K a equivalently all these characterize the behaviour of the system in frequency domain. The only thing is this that we have to absorb their effect when the effect is reflected in time domain because we can visualize the behaviour in time domain better. We know that whether he is tracking a step input if it is not tracking the step input how much is the transient error. Probably we can visualize the dynamic behaviour of the system in time domain better than that in frequency domain. But as I have said earlier the frequency domain design is simpler so it means somehow we should find out the way of relating the time domain to frequency domain indices so that we can have the advantages of both.

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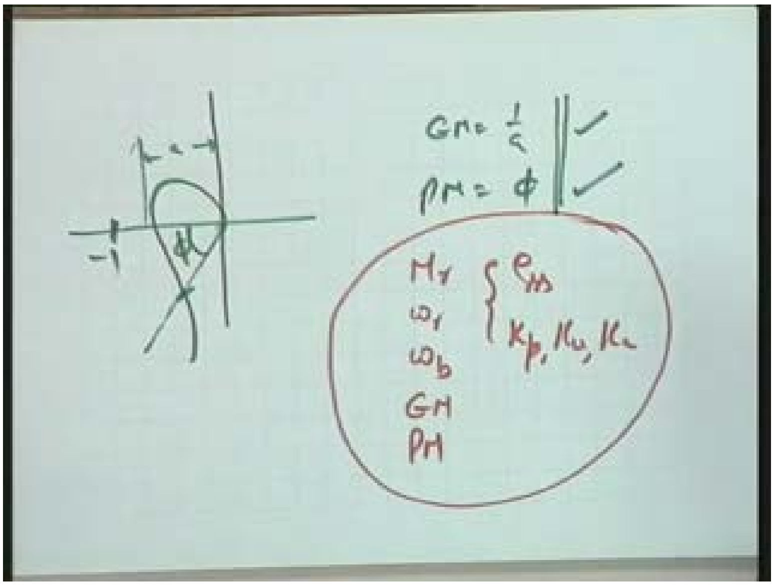

We can have the advantages representing a system in time domain and on the other side we should have the advantages of designing a system in frequency domain. So the link between the advantages of frequency domain design and appreciation of the performance in time domain can be obtained if I give you linking relationships between the time domain indices and the frequency domain indices; that way the appreciation will be better.

If you recall which definitely you would have got that particular information in signals and systems course if I have the frequency response of a system through Fourier transform its time response can be explicitly obtained. So it means at least this much I know that time domain behaviour and frequency domain behaviour are correlated. The correlation is through Fourier transform but again the Fourier transform evaluation is so difficult that may be the advantages of the correlation are lost.

So you see that since the computer is available to us if I have a frequency domain data I can come to original transfer function and simulate that transfer function in time domain and get the total response in the time domain. So inversion of Fourier transform integral or obtaining the time response from the frequency response data using Fourier transform may turn out to be counterproductive because of the complexities involved because it is a complex relationship you see and I ask myself as to what is the advantage I am getting. If through that complex relationship I have to establish a relationship why not go to the transfer function and simulate it and see how the system is going to behave in the time domain.

What I want is the approximating relationship so that quickly I can get an idea as to the values of M r, omega r, omega b, phase margin, gain margin or whatever I am choosing do they make sense when I translate the system into the time domain. If roughly they give me certain idea the exact relationship the exact time domain behaviour can definitely be obtained by complete simulation by accurate simulation exercise. So since I am interested only in the approximate relationships and I know that, well, a system of this nature omega n squared over s into s plus 2 zeta omega n you know it is a second-order system a standard second-order system you know you have seen so far through the root locus design in other methods that in most of the cases the final design you have got has got two dominant poles and other poles either they are dominant non-dominant or there is a 0 close by.

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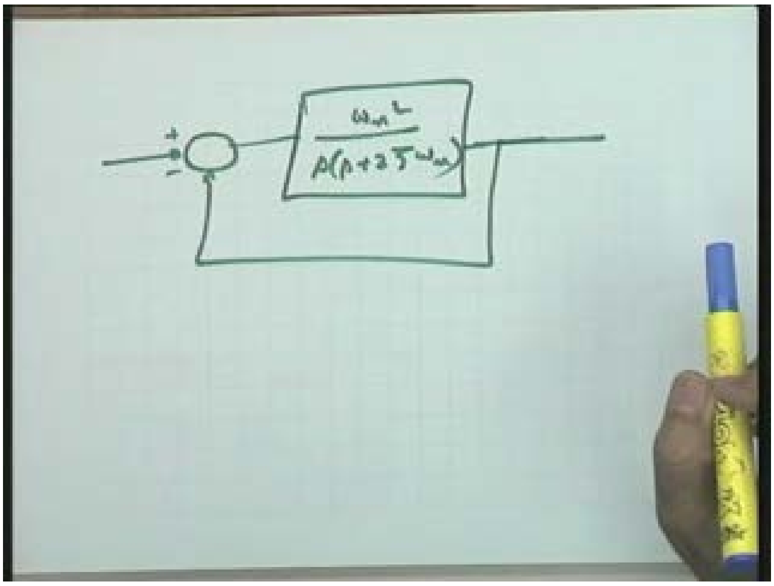

You see, in many cases you have been able to obtain such a situation and you have seen that the system behaves that way. we cannot guarantee surely, it means if it does not happen that way it means the intuitive feeling you have got by comparing your system with this system in that case is not that correct. However, you see, you are now going to accept the final design till you have confirmed it by final simulation. So making this approximation that your closed-loop system is going to behave like a system with two dominant closed-loop poles I think you are not going to make big sacrifice in making this approximation. After all if this approximation turns out to be correct in your design it is a happy situation the happy situation being the trial and error will be reduced because in one shot you have got the design. But suppose it does not turn out to be and you are just viewing this relationship with respect to the standard second-order system the only thing is that it will show up in simulation and you will be able to realize that may be the correlations you have between the time domain and frequency domain are not correct and by trial and error you will be able to tune those correlations, you will be able to get better correlations or you will be able to adjust the frequency domain parameters frequency domain specifications in such a way that the final simulation gives you the correct result.

So please, I conclude my statement with this as far as comparison of this is concerned I am at the stage making a big approximation that the closed-loop system you are going to design in frequency domain will behave equivalent to a second-order system or will have dominant pair of roots. If it has dominant pair of roots in that particular case you can quickly visualize how your system will behave in time domain. If this approximation is not correct there will be a big gap between your visualization and exact simulation. However, a trial and error will take care of this gap. I hope my point is well taken.

If that is the case then help me please what is the correlation between the time domain behaviour and the frequency domain behaviour assuming that our final system is going to behave this way. If I take this as the final system you know that your closed-loop transfer function M(s) becomes equal to omega n squared over s squared plus 2 zeta omega n s plus omega n squared this is your M(s) the closed-loop transfer function of a second-order system where zeta and omega n parameters have taken and we know the physical interpretation of zeta and omega n in terms of dynamical behaviour of the system we know that. The only thing we are saying we are assuming is that our closed-loop system is going to behave like a second-order system.

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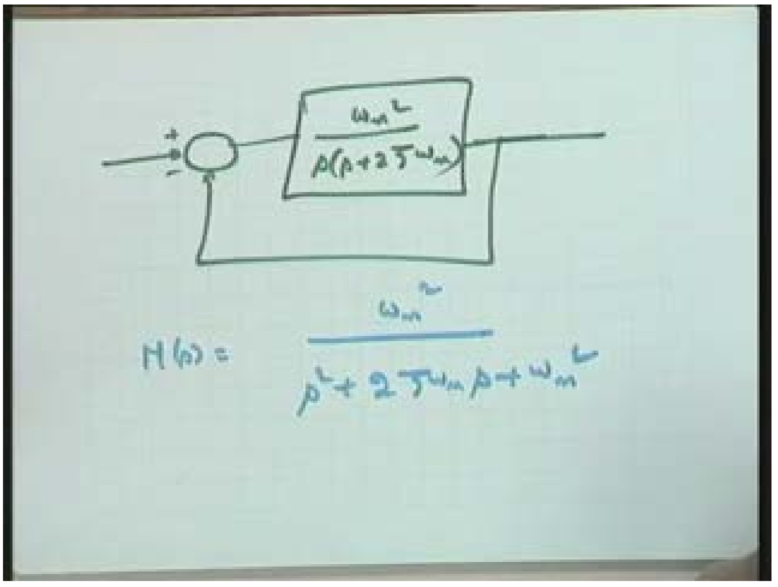

However, this approximation, please note, will not enter your design, I am re-emphasizing this point, this approximation is not re-entering your design you see, if that is the case you have to be extra careful. In the frequency domain design this approximation is not entering this approximation is only for quick visualization of the behaviour of the system which you know in frequency domain, a quick translation of that behaviour in the time domain that is all, the approximations we are going to make for that purpose only.

If that is the case in that case I will like you to help me: M(j omega) becomes equal to omega n squared over help me please minus omega squared plus j 2 zeta omega n omega plus omega n squared. Well, if I take normalized frequency u equal to omega by omega n M(ju) or M(u) whatever way you call it is it equal to 1 minus u squared plus j 2 zeta u taking in terms of normalized frequency please; omega n is the parameter of the system now, it is a fixed parameter of the system, the natural frequency. So I am taking u is equal to omega by omega n in that particular case the magnitude becomes this.

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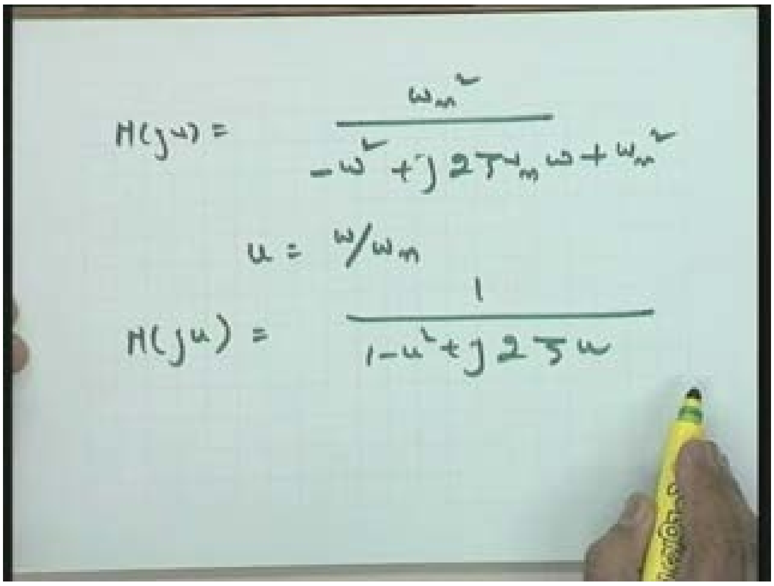

Yes, help me, how do I get the frequency at which the resonant will occur at the peak will occur of the closed-loop frequency system. now you imagine this is the closed-loop frequency response of the second-order system (Refer Slide Time: 23:34); I am interested in getting this value omega r, M r, as well as omega b these are the three parameters I want to get for this system. Give me the guideline please.

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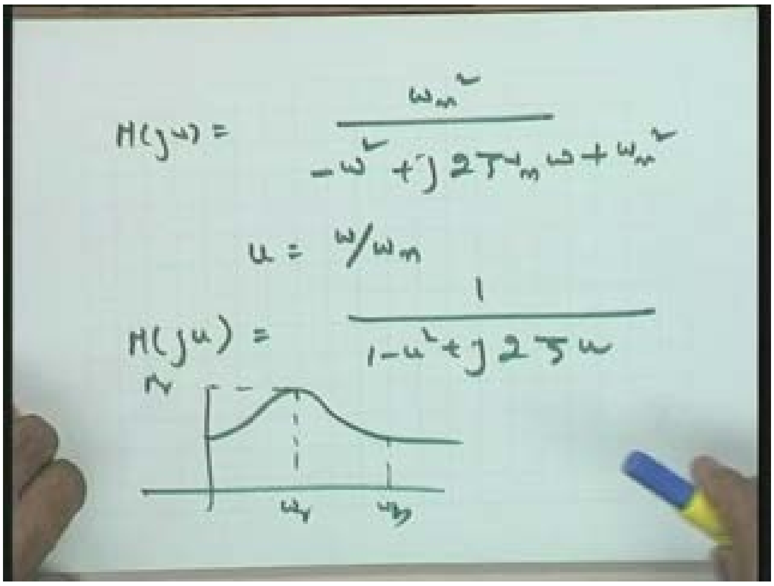

How do I get it, this value? Shall I take derivative of the magnitude with respect to u; will it be alright?

You see that first you take the magnitude the magnitude M(ju) is going to be equal to 1 over 1 minus u squared whole squared plus 2 zeta u squared under the root. This is your u please M. This M is the magnitude of the second-order system. I am going to set dM by du equal to 0 that will give me u r the normalized frequency at which the resonance occurs the peak occurs and that frequency if I substitute in M I am going to get M r, please see, I am going to get the values of u r and M r from this particular equation a second-order equation it is a simple mathematical step I want to save time here. Please see your u r turns out to be equal to 1 minus 2 zeta squared and therefore your omega r is equal to omega n 1 minus 2 zeta squared it is the resonance frequency.

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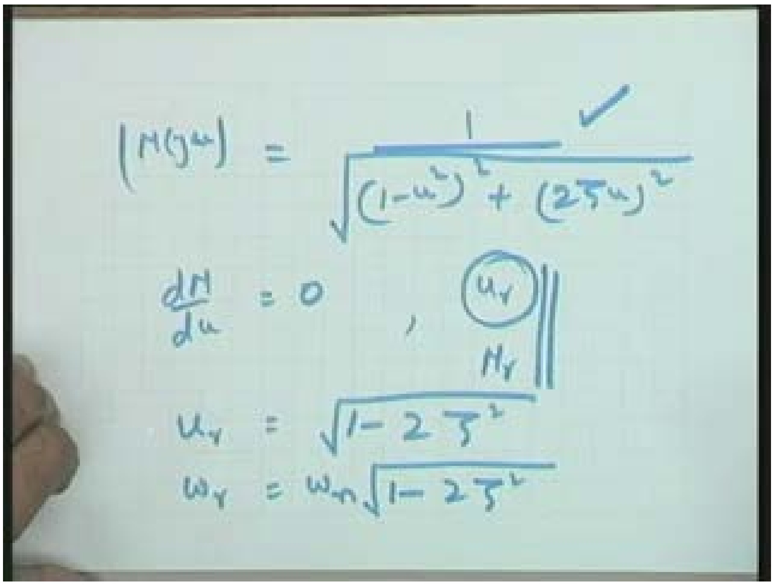

Now, corresponding M r by substituting this value of omega r I get I write here again omega r 1minus 2 zeta squared and M r just substitution gives me 1 by 2 zeta 1 minus zeta squared under the root. These are the values of omega r and M r I am getting as far as these conditions are concerned.

Now you see that, as I said, you can visualize, you can get a feel of the time domain behaviour. this particular expression you can see (Refer Slide Time: 25:50) is a direct relationship between zeta and M r and hence it is a direct relationship of resonance peak in frequency domain and peak overshoot in time domain because you know that peak overshoot in time domain is an explicit function of zeta only. So it means the quantitative index omega r which you have in frequency domain gives you a feeling of the time domain behaviour in terms of zeta.

You see that I get this plot again with respect to one so if I get it with respect to I can say typically this is for zeta equal to 0.2 and let us say as zeta increases I take this zeta is equal to 0.5 and so on. So you see that for different values of zeta I can make the sketch and I know that the peak overshoot for a second-order system is a function of zeta only. So, in frequency domain if you have got the behaviour in terms of M r you can immediately visualize as to how the system will behave in time domain because you know zeta and hence you know peak overshoot. So it means M r becomes a representative of the relative stability index in time domain because it gives you the values of zeta and hence M p.

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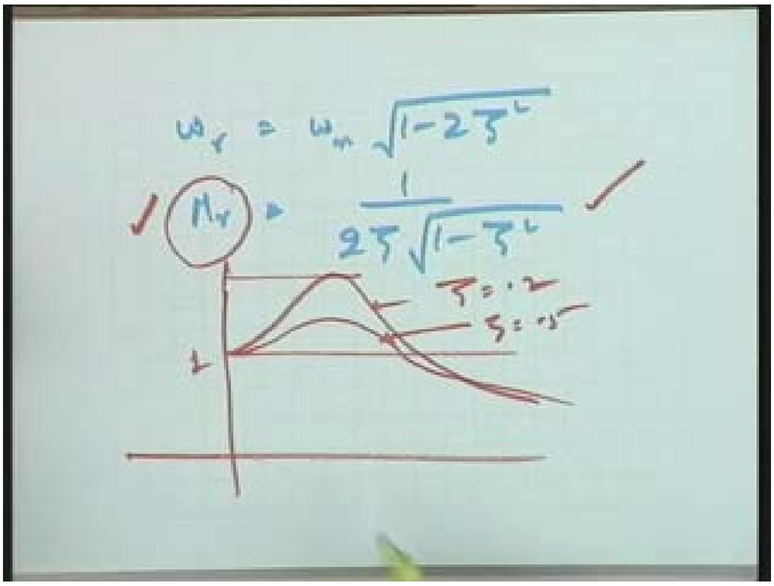

One point may please be noted that this actually is defined for zeta greater than 0 less than 0.707 otherwise the value becomes negative. So, for this range zeta between 0 and 0.707 these equations are valid. But does not matter you see, for zeta greater than 0.707 your system behaves very close to a critically damped system that is it gives you a behaviour close to zeta is equal to 1 close to a behaviour which does not have M r at all so it means for any value of zeta greater than 0.7 equivalently you can say that as if there is no overshooting here there is no resonance peak it is flat you see you can imagine you can consider after all it is an approximation you are taking. So it means if you are getting omega r close to 1 that is if you are getting as if there is no overshoot over here in terms of time domain you can say your zeta is greater than 0.7 and any value greater than 0.7 is very close to critically damped system.

You have now to know exactly whether the overshoot is 5 percent or 10 percent or 2 percent how does it matter if the overshoot is in the range of 2 to 5 percent it is equivalent to critically damped system as far as the overall behaviour of the system is concerned. You are more interested to see whether the system is going to the verge of instability or not that is more risky; whether it is 40 percent or 50 percent that is more risky and we are interested to know that behaviour; 2 percent or 5 percent will anyhow, will show up when I go to the system simulation.

Just see; if M r is exactly known to me what is omega r representing; for a given zeta your omega r is representing omega n and for a given zeta it represents omega n and hence it represents the rise time or settling time of the system. So you can say that if I give you omega r and M r as the two indices you can obtain the values of zeta and omega n and hence you can obtain the peak overshoot rise time, settling time and everything is in your hand with the only proviso that it is valid only if it can be approximated by a second-order system; if not you see you get it by simulation there is no other go because you will really not like to go by Fourier transformation to get the time domain because simulation will turn out to be an easier approach than getting it through the Fourier transform.

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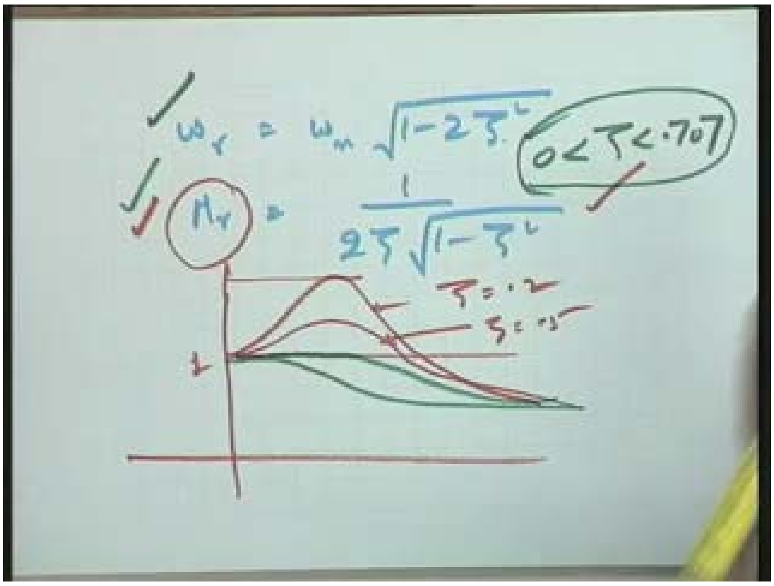

You get the first feel from here you see, it is just the designer's feel, anyhow you are not going to give the results to the user without extensive simulation so the extensive simulation will show up whether the correlation which you have realized using the second-order function is or not if not then that simulation only will guide you please note that simulation will only guide you as to what should be the value of M r and omega r in your design; no quantitative help can be given to you; this simulation and your own experience is going to guide you that in the next trial of design what values of M r and omega r should be taken.

Thus, I say that M r and omega r they become two indices in frequency domain representative of the time domain behaviour. Now, as in time domain there are more than one indices representing zeta and omega n so is the case here.

Help me how do you get I mean today I am not getting your help I want your help please. How do you get omega b for a second-order system, yes, omega b? Give me the guideline, equations I will simply write and then immediately go to the value of omega b. I am interested in the value of bandwidth in terms of the zeta and omega n. You know that your M magnitude is equal to 1 over 1 minus u squared whole squared plus 2 zeta u squared under the root. Yes please, what do you suggest?
[Conversation between student and professor……..] ((root equal to 1 by root 2)) ((00:31:27 min)) yes, here also though the equation looks a little complex but evaluating is not difficult you see; you just take the magnitude is equal to 0.707 and manipulate this equation and substitute back u is equal to omega b over omega n in that particular case u b equal to.

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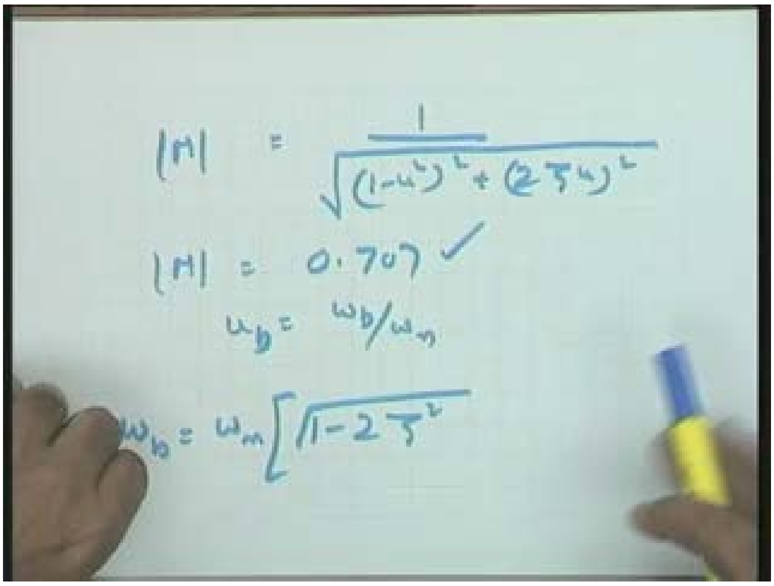

Get the value of u b from here and u b becomes equal to omega b by omega n and that gives you omega b is equal to omega n I am giving you the complete expression please 1 minus 2 zeta squared sorry sorry I should rewrite it just I have to translate it from my notes; it is omega b is equal to omega n 1 minus 2 zeta squared plus 4 zeta 4 minus 4 zeta squared plus 2 under the root it is a big expression but does not matter; I am interested in the relationship its physical significance in time domain and it gives that value.

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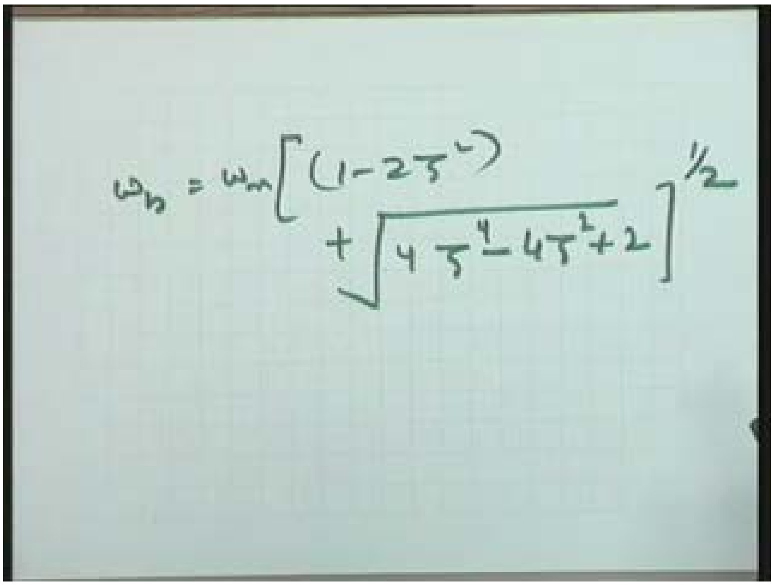

You can just see now; omega b again as I said is a representative of rise time. For a given zeta omega r relates zeta, for a given fixed zeta omega b is related to omega n and therefore the rise time or settling time requirements can be translated in terms of omega b. Of course in frequency domain with respect to noise filtering characteristics omega b has got its own interpretation also but when you say it in terms of time domain it gives you a feel as to how the system is going to perform what is going to be the rise time or settling time because using these relationships you can get the value of omega r and omega b. So you can see that I really cannot give you the values of M r, omega r, omega b independently because they are correlated. So it means if I am giving you more than two specifications to represent the dynamic behaviour of the system in frequency domain these specifications have to be consistent.

Like in the case of time domain the settling time and raise time cannot be independently given you see; you can give me anything, you see it has to be compatible with the system they have to be consistent. Similarly, in this particular case if I give you specifications more than one, so, for a particular behaviour in that particular case the specifications have to be consistent.

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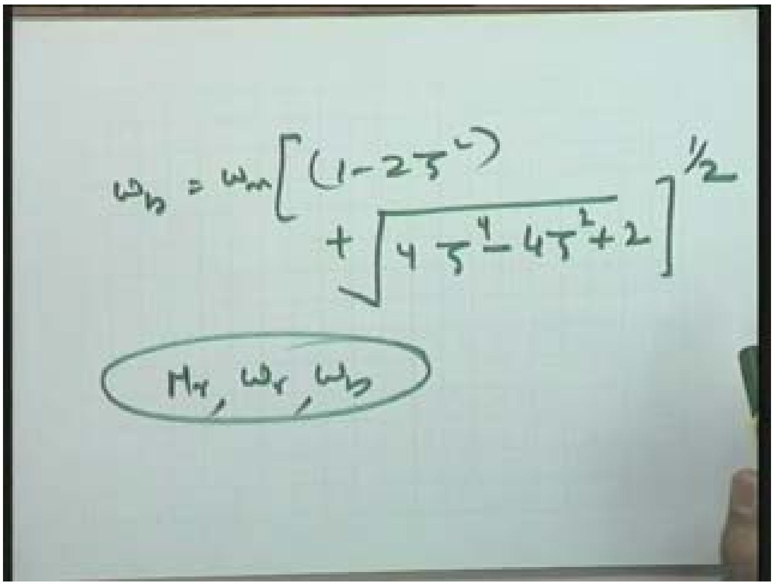

Therefore, in this case M r and omega b may turn out to be better set of specifications because omega b visualization in term of noise filtering characteristics is better. So if I say I have to define these two may be M r and omega b I will put in my design algorithm and as I check if the omega r specification is also given I will check whether that specification is also satisfied or not. But as far as the design algorithm is concerned I will like to use M r as a representative of damping and omega b for a given damping rise time or settling time these two specifications I will embed in my design algorithm and omega r can be checked, well, in addition the specification is also given.

In addition to this you see if you have more methods different methods of specifying the performance may be the design becomes simpler. Now, I have given you that the gain margin and the phase margin these two also are important characteristics important characterization variables of a closed-loop system and the advantage in gain margin and phase margin is this that you can obtain these indices from the open-loop frequency response data directly. I am explaining my point again. You see; as far as M r, omega r and omega b is concerned these are the indices you can obtain only from the closed-loop frequency response.

We are yet to see, as to, for a given open-loop frequency response data how do I get the closed-loop frequency response. If I have the computer available there is no problem I will immediately take G over 1 plus GH and evaluate using the computer. But if the computer is available if I am working on the computer may be all these design exercises will take a different turn. Taking this design this way simply means pre-computer training so that the subject becomes clear so that we have a feel of the design and the final design can be taken using CAD software. So it means I will really like to rely on these methods of indices these methods of characterizing the system behaviour (Refer Slide Time: 36:16) I need your attention on this point may be it comes today in the other class M r, omega r and omega b become different good methods of characterization if I give you methods of getting the closed-loop frequency response quickly because that is the point I have normally the open-loop frequency response data.

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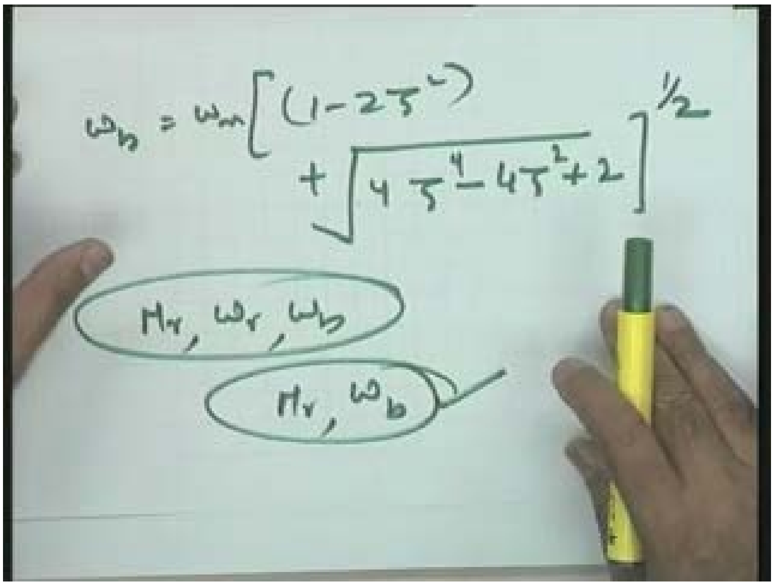

Therefore, let me see if I can characterise the system directly using open-loop frequency response data, the answer fortunately is yes because the Nyquist stability criterion using either the Nyquist plot or the bode plots gives you the information on the closed-loop behaviour using directly the open-loop frequency response data. So you see this point (Refer Slide Time: 00:36:57) this is your gain margin 1 over A and here is your phase margin. You now please relate the gain margin and phase margin for a standard second-order system.

What is the open-loop frequency of a standard second-order system?
G(s) is equal to omega n squared over s(2 zeta into s plus 2 zeta omega n) this is your open-loop frequency data open-loop transfer function of a standard second-order system. Come on please help me, I want the phase margin and gain margin for this standard second-order system directly from the open-loop frequency response data.

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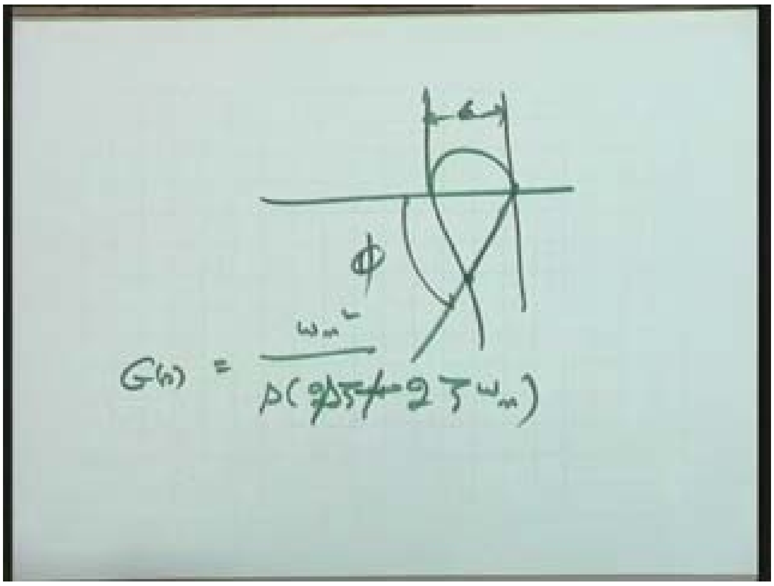

Assume that this is the plot of this system. You know that the plot of this system will not be like this, is it true the statement that if I say that the plot this system is of this nature it is a wrong statement. It is a type-1 system a second-order system what will be the nature of the plot please in this case for a standard second-order system?

This only (Refer Slide Time: 38:09) is it not? It will be asymptotic to this axis it will not cross that we have seen; it is a standard second-order system omega n squared over s(s plus 2 zeta omega n). So, as far as this system is concerned it will not cross the axis that we have seen that the polar plot of this system for omega is equal to 0 to omega is equal to infinity is going to take this shape.

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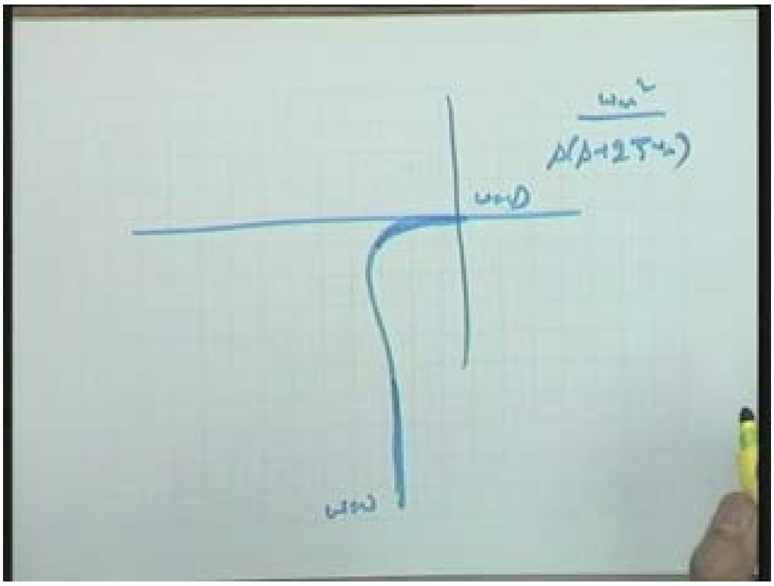

Now, in this particular case phase margin only can be calculated and the phase margin as you know will be given by this, this point (Refer Slide Time: 38:44) this point is gain crossover frequency please see it is the gain crossover frequency let me call it omega c or omega gc if you want; the gain cross over frequency is omega gc at which the gain is equal to unity and this angle measured positive with respect to this axis is your phase margin and for a standard second-order system correlations with respect to phase margin is possible but correlation with respect to gain margin is not possible for a standard system because gain margin is equal to infinity for all cases. The response is of this nature.

In that case I want you to give me the relationship for the phase margin please: G(j omega) equal to omega n squared over j omega j omega plus 2 zeta omega n no no no omega n squared s plus no that was right j that was right j omega plus 2 zeta omega n I am sorry that was right. This is your open-loop frequency response. get me the value of the gain crossover frequency please for this, how do I calculate as you have said in the earlier case I set the magnitude is equal to 1 in this particular case and the magnitude G(j omega) is equal to omega n squared over omega plus 2 into 2 zeta omega n squared plus omega n squared under the root. This is your magnitude.

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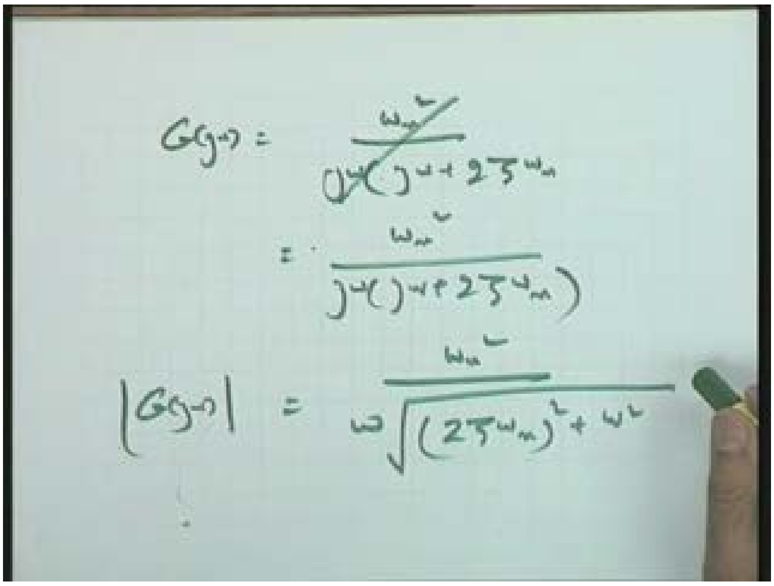

Now I want to set magnitude equal to 1. the frequency which I get by setting magnitude is equal to 1 is the gain crossover frequency and I give you here the value of the frequency assuming that you will be able to solve this equation. A little involved result but solving this equation is straightforward.

omega gain crossover frequency turns out to be omega n k a factor k is coming and this k is a function of purely zeta I give it from my notes please 4 zeta 4 plus 1e under root minus 2 zeta squared this is the value of the omega gain crossover frequency for a standard second-order system please.

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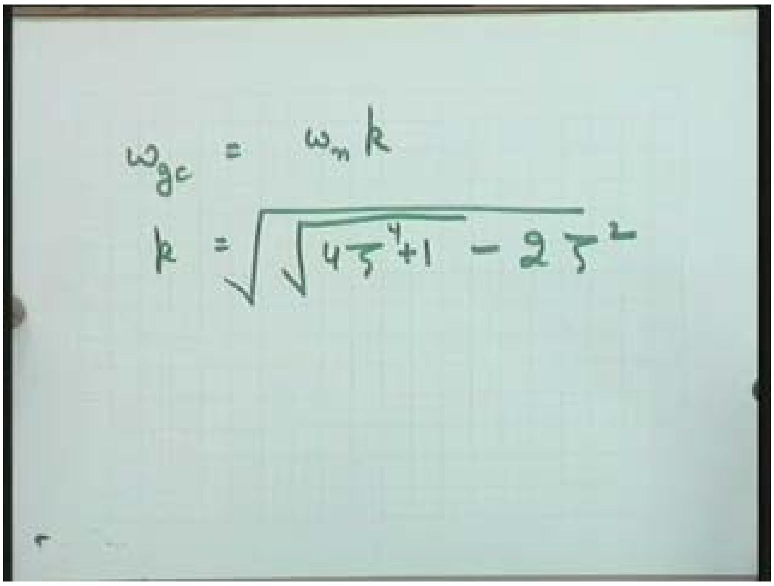

Now you can just see that omega gain crossover frequency……… the gain crossover is a function of zeta n omega n but the only good point in this particular case is that you are getting it from the open-loop frequency response data directly without plotting the closed-loop frequency response. Now you can give me the phase margin also.

What is phase margin equal to?
At this particular point I have to get the value of the phase margin. is it alright that please see this equation: 180 degrees plus the phase angle of G(j omega) I am writing as the phase margin: G(j at omega gc) is the phase margin. please see that this is the point phase of omega j gc what is this value this is the value with negative sign. So let us say if it is minus 120 in that particular case your phase margin is 60 degrees; you know that for a stable system the phase margin is positive. So this angle (Refer Slide Time: 42:23) this angle at this particular point of the standard second-order system which you have taken if this is minus 120 degrees your phase margin is 60 degrees. You can measure it from the positive direction you will have to subtract 360 degrees out of it you will get the same answer. So the phase margin becomes equal to 180 degrees plus phase of G at the frequency omega gc.

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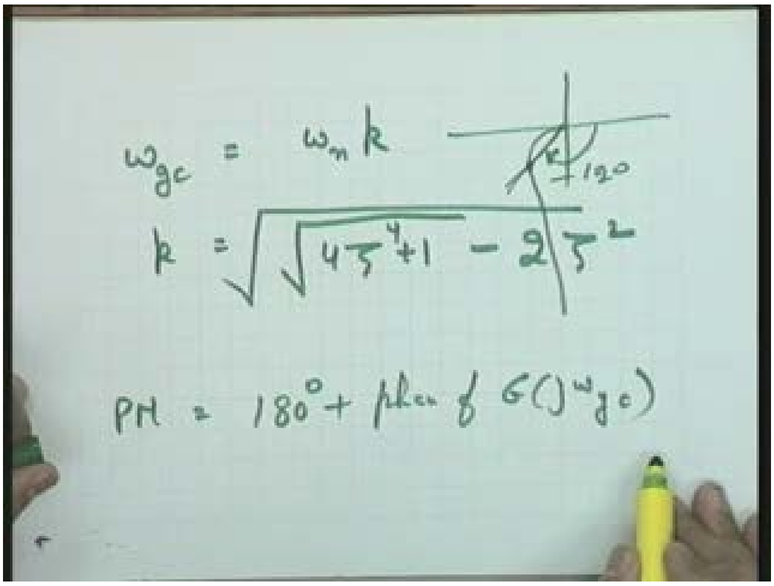

So again a simple exercise; I want you to substitute omega gc in G calculate this angle and give me the result for the phase margin. The result for the phase margin turns out to be equal to I give you the final result it is tangent inverse 2 zeta upon k. I need your comments now assuming that you will be able to verify these results the calculations.

Now look at this please; in this particular case you have the phase margin as a function of zeta only because k is a function of only zeta. It is a function of zeta only as you have seen and rather we can even make an approximation, in this particular case if you are working in low frequency range let me say it is zeta verses phase margin in degrees it turns out to be this expression please. For zeta roughly less than 0.5 the phase margin is a linear function and it turns out to be 100 into zeta roughly the phase margin turns out to be 100 into zeta if your values of zeta are less than 5 otherwise you have got the exact expression for a given value of zeta you can calculate the phase margin and vice versa. So you can find in this particular case now the phase margin and the gain crossover frequency becomes an equivalent set which represents the frequency domain behaviour of a standard second-order system.

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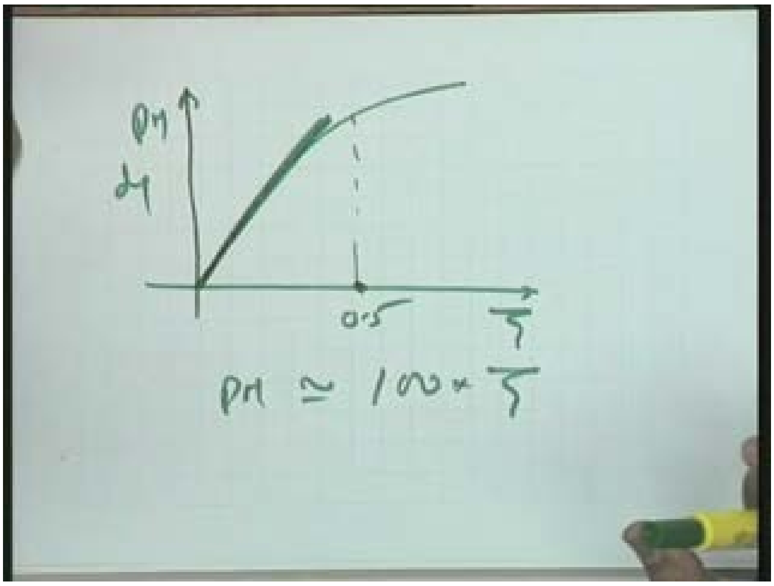

If you assume that a closed-loop higher order system is going to behave in a way standard second-order system behaves then may be the closed-loop behaviour of the system can be adequately represented in terms of the phase margin and the gain crossover frequency.

Gain margin here I am not taking because for a standard second-order system gain margin is equal to infinity. But if you are working with a higher order system naturally there will be a gain margin and this additional specification on gain margin can be given because you know intuitively though we are not deriving any relationship with zeta n omega n intuitively we know that larger the gain margin more stable is your system. So you see after all if any additional specification is given it has a meaning with respect to relative stability if the gain margin is not infinity because the system being a higher order system and it will cut the negative axis in that particular case intuitive feeling is this that I will like a sufficient gain margin in the system, the system should not be close to instability.

Therefore, as far as the total specification is concerned I can build the phase margin and the gain crossover frequency omega gc in the system specifications I can build that and in addition to phase margin and omega gc once the design is complete I can check what is the value of the gain margin intuitively feel that what should be the value, a larger value is more appropriate so that intuitive feeling will guide me whether my design is or not and the final simulation is going to confirm because the total behaviour will be available in the final case.

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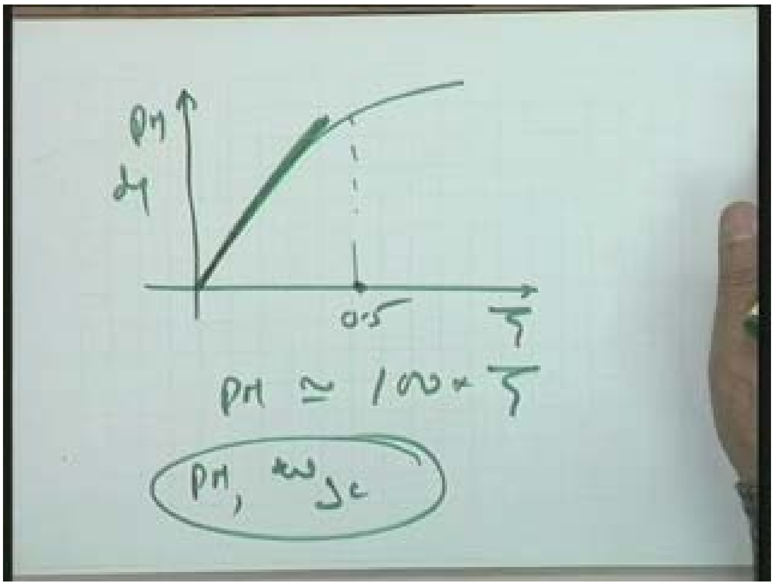

So you see that there are equivalent roots available. I can say that there are many specifications and which specification you prefer it depends. I personally feel depending upon my design experience that omega r as a representative of zeta M r sorry omega b as a representative of bandwidth is a reasonably accurate set of specifications giving the dynamic behaviour of the system in frequency domain. Do not complicate your design algorithm. you meet these two specifications on M r and bandwidth and then you see whether your other specifications are adequately satisfied or not.

Equivalently in many design problems we will solve we will take phase margin and bandwidth that is also fine because phase margin is a representative of zeta and bandwidth is a better characteristic because the bandwidth really tells me what will be the noise filtering characteristics of the system. However, if I am working with open-loop frequency response and the closed-loop frequency response I am not obtaining in that particular case may be phase margin and gain crossover frequency omega gc is a consistent set of specifications.

Now, if omega b is also given you get these specifications using your design algorithm get the closed-loop frequency response of the system and you can check whether your omega b which is given to you in terms of noise filtering requirements of the system is satisfied or not. So, for anything more than these two (Refer Slide Time: 47:45) there is no constrain, the only thing is that after the design is complete you will see whether that is also met with or not. Hopefully if the compatibility is there the specifications will be met if not then little retuning or retrial will satisfy the additional requirements as well. So this becomes the dynamic behaviour or representation of the dynamic behaviour of the system the steady-state behaviour of the system of course can be represented in terms of K p K v and K a. This represents the steady state behaviour of the system.

Hence a design problem will consist of a typical design problem M r, omega b, K v let us say or phase margin, omega b, Kv or phase margin, omega gc, K v this may be a typical set of specifications which we can accommodate in our design algorithm. Once the design is done I know intuitively what is the time behaviour because I can correlate this in terms of zeta n omega n and after that I can see whether other specifications if given are met with or not if they are met then translate your total thing into time domain and see whether time domain behaviour is satisfactory, if yes, fine, otherwise that simulation should give you the guideline as to what type of tuning of the specifications should be done so that the required behaviour obtained no other quantitative guidelines are available.

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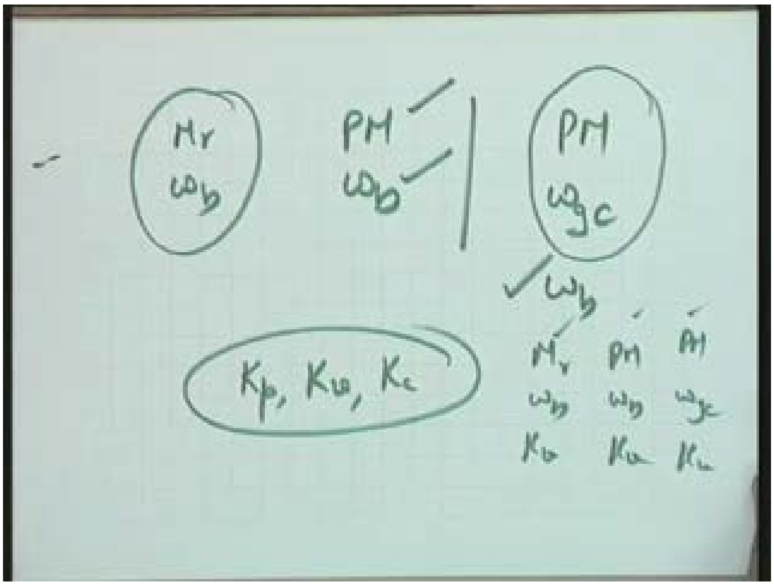

Thank you.