Control Engineering
Prof. Madan Gopal
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture - 38
The Nyquist Stability Criterion and Stability Margins (Contd.)

As I told you Bode plot is an extremely important graphical tool as far as frequency domain design is concerned. Our today's concern is to see what are the different ways the approximate Bode plot could be constructed and then how to use the Bode plots for stability analysis particularly the stability margins; the gain margins, phase margin you will be able to view on the Bode plots. So it is not just get an alternative on how to study the phase margins but as you will see the Bode plot will be the most important graphical tool available with us when we enter into the frequency domain discussion.

I had started with it and I give an idea as to what are the access, the frequency axis is a log scale and that is why we use a semi-log paper, the vertical axis for one plot it is magnitude and decibels and for the other plot it is phase angle in degrees. And I told you that there are various building blocks and if we know the Bode plot construction for the basic building blocks and after that the assembly of all those blocks, algebraic addition of all those blocks will easily give us the Bode plot the magnitude plot as well as the phase plot.

Let us get started with the basic building block. Recall what was the base first building block. The first building block we had taken was the gain K. So, looking at the gain K which may appear in any transfer function you can see that if I convert this into magnitude the decibels corresponding to gain K is equal to 20 log K and this remains this in all frequencies so naturally on a semi-log sheet……. Now, as I told you yesterday, when I take this axis it is definitely for me on a semi-log paper and on this it is db so it is going to be a horizontal line depending upon the gain K.

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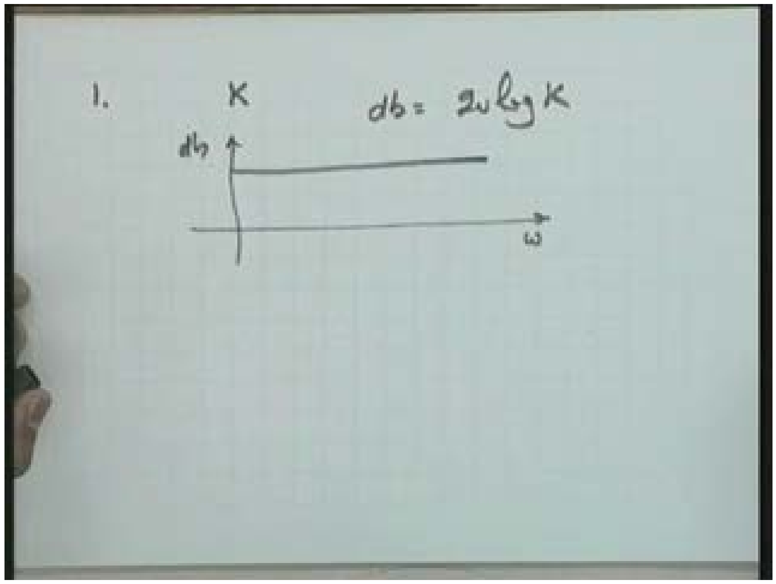

You will please note that if your gain K is greater than 1 this line will come above this particular axis which you have taken (Refer Slide Time: 3:05) and if it is less than 1db will become negative and your line will come below. So, db is a log of K 20 log K so the line coming here or here will depend upon the magnitude of K relative to unity.

Phase angle, well, it remains….. what is the phase angle in this particular case it is 0 so 0 degrees phase angle will come; since I have made this axis let me tell you that our primary concern in studying the Bode plot today is regarding stability and as you have seen, that from the Nyquist stability criterion I am more interested in 180 degree axis particularly minus 180 degrees let me call it. So when I make the phase plot please note that the axis normally taken the vertical axis normally taken is with respect to minus 180 degrees and not with respect to 0 degrees because this is our primary concern as for as stability analysis is concerned. So the vertical axis in the magnitude plot is going to be 0db and the vertical axis in the phase plot the origin of the magnitude plot is going to be 0 and the origin of the phase plot is going to be minus 180 degrees.

How about this axis please as far as the origin is concerned? (Refer Slide Time: 4:30) Will it be 0 radians per second as for as this excess is concerned, I am telling you know about the axis, I will repeat my point may be I have made some wrong statement. This is the Bode magnitude plot, the frequency on the log scale and magnitude on the linear scale and I am telling you that for the problems we will be confronted with 0db as the starting point on the vertical scale is nice. Similarly minus 180 degrees starting point on the vertical scale will be most appropriate with respect to stability analysis.

Help me please; what will be the frequency at this particular point? Shall we start at 0 radians per second or any other frequency?
Please see that though omega is marked over here it is a log scale, 0 is not defined. So in this case this point may please be noted that since you are using the log scale this particular frequency will start from a low value to a high value. the advantage, one of the other advantages

of the Bode plot you may please see that, as a control system, you know a control system is a low pass filter so it means you are really as far as the useful signals are concerned you are interested in the low frequency range as far as the analysis of the system is concerned but you want to see the effect of high frequency noise as well. So it means in your graphical representation of the performance of a system you are interested in both the low frequency range as well as the high frequency range because the total system has to be seen in the low frequency range with respect to performance, in the high frequency range with respect to disturbance rejection characteristics.

So, if you go for a Nyquist plot what will happen is, since a very high frequency is to be accommodated the total plot in the low frequency range will get compressed obviously because of the limitation of the graph sheet. Since the high frequency range you have to accommodate because you want to study the disturbance rejection characteristics so you may choose a suitable scale the low frequency region will get compressed and you may not be able to study the characteristics of the low pass filtering characteristics of the system adequately. So what will happen in the log scale you see, the low frequency range because of the log operation will get elongated and therefore you are able to effectively study the low frequency region as well as the high frequency region on a log scale. So this is an additional advantage of this log scale as far as control system study is concerned because we are able to view the performance of the system over a wider range of frequency which is of interest to us.

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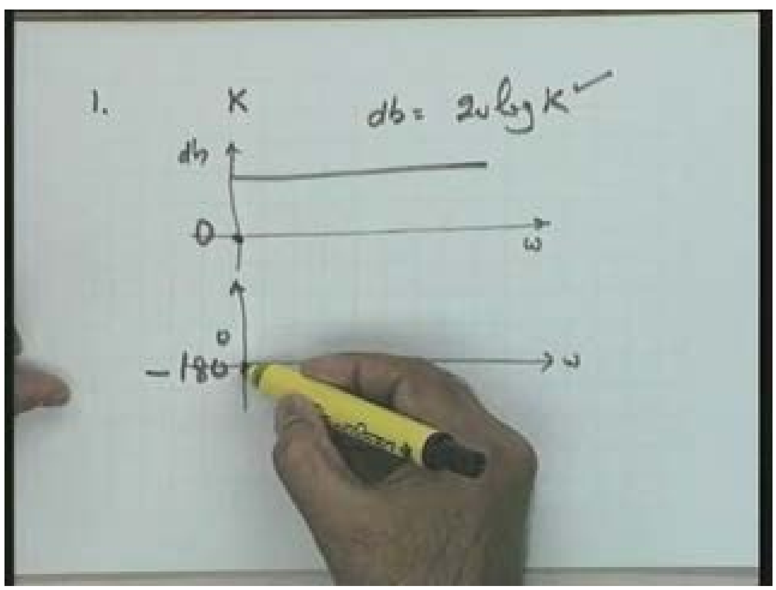

So this point may please be noted that at this particular point when I take minus 180 degrees on the vertical axis on the horizontal axis it could be let us say for example 0.01. Or if 0.01 is too low in that case 0.1. Why do I say 0.01 is too low; the reason being, if you see this particular graph sheet a typical graph sheet (Refer Slide Time: 7:52) you find that there are three decades available; I will define what is a decade multiple of 10 so you see that if you take 0.01 and the maximum as you will see shortly is if 0.01 is taken then 1, 10, 100 so 100 will be the maximum which can be seen on this graph sheet please.

0.01, this is a multiple of 10 as I am going to explain to you so this will be 0.1 and another multiple of 10 it is 1 and another multiple of 10 it is rather 10. So it means the maximum you can view in this particular case is 10 radians per second and therefore 0.01 may turn out to be too low you may not be able to use your high frequency range adequately. So deciding this you see, whether it is going to be 0.01, 0.1 or even 1 it depends upon the total high frequency range you want to study. So it means this is one important point when you make up your Bode plot the decision appropriate decision should be taken right in the beginning otherwise what may happen is that you may have constructed the plot half through half way through and you find that your plot is not adequate because it is not representing the total frequency range you are interested in. This is very important you see marking this axis as well as the horizontal axis; the horizontal axis first find out the total range and the take an appropriate decision.

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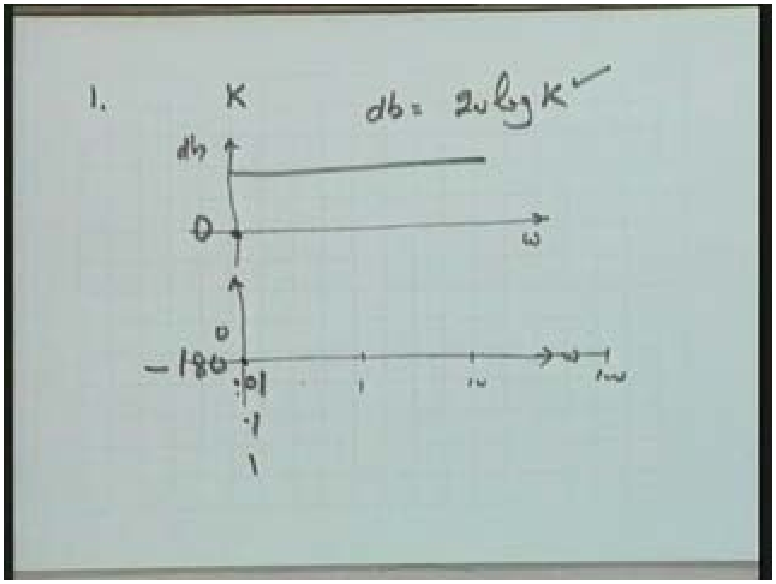

Therefore, coming back to the first building block I find that the first building block is simple constant k and it does not create any problem; a magnitude line horizontal and a zero phase line. Go to the second building block please; the second building block let me take 1 over j omega though I told you in general it is j omega plus minus N. Let me get started with only 1 over j omega and then I will see what happens if the value if the type of the system increases from 1 to 2 and so on.

Hence, in this particular case please find the phase angle picture is very clear it is minus 90 degrees phi equal to minus 90 degrees so it is going to be horizontal line for all frequencies in the phase plot I need not plot it. Let me look at the magnitude please; the magnitude is db equal to 20 log omega minus 20 please see; db is equal to minus 20 log omega.

Now what is this log?
You see, if I take omega verses db you find definitely that it is a nonlinear function omega verses db. But if I take log omega verses db in that particular case you find that you have got an equation of a straight line with a slope of minus 20db per unit of log omega. I need your attention

please; minus 20dbs per unit of log omega. This, now, as I told you yesterday that it is this contribution from Bode which converts which makes the construction of the frequency response plots very simple. This total plot becomes simply a straight line of minus 20dbs per unit of log omega.

Now what is a unit of log omega?
You know that unit of log omega will come by log 10…. factor of 10 is equal to 1 it gives you and therefore log omega 2 by omega 1 is equal to unity gives you omega 2 equal to 10 omega 1. So it means when the frequency changes by a factor of 10 what you have you have one unit of log omega, this I am telling you because what we have we are going to have this particular semi-log paper in which the factor of 10 is available to me.

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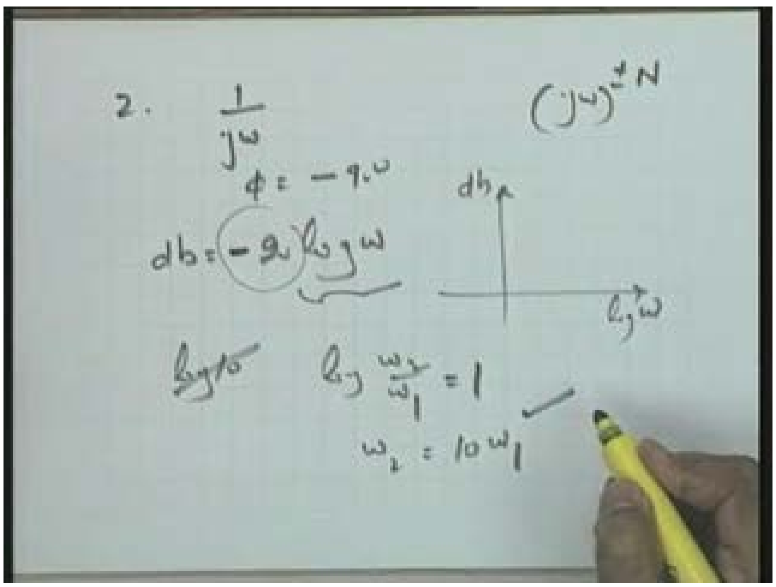

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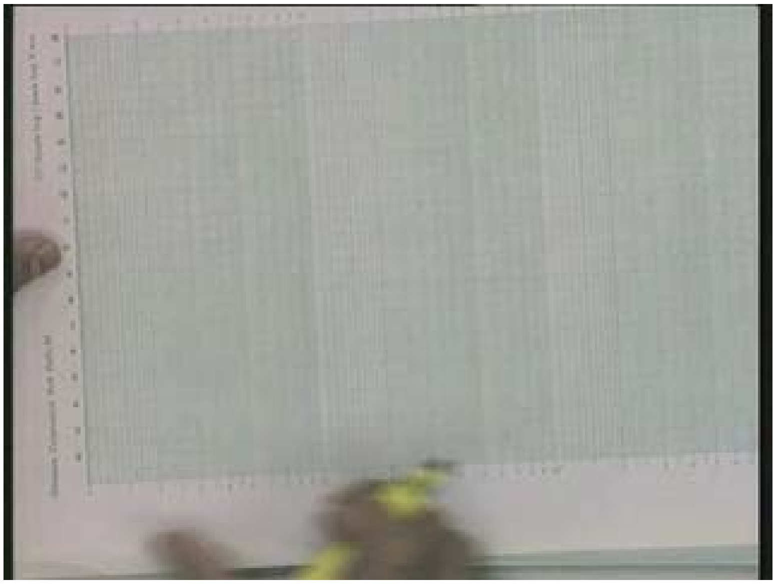

This is the factor of 10…. 1, 10, 100, 1000 and so on. So it means this is a unit as far as log omega is concerned and you find if it is visible hopefully it should be visible then after a unit of log omega you find there is a change in colour on the semi-log paper. So, if you change the frequency by a factor of 10 it means on a log scale you are changing by a unit of log omega and this is referred to as a decade of frequencies. Thus, it means, I can say that if I am using a semi-log paper in that particular case db equal to minus 20 log omega is a line is a straight line with a slope minus 20dbs per decade, decade of frequencies.

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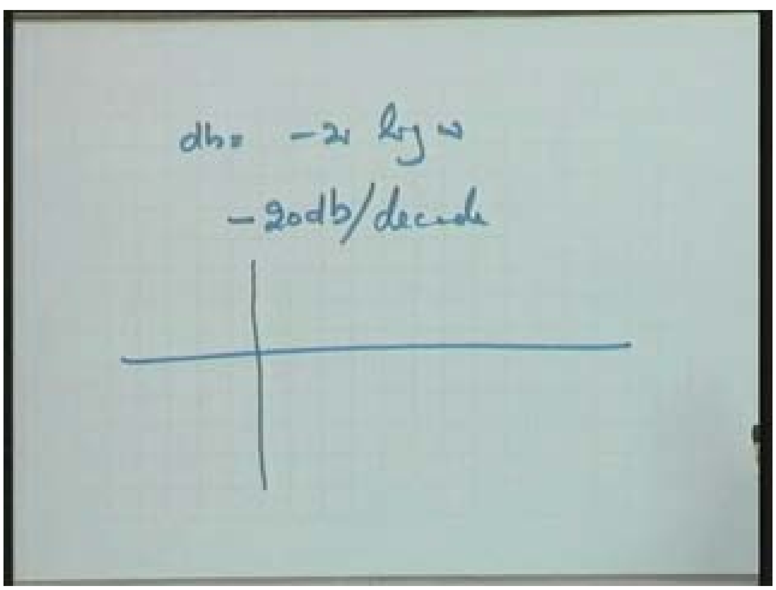

Now let me say how do I make the sketch. Magnitude sketch I want to make, the angle sketch is very simple it is minus 90 degrees. This is omega. Now please see, semi-log paper I am referring to now and this is db (Refer Slide Time: 12:54) so minus 20dbs you see that one thing what is the frequency range of interest; please see your factor is 1 over j omega and you find that this particular factor is going to contribute for all frequencies; it is significant for all frequencies right from very low frequencies omega is equal to 0 to a very high frequencies. So it means this particular factor I am interested in the overall range of frequencies let me say that this is 0.1 or any value you may take.

What is a decade? Decade is 1 over here and what is minus 20dbs per decade?
Let me take minus 20dbs here let us say 10 minus 10 minus 20dbs, let me pass a line through here, this is a line of minus 20dbs per decade. So it means I have to take a decade of frequencies and at this decade put it down by minus 20dbs, join this to this particular point and you have a line of minus 20dbs per decade.

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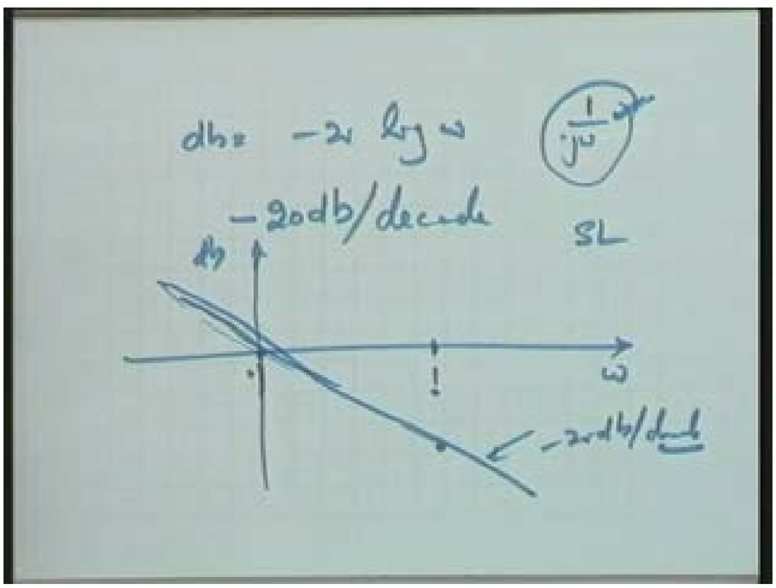

[Conversation between Student and Professor……. 1 over j omega is here ((…..13:51))] yes, 1 over j omega not j omega because it is minus 20dbs per decade I am taking. After a decade of frequency the magnitude goes down by minus 20dbs. You see that we have a line now, this line is effective in the entire region, up or downward why it should be shifted up means that it is going to be effective for frequencies lower than 0.1 as well.

[Conversation between Student and Professor……. 14:20……]…... at 0 it should be 20, yes, there is no 0 is one of one thing, sorry, though…….. at 0.1 if you are taking it is equivalently speaking if you have a paper here then you take a decade of frequencies here and j just take 20 here it depends; you want on the semi-log paper a decade; if you start from here so how do you take 20 in that particular case. To take 20 you need a decade of frequencies after a decade you take a 20 this way or minus 20 this way it is the same line a line of minus 20dbs per decade. Or equivalently speaking in this case you have to take that this particular line you can equivalently take it this way: db equal to minus 20dbs per decade or minus 20 log omega. I want a starting point please see.

Help me, what is the value….. at omega is equal to 1 this is the line I have given you. as far as minus 20dbs per decade is concerned as I suggested that let me take this point and take 20 up that way; as far as the line is concerned you take any point on this axis any point on this axis you mark a decade on this side, you take minus 20db down, you mark a decade on this side you take 20db up it is going to be a line of minus 20dbs per decade. But you see you have to tell me the plot of 1 over j omega from which point will this line pass that is very necessary.

As far as drawing the line of minus 20 degree per decade that is simple. Yes, omega equal to 1 this is what I was going to see. now you see that first is drawing a line of minus 20dbs per decade, you have to be very clear that this is your particular starting point, if you are taking higher frequencies you go minus 20db down, if you are taking lower frequencies down you go plus 20 that is 20dbs up this is equivalent; your line will be minus 20dbs per decade.

Now, from where will it pass?
Just see, at omega is equal to 1db is equal to 0. So it means you actually now will do the following. Whatever may be your frequency scale let me take it as 0.01 here, it is 0.1 here, it is 1 here and it is 10 here. Let me say this is the frequency scale I have taken. Now, at omega is equal to 1 it is minus 20dbs per decade so it means you will have to take the line passing through this point and since you have to draw line passing through this point why to make it pass though the origin and then take parallel you can straight away draw line which passes through this point and has a slope of minus 20dbs per decade. You will go to 0.1 and take a plus 20 here and let it pass through here. You will go to 0.1 and let it pass through here. Equivalently you see that in this particular case, yeah in this particular case this is good enough, this will be the right point, this will be write strategy that you take the point omega is equal to 1 and from that point let the line of minus 20dbs per decade directly pass and that you can take by going one decade down and with respect to this you take plus 20dbs up.

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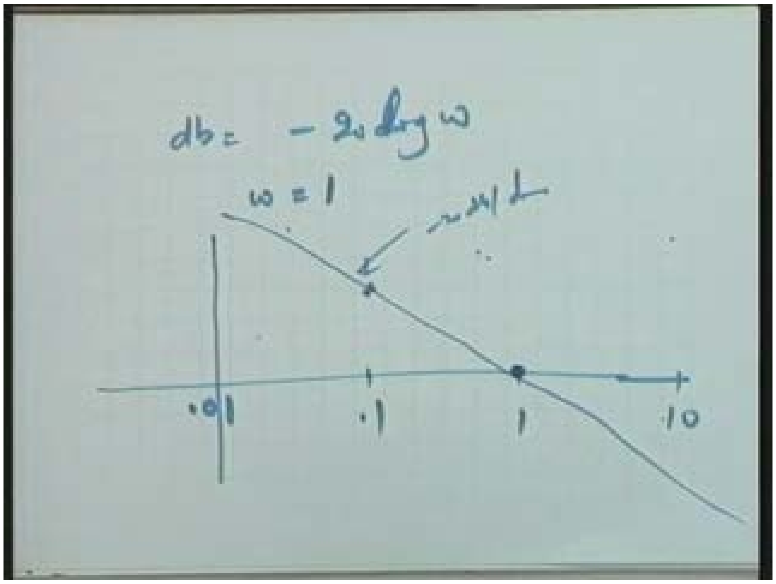

Now help me please; let me retain this particular plot; help me now, what is 1 over j omega squared?
1 over j omega squared identically now it is going to be db equal to or let me just change the colour so that you know that a different situation is being discussed now.

[Conversation between Student and Professor…… 18:04]……..minus 40dbs per decade; exactly same thing now because your magnitude is going to be db equal to minus 40 log omega. Again you will pass it through omega equal to 1 so it is going to be now a line of minus 40dbs per decade and let me tentatively take this line. And similar is the situation if you take this to be in the numerator; numerators normally we do not come across so let me not make the issue complex but you see, as far as the Bode plot construction is concerned the slope will change that is all; instead of minus 20dbs per decade it will become plus 20dbs per decade that is the slope will become this way or plus 40dbs per decade as the case may be.

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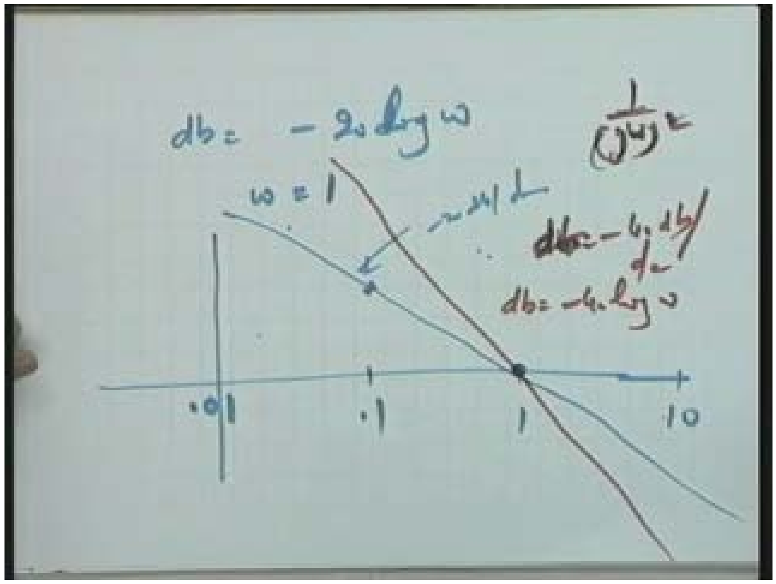

Now let me mix up the two and let me say that I have a G(s) equal to K over s or K over s squared. In this particular case please see, one aspect is this that you plot for K, you plot for 1 by s and adopt the two. But you see that this K by s factor could be taken as a single building block; it works better it works quicker. As far as the phase angle is concerned there is no problem it is again minus 90 degrees.

Let us see the magnitude: G(j omega) is equal to K over j omega. So db as you see in this particular case is minus 20 log omega plus 20 log K. So it simply shifts your point you see. Now again this is a line of minus 20dbs per decade; you can see that, you can put it this way y is equal to mx plus c. The only thing is that now constant has changed in your standard straight line equation with log omega is equal to x it is y is equal to mx plus c. So it is again minus 20dbs per decade but the only thing is this that the point from this line is to pass as shifted. Therefore, it is better to plot it directly this way instead of adding the two.

Help me, where is the point? What is the point?
You take omega is equal to 1 you find that your db is equal to 20 log K. So it means one point is this that if I take this axis omega is equal to 1 20 log K magnitude you take it can be down or up depending upon the value of K whether K is greater than 1 or less than 1. Now this is the point from where the line of minus 20dbs per decade should pass. so I will suggest that you go this way it depends how……. if you are comfortable with drawing a parallel line fine otherwise you take a point; take a point correspondingly at the location with the same db one decade below and make it 20dbs up and let it pass through this point. You see that you can take a corresponding point with the same magnitude on the other side and then take it 20dbs up. So, in that particular case you find that from this particular point if you make a line in that case minus 20dbs per decade line could be drawn.

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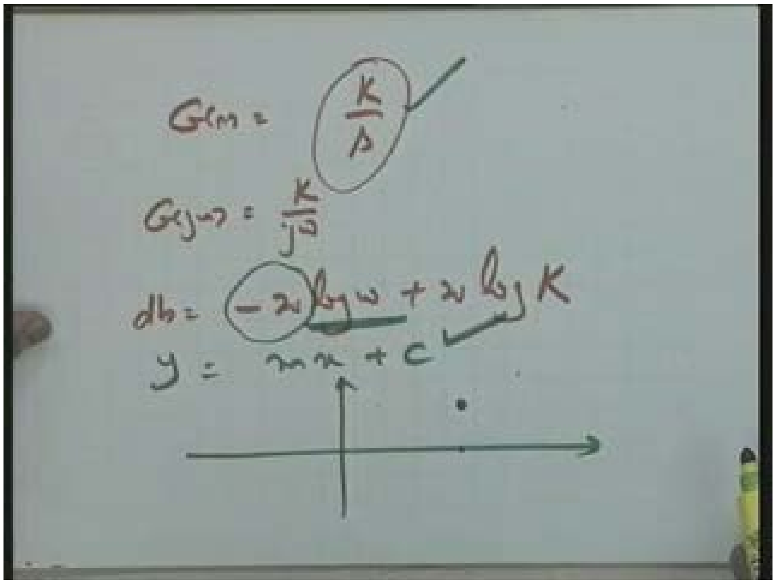

Equivalently you see that if you take omega is equal to k you find that db is equal to 0 so it means your line has to cut this horizontal axis at omega is equal to k. I leave it to you, you see what method you are more comfortable with; either you locate this particular point and pass a line through this (Refer Slide Time: 21:35) or you take omega is equal to k point on this axis and pass a line of minus 20dbs per decade through this.

Now a question please; let me change a colour here: k by j omega squared can you help me; this is okay that it will be minus 40dbs per decade I am sure you know it but help me about a point through which it pass, the point through which it pass. You can write this equation; basic equation you should write; the basic equation is: db equal to minus 40 log omega plus 20 log K [Conversation between Student and Professor – Not audible ((00:22:15 min))] yes, please see that. This is the point, it needs your attention please. if identically you increase it you see your equation is now db is equal to minus 40 log omega plus 20 log K so either you take omega is equal to K there is no problem,…….. no, omega……. No….. the same thing will be applicable there as well.

Omega is equal to k under root if you take in that particular case you can pass it through this particular point. Or if you take omega is equal to 1 rather then it will remain the same because 20 log K is the point through which the line of minus 40dbs per decade will pass. So it means either you take omega is equal to 1 locate the point 20 log K and now the line is minus 40dbs per decade or omega is equal to under root k as has been told and your line should pass through that point. I hope it is clear and it may increase to 3, 4, 5 but I told you normally we come across type-0, type-1 and type-2 systems.

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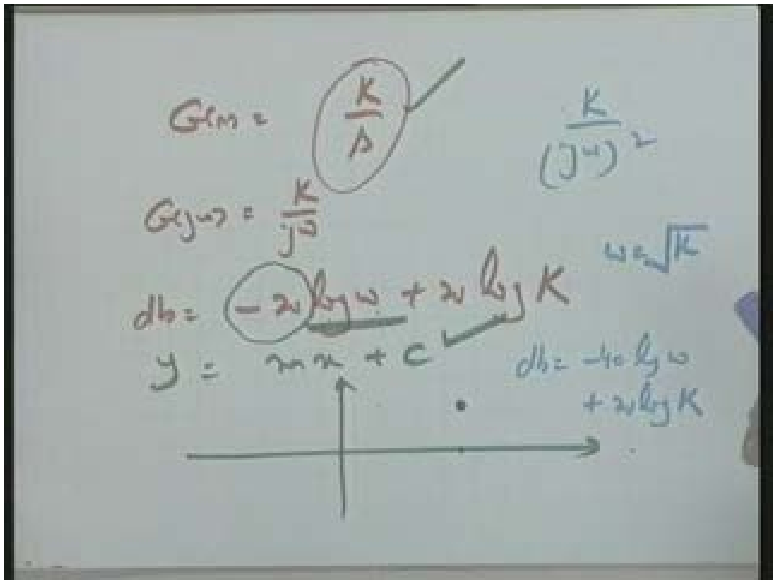

Interestingly I think let me make a mention here itself; the Bode plot if I give you, you see, if you are working with a type-0 system, in the low frequency range you will find you get a horizontal line. It will simply mean that there was only a constant k and j omega was missing, it is j omega to the power of 0 if in the low frequency region you get a line of minus 20dbs per decade. As you will see, other factors will add up in the high frequency region as I will convince you that the other factors of the transfer function will add up in the high frequency region. So interestingly the Bode plot just the very sight of the plot the magnitude side of it tells you that whether you are handling a type-0 system, a type-1 system or a type-2 system. A 0dbs per decade line, a minus 20dbs per decade line or a minus 40dbs per decade line in the low frequency region tells us whether the system under consideration may be the transfer function model is not given to you but still you can tell me which type of system you are analyzing.

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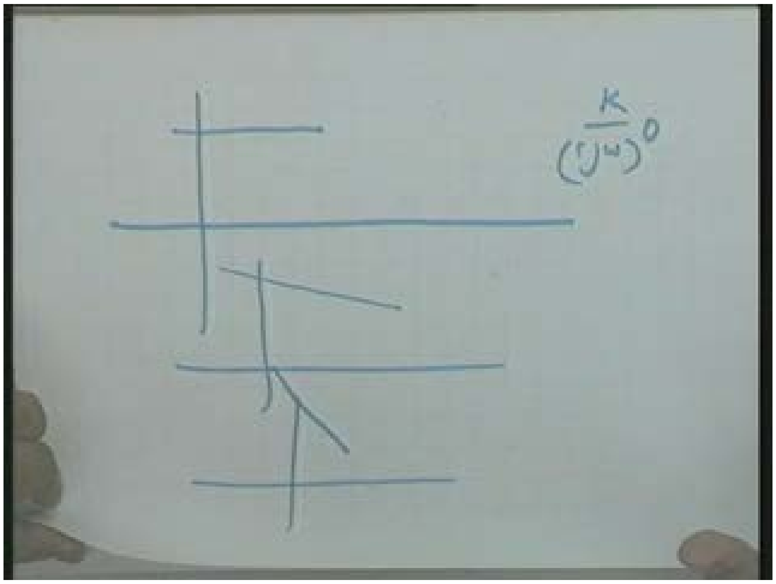

Let me go now quickly go to the other building block. With this introduction I think the speed can be increased now. G(s) is equal to 1 over 1plus j omega T a pole, a simple pole or a simple lag I can call it. By simple I mean a first-order I will make it quadratic when it is a second-order factor. Now here you see that as for as the magnitude is concerned this can be written as 1 over one plus omega squared T squared under root angle minus tangent inverse omega t.

Let us first look at the magnitude: db equal to what please? db in this particular case you find is 10 with the minus sign log 1 plus omega squared T squared. I think now some of the simple derivations I like to miss so that I can conclude the discussion on Bode plots today but those derivations…….. I will give the guidelines it will be very simple.

This is your plot without any approximation please (Refer Slide Time: 25:31) it is db is equal to minus 10 log 1 plus omega squared T squared. I hope this is well taken. Come on, now you divide this into two frequency ranges. I take a frequency range….. let us say that this is the point where omega is equal to 1 over T. Now, when omega T is less less than 1 that is you are working in this particular range because this can be neglected. If omega T is less less than 1 this can be neglected and the other is omega T is greater greater than 1 in that particular case one can be neglected and this is my frequency range.

I want you to give me a sketch for these two cases omega T is much less than 1 and omega T 1 is much greater than 1. Now, when omega T is must less than 1 you find it is a 0db line there is problem here. But let me go to omega T much greater than 1.

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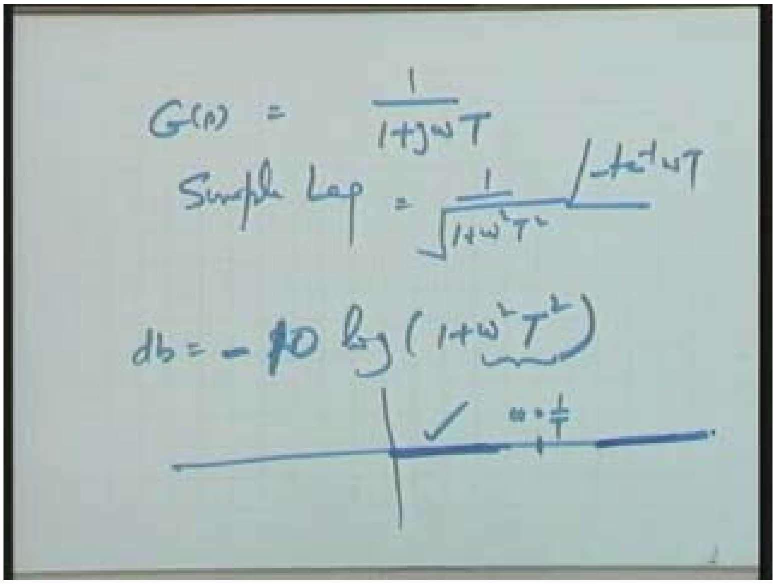

I like to write to this equation again: db is equal to so that you know that this is the exact equation.

Case 1 omega T is less less than 1 in that case db is equal to 0 I is your equation.
Case 2 omega T is greater greater than 1db equal to minus 20 log omega T is your equation you can see. db is equal to minus 20 log omega t. so you see that for omega T greater greater than 1and this, this is a straight line now. For various values of omega if I take omega verses omega T this is a straight line of minus 20dbs per decade for the frequencies under consideration.

Now what will happen at omega T is equal to 1?
At omega is T is equal to 1 it passes through db is equal to 0. So it means this equation passes through 0db line if I extend it to omega T is equal to 1. But otherwise in the large frequency range it is a range of minus 20dbs per decade of frequencies. So it means if I make a sketch of this, (Refer Slide Time: 27:47) this is omega equal to 1 over T or omega T is equal to 1 this let me call it as my corner frequency. This is the corner frequency. So, at the corner frequency you just take the two ranges: one range you take on this side and the other range you take on this side. So, if I extend these ranges then in this particular range the plot is this. If I just assume that this approximation is valid up to omega is equal to 1 over T in that case it gets extended to this point and if I assume for this range this approximation is valid again up to omega is equal to 1 over T in that particular case my line of minus 20dbs per decade passes through this particular point omega is equal to 1 over t. So I get now this as the total plot omega is equal to 1 over T as my corner frequency and this is the equation I am getting.

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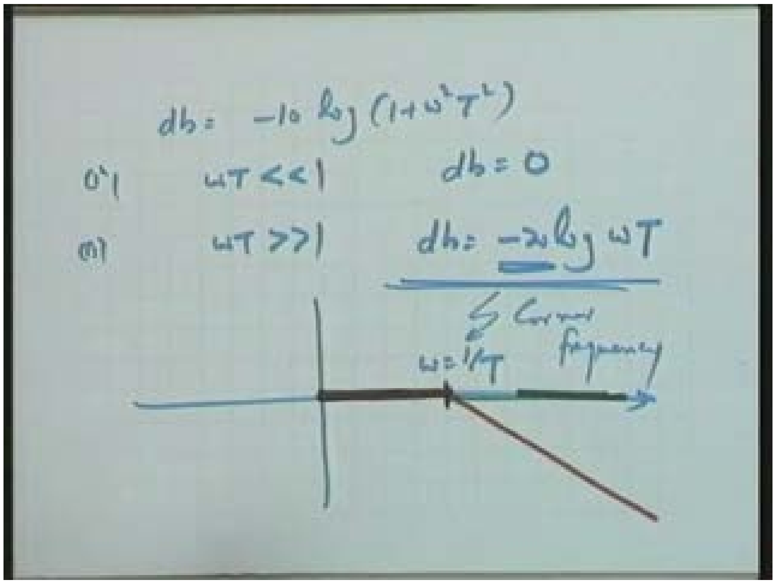

Now help me please; what is the phase angle plot? Please make a sketch.
[Conversation between student and Professor…………….28:56….] yes, that we are going to see. What is the actual plot and what is the amount of deviation I am going to see this. but help me please in this particular case what is the corresponding phase plot and let me see what are the next building blocks appropriately I should take. Help me about the corresponding phase plot please. These are exact plot of minus 20db per decade here………yes, let us see how you can see even the actual plot construction also, let me see.

Now please see as he said what is the derivation with respect to the actual plot db equal to minus 10 log 1 plus omega squared T squared is the actual value. Now this actual value you just store it and case 1db equal to 0; case 2 I am writing in this form db equal to minus 20 log omega minus 20 log T I am writing this as the second equation. For omega T greater greater than 1 my question is db is equal to minus 20 log omega minus 20 log T. Now you can see you just give me a sketch of this line it is y is equal to mx plus c; it passes through the point omega is equal to 1 over T because db will be 0 and the line is a minus 20dbs per decade. I hope you are getting my point assuming that the approximation is valid up to the point omega is equal to 1 over T. In that particular case this line is an accurate approximation otherwise there is an error between this and this.

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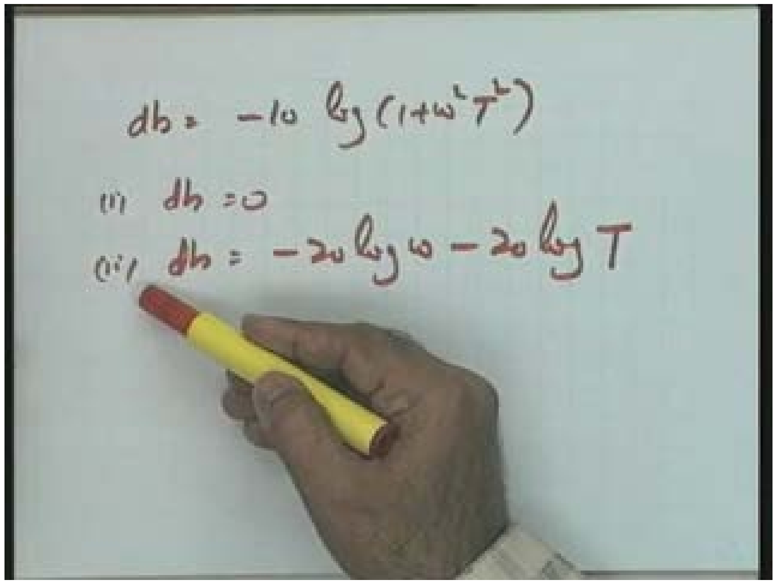

What I have to do is now I have to calculate the error between the two points between the actual point and this curve. This point may please be noted that if the approximation is valid in this particular case up to omega is equal to 1 over T you have this line. If in this particular case the approximation is valid up to omega is equal to 1 over T you have this line so these two independently are accurate plots provided the approximations are valid.

Now let…….. yes, [Conversation between student and Professor…………….30:59]……. ((what will be the valid because actually at omega d is equal to 1 it should be minus 3db)), yes. Now, since they are not valid I have to calculate the error as he has pointed out. If I calculate the actual plot you see you just take various values of omegas and make an actual plot of this; the actual plot of this turns out to be this. I am making an approximation. So the green one is an actual one plotted from here and this red one is from here and this red one is from here. This is approximation extended up to omega is equal to 1 over t, this is an approximation extended again up to 1 over omega is equal to 1 over T.

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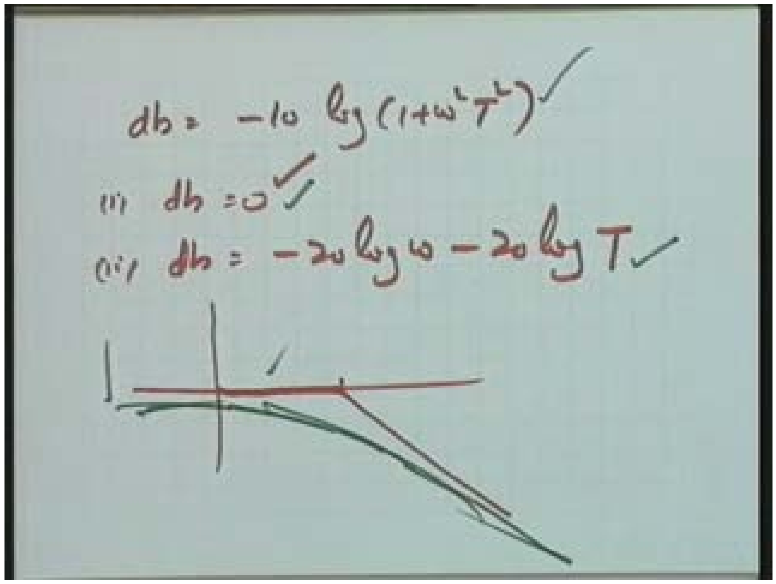

Now the question is this; if I have to make this particular green plot by using the calculations from this particular equation then the advantage of the Bode construction is lost; the power of the method is this that if you give me the error at one or two crucial points between the green and the red plots I will be able to construct an approximate green plot which will be quite close to the one one can obtain using this particular equation and the answer has already been given by one of you that if you do the calculations at this particular point at omega is equal to 1 over T this gives you minus 3db.

You can do this calculation please; at omega is equal to 1 over T it is going to be minus 10 log 2 and minus 10 log 2 is minus 3. So it means at this particular corner frequency as I have named it the actual value is 3db down. You see; what normally is done is that couple of more points are taken and the normal practice is this that the one point which is taken here is at a frequency which is half of this frequency. So, if this omega is equal to 1 over T I take omega is equal to 1 over 2 T here it is a normal practice so you can take it more than that or you can take it less than that. And another point which is normally taken is two times this frequency which I take 2 by T here.

So if you put omega is equal to 1 by 2 T and calculate what will be the error; the error will be between this and this, you will find this error turns out to be minus 1db, you can please check. Put omega is equal to 1 by 2 T and take the difference between this and this the error is minus 1db; you put the value omega is equal to 2 by T and take the error between this value and this value now you will find again the error is minus 1db approximately.

So, at frequencies 1 by 2 T and a frequency 2 by T the errors are minus 1db here and minus 1db here and these three points are found quite adequate to make the green plot that is to make the correct plot the accurate plot for this particular system which takes care of the approximations and this frequency omega 2 by omega 1 is equal to 2 a factor of 2 on this side or a factor of 2 on the other side is refer to as an octave of frequency like a decade is a factor of 10 an octave is a factor of 2.

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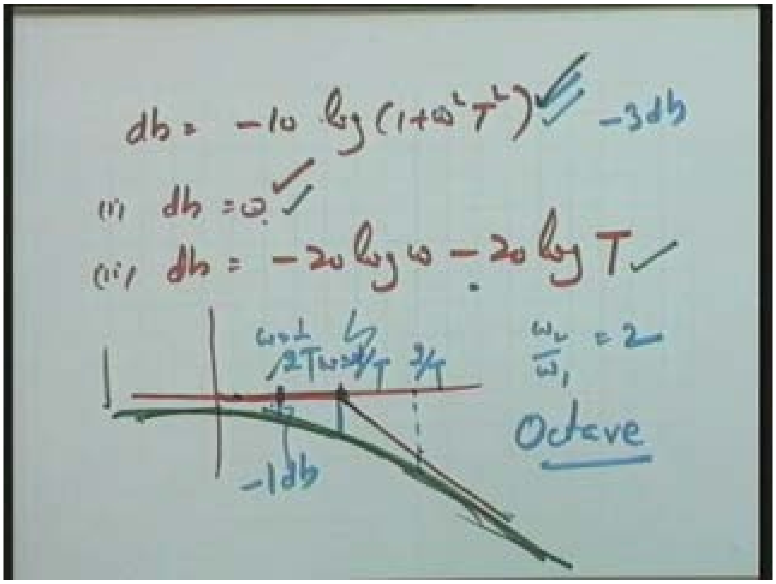

Now I sum up and wait for your question please. I sum up now the factor G(s) is equal to 1 over 1 plus J omega T this is my factor and the method is the following magnitude plot. I take omega is equal to 1 over T as the corner frequency and I make this line and this line a line of minus 20dbs per decade. You will please note one point that if you calculate this at omega is equal to 2 by T this will turn out be minus 6db please this magnitude and the hence minus 20dbs per decade is same as minus 6dbs per octave. This may be helpful in case the graph paper is not available for a decade; one octave on the other side also you can take to make a line of minus 20dbs per decade. Minus 6dbs are octave and minus 20dbs per decade are identical they are same.

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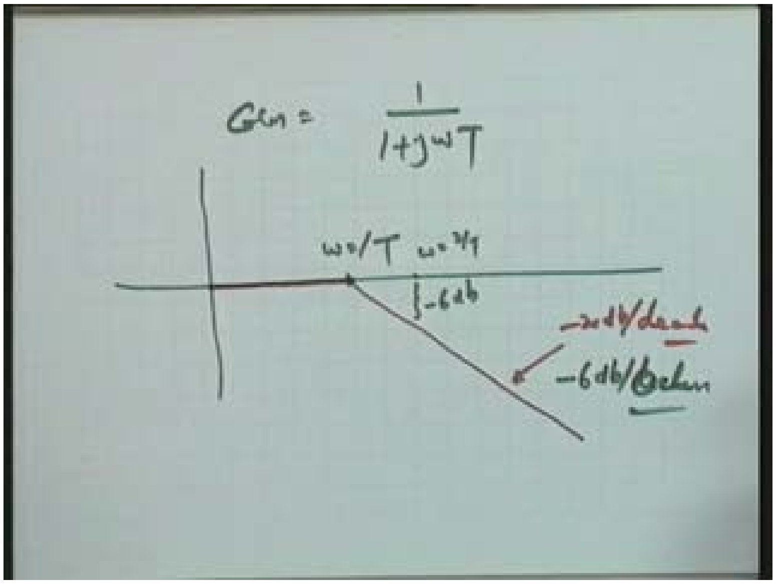

This is your corner frequency and this is called asymptotic Bode plot please. Now, from the asymptotic Bode plot the actual plot will come in the following way; just mark a point 3db down, take one octave below, mark a point 1db down, take one octave above mark a point 1db down and join these making asymptotic to the 0db line and asymptotic to minus 20dbs per decade line this becomes your accurate Bode plot without any calculations.

[Conversation between Student and Professor……….36:17…….((Sir ,what is minus 3db comes sir))] minus 3db comes actual value is db is equal to minus 10 log of 1 plus omega squared T squared this is the actual value corresponding to the green curve. An approximate value we have taken 0db is equal to 0 so this omega is equal to 1 over T gives you minus 3db that is all and identically for omega…….. this is the simple calculation; at omega is equal 1 by 2T or omega is equal to 2 by T I am subtracting it from the asymptotic value. So this way we get the green curve.

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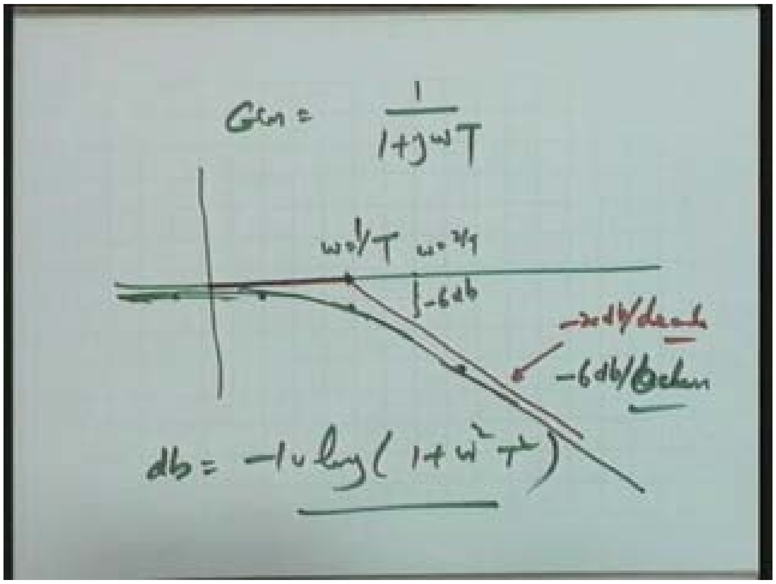

Now look at the zeros, yes, zeros we do come across; G(s) is equal to 1 plus j omega T; now 1 over j omega T if I take please help me; now I think I have to speed up with this introduction; 1 by T here so it is going to be the asymptotic plot, this is your line minus 6dbs per octave or minus 20dbs per decade. Now you can check the errors are going to be plus. You take 3db up, you take 1db up at one octave below the free corner frequency, you take 1db up at one octave above the corner frequency, make a line asymptotic to the two you get this particular thing. I hope you are getting my point.

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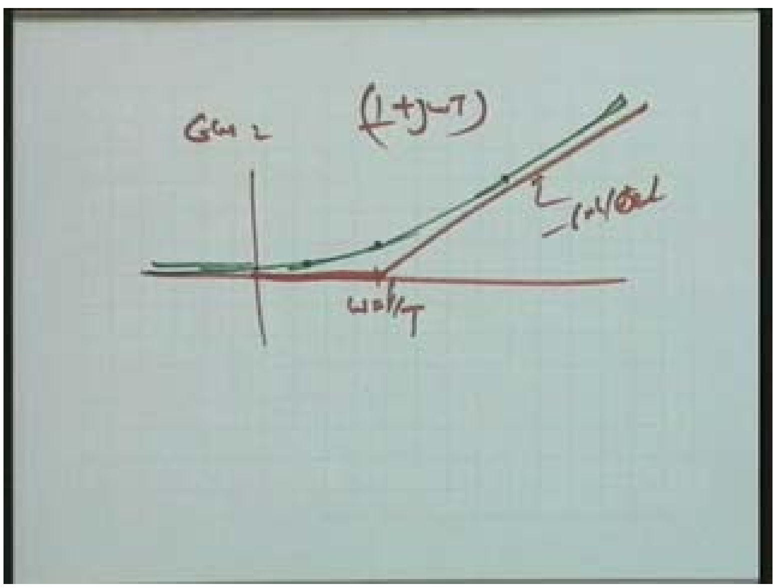

Now I can just keep on extending the cases 1 over 1 plus j omega T squared a double pole. Come on please, orally I think we need not even draw. It is minus 40dbs per decade as the line; at the corner frequency omega is equal to 1 over T note the word corner frequency it is omega is equal to 1 over T when your factor is in the pole in the time constant form. 1 plus j omega T is the form of the factor we have taken. Now minus 40dbs per decade and how about the errors; please see that the errors will get algebraically added up. It is minus 6db at the corner frequency minus 2db at one octave below and above the corner frequency.

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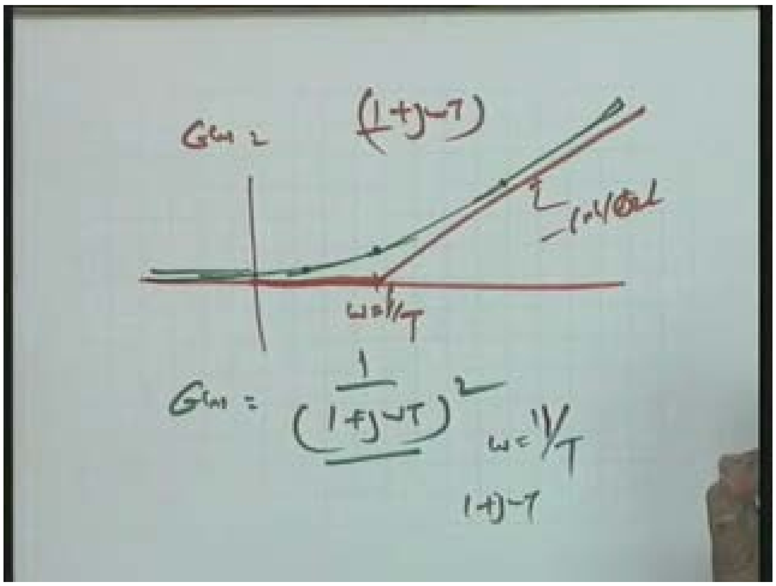

I am left with couple of more…… oh yes, angle for this please, angle for this I have to take. G(j omega) equal to 1 plus j omega T. now please see that if you see different books as far as the angle plot is concerned different methods will be given; at least five seven different methods you will be able to get, different methods means actually different approximations.

What I feel is the following: angle of G(j omega) equal to minus tangent inverse omega T and if I make a sketch omega is equal to 0 gives me 0 and omega is equal to infinity gives me minus 90 degrees and at omega is equal to 1 by T I get minus 45 degrees. So you see that the angle plot is the following please: skew symmetric with respect to the corner frequency is a tangential function.

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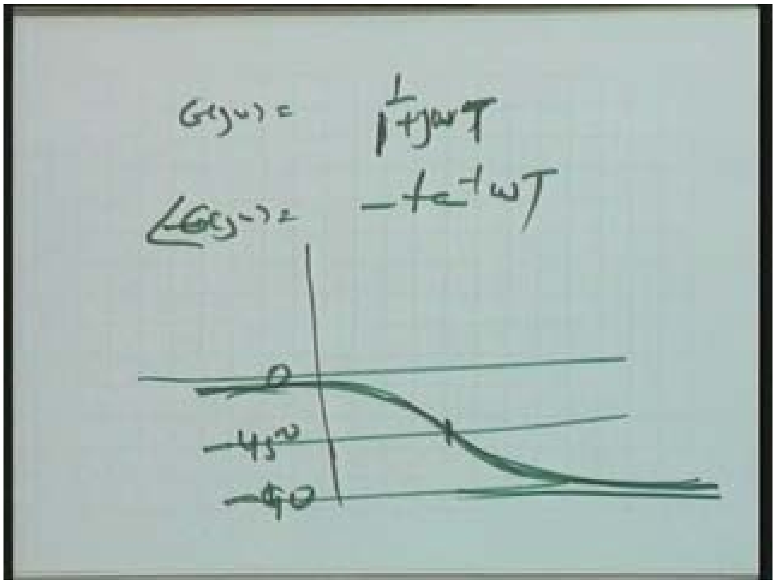

Now I say that many books will give different methods. You see that they approximate it by the one that you make a template and keep on using it you want to avoid calculations. The other, well, you just approximate it omega is equal to 1 by T is here; go one decade below so it is omega is equal to 1 by 10 T draw a vertical line, go one decade up above omega is equal to 10 by T draw a vertical line, join these two lines pass through passing through this point this red straight line may be taken as the asymptotic approximation of the angle plot. So, that way you see, if you are getting a factor of double pole in that case it will become 0 minus 90 minus 180 degrees so this will be a straight line passing through minus 90 degrees line and at omega is equal to 1 over T. So one decade below, one decade above, join these two points (Refer Slide Time: 40:34) it is a reasonably correct approximation but this approximation can give you an error to the tune of 6 degrees and 6 degrees error in phase margin calculations is an appreciable error and fortunately the calculations of this factor minus tangent inverse omega T is not that clumsy. You see, after all you have to simply press the calculator buttons if you are not sitting on the computer assuming that you are really not……….. if you are sitting on the computer terminals then no approximation is involved. We are assuming that it is a paper and pencil design and with it also the calculator can be added to the tools and if that is the case probably the angle calculations is not very difficult and all the angles get added up so my summation my view point is this that the approximation in the angle plot should not be made.

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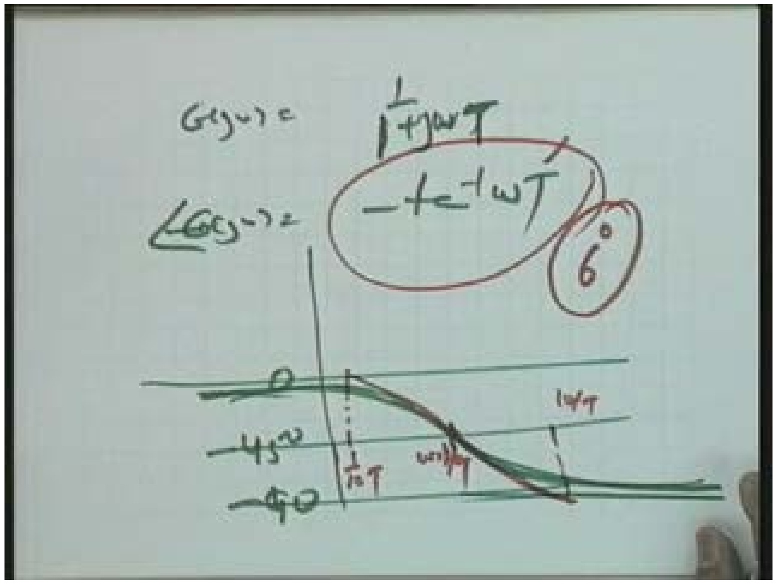
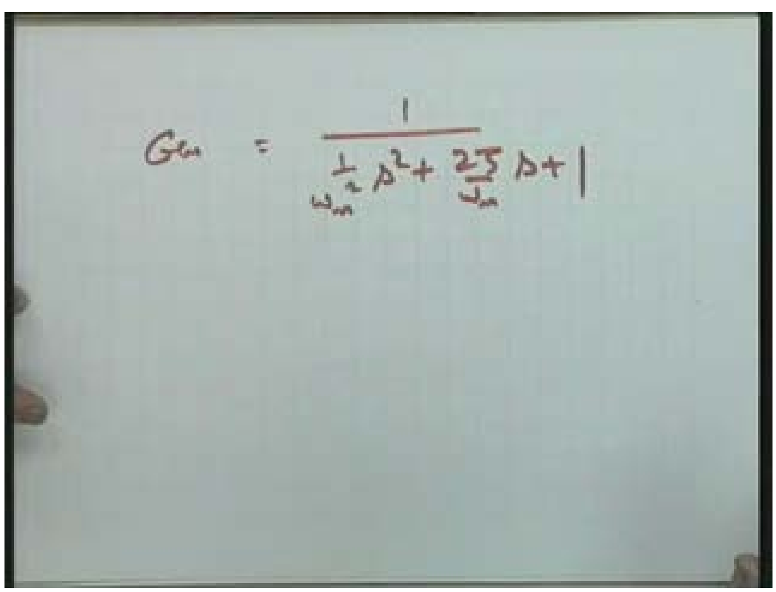


Otherwise yes it is a straight line approximation and keep on adding up the various straight lines for various plots because angles are getting added and a suitable asymptotic approximation a suitable straight line approximation for the angle plot also you will get. But I think you can avoid that. Now, one more factor is there and that factor is G(s) H(s) or only G(s) is equal to 1 over 1 over omega n squared s squared plus 2 zeta omega and s plus 1 this is one more factor. Please see; before I go to this factor I will give you the idea how to take up the phase margin and gain margin on the Bode plot because even if I do not get time for this I will tell you to study of your own but gain margin phase margin idea I have to give.

(Refer Slide Time: 42:20)



Please help me; if I take a typical, okay let me take a…….. I will come back to this let me retain this if get time I will come back to this. Take a typical function G(s) is equal to k over s(1 plus sT 1) (1 plus sT 2) please make a sketch of this, the magnitude plot. The magnitude plot in this case you will help me please how do I get started? You please see that omega one is low frequency plot let me first take K is equal to 10 typically let me take K is equal to 10 so 10 by j omega is one factor and this is going to give you a line of minus 20dbs per decade passing through omega is equal to 10 or an appropriate magnitude over here corresponding to omega is equal to 1.

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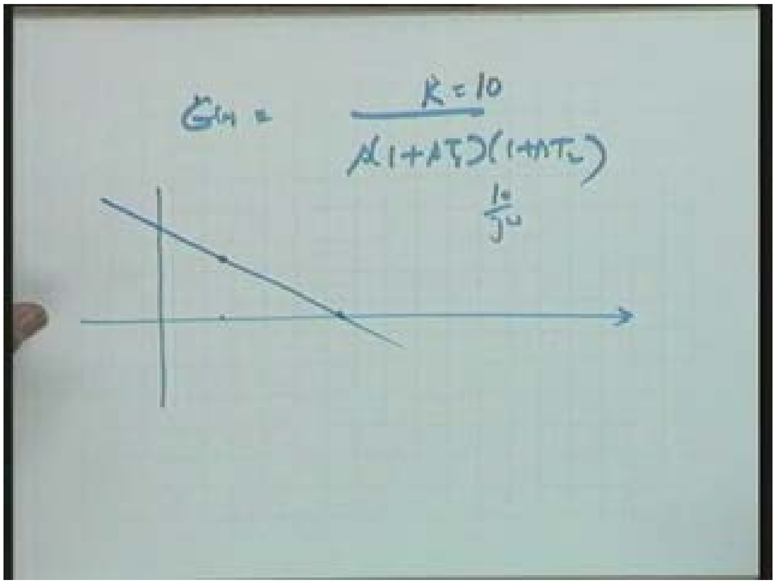

Take the next corner frequency please. Now please see that this plot is valid for the entire frequency range. Now you can take 1 over 1 plus sT 1 what is the plot of this assuming that omega is equal to 1 T 1 lies over here. You see; in this particular case let me change the colour over here this is your plot as far as omega is equal to 1 T 1 is concerned (Refer Slide Time: 43:35) as far as this pole is concerned.

Now if you adopt these two plots please note that it is equivalent to changing the slope of the original line which was otherwise valid for the entire frequency range changing the slope of this original at the corner frequency by a factor of minus 20dbs per decade. So this original line was earlier minus 20dbs per decade; from this corner frequency the slope changes by another minus 20dbs per decade so at this particular corner frequency you change this slope by minus 20 so its total becomes minus 40 and hence the addition of the 2 is a line of minus 40dbs per decade passing through the corner frequency taken on this particular line.

Same argument is valid for the third case. Let us say omega is equal to 1 by T 2 is lying here omega is equal to 1 by T 2 and this is the minus 20dbs per decade plot. Now, if you adopted these three it is…….. you see after all up to this point it does add up anything it is a 0db line as far as asymptotic plot is concerned (Refer Slide Time: 44:45) so it means what I have to do is I have to take this particular point on this line and adopt the slope by another minus 20dbs per decade so it becomes minus 60dbs per decade and this become my line and hence this becomes the total magnitude plot for this particular system asymptotic.

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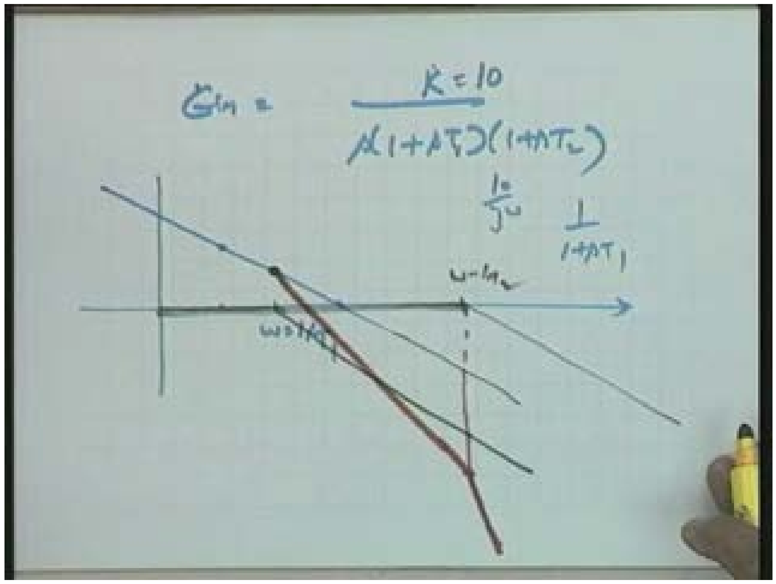

Thus, you see that procedure will be; define the corner frequencies and at each corner frequency keep on changing the slope by a factor of minus 20db if the pole is in the denominator plus 20dbs if the factor is in the numerator because the numerator one is a 0 which contributes a slope of plus 20dbs per decade.

Now how about the approximations?
The approximations can be made like this the corrections can be made like this: take the corner frequency, take a point 3db down why down because the slope changed by minus 20dbs; if there was a change in this slope by plus 20dbs I would have taken this 3db up. So in this case it changes by minus 20dbs. Now it is not the slope at this particular line but whether this error will be up or down depends upon the net change at the corner frequency whether it is with a negative sign or a positive sign. In this particular case it is with a negative sign so I take it down and here 1db one octave below is here and one octave above is here.

Note interestingly so if I take now another point it is 3db down here; one octave above okay is here, one octave below is here, one octave below could be here also does not matter it is just the algebraic addition; it does not mean that one octave below cannot cross this particular point or it cannot be at the same point and if it is at the same point it becomes 2db down please see that it is just an algebraic addition; one octave above because of this factor has come over here one octave below because of this factor can come at the same point (Refer Slide Time: 46:54) in that particular case net error at this particular point will become minus 2dbs and hence a plot passing through suitably all through these points is the accurate plot of the system.

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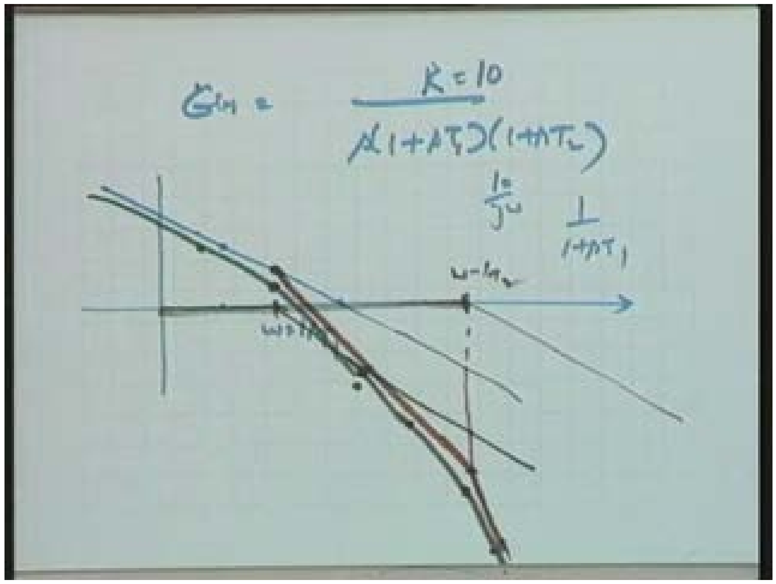

Help me please; what is the gain crossover frequency. Now recall the Nyquist plot, Nyquist plot for this particular system, yes it will be this way is it not? For this particular system the Nyquist plot is going to take this shape. The frequency at which the magnitude is 1 we have said that it is our gain crossover frequency. So naturally the green line intersecting this particular point is your gain crossover frequency.

Now you said that the phase angle with respect to this minus 180 degrees line is your phase margin. Can I say, you see looking at the time I do not want to refresh it or do not want to draw fresh graph but I hope you will not get confused with this if I draw a Bode plot right here. This is minus 180 degrees line, I am now making a phase angle plot; phase angle plot in this particular case please help me; what is the total frequency angle range; can I say that it will be minus 90 to minus 270 is it visible? I think it is visible from here also minus 90 to minus 270 from the polar plot otherwise you can say that for omega tending to 0 it is minus 90 and for omega tending to infinity it is minus 270 it is very clear from here. So it means I expect the plot, this is minus 90 this is minus 270 I expect the magnitude the phase angle plot to be of this nature asymptotic to minus 90 line on this side asymptotic to minus 270 line on this side and this is visible from here also.

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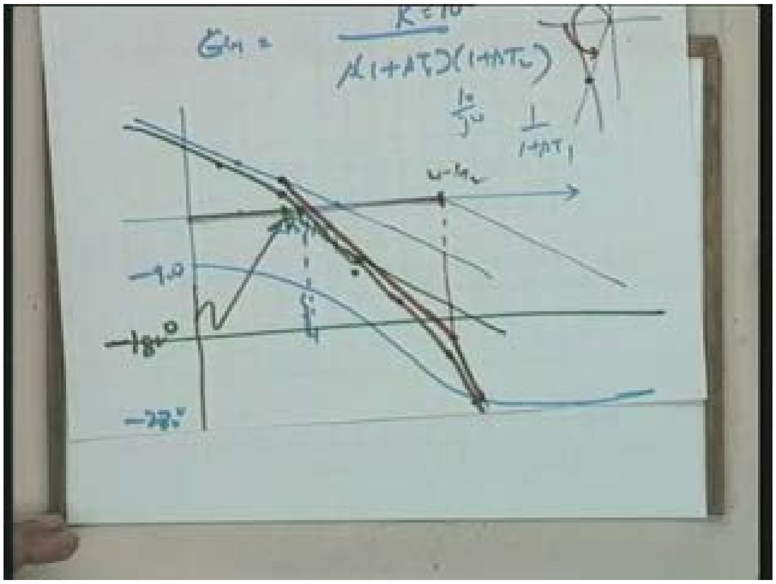

Can you tell me please what is the phase margin?
Can I say that this is the phase margin because this is the angle which can be added to this system to make the phase angle minus 180 degrees so this becomes the phase margin of the system please see.

[Conversation between student and Professor………………49:12………] phase plot I am saying the phase plot in this particular case phase angle 5 minus 90 degrees minus tangent inverse omega T 1 tau 1 minus tangent inverse omega tau 2; let omega go to 0 it is minus 90 degrees, let omega go to infinity it is minus 270 degrees. So I have an entire range of frequencies. So in the lower range of frequencies it becomes tangential to minus I do not have omega is equal to 0 so I am going to make it tangential to minus 90 line and in higher frequency range I am going to make it tangential to minus 270 degrees line; this is my vertical axis as far as the phase angle plot is concerned.

Now what are the different points here?
I am saying that you calculate instead of taking the straight line asymptotes because they are going to give you lot of errors. So you make a couple of calculations in between and draw a rough plot that gives you the phase angle plot and at the gain crossover frequency if I calculate the phase angle I say that this is the phase margin and this if you take it this way it is a positive phase margin if you take it this way it is the negative phase margin because this much of angle can be added to make it minus 180 degrees so this is the phase margin on this particular axis which you have to now take with respect to minus 180 line that is why the importance I told you when you take the phase angle plot you make your plot with respect to minus 180 degrees line.

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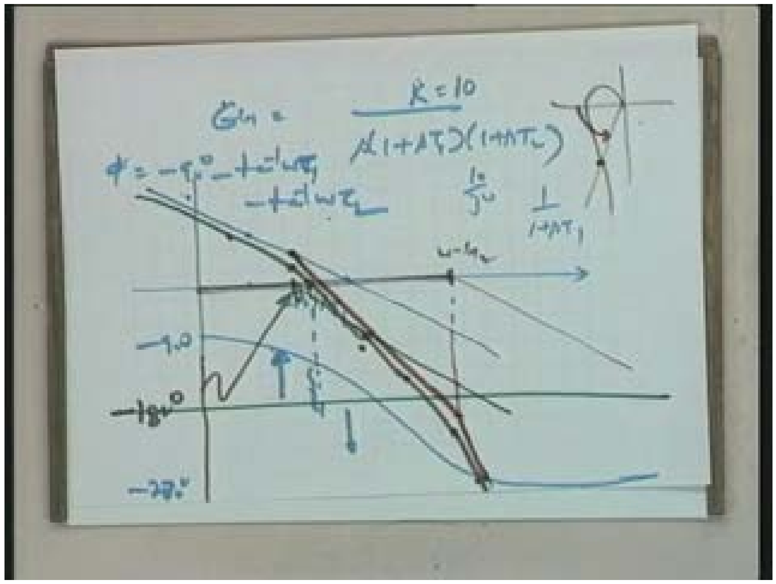

Can you now tell me what is the phase crossover frequency this will become the phase crossover frequency?

This will become the phase crossover frequency the frequency at which the angle becomes minus 180 degrees. This is the corresponding point here. Help me how to get the gain margin what is this; this is 1 by a and 1 by a makes it unity and unit here means 0db 20 log 1 is equal to 0 so if I take this vertical line (Refer Slide Time: 51:09) please see that this much is the gain margin; this much of gain I can add to the system, this much of gain I can add, this much db is the gain margin, this much of gain I can add to this system before the system becomes unstable or before at this particular angle this angle will be minus…. you see this angle will be minus 180 degree magnitude will be 0dbs it means it is equivalent to the Nyquist plot that passes through the minus 1 plus j0 point. So intersection of this you have to take this much of db you have to take and this gives you the gain margin, this gives you the phase margin and the plot of this type of factor (Refer Slide Time: 51:55) please do read I will be taking up a numerical example in the tutorial class so that this gets covered and we start with a new topic next time, thank you.