Control Engineering
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Lecture - 37
The Nyquist Stability Criterion and Stability Margins (Contd.)

I think we can now resume our discussion on the Nyquist stability criterion. The use of the criterion for study of absolute stability has already been taken. Now let us take how to use the criterion for the study of relative stability. You already know what relative stability is. Let me take you to the time domain; let us say this is a time domain plot (Refer Slide Time: 1:17) and the closed-loop this is sigma j omega s plane I am taking these are the two closed-loop poles the dominant poles I have taken.

Now if you take sigma and j omega axis here and the two poles if they are taken close to the j omega axis you know that the transient in this particular system will die faster compared to this particular case and therefore you can say that the relative stability of the system is guided by the distance of the real part of these poles with respect to the imaginary axis. You already know that this real part guides the envelop of the oscillating response. So we know that the closer the dominant poles are to the j omega axis poorer is the relative stability of the system.

Let us interpret this in frequency domain. In frequency domain, if I consider this system and make a Nyquist plot of it this is now the GH plane that is G(s) H(s) plane where GH is the open-loop transfer function of the system. So this is real part, this is imaginary part, this is minus 1. When I say the Nyquist plot primarily you see it is the polar plot because its mirror image about the real axis and then closing the path will give you the complete Nyquist plot.

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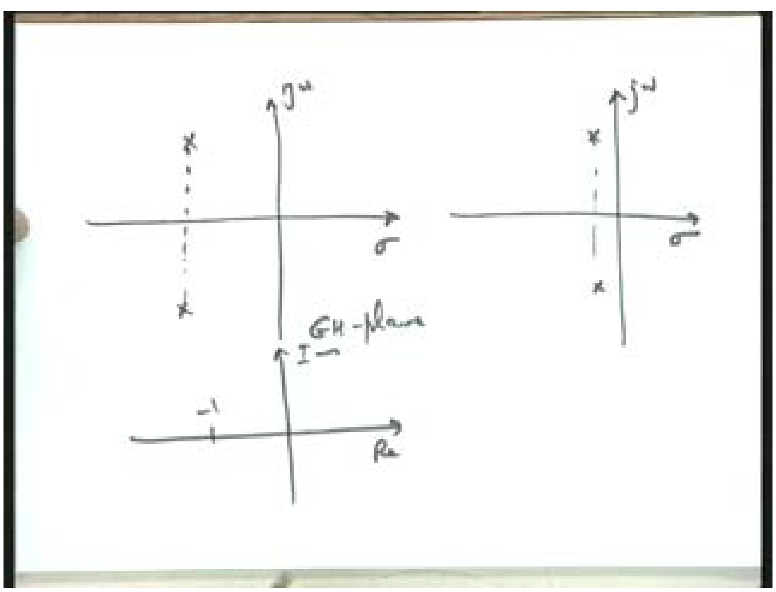

In this particular case please see I expect that the plot will be of this nature. For example, just I am giving you an example, the plot will be of this nature while in this particular case I expect

this is real, this is imaginary, this is your minus 1 point the plot will be of this nature. What is the different between the two; how do I evaluate which system is better?
You please see that the enclosure of the minus 1 point by the Nyquist plot is the index of absolute stability. Now you can, I think, initiatively feel that closer this plot is to the minus 1 point (Refer Slide Time: 3:30) more is the risk of system going towards instability as is the case over here that is here the index was the distance with respect to imaginary axis.

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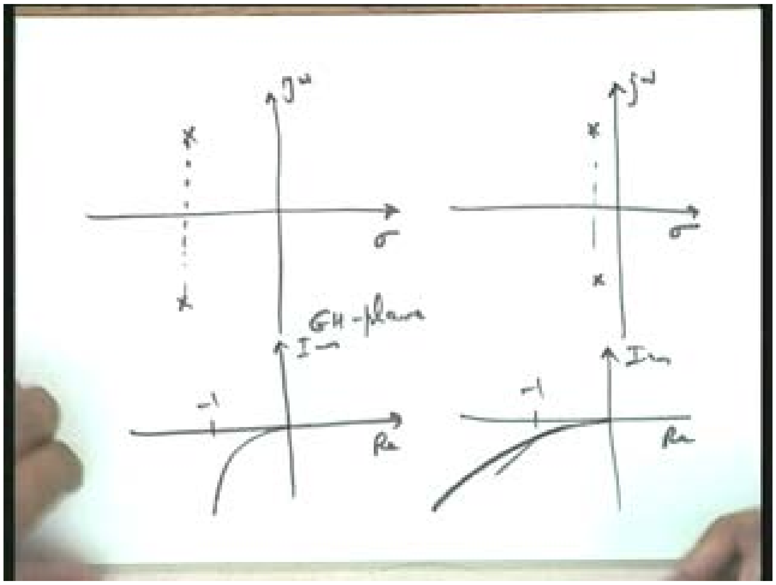

Now here I can say that the polar plot portion, you see that if I give you a complete Nyquist plot you will realize that primarily we are interested in the polar plot; this is real part, this is imaginary part, this is minus 1 plus j0 a typical system, this is omega is equal to 0 plus, this is omega is equal to plus infinity, I have omega is equal to minus infinity and going to omega is equal to 0 minus and this is getting closed.

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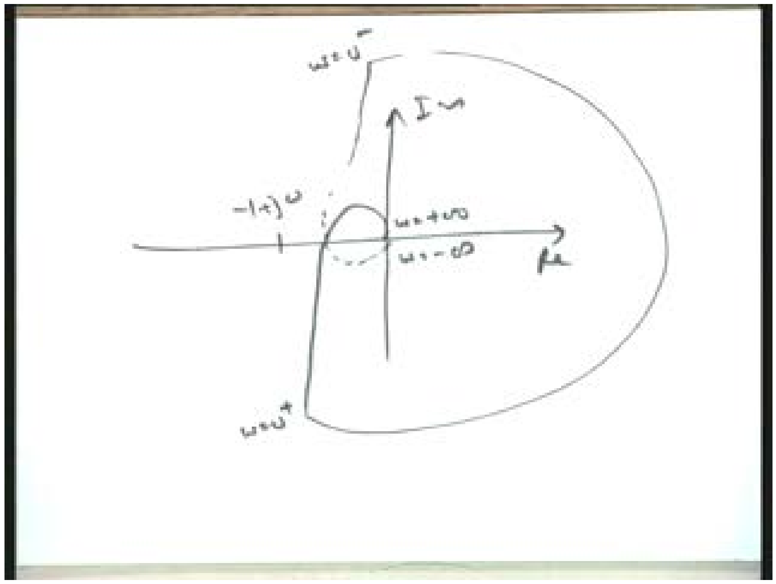

So you can see that the primary contribution to this particular plot is given by the polar plot, it is the frequency response of the system which you have got from omega is equal to 0 to omega is equal to infinity. So you can very well see, in this particular case, if the system goes this way (Refer Slide Time: 4:38) this is more closer to the minus 1 point and therefore there is more risk of system going to sustained oscillations that is the system is more prone to instability.

If you increase the gain so that it goes this way and naturally the system has become unstable because now minus 1 plus j0 point is enclosed provided I am assuming that the P is equal to 0, please see. Stability is definitely with respect to the number of open-loop poles in the right-half plane. So if I am analyzing the stability with respect to the Nyquist plot given to you I assume that P is equal to 0 and therefore this system is more stable, this system is more closer to instability or sustained oscillations and this system becomes unstable. So I can say that the relative stability is given by relative distance of the plot the polar plot or the Nyquist plot with respect to the point minus 1 plus j0.

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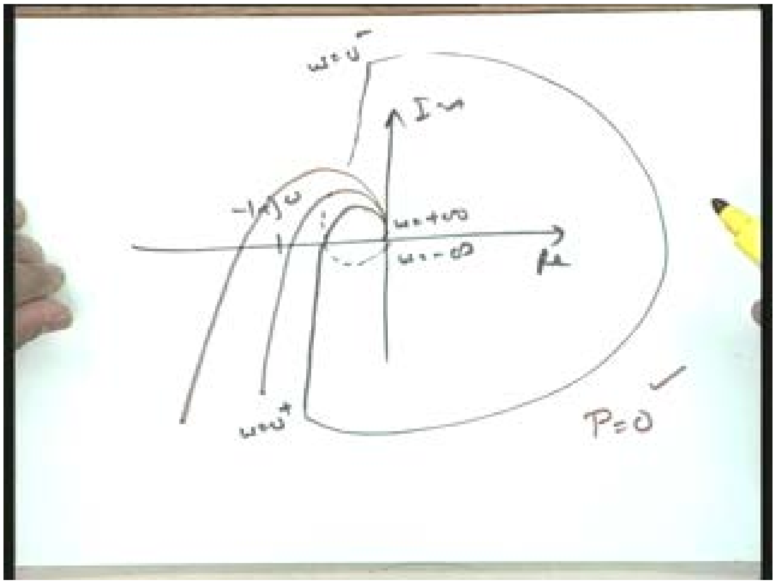

Now how to measure relative distance?
Let us say that this is minus 1 point, if the plot intersects at this particular point, well there is a big distance as far as the real axis is concerned. So it means I consider this to be the intersection point and let us say that this is the magnitude a so it appears lower the magnitude a more is the stability achieved because the intersection is closer to this and hence we are far away from the minus 1 plus j0 point. See this particular situation. So naturally we are far away from minus 1 plus j0 point if this intersection is smaller this magnitude is small.

But you see the situation; after all this is a vector, it is not only a magnitude it depends up on the phase angle as well. Let me take this intersection but let me assume that the plot is like this, after all this is also a possibility, this is also a polar plot of a typical system. So in this particular case though the intersection is same though we are far away from the minus 1 plus j0 point as far as the intersection point is concerned but still you find that the green curve is more prone to instability because it is closure to minus 1 plus j0 point so it appears that the distance the relative distance of the polar plot with respect to minus 1 plus j0 point cannot be given by just the intersection with the negative real axis the phase angle is also important and that is why two indices are used.

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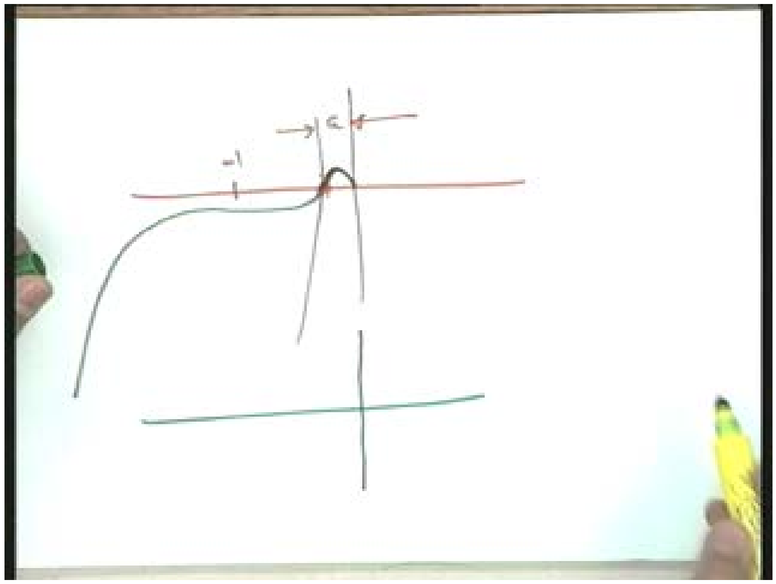

I am giving these two indices; two indices are used to measure the distance and the indices are this is minus 1 point this is minus 1 point 1 is this point of intersection let me call it (a). (a) is the magnitude, this is one index and the other index, you see the phase angle, what is done is you see that you just draw a unit circle with this origin as the centre that is you locate a point here on this particular curve where the magnitude is 1, this particular angle is also a measure of closeness of the plot to minus 1 plus j0 point.

I again repeat; the magnitude point on the plot which is equal to unity is located which is equivalent to saying that a unit circle passing through minus 1 point wherever it intersects your polar plot is the critical point you want to consider. So this particular point is taken and this particular point you draw a line over here this angle (Refer Slide Time: 8:29) is the angle which gives you a measure of the distance with respect to minus 1 plus j0 point.

Yes please,………but if it not intersect at all, it may not intersect at all, okay those points also we will see. I will keep your point in mind and I will answer this. So if I consider this very plot which I have given you where there is an intersection over here in that particular case you find that this distance and this angle the two you see you can claim that, well, even these two may not be sufficient so naturally the complete plot is a more better description of its distance with respect to minus 1 plus j0 point. But if I take the complete plot I cannot set the complete plot into suitably I to my design algorithm. So after all I want a simple design algorithm therefore the complete plot has been quantified into two indices: one is the intersection with this axis and the other is this particular angle where the magnitude is 1.

Now you just see, if I increase the magnitude, please see, angles will remain the same, if I increase the magnitude what type of response do you get, well, this is let us say at a value of K higher than…. this is K 2 which is greater than K 1 all the angles will be remaining the same but only the magnitudes will be changing, keep all the polar vectors only change the magnitudes, and if you further increase your magnitude the plot will be like this that is it is the same type of plot the only thing is that its magnitudes change and therefore the plot goes this way. So it means there will be an increase of gain which will slowly take it through the minus 1 plus j0 point.

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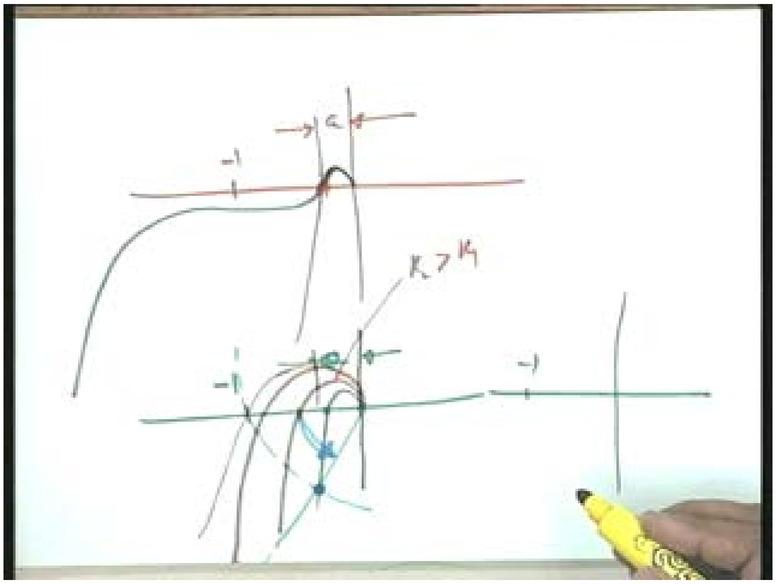

Can you tell me by what factor can I increase the gain so that the plot passes through minus 1 plus j0 point. I repeat my question; by what factor can I increase the gain so that the plot passes through minus 1 plus j0 point? it is 1 by a obviously, please see, if (a) is the intersection here so you want this magnitude at this particular frequency this is one vector minus 180 degrees. So this will pass through minus 1 point it means the K is to be elongated by this much factor which equal to 1by a. So it means if your original polar vectors all of them are multiplied by 1 by a in that particular case the plot will pass through minus 1 plus j0 point and this 1 by a factor is referred to as gain margin GM it is a very very important term for us. So gain margin is 1 by a it means this much margin is available to you before you drive the system to marginal stability or towards instability. So this is the gain margin.

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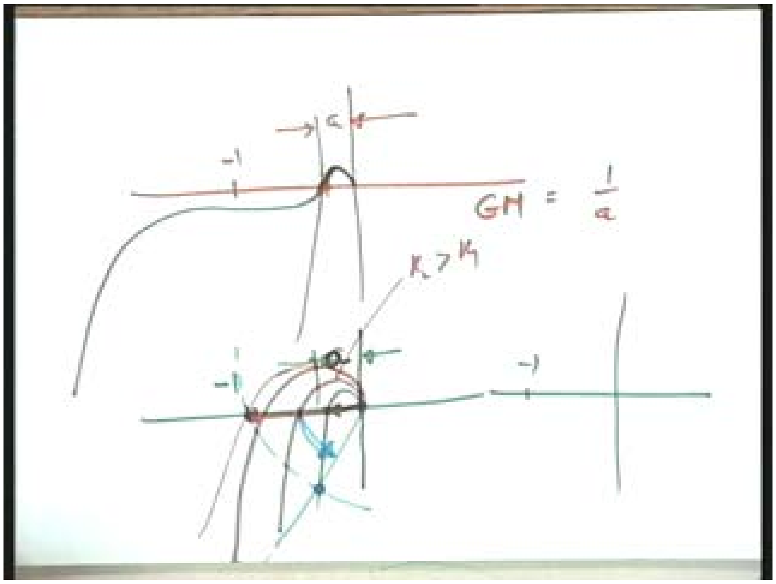

Similarly, please see, let me take this sketch again, this was a typical plot I am taking, all the specific point I will answer later. you see this is the point, this is the magnitude a (Refer Slide Time: 11:41) and let us say that this plot corresponds to G(s) H(s) equal to (1 plus s T 1) s into 1 plus s T 2 some function I have taken: 1 plus s T 1 okay you can take even some K 1 over s into 1 plus s T 2 is the transfer function for which the polar plot is given to you; omega is equal to 0 to omega is equal to infinity. You will note that I have taken P is equal to 0 that is the right-half plane poles is equal to 0.

Now you can see if I increase the gain by a factor of 1 by a in that particular case the system will pass through minus 1 plus j0 point and hence will become marginally stable at the most marginally stable and therefore corresponding to this particular situation I can say that the gain margin of the system is 1 by 8. Consider the phase margin; you see this is the point, take this angle, you see how much angle you can add, how much phase lag, you see phase lag is the term I will use which creates in stability. You have seen dead time introduces phase lag which is a problem which creates instability. So you see that the question is how much phase lag can be added to the system so that the system this particular point passes through minus 1 plus j0 point in that particular case that I will say is the phase margin and you can see that this much is the additional phase lag which you can add to this system. So the phase margin is positive.

Suppose this phi it means plus phi is the phase margin because by dead time or because of any other factor in the loop if an additional phase lag of minus phi appears into the system the system still remains stable that is it just goes to the verge of instability. You will please note that this phase margin is positive because this much of phase margin still you have, if this much of phase lag is added to the system in that particular case the system goes to the point of instability and hence these are the two measures the gain margin and the phase margin which represent the measure of relative stability using the Nyquist stability criterion, very important measures.

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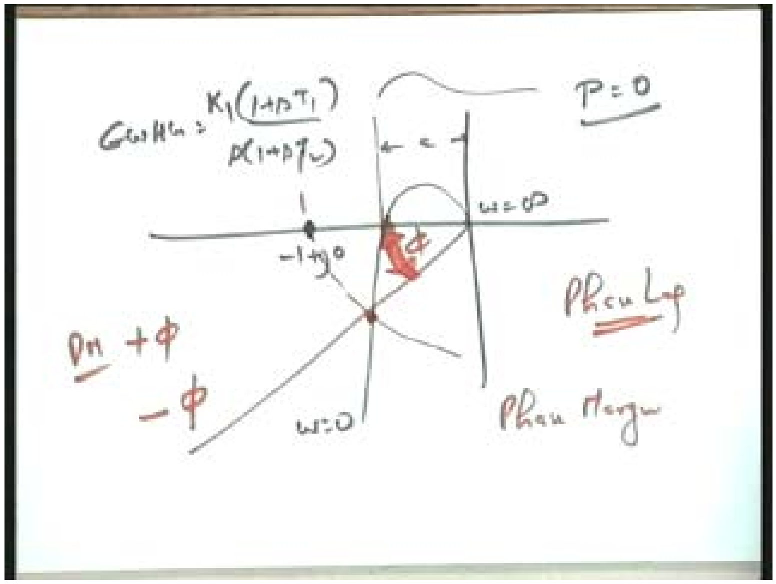

Let me answer his point. He said that, well, the plot may not intersect, I can even give you the example, you know that example where the plot will not intersect, this is the example, yes, what is the polar plot of this system? The polar plot of this particular system as you know is this; this is your minus 1 plus j0 point. Now you increase the value of K, any amount you increase the value of K it has to become asymptotic to this particular line and since for every value of K the plot is asymptotic to this particular line in that particular case you can say that the plot will never intersect. Hence you can say the gain margin in this particular case is infinity.

But you have not to be very happy that the gain margin is infinity, it simply says that for such systems where the intersection is not there phase margin is the prime index of relative stability of a system because it tells you how close it goes to minus 1 plus j0 point. So your gain margin is infinity in this particular case and the phase margin you can see will depend upon the value of K. So this much is the phase margin here the phase margin is reducing, phase margin is reducing, phase margin has further reduced (Refer Slide Time: 15:35) as you keep on increasing the value of K while in every case your gain margin is equal to infinity.

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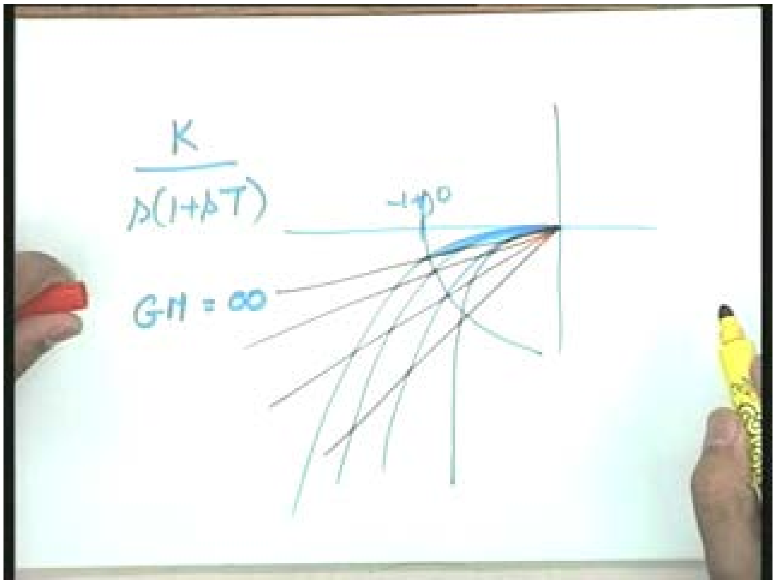

[Conversation between student and Professor…….Sir,….. also not intersect ((00:15:43 min)) this may not intersect, yes, come on, cite some example, ((constant is could be included or)) ((00:15:54 min))] this is in hypothetical case, come up with the case, this question was there, yes, I know that this is a physical system, this is just a hypothetical case and if you come up with the case I think we will evaluate with that particular case otherwise we do not come across such situations where this does not intersect because omega is equal to 0 to infinity it will pass through this.

Thus, now let me say that this discussion is complete. [Conversation between student and Professor………. Excuse me sir, yes please….. Sir, if the numerator and denominator degrees are equal then that case at s equal to infinity it will not tend to origin it tends to some constant in that case the circle might not intersect the plot)) ((00:16:23 min))] okay let us take an example.

I take an example G(s) H(s) equal to……. Well, you are referring to a type-0 system that is all, I can tell you this thing that you are refereeing to a type-0 system so let me take 1 over (1 plus s T 1) (1 plus s T 2) Sir, talking about the case where numerator and denominator are equal…. 1 pulse s T 1 divided by 1 plus s T 2 come on let us quickly get the rough plot of this; 1 plus j omega T 1 over 1 plus j omega T 2, help me please, what is the value at omega is equal to 0 this is your starting point (Refer Slide Time: 17:21). When omega tends to infinity when omega tends to infinity what is your point in that particular case it is j omega T 1 over j omega T 2. So in this particular case there is certain magnitude in that magnitude you can see this T 1 by T 2 angle will be 0 and magnitude you can take as going to certain finite value.

Now it depends about the intersection with this axis……… come on, think of the intersection with this axis or should it go this way or it is the simple plot? It depends, yes, could you help me please; I think this may be simple plot. In this particular case if I take 1 plus j omega T 1………

it does not intersect with the imaginary axis does not intersect with the imaginary axis in this case this is a simple plot and if I take the phase margin in this particular case the phase margin definition, saying that phase margin is equal to any value……

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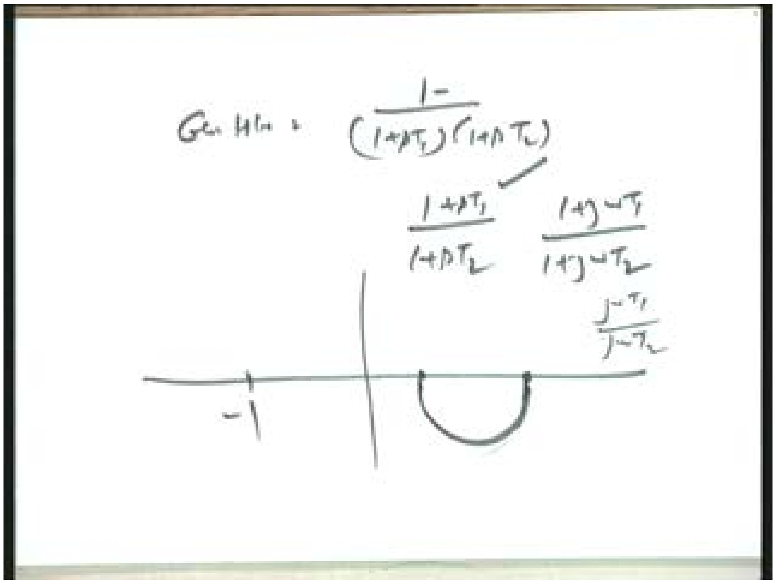

[Conversation between Student and Professor – Not audible ((00:18:44 min))] well, I mean, the only thing in this particular case is this that the gain margin and phase margin the system is stable for all values of T 1 and T 2 the only thing it can be said that…. yes, I think I will study this point again, it has come to me, sir we have an example, yes come on……… s plus half divided by s plus 1 into s minus 1 s plus 1…….. no, this case I will not take, it is a case of an open-loop unstable system I am going to comment on this; open-loop unstable system will not fit into the definition of the gain margin and phase margin I have given. I will explain this point very shortly.

This example seems to be quite alright that here we have a type-0 system and in the type-0 system we have a numerator 0 and in this particular case if I vary T 1 and T 2 so the system is stable for all values of T 1 and T 2. The relative stability we are studying with respect to certain parameters, the only reply which immediately comes to me, you see, though the quantitative reply will come later the immediate reply in this particular case which comes to my mind is this that since we have taken this example and T 1 and T 2 are the two parameters; we are studying the relative stability of the system with respect to the parameter of the model whether it is gain or whether it is any other parameter and we say that the parameters of the model can be changed because this much of gain margin or this much of phase margin is available.

So in this particular case if you study the stability of the system it will turn out to be stable for all values of T 1 and T 2. Therefore it appears the term gain margin and phase margin for such a system lose meaning. This is the tentative reply I am giving you. But mathematically or quantitatively I think I will be able to answer you next time. But if you have any point please come up with that. This is the thing. From the plot which we have got it appears that for all values of T 1 and T 2 the system is stable so you are studying the relative stability of the system with respect to the parameters of the system that is how the parameters of the system can drive the system to the verge of instability.

Therefore, in this particular case since the parameter of the system can never drive this system to instability so may be the indices the gain margin and the phase margin lose their meaning or you may say that, well they are infinite but there it might just turn out be a mathematical definition. For example, gain margin is equal to infinity is it not a mathematical definition only because that type of system a type-0 system we had taken type-1 system we had taken gain margin is equal to infinity does not really mean that we have a very good stable system. Naturally the phase margin was important there. Similarly, in this particular case it appears that the system is stable for all parameter values of T 1 and T 2 and therefore the definition is to be reviewed. I think we can keep this point pending unless there is an additional point from your side. Hopefully the answer will be right which I am giving.

Now let me answer his point. His point is this; he has given this example: 1 over s plus half s plus 1 into s minus 1 please see that when you have an open-loop pole. Open-loop pole is unstable; in this, the open-loop system is unstable in that particular case your gain margin and phase margin definitions change because in this case now you see for stability of the system itself your system has to encircle the point minus 1 plus j0. One encirclement you will require because there is P is equal to 1 so the gain margin and phase margin definitions which I have given you earlier they do not carry over straightaway to a system which is open-loop unstable because in this particular case this plot if I make….. let us assume though in this particular case it will not be a plot for this let me put some s also because then it may take this shape; so let me assume that it is this shape in that particular case the system will be stable only if there is one encirclement in the counter clockwise direction and that encirclement corresponds to the open-loop pole in the right-half plane.

Not that the definitions of stability margins, not that the definition of gain margin and phase margin cannot be extended to the situations where the open-loop system is unstable but normally we do not do it if the open-loop system is unstable we go to the complete Nyquist plot, we go the root locus analysis, we go to the other methods of relative stability analysis, we normally do not use the stability margins as an index of relative stability. I am explaining this point not that it is not possible to define but it loses its advantage you see.

For every case if there are two poles for that case you will have to take special care that is for the two poles there have to be two encirclements of the minus 1 plus j0 point for the system to be stable and hence let us conclude that the stability margins the definition of gain margin and the phase margin which I have given these definitions are applicable to open-loop stable systems. We limit our discussions to this and this limitation is not a big limitation because most of the industrial systems we work with are open-loop stable systems. So if you come across an open-loop unstable system take special care of that system that is the only point.

Now let us just finally take up the definition of the gain margin and phase margin, the special cases also we have taken where there are no intersections we have taken. Now let me take a general case; let us say a typical plot let me take this as minus 1 plus j0. Couple of more definitions could also come. You will see that this is the point at which the angle is minus 180 degrees, this is the point at which the angle is minus 180 degrees so let me call this as omega PC omega phase crossover. This is omega phase crossover that is it is the phase crossover frequency, it is the frequency at which the angle is minus 180 degrees. these terms will be very often used and hence should be clear phase crossover frequency.

Similarly, let me take the point at which the magnitude is 1 unit magnitude and let me call this frequency as omega gc gain crossover frequency. So I have to determine, this is for you to determine the two frequencies: the gain crossover and the phase crossover and this you can determine (Refer Slide Time: 26:10) this particular plot the Nyquist plot or the polar plot is available to you either from the experimental data or from the data derived from the open-loop transfer function of the system. So these two definitions please.

Now the other definition; take the magnitude of G(j omega) G(j omega) H(j omega) magnitude at omega equal to omega phase crossover. Analytically also you can obtain the gain margin phase margin please see. At this particular frequency take the magnitude and say that this magnitude is (a) in that particular case your gain margin is equal to 1 by a.

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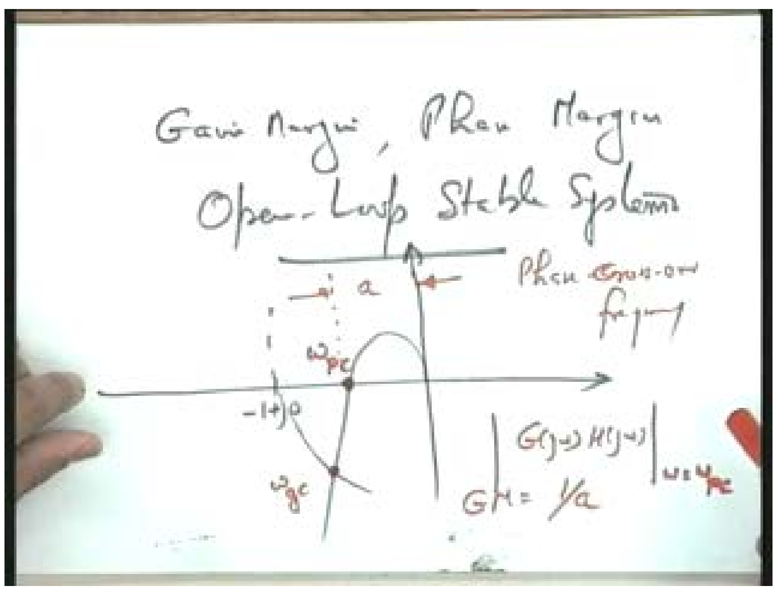

Thus, this is the gain margin please, the original gain of the system for which this plot has been made, and this point may please be noted. It is not the gain of the system; it becomes the gain only when the original plot has been made for a unity gain otherwise it represents the factor by which the original gain of the system can be changed.

So lastly let me explain the phase margin and here I will draw a line; I will measure this angle and this angle I will call the phase margin. This is the margin (Refer Slide Time: 27:30) by which the phase can be increased and now you can see that, tell me know, I think if the definition is clear and if it is clear we are working with open-loop stable systems, tell me, for a stable system the gain margin will be positive or negative for a stable system, the gain margin will be greater than 1 or less than 1?

Please see that this point may please be noted very carefully; the gain margin will always be greater than 1 for a stable system and it will be less than 1 for an unstable system surely. You see that it crosses this. Now surely it is unstable because we are working with open-loop stable cases. So it means if it crosses this and still you apply your definition of the gain margin so if it is less than 1 so it means the gain has to be, I mean qualitatively what is the meaning of the gain margin is less than 1 so it simply means that you have already increased the gain to an extent that the system has become unstable; now you decrease the gain by this factor if you want to bring the system back to the stability region. This point may please be noted. Your system has already been driven to instability region because of higher value of gain you reduce the value of gain so that the system comes back to the stability region. So the gain margin is greater than 1 for stability less than 1 for unstable systems.

Similarly, you can see that the phase margin………. for this particular case if you see the unit point the unit circle is here it crosses this here as far as the this plot is concerned and this will become your phase margin. So it means if I go this way I measure this angle in the clockwise direction and the angle is negative, if I go this way (Refer Slide Time: 29:27) I measure this angle in the counter clockwise direction and the angle is positive.

So you can now see that keeping this convention that the angle will always be measured with respect to minus 180 degree axis and it will be positive if I go in the counter clockwise direction it will be negative if I go in the clockwise direction then I can say that if the phase margin is positive it is an indication of a stable system and if phase margin is negative it is an indication of an unstable system. So these are the qualitative indices and you will find that the phase margin and the gain margin they become two very important performance specifications of a system for design in frequency domain. That is why it is said that the design in frequency domain is centered around the gain margin and the phase margin.

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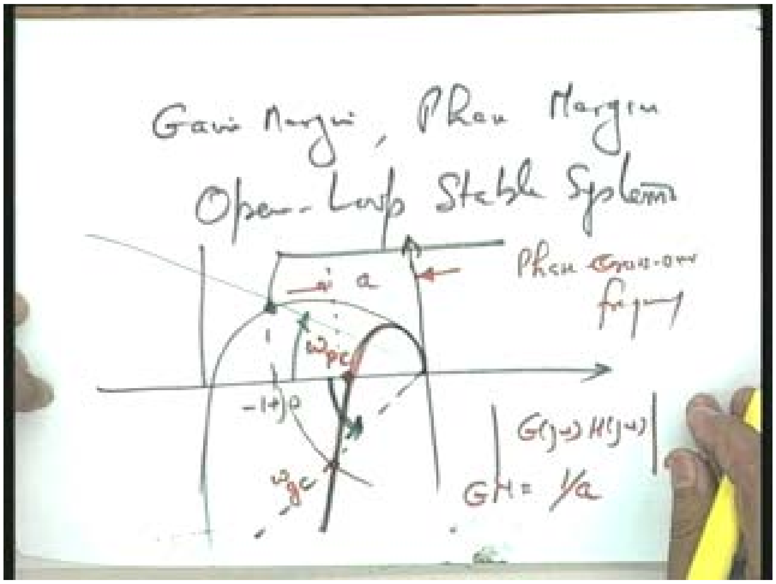

Now, under the Nyquist stability criterion the last point which is left is the use of Bode plots for gain margin phase margin analysis; for stability analysis use of Bode plots. Now, in this particular case please note that the Nyquist plot requires plotting of G(j omega) H(j omega) which as you know is magnitude G(j omega) H(j omega) angle G H(j omega); it means you require a table omega the magnitude and the angle for frequencies from 0 to infinity this will give you the polar plot. So it means lot of calculation is involved. And if this much of calculation is being done in that particular case it is all the time better to go to the computer and immediately get the polar plot of the system and hence the Nyquist plot of the system.

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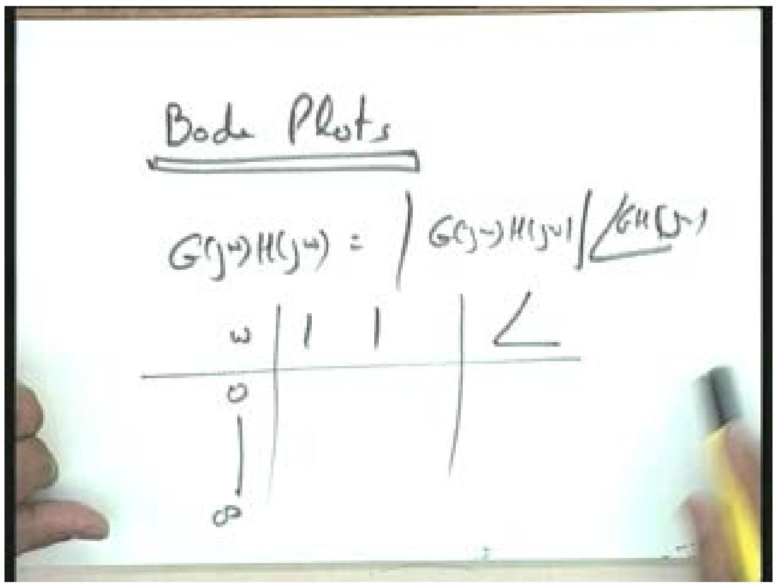

Fortunately we have a method; Bode suggested this method by which this particular plot the frequency response data can be displayed on plot without hardly any calculation, without any calculation it will become a straight line asymptotic plot and you will not require this calculation omega, magnitude and phase angle and all your conclusions about the system stability, the gain margin and phase margin will be easily interpreted on the Bode plots. So the contribution of Bode is an extremely important contribution because the frequency response design the frequency domain design is done on the Bode plot and not on the Nyquist plot. The two ideas are getting mixed you see. The basic idea is given by Nyquist the Nyquist stability criterion but the use of the idea has given by Bode in terms of the Bode plot.

Therefore, now onwards you will hardly come across Nyquist plot in our discussion because a tool which is easier than the polar plot will become in our hand as soon as I give you the constructional methods and analysis methods on the Bode plots. So now let me introduce the Bode plot to you and I introduce it with respect to this transfer function G(j omega) a typical transfer function (1 plus j omega T 1) j omega (1 plus j omega T 2) and other factors here as well as here. Take up these factors; simple factors I have taken but all other factors can easily be taken.

Therefore, now you see that I have taken this to be a type-1 system, a square will come if I consider a type-2 system and so on. In general it will be an nth power giving you a type-M system. So magnitude and phase is to be calculated. But you see what will happen if I take the log of this particular equation. If I take the log on both sides you see that these multiplicative terms they become additive. So in that particular case I can say log of base 10 G(j omega) H(j omega) is equal to log base 10 K plus log base 10 (1 plus j omega T 1) minus log base 10 j omega minus log base 10 (1 plus j omega T 2) and so on. This is the basic idea.

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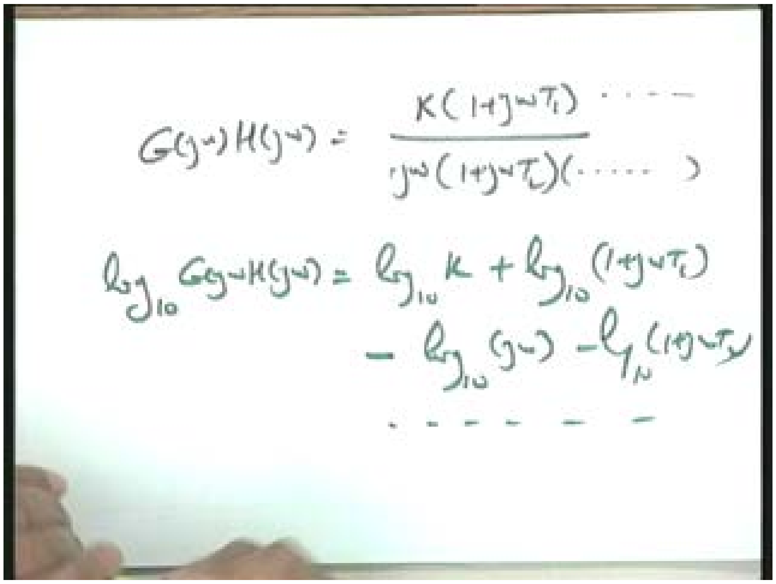

Now we can say that it is known to us what is their beginning but he said that, well, if we take log, not only this but one more contribution that he will take appropriate scales on the magnitude on the vertical and the horizontal axis so that all these individual contributions become straight lines please see. not only the logs, additional contribution from Bode is this that the plot of all these terms are more or less straight lines and therefore addition of all these straight lines becomes easier and hence the overall plots is easily available to you. So this is the major thing that the multiplicative terms become additive terms if you take the log operation on the transfer function.

What I will do now, I will take the magnitude and phase separately; the magnitude of this as you see is K 1 plus j omega T 1 magnitude j omega magnitude 1 plus j omega T 2 magnitude. I hope you will not mind if I do n't write additional terms; when I take the general case I will be able to write the additional terms also quickly without any difficulty. This is magnitude, how about the angle please? Angle G(j omega) H(j omega) is equal to…….. help me please, as far as the angle contribution of K is concerned it is 0 so it is minus it is plus tangent inverse omega T 1 minus 90 degrees minus tangent inverse omega T 2 this becomes the angle. So take the log of the magnitude. Since the angle is already a linear term you can see that the angle is already a summation of various terms. Let me take the log of the magnitude: log 10 I can take, no, forget about it because it will be 10 all through in my discussion so: G(j omega) H(j omega) magnitude is equal to log of K plus log of (1 plus j omega T 1) magnitude minus log of j omega magnitude minus log of 1 plus j omega T 2 magnitude.

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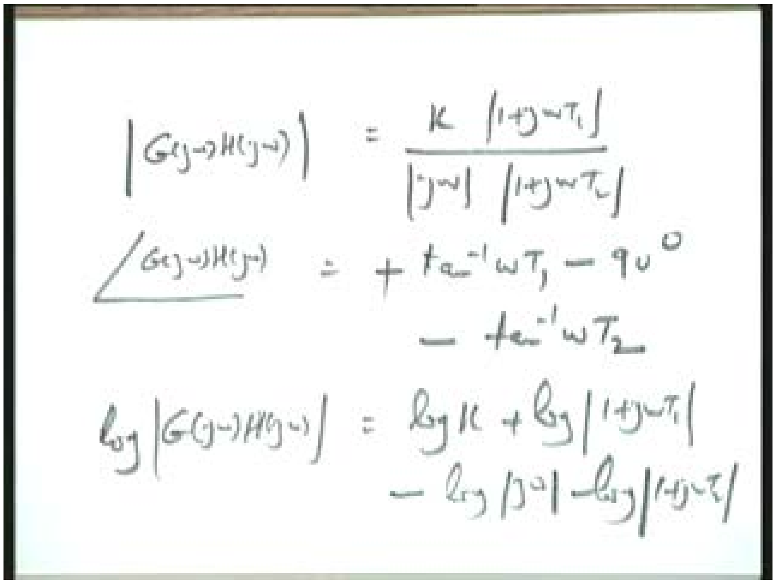

Now you see, I do not find, at least I do not find any justification in using the units in control systems, for this magnitude the unit is decibel, I will define the decibel. Decibel is used, it is the unit abbreviated as db and if you have a magnitude M unit-less magnitude M then 20 log to the base M 10 of M is equal to the magnitude in decibels and this is what we take, the log of magnitude has been taken, the unit we take is decibels; now you can say that why we multiply by 20, the only reason probably comes to my mind is that, in communication engineering right from the beginning from bel we went to decibel the decibel unit depending upon the power of communication was a suitable unit used so we are continuing with it and there is no reason to revert back because reverting back also does not give you any advantage that is probably the only reason otherwise you could say that log of 10 log base 10 of magnitude M could directly be the magnitude; this 20 factor does not give you any advantage quantitatively other than this that you are using a well-known unit this is my view point and as it has been said in the literature.

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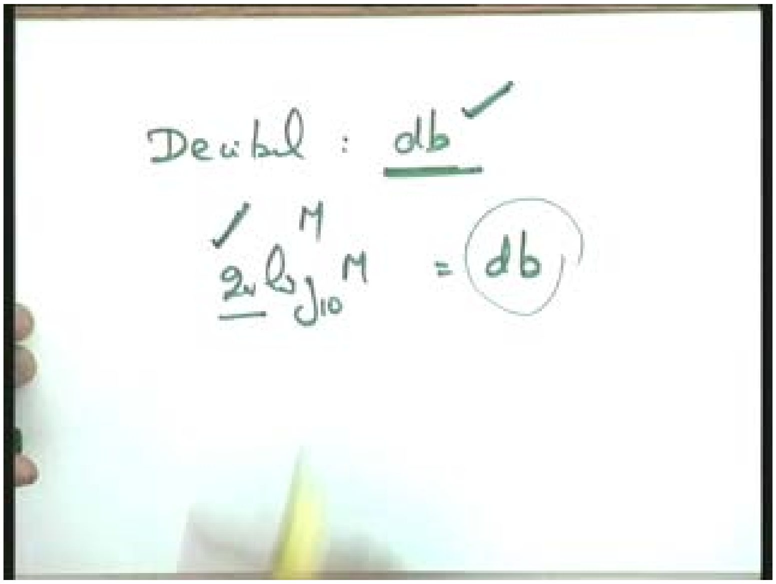

One more point please; one question which comes to my mind is that why do we take log to the base 10 why not a natural log. You see that even the natural log will break it up. Again the argument probably is the same. The natural log also will break it up and will convert the multiplicative terms into the additive terms and you can develop the entire thing in terms of the natural log but since the unit of this is decibels and Bode has developed all along in terms of these units even reverting to the natural log does not give you any specific advantage. So I say that the base 10 and the multiplication by 20 taking it to decibel is just because of historical reasons and there is no other qualitative reason which can be attributed to it.

Therefore, if we agree to this that we will also continue with this in that particular case please see the angle becomes this and the magnitude 10 I can drop altogether now throughout my discussion because l o g will mean that the base is 10. So I can now multiply it by 20, this unit multiplied by 20, this unit multiplied by 20, 20, and 20 and so on and this is the decibels.

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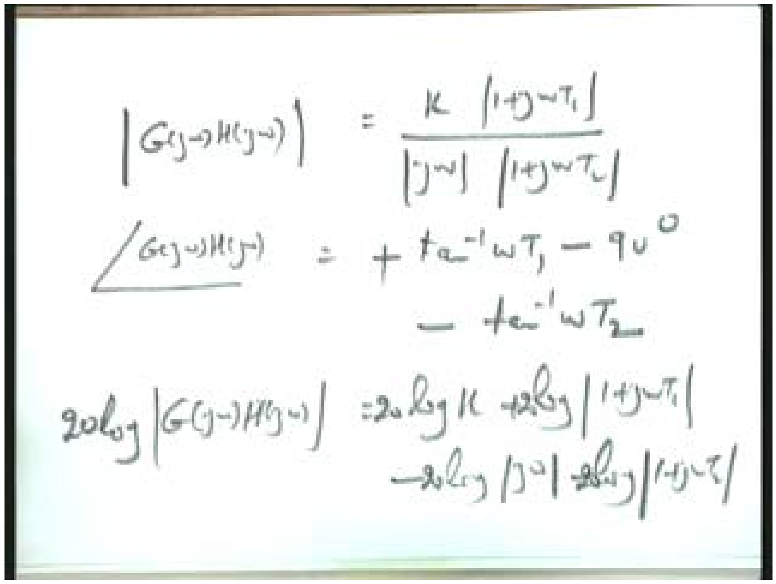

So it means what is the Bode's contribution, what is the Bode plot?
The Bode plot now will consist of two plots instead of one as in the case of polar plot. you see, first let me not take the Bode plot let me take the possibility of the plots the possibility of the plots could be you take omega the frequency on this side and decibels on this side you call it a magnitude plot, you take omega the frequency and the phase angle phi on this side you call it a phase plot.

Now you still if you plot it this way you do not get any big advantage, you see. Though the multiplication has been converted into addition in addition to this you do not get any advantage.
However, as you will see as I will show to you if instead of omega if you take log omega as the horizontal axis, if you take log omega as the horizontal axis then all the individual terms I have given you in the previous slide they are actually straight lines with respect to log omega and not with respect to omega. So it means their plots are going to be simple, making the plots are going to be simple so log omega is on this axis and decibels.

Now what will be the scale on this side?
The scale on this is going to be linear and not log because the log operation has already been taken and the magnitude now decibels can be taken with respect to linear scale. So this scale is linear on this axis, this scale is log omega scale, here phase is linear on this axis and here omega is the log omega scale. In that particular case you can see that the asymptotic plots or the approximate plot in both the cases phase as well as this becomes the straight line asymptotes.

You could see that, well, this point comes to my mind that why do you take the log operation first. You take the terms and you can use a scale which is a log log scale or before I answer this let me take this semi log paper, I have checked, it cannot be projected it is not visible is it, that is fine; in that particular case you will please note that this is the semi-log paper in which the vertical axis is a linear scale is a linearly divided scale and the horizontal axis is the log scale. So if you want log omega you have now to take the log operation on omega, you simply insert your omega on this particular axis it will appropriately take a point corresponding to log omega if linear scale were used. So it is a non linear scale it is a log scale as you see and in this particular case the advantage is this that every time log operation need not be taken. You have not to take the log operation every time; you can directly take the value of omega in this particular thing.

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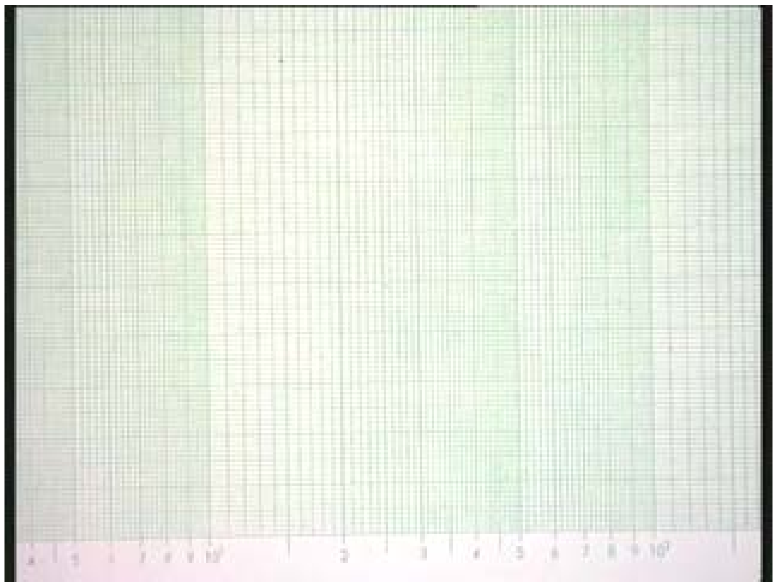

Therefore, it appears that a question can come in anybody's mind that if one could take the frequency this way so that the log of that frequency can automatically be taken on this particular scale then why not take a log log paper so that the magnitude which is available to me the log of that magnitude with a factor of 20 can be taken on this particular scale hence this also could be a log scale where the real magnitude without taking the log operation could be taken over here and automatically log of that will be taken and db will be calculated.

This normally we do not do because the phase plot cannot be fitted into this particular format. Phase plot is in absolute value. There since the phase plot is already additive you are not going to take the log operation on the phase. Since the phase plot cannot appropriately be fitted into the format of a log log paper it is most appropriate it is most convenient as far as the Bode's plot study is concerned that we take a semi-log paper wherein the vertical axis is your linear scale and the horizontal axis is the logarithmic scale.

So now onwards in my discussion when I take the horizontal axis please see I will be writing omega all the time instead of writing log omega, hence that will automatically mean I may not write again and again that the axis I am marking I am marking on a semi-log paper, I will see to it that when I take these frequencies I appropriately make it non-linear and not linear that non-linearity may not be true because it will be approximate however omega verses db I may be showing on my plots here but they will definitely represent the axis on a semi-log sheet. If that is the case….. now in this particular case please see that I find the following building blocks.

If I know the magnitude and phase angle plots of these factors in that particular case I can simply add up all of them to get the overall magnitude and phase angle. Please help me; I have not taken a complete transfer function; if I miss any building block you have to help me. I take the first building block as the magnitude K, the second building block I am taking as j omega plus minus N general I am taking, it could represent a 0 at the origin of certain order, it could represent and a pole at the origin though you know that practically 1 by j omega 1 by j omega squared and 1by j omega raise to the power of 0; type-1 type-0 type-1 type-2 systems are the most practical cases that we know but we can say that this is the most general case.

Third case: (1 plus j omega T) plus minus m let us say this is a first-order factor in the numerator or in the denominator, this can happen both ways; it can come in the numerator as well as in the denominator. Fourth building block; please help me; the fourth building block the general type of transfer functions which we have taken can I take this way? It is (1 by omega n squared s squared plus 2 zeta by omega n s plus 1) raise to the power of plus minus r even if you want to make it more general though multiple factors we hardly come across. So it is a second-order factor and I have written it in this form which is the time constant form, s I will replace as j omega. So this s is equal to j omega and when I replace I will get the appropriate factor. It is a second-order factor which can come both in the numerator and the denominator, this is another block which I can get, the fourth building block.

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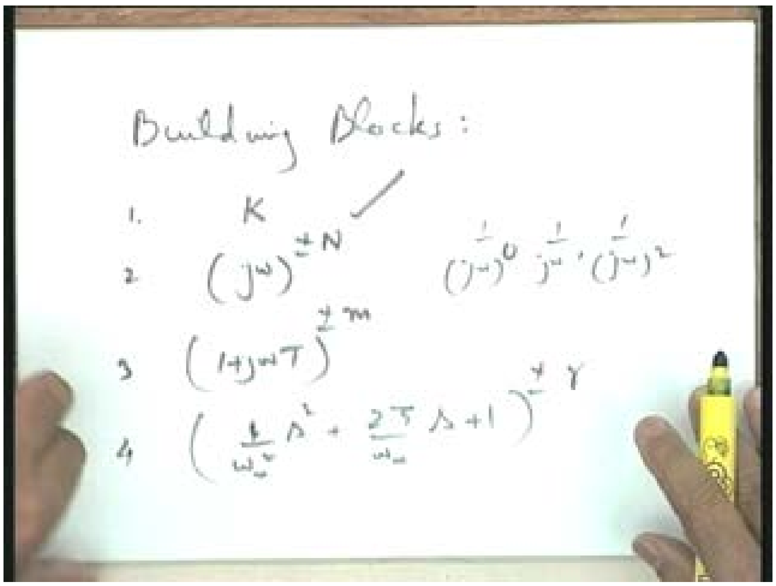

Anything which I missed which you think you may come across as far as the Bode plot construction is concerned, anything which I have missed please do help me? There could be something let me list it here. Yes, I could list here probably the delay also: e to the power of minus j omega tau d I told you. In process control systems the delay is quite frequently available and therefore this in our models may come quite often and therefore let me take this also as a building block: e to the power of minus j omega tau D where tau D is the delay.

One more point please; let me not take a separate building block let me introduce here itself and tell you that 1 minus j omega T or 1 over 1 minus j omega T is a possibility you cannot rule out. What this corresponds to? This possibility will mean that the 0 of the system is in the right-half lane; the system is stable but the 0 of the system is in the right-half plane, this possibility simply means that the pole of the system is in the right-half plane. I am not listing them as a separate building block because it is so easy to convert the construction of this building block into this form because the magnitude will remain the same the phase angle will simply get reversed, the minus sign will come. That is, the only change that will come if instead of 1 plus j omega T I have the factor 1 minus j omega T. So I will take up these building blocks quickly and then a composite example, use of stability analysis using the Bode plot this is the thing which is left.

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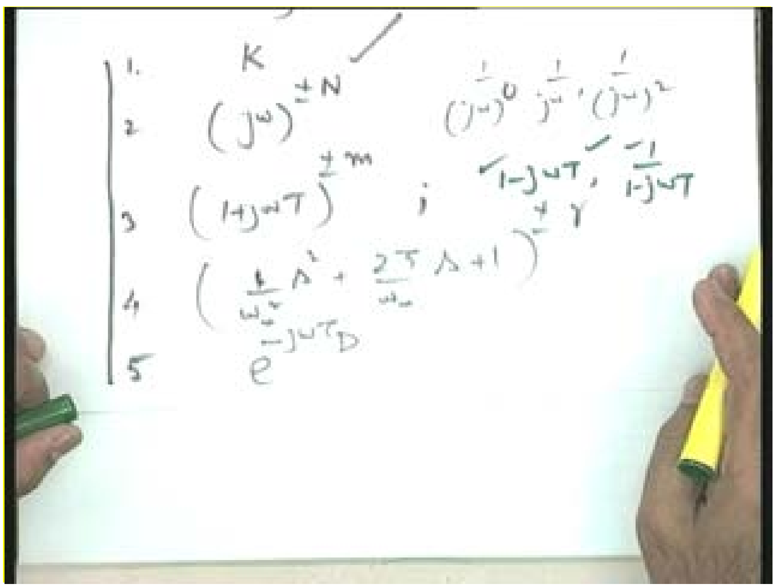

I am sorry we could complete it today so one or may be half more lecture on the subject next time, thank you.