Control Engineering
Prof. Madan Gopal
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture - 36
The Nyquist Stability Criterion and Stability Margins (Contd.)

Before I take the application of Nyquist stability criterion to control systems I think a quick revision of the criterion basic concept will be in order. The idea was this that in the s plane you consider the total contour which is the right-half plane including the imaginary axis. This is the total contour and let me call this as Nyquist contour (Refer Slide Time: 1:22) because we are interested to see whether closed-loop poles of the system or zeros of 1 plus G(s) H(s) just lie in this region enclosed by this particular contour or not which consists of the imaginary axis as well as the entire right-half plane

When I go for mathematical mapping what I will do is this particular contour will be mathematically modeled as an infinite as a circle of infinite radius so what I will do is R is the radius and R e to the power j theta will become the value of s so s for any point on this dotted line is R e to the power of j theta where R tends to infinity and theta varies from this angle of plus 90 degrees to minus 90 degrees this will be the s point when I take up mapping with respect to G(s) H(s) point.

You see that I think at this point itself let me make this comment; you see after all G(s) H(s) is a physically realizable transfer function and I made a comment on G(s) H(s) that normally the power of s will be power of Ns will be less than the power of the denominator del s. Please help me, what will be the mapping of any point on this dotted semicircle? Well, you know that R tends to infinity with this point in mind that the power of del s is more that of power of N(s) can you help me please what will be the mapping of this particular point? I repeat my question G(s) H(s) is the open-loop transfer function we have; I mean it is an in between question an injection in between, I hope I will get the answer for this; N(s) is the numerator polynomial and del s is the denominator polynomial. N(s) is let us say mth order and del s is that of nth order where m is less than equal to n so typically let us take m less than n and equal to n will follow.

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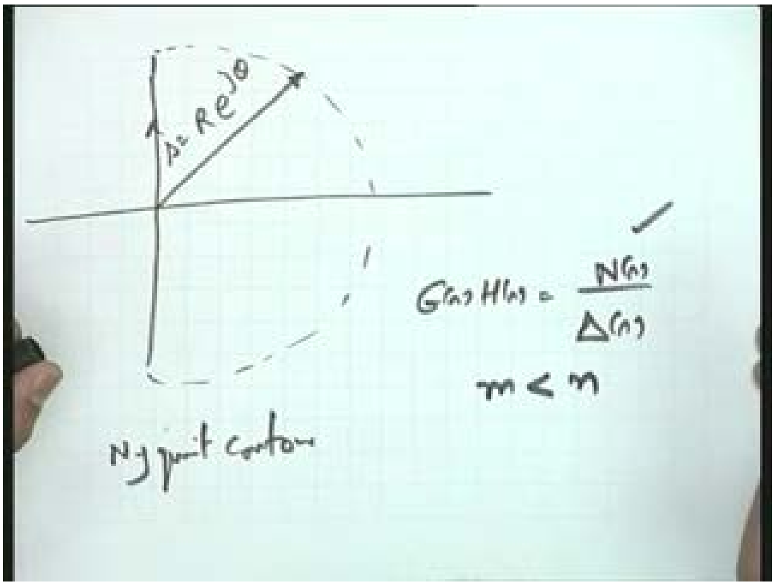

Now in that particular case if I substitute s equal to R e to the power of j theta with R tends to infinity can you give me the value of G(s) H(s) for a typical point s corresponding to this particular description?

[Conversation between Student and Professor – Not audible ((00:03:47 min))]

It is 0 please see. So any point s since n is more than m as the limit s is equal to R e to the power j theta will R tends to infinity this entire infinite semicircle maps in to a point zero that is it is the origin G(s) H(s) plane. Now you see if m is equal to n you can identically follow this point in that particular case this infinite semicircle will map into a constant point if m is equal to n. That is if numerator power is equal to that of the denominator power in that particular case the entire semicircle will map in to a point on the real axis a constant value. So it means if I am taking this as the total contour though it appears how will I do the mapping of this particular semicircle when the radius is infinite but we see that the physical nature of the problem helps us and it tells us that the entire semicircle is either mapped into a constant point on the real axis or in many cases it maps in to the origin of the G(s) H(s) plane. So it means affectively mapping of this total contour means what means starting from omega is equal to 0, going to omega is equal to infinity on the imaginary axis, your s is equal to j omega so omega is equal to 0 to omega is equal to infinity and the counter part is minus infinity to 0. So what I can do is I can take this as omega is equal to 0 plus, this is omega equal to plus infinity (Refer Slide Time: 5:32) just to identify various points you come to this particular point through infinite semicircle call it omega is equal to minus infinity and this is omega is equal to 0 minus omega plus…… 0 plus and 0 minus both join at the same point which is the origin of the s plane.

Why do I differentiate this why not only omega is equal 0?
The reason being as you will see that omega is equal to 0 to infinity plot is a mirror image of omega is equal to minus infinity to 0 plot. So it means effectively the mapping will be done only for this part of the Nyquist contour because this part is just a reflection around the real axis and as far as this part is concerned this maps into a constant point. However, this is just the technique and the technique will become clear as to how we do the mapping. But it is clear that the Nyquist contour is the entire right-half plane and the Nyquist criterion tells us

whether there are any closed-loops zeros of 1 plus G(s) H(s) which become the closed-loop poles of the system whether there are any closed-loop poles of the system inside this particular region.

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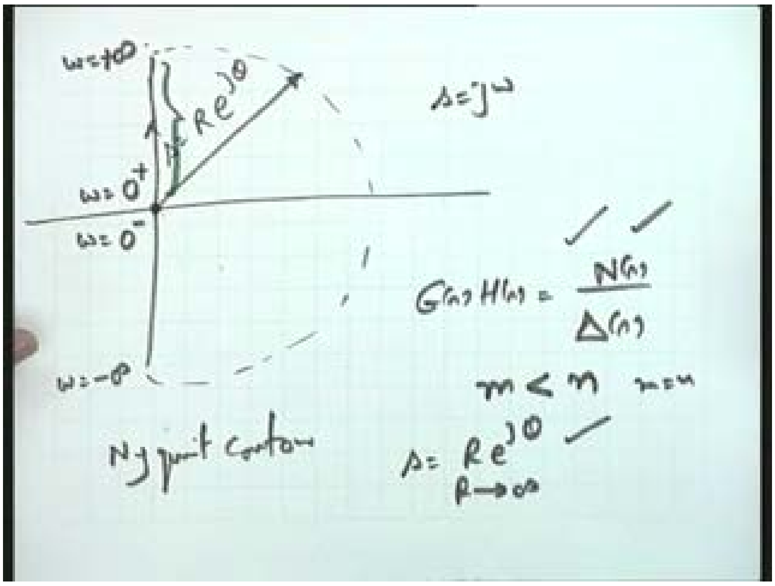

The criterion which we finalize last time is the following that is I take the map of the function G(s) H(s) when s moves along the contour I have just now given called the Nyquist contour so you make a plot of G(s) H(s). This is the total exercises you will have to do. Make a plot of G(s) H(s) as the point moves the representative point s travels along the Nyquist contour in the s plane and let us say that this is the plot I get.

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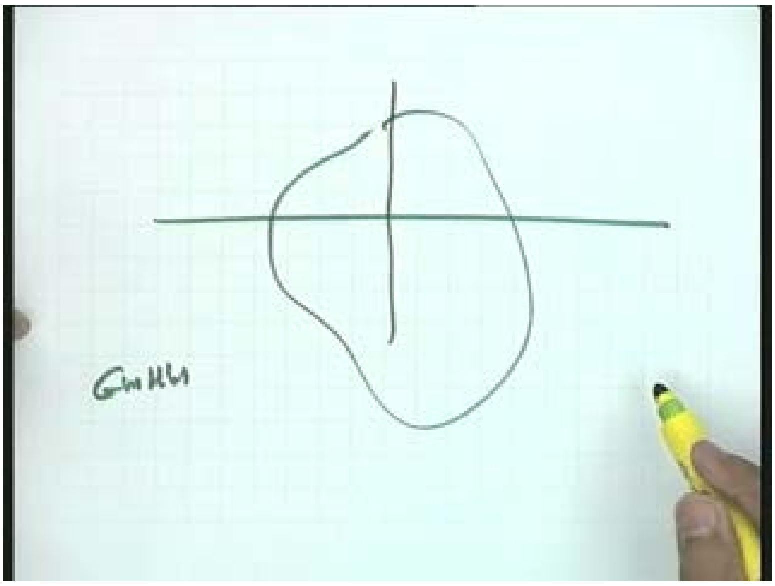

As I said I am not interested in the exact quantitative nature of the plot only the qualitative shape will do. This is real axis, this is the imaginary axis of the G(s) H(s) plane (Refer Slide Time: 7:33) let me call it and the crucial point is minus 1 plus j0 because the shifting has been done; instead of the plot 1 plus G(s) H(s) now I am making a plot of G(s) H(s) only. This is the crucial point minus 1 plus j0 and I count the number of encirclements of this particular point minus 1 plus j0 by this particular plot and if the total number of counter clockwise encirclements is equal to N this is the criterion if the number of counter clockwise encirclements is equal to N then N is equal to P minus Z where P is equal to number of open-loop poles in the Nyquist region enclosed by the Nyquist contour and Z is the number of zeros which is equal to the number of closed-loop poles of the system.

In this particular case let us see the contour is like this, this let me call as the Nyquist plot now. So it means the two terms now I have defined a Nyquist contour in the s plane and a corresponding Nyquist plot in the G(s) H(s) plane which is a mapping of that contour by the function G(s) H(s) which is define to you which is given to you for an open-loop system. In this particular case, for example, you see that the point minus 1 plus j0 is not enclosed, is not encircled and therefore this particular Nyquist plot corresponds to a system which is a stable system though G(s) H(s) has not been given to you but this corresponds to a system which is a stable system.

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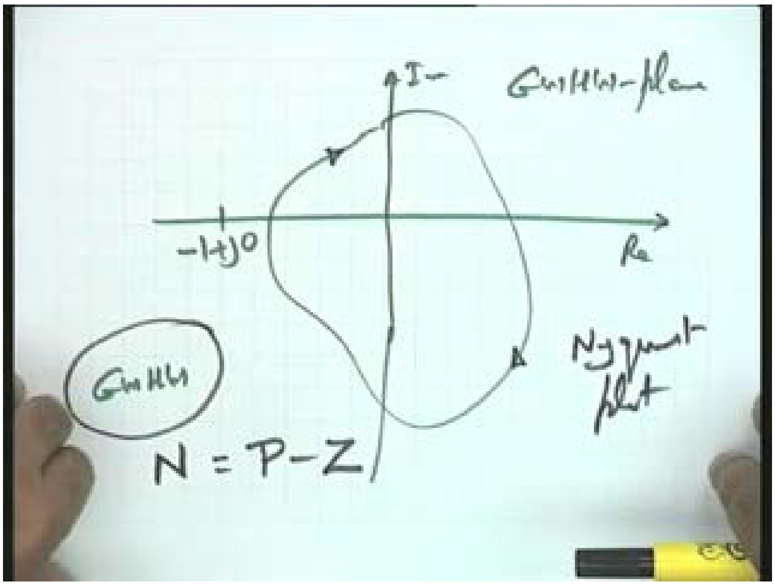

Take up some typical plots let us say this is minus 1 plot I take this plot, this is the direction of the plot (Refer Slide Time: 9:29) let us say this is real, this is imaginary I am giving you a count of N only; in this particular case please help me there is one encirclement of minus 1 plus j0 point and that is in the clockwise direction. So you see you cannot comment on the stability just by determining N because you require the information on P as well. Recall the statement I gave last time that normally P is equal to 0 normally in most of the situations P is equal to 0 meaning thereby we are handling open-loops stable systems: P is equal to number of poles of the open-loop transfer function which are there in the right-half plane.

Normally you will come across situations with P is equal to 0 but in general once you know N you require the information on P to get the conclusion on the Nyquist stability criterion. So in this particular case you find that you have N is equal to 1 is it N equal to 1 please? It is N equal to minus 1 because N has been defined as number of encirclements in the counter clockwise direction so N is an algebraic quantity and that point may please be noted. Take another sketch you see this is your minus 1 point.

I make this sketch. You see that I start from here this goes like this, this plot goes like this, now in this particular case please help me what is the value of N? You see that one thing is there that you can count the number of encirclements. I give you a clue you can proceed like this; take your minus 1 point, just draw a line from this particular point in any direction you like, any direction, one line you draw from here and just see how many times this line has been intercepted. The net number of interceptions of this line is equal to the number encirclements. mind the word ‘net’ I am saying because this interceptions will be equal to the algebraic sum because you will take the counter clockwise and clockwise directions as well so take the net number of interceptions and the net number is going to be equal to the value of N with proper algebraic sign attached to it.

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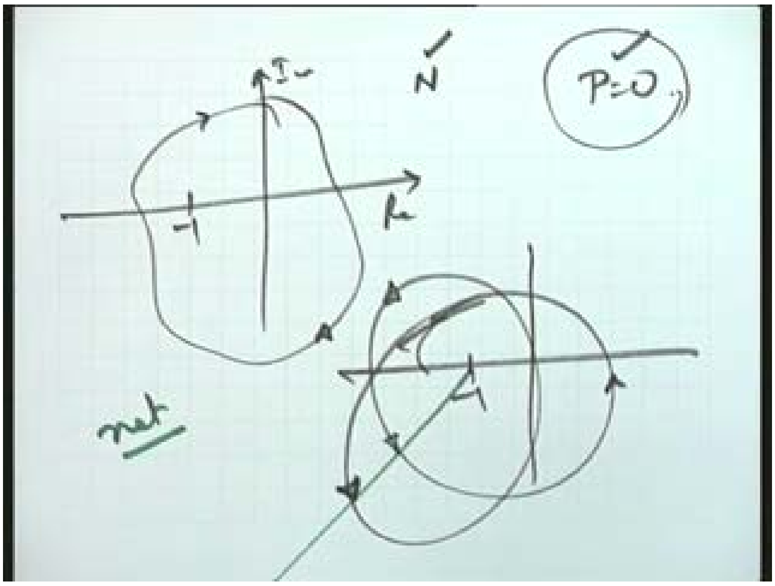

Therefore, you see that it depends upon the type of sketch I get; I get the value of N, I get the value of P and from there the Nyquist stability criterion can be employed to give a conclusion on stability.

Now I think it will be better if I take couple of examples. Some of the questions you may have in your mind those questions will also naturally come up through those examples. I take a feedback system with open-loop transfer function G(s) H(s) is equal to (T 1 s plus 1) (T 2 s plus 1) this is the feedback system I have taken. Now please note; when I apply the frequency domain methods normally time constant form of the transfer function is more convenient to use.

You will recall, in the root locus method the more convenient form is the pole zero form because I wanted you open-loop poles and open-loop zero to get started with the root locus construction. So this is the pole zero form of the transfer function which is more convenient for the root locus construction and this is the time constant form of the transfer function. As you will see it is more convenient for any frequency domain method. It is quite obvious that the sinusoidal transfer function and the angle calculation becomes straightforward when the transfer function is written in the time constant form. Conversion of this form to form is straightforward.

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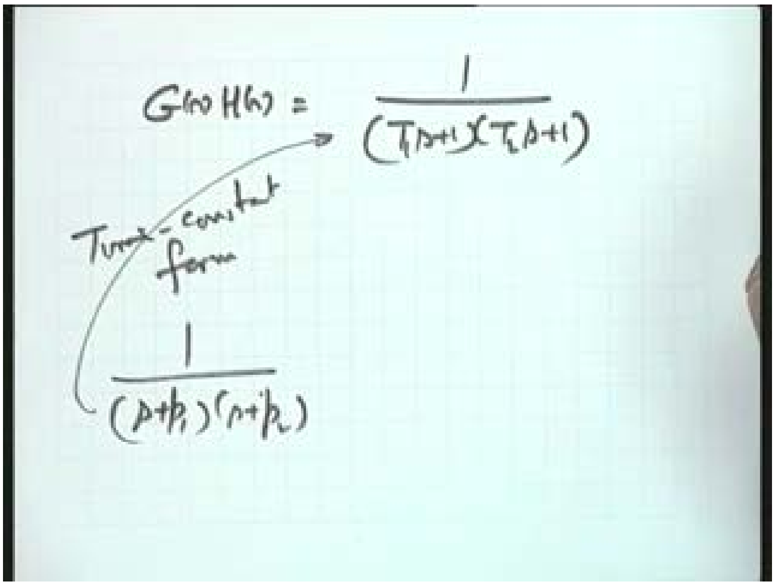

Therefore, you see that you take this as the first step of any frequency domain analysis or design method. If the transfer function is given to you in the pole zero form, do convert it first of all into the time constant form so that the analysis becomes convenient. In this case straight away I have given you the transfer function in the time constant form. So from here the next step is to get the sinusoidal transfer function. G(j omega) H(j omega) equal to (1 plus j omega T 1) (1 plus j omega T 2) these are the two complex quantities and this is the sinusoidal transfer function.

Why did I take the sinusoidal transfer function?
Please note: I want to do the mapping of the entire Nyquist contour under the function G(s) H(s). We have already observed that this mapping effectively means the mapping of the positive half of the imaginary axis because once the mapping of the positive half of the imaginary axis is done the complete mapping is straightforward negative half being mirror image and this being a constant point and mapping of the positive half of the imaginary axis please note my statement is equivalent to the frequency response of the open-loop system. After all, positive half of the imaginary axis means omega is equal to 0 to omega is equal to infinity this is a very important point and I want you to appreciate this point.

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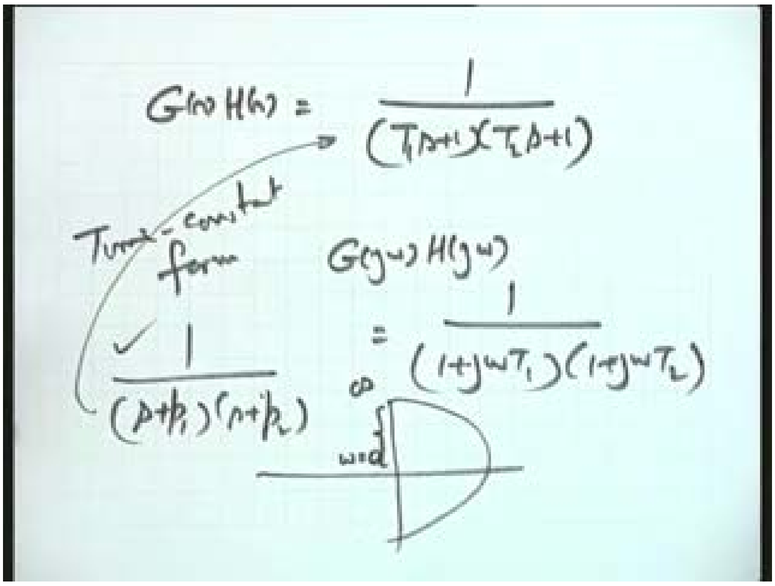

Well, you see G(j omega) H(j omega) is G(s) H(s) the function the mapping function with s is equal to j omega when the mapping is being done only on the imaginary axis then this is the mapping function. You see that the magnitude of this function is G(j omega) H(j omega) and the angle of this function is G(j omega) H(j omega). With this information given to you you can draw any point, you can sketch; you can locate any point corresponding to an s point on the imaginary axis. But I know that this is the amplitude ratio of the open-loop system when subjected to a sinusoidal input; this is the change in the phase angle of the sinusoidal input to the open-loop system. So it means when I know this magnitude and this angle for any value of frequency naturally I know the output of the system when subjected to a sinusoidal input.

So you see I can say that G(j omega) H(j omega) which as per Nyquist criterion is the mapping of the imaginary axis under the mapping function G(s) H(s) equivalently speaking it is nothing but the frequency response of the open-loop system. So it means actually this mapping if G(s) H(s) function is not available to you does not matter. I can get these magnitudes and this angle experimentally and therefore the Nyquist stability criterion can be applied on experimental data as well. Recognizing this, the open-loop frequency response of the system is nothing but the mapping of the positive half of the axis.

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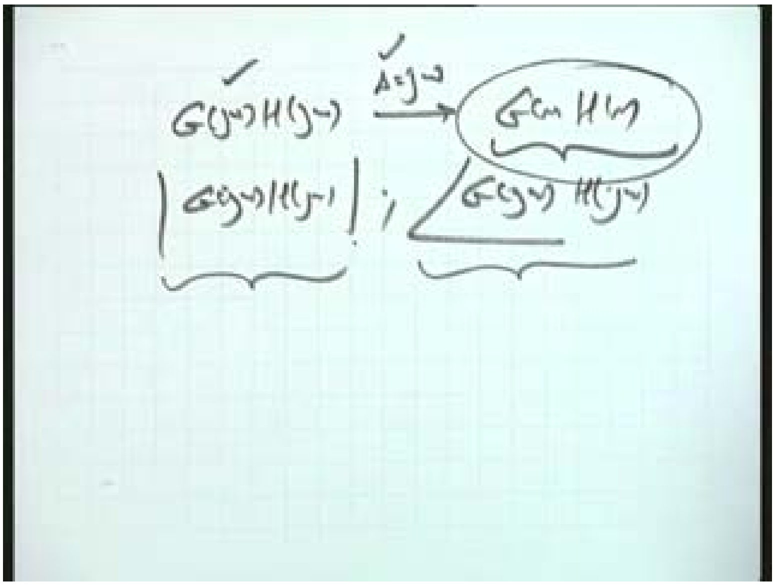

So this link between the mapping function the mapping of the positive half and the open-loop frequency response should be very clear you see because this links you with the actual hardware, this links you with the experiments which you can conduct on this system. Coming back to the original problem the open-loop system is given by a transfer function whose sinusoidal form is 1 over (1 plus j omega T 1) (1 plus j omega T 2).

I am interested in a rough sketch. I am really not interested in the total quantitative. You see, if the total plot is needed probably the power of the Nyquist stability criterion is lost. I am interested only in the encirclements of the minus 1 point. Normally you see this is the guideline; you may or may not agree with this guideline but my experience shows that if you have these four points the rough sketch you can easily make; the four points are: the magnitude of G and the angle of G at omega is equal to 0, the magnitude of G and the angle of G at omega is equal to infinity as far as the frequency response is concerned naturally the other part will automatically follow, intersection with the imaginary axis and intersection with real axis. If you have these four points my experience shows that in many cases the required Nyquist plot can be made and you do not require any additional point unless you have specific question in mind. If only stability is to be investigated in that particular case these four points are good enough.

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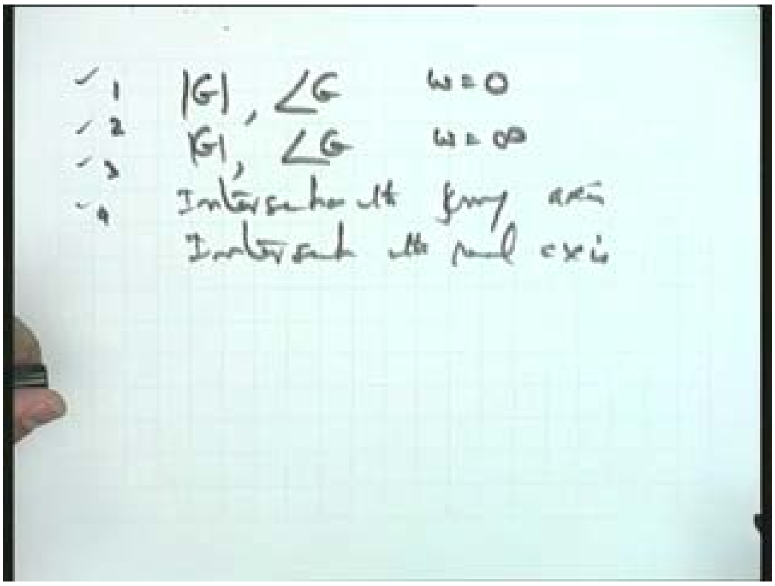

Please tell me now; let us see one by one what are the values of these four points as far as the given function is concerned: (1 plus j omega T 1) (1 plus j omega T 2) help me with the first point please.

What is the magnitude and angle when omega is equal to 0?
You find in this particular case the magnitude is 1 and angle is 0 so you have one point at omega is equal to 0.
Number 2: what is the magnitude and angle when omega is equal to infinity?
Please help me, corresponding to omega is equal to infinity I am writing straightaway infinity zero angle minus 180 degrees at omega is equal to infinity. You can just see at omega is equal to infinity I am approximating this as 1 by j squared omega squared T 1 T 2. So, instead of taking omega is equal to infinity take omega tends to infinity; if omega tends to infinity the plot goes to the origin being asymptotic to the minus 180 degrees line, it gives you the shape as to how to approach 0 because omega is equal to infinity means you have to make it asymptotic to some point and just take it to the point which it will take when omega tends to infinity. So from here I find that for large values of omega this angle is minus 180 degrees so I find that the plot tends to the origin at omega is equal to infinity being asymptotic to the minus 180 degree line; this is very important as far as the shape of the polar plot or the Nyquist plot is concerned.

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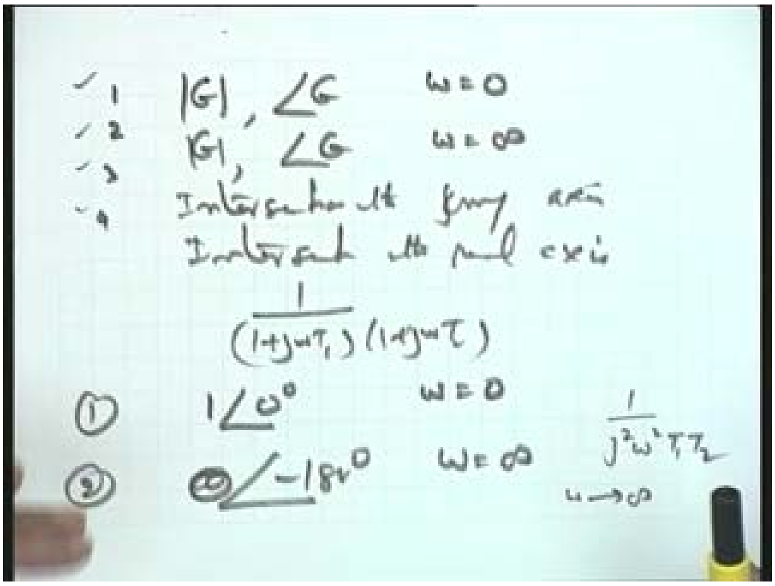

The third point I have to take is intersection with the real and imaginary axis. Though you can find in anyway, you see, you may not you may just see that you will take a different method but I find that converting this into x plus j y type of vector gives me this answer very quickly; I will set the imaginary part equal to 0 to find the intersection with the real axis and then in magnitude will also immediately follow. I will set the real part equal to 0 to find the intersection with the imaginary axis that way the frequencies at which the intersections take place and the corresponding magnitudes will become available to me. though the answers are available with me here but I think I will save time because this is a very clear picture; I need not give you the intersections with the imaginary and the real axis so I give you a hint; my method of working I converted it into this type of vector and then set x is equal to 0 to get one answer and set y is equal 0 to get the other answer. And if you follow this or any other method I say that you have got what is called the polar plot of the frequency response. I am still not working on the Nyquist plot you see I am not still applying the Nyquist criterion; this is the polar plot of the frequency response of the open-loop system.

Start here (Refer Slide Time: 21:52) omega is equal to 0 this magnitude is 1; I will get the intersection here some value of omega, I will get the intersection, in this particular case the real axis is only at this point as you will see there is no other intersection so I get the plot as this. These magnitudes I leave to you, you can easily calculate and this is good enough you see. Make omega is equal to infinity plus infinity as the point; it is plus infinity because I am going to take the mirror image and let me take this omega is equal 0 plus because of the requirement of the mirror image please. So omega is equal to 0 plus omega is equal to plus infinity are the points and this is the polar plot of the frequency response I have got.

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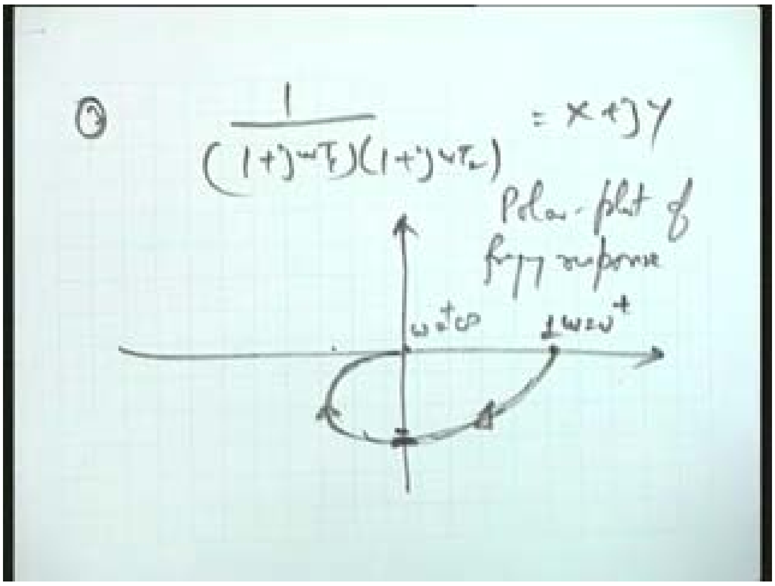

Though with experience you see slowly you will find that you can conclude stability right from here because you can visualize the total Nyquist plot in mind. You need not bring it on paper but just initial in one or two examples it will be interesting to bring it on paper as well. So omega is equal to 0 plus 2 omega is equal to plus infinity so let me make the Nyquist contour here. So omega is equal to 0 plus I started from here I have reached plus infinity. Now please help me, give me the mapping of R e to the power of j theta and for that I take up the original G(s) H(s); I wanted this sinusoidal function only for the purpose of mapping of the imaginary axis.

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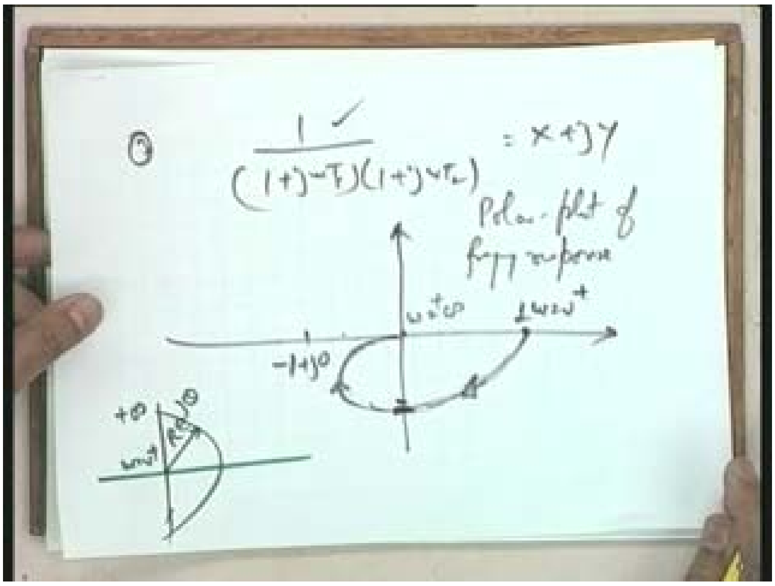

Let me go to original G(s) H(s) which is equal to 1 over (s T 1 plus 1) (s T 2 plus 1) with s equal to R e to the power of j theta. I need your attention here please. One or two examples I will fully describe and then quickly the analysis will follow.

Substituting here I get the function as 1 over R e to the power j theta this is s into T 1 plus 1 T 2 R e to the power j theta plus 1. Now if R goes to infinity in that particular case this function becomes equal to T 1 T 2 R square e to the power of j 2 theta. Please see; R is a very large value let it not go to infinity; R is a very large value; it is T 1 T 2 R square e to the power of j 2 theta.

Help me, what is the magnitude and angle as far as this function is concerned?
I again need your attention on the Nyquist contour. On this R e to the power of j theta when I take s equal to R e to the power of j theta what is theta; theta varies from plus 90 degrees through 0 degrees to minus 90 degrees please; theta varies from plus 90 degrees to minus 90 degrees through 0 as far as s points are concerned. And what is this function now? The magnitude is 0 and how about angle; angle is e to the power of minus j2 theta.

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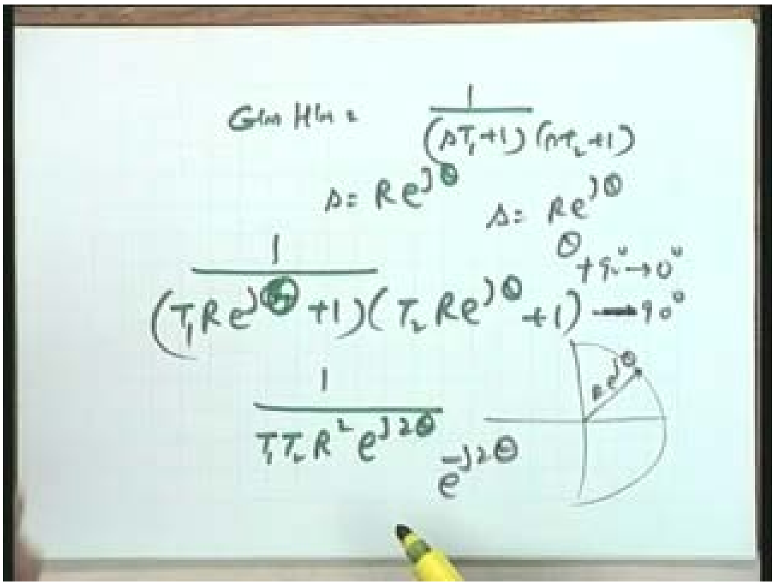

So now you see that as far as the map is concerned the magnitude is 0 but theta varies from minus 180 degrees to plus 180 degrees through 0 degrees. I need your attention here minus 180 degrees to plus 180 degrees through 0 degrees and the magnitude tends to 0. Now help me how can I make the sketch? I think I can go back to this particular plot (Refer Slide Time: 25:29); how can I make the sketch?
This sketch; the best way to illustrate is this; may be this particular sketch you are approaching omega is equal to plus infinity can I take it this way that this is the point corresponding to omega is equal to plus infinity (Refer Slide Time: 25:46); this is the point corresponding to omega is equal to minus infinity. Well, this particular contour tends to the origin.

You see that I am making a contour around the origin; this contour the angle tends to this origin through asymptotic to minus 180 degrees and comes out of the origin asymptotic to plus 180 degrees. It is a mapping of s is equal to R e to the power of j theta because theta angle is also important and theta angle shows up this way that you enter the origin asymptotic to minus 180 degrees and come out of the origin asymptotic to plus 180 degrees. I hope this point is clear as far as the mapping is concerned.

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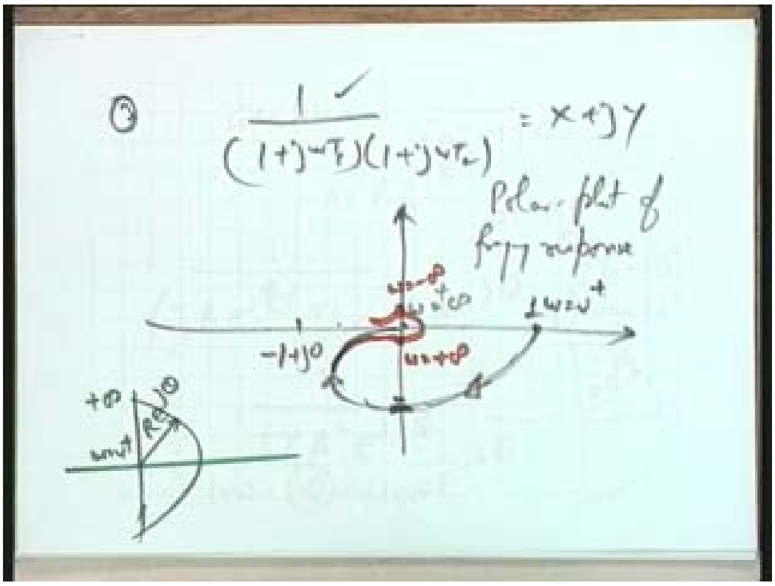

But now the question arises is it not total mathematical jugglery? When I am interested in minus one plus j0 point whatever shape it may take around the origin it is not going to concern the stability analysis and therefore I can just take this into the origin and come out of the origin as far as the Nyquist stability criterion is concerned. But you should see how the mapping is going on. The mapping of the total point is going on this way, the way I have illustrated in this example. But I am further saying that since this particular point is far away from the minus 1 plus j0 point the critical point of interest to me whether I do the mapping this way or I just take origin as the total map of this particular contour it is not going to affect the stability analysis and therefore from now onwards in all my examples I will avoid making this contour I will be taking origin only; the only thing is that this angle I will determine so that I must see whether it approaches the origin this way or it approaches the origin this way or it may even approach the origin this way which way it approaches the origin because in the process of approaching the origin there is a possibility of encircling minus 1 point.

So asymptote I will definitely determine because this is the asymptote which will tell me in which direction it is going to be tangential to since the possibility of minus 1 point coming under the grip is there and therefore this angle I will determine but the contour around the origin I need not determine because that is not going to affect the stability.

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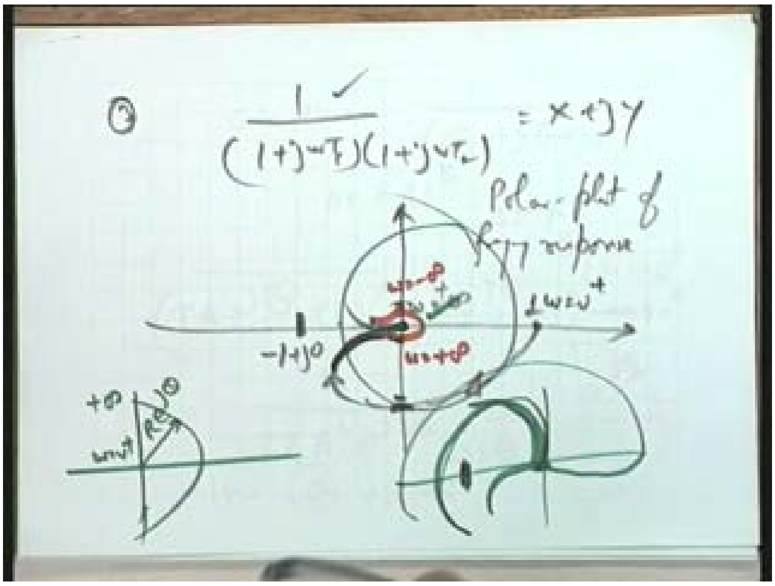

Hence now I am very close to the answer as far as this question is concerned; this G(s) H(s) question is concerned. Recall the polar plot. The polar plot is the following: the polar plot…… whenever I use the word polar plot please note that I am making the actual frequency plot. Omega is equal to minus something has no physical meaning you see, frequency is positive. When you are taking omega is equal to minus something you are actual taking the mirror image from the point of view of mapping the negative axis which is required by Nyquist stability criterion.

So now the total plot becomes omega is equal to 0 plus here, go in this direction let me say omega is increasing and I now take omega is equal to plus infinity; on this side let me write omega is equal to plus infinity, on this side let me write omega is equal to minus infinity and I am at the same point. Make a mirror image, omega is equal to minus infinity you are going this way, you are going this way (Refer Slide Time: 29:29) omega is equal to 0 plus also should have been taken on this side this is omega is equal to 0 minus this gives me the complete map of the Nyquist contour; complete map of the Nyquist contour is available; consider minus 1 plus j0 point a point in line in any direction there is no intersection and hence the system under consideration is a stable system.

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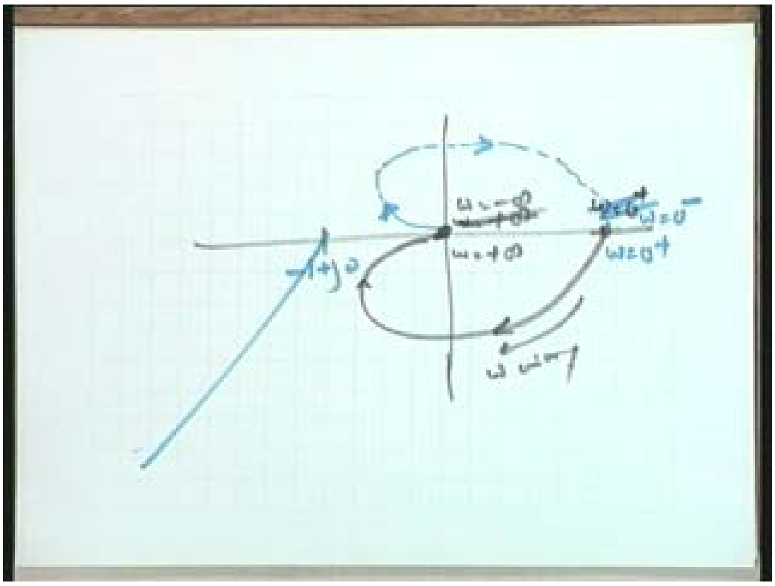

Now you see that you see though this particular problem has taken little more time but actually any problem means you are only interested in the frequency response of the system that is all. Once the frequency response is available you can quickly get the answer to the problem.

One more example which will give you, you see there was a question last time which will provide the answer to that question as well and that example I take Gs a unity-feedback system for example I may take or G(s) H(s) you may write this is s(T s plus 1) this is a type-1 system and there was a question if the contour passes through the singularities or the pole or open-loop pole in that case what will happen that we will see through this example. But before I take that let me take the polar plot of this because polar plot is not affected by this question. Let me first make the polar plot and then I will take the map of the total Nyquist contour and hence I will sketch a Nyquist plot out of it.

The polar plot now you can help me quickly please is obtained from G(j omega) H(j omega) G(j omega) in this particular case equal to 1 by j omega one plus j omega T. help me with four points. I need four points as I told you. the first point omega equal to 0 what is the picture please omega equal to 0 the magnitude is infinity and how about the angle minus 90 degrees; you see that it comes as infinity minus 90 degrees. I will comment on this point. As far as this way…….. I mean if I just substitute omega is equal to 0 in this particular case yes the magnitude is infinity and the angle is minus 90 degrees.

Number 2 omega is equal to infinity if I take you see that omega is equal to infinity, it could be equal to minus 90; omega is equal to 0……. j gives minus 90 [Conversation between student and professor………..32:10] yes, what is question please; when I take omega is equal to 0 the magnitude is infinity angle is minus 90 it is correct, it is correct I think there is no problem. Now you take omega is equal to infinity in that particular case please help me what is the magnitude and the angle? 0 and minus 180 degrees so I have got these two points.

Now third point I need and that third point will solve this question mark also. I want the intersections with the real and imaginary axis. Please do find the intersections with the real and imaginary axis as far as this is concerned. I think I have this answer or you can quickly, probably you can quickly give me the real and imaginary part you see. May be you will be quicker in comparison to my getting the answer from the notes. Well, I think I have got the answer from the notes. Anyhow you will be able to get convert this into real and imaginary part and the real and imaginary parts are minus T over 1 plus omega square T square minus j 1 over omega 1 plus omega square T square this is this function G(j omega) in the real and when converted into this particular vector.

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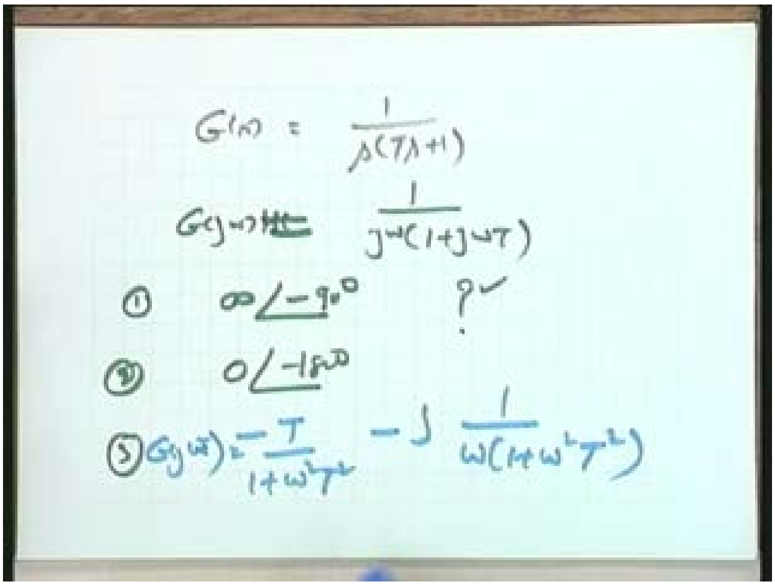

Now you see that what is the intersection with the real axis or with the imaginary axis you find that there is no intersection. If at all there is an intersection it is at omega is equal to infinity as it shows over here. So tell me what is happening at omega is equal to 0? From here it is clear that omega is equal to 0 gives you omega is equal 0 gives you minus T minus j infinity. So you see this gives you additional information. Actually this omega is equal to 0 picture it means it goes to infinity along the asymptote drawn at a line minus T0 I need your attention please converted into this vector and if I substitute omega is equal to 0 in this particular vector I find it is minus T minus j infinity vector.

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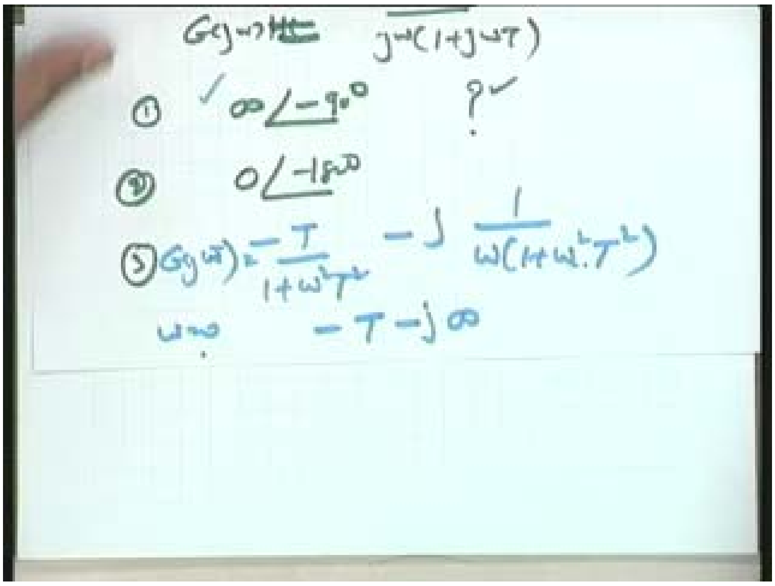

So it means actually if I make a sketch of this the sketch is the following: though this does not matter you see if you would not have realized you would not have got this information then also the stability answer would have been same. I take this point as minus T point. You see that this is the plot this is only information I want to give this is omega tends to 0 omega tends to 0 is a line which is tangential to this particular line minus T line which now you can see is equivalent to saying that it is infinity angle minus 90 degrees; both the information are same you see the only thing is that this is a more complete information, it is not tangential to this axis rather it is tangential to the axis drawn at the minus T0 line. Is it alright please? I hope this point is clear. When I convert this into real and imaginary parts I find that omega is equal to 0 gives me this point.

And what is omega is equal to infinity?
Omega is equal to plus infinity gives me this point the origin and it becomes tangential to this line and it is very obvious from this particular vector that the mirror intersects the real axis nor the imaginary axis but for the points at 0 at infinity. But for the points at 0 at infinity knowing other intersections are there and therefore this is the complete polar plot of the given system. Any questions please? If I really have confused you please do come up with the question. I just wanted to give this as the additional point, this is the additional point, this is the line plot which is asymptotic to minus T0 line instead of being asymptotic to the imaginary axis. Come on please, if this is okay, I hope this is okay because there has been no questions from you.

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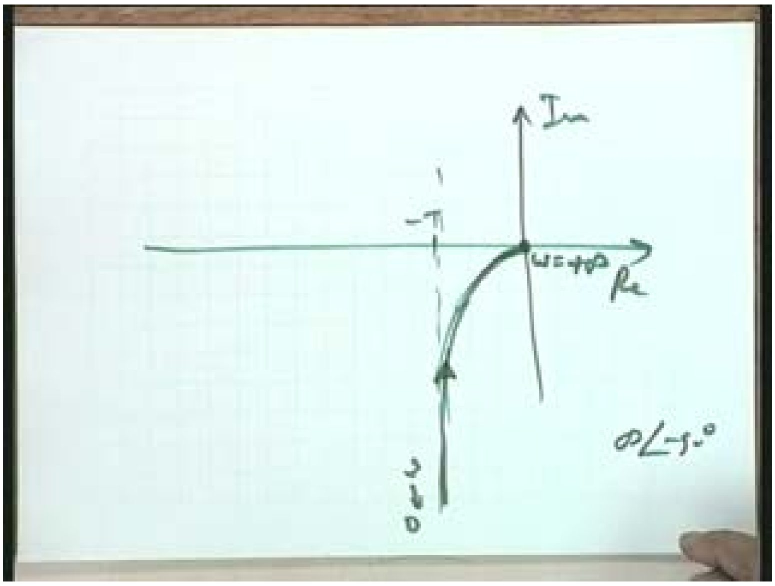

Now you give me the total Nyquist plot. This is your Nyquist contour (Refer Slide Time: 36:30) you started with omega is equal to 0 plus and you have gone to omega is equal to plus infinity. No problem as far as this portion is concerned because the Nyquist plot of this portion is given by this. Now you go along this contour. I leave this as a simple exercise to you that e to the power of j theta mapping is in origin itself, you can take at appropriate angle we need not worry about this. So you have come to omega is equal to minus infinity and this minus infinity point to omega is equal to 0 minus is a mirror image so I have come to omega is equal to 0 minus as well.

[Conversation between Student and Professor – Not audible ((00:37:12 min))]

Yes, because this plot you see this is a general statement I gave that from omega is equal to 0 to infinity to omega is equal to minus infinity to 0 any function G(j omega) H(j omega) is a mirror image of each other with respect to the real axis. So in this particular case, this is a general statement not in particular to this statement but this is a general statement which is applicable to any G(j omega) H(j omega) which can easily be verified mathematically also.

Now the only problem in this particular case is the zero point you see. There is a pole open-loop pole at this particular point and if you take the mapping of this particular point it gives you infinity. So what I do is this that I do an indenting. So what I do is I take this as the Nyquist contour where this particular pole (Refer Slide Time: 38:12) has been indented has been avoided by taking a semicircle with radius is equal to epsilon where epsilon tends to 0. So it means if there is an open-loop pole on the imaginary axis in this particular case it is on the origin this could be at any other point also; if there is an open-loop at the imaginary axis I bypass this particular pole in the Nyquist contour by taking a semicircle of radius epsilon where epsilon tends to 0.

Please see, it really does not affect your stability analysis, it does not affect because you are interested in the closed-loop poles of the system. This is an open-loop pole when the logic behind bypassing is this that, well, this open-loop pole has been taken at s is equal to 0. In a real life physical system getting a pole at s is equal 0 is an idealization of the physical situation; there will be some positive damping present in the system; actually this pole will be close to 0 in the left-half of the s plane and therefore I can detour it by taking a small semicircle with epsilon tends to 0.

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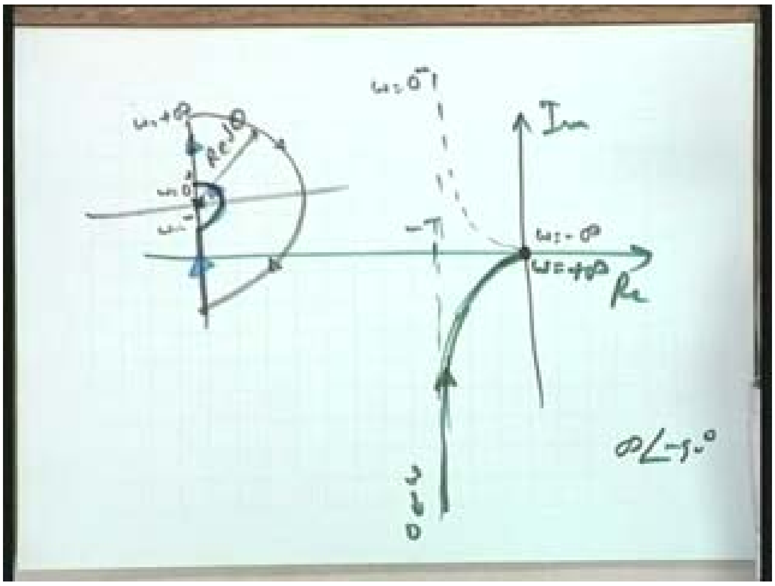

So, for me if there is an open-loop pole on the imaginary axis the Nyquist contour is given by this where this is a semicircle given by. You see that there will absolutely no problem if the detouring is done this way, absolutely no problem. The only the problem will come, you will have to now suitably apply the Nyquist stability criterion. If you do the detouring this way in that particular case one of the open-loop poles has been included in the region enclosed by the Nyquist plot Nyquist contour.

In that particular case when you take N is equal to P minus Z corresponding to this P is equal to 1 will have to be taken. If you do the detouring like this in that particular case since this particular pole has not been taken in this particular region in that particular case this particular pole will not contribute to the number P over here. So the two methods detouring this particular pole this way or this way will lead to exactly the identical result as far as the closed-loop stability is concerned the only thing is that the stability criterion will have to be applied with caution. In this particular case this will not be included in this region, well, in this particular case this is included in this particular region. This is an open-loop pole and we are interested in this stability of the closed-loop system please.

[Conversation between student and professor…((40:50))….. Sir, when N obtained is different in this case, yes, in the two cases N obtained will be different for the same system, stability answer will be same. For the same system the answer on stability will be same but the N obtained in the two cases will be different. So now I take for our discussion always this particular picture. If I take this picture (Refer Slide Time: 41:08) this will not come under P and now please help me, for the system under consideration what is the mapping of this semicircle come on. Make an attempt and then give me the answer taking this as epsilon e to the power of j theta take the semicircle.

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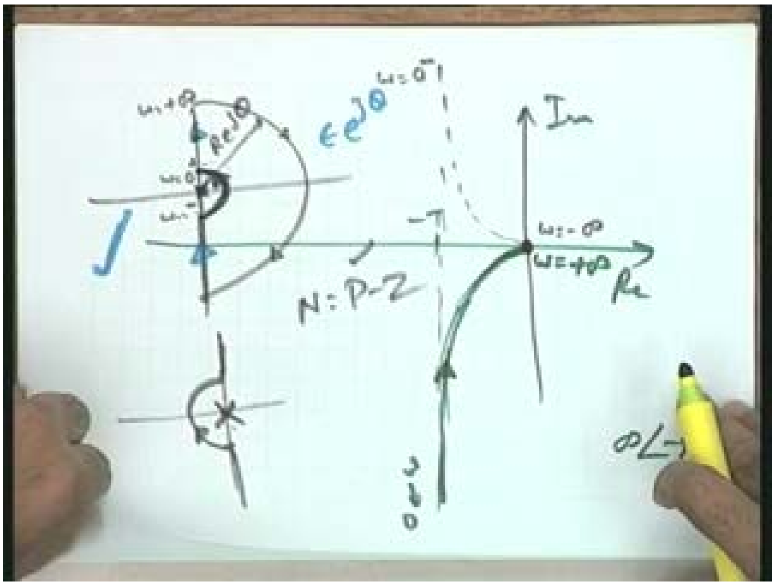

Like the infinite semicircle we had taken I take the semicircle now the infinite decimal semicircle is given by s is equal to epsilon e to the power of j theta where theta varies from minus 90 to plus 90 through 0 degrees please; minus 90 to plus 90 through 0 degrees. Come on please let us do the substitution: G(s) H(s) corresponding to this becomes 1 over K over I had taken in general epsilon e to the power of j theta T s plus 1 T epsilon e to the power j theta plus 1. This is equal to, yes please, K over…… I think this can be neglected with respect to 1 epsilon e to the power of j theta equal to K by epsilon e to the power of minus j theta. K by epsilon e to the power of minus j……..

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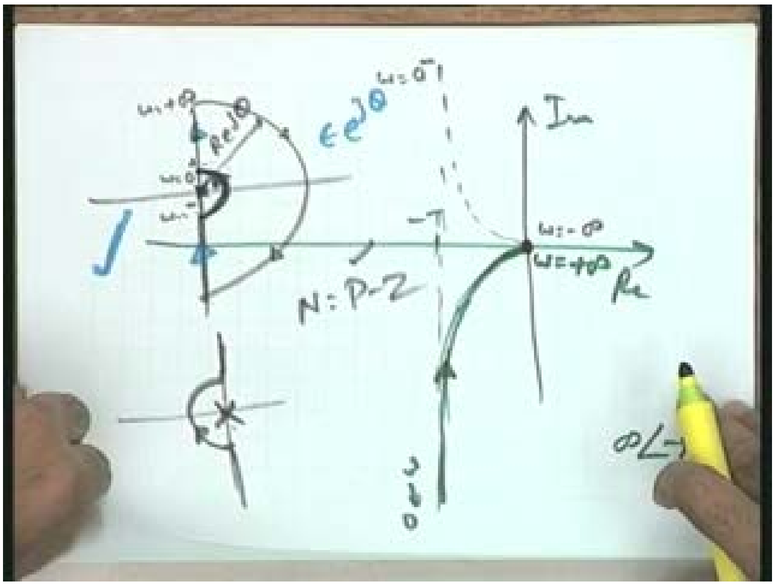

Now you tell me, this is an infinite magnitude K by epsilon with the angle now changing from plus 90 degrees to minus 90 degrees through 0 degrees. The infinite decimal semicircle has been modeled as an infinite semicircle now because the radius has come out to be infinity but how about the angle, the nature of the angle has now reversed, the infinite this particular semicircle is changing from minus 90 to plus 90 now it will be changing from plus 90 to minus 90.

You can see that actually where you have come now, you see that in this particular case you have come to omega is equal to 0 plus you are going to omega is equal to from 0 minus to 0 plus you are going through this particular circle which has a radius R tending to infinity and this completes your Nyquist plot. It is the basic understanding you see. As far as Nyquist stability is concerned you will quickly get this information from the polar plot itself. You see that detouring of the origin which you have taken because of pole at s is equal to 0 has resulted in this particular plot.

You see, the polar plot, you started with omega is equal to 0. Now you see the total sketch, you started with omega is equal to 0 plus it is the frequency response of the system and you went up to omega is equal to plus infinity. This is the experimental data or the data obtain from the open-loop model. Omega is equal to 0 to minus infinity to 0 minus is a mirror image and now this has been modeled (Refer Slide Time: 44:01) as this infinite semicircle and therefore if you take a minus 1 point this does not cut here and therefore for all values of K.

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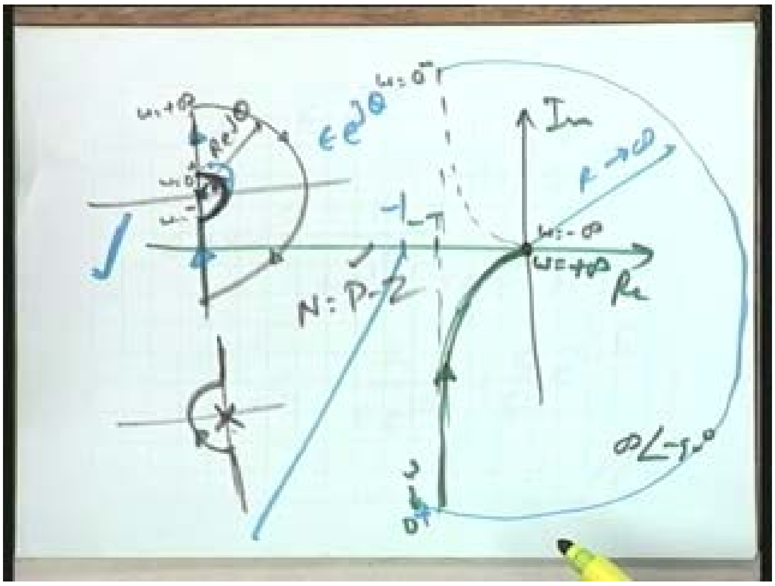

You see that you keep on changing the value of K the only thing is that its magnitude will change but still it will not pass through the minus 1 point. So as far as this problem is concerned K over s(T s plus 1) it is stable for all values of K because the corresponding Nyquist plot will not come nearer will not cross or will not encircle the minus 1 point for any value of K. This is the conclusion obtained from Nyquist stability criterion as far as this type one system is concerned.

Any question please? I know that lot of time has been spent on this but the relative importance of this also cannot be denied and therefore we can come out of this loop of Nyquist stability criterion only if you answer one question fully though in the tutorials we will take many problems one question fully I want you to answer and I take a type-2 system as a question for you, yes please.

G(s) H(s) equal to 4s plus 1 divided by s squared (s plus 1) (2s plus 1) answer this question and we come out of the Nyquist stability criterion. Come on please four points you get for me, at least roughly the idea you give me. Omega is equal to 0 what is the….. let me first write the transfer function plot: 1 plus j4 omega divided by (j omega) square (1 plus j omega) (1 plus j2 omega) omega is equal to 0 please….. infinity minus 180 degrees so it is asymptotic to minus 180 degrees line as you find. Omega is equal to infinity. Now if you take omega is equal to infinity what will happen j4 omega divided by j omega squared 0 minus 270 is correct, whether all of you have got it please? For omega is equal to infinity since you have given let me not write this 0 minus 270 degrees is the angle. Is it okay please? Yes, which is same as 90 degrees of course no doubt.

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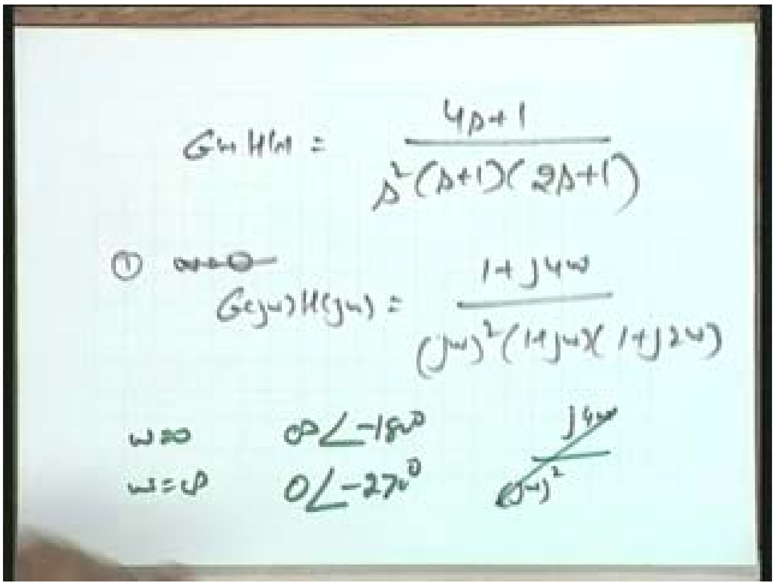

Now intersections with the imaginary axis, intersections with the imaginary axis now you see, well, you will have to make it into vector x plus jy vector and from there you get the intersection. Since I know the result I will give you the nature of the general plot. The nature of the general plot is like this: this is your plot please (Refer Slide Time: 47:15). Omega is…. this is minus 270 so I am reaching at this particular point omega is equal to plus infinity this is omega tends to 0 plus this is omega tends to 0 plus. So you have got the polar plane. So it means in this particular case as far as the Nyquist contour is concerned omega is equal to 0 plus to omega is equal to plus infinity has been scanned.

Now look at this particular portion. Again a very simple test it is the origin itself. Now come to the origin there is a double pole here. Again you see double pole can be avoided by taking a detour. In this way now we have decided that we will take the detour this way only so double pole can be avoided. Help me please; what is the mapping of this semicircle epsilon e to the power of j theta, what is the mapping this should be clear from your G(s) H(s) function. I want to get one answer to proceed forward you see. In this particular case what is the mapping? The magnitude is infinity it is obvious, how about the angle; when epsilon tends to 0 in that particular case please see the only thing is that 1 by s squared is the guiding factor, this is the only clue all others you can neglect. Now e to the power of minus j2 theta will come in the numerator. [Conversation between Student and Professor – Not audible ((00:48:38 min))] So it means it is plus this is if this changes from minus 90 to plus 90, this is plus 180 to minus 180. So it means as far as the mirror image is concerned mirror image I can quickly make. So it means….. I am sorry this really is not the mirror image anyhow I am sure you can make a better image omega is equal to minus infinity omega is equal to 0 minus I have already reached, I have to do the mapping of this. The mapping of this says that it is from plus 180 degrees to minus 180 degrees with radius infinity and therefore this is your Nyquist plot; from plus 180 degrees to minus 180 degrees this your Nyquist plot.

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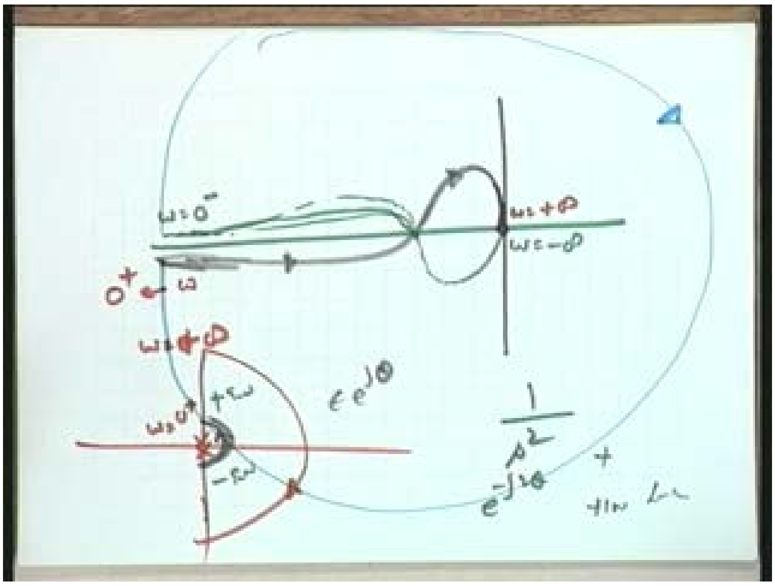

Now you have to see, now in this particular case the numerical answer will come very clearly because it will depend upon the magnitude; whether the point minus 1 lies here or here (Refer Slide Time: 49:49) and if I add K to the system K s square 4s plus 1 into the function s plus 1 into 2s plus 1 you can very easily see that by K the system can be destabilized or stabilized because the plot can be contracted this way or can be expanded this way depending upon the value of K K is going to change the magnitude. So in this particular case for K is equal to 1 the stability answer will depend up on the location of minus 1 point.

Let us say that it is located here. I leave this as an exercise for you to see whether it is located here or here. if it is located here (Refer Slide Time: 50:31) in that particular case just draw a line; one will intercept here, another intercept here and you find that if it is located here there are two clockwise encirclements two clockwise encirclements in this particular case if it is located here. So it means N equal to minus 2 see this point please. Algebraic sign of N you have to take note of: N is equal to minus 2 in this particular case since there are two clockwise encirclements and recall the transfer function open-loop transfer function.

Help me what is P equal to capital P equal to for the given transfer function? It is 0 because of the type of detouring we have taken. So it means N equal to P minus Z. So, minus 2 equal to minus Z or Z equal to 2 if your magnitudes are such that the minus 1 point comes over here in that particular case your system under consideration is unstable with the two poles in the right-half plane, with the two closed-loop poles in the right-half plane or with two zeros of 1 plus G(s) H(s) function in the right-half plane, the statements are equivalent.

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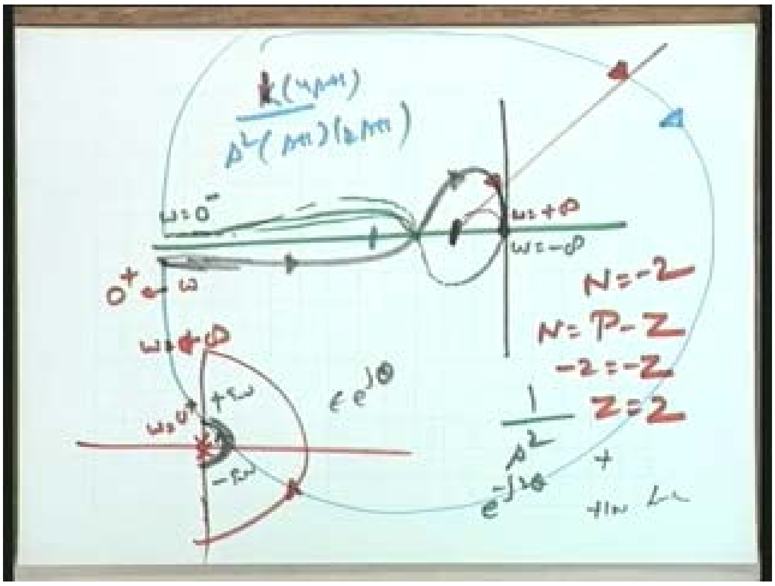

Consider that your magnitude K is such that the point lies here. Come please help me what are these; just let draw any line. You see in this case as you have very clearly identified N is 0. In this particular case N is 0 because of the net encirclements one is in this direction the other one is in this direction. You can see otherwise, physically it is clear that the point minus 1 is not encircled. But it is okay even if you draw the line in that particular case also you find that the criterion meets the requirement N is equal to 0, T is already equal to 0 therefore Z is equal to 0 and hence the system under the consideration is stable. So you can find that the value of K can guide the stability; it depends whether the total plot is going to encircle the minus 1 point or not, thank you very much.