Control Engineering
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Lecture - 35
The Nyquist Stability Criterion and Stability Margins

Well friends, from today onwards we enter into a new phase of our discussion. I like to say the basic concepts about the hardware, about the performance a control system is required to achieve, about how to design a compensator to achieve that performance. I like to say that the basic concept involved in the complete process of design has already been taken. The only thing is that one of the roots has been taken the pole-zero root or the root locus method for design has been discussed.

Today onwards in the remaining six hours, about 6 hours I think we are left with, in this period we are going to take up the new formalism of analysis and design called the frequency domain formalism. So you will find that the basic concepts remain the same the only thing is that the interpretation and hence the analysis and design techniques will change. Though the sequence is not this way may be the frequency domain was developed earlier than the root locus but nevertheless the presentation of the subject has been done in a particular way so that the idea of the transient performance becomes clear. You will realize that it is very easy to appreciate the transient performance in terms poles and zeroes in terms of the root locus plot because you are able to visualize the transient performance directly in terms of closed-loop poles.

You will find that in the frequency domain formalism though the analysis and design techniques are really very good but the interpretation of the results is indirect it is not direct as direct as was there in the case of the root locus method. I will say that as far as present stage of analysis and design of control systems is concerned you consider root locus method, you consider frequency domain or you consider other advanced methods which you have not studied like the state feedback pole placement like the optimal control. Probably the most often used is the frequency domain formalism. The reason is this that this gives as it is said the Robust control.

The dependence of the frequency domain methods on the model of the system is not that stringent as that of any other method including root locus and we know that the model of a system is hard to obtain so naturally you will like to rely on the methods which do not crucially depend on the model of the system. I like to say that this is the main important reason of course the other reasons will follow that you will find that the analysis and design in frequency domain is easier comparatively. So these are some of the other advantages. But as I view, the most important advantage of this formalism is the dependence on the model; the model may not be very accurate still you can get suitably good results implementable result while all other methods heavily depend upon the accuracy of the model.

So I will like to introduce…… you would already know what is frequency response or what is frequency domain. I will like to introduce just make you recall what is frequency response through this particular simple block diagram. Let us say this is the process a system you already know and this system is subjected to an input which is sinusoidal in nature. Let us say r(t) is equal to R 0 sign omega t this is the input. Now the output as is known will have the transient as well as the steady-state component. If I consider this system to be a stable system in that particular case the transient components will definitely die out with time. So it means we will be left with only the steady-state component. So if I concentrate only on the steady-state component of the response I say that y(t) will look like Y 0 sin (omega t plus phi). This will be the nature of y(t) so it means steady-state response of a linear system described by the transfer function G(s) is also sinusoidal for a sinusoidal input the only change is in the amplitude and the phase angle.

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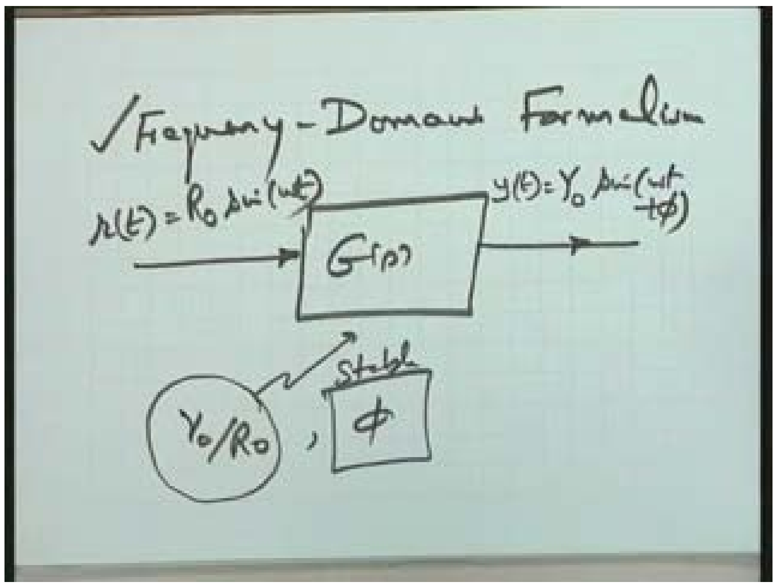

So if I give you the amplitude Y 0 or if I give you the amplitude ratio Y 0 over R 0 and the phase angle phi for different values of omega the sinusoidal frequency you have got the complete input output picture of the system as far as steady-state sinusoidal response is concerned. So you see that in that particular case I will say that the frequency response of this system is given by the variation of Y 0 over i 0 R 0 verses frequency and the variation of phi verses frequency. Because if this information is given to you, you give me any sinusoidal input you will be able to get the sinusoidal response of the system at steady-state.

So I have defined in the process what do I mean by frequency response. So the frequency response for me for a given function G(s) is the amplitude ratio i 0 R 0 verse omega it is you can say is the magnitude plot or magnitude response and the other is phi verses omega. With this information you give me any sinusoidal input and I will you the sinusoidal output of the system.
Let me not go to the details. You know that this Y 0 over R 0 is nothing but the magnitude of G calculated at s is equal to j omega. So I have now to excite the system.

What I want to say is that the frequency response of the system is completely contained in the mathematical model of the system, in the transfer function model of the system. The amplitude ratio is nothing but the transfer function G evaluated at s is equal j omega similarly phi is nothing but the angle of G(j omega). So now let me put it the other way that the magnitude G(j omega) and angle G(j omega) verses omega constitutes the frequency response of the system for me.

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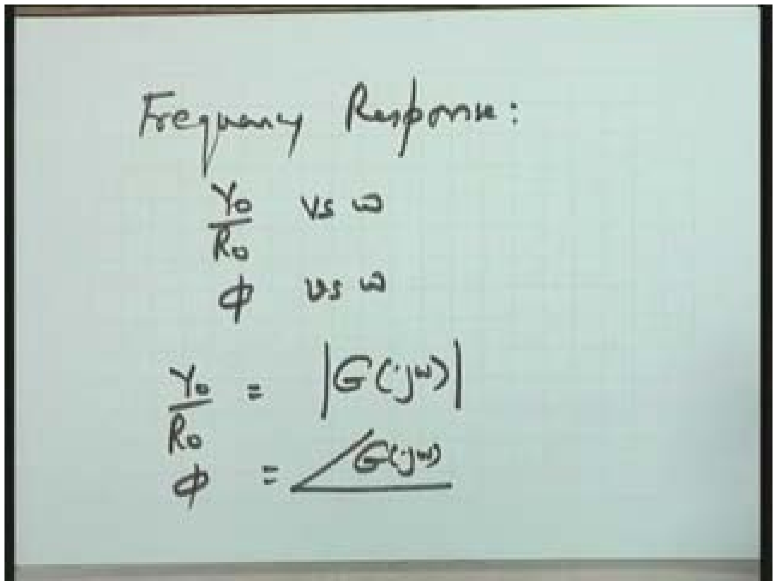

However, this point may be clear at this juncture. I know that you already know it but I like to repeat this. I said G(j omega) verses omega angle G(j omega) verses omega. You may plot them and you can get frequency response plots the graphical picture of the frequency response. Let us say frequency response plots are available. I made a mention that this information gives you the output of the system for any sinusoidal input at steady-state. So, on the face of it it appears as if this particular mathematical model which is the frequency response of the system is only a steady-state mathematical model of the system, is only a model which describes the performance of the system at steady-state. Fortunately the situation is that.

Actually it is a complete characterisation of the system including the transient response because you know given the frequency response through Fourier transforms you can correlate this to time response and hence from the frequency response you can get the response of the system to any time function the only thing is that the time function will have to be broken up into superposition of frequency signals either through Fourier series if it is periodical signal or through Fourier transforms for a non-periodic signal.

So at this point it should be very clearly understood that though all through our discussion now onwards our concentration is going to be on frequency response plots. But it should be clear that the frequency response plots though they have been experimentally or otherwise obtained from the sinusoidal response of the system at steady-state it actually constitutes the complete mathematical behaviour of the system, complete transient as well as steady-state behaviour of the system because Fourier transforms is a correlation is a link between the transient response of the system and the steady-state frequency response of the system.

I will be coming to this point at a later stage when I want to get the transient behaviour of the system, when I want to interpret the frequency response into time domain. At this juncture I will like to proceed further with this comment that the frequency response plot of a given system is a complete characterisation of the system both the transient and steady-state, this should be clear. And one more thing; as far as the frequency response plot is concerned experimentally it is so easy to obtain experimental data that suppose you are working suppose the mathematical model is not available to you you see the mathematical model is not available G is not available to you what you finally want in terms of control system analysis and design is the magnitude of the amplitude ratio verses omega and the angle of the amplitude ratio verses omega this is what you need, so this you can easily obtain experimentally on a given system.

If the hardware is given to you but the mathematical model is not known to you you cannot proceed with the root locus method please see, you will have to first identify the mathematical model of the system. The mathematical model is nothing but the pole-zero model is nothing but transfer function model of the system. But if you go for the frequency domain formalism if you are given the model obtained by physical laws or obtained by any other method you can obtain the frequency response this way or equivalently you subject your system to sinusoidal testing and sinusoidal testing is easy hardware-wise also; subject your system to sinusoidal testing, obtain the magnitude and the phase angle, use this frequency response data to get the frequency response plots and these plots which have been obtained directly from the system hardware is a complete characterisation of the system. You really do not require the transfer function model or any other model for analysis and design. This is a very very important attribute of the system because the modelling of a system is a very difficult exercise.

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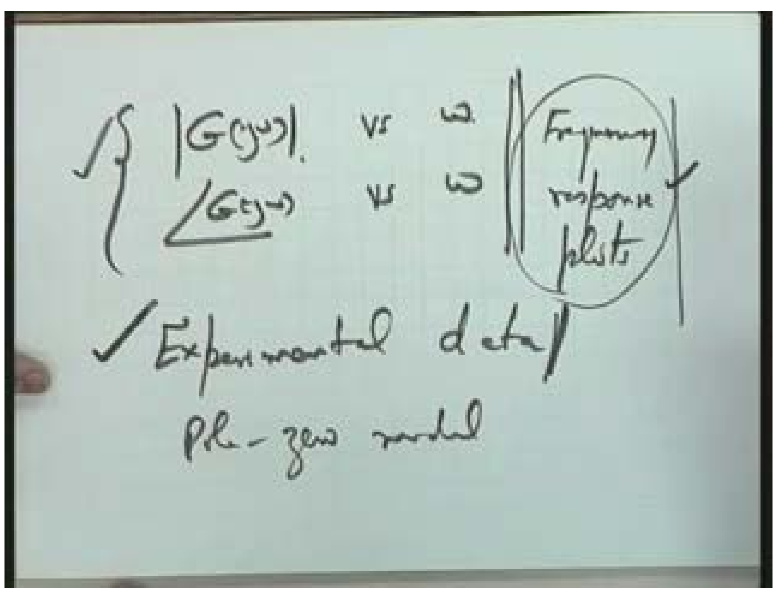

Even through frequency response, for example, suppose you have got this experimental data which is a frequency response data and you want to use the root locus method what will you do? This frequency response data will have to be suitably fitted into a pole-zero model and that pole-zero model will be used for root locus design. But it should very clear that this fit cannot be one to one fit you are definitely bound to make errors. So the frequency response data which you have got you are getting a pole-zero model because you want to use root locus method. However, in the process you have definitely made errors because a complete mathematical model is not possible, a complete fitting will not be possible so you are working on an approximate model when you are going for a root locus formalism. But in the frequency domain formalism since this step in between is not needed you are directly using the raw data for your analysis and design.

So I hope this point is clear that as to why is this particular approach the frequency domain approach is so important for the control engineers. It is a really very important….. it is extremely important for the control engineer. Not only this you see you see, some of the concepts of the performance of the system are well understood from the communication theory. Communication engineers you see are happy with the sinusoidal transfer functions because for them the input is also sinusoidal in nature or superposition of sinusoidal signals. A control engineer naturally is not necessarily working with sinusoidal input. When we are taking frequency domain formalism it is absolutely not in our mind that our controlled system will be subjected to sinusoidal input.
It may never be subjected to sinusoidal inputs but still we are going for frequency domain formalism because of its advantages.

The noise characteristics of a system can best be understood in frequency domain. So naturally if I have frequency domain formalism I can see the noise filtering characteristics more appropriately then its equivalent affect in time domain. So the bandwidth will be defined, the bandwidth equivalent is rise time but the rise of a system really is not a true representative of the noise characteristics of the system as far as our understanding is concerned.

Well, bandwidth we immediately appreciate as to what type of noise characteristics or noise filtering behaviour the system is going have. So it means there are some characteristics of control system behaviour which are better understood in frequency domain. So I like to conclude my statement introductory statement with this particular view point that the root locus formalism is better because the transient response is well appreciated directly in terms of closed-loop poles though the root locus method relies on the accurate mathematical model. The frequency domain method is better because it gives us the noise filtering characteristics, it can work on less accurate mathematical model or experimental model and secondly, it really gives us easier methods of analysis and design.

So can we say that the two methods complement each other? Otherwise we can say that, well, there is not design method…….if one design was fool proof the theory would have stopped there, we still continue with the research because no design method is fool proof. so I will say, I will like to comment that the two methods why only two even the other methods which have been developed which we will not be able to take up in the class all these methods complement each other and therefore a control system designer has to be conversant with all the methods of analysis and design which have been developed so far so that he can develop some new methods as well. I expect some new results from some of you if you take up control systems as your major area of professionalism.

With this now I think I can go to the basic steps of the frequency domain analysis and design. I will like to first take up the stability then I will like to explain how to specify the performance and then the design. You see these are the three steps we have taken up in the root locus method: the stability, the performance and the design. In the root locus method the stability could be determined by Routh-Hurwitz criterion or directly by the root locus. Here a stability criterion called the Nyquist stability criterion is available to us developed by Nyquist a very important criterion rather the frequency domain formalism is centred around this Nyquist stability criterion. This is the key feature of the total analysis and design method. It is a very important point, the backbone total of the frequency domain design methods.

In the performance, well, in the time domain we had the rise time, the settling time, the peak overshoot and so on. In the frequency domain we will have the frequency domain's performance specifications. For example, one example I have already given the bandwidth the resonant peak of frequency response as you will see the resonant frequency and so on. That is some of the frequency domain features. Here I said that well some of these features which have correlation with the time domain features are to be interpreted in the time domain indirectly this is the only limitation of the method.

The design, yes, we had the lag compensator, the lead compensator, the lag-lead compensator in the time domain or with the root locus method. You will find that all these basic compensators for which the Op-Amp circuit has been given to you works satisfactorily when we go to the frequency domain design also and the design method becomes easier. In the time domain you always have been raising a question to me whether in the tutorial class or in the theory class as to what happens to the dominance condition; if the dominance condition is not satisfied how do you proceed. So you can appreciate, more or less you are entering into a trial and error design cycle and when you repeat your questions to me again and again I end up with the answer that if you cannot achieve this, well, I do not have any other tool with me. In that particular case you have to either accept the design or reject the design because there is no other tool with me other than the trial and error which you can carry out.

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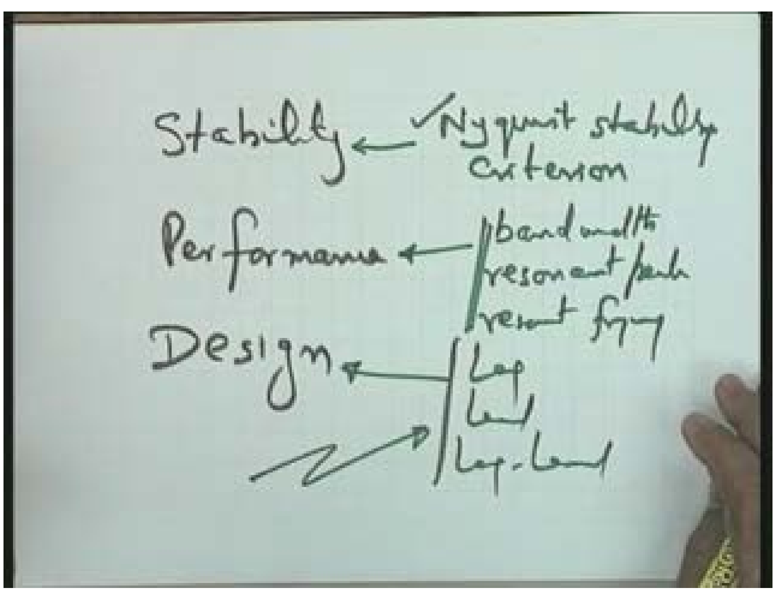

But you will find that when you go to the frequency domain the trial and error though they are but is reduced considerably. All those questions which you had been raising to me will get reduced when we go lag compensator, lead compensator or lag-lead compensator in frequency domain. So these are the basic features of the frequency domain design analogous to the root locus design which we are going to take up and therefore without spending more time I get started with the first feature, the Nyquist stability criterion.

The complete rigorous proof I will avoid for want of time but I think I will be able to make you appreciate the basic idea behind the criterion. It depends upon complex variable theory particularly the principle of argument. I like to explain the criterion this way. Let me take up, yes I really need your attention on this point because the rigorous mathematics is being avoided, only intuitively I will make you understand this particular criterion and its application. The application will be straightforward but try to understand the basic concept behind it.

I take this as the closed-loop system, a general system, a general single loop system. In that particular case I know that Y(s) over R(s) is equal to G(s) over 1 plus G(s) H(s) is the closed-loop transfer function of the system. Please see, help me.

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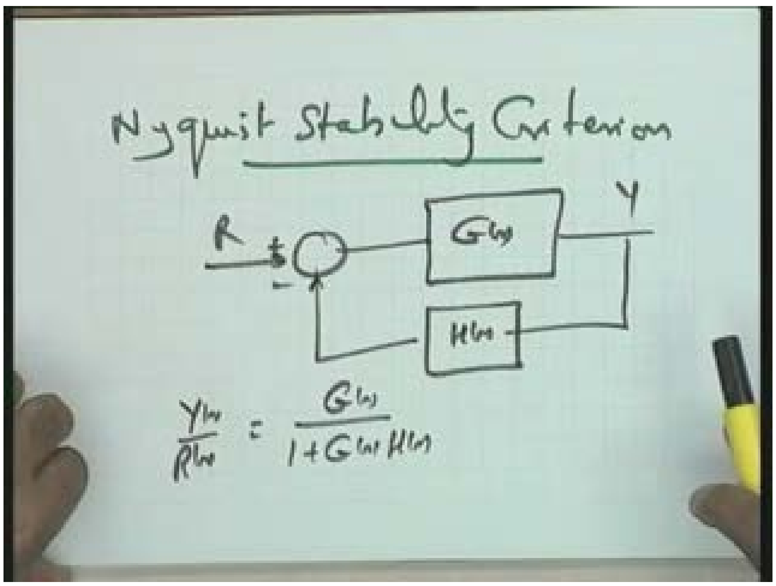

The denominator of this function is 1 plus G(s) H(s). If I say, see my statement please; if I say that there is no point in the right-half of s plane say this is the s plane sigma j omega (Refer Slide Time: 20:47) this is the region I am considering including the imaginary axis. If there is no point in the right-half of the s plane including the imaginary axis for which 1 plus G(s) H(s) equal to 0 in that particular case your closed-loop system is a stable system. You see that I am twisting the characteristics equation statement a little because I want to apply this in this way when I come to Nyquist stability criterion.

I repeat this point that this the s plane I have considered; in this s plane all the s points are there. You consider the region the right-half of the s plane including the imaginary axis. The marginal stability case, initially let me talk of, let me take this as unstable case so that by stability will mean that all the pole all the closed-loop poles lie in this particular region the left-half of the s plane this is my stability definition, marginal stability I am taking it to the unstable region. So my statement which I gave I repeat here that if there is no point on the imaginary axis or in the right-half of the s plane for which 1 plus G(s) H(s) equal to 0 in that particular case I say that the system is stable I hope this point is well taken okay?

Let me talk of G(s) H(s), what is G(s) H(s)?
G(s) H(s) is the open-loop transfer function which is known to me. It consists of the process transfer function, may be the compensator transfer function, the sensor transfer function or any transfer function in the loop but open-loop the loop is broken. So it means G(s) H(s) is known to me. so this G(s) H(s) let me assume for the present is a ratio of two polynomials the N(s) polynomial and del(s) polynomial and all the G(s) H(s) we have encountered so far are really the ratios of the two polynomials except the case of dead time where e to the power of minus s tau d we have taken.

So initially for convenience though the result will be applicable to other functions including dead time initially for convenience let me take G(s) H(s) to be a ratio of two polynomials: the N(s) polynomial and del(s) polynomial. So you will please note, I say that the zeros of G(s) H(s) and the poles of G(s) H(s) are known to me, please see that. Because this is an open-loop function; the process, the sensor, the compensator and all these transfer functions can be multiplied together to get me the value of G(s) H(s) and hence the zeros of G(s) and poles of G(s) H(s) are known to me, this is nothing to investigate, this is a data given to me. This point may be stored in your memory that particularly I am referring to the poles of G(s) H(s) known to me.

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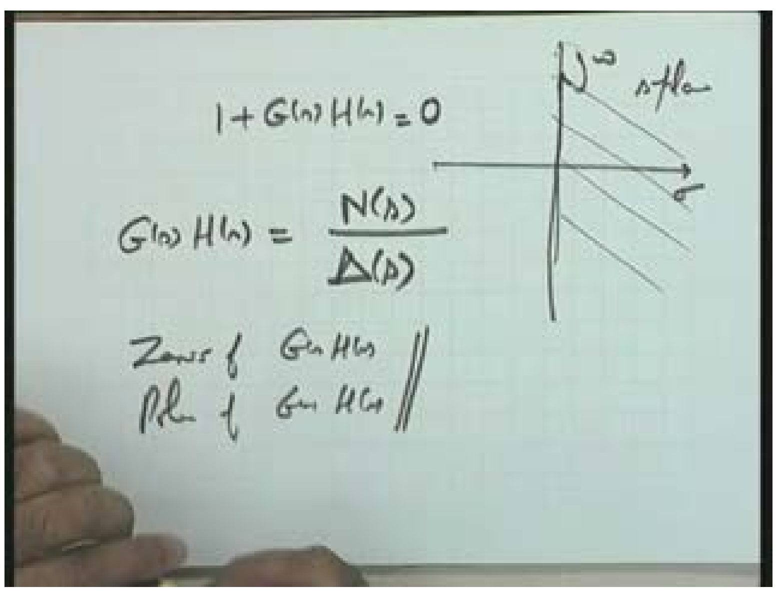

Therefore, you see that if the poles of G(s) H(s) are in the left-half plane the system is open-loop stable and if some of the poles of G(s) H(s) are in the right-half plane then the system is open-loop unstable. But a system may be open-loop stable or open-loop unstable it does not matter because I want to study the stability under closed-loop. My question is that the system under closed-loop should be a stable system because a feedback system or feedback principle has got the capability of stabilizing an otherwise open-loop unstable system. So you see that if you are given an open-loop unstable system it does not matter. It means your plant is open-loop unstable and you want to study the stability properties when the loop has been closed that is under feedback.

Whether the poles of whether the roots of delta(s) or the poles of G(s) H(s) are in the left-half plane or in the right-half plane, whether it is open-loop or open-loop unstable the point I am reemphasizing that this information is known to me and I am interested in the information on the closed-loop system.

Now I am interested to study the function 1 plus N(s) over delta(s) please which is equal to as you see delta(s) plus N(s) over delta(s). I need your help here please. suppose delta s is an nth order polynomial so this is equal to delta(s) plus N(s), the nth order polynomial I assume has been factorised to give you N poles (s minus alpha 1) (s minus alpha 2) and (s minus alpha n).

Can you give me what is the numerator polynomial; what is the order of the numerator polynomial?
I should be able to get the answer from you. I repeat my question: the denominator is known to you it is an nth order, the numerator is not known to you in terms of factored form though N(s) and del(s) are separately known to you but delta s plus N(s) in factored form are not known to you because this is nothing but the characteristic equation of the system which you want to investigate. So my question was to you: what is the order of the numerator roots? N only. This point may very carefully be noted.

The order of N(s) has to be less than or equal to delta(s) for physical realisability of G(s) H(s); after all G(s) H(s) is a physical system and N(s) by delta(s) is the pole-zero form of the physical system. So the order of N(s) has to be less than or equal to delta(s). In most of the control systems we come-up with a situation where the order of N(s) is delta(s). But to give it a complete mathematical picture let me call it less then equal to delta(s). So, if that is the case delta(s) plus N(s) naturally at the most can be of order N and hence this can written as (s minus beta 1) (s minus beta 2) (s minus beta n) divided by (s minus alpha 1) (s minus alpha 2) (s minus alpha n) this is 1 plus G(s) H(s). A very interesting point please. The principle of argument will follow from this point: 1 plus G(s) H(s) is in this form, it is a general one.

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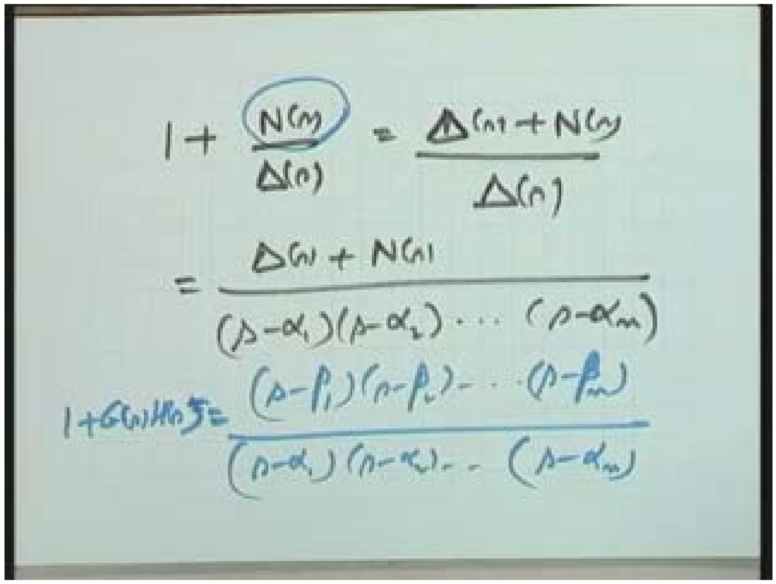

Recall the point; the point is this that if there is any s in the right-half the s plane or on the imaginary axis so that 1 plus G(s) H(s) is equal to 0 in that particular case the system is unstable. This is what is our conjecture is on the basis of which we are going discuss this mapping. So when I say any s consider this as the s plane (Refer Slide Time: 27:49) sigma j omega consider any point s here or anywhere; you see this is a complex variable sigma plus j omega.

If I substitute in the expression: G(sigma plus j omega) H(sigma plus j omega) I can say that this gives me point W which is again a complex variable and let me call this point as u plus jv and therefore if I take what is called another complex plane the W plane whose real axis is u and imaginary axis is jv can I say that this point maps into a point in the W plane under the function 1 plus G(s) H(s) so it is a mapping. And for a rational function though we do not go to deep mathematics it is actually a one-to-one mapping. So the function we are considering it is a one-to-one mapping; for every point here there is one and only point here in the W plane. So it means, actually let me now change the language change the language now; I say that I really want to see the mapping of the entire right-half of s plane under the function 1 plus G(s) H(s) to see whether the origin is covered by this mapping equivalently.

I want to consider the imaginary axis and the right-half of the s plane completely and I want to map this on to the W plane using the function 1 plus G(s) H(s) to see whether the origin of the W plane is covered or not. If the origin of W plane is not covered the system is stable and if it is covered the system is unstable. This is the equivalent statement of what I have been giving over earlier so it means I am going to consider the mapping 1 plus G(s) H(s).

And since every map…… I need your attention here, every map corresponds to a point in the W plane I will like to say that any contour here a contour is nothing but a connection of points will give me a contour in the W plane surely because there is a one-to-one mapping. Since there is a one-to-one mapping a contour in the s plane a connected graph in the s plane will give me a contour or a connected graph in the W plane under the function 1 plus G(s) H(s). So I am actually mapping from s plane to W plane under the function 1 plus G(s) H(s).

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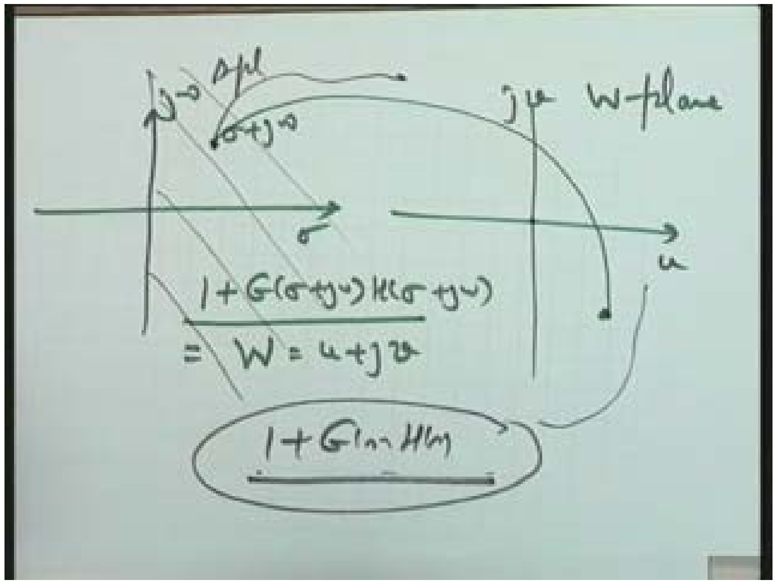

One point may please be noted and that point is this: what is after all your interest you are really not interested to see that under this map what is the magnitude of each and every point correspondingly in the W plane, you are really not interested. Your interest is to see only the covering of or only what is the behaviour of the system around the origin of the W plane; whether the W plane origin is covered or not. So you see that if actually a one-to-one mapping is to be obtained it becomes a very difficult preposition and Nyquist stability criterion will lose its importance. We are really not interested in a one-to-one mapping quantitatively; we are interested in a one-to-one mapping qualitatively only because our interest hinges around the origin of the W plane and this we see any contour in the s plane will map into a contour in the W plane, this also should be clear.

If this is appreciated then a question will follow; the answer to this question will convince me whether my point is well understood or not. I take just tentative contour in the s plane and let me traverse this contour in the clockwise direction, help me what will happen in the W plane or 1 plus G(s) H(s) plane. This is the contour any tentative contour I have taken. The only thing I do is this that one of the zeros at s is equal to beta 1 I enclose in this contour. all other zeros let us say beta 2 pole alpha 1 another pole alpha 2 are outside this contour. See my point please; as I said that any contour in the s plane will give me a corresponding contour in the W plane without justifying as to why did I take this I take a contour and this is a closed contour (Refer Slide Time: 32:24) with the only point that a 0 one of the zeros of the function 1 plus G(s) H(s) is enclosed in this contour and all other zeros and poles lie outside this contour.

Now, as I said, I am not interested in the quantitative mapping I am interested only in the qualitative mapping and comment on the qualitative mapping of this contour to the W plane contour under the function 1plus G(s) H(s) whose poles and zeros are shown to you. This you know is (s minus beta 1) (s minus beta 2) and so on (s minus alpha 1) (s minus alpha 2) and so on. This is your function 1 plus G(s) H(s) and I want you now to comment as to what will be the qualitative nature of the contour in the W plane when s plan changes from point to point all along this particular contour.

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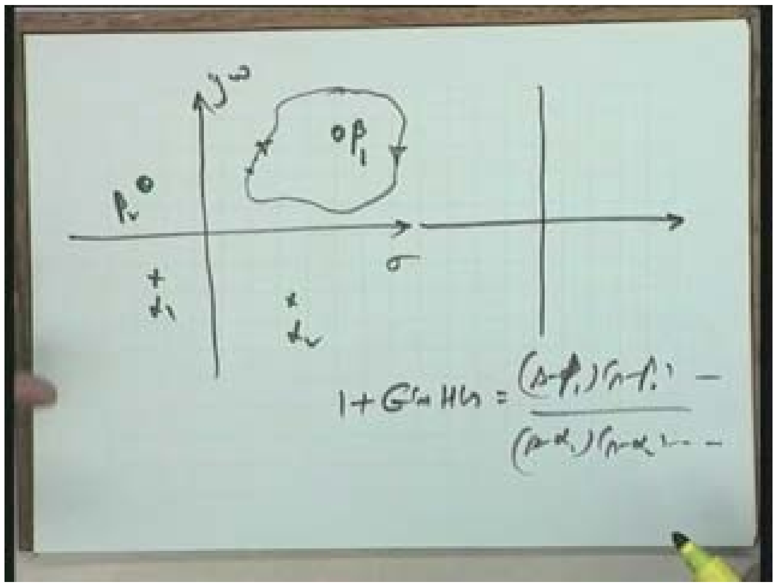

Okay, let us get started. Let us first take beta 1 itself. You see after all if you take s if you take s is equal to s 1 what will you do? You will substitute s is equal to s 1 in this so it means (s 1 minus beta 1) (s 1 minus beta 2) are the numerator phasers; (s 1 minus alpha 1) (s 1 minus alpha 2) are the denominator phasers, multiply the numerator phasers divided by the multiplication of the denominator phasers that gives you the magnitude of the corresponding point here.

Now how about the phase angle?
Adopt all the angles of these phasers, adopt all the angles of these phasers take the difference and that is the angle of this point here. So the s is equal to s 1 point using this particular function 1 plus G(s) H(s) can suitably be converted into this particular point W 1 here (Refer Slide Time: 34:19) this is the W 1 point and how do you get it?
You just take up the phaser, this is your phaser s is equal to s 1 (s 1 minus beta) phaser similarly this is (s 1 minus beta 2) phaser this is (s 1 minus alpha 1) phaser this is (s 1 minus alpha 2) phaser and so on. Since graphically you are working with you are working with a graph you need not do the calculations you just measure all these phasers, multiply the numerator phasers, multiply the denominator phasers, divide and that gives you the magnitude; the angles minus these angles gives you this angle and hence there is a corresponding point. So this way if I keep on travelling along this particular contour in the clockwise direction I will get the corresponding points over here and I will definitely get a closed contour in the W plane.

Now I say that, well, quantitatively I am not interested in…… now qualitative picture just see; look at individual phasers, what will happen with the (s minus beta 1) phaser? (s minus beta 1) phaser as you travel along this particular contour contributes an angle of minus 2 pi you can see;

start form this point, traverse along this and the total angle contributed is minus 2 pi.

Come on look at the (s minus beta 2) phaser (Refer Slide Time: 35:49) start from here and go back to this and come back to this the total angle contributed by (s minus beta 2) phaser is 0. Because if this is positive there is a corresponding negative angle, please see. If this side is positive there is a corresponding negative angle. So you see that the (s minus beta 2) phaser if you take as s traverses along this particular root the net angle contributed by (s minus beta 2)….. this is the crux of the total Nyquist stability criterion, please see. That, if there is a 0 enclosed in it it is going contribute an angle of minus 2 pi and if all others poles and zeros phasers outside do not contribute the net angle as far as this particular contour is concerned so qualitatively I can say that whatever may be the shape you see when you travel along this particular root the contour is going to encircle the origin once and only once and that to in the counter clockwise direction because the angle is minus 2 pi. See this point please; on this I am going to base the criteria and the criterion will become very simple.

You see that if there is a 0o enclosed here (Refer Slide Time: 37:11) by this s plane contour which you have tentatively taken in that particular case I find in the corresponding s plane contour there will be a contour shape may be anything I am not going to do the calculations because I am not interested in the shape but this is sure that this particular contour will enclose this particular minus 2 pi minus 2 pi will be sorry what is the angle please? Help me please; this if you travel from this side to this side this side angles are minus 2 pi sorry clockwise direction because this angle if you are taking the net angle is this….. this way the angles are taken negative and minus 2 pi so it means this is going to the encircle the origin in the clockwise direction once.

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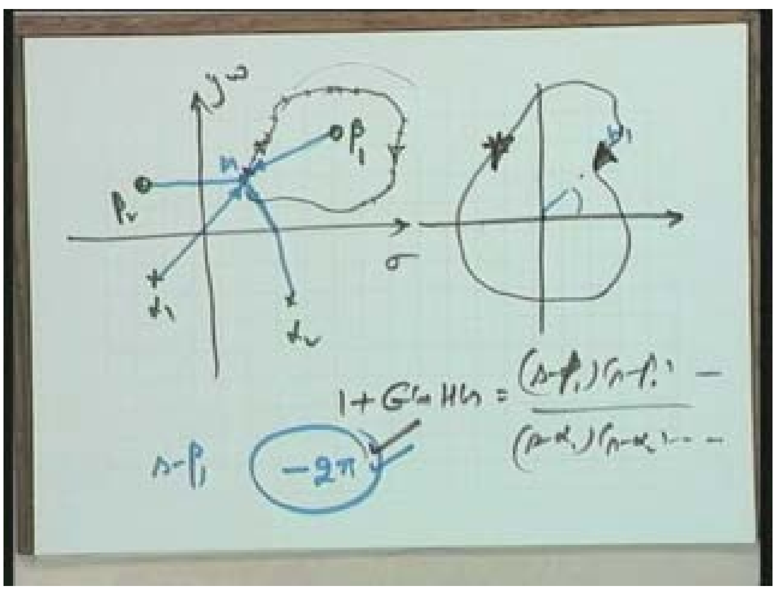

Now you just imagine that…. repeat this performance and say that there is pole now in the origin. In this particular enclosed path there is s pole and not a 0. All other zeros and poles are outside this particular contour, all other poles and zeros are outside this. Help me please, what will happen to the encirclement when you go along this particular contour?

It will become anticlockwise. Yes, please see that now also the angle is minus 2 pi but since it is a pole the net angle contribution is reverse of that and therefore it will become plus 2 pi as well as net angle contribution to 1 plus G(s) H(s) is concerned. The net angle contribution to 1 plus G(s) H(s) I hope this point is very clear. Angle is minus 2 pi as you traverse it this way in the clockwise direction (Refer Slide Time: 38:57) but since this particular pole is in the denominator of the function 1 plus G(s) H(s) so the net angle contribution is plus 2 pi and therefore the net encirclement of the origin is going to be 1s in the contour clockwise direction. And now extend this particular situation; this is your contour, it has got two poles and a 0, two poles and a 0. So in that particular case please see that two counter clockwise and one clockwise the corresponding thing will be that there is one net rotation in the counter clockwise direction because the two counter clockwise rotations are given by poles and one clockwise rotation is given by 0 and therefore the total counter clockwise rotations are 2 minus 1 is equal to 1.

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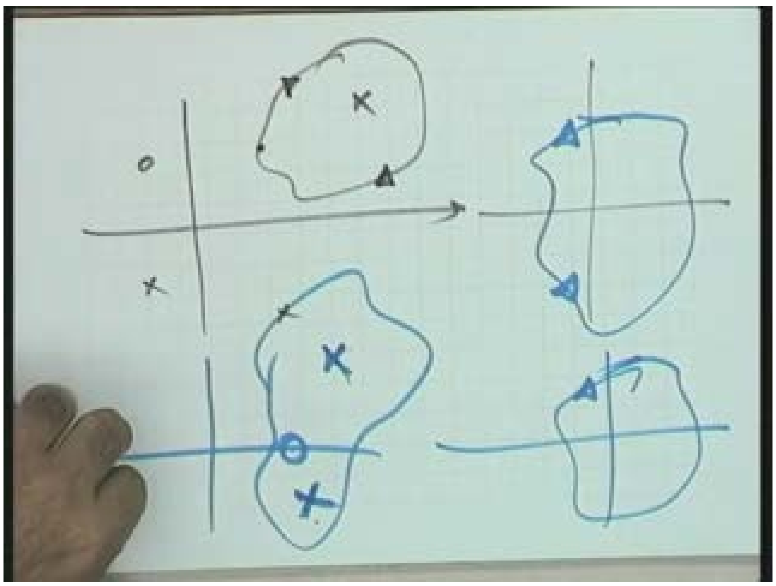

How about this? Please see; what I want to say is this, now I think this the picture should be clear, through this the picture should be clear now and I hope you will be to state the Nyquist stability criterion now.

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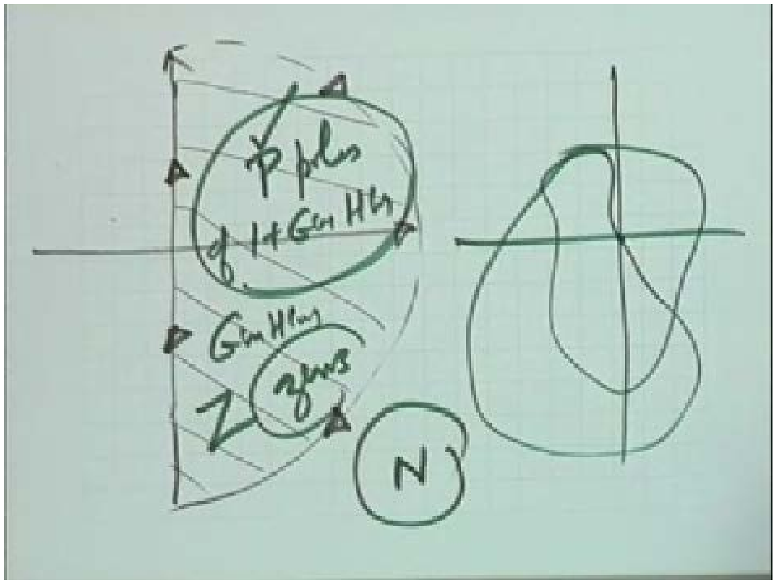

I will leave it open and try to get the criterion from you as to how to test this stability. What I want to say is this that the s plane contour which I have thought of is now the complete right-half plane though it is an infinite contour. How to handle this situation I will come to this later. The complete region and therefore this is the contour which I want to travel in the clockwise direction. And suppose you have got the map this is the corresponding map, it encircles to twice. Thrice we do not know. Suppose you have got the map so please tell me as to how to check the stability.

Suppose in this particular contour (Refer Slide Time: 40:54) just suppose that there are how many P poles of 1 plus G(s) H(s) you will please recall that these P poles of that 1 plus G(s) H(s) are nothing but the poles of G(s) H(s) which are known to you and hence the value P is known to you. Capital P I am taking P poles of 1 plus G(s) H(s) these are not closed-loops this point may please very well understood, these are not the closed-loop of the system; the function under consideration is not Y(s) by R(s) but the function under consideration is 1 plus G(s) H(s) so it means if there are P poles of 1 plus G(s) H(s) in this particular region and let us say Z zeros what are the zeros of 1 plus G(s) H(s); are you able to appreciate this point that zeros of 1 plus G(s) H(s) are nothing but the closed-loop poles of the system? Try to see this point.

Your zeros are not known to you; zeros of 1 plus G(s) H(s) you can work out in your own way and you will find that zeros of 1 plus G(s) H(s) are nothing but the closed-loop of the system and poles of 1 plus G(s) H(s) are nothing but the open-loop poles of the system. So if there are P poles a value which is known to you and Z zeros a value which is not known to you and you have got the 1 plus G(s) H(s) map of the function in that…….. let us say that there are N encirclements of the origin in the counter clockwise direction.

Can you give me the equation please? I want the equation from you.
There are P poles of 1 plus G(s) H(s) in the right-half plane and Z zeros and there are N encirclements of the 1 plus G(s) H(s) contour of the origin by the 1 plus G(s) H(s) contour can

you give me an equation between N, P and Z? It is N equals to P minus Z that is fine. This is what the Nyquist stability criterion is. How to use it and how to interpret it we are going see.

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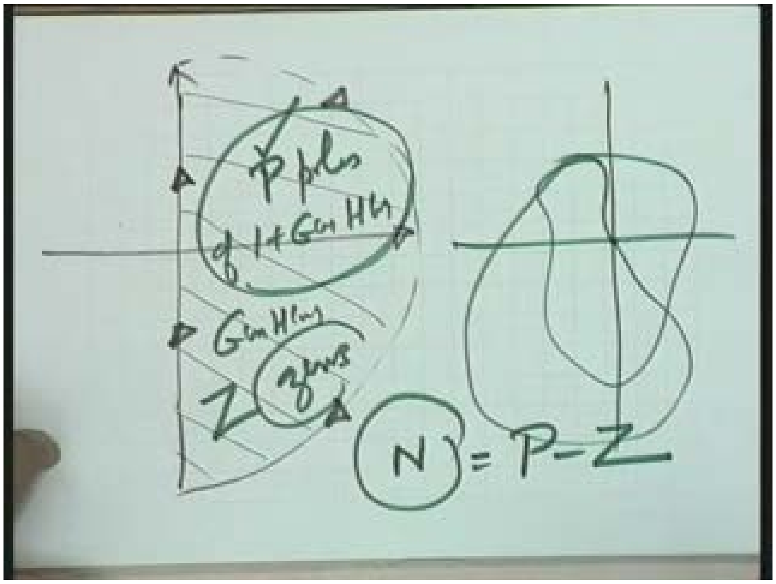

But the Nyquist stability criterion is this and the importance you will see that the total frequency domain design is based on the Nyquist stability criterion. It says N is equal to P minus Z that is the total counter clockwise encirclements of the origin of the W plane contour is equal to P the number of poles of 1 plus G(s) H(s) in the right-half plane minus Z the number of zeros of 1 plus G(s) H(s) in the right-half plane. This should be clear now.

Now you see that the only thing you have to do is now get a map of….. write this particular contour getting a map of this particular contour and counting N is the only thing left, counting N. [Conversation between Student and Professor………yes please, ((what is the poles and zeroes)) ((00:43:57 min)) on the contour, yes it is very very important point please though I am going exemplify it you see when I take this contour his point is this, can you tell me what will happen if there is pole or a zero pole particularly? Zero will not create any problem but if there is pole on the contour………. in that indent the contour……..you are going to get the corresponding infinity and the corresponding mapping does not exist so initially when I am describing this particular situation let me describe this under the assumption that there is no pole on the contour. I will extend it, what will happen when there is pole on the contour? It is because pole is that of an open-loop system and you know beforehand whether it lies on the contour or not. You know beforehand that it does not create any problem as far as pole is concerned. Zeros will not create any problem but pole will.

In the beginning let me say that I am talking of only those systems for which there is no pole on this particular contour and this information is known to me, for this there is no problem. You see the corresponding encirclements and these encirclements will give you the value of N. The value of N is known to you and the value of P is known to you and hence the value of Z is known to you.

[Conversation between Student and Professor………(45:20) Sir, there is zero on the contour, yes is that including Z. If there is 0 on the contour what will be its mapping? The contour will pass through the origin, the contour in that particular case will pass through origin and this actually is the case of marginal stability so this case also will come up please. That is, actually if this contour passes through the origin this case corresponds to as you have seen in this particular case the zeros of the system, zeros of sorry sorry sorry sorry.

Now tell me please? Zeros of N(s) are known to you; zeros of N(s) on the contour do not create any problem; the zeros of N(s) because it is the zeros of 1 plus G(s) H(s) which are there (s minus beta 1) (s minus beta 2) and so on. So, as far as this statement that a 0 on the contour has no meaning because the zeros of the closed-loop system you do not know, the zeros of the function 1 plus G(s) H(s) you do not know and the zeros of G(s) H(s) are only known to you and the zeros of G(s) H(s) do not create any mapping problem.

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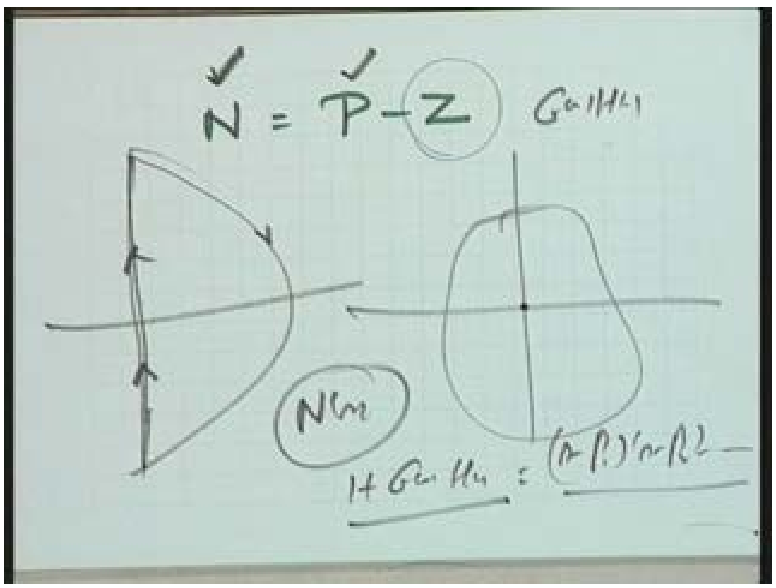

Sir, what will be the problem by a 0 1 plus G(s) H(s)? That is an analysis problem that we will see; that is, you see this statement at last let us revise that the Nyquist contour passing through a 0 of G(s) H(s) does not matter, this I am revising. This is another point that is if this contour passes through the origin of the 1 plus G(s) H(s) contour this is a point I am going to explain, this is a different point. So it means we are left with two points now to be explained: one: if there is a pole of G(s) H(s) on the contour because you get corresponding infinity and second; if this contour passes through the origin. But for these two points everything is all right, please see.
These are the two points which need explanation. But if there is a 0 of G(s) H(s) on this particular contour that does not create any problem that should be clear that is what I wanted to revise.

So your singularities or the problem cases are there when there is a pole of G(s) H(s) on the Nyquist contour let us call this as a Nyquist contour (Refer Slide Time: 47:32) this particular contour which you have taken I name it as Nyquist contour. So, if there is a pole of G(s) H(s) on this yes there will be a problem and the second point of interpretation is this that if it passes through the origin. Keeping these two points pending assuming that none of these two conditions exist let us see what is the interpretation of Nyquist stability criterion. It says that N is equal to P minus Z you count N from here, P is already known to you therefore from this equation you have Z. So in this particular case you find that these many zeros of 1 plus G(s) H(s) are in the right-half plane and hence not only instability but some additional information about instability is also given as was given by Routh stability criterion.

You recall, it gave you the number of poles in the right-half plane. This information is available in the Nyquist stability criterion also; it gives you information on the number of zeros of 1 plus G(s) H(s) in the right-half plane which is the equivalent to number of closed-loop poles in the right-half plane. So it means the procedure is this that you take this contour, map it into this point and see the number of encirclements on the origin and apply this particular algorithm to check whether the system is stable or not. If it is unstable how many closed-loop poles lie in the right-half plane? If this is clear then help me please equivalently; if you have an open-loop stabled system what is the condition of stability of the closed-loop system? I repeat my sentence for you to observe; for an open-loop stable system means P is equal to 0 what is the condition of stability of the closed-loop system the case most often we come across; I am making the statement because this is the case we come across most often for what is the condition of stability of the closed-loop system for an open-loop stable plant?
Yes, I should get the answer from you I will wait please. [Conversation between Student and Professor….. 0 should be under left-half, no, in terms of encirclements what should be the condition on the W plane contour if your original system is open-loop stable that is the open-loop system is open-loop stable? There should be P encirclements…….. there should be? When I say open-loop stable means P is equal to 0 this is what I want you to identify. There should be no encirclements of the origin that is why I said that I repeated the statement open-loop stable system. An open-loop stable system is capital P is equal to 0 note this point please.

You will…. 90 percent cases will be of this nature of the statement that is why I repeat Nyquist stability criterion should be clear but most often we will come across the statement that for an open-loop stable system P is equal to 0 so it means if you have case for which P is equal to 0 in that particular case the system is stable and you know that for stability Z is equal to 0 and Z is the closed-loop pole in the right-half plane; Z is equal to closed-loop poles in the right-half plane. For stability Z has to be equal to 0 so it means, the statement is: for open-loop stable system the stability requires that the Nyquist contour the Nyquist plot or this particular plot in the G(s) H(s) plane let me call it the Nyquist plot, the Nyquist plot in the W plane does not encircle the origin. Nyquist plot in the W plane does not encircle the origin. That is for example this could be the possibility.

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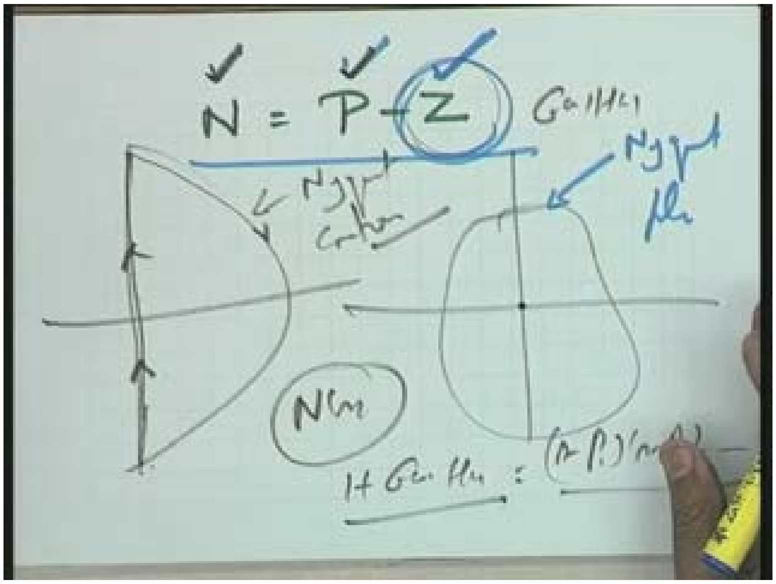

If this is the contour (Refer Slide Time: 51:34) P is equal to 0 there is no open-loop pole in this particular region, your plot may be of this type, it cannot encircle the origin, your plot may be of this type. What is this plot? This plot is a plot of 1 plus G(s) H(s) function. Please see that. I am looking at the watch. 1 plus G(s) H(s) plot is difficult to make. I hope you will not mind if instead of making 1 plus G(s) H(s) plot I make only G(s) H(s) plot and encirclements of the origin instead of looking at the encirclements of the origin I look at the encirclements of the point minus 1 plus j0. That is, I shift the entire plot by one unit towards left. So in that case the concluding statement an open-loop stable system is stable under closed-loop operation if and only if the plot of G(s) H(s) does not encircle the minus 1 plus j0 point. This is the crux of the total criterion and our additional lectures are going to be based on this criterion only.

How about the other case you see? If there is a case of open-loop unstable system tell me N is equal to P minus Z what is condition of stability? Statement please if you want to give me the right statement. An open-loop unstable system with P is equal to 1 let us say a situation of this type s plus 2 over (s plus 1) (s minus 1) you find that there is one pole in the right-half plane, it is open-loop unstable system, it means capital P is equal to 1, tell me what is condition on G(s) H(s) plot for stability?
[Conversation between Student and Professor – Not audible ((00:53:30 min))] It should encircle minus 1 pus j0 point once, it is not enough, once in counter clockwise direction, statement once is not enough please. For this system to be stable under closed-loop operation the G(s) H(s) plot should encircle minus 1 plus j0 point once in the counter clockwise direction this is what is the Nyquist stability criterion, thank you very much.

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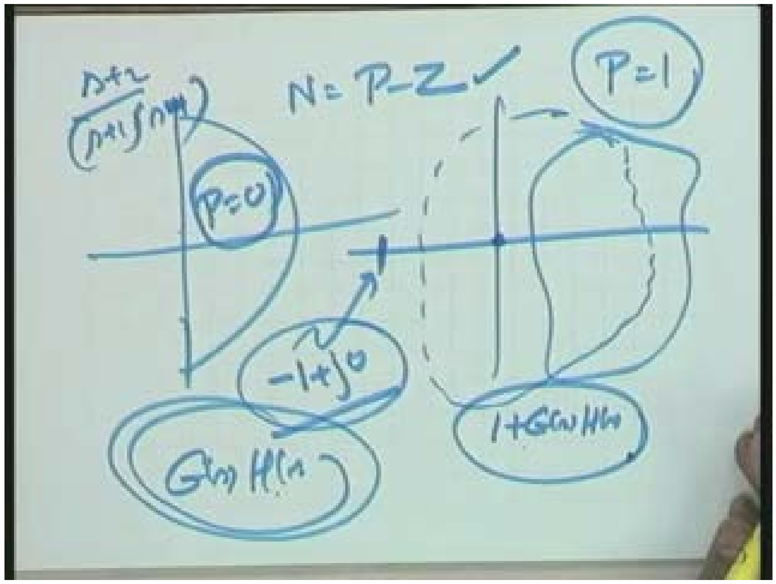

The applications will follow in the next lecture.