

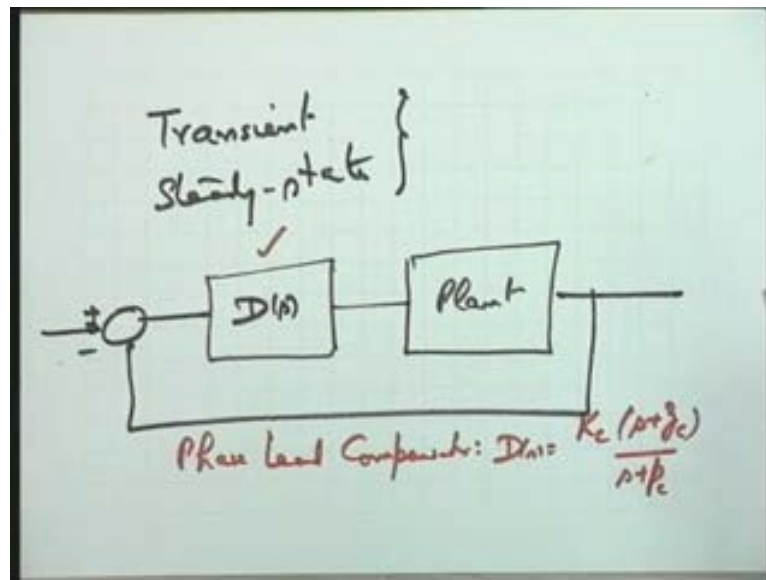
Control Engineering
Prof. Madan Gopal
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture - 34
Compensator Design using Root Locus Plots (Contd..)

Yes friends, let us take up the last phase of our discussion on root locus design. As I told you last time may be couple of full design examples we could take up today so that all the aspects of root locus analysis and design we have discussed so far are taken care of. I will just try to revise first as to what is the compensator and what are the types of compensators we have decided. The compensator is a form of a controller PI, PD or PID controller and the requirement is to compensate for the deficiency in the performance. The performance has been listed under the two subheadings: transient performance and steady-state performance. So it means specifications will be given on transient and steady-state performance and all other specifications will be tried will be seen only by simulation.

The objective is to meet both the transient and the steady-state performance on a system, performance requirements whatever that has been imposed by the user. We have seen that the structure of the system is going to be like this (Refer Slide Time: 2:08) this is the plant and in cascade I put a controller or a compensator. Let us say that the $D(s)$ is the transfer function of the compensator, this is the structure we have taken though minor feedback loop is also possible and naturally if I put a minor feedback loop it can also be converted into a structure of this form so that the design can be taken up. Now what are the different values of $D(s)$ we have seen. The different values of $D(s)$ you see let me revise for you: one is phase lead compensator. In a phase lead compensator if you recall the transfer function $D(s)$ is of the form $K_c a$ gain $(s + z_c)$ over $s + p_c$ that is a pole and a 0.

I told you that there are three parameters of the compensator: the gain K_c , the pole and a 0 and if you recall an Op-Amp circuit given to you the transfer function of that Op-Amp circuit resulted in this type of model. There were three values the z_c p_c and K_c which were to be decided from the different values of resistors and capacitors in that Op-Amp circuit.

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So, if I say that the phase lead compensator is given by this; I repeat it here: $K_c \frac{(s+z_c)}{s+p_c}$ over $s+p_c$ the pole-zero structure of the phase lead compensator is of the following type: there is a 0 here at minus z_c and there is a pole far away at minus p_c . Actually the compensation I hope you definitely remember it, is given by this derivative action the 0 at z_c is equal to minus that s is equal to minus z_c corresponds to the derivative action and the total root locus plot is being pulled to this to the left-half of the plane because of this derivative action (Refer Slide Time: 4:11) because of the 0 at s is equal to minus z_c .

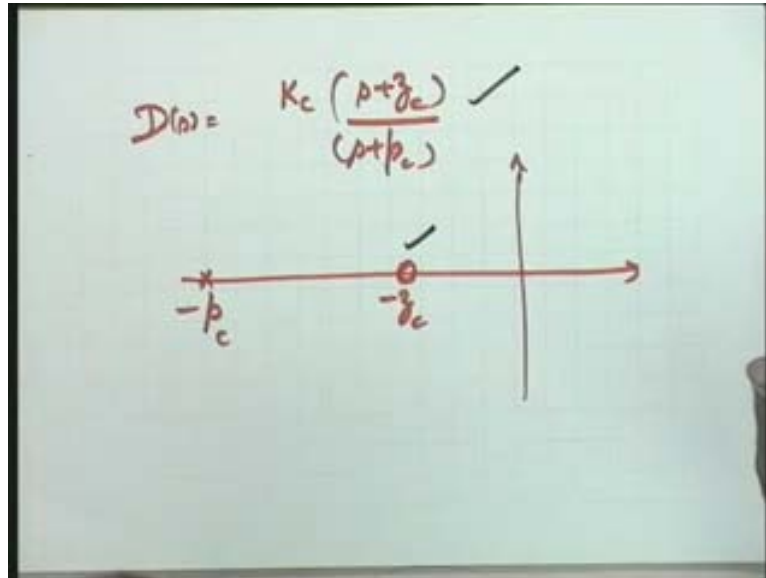
Additional pole is coming because of the realization and the noise attenuation requirements. So, if the noise attenuation is required high frequency noise will be attenuated because of this filter because of this pole at s is equal to minus p_c as we have seen. So you see in this particular case what is the requirement of the lead compensator or how do you decide where should you put the lead compensator. See that the 0 at s is equal to minus z_c is going to pull the total root locus to the left so does it not say that if there is a deficiency in the transient performance of the system better you go for a lead compensator because putting it or shifting it to the left means improving the stability of the system, improving the transient performance of the system.

A root locus plot nearer the $j\omega$ axis is an indication of the situation close to instability, close to oscillations. A root locus plot far away from the $j\omega$ axis guarantees good margin of stability. So a 0 at s is equal to minus z_c pulls the root locus plot to the left and therefore you should take up the phase lead compensation as and when there is a requirement of transient performance accuracy. I think right at this stage it will be appropriate if I take an example then the attributes of the phase lag compensator will be taken later.

I give a comment here that the PD controller which we have already read which we have already discussed at so many places in our discussion in this classroom is a special case of a lead compensator. If I make p_c 's if I remove this particular pole (Refer Slide Time: 6:05) this is the transfer function of a PD compensator. So I am introducing a pole over here to take care of the additional noise filtering requirements. So, if I ask you for a design of a PD

compensator you can forget about this pole so in that particular case I want you to place the 0 appropriately so that all the performance requirements are satisfied.

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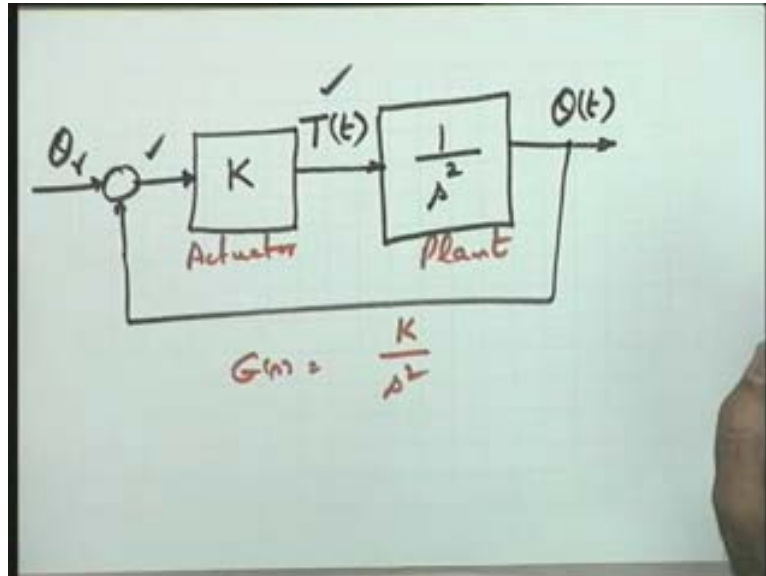
A simple example simple from the point of view of calculations but otherwise it is a practical example. I take the example of attitude control of a satellite: $1/s^2$ is a model I think we derived this model in one of the tutorial classes but does not matter the attributes of the model will be clear right in our discussion today itself. $1/s^2$ is the model of this and you see that it is a type-2 system, the input to this is a torque $T(t)$ and the output is the attitude angle $\theta(t)$. So please note that since it is a type-2 system comment on its steady-state performance, the steady-state performance of this system when the loop is closed is going to be satisfactory, is going to be good because the steady-state error to step inputs will be 0, the steady-state error to ramp inputs will also be 0 and there will be a finite amount of steady-state error to acceleration inputs this being a type-2 system.

You see that when you are given a plant some initial comments you should be able to make right at this particular stage itself. So in this particular case the most important thing as far as compensation is concerned will be with respect to stability, with respect to transient performance because a double at the origin is going to create lot of instability problems. Initially let me take a very simple situation: a constant K and it is a constant of an actuator which actuates so that the electrical signal here is converted into torque (Refer Slide Time: 8:15) this is actually taking care of firing of the thrusters. So it means thrusters are fired suitably so that a torque T is generated which is proportional to this electrical signal which again is proportional to the error between the commanded signal θ_r and the actual position θ .

So actually you see that the total hardware need not be given to you again and again, the position should be very clear now that a suitable hardware is required over here so that θ_r the commanded position is compared with θ the actual position and error in radians will be generated that error naturally will be converted to an electrical signal and that electrical signal is converted into a torque and all this is clubbed into a single constant K and this is the

plant so let me call this as actuator and here is the plant. It is a feedback system and therefore I can say that the open-loop transfer function of this feedback system is K/s^2 .

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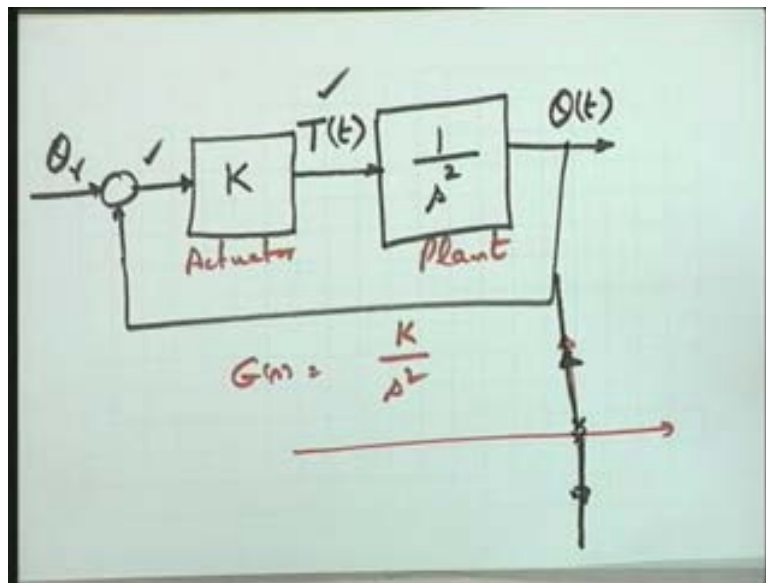


How about the closed-loop?

The closed-loop behavior will be given by the root locus sketch as K varied, you can immediately draw the root locus sketch. In this particular case please see that the root locus sketch is given by these lines on the imaginary axis. So it means for all values of s the root locus plot shows that the system is oscillatory in nature. So, before you go for a..... though you see I am going to illustrate phase lead compensator but naturally you see given a situation you will have to take this decision whether you have to go for a phase lead compensator or a phase lag compensator. The decision making problem is also the problem of the designer. So at this point you see I take a decision that probably phase lead compensator is the choice because phase lag compensator has to improve the steady-state accuracy and steady-state accuracy improvement is not required in this case.

Phase lag you can just imagine it is nothing but adding one more integrator to the system. Adding one more integrator means making a type-3, naturally you will not like to go for that because it will be a highly unstable system. So the choice in this particular case is phase lead and the idea is this that this particular root locus plot should be pulled on to this side.

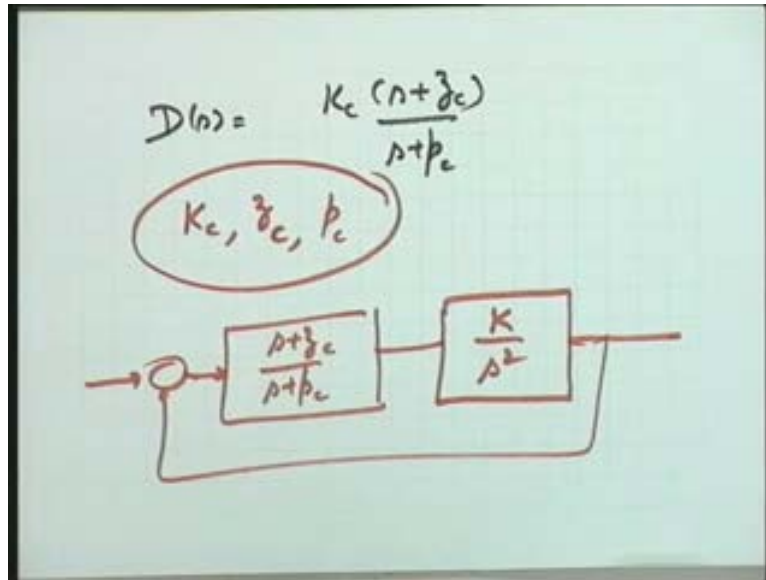
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Therefore now I take..... any question please? I take this, this K is being replaced by $D(s)$ is equal to $K_c (s + z_c)$ over $s + p_c$ this is what I need. You see that the design procedure is really not a procedure which can be followed step by step. All of you for this particular problem will be giving a different design but all the designs may be acceptable and therefore what you have to concentrate on is the concept behind the design and not the exact procedure and my effort will be to illustrate the concepts and this example has been taken for that particular objective. So see that this K has been replaced by this particular compensator $K_c (s + z_c)$ over $s + p_c$ so there are three parameters K_c , z_c and p_c which you can manipulate, there are three parameters.

You see that the compensator design becomes more logical; the steps become more logical if I reformulate my problem like this. **I am sure you will appreciate my point and raise a question if it is not clear.** I say that the plant only from the point of view of steps in the design procedure I said that the plant is given by K by s squared where K is an adjustable gain and the compensator is given by $s + z_c$ over $s + p_c$. This is the structure I take as far as the design problem is concerned.

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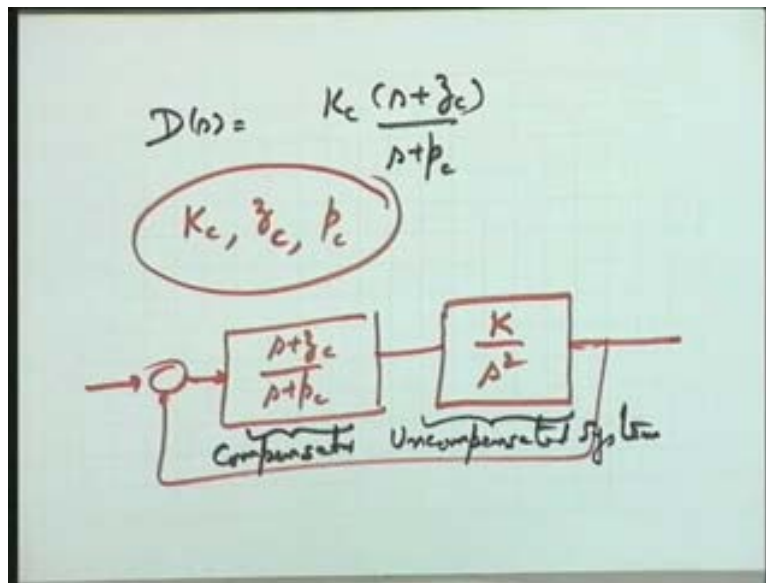


So it really does not matter because if the plant gain is fixed, in that particular case whatever decision on K you take that naturally will be implemented by a suitable amplifier or an amplification stage in the compensator itself. The point is this that there are three parameters which are under your control; how do you implement those three parameters how do you realize those parameters it depends upon the hardware.

So when I put a K over here it necessarily does not mean that this K means that you will make an adjustment in the plant, this K really does not mean that though adjustment in the plant may partially realize your gain. For example, you require this K to be 100 you may require you may realize partially if some adjustment is possible; we do not know what type of plant we have, we do not know what type of hardware we have; may be this type of gain if you got 100 partially may be realized by the plant and then partially it will be realized by an amplification stage in the compensator. Or in addition to this particular compensator you may introduce one more amplifier in this stage.

So what I want to say is this that the implementation of the total scheme depends upon the hardware and I take the design procedure in such a way that there are three parameters $K, z_c,$ and p_c and I will call this system as an uncompensated system in my design terminology. Though you can see now actually in uncompensated system is a partially compensated system because one of the decisions you have already taken about the gain and that gain K is also a parameter of the compensator so I think I have quite emphasized or may be over emphasized my point this is only to set the design sequence in order that the uncompensated system is given by K into a transfer function where K is an adjustable parameter and a compensator is given by s plus z_c over s plus p_c where its gain has been taken as unity, where the gain of these $D(s)$ has been taken as unity for the purpose of convenience of design procedure.

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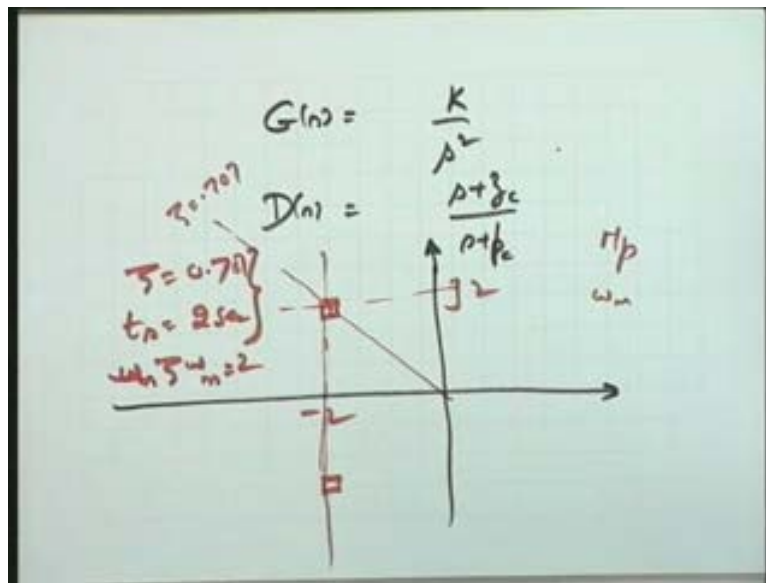


So in this particular plant which is the plant of an attitude control system for a satellite the uncompensated system I will say is K/s^2 this is an uncompensated system and $D(s)$ the compensator is $(s+z_c)/(s+p_c)$. Now you please help take the decisions and you will find that these decisions the design experience are so varied; you pick up any book any literature on the subject you will find that the author will give his own view point there but the basic idea will remain the same. So the sequence which I give may be different than the sequence you see from any book you pick up, including my own book where the sequence may be different.

But see the concept behind it, the concept is this that I want to pull this particular root locus to the left so that transient performance requirements are satisfied. So it means first of all I will have to look at the transient performance requirements. Let us say that the problem under consideration demands the requirements to be zeta equal to 0.7 and settling time equal to 2 seconds. These are the performance requirements the user has given you, you have to meet these requirements.

So, from here the design procedure starts: zeta is equal to 0.7, t_s is equal to 2 seconds or in whatever form that is the transient performance requirements may be given to you they may be given to in terms of M_p and ω_n for example, the only thing is this that I want these performance requirements to be translated into desired dominant closed-loop poles. So if zeta is given, t_s is given you can calculate ω_n or t_s is equal to 2 means tell me what is τ ω_n equal to? If t_s is equal to 2 what is zeta ω_n ? It is equal to $2 \dots 4$ by zeta ω_n is equal to t_s so zeta ω_n is equal to 2 so you take this as minus 2 and the closed-loop pole locations turn out to be minus 2 plus minus $j2$. ω_n is equal to 2 or draw a zeta line whatever way you like; you see here (Refer Slide Time: 17:12) this is your zeta line, zeta equal to 0.707 let me take it exactly as given in my notes. So this is the zeta line I have taken, zeta ω_n is equal to 2 so this is the vertical line, I get this as the intersection which gives me minus 2 plus minus $j2$ as the desired locations of the closed-loop poles. I hope this point is well taken.

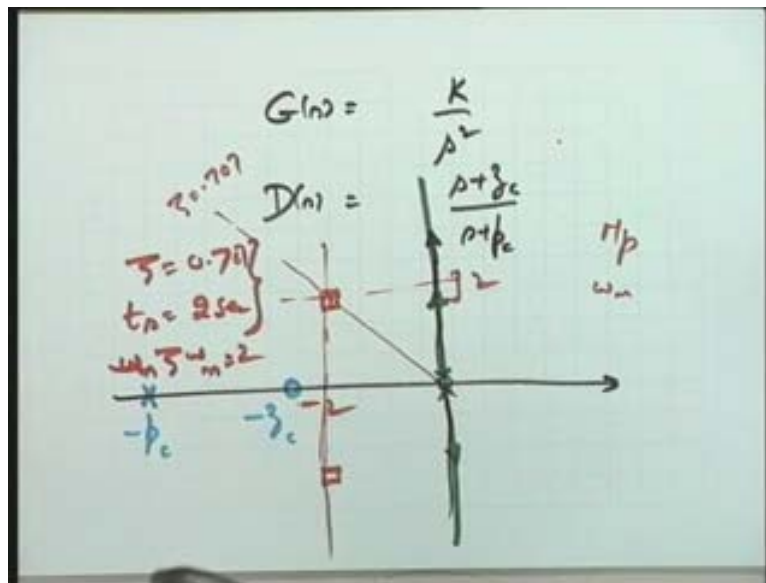
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Any transient performance specifications may be given to you. The very first step will be, you can write the steps in your own way, once the steps are followed very well I am sure you will be able to give me any design so translate it into these locations. now you know that your root locus plot for an uncompensated system, note the word uncompensated, the gain K which is an adjustable parameter has been shifted to the uncompensated system though you may realize this value of K using the compensator only. So you find that this green curve represents the root locus plot of the uncompensated system. The requirement is this that to pull on this particular side (Refer Slide Time: 18:19) so that the root locus plot passes through this particular point if it does it means your requirement of the satisfaction of the transient accuracy will be satisfied.

Any suggestion please on the basis of discussions we had? What do you suggest? A PD compensator means first of all trying to place a 0 somewhere suitably. You see, you have to place a 0 and a pole. So once you have translated this into these closed-loop poles the other primary step in the design is to place the 0 and the pole. Let me tentatively place it here. Let us say minus z_c and minus p_c I do not know their locations. I am going to give you guidelines as to how to locate these poles and zeros but sure that their sequence or this will not be inter-changed. This 0 is going to be closer to the imaginary axis compared to pole this at least is very well set because otherwise it is not a phase lead compensator otherwise it is not a derivative control. So this sequence is very well said that the 0 will be closure to the imaginary axis compared to pole and keeping that in mind I have placed a 0 here and a pole here but I do not know the distance between the two and location of one of them. So, if I gave you the location of one of them and the distance between the two is means the 0 and the pole are located.

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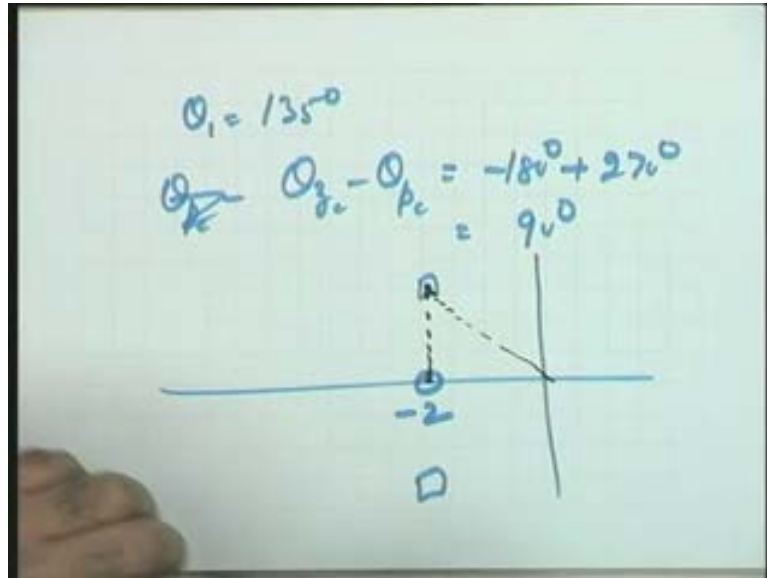
[Conversation between student and professor.....20:00 come on, please see, the second most important..... yes please.... that is, that is that is coming, anyway in this exercise..... I have....., yes you have very rightly pointed out. Since I know the answer so naturally I have placed the 0 over here but initially you imagine as if it is there on the entire left-half plane right axis the only point is this that the 0 is closer to the imaginary axis compared to the pole; his point I will take care of in my discussion, it is naturally coming in my discussion.

Now you see that as far as..... this is the location, you want that the overall root locus when the pole and the 0 have been placed should satisfy the angle criterion at this particular point. So what is the net angle contribution by these two poles you can calculate; let us say this is theta 1 in that particular case 2theta 1 is the net angle contributed. Let us say that the angle contributed in this particular case here is theta z and the angle z c you may call it and the angle contributed is theta p c. So please note that I really do not know the locations but this is at least clear that theta z c minus theta p c minus 2theta 1 should be equal to minus 180 degrees for example and you know theta 1 because you know this location, you know the open-loop poles and hence you know theta 1 and therefore you know theta z c minus theta p c that is you know what is the net angle which this pair should contribute. So it means the only requirement will be to place one of them at appropriate location and once you know the angle the location of the other particular the location of the pole will automatically follow if I am able to place the 0 at the appropriate location.

In this particular example the value of theta z is 116..... oh I should not give that theta; the value of this is 135 theta 1 is equal to 135 degrees. Therefore I know that theta p c minus or theta z c let me call it, theta z c minus theta p c is equal to minus 180 degrees how much please plus 270 degrees equal to 90 degrees. This is 90 degrees. Now if this 90 degrees is to be realized by suitable pole-zero pair so you give me the location of one. Now his point comes where do I place the 0 for example? Pole location will automatically follow. Let me say..... to the arguments to follow later I give you a 0 at this particular location at minus 2 I place the 0 at minus 2 the reasons to follow: in that particular case what will be the location of the pole please if I place the 0 at minus 2? Minus 2 location if you place what will be the location of the pole in this particular case?

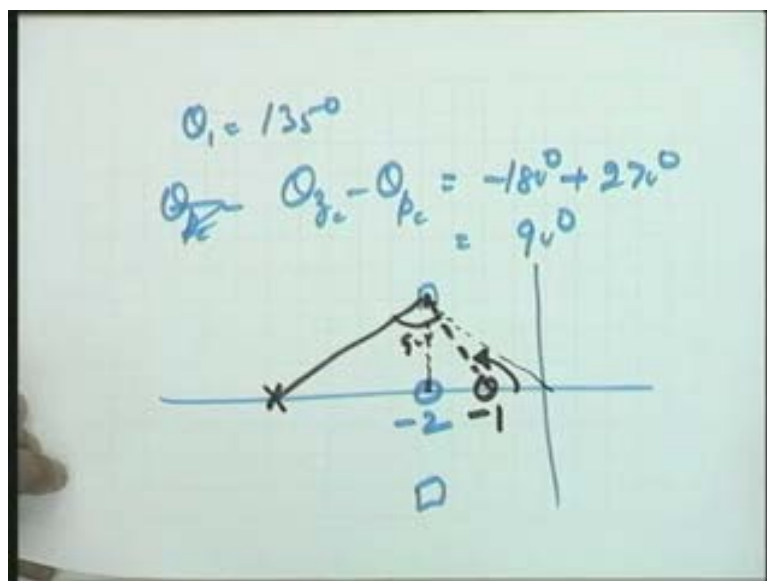
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 Now what is the implication of this will become clear.

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Now let me shift this 0. Suppose I place it at minus 1, if I place it at minus 1 again its implications to follow: let us say that this is the angle I can calculate and I can put this as 90 degrees and this is the position of the pole I am going to get (Refer Slide Time: 23:34). Let me analyze this that is the location of the 0 at minus one and the corresponding pole so that 90 degrees angle is achieved and then let us see where we should place the 0.

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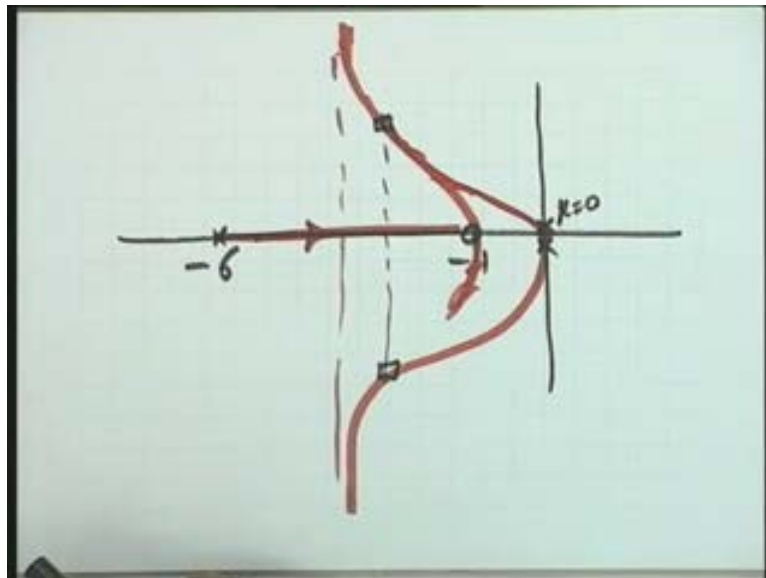


If I analyze this I make the final root locus sketch with this choice. This is double; this is a minus 1 over here. This was minus 2 plus minus J2 locations here, the location of the pole turns out to be at minus 6 please; draw the root locus diagram first. I have made a tentative decision. You found that at minus 2 it was not feasible so I have shifted it to minus 1, I have

made a tentative decision and the argument as to why this decision has been taken is yet to follow.

So in this particular case the sketch is like this (Refer Slide Time: 24:29) which you can easily sorry sorry sorry which you can easily check by applying simple root locus rules. This is the situation that is a root locus branch starts at K is equal to 0 this has to pass through this point because the angle criterion is satisfied, you cannot imagine a root locus not passing through this point you have satisfied the angle criterion, poles and zeroes have been taken in such a way so as to give you 90 degrees and therefore the angle criterion is satisfied and hence it has to pass through this point. So if it passes through this particular point then in that particular case there are three branches: one is going this way, the other is going this way and how about the third one please the third one is obviously given to be this one so these are the three branches. So it appears in this particular case as if your design requirements have been satisfied because you see your root locus branches pass through these two points which correspond to the transient performance but for one point how about the dominance condition. if the dominance condition is not satisfied then zeta is equal to 0.707 you have taken but the corresponding peak over shoot will be different than that which you get by the second-order correlation between zeta and M_p because of the effect of the third pole coming on the system. So it means the root locus will be complete only if you have understood and you have seen the effect of the third pole on the system.

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In this particular case and to get the third pole what do you require?

Please see you will have to calculate the value of K by the magnitude criterion. So I can now say that this is the third important step in your design, the second step you have taken but for the location of this particular 0 where do I locate, the third important step is that at this particular point you calculate the value of K by the magnitude criterion and the value of K turns out to be 16 in this particular case. So corresponding to the value of K you will try to locate the third pole and the third pole will be located on this particular this particular segment and let us say that this is the location of the third pole.

Now you know that if this particular third pole lies in the region close to the complex conjugate poles the dominance condition is not satisfied and therefore you will have to see the effect of the third pole by simulation. The requirements are always given this way that the peak overshoot is less than equal to that quantity.

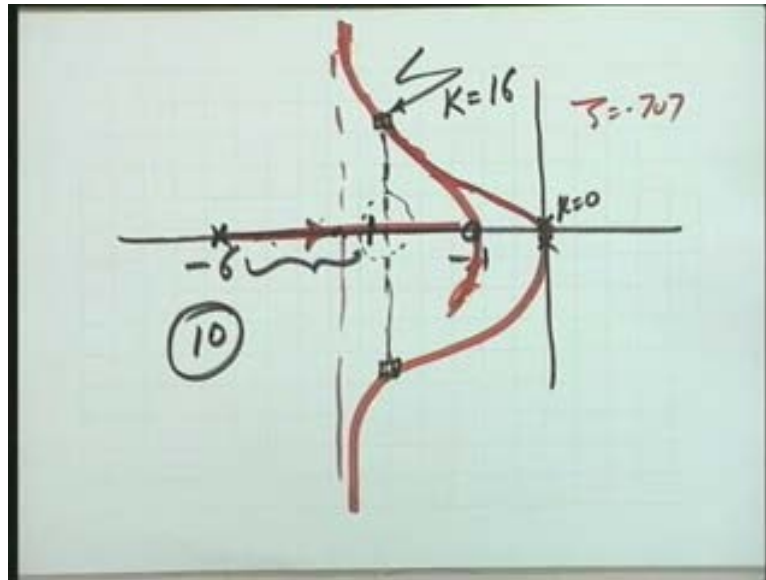
I have taken zeta equal to 0.707. So you can see, if the dominance condition is not satisfied as is evident in this particular case the dominance condition will not be satisfied then you have to simulate it and see the effect of the third pole and if the effect of the third pole is to give you the peak overshoot and the settling time in the acceptable range in that particular case your design is complete otherwise there is no other clue to be given to you other than that you place your 0 here and there by trial and error; I have placed it by minus 1 and I can shift it to this side, I can shift it by trial and error on this particular axis so that the dominance condition is satisfied if not completely at least the simulation experiment gives me acceptable performance. So there is no other clue other than this that this 0 can be adjusted on this particular axis suitably (Refer Slide Time: 28:19) so that the dominance condition is fairly satisfied so that these particular two complex poles are truly a representative of the transient response of the system. Some clue can follow and that clue is the following.

If possible, in this particular case it is not possible, if possible you place your 0 in this side that is if I take this as the vertical line (Refer Slide Time: 28:49) on the left hand side of the vertical line can you give me a reason? I am saying that.... now, only clues I am giving which may not work in this example, in general the clues are the following: if possible place your 0 in the region to the left of this vertical line and not to the right. The reason I hope is obvious. If the 0 is there on the left a root locus branch will terminate on the 0 so it means the third pole will lie to the left of this particular 0 and hence the dominance condition has a fair chance to be satisfied.

In this particular case since 90 degrees requirement was there you could not place it but in general considering any problem where it is possible where you are looking for a suitable location for 0 in that particular case look for a suitable location in the vicinity of this particular region down below and try to go to this side and not to this side if it is possible if not you have to go to this side as we have taken in this particular case. Shifting it on this side makes it makes it more appropriate because the dominance condition will be satisfied.

One more point you see, what should be the factor between pole and 0 that of course is guided by this particular thing but otherwise high frequency attenuation will guide this particular factor as to how much of filtering you require, how much of filtering you require and this factor of the order of 10 normally gives you a better filter design. All these things may be contradictory but all these things you have to keep in mind. You see, after all it is a high frequency attenuator. This particular pole which you are placing at minus 6 (Refer Slide Time: 30:39) is going to attenuate the high frequencies; a reasonably good attenuation will come if a value of 10 is there as the ratio between the 0 and the pole this is again a guideline. However, this may contradict with the angle requirements and you have to suitably take care of this particular point as well.

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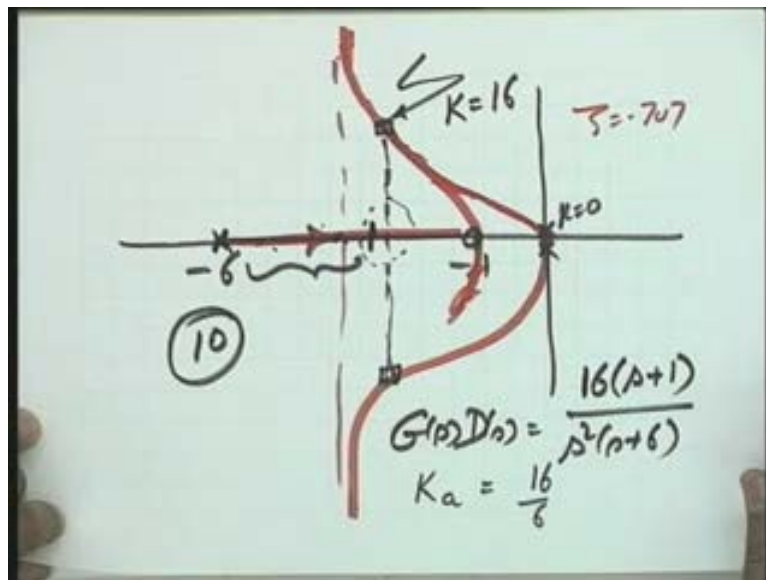
So you see that as far as the design procedure is concerned only certain guidelines have been given to you and no quantitative method can be given in this particular case. And as far as steady-state requirements are concerned, well, steady-state requirements you need not check because it is a type-2 system the steady-state requirements on ramp and step signals will always be satisfied because K_v is equal to infinity and K_p is equal to infinity and therefore steady-state errors are 0.

However, if acceleration error is to be calculated the acceleration error can easily be calculated from this particular relationship.

What is $G(s)D(s)$ please, help me?

If I accept these as the values it is K which is 16 into 0 has been placed at $1/s + 1$ divided by s^2 into $s + 6$. This is your open-loop transfer function of the compensated system. And what is the value of K_a please? K_a in this particular case turns out to be 16 upon 6 this is the acceleration error constant. So, acceleration error is going to be 6 upon 16 radians which is a finite value which naturally will be acceptable because we know that acceleration input itself is a very difficult input and in many situations we are satisfied with the ramp and step input steady-state error analysis. So in this case acceleration error constant is 16 by 6 which gives you corresponding acceleration error as 6 by 16 radians.

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I think you see, if there is any point which comes to your mind as to how to take the decisions please do tell me if any point..... In general you will please find, I have taken this to be $G(s)D(s)$ is equal to $K(s + zc)$ over $s^2(s + pc)$. You find that the gain the low frequency gain in this particular case and the high frequency gain ratio is guided by zc by pc that is why I said that zc by pc should preferably be of the order of 10 to give you a full good filter design; this is the requirement based on the good filter design.

Coming to the phase lag network **I have to look at the watch also you see and I have to declare it complete today and therefore I need your attention on the phase lag network now:** $D(s)$ is equal to.... again if you do not mind K I can take to this plant and therefore K into the transfer function becomes the total transfer function of the compensated system. So, for the lag network also the transfer function is of this form: $s + zc$ over $s + pc$. This is the lag network model. But the pole-zero pair now is of this nature.

Help me please; a pole close to the origin not necessarily at the origin. If you place a pole at the origin in that particular case it is the special case and that is the case of a PI controller. You know that a PI controller will mean that the pole is placed exactly at the origin. So let me place, let me add some extra degrees of freedom into my controller. I can of course place my pole at the origin but to get extra degrees of freedom I place the pole at s is equal to minus pc where pc is equal to 0 is a special case and a 0 I place at minus zc which is close to pole, as you will see the reasons will become clear why do we take close to poles.

One of the qualitative reasons I gave you that this particular pole is going to destabilize the system, is going to deteriorate the stability of the system, **the effect of the stability will become** the effect on stability will become compensated if there is a 0 close by. This is a qualitative statement and quantitatively this will become very clear when we just now take up an example. So this again a pole-zero pair as you find with a pole near the origin and a 0 following there and the ratio of zc and pc also is suitably guided and we will see how to take up this particular thing into consideration.

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$$G(s) D(s) = \frac{K(s+z_c)}{s^2(s+p_c)}$$

Phase Lag:

$$D(s) = \frac{s+z_c}{s+p_c}$$

The selection mode, how do you make the selection? You see, primarily you are adding a pole to the system; primarily you are adding an integrator in the system as far as the phase lag compensator is concerned. So it means the prime objective of design by phase lag compensation is the steady-state performance improvement. So it means as far as the transient performance is concerned it is going to be deteriorated rather compared to that of the uncompensated system. At the most you can say that you will suitably control the values of z_c and p_c so that the transient performance is not deteriorated but it will definitely not improve at least so that is why I said that I will place a 0 close by so that the negative effect of pole on the transient performance of the system is compensated.

Therefore you must set these requirements in your mind; you have to take a decision as to whether you want to go for a phase lag compensator or a phase lead compensator. I will say that a phase lag compensator is possible if and only if your uncompensated system is meeting the transient performance requirements. Please note that the gain adjustment in the system is coming under the uncompensated system itself. Gain adjustment is possible, so you see that by suitable gain adjustment which naturally..... actually it is a proportional control but I have shifted it to the state to uncompensated range so if by suitable gain adjustment you are able to meet the transient performance of the system in that particular case you can think of adding the phase lag compensator to improve the steady-state accuracy because you will be able to control z_c and p_c suitably so that the transient performance of the uncompensated system is not deteriorated.

I think it should be clear. If the uncompensated system that is merely by gain adjustment you are not able to meet the transient performance requirements in that particular case you have to pull the root locus to the left and hence you have to go for a phase lead compensator. The decision is in your hands you see. You can make the decision by simply studying the uncompensated system with respect to only gain adjustment. I think again I will like to illustrate this through an example.

Can I take an example of K by s squared in this particular case?

For a phase lag compensator I am looking for an example and I should not take the example of K by s squared because it is ruled out. For the plant K by s squared you will not yet add a compensator which is a phase lag compensator and hence a type-2 plant it is obvious now, for a type-2 plant you never add a phase lag compensator because it will highly destabilize the system. So let me go towards softer plants and by softer plants I mean with respect to stability type-1 or a type-0. So there is a scope of adding a phase lag compensator to type-1 or type-0 plants.

Let me take an example of type-1 because type-0 plants are more easy to control you see. So, if we are able to discuss with respect to type-1 any example of type-0 you will be able to handle very nicely. $G(s)$ I take K over $s(s + 2)$ you can imagine this to be a motor servo a tracking system for example. Position tracking is to be done and this is the simple model of the DC motor I have taken; a loop is to be closed to give you a closed-loops position control system. K I have embedded in the uncompensated system.

Look at the performance. The performance requirements are the following please see zeta equal to 0.45 I am taking and K_v requirement is 20 steady-state, K_v is the ramp error requirement. You see, in this particular case it is a type-1 system, you should be clear that the step error requirements need not be given because the error to step inputs will be 0. Ramp error will be finite and hence K_v may be specified in this particular case. In the earlier case the steady-state requirement was not specified obviously because the system under consideration was a type-2 system. Suppose it were specified, in that particular case you see, once the transient requirements are satisfied whatever K you have selected you can best see whether your steady-state requirements are satisfied or not. If it is not satisfied you cannot do anything you see as far as the phase lead compensation is concerned.

I will put this question to you as to if the steady-state requirement using a phase lead compensator is not satisfied what you propose? **This is my question to you and I will end my discussion with that question only.** Please keep that question in mind. You recall that phase lead compensation, the steady-state error requirements were not specified because it was type-2. But suppose it is type-1 and steady-state requirement is specified and it does not meet that requirement then what you do because any adjustment of pole and 0 will only marginally change that requirement will not satisfy that requirement. So this is my question to you please you have to answer it.

I now take you back to this particular problem: $G(s)$ is equal to K over $s(s + 2)$ is my plant which is a motor servo, zeta is equal to 0.5 K_v is equal to 50. First of all look at the steady-state requirements of the uncompensated system. Help me please; what is K_v of the uncompensated system? Give me the answer please quickly. K_v of the uncompensated system in this particular case is K by 2. This is the K_v of the uncompensated system. Now let us see, keep this in mind, I want K so that this performance requirement is satisfied.

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$$G(s) = \frac{K}{s(n+2)}$$
$$\zeta = 0.45 \quad K_v = 20$$

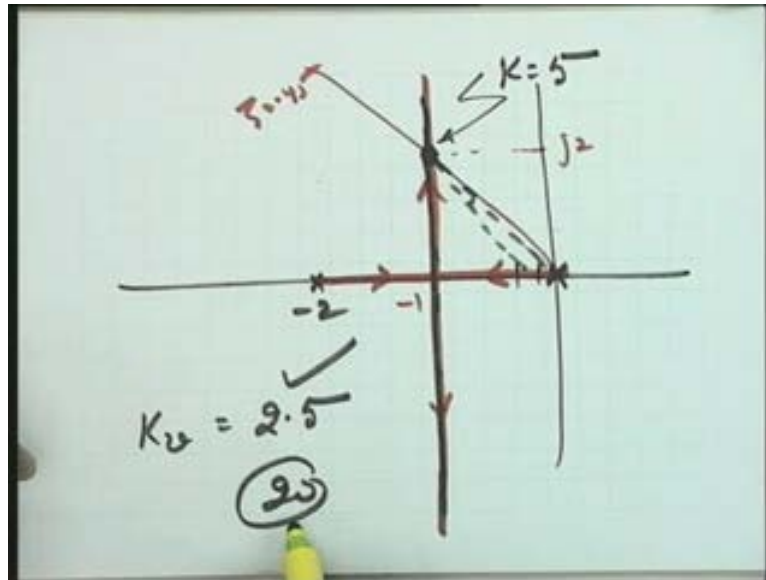
$$K_v(\text{uncompensated}) = \frac{K}{2}$$

I go for the root locus of the uncompensated system; a pole here, a pole here, breakaway in this case a particular example has been selected so that the root locus plot is easy to make. Please locate the closed-loop poles zeta is equal to 0.45 line has been drawn over here. This zeta is equal to 0.45 line if I draw over here it gives you the pole locations as minus 1 plus minus $J\sqrt{2}$ these are the locations please. Now this particular point (Refer Slide Time: 42:22) is on the root locus of the uncompensated system and it satisfies the transient performance requirement, please see.

Let me first of all get the value of gain K for this particular case. The gain K in this particular case comes out to be 5. So it means K is equal to 5 satisfies the transient performance requirement, let me simply check the steady-state accuracy; K_v in this particular case will turn out to be 2.5 because K by 2 is your K_v which falls short of 20 by a large factor. So it means in this particular case the uncompensated system satisfies the transient requirements but not the steady-state so what will you do? I really want now, you please see, I want to add the compensator pole and 0 to satisfy the twin requirements: One: the K_v should be boosted from 2.5 to 20 and second the compensated root locus plot should also pass through this point, these are my requirements; **if you can satisfy** and the third of course will be the dominance condition that any how you will have to see.

If this requirement is satisfied (Refer Slide Time: 43:36) that the compensated root locus plot passes through this point and the K_v is boosted to 20 in that particular case the requirement is satisfied. So what I do in this case, I place the 0 and the pole very close to the origin so that the net angle contribution due to both 0 and pole is negligible. Because if the pole and 0 angles are negatives of each other so the net angle contribution is given by the pole-zero pair and if this net angle contribution is 1 to 5 degrees in that particular case this pole-zero pair is not going to disturb the root locus. The compensated root locus will also pass through this particular point. So this is one point that these should be placed so that this particular point (Refer Slide Time: 44:27) remains on the compensated root locus.

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How about the other requirement the requirement of the K_v ?

Look at the K_v please; help me please how to meet the K_v requirement?

Your $D(s)G(s)$ will become $K(s + z_c)$ over $s(s + 2)(s + p_c)$. Look at K_v please. K_v is equal to $K z_c$ over 2 into p_c . So you see this, this you want equal to 20. If you want this requirement to be satisfied it gives you a factor z_c by p_c should be boosted by a factor of about 8. In that particular case the K_v requirement will also be satisfied. You see that K is equal to 5 interestingly (Refer Slide Time: 45:23) not only the angle criterion the magnitude criterion at this particular point also will give you the same value of K practically because these two distances will be nearly the same. One distance is in the numerator and the other distance is in denominator. So the compensated root locus at this particular point will roughly have the same value of K equal to 5 and the angle criterion will also be satisfied and therefore the system K_v will be given by K by 2 into z_c over p_c . So it means it gives me a clue as to how z_c over p_c could be taken. So z_c over p_c should be taken as a factor of 8 so that the steady-state requirements are satisfied.

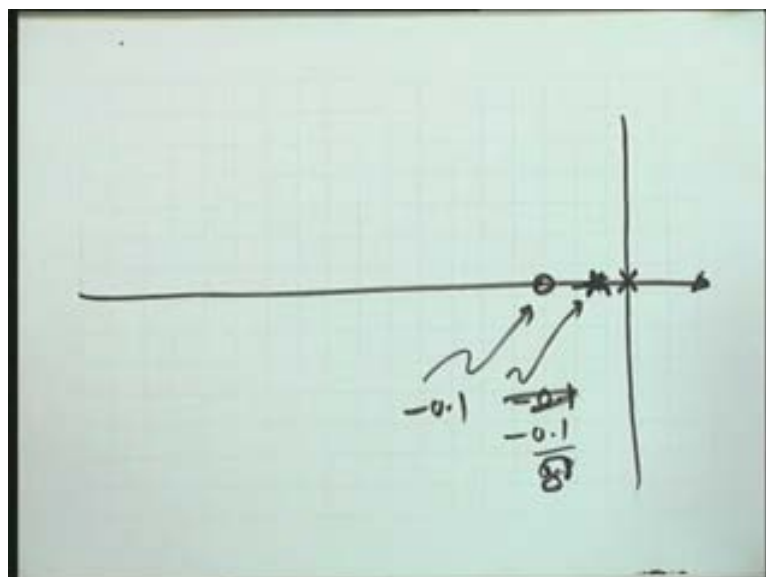
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$$D(s)G(s) = \frac{K(s+z_c)}{s(s+p_c)}$$

$$K_v = \frac{K z_c}{s+p_c} = \underline{\underline{20}}$$

Now with this choice of **K v and p c** z c and p c I have to place the 0 and the pole so that all the requirements I have raised are suitably satisfied. In addition, the dominance condition. Now I am making an exploded version here so that the dominance condition requirements are clearly known. I am placing the pole at minus 0.1, one pole is there, I am placing this pole at minus 0.1 the reason will become clear, this reason comes from the dominance condition please. Help me, where will be the 0 placed? Minus 0.1 [Conversation between Student and Professor – Not audible ((00:46:49 min))] minus 0.8 that is right. I will rather put it this way please see, I think the 0 is taken as minus 0.1, pole I will shift it on this side, minus 0.1, the pole is at minus 0.1 divided by 0.8. This is the pole. This is..... divided by 8 divided by 8 this is the pole-zero pair I have taken.

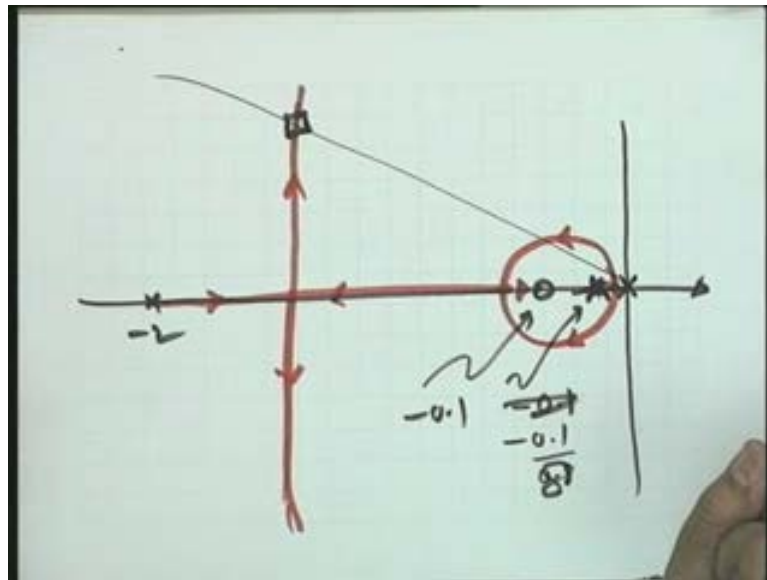
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The location as to why do I take 0.1 will become clear soon. Now in this particular case the third pole turns out to be at minus 2 so let me just put it far away. This is not too scale as I am

telling you so that the root locus plot in this region becomes clear. In this region now the root locus plot, please make the root locus plot, there will be a breakaway point here (Refer Slide Time: 47:41). You can check you see in this particular case there will be two breakaway points: One breakaway point here roughly I am taking here and one breakaway point here and one breakaway point here. A root locus branch starts over here goes this way terminates at this point, another root locus bands starts from here goes this way and terminates at infinity, other root locus bands starts from here goes this way and terminates at infinity. This is your root locus sketch which can easily be examined and your zeta is equal to 0.5 line this was your original point which is there on this particular plot as well, which is there on this particular plot.

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Now you just look at the dominance condition. Where is the third pole? Help me please; what is the location of the third pole in this particular case? Can you tell me, can the third pole be a complex pole? Can the third pole be a complex pole please? You see, the poles have always to be complex conjugate so in this case it is a third-order system so the third pole cannot be a complex pole, it has to be a real pole only so that real pole as can be checked will lie in this vicinity in this particular region (Refer Slide time: 48:58) the real pole for this value of K is going to lie in this particular region and since the third pole is in this particular region there is a 0 close by and hence this 0 is going to nullify the effect of the third pole and hence the dominance condition is more or less satisfied though the third pole is quite close to the imaginary axis, the dominance condition is satisfied and hence this is the clue only. If it is not satisfied, if this is not sufficiently close to this particular 0 you can make an adjustment of the pole-zero locations so that the third pole is sufficiently close to this particular 0 and this pole-zero adjustments naturally you are going to do using the CAD software not by the paper and pencil design.

This is only the method, the guidelines given to you, the third pole will lie somewhere over so by suitable adjustments here and there you can see that your dominance condition is more or less satisfied and finally it will turn out with the help of simulation as to whether your requirements on the transient and steady state requirements are satisfied or not. So I conclude and want an answer from you that if the transient performance of the uncompensated, by

uncompensated I mean only gain adjustment is allowed, if the transient performance of the uncompensated system is satisfactory in that particular case you will like to go for a lag compensator and you place your poles and zeroes so that steady-state performance is satisfied.

If the transient performance is not satisfactory you go for a lead compensator. That is you push it so that your transient performance is satisfied and then check whether your steady-state requirement is satisfied or not, if not you see you cannot say anything, the lead compensator can give you only that much. Now you tell me, please see a situation where transient performance is not satisfactory and the steady-state requirement is not satisfied when the phase lead compensator has been designed what will you do, what is your clue to that? You put a lag in cascade that is you go for a lag lead compensator.

You first design a lead compensator please see. You see that now the standard procedure will be like this. This is the standard final procedure. You simply investigate your open-loop system very carefully, you do the gain adjustments, see the transient performance, if the transient performance is satisfactory the choice is lag okay then you will not required lead the decision has been taken. If the transient performance is not satisfactory you go for lead and make the transient performance satisfactory the lead compensator has been designed; now you check the steady-state error.

If the steady-state error is satisfactory your design is complete if it is not then the lead compensated system should now be treated as an uncompensated system and enter into phase lag design module. If the steady-state requirement is not satisfied in that particular case lead compensated system should be taken as an uncompensated system as far as the module of phase lag compensation design is concerned because the phase lag compensation will not disturb the transient performance realization which you have got with the help of phase lead compensator. So you can enter into lag module and design a lead lag compensator so that the transient and steady-state accuracy requirements are all satisfied. So the decision is yours; it could be a lead, a lag or a lead lag compensation as far as a final design of the system is concerned; Op-Amp realization of both lead lag and lag lead has been given to you because a lag lead will simply require an adjustment of those parameters which can be done and with this I conclude my discussion, thank you.