Control Engineering
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Lecture - 33
Compensator Design using Root Locus Plots (Contd..)

Well friends, with the root locus sketching guidelines given to you with couple of examples also how to make a complete root locus plot now I think we have come to a stage where the design problems could be taken up. So as I have mentioned earlier root locus method provides a very powerful method of design, it is being practiced in industry today, many of our industrial controlled systems have been designed using either the root locus approach or the frequency domain that is the approach based on Bode plots. So, one of the two important design techniques is being taken today so I need your attention on this.

So to take up the design problem I think the best way will be to take up an example. So let me say that I have a plant and this plant model I could take let us say 5 over s(1 by 6 s plus 1) (1 by 2 s plus 1) it is a plant model, you can imagine this to be the model of a field controlled motor. You can assume as if the problem being considered is a motor servo. So in this case let me say that this is the output and this is the manipulated input to the system (Refer Slide Time: 2:20). So naturally in any design problem whatsoever may be the performance requirements you will first of all like to examine whether by pure gain adjustments your requirements of performance are satisfied or not. So naturally the very first thing I like to examine the very first diagram I like to examine is the system diagram where this amplifier is a simple proportional controller.

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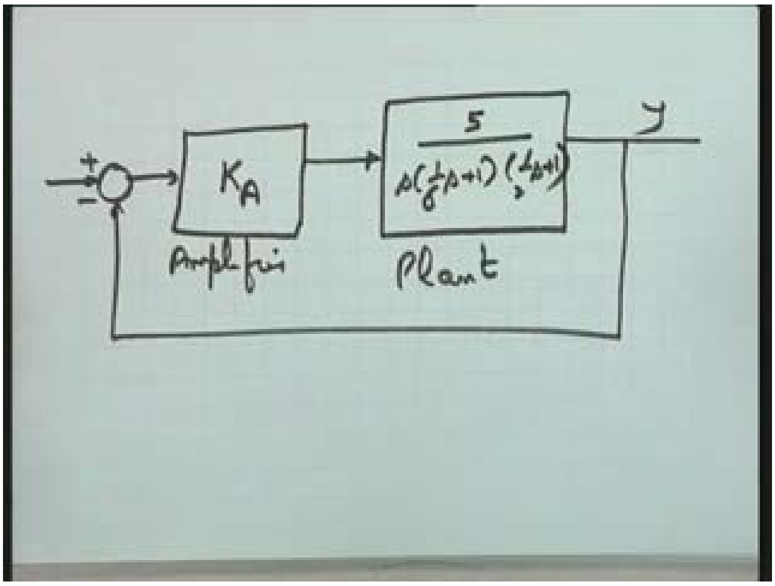

I like to see whether the K A satisfies this requirement or not and what are the requirements;
the requirements as I said, normally the requirements on sensitivity or robustness are not quantified, we only quantify the requirements on steady-state accuracy and transient accuracy and then check whether robustness requirements are satisfied or not. This is what we have decided that this will be our design strategy; we will not include the robustness requirements

directly into the design. So with that logic I can say that the transient performance requirements and the steady-state performance requirements will be the prime requirements the user will impose. And once I have completed my design with respect to these requirements I will see whether robustness, disturbance rejection and other requirements are okay or not.

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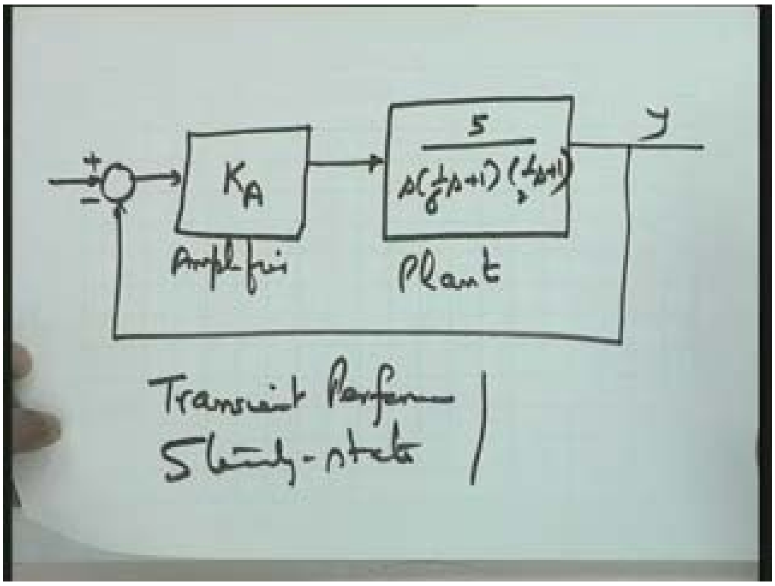

The transient requirements you know are given in terms of relative stability and the speed of response. These are the transient performance requirements we know. Relative stability is related to peak overshoot and therefore this requirement can be specified in terms of either zeta or in terms of M p. The speed of response requirement can be specified in terms of settling time, in terms of raise time or even in terms of omega n because these are correlated through second-order correlations. However, when you specify these requirements with respect to second-order system the dominance condition must be examined and suitable action has to be taken so that either the dominance condition is satisfied and if dominance condition is not satisfied then suitably the effect of the third pole or other poles on the performance of the system is taken care of that you have to take into consideration while you take up a design problem.

So let me say that for the system under consideration the requirements are this zeta the steady-state accuracy requirement is given in terms of K v equal to 5. Please see that how do you proceed further. You see that the total design environment let us try to create because the design is really not a sequential method step by step analytical method, many of the steps I may be giving to you you may disagree with that, you may come forward with better sequence with better steps because there is definitely an art included in the design and therefore your contribution in today’s discussion will be welcome and the starting point is a motor servo where the amplifier manipulation, where the gain manipulation is being considered.

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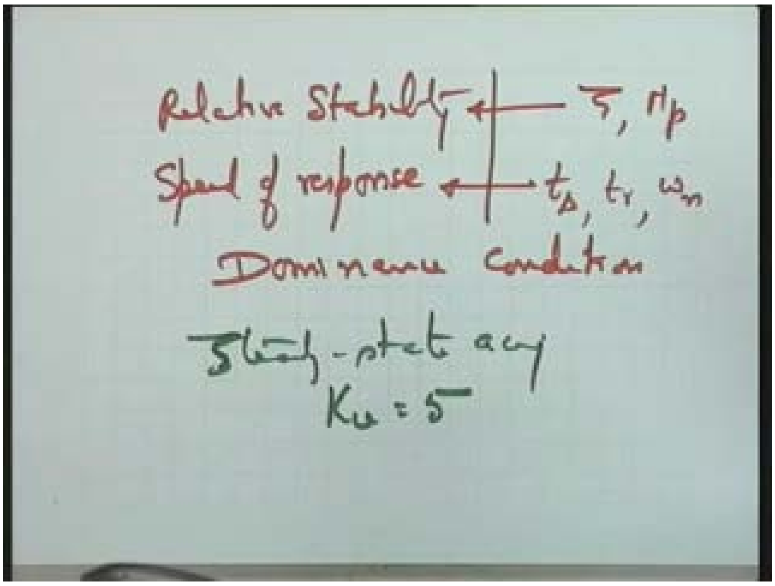

Let us say that the prime requirement of design is the steady-state accuracy. You please see that the system under consideration is a type-1 system, is it all right please; and a type-1 system will perform with zero steady-state error for a step input. So there is no point in specifying the steady-state accuracy to step inputs that is K p in this particular case will definitely be infinity. Therefore the steady-state error to step input will be 0. So if I specify the steady-state accuracy for such a system it has to be in terms of the K v or the K A.

Now if I go for an amplifier design if I take up only a constant here tell me what will be the value of K A for any value, what will be the value of K A the acceleration error constant for any value of amplifier gain, help me please. For any value of amplifier gain what will be the value of the acceleration error constant it will be 0. I mean all these things you should you see should become our a b c d they should really immediately be translated into the performance.

So in this particular case you see it is obvious, the question might come to your mind as why did I specify directly K v. K v is equal to 5 though I know that K p and K A are also the two other error constants I have introduced to you. So you see that for this particular problem both K p and K A have no meaning if I go for a pure gain adjustment because K A is equal to 0 so in this particular case the steady-state error to acceleration input is going to be infinite
so it means I know that my system is not going to perform very well if the parabolic input comes up and may be the user says it is okay his system is not going to receive parabolic inputs and as far as position inputs are concerned, as far as constant inputs are concerned it is guaranteed that for any gain the steady-state error to constant inputs will be 0 so no need of giving any specifications (Refer Slide Time: 8:07) therefore if I want to specify quantitatively the steady-state performance of this system it has to be specified in terms of steady-state error to ramp input and one of the ways of specifying is to give K v or directly to say steady-state error to ramp inputs is equal to 0.2; I want K v greater than equal to 5, steady-state error e ss less than equal to 0.2.

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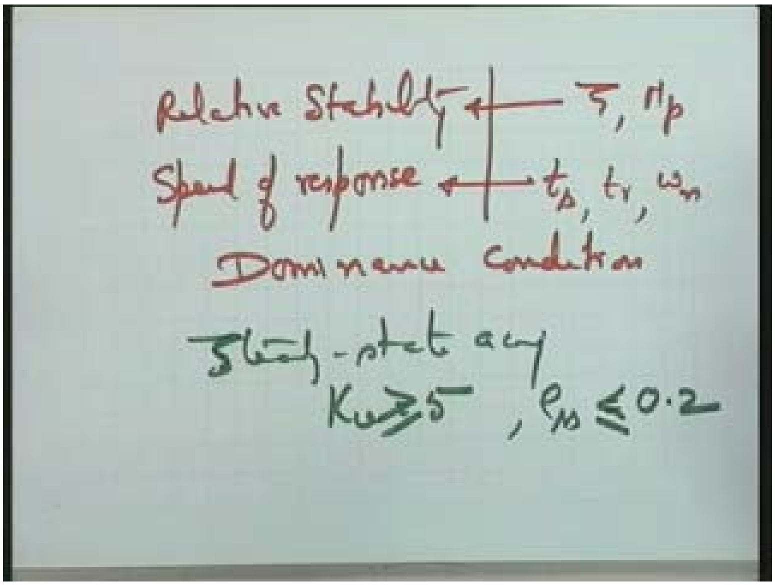

If it is a position servo it is 0.2 radiance so this itself is sufficient an error and you will like to reduce even this particular error. To get started let me assume that K v greater than equal to 5 is my requirement. Now if I go for an amplifier design you see I have only one parameter. So if I satisfy the requirements on K v in that particular case may be K A will be set to satisfy K v requirements let us say how.

What is G(s) equal to?
G(s) equal to please see 12 60 so 60 K A divided by s (s plus 3) (s plus 6). Putting it in the pole-zero form this is the open-loop transfer function. In one of the tutorials I discussed this problem but this was not completely discussed there so initial portion I am repeating so that we can continue with the design of this particulars motor servo problem.

So I say that G(s) equal to 60 K A divided by s(s plus 2) (s plus 6). So K v in this particular case I find, please see, K v is equal to how much help me please K v is equal to 5K A is it all right or [Conversation between Student and Professor – Not audible ((00:09:59 min))] yes, I think it is okay K v in the limit equal to limit s tends to 0 sG(s) so sG(s) this s goes this s goes 12 5 K A is the value of K v you are getting.

Now if your K v requirement is 5 so it means this requirement is satisfied by a K A equal to 1 or this if I say K by s into s plus 2(s plus 6) where K is the root locus gain then this requirement is satisfied by K is equal to 60, K is the root locus gain.

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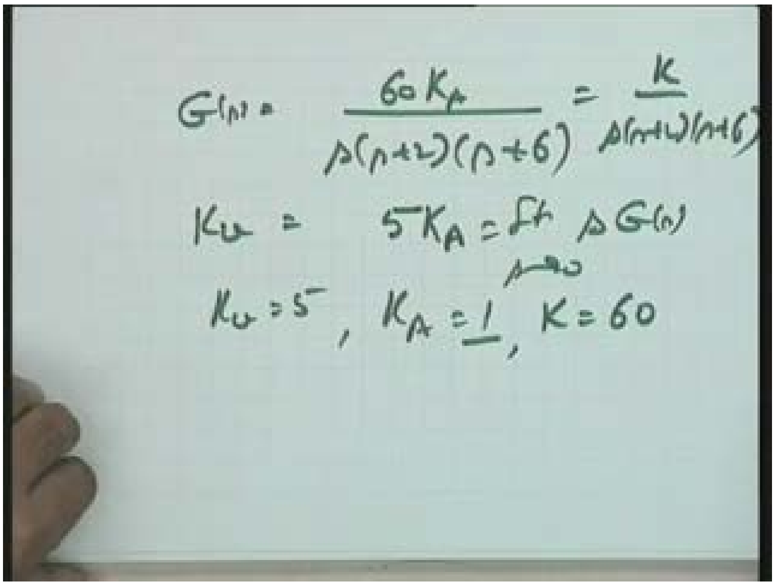

So now with K is equal to 60 or K A is equal to 1 I am happy that my steady-state requirements are satisfied and K A is equal to 1 is a reasonable choice from the point of view of implementation, no saturation problems will occur. But let us see the root locus plot for this system with K is equal to 60.

You see that this is one point K is equal to 0 minus 2 and minus 6. Let us make a rough root locus sketch. I will not give you those rules now what is the centroid, what are the angles what is the breakaway point all these things I am sure you will be able to calculate. I give you a rough root locus sketch and go ahead with my design statements design comments.

So in this particular case the root locus is expected to be of this form: one branch going this way, second branch is here and these are the directions these two branches are going to take. This is your root locus.

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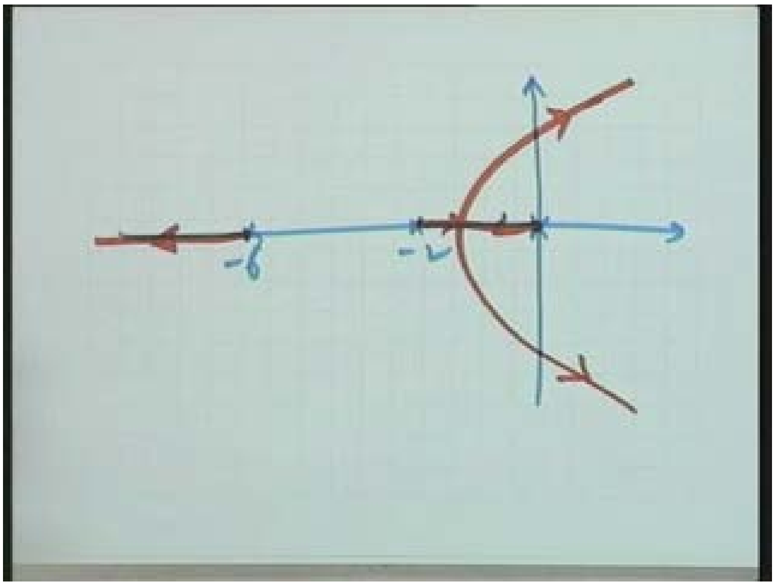

Now you have decided K is equal to 60 is the point which meets your steady-state accuracy requirements. Now please see that I will like K is equal to 60 point to be located on the root locus plot so that I can evaluate the transient performance of the system. Once the steady-state performance has been set by selecting K is equal to 60 how do I check the transient performance please see I will have to go root locus to determine the dominant poles these dominant poles will give me the values of zeta and omega n and hence will give me the values of peak overshoot settling time in all that. Therefore you will apply your magnitude criterion, I am missing all these details now, you will apply your magnitude criterion and locate a point for which K is equal to 60. In this particular case these are the three distances because there is no 0 involved.

Let us say I have located this point (Refer Slide Time: 12:34) which corresponds to K is equal to 60. So what is this point corresponding to? this particular line gives you the value of zeta so it means if this angle is theta in that particular case the value of zeta is given by cos theta. So in this case now I take the help of my notes I get zeta equal to 0.105. You had only the steady-state accuracy requirement set by taking K A is equal to 1, I find zeta equal to 0.105.

How about the closed-loop poles?
These are the closed-loop poles of the system and corresponding to these closed-loop poles this itself is omega n the natural frequency. In this particular case for the natural frequency the calculations I have with me is 2.85 and therefore settling time t s; please see that settling time now you can calculate 4 by zeta omega n or since the graph is available so please see that zeta omega n is directly available to you as this particular portion on the real axis. The real axis portion of the complex conjugate roots is nothing but your zeta omega n product. Hence this zeta omega n product can be directly read from here and 4 by zeta omega n is the settling time with respect to 2 percent tolerance, in this particular case this turns out to be 13.36 seconds.

Now you see that the 0.105 and this particular settling time (Refer Slide Time: 14:16) they have a meaning only when the dominance condition is satisfied. So in this particular case as I had mentioned in the tutorial class also please see, you need not check the dominance condition by taking a third pole over here because after all this is going to be less than 1 and your pole has to be at a distance more than minus 6 so the dominance condition is satisfied in this particular case and therefore you need not locate the third pole for the value of K is equal to 60 in this particular case you can save time. Otherwise if this was closer to this (Refer Slide Time: 14:51) in that particular case you would have located the third pole again by the magnitude criterion and see whether the dominance condition is satisfied or not. In this case this zeta omega n as you see, 4 by zeta omega n I am giving you as 13.36 so naturally zeta omega n is less than 1 and therefore the third pole is at a distance more than minus 6 the dominance condition is satisfied, the performance of this third-order system is amply represented by these complex conjugant poles and hence zeta is 0.1 and settling time is 13.36 for the third-order system.

Now you see that for a general industrial application the user is not going to agree to this design (Refer Slide Time: 15:39) this will be a highly oscillatory system with lot of settling time so both the speed of response and the relative stability are poor in this case. So it means only amplifier gain K A could not meet your requirements on the steady-state accuracy as well as the transient response.

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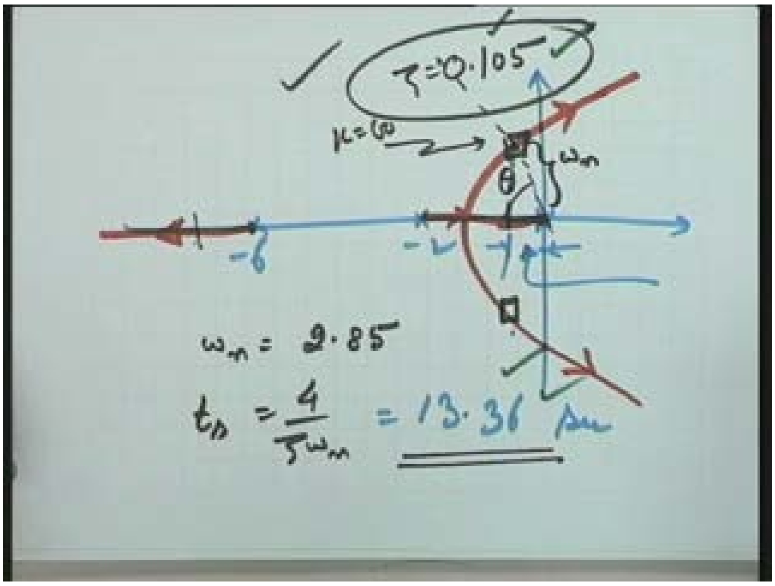

[Conversation between Student and Professor – Not audible ((00:15:58 min))] You see that since your question is coming logically in the discussion could you kindly wait? It is coming I am sure because in the next slide itself you will find that the dominance condition is not satisfied. At least it should be clear at this particular stage that the amplifier gain if I meet the steady-state requirements the transient requirements are not satisfied.

Let me reverse the problem. Let me say that, well, the transient requirements the user has given me: zeta equal to 0.6 and settling time t s is less than equal to 4 seconds. I can meet one of the things you see with only K A and let us say by meeting one of these parameters by selecting a suitable K A whether the other requirement is satisfied or not this is what I have to check.

Now in the process I am giving you the procedures also. Now you see that…… and I have only K the root locus gain in my control and by K I mean indirectly I am controlling K through K A because K is equal to 60 into K A.

Let me go back to the root locus plot I had made with K as a variable parameter: minus 6 here and this is the root locus. Now see the procedure, through these trivial cases you see I am setting the procedure of design into your mind. You see zeta equal to 0.6 is there, t s is 4 seconds. Suppose you are given zeta equal to 0.6 so how do you try? You can try one because K A has got only one degree of freedom you cannot try more than one design requirements with only K A.

Let us say I tried zeta equal to 0.6 in that particular case as you know I will draw a line corresponding to zeta equal to 0.6 and this line as you know this theta will be cos inverse zeta. I now get this point. If this is a closed-loop pole the requirement of zeta equal to 0.6 is satisfied. Well, this you will recall my statement, I gave yesterday only that this point necessarily need not be on the root locus and hence angle criterion is to be tried. But assuming that that you will do you see and you have located this particular point which satisfies the angle criterion, this point by trial and error along this line (Refer Slide Time:

18:30) you have satisfied the angle criterion so naturally this becomes the closed-loop pole zeta is equal to 0.6 value and hence the relative stability requirements have been satisfied.

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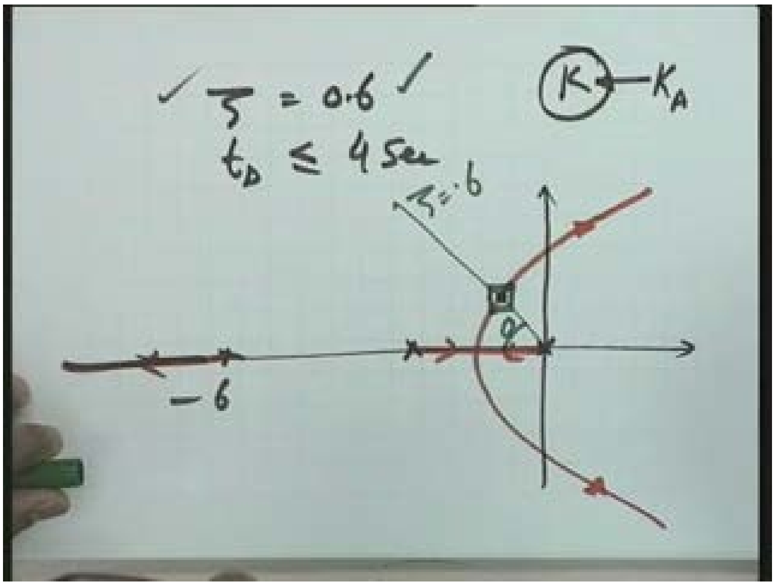

Now, how about the settling time. you can check it only, you see you cannot even simultaneously satisfy these two requirements by only one parameter. You can check it by taking this value, finding this value of zeta omega n and 4 by zeta omega n gives you the settling time, in this particular case it is 5.3 t s equal to 5.3 seconds. It is at least close to the value given 4 seconds, 13 seconds for the system you settled down is too much a value you see that really will become a poor system as far as the speed of response is concerned. So 5.6 is the sec…. you may convince the user, look 4 seconds you have asked, go and examine whether 5.3 seconds is acceptable to you or not because it is at least close to that value provided you are able to give him the steady-state accuracy.

The user also has to rethink of the specifications once a suitable design has been made and there is a scope of rethinking as far as meeting this requirement of t s is concerned 5.3 but provided the steady-state requirement is satisfied. Now, before I go to the steady-state requirement let me give you one more thing. Suppose instead of going for zeta I take t s requirement first in that particular case do not take this you just draw a vertical line, vertical line which meets the requirements on t s.

If I give you t s you know zeta omega n. Draw a vertical line this gives you point of intersection. Now this point of intersection may not lie on the root locus plot and therefore along this line you may trial and error because zeta omega n is fixed by this line. So along this line if you make a trial and error you are able to locate a point which satisfies the angle criterion and hence your speed of response requirement is satisfied.

Now how about zeta?
Join this line to the origin (Refer Slide Time: 20:45) get this angle and see whether your zeta is close to 0.6 or not. So you see that if zeta or t s requirement is given you can either first set the value of zeta and get an amplifier gain or root locus gain or you can draw a vertical line to get the value of the root locus gain to satisfy the steady-state to satisfy the settling time

requirement. But I have seen…… in this particular case from the first design you have seen that t s is equal to 5.3 okay you may say well, as a designer you will go back to the user and you will convince him can you accept this design but before we do that let us see the steady-state accuracy. Now, for that I require the value of K v.

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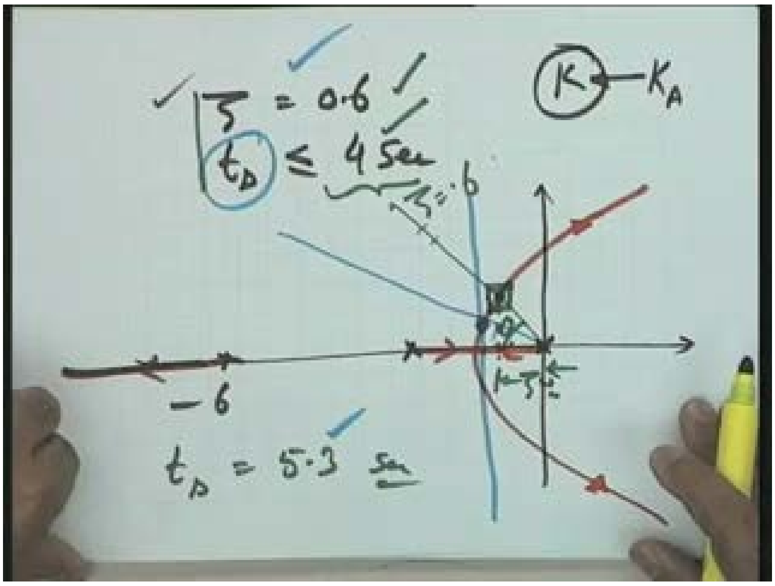

Now for K v I know that it is limit s tends to 0 sG(s) and with oh yes I am sorry before I go to K v I require calculating this K (Refer Slide Time: 21:39). At this particular value the point of intersection which you have taken what is the value of K? You will apply the magnitude criterion for the value of K and the value of K in this particular case turns out to be 10.5. It was a different calculation in the tutorial class but does not matter, as I told you my calculations are also rough calculations and not obtained from the computer. So this is a rough sketch and the final design if I have to give to the user for this particular system I will use the CAD software and give him the final design.

So, as per the calculation available with me the value of K at this particular point is 10.5. So correspondingly K A value can be calculated it will be 10.5 divided by 60. But let me calculate the K v value it is going to be 60 into K A is 10.5 divided by yes help me please? 12, is it all right? Yes, it will be 12 that is fine; 10.5 divided by 12 which is equal to 0.875 you find that the K v value is too low and the steady-state error in this particular case is going to be 1.14 radians; do convert it into degrees and see that you will definitely not your user is not going to accept such a design.

You see, the units also must be kept in mind, it is a position servo system we have thought of and this 1.14 is in radians, in degrees it is too much a value and therefore the steady-state accuracy is not acceptable. So though you might have convinced the user in terms of settling time but he is not going to be convinced as far as steady-state accuracy is concerned and hence I find that mere gain adjustment unfortunately has not satisfied the requirements.

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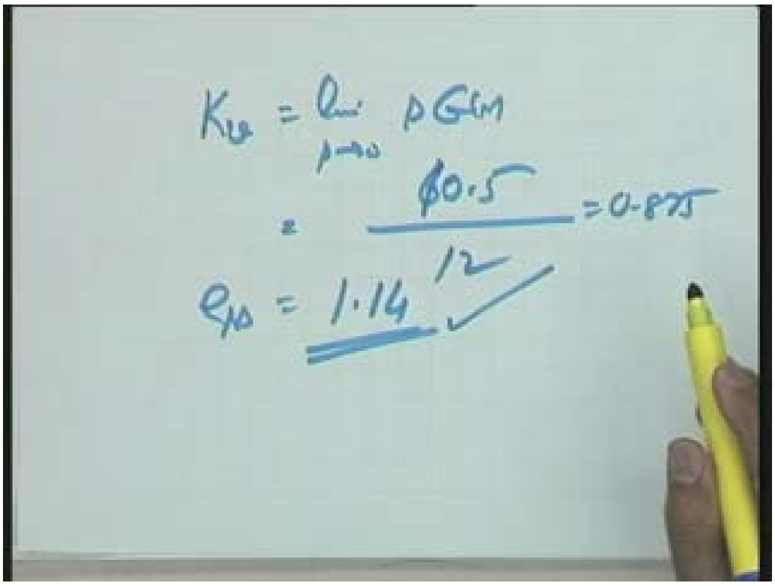

Now please see what are your requirements, what do you want? After all….. okay let me take a fresh sketch I think and I will get this guideline from you as to what are our specific requirements and how do we meet them. Minus 2 minus 6 this is the root locus sketch (Refer Slide Time: 24:02). Now you see that if the value of zeta I take as 0.15 that is if I take K is equal to 60 the complex roots lie here. If I take the lower value of zeta that is the higher value of zeta that is if I reduce the gain the value lies here if I reduce the gain but the steady-state accuracy drastically goes down.

So what you really want?
You want the transient accuracy to be preserved without losing the steady-state accuracy or vice versa. So what I suggest is this that without giving you the reason as to why do I suggest let me say that to this particular plant I introduce a 0 at this particular point between the two poles let us say at minus 3, we are yet to see as to why but I am introducing a 0 at this particular point. Suppose I will change the hardware of the system in such a way that the system has got a 0 at s is equal to minus 3.

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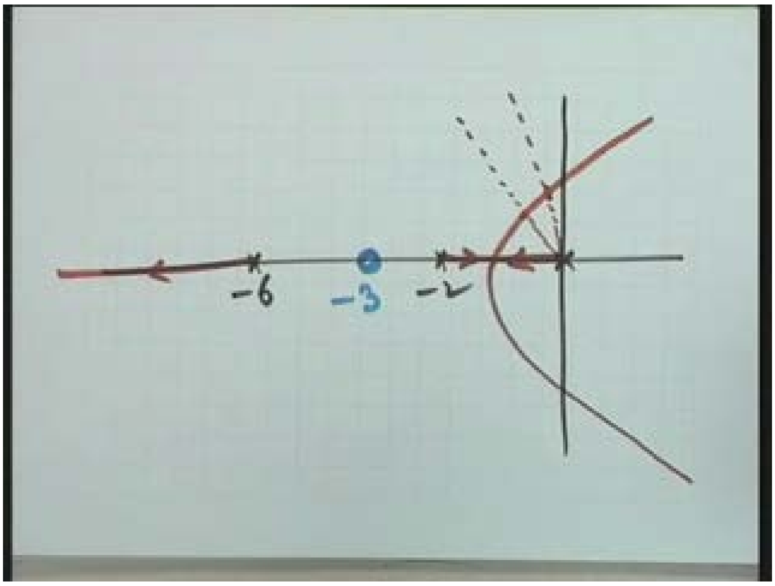

So help me as to what is going to be the effect of this s is equal to minus 3 on the root locus plot. You will find that the effect is a stabilizing effect; a 0 here let me take…. oh I am sorry this will not appear here (Refer Slide Time: 25:25) yes help me please, you will find that this is one root locus branch; you should be able to make these sketches now very quickly you see. Yes what do you expect in this particular case? Help me; one of the root locus branches has gone this way from minus 6 to minus 3. Now these two branches K is equal to 0 to K is equal to K is equal to 0 here and K is equal to 0 here are the two starting points what do you except in this case.

Well, it is very difficult to say because it has to go to infinity. Now which way it goes to the infinity depends so naturally you require the asymptotic angles and the centroid. If I had given you this information then you would have immediately given otherwise you see it is very difficult to make a guess because they have to go to infinity but there are infinite directions for it to go to infinity and therefore that guess is difficult and you find that now I have drawn the asymptotes the asymptotes turn out to be like this and therefore you find that the root locus plot becomes like this (Refer Slide Time: 26:34). You find the effect; it is a tremendous effect on the root locus plot. The total plot has been pulled to the left and pulling the plot to the left means stabilising the system in general for all values of K. Actually these plots are nothing but the complex conjugate poles dominant poles over here and these are the closed-loop poles and the closed-loop poles are going to guide the dynamics of the closed-loop system. So once you place your 0 at this particular point you find that the total plot has been pulled this way and hence it is a tremendous stabilising effect on the system.

You recall that the 0.105 damping ratio was very poor the system was oscillatory, so to make it a better damped system I have added a 0 so it means can we say that this particular 0 is equivalent to adding damping to the system or if you take in your own terms it is equivalent to a PD compensator proportional derivative controller because after all what is a proportional derivative controller, a proportional derivative controller gives you a 0 in the forward path transfer function of the system. So it means the word proportional derivative controller has not equivalently been translated into a 0 in this particular root locus sketch. Now, in this particular case let me take up zeta is equal to 0.6 requirements zeta equal to 0.6 requirements I am taking now.

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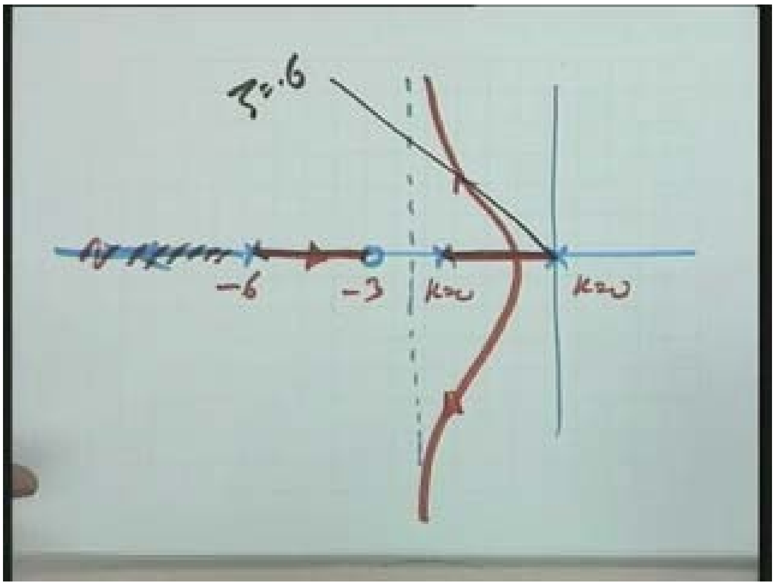

You see that earlier zeta is equal to 0.6, in the earlier root locus plot if you recall you had taken the zeta is equal 0.6 was not lying on the root locus. Now you see that this requirement has been obtained over here and this becomes the closed-loop system and here if you do all the calculations K turns out to be equal to 16, K turns out to be equal to 16 and corresponding to K the K A can be calculated and if you see zeta omega n this is your zeta omega n (Refer Slide Time: 28:45) and hence settling time can be calculated; as per the calculation with me settling time is 1.96. Zeta is equal to 0.6 and settling time is 1.96.

Let me look at the steady-state accuracy requirement: K v equal to how much please, help me for K is equal to 16 K v will be equal to 16 [Conversation between Student and Professor – 00:29:07 min] 12 so 16 by 12 is your K v, K is equal to 16, no, 0 is also contributing to K v now, no, please see that is why I thought why it is becoming different compared to the value I have I have written down.

Now your G(s) the forward path transfer function becomes s plus 3 K is equal to 16 divided by s(s plus 2) (s plus 6) your K v now becomes 16 into 3 divided by 12 and is equal to 4. Please see that the compensator is changing the K v also and this relationship you have seen that when we had a 0 now K v is equal to 4. In the earlier design K v was equal to 5, now you have at least reached closer to what the user has said. The user requirements are more or less satisfied you see; your t s is equal to 1.96 you wanted less than equal to 4 seconds; your zeta equal to 0.6 you have already met, your K v was less than equal to greater than equal to 5you have achieved 4, after all you have achieved 4 by just one time adjustment I have simply said that you make your K v this 0 at equal to minus 3; after all there is a flexibility of changing this particular 0. So now it is a trial and error adjustment.

We have seen that adding a 0 has a zero potential of changing the design; by trial and adjustment here and there now it is possible to meet the requirements on zeta settling time and the K v by placing the 0 appropriately and this appropriate you see once you have got this idea here the CAD software becomes very useful because the trial and error becomes quicker there whether it has to go this way or that way that depends. [Conversation between Student and Professor – Not audible ((00:31:09 min))].

Yes, yes, the last point K is equal to 16 in this particular case I have got over here and the value of this particular point can be calculated. Now you see that in this particular case now you find that the third pole lies in the region minus 6 to minus 3 and there is a great risk of the dominance condition being violated. You see this point? This point was raised by one of you that if the dominance condition is not satisfied what to do. There is nothing we can do but for trial and error. But some guidelines if we remember in that particular case the trial and error gets reduced.

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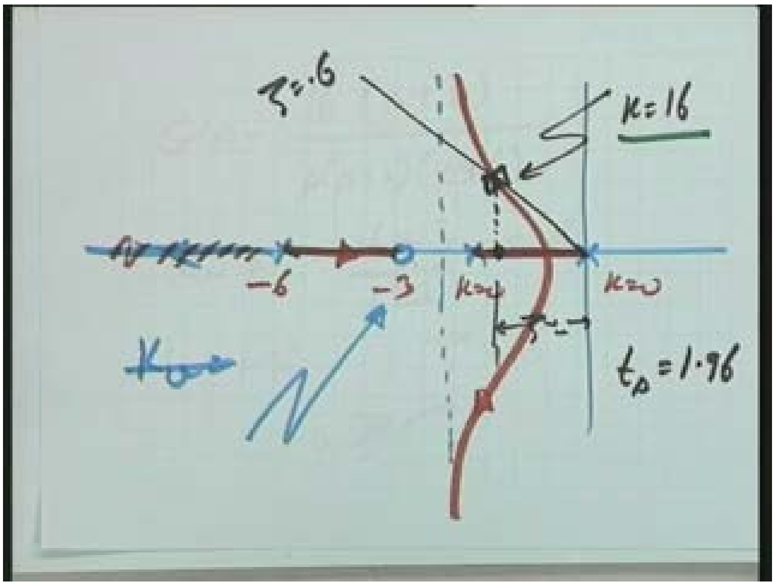

What is the trial and error?
One trial and error you see is this that: if the third pole lies sufficiently far away if it is not six times okay then let it be at least three to four times; in that particular case hopefully your design may be acceptable. In that particular case instead of designing for zeta is equal to 0.6 you may design for some other value. You see, after all a third pole makes the process sluggish, a third pole in the system will make the process sluggish so it means compared to t s is equal to 1.96 which now you have got without considering this particular thing now if you simulate what will happen after all it will become more than 1.96 and it is still within the margin because t s less than 4 seconds is acceptable to you. So, if the dominance condition is not satisfied, not that everything has been lost the only point is this that you have to re-examine it very carefully. This third pole is going to make the system sluggish and you see whether the true simulation gives you settling time which is acceptable to or not.

One more point: you see that fortunately in this particular case there is a 0 available in this particular region. You see that if your third pole, I need your attention at this particular point if your third pole lies close to this particular 0 that will depend upon……… you will have to locate I do not have this value available here; for K is equal to 16 you have to locate the third pole, if the third pole turns out to be……, I need your attention please; if the third pole turns out to be closer to this 0 in that particular case this pole-zero pair will anyhow nullify the effect of this pole and hence the dominance condition will be satisfied. So dominance condition does not mean only that the pole is far away, the dominance condition means that either the pole is far away compared to this real part or there is a 0 close by so that the residue at this particular pole becomes negligible. There are two possibilities existing in this design, you have to carry out the complete design. One is this that if you are not lucky enough to get this particular pole close to this 0 or far away then there is a margin available. This third pole may increase the settling time which still may be acceptable to you.

[Conversation between Student and Professor…… the zero in this case if the pole is close to the 0……..34:23] if the pole is close by to the 0 yes the effect of 0 as well as the pole both get nullified, the effect of 0 will….. it is more or less cancellation of pole m 0. You can imagine that if the pole was sitting on the 0 in that particular case the pole-zero in the transfer function will not appear so neither the effect of 0 nor the pole but exact cancellation is not possible they will be close by so peaking effect of the 0 as well as the effect of the pole in making the response sluggish will get nullified. Nullified you see is a just ideal term, it will get reduced, yes, but by simulation you have to examine.

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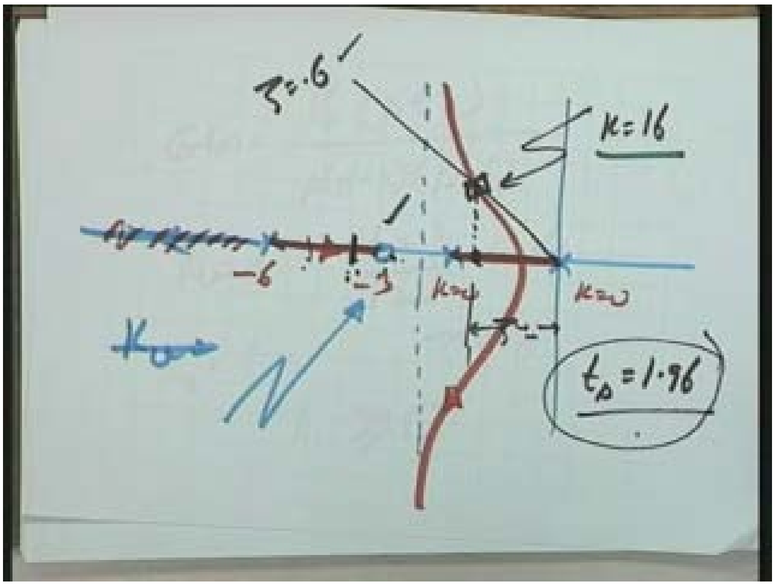

So when such a situation comes you see that is why we say that a rough design is necessary. now if all these trial and error you make with a paper and pencil probably there is no need as far as today’s advancement in CAD software is concerned, you have to go to the computer to complete this particular design. But initially you can go ahead with this particular diagram, make a rough idea as to where these trial and error points should be there, make trial and error using the CAD software and get your final design. Is it okay please? I hope this is clear.

Therefore, that way it appears with the help of a PD design, with the help of the 0 only I am able to get the requirement satisfied on both the steady-state and the transient accuracy. Hopefully if not then at lease within certain tolerances my requirements on both the steady-state accuracy and the transient response transient performance gets satisfied.

Now a definition, a very important term used in the literature: you see that this D(s) by a 0 means s plus z compensator z c has been added to the system. It means, as you have seen in the process of calculating the K v we have added s plus 3 to the system. Now come to the physical realisation. Physical realisation of D(s) equal to s plus z c, you see, as we will go to the frequency domain it will become more clear that such a controller is prone to noise and therefore pure differentiators we normally avoid. So adding a high frequency attenuator, adding a filter is also necessary as far as the complete acceptable design is concerned and therefore an actual D(s) used in practice is s plus z c over s plus p c where this pole at s is equal to minus p c gives you the filtering effect.

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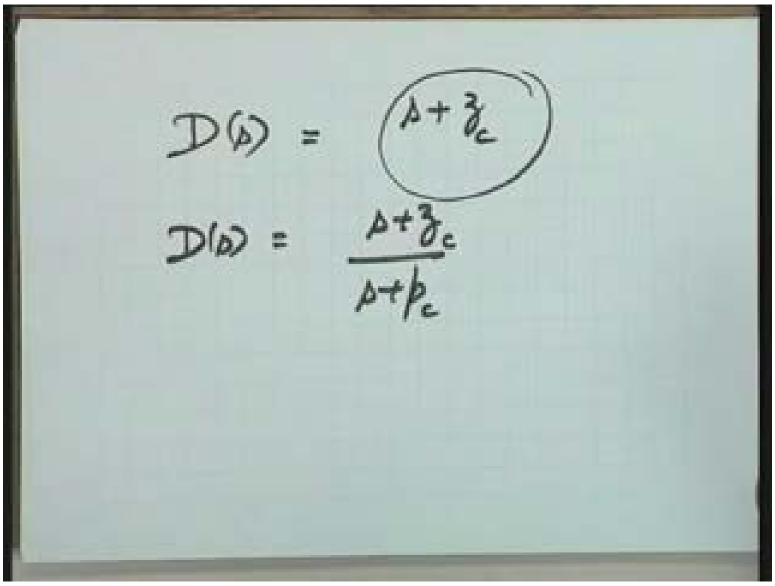

[Sir, give….. then what……….37:12] filtering effect that is high frequency signals will be filtered out. […….Sir this pole is very far from the region] yes, now let us see where do I place the pole. Now you see that if I place the pole in this particular region (Refer Slide Time: 37:26) the whole design gets disturbed. In this particular case all these requirements which you have satisfied, the purpose of the pole is really not to help in the design, your performance requirements on the steady-state and the transient accuracy are satisfied, your performance requirements are satisfied so what you will do is you place the pole so that as far as the root locus sketch is concerned it is more or less the same and therefore I place the pole far away may be at minus 10, minus 12 so it just depends upon the realisation requirements.

Many a times you can say that far away means minus infinity, if I place it at minus infinity what will happen when you realise it using Op-Amp circuit or even when you realise it using a digital computer numerical accuracy requires some good numerical coefficients coming into the system you see. So at this juncture I leave it to the realisation problem. Practically this should be as far away as possible so that the effect on this particular root locus is minimised and the far away point should be taken care of, the filtering requirements and the realisation requirements should dictate the third pole which should be added and you see that the third pole when added will add one more root locus plot and hence one more pole will be there and your overall root locus diagram may look like this; overall root locus diagram with the pole added.

So 0 minus 2 minus 3 is a 0 minus 6 a pole has been taken at as minus 10; let us make a root locus sketch. one going this way, this is this way, now you see it is not the vertical axis it has come this way and your root locus diagram……. since I know the result I have drawn it this way but you really require the rules to apply that.

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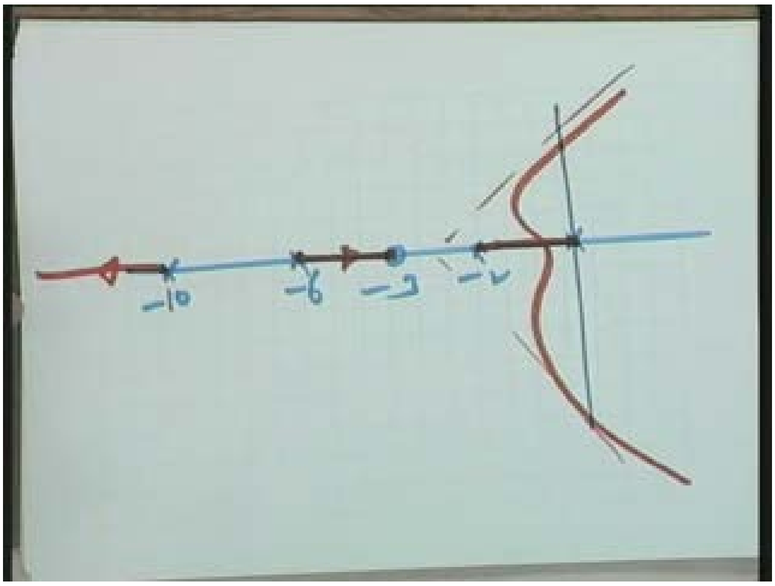

The only point in this particular case the only claim is this; around zeta equal to 0.6 point the root locus is not disturbed it is taken this way so naturally it means the next pole has a destabilizing effect. it has tilted the root locus this way but the only claim is this that you should select a pole here, you should select a filter so that in the region of your interest the root locus is not disturbed let it be pulled this way it does not matter. So it means zeta is equal to 0.6 continues to lie on the root locus intersection of this and the K v requirement is equal to 4 continues to get satisfied more or less and therefore now you have an additional freedom you have to adjust this pole-zero pair in such a way that the complete root locus diagram is like this and your requirements on both the steady-state and the transient accuracy are satisfied.

So in this particular case giving with the values please zeta turns out to be 0.6, settling time is 3.33 within tolerance and K v equal to 1.7 and 1.7 has become poor but it has the flexibility I have given you just one value. So it means minus 10 and minus 3 this is the design procedure, I am going to give you the design procedure in the next class through an example as to how to adjust the values of minus 3 and minus 10 so that your requirements on steady-state accuracy and transient accuracy are satisfied.

In this particular case placing the pole at minus 10 has disturbed your steady-state accuracy though the transient accuracy is within the limits provided the dominance condition is more or less satisfied. K v has gone down and therefore the steady-state error will go up and therefore I have to adjust these two values appropriately so that the steady-state requirement is also satisfied. This I assure you that well systematic trial and error procedure is available by which this pole and zero both can be……. how many total parameters are there, please see; the total parameters are the zero location, the pole location and the amplifier gain. I still assume that the amplifier gain is available for design.

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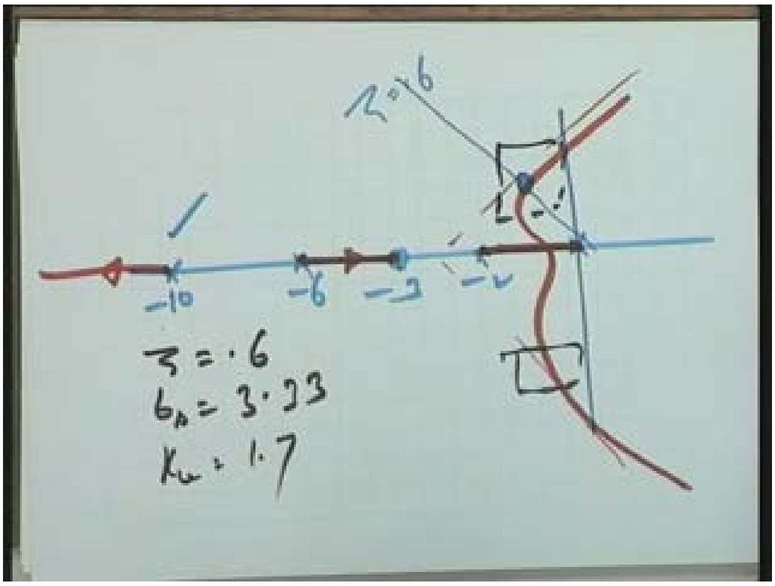

So it means you can say that D(s) is equal to s plus z c s plus p c so it means z c and p c are the two parameters and the amplifier gain K A; these are the three total parameters available for design and with these three parameters using this type of network you will find that it is possible to get the steady-state and the transient requirements met.

Just a definition; if you take a 0 at minus z c and a pole at minus p c and this is your closed-loop pole can you tell me what is the net angle contribution of the pole-zero pair of D(s) on the closed-loop pole location; can you help me please; is it lead or lag, is it positive or negative? I think it is clear, this is plus (Refer Slide Time: 42:40) this is minus so it is the net positive angle which is being added and it is because of this that this compensator which compensates for the deficiency in the performance is referred to as phase lead network or phase lead compensator is the name given to this.

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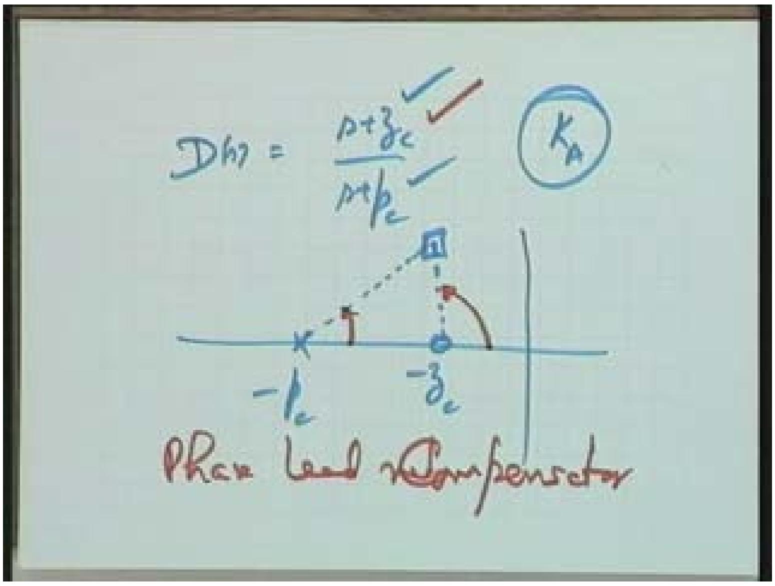

A phase lead compensator is a compensator which adds angle……. in the frequency the meaning of the word rather 'phase lead' will become more clear when we go to the frequency domain. But at this point I think the name why the 'phase lead' is coming should be clear that it is coming because of the lead angle it provides.

Well, as a passing remark on this I think I will like to give you a circuit and leave this as an exercise for you to see that this in fact is the Op-Amp circuit for a phase lead network, please see this. This transfer function of this will turn out to be the transfer function of lead network which I have given. I give you the transfer functions since it is available in my notes: D(s) is equal to minus R F(s R 1C plus 1) divided by s R 1 R 2 C plus R 1 plus R 2. You find that it is nothing but s plus z c over s plus p c by suitable design of the parameters R 1 R 2 C and R F.

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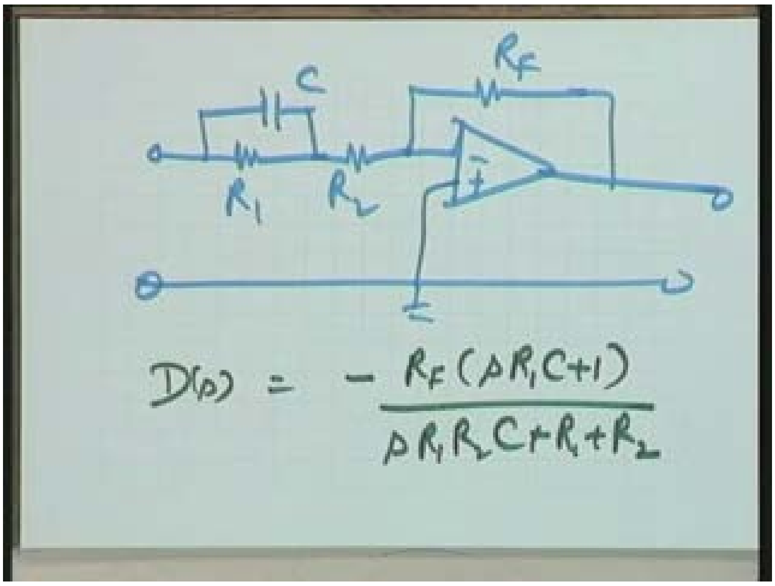

This D(s) can be put in the form: D(s) is equal to minus K c, the minus you can just see can be taken care of by just putting one inverting amplifier. If you have not noted I repeat what I gave you in this slide again. This (Refer Slide Time: 44:35) is the transfer function I get for this particular Op-Amp circuit and rearranging this transfer function in the format we know it is minus K c it is (tau s plus 1) divided by alpha tau s plus 1 where tau s is equal to R 1 C K c equal to R F by R 1 plus R 2 and alpha equal to R 2 divided by R 1 plus R 2. So both poles and zeros and the gain all the three parameters can suitably be adjusted by adjusting the values of R 1 R 2 C and R F. So this is a typical Op-Amp circuit which is used to realise a phase lead network please.

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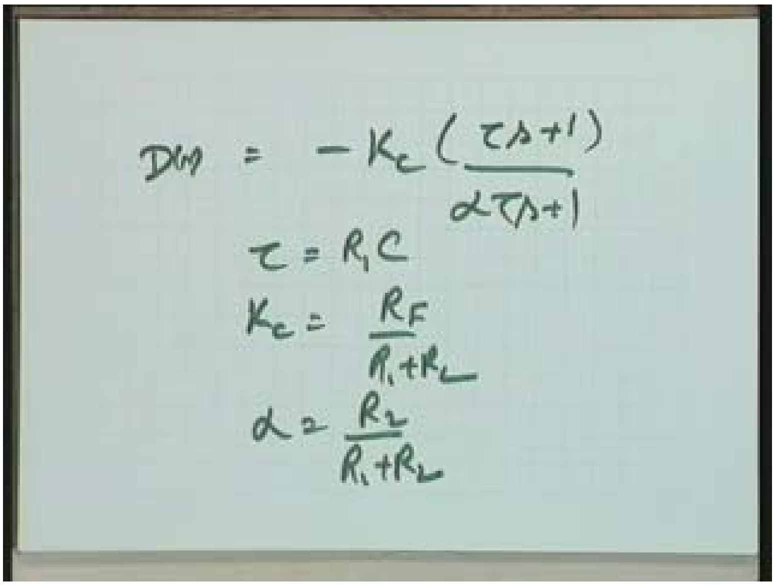

Is it okay please; any question on this? This I hope you can very easily examine. In this case the design freedom is there not that it is one-to-one relationship you know three parameters using these three….. and here there are four selections C R F R 1 and R 2 so depending upon the suitable values of these network these particular values of devises from considerations other than implementing the value of D(s) you have to suitably select the value, the design freedom is there; you have more freedom than the values required and hence one of the values could be taken and see other appropriate values of resistors and capacitors is taken.

Let me quickly go to the other aspect also so that next time I have the design examples only and the other aspect I take is the phase lag design or the integral control. You recall this diagram minus 6 and tell me if I place a pole at the origin what will happen to both transient and steady-state accuracy. I am drawing my sketch over here, you give me the answer to my question please.

If I place a pole at the origin what will happen to both the steady-state and the transient accuracy?

First of all comment on the steady-state accuracy please if I place a pole at the origin. K v equal to 0, K v is equal to 0 meaning thereby that your steady-state requirement has been drastically improved. [Conversation between Student and Professor – Not audible ((00:47:03 min))] steady-state requirement; K v is in finite steady-state otherwise it is 0. You see any specification you give the steady-state requirement is satisfied if I place a pole at the origin.

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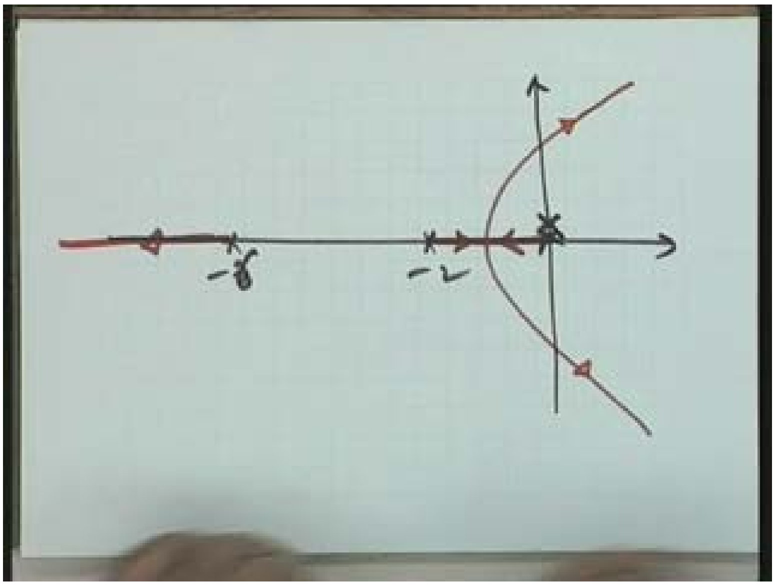

You see I am not to worry about the steady-state requirement at all. But you see you have to keep in mind you may be over doing something, the user does not ask you K v is equal to infinity but you have given him K v is equal to infinity fine provided you have not disturbed the transient too much. Otherwise he does not ask you for K v is equal to infinity he says that K v greater than equal to 5 will suffice. But we have over done it as far as the steady-state requirement is concerned and let us see what is the price we are paying.

Come on tell me what is the price we have paid?
Yes, anyone who has made a sketch of this. What is the price we have paid by this selection? This goes over here, no, this does not go there, yes tell me where will it go. This is there on the root locus so there will be a breakaway point here. Actually as I told you that asymptotes are required for you to make an idea without the asymptotes it was becoming difficult for you, now may by you can give an idea.

This (Refer Slide Time: 48:23) is as far as the two root locus branches are concerned and here these are the two; you have destabilised the system completely for all values of K. In the process of giving K v is equal to infinity you have destabilised the system completely. Now if you have given this it means to stabilise it, you pull these two branches back into the left-half plane and to pull it back into the left-half plane what I do is I place a 0 close by close to the pole I had introduced. you see, the pole I have introduced to change the type number of the system but this particular pole has drastically destabilised the system and now I want to stabilise it again I want to pull it back and I can pull it back by placing a 0 close by.

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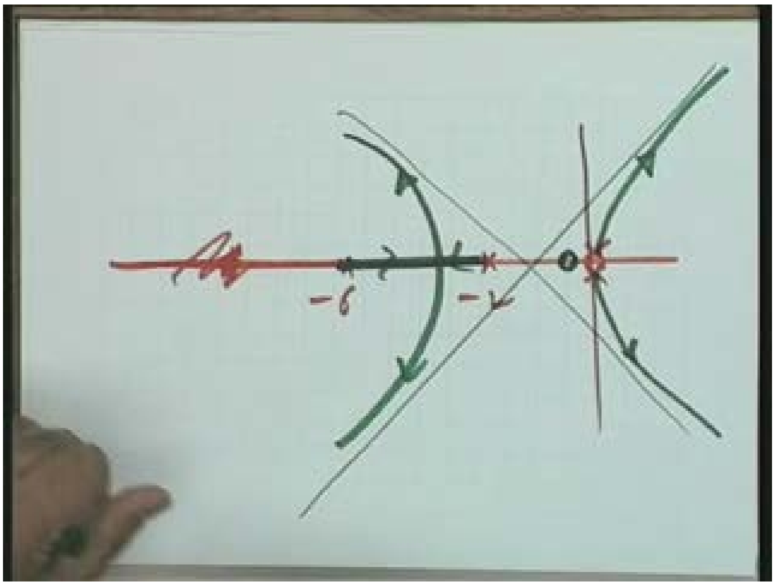

Help me please; if I place a 0 close by what is going to be the root locus sketch. A 0 close by a minus 2 here and a minus 6 here (Refer Slide Time: 49:27) again it may be difficult to guess it but at least it is clear that this is the portion on the root locus branch, this is the portion on the root locus and how about these axes the axes are going to be like this and if I make a sketch in this case the root locus sketch is going to be like this. Well, very difficult to plot it without the rules or without taking the help of the computer.

So you find that the effect of the 0 is to pull it to this direction and hence the possibility of getting zeta equal to 0.6 has become possible; zeta is equal to 0.6 that possibility now exists because you see that once you are able to realise zeta is equal to 0.6 you are not to worry about K v K v you have already done it you see your K v is equal to infinity. So the only thing is this, this zero adjustment you have to make here and there so that your zeta requirement and settling time requirements are satisfied. This is what you have to see the zeta and settling time requirements are to be satisfied.

So though I have some calculations with me looking at the time I like to conclude. You see that if I put this 0 over here though the comments on dominance and other requirements are the same you see let me not repeat that. Now look at this particular point; it means I want D(s) to be of the form s plus z c over s which you know is nothing but a PI controller which is nothing but a PI controller the proportional integral derivative controller because it is nothing but 1 plus z c over s 1 plus 1 over t i s which is nothing but a PI controller. So it means with the help of a PI controller it is possible to give the K v equal to infinity and by suitable adjustment of the 0 maybe we can satisfy the transient requirements. So the PI requirement is satisfied.

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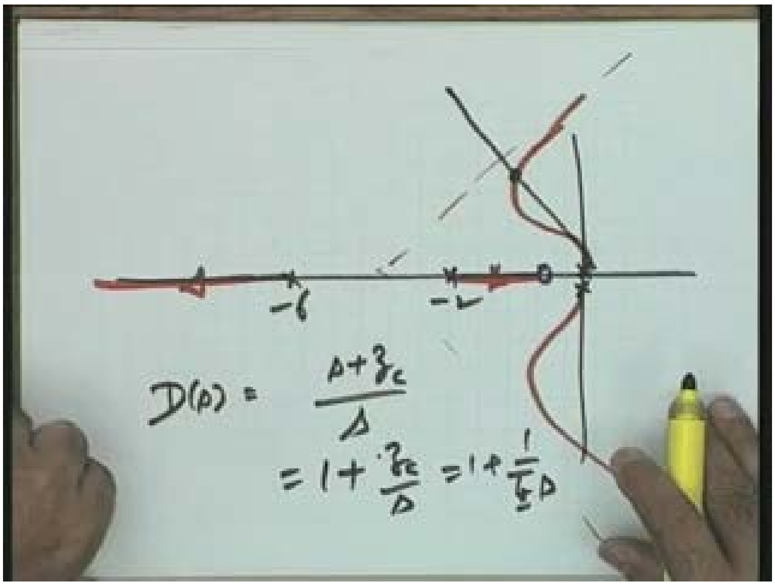

Now the question is this, the last point I want to mention, what is the need of jumping to this particular requirement of K v is equal to infinity when the user does not need it?

The pole at this particular point has totally destabilised the system. So the design requirements can be appropriately met with this way that this was your original pole you make your pole and a 0 here you see, these are the compensator poles and zeroes, the compensator pole instead of making it at the 0 I am placing it close to 0 you see this point please. The compensator pole instead of fixing it up at the 0 I am placing it close to 0 and this is your compensator 0and therefore D(s) becomes equal to s plus z c over s plus p c.

Can you tell me what is the net angle contribution; is it positive or negative?

It is negative and because of this this type of compensator is called a phase lag compensator.

[Conversation between Student and Professor Sir, about dominance…….. yes …two poles were at origin they always dominate 52:48] the two now you see that the dominance is not with respect to these two poles. Now these are the closed-loop poles, you have to see dominance with respect to the closed-loop poles these are the open-loop poles they have lost their meaning once the loop has been closed. The dominance has to be seen with respect to closed-loop poles only.

You see that as I said I am going to give you the complete picture with respect to the two design examples. Here you can see, instead of placing the pole exactly at the origin I place it nearby and if I place it nearby in that particular case the requirement of meeting the steady-state accuracy requirement as well as meeting the transient requirement may be satisfied and I add flexibility to my design. I can select now the pole, I can select the 0 and I can select the gain also. The last point please see it is for you to check that this system which I am giving you is actually a phase lag controller, please check this, make a suitable manipulation of the transfer function of this…. There is a resistor here, a resistor, a resistor, a capacitor these are the total elements in the system, take input output ratio, make suitable adjustment of the transfer function and see that it fits into this type of system.

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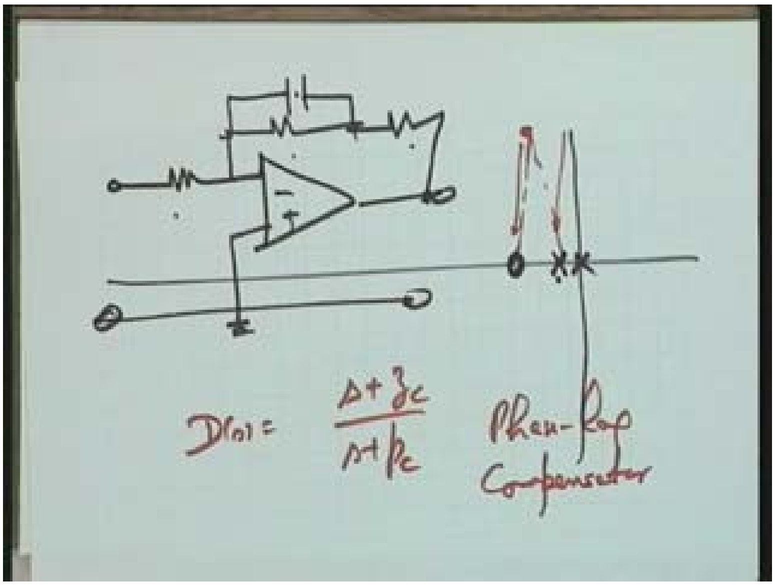

I conclude my discussion over here and the next example next class is going to be two design examples, thank you.