Control Engineering

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Lecture - 32

Compensator Design using Root Locus Plots (Contd…)

The rules for sketching, the root locus general guidelines for sketching. I think, more or less all the rules were covered but for two. Let me give those two rules quickly and then go on to the complete examples and through these examples I think all the rules will be properly revised. The two pending rules the numbers I do not know what was the list you see but let us not worry about the numbers, the two pending rules were the angle of departure from a complex pole or angle of arrival at a complex zero.

If you go through your notes you will find a root locus of this type coming in your notes because I had taken this example. Let us say that these are the two poles and here is a 0. Now, if you apply all the rules of root locus sketching you find that this total thing is on the root locus plot this segment and if you apply your rules for breakaway or break in you will find that there is a break in point over here an approach point over here and the root locus starting over here breaks into the real axis and these are the two root locus branches: one going this way and terminating at the 0 and the other going this way and terminating at the infinity. Let us say K is the root locus gain K is equal to 0, K is equal to 0 and infinity here K tends to infinity here this is a root locus sketch which is already there in your notes.

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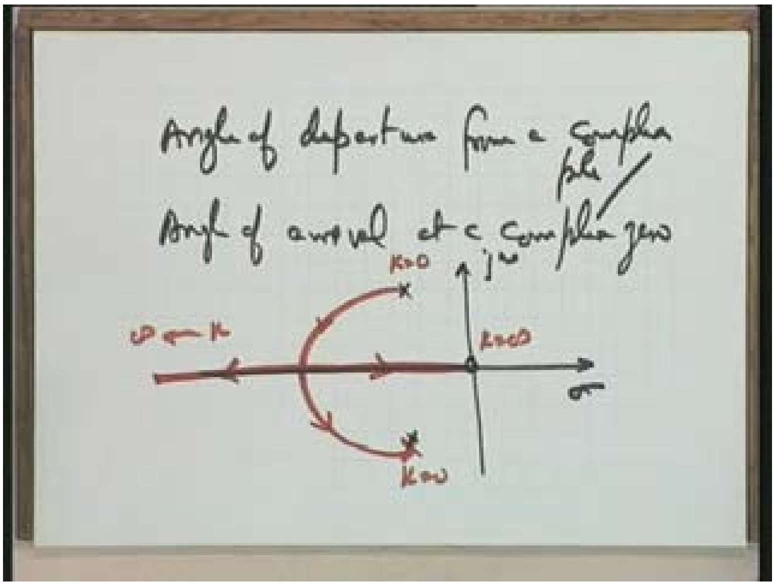

Now the question is this that well, in this particular sketch fortunately it can be proved rather I will like to leave this as an exercise to you to prove that this complex root locus portion is a circle. So if it is circle you can easily make the root locus diagram. But otherwise in general you will very much like that the root locus when it leaves this particular complex poles does it leave in this direction, in this direction, in this direction or in this direction. For example, it could go this way, it could go this way or any direction it can take in general as I am telling you that for this particular numerical example you can prove in general that the complex root branches are circle with this as radius and hence you do not need the direction. But since I have taken this very example let me now exemplify the direction through this example itself. The process is like this: it says that you get the angle contribution at this particular pole let us call this as theta 1 due to this 0; you get the angle contribution because of this pole at this point and let me call this as theta 2, the net contribution at this particular pole is how much please, it is theta 1 minus theta 2 because theta 1 is because of a zero and theta 2 is because of a pole so theta 1 is positive and theta 2 will be negative being in the denominator. So the net angle contribution at this particular pole because of the other poles and zeros in this particular case is theta 1 minus theta 2 and the rule says that the root locus will leave this particular pole at an angle phi is equal to plus minus 180 degrees into (2q plus 1) plus phi p let me call it plus phi where phi is the net angle contribution due to all other poles and zeros at the complex pole under consideration.

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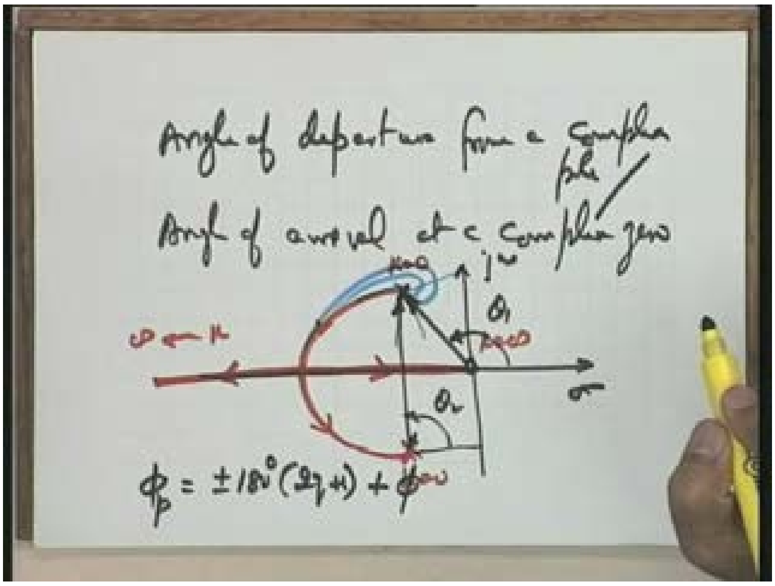

I repeat the rule please.
Sir, what is theta 2? Theta 2 in this figure is naturally 90 degrees but otherwise yes it is this particular pole I have taken I have drawn a vector to this particular point under consideration and this is the angle contribution of this pole at this point. So I have drawn a horizontal line and this is the angle. In this particular case it is too obvious because these are the complex poles this particular angle is theta 2 equal to 90 degrees.

In general, let me take a general situation so that the specific situation is avoided. This is complex sphere, here is pole, here is a 0, another pole and you want….. in this particular case now there is no circle business. You cannot say in general that the locus leaving this particular point is a circle. Now what you need is that a root locus will start from K is equal to 0. I really do not know which direction it will go. I want the direction of take off from this particular point. It could be any of the directions as far as total 360 degrees angle is concerned. And now the direction I say suppose it is phi p how do I determine how do I write this phi p, please see that the phi p angle will be written this way, draw a horizontal line, draw an angle phi p. So if I determine the angle of departure at phi p so it means the root locus will leave this way at this particular point. Now this angle of departure phi p is given by this formula. I am repeating this formula for your benefit; phi p is equal to plus minus 180 degrees (2q plus 1). You can take q is equal to 0 also because q is equal to 1 means a shift by 360 degrees which does not make any difference as far as the angle is concerned. I am simply writing the general formula otherwise q is equal to 0 will suffice plus phi; what is phi? phi is the net angle contribution at this particular pole due to all other poles and zeros. Note this point please: phi is the total angle contribution at the pole under consideration due to all other poles and zeros.

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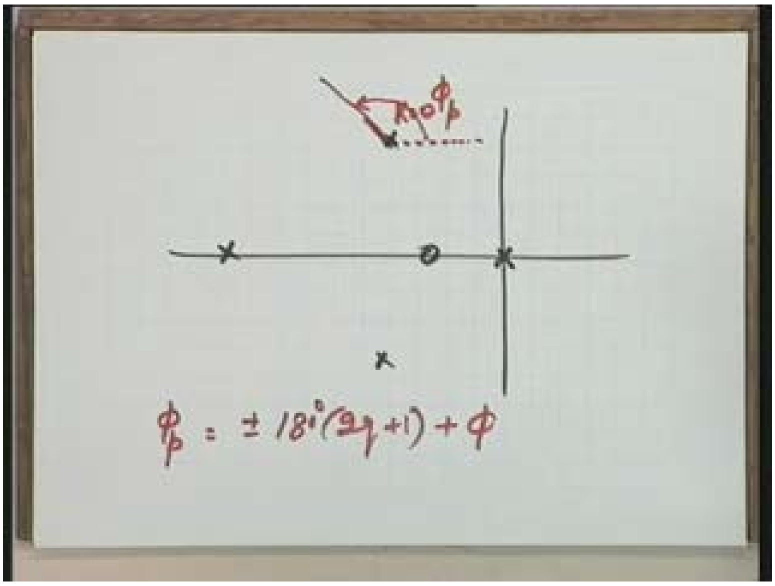

Now let us see how to determine phi?
Consider this particular pole, draw a vector here, this is the angle contribution due to this pole. However, please note, since this is a pole the angle will come with a negative sign. Take this 0 this is the angle contribution (Refer Slide Time: 7:31) due to this zero the angle will come with a positive sign. Here is the angle contribution due to this pole and lastly this is the angle contribution due to this pole which again obviously is 90 degrees.

So calculate the total angle taking suitable algebraic sign into consideration; positive sign with the zeros and negative sign with the poles. Let us say that total angle turns out be phi so in that particular case the root locus will leave this particular pole at an angle phi p given by this formula. I hope this…. and this will be exemplified. Let me not give an example here. I am going to take a comprehensive example in which all the rules will be revised so naturally this rule will also suitably come up there.

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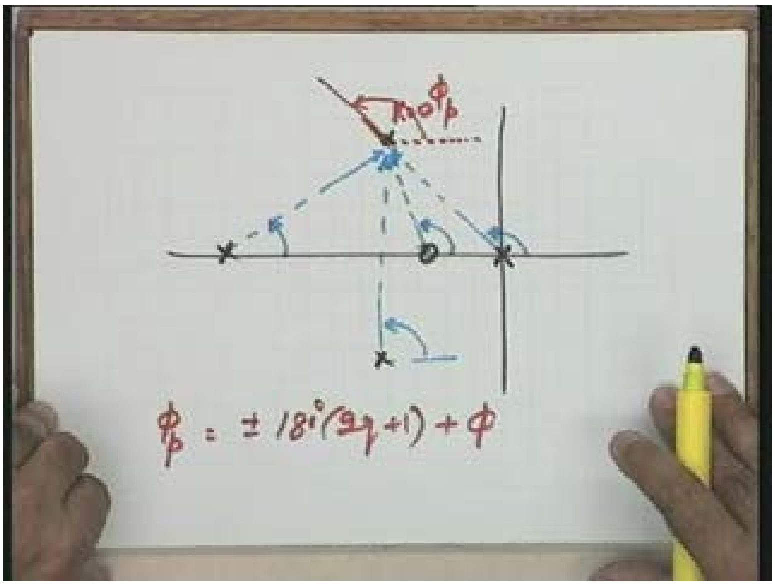

Similarly you may have a situation of this type. Let us say you have two complex zeros and let us say these are the two poles here; a typical situation I take this way. Now you see that a root locus starts from K is equal to 0, K is equal to 0 there are two zeros available so naturally it will terminate at these two points. I do not know the root locus shape but at least I know that the termination will be at these two points whatever directions it may take and these portions of the root locus branches will be determined by other rules which I have already given to you.

Now I want to concentrate on the rule as to which way will it approach the 0. will it approach the way I have already drawn or will it approach this way or will approach this way which is the way it will approach this 0 and that is given by the following relation: phi z equal to the angle at which the root locus branch will approach a complex zero is equal to again plus minus 180 degrees (2q plus 1) minus phi I take where now what is phi it is the same thing; the net angle contribution due to all other poles and zeros at the zero under consideration.

In this particular numerical for example you see you just take this 0 so the net angle contribution is double of this because there are two poles. Suppose this is theta 1 so the net angle contribution at this 0 is minus 2 theta 1. You join this 0 with this and make this vector and the net angle contribution due to this 0 is theta 2 which is equal to 90 degrees and therefore in this particular case your phi z the angle at which it will leave this complex zero is equal to say 180 degrees minus or plus sign you can take minus phi, phi is (theta 2 minus twice theta 1) theta 2 is 90 degrees in this particular case with a positive sign minus twice theta 1 this being a pole appears in the denominator so twice theta 1 will appear this way this is the net angle contribution.

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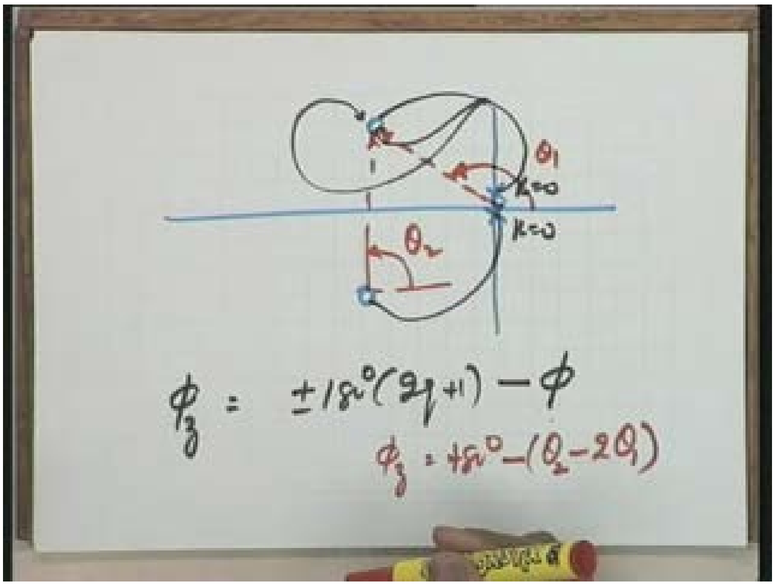

This is the way it will leave. So it means if I have the phi z angle available with me I will draw the phi z angle like this: this is phi z sorry this is phi z (Refer Slide Time: 10:50) if it approaches the way that I have drawn earlier; if it really approaches through this particular way then the phi z the angle at which it approaches the origin approaches the complex zero is given by this particular direction. So this becomes the angle at which the root locus branch approaches a complex zero.

Lastly, you will recall an example we had taken: G(s) equal to K over s (s plus 1) (s plus 2) for example take this way so s is equal to 0, s is equal to minus 1 s is equal to minus 2. You apply your rules you will find that this is one branch going to infinity. You find that this segment lies on the root locus and hence there has to be a breakaway point. So these are the directions and therefore the root locus most appropriately will go this way that is asymptotes angles etc you will determine.

The last rule which I want to give you is the following that you can determine the point of intersection with the imaginary axis using the Routh-Hurwitz criterion. In this particular case what you will do for example, the characteristic equation of the system is (s squared plus help me please 3s plus 2) plus k equal to 0. This is s cube plus 3s squared plus 2 s plus K equal to 0 is the characteristic equation of the system given G(s).

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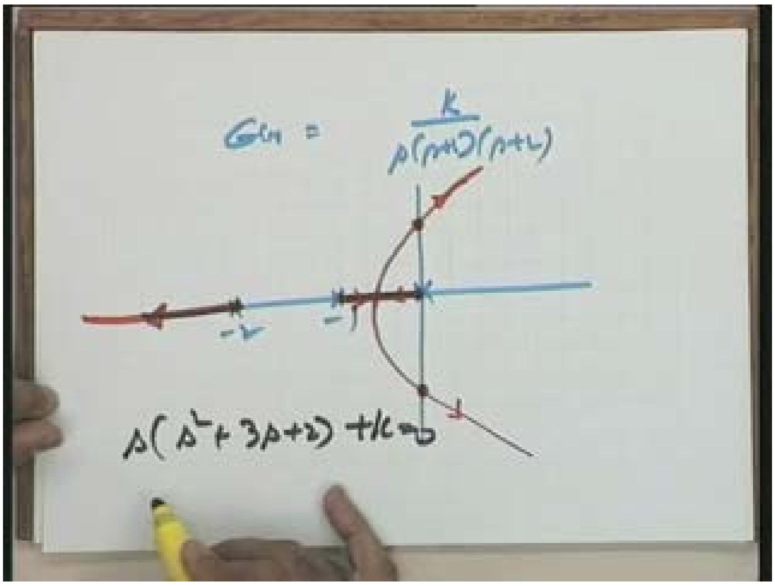

Make a Routh table; I am repeating this because in tutorial class there was a little confusion. You were not remembering this point that is why I am repeating. If you make a Routh table it is 1 2 3 K. So continuing with this it is 6 minus K divided by 3 K. Now you know that the system will be stable for K greater than 6 for K less than 6. The system in this particular case is stable for K less than 6 so at K equal to 6 you find there is a case of the row becoming all zero.

So, if I take K equal to 6 this element becomes 0, this missing element is automatically 0 which corresponds to a situation that an auxiliary equation is the factor of the original characteristic equation and auxiliary equation you make from the previous row looking at the previous row you get 3s squared plus 6 equal to 0 is the auxiliary equation or equal to A(s) is the auxiliary polynomial. 3s squared plus 6 because K is equal to 6 gives you all zero row, you have to go to one pervious row and 3 into s squared K into s to the power of 0 the even order of s will give you this polynomial and therefore the roots of this polynomial or the roots of the original characteristic equation as you find that the roots are s squared plus 2 equal to 0 or s is equal to plus minus J under root 2. These are the roots.

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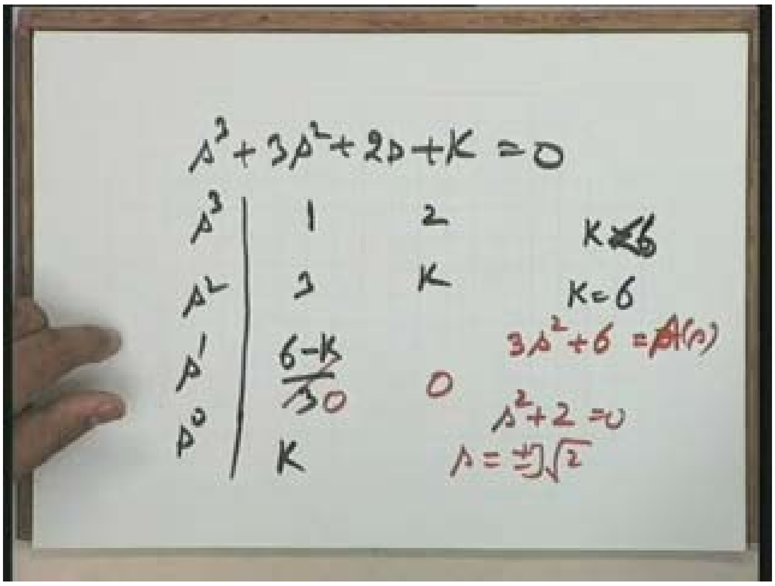

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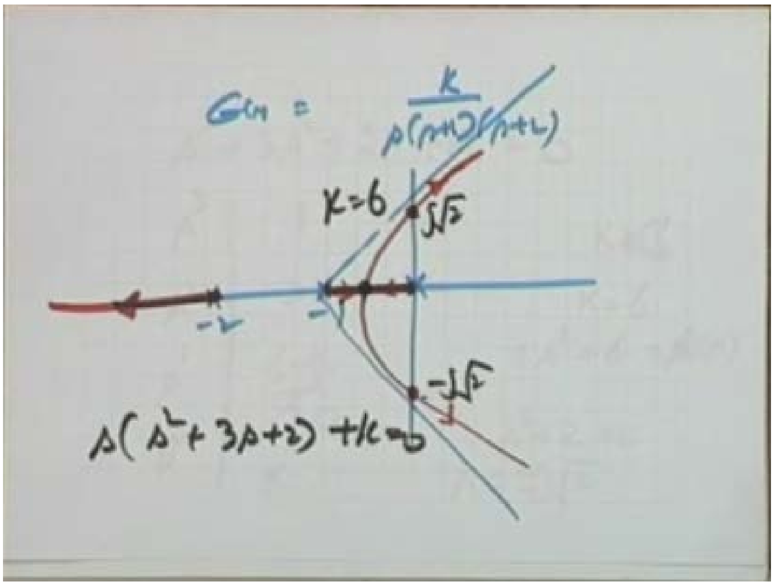

So in this particular case it is obvious going back to the original root locus so at this particular point J under root 2 minus J under root 2 and K is equal to 6 is the point coming over here. So in this particular case now you find that if you have K is equal to 6 and J under root 2 minus J under root 2 as the points available so it is great help to you, it gives a method of sketching the root locus a rough root locus because what you had were the following you had this, for example, these are the asymptotes you know.

This is the breakaway point (Refer Slide Time: 14:58) you have calculated so this breakaway point, these asymptotes and these additional two points give you a sufficient information to make a rough sketch of the system. At this way all the rules can be utilized to make a rough diagram and from that rough diagram the analysis and design can follow. This is what is the total quota of rules and you have now to remember these rules you see. If you solve a couple of examples I am sure that these rules will get set in your mind.

Now I will like to give you a few examples three examples are in my mind, three different examples, all the three examples will exemplify something new. First of all I take a comprehensive example to revise all the rules and this example is given a unity-feedback system with open-loop transfer function K over s into (s plus 3s) (s square plus 2s plus 2) this is the example please; apply all the rules on this example and let us see what type of root locus do we get.

So the first step is; take the open-loop poles and zeros on the s plane. So, in this particular case you find that s is equal to 0 s is equal to minus 3 and help me please what are the locations of the open-loop poles due to the factor (s squared plus 2s plus 2). Yes, what are the factors please? [Conversation between Student and Professor – Not audible ((00:16:46 min))] minus 1 plus minus J so let us take minus 1 over here and I am sorry this is not to scale but does not matter this is the situation as far as complex poles are concerned though the pole-zero configuration which I have given you is not to scale.

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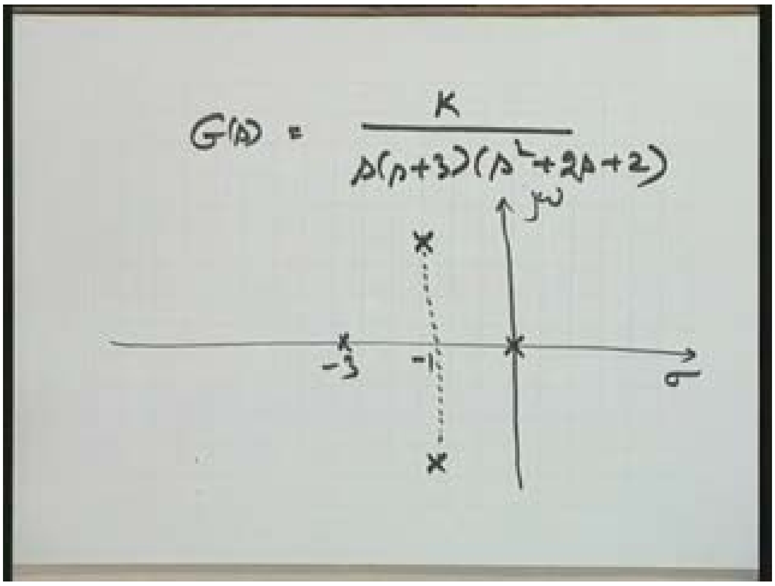

Now you just see in this particular case one two three four there are four poles and therefore there will be four root locus branches K is equal to 0 K is equal to 0 these are the starting points (Refer Slide Time: 17:18) so in this case I know that this particular portion will be on the root locus. I want you to make an attempt first of all to set an idea as to what type of root locus you expect.

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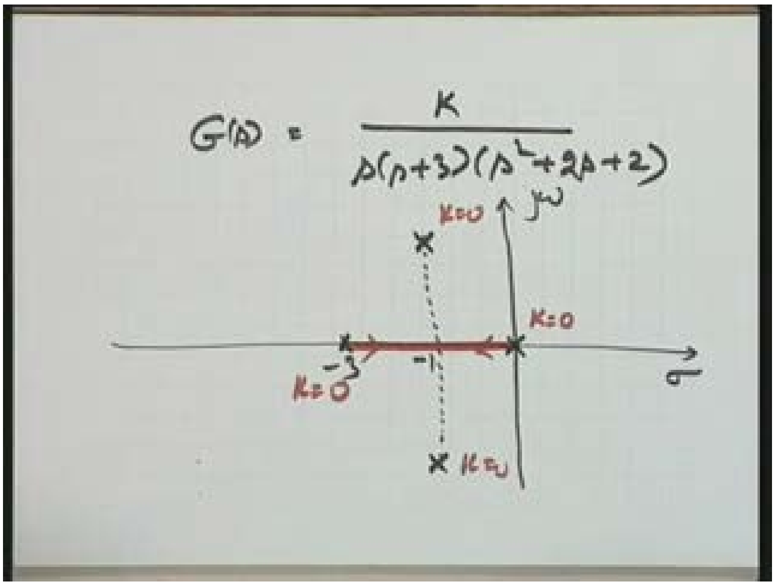

We will make the calculations later and for that I make the pole-zero diagram so that it is properly scaled. So, if I take this 1 over here 3 one two three one two three roughly, hopefully this is a better picture compared to the earlier situation.

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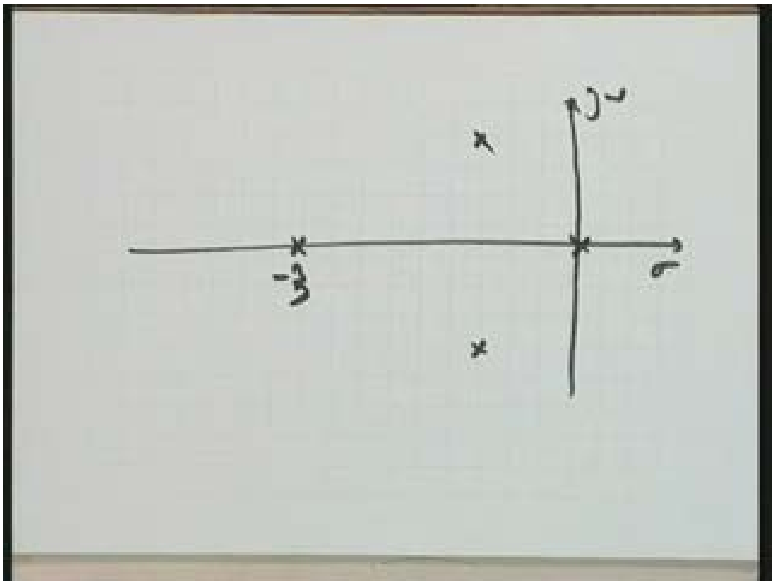

You find that in this particular case this total is on the root locus. So naturally one branch starts from here K is equal to 0, other branch from here K is equal to 0, one more branch K is equal to 0 and a K is equal to 0. This being complex poles one thing I want that the direction at which the roots are going to depart should be known to me because there are three there are so many possibilities and out of those possibilities one of the possibilities should be set and this is one of the….. I may not be applying the rules in the order in which I have given you.
Now let us set the directions phi p. How do you calculate the phi p please see; this angle you will take up, this angle and this (Refer Slide Time: 18:49) and all these will be with negative signs so the phi p angle is going to be plus minus 180 degrees minus this angle minus this

angle minus this angle and the result available with me which may be wrong because after all it is rough graphical calculation you can check it but I give you the numerical values which I have got it is minus 71.6 degrees is equal to phi p. Applying the formula I have given you the phi p comes out to be this way and therefore now I know that if this is the angle I take, well, roughly this branch goes this way and by symmetry rule this branch goes this way. At least the angle at which they depart becomes known to me; it is minus 71.6 and by symmetry I am taking the other angle. Do raise the question to me please directly if there is any doubt.

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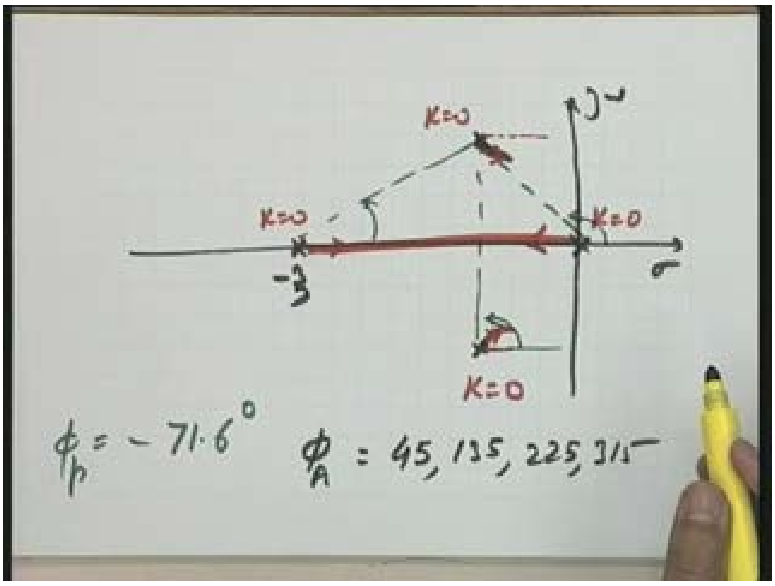

Now I was asking you as to what is the rough sketch you can make. Probably before you suggest the rough sketch let us see the angle of asymptotes and the centroid. From that point onwards am sure the rough sketch will automatically emerge in your mind. the angle of asymptotes formula you apply and give me, some of you might have calculated, please give me the angle phi A, yes anyone who has calculated, otherwise I write from my notes 45, 125, 225 and 315 these are the four angles you are going to get because n minus m is equal to 3, q will be equal to 0 1 2 and 3, I will give you the formula if you want. Therefore, phi A equal to (2q plus 1) 180 degrees over n minus m q is equal to 0 to n minus m minus 1. So this formula application of this formula will give you 45, 135, 225, and 315

[Conversation between Student and Professor – Not audible ((00:21:05 min))] n minus m is 3, n minus… 4 minus 1 4 minus 1 is…. yes yes 4 only yes I am sorry am sorry yes n minus m is 4 so n minus m minus 1 is 3 so q will be 0, 1, 2, 3 so four angles you are right and the four angles I have given you. Now let us see minus sigma A the centroid this is sigma real parts of poles. In this case no zeros so let me not write it, divided by 4 four poles n minus m is 4, now I will not make an error. Real parts of poles you can check: minus 1 minus 1 minus 3 so total is minus phi by 4 so you get minus 1.25.

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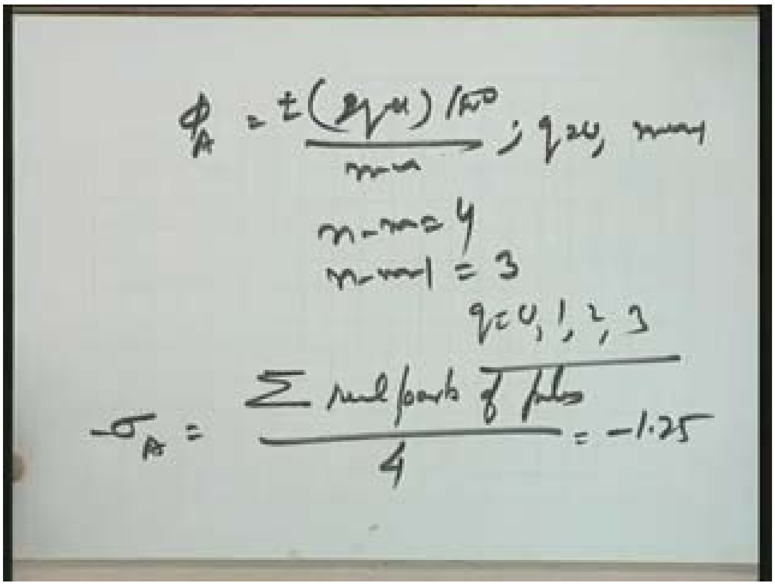

Now it means your asymptotes now are available at minus 1.25 (Refer Slide Time 22:07) somewhere here angles, yes, are you getting an idea now with these asymptotes before you as to what type of root locus are you expecting. After all these asymptotes are the four directions for the root locus branches, there is no zero available. So the four root locus branches will terminate on these asymptotes only. So, though you see it is to be proved but it appears that this root locus branch may go this way, this root locus branch this way to infinity (Refer Slide Time 22:47) and there may be maybe I say a breakaway point somewhere here so that one root locus branch goes this way and the other goes this way and the root locus plot is complete. This is intuitively this feeling comes. From looking at this particular picture intuitively it should be clear that maybe the root locus will go this way only and therefore to see that the next rule I have to apply is the breakaway point calculation.

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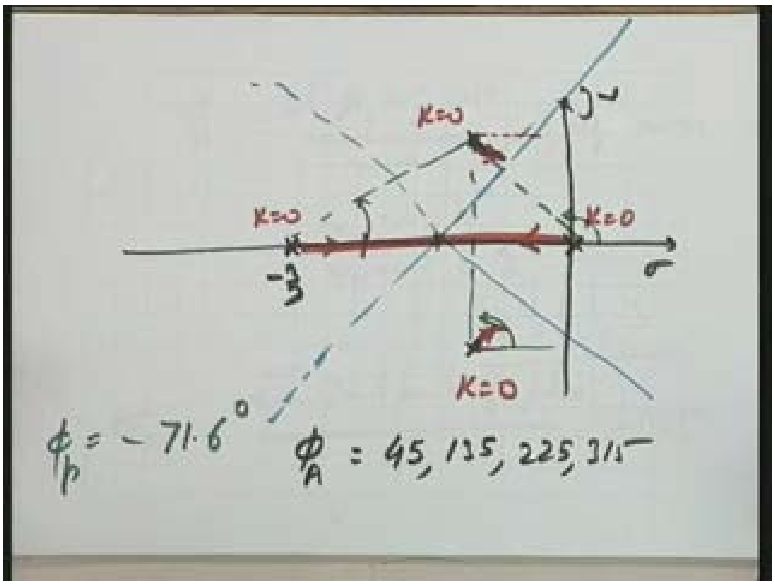

Therefore, if I calculate the breakaway point in this particular case you recall the relationship for the breakaway point this is your G(s) (Refer Slide Time 23:25) or F(s) whatever you call it, your breakaway point is dK by ds is equal to 0. What is K? K is in this particular case is minus sign you can take but minus sign is not important, s(s plus 3) (s squared plus 2s plus 2) this is your K. 1 plus G(s) equal to 0 will give you this value of K taking all other terms on the other side. Now you have to set dK by ds is equal to 0. So in this particular case you find that dK by ds will turn out to be a third-order polynomial. So, solving a third-order polynomial is difficult. So I hope with the calculator available with you you will be able to get a roughly a value of you see of one real value, you expect to one breakaway point you just see that.

From the root locus plot it appears you expect one breakaway point, the other two points which will turn out to be roots of the equation dK by ds is equal to 0 are hopefully not the breakaway points as it is intuitively clear from the root locus plot. However, if you are sitting on the computer terminal you will immediately check for the other breakaway points. But if you are doing it by hand calculations then this guideline is also good enough that you try for one real root of the equation and that location of the real root also roughly you know that it should be somewhere in this particular region and hence the breakaway point be calculated.

The answer available with me is for example the complete answer is minus 2.3 minus 0.725 plus minus J 0.365. You see that these are the roots of the equation dK by ds is equal to 0 which you find that this is the third-order equation. So this is one root of that equation minus 2.3 and these are the other roots and you can see now these two roots are not the breakaway points because they do not lie on the root locus branches. You can even apply the angle criterion on these two points; you will find that the angle criterion is not satisfied. You see, after all if we are making the rough root locus sketch we are not saying that, well, the computer is so costly that we cannot afford the computation.

The idea is this that we want to get a quick glimpse of the total problem. Even if there is a little error here then it does not matter because the final design definitely you are going to makes it using the CAD software. So a rough calculation is necessary so that when you see when you design it on the computer so there is no error there because the rough idea of the total problem is available with you. This concept will follow in the examination also please see. I will not worry about the very exact values so I will worry about the logical sequence you have followed as far as the analysis or design problem is concerned.

In this particular case for example you find minus 2.3 appears over here this is the breakaway point (Refer Slide Time: 26:34) and hence logically the root locus branches will go this way. In this particular case I have taken a little more comprehensive example. But from the point of view of illustrations simple example could have been taken where the calculations are less. But here all those points which I want to explain hopefully will become clear all the rules will come up in this particular example.

Sir, you said is not a breakaway point because it does not lie on the locus but that is only from intuition it could lie, yes that is what I said that by intuition…… because I expect that this will go this way. If you say it could lie the only check is this that take the point and apply the angle criterion. After all the point you have calculated my intuition says that it will not lie, if you do not agree quantitative evaluation is possible, take a point and on that because angle criterion requires all the open-loop poles only, take that particular point and apply the angle criterion and satisfy yourself.

Now these root locus two branches we have already calculated, go to the other two branches. As I said my intuition says though you can check that it should go this way. Now, to make a rough sketch without going to any calculation one point I know is the angle of departure. If in addition, I know this point, where it is going to cut the imaginary axis may be the rough sketch which I am going to make will be more accurate. And the point where it cuts this particular axis can easily be calculated using the Routh-Hurwitz criterion. Since I have already explained the Routh-Hurwitz criterion to you let me not redo it on this example let me give you the result. I find that as far as this point is concerned it is j 1.1 and corresponding K is 8.16 coming from my notes please.

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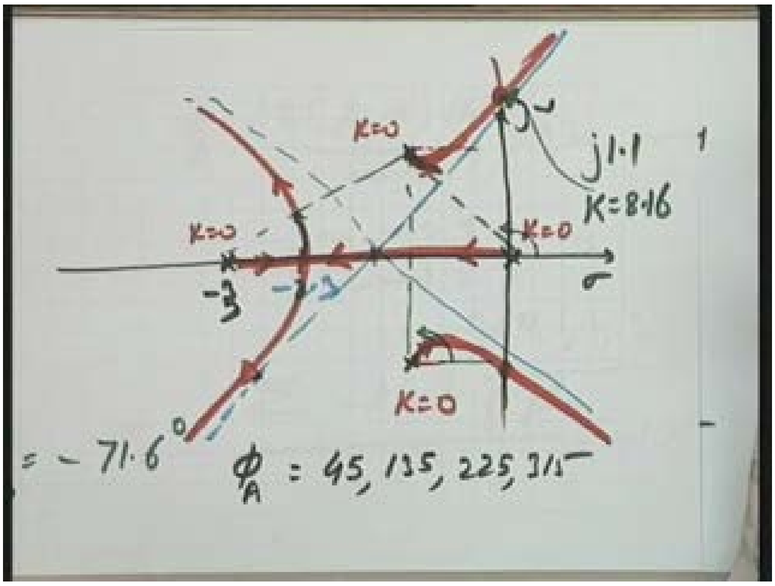

K is equal to 8.16 starting from K is equal to 0, K is equal to 8.16 makes it oscillatory and larger values of k will make the system unstable and the corresponding roots of the auxiliary equation which I have found are plus minus j1.1. So if this point is also available an angle of departure is available in that particular case a rough root locus sketch can be drawn.

Any question please?
Now if this is okay, yet my discussion on this example is not clear, not complete. Let me say how do I use this information. I have taken this as minus 1, this as minus 1, this as minus 1, this as minus 3 and let me say that these are the, there is no pole here sorry, this is the root locus sketch you have already made. but please note that you cannot take any quantitative information from this sketch unless the quantitative information is required only at these points for which you have made exact calculations otherwise it is a rough sketch, any quantitative information you want to take, you see you have to test the validity of the sketch by application of the angle criterion.

For example, I want you to give me the value of K, recall, the open-loop transfer function was K s(s plus 2) (s squared plus 2s plus 2). I want you to give me the value of K so that the closed-loop system has got damping equal to 0.6. Come on please, help me, how will you do it. you know that damping equal to 0.6 is a line corresponding to this zeta equal to 0.6 line it is called where theta is equal to cos inverse zeta. So you make a line in the s plane.

Now my design problem starts please. It is the design problem now I am making, it is a simple design problem, only gain adjustment, the PI PD other controllers I have not taken, only the amplifier gain can be manipulated and my question to you design question to you is this that give me a suitable gain K, adjustment will be done using amplifier so that the closed-loop system has a damping of 0.6 so the starting point is this that I draw a damping line, see the procedure please, so this damping line intersects this particular root locus at this point so this is a candidate as far as the root locus point is concerned but not necessarily a root locus point because I really do not know whether this point satisfies the angle criterion, it is surely a candidate because a rough sketch was made. Now, how to check whether this is a point or not. So, in that particular case at this particular point I will apply the angle criterion.

If the total angle contribution at this particular point due to all these poles is an odd multiple of 180 degrees in that particular case this point fortunately is a point on the root locus. If it does not then your rough sketch does not pass through this point and you have to make an adjustment in your rough sketch so that this point or a point close by is a point on the root locus.

Where do you try?
You cannot try here and here because you want zeta is equal to 0.6 so it means your trial of points is dictated by this line only. So you take a point here or here depending upon whether angle is plus or minus, whether you want to increase or decrease the angle that will become clear when you make the root locus sketch from here when you apply the angle criterion. So your trial point is along this line because you want a point on the root locus for which zeta is equal to 0.6. So make a trial point and see that the angle criterion is satisfied. So let me save time over here; let me assume that all the trials have given you this particular point and now let me go to my notes.

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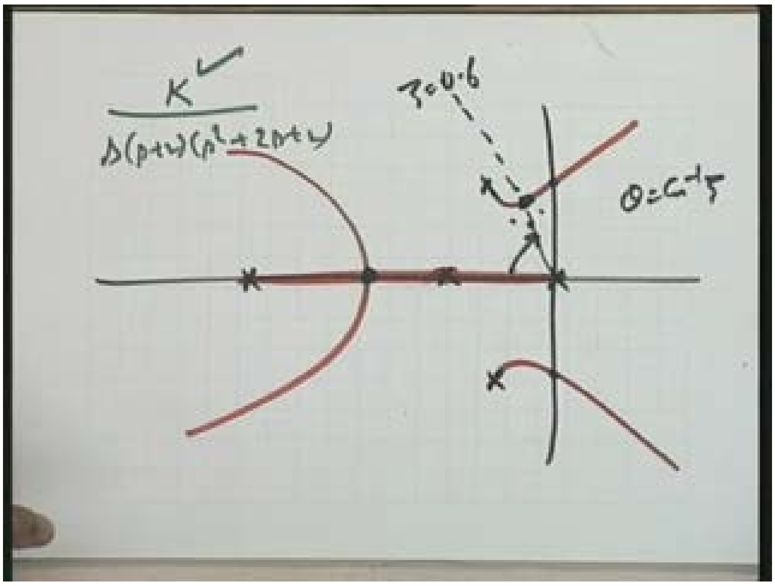

This particular point, well in my notes the calculations have been made for zeta is equal to 0.5 the point is, this particular point is minus point 4 plus minus j 0.7 plus minus j it means these are actually the closed-loop poles of the system. See this particular point: zeta is equal to 0.5 this particular point gives you closed-loop poles of the system and the corresponding value of K as per my calculations is 2.91. Help me please how do you calculate k, I have already given you the idea. The K calculation will be done by applying the magnitude criterion. At this particular point this distance into this distance into this distance into this distance is the value of K because there are no zeros (Refer Slide Time: 33:54). In general, K is equal to s plus p 1 magnitude s plus p 2 magnitude divided by s plus z 1 magnitude s plus z 2 magnitude and so on.

Now this is the point under consideration so s plus p 1 is this distance drawn to scale the scale of the graph which you have already taken. So do not do this calculation otherwise you can do these calculations, you substitute your s value over here and do it analytically but graphically it will go better. After all you have to spend this much of time in analytical calculation better you go to the computer. So graphically a quick answer can be obtained by taking this value into this value into this value into this value. Since there is no 0 here the denominator will not have any value so denominator is 1 otherwise in the denominator corresponding magnitudes would have come so the value of K can be calculated over here and as per the nodes here the value of K turns out to be 2.91. So the value of K is 2.91 and the corresponding closed-loop poles are minus 0.4 plus minus J 0.7. Now this corresponds to the value of zeta equal to 0.5.

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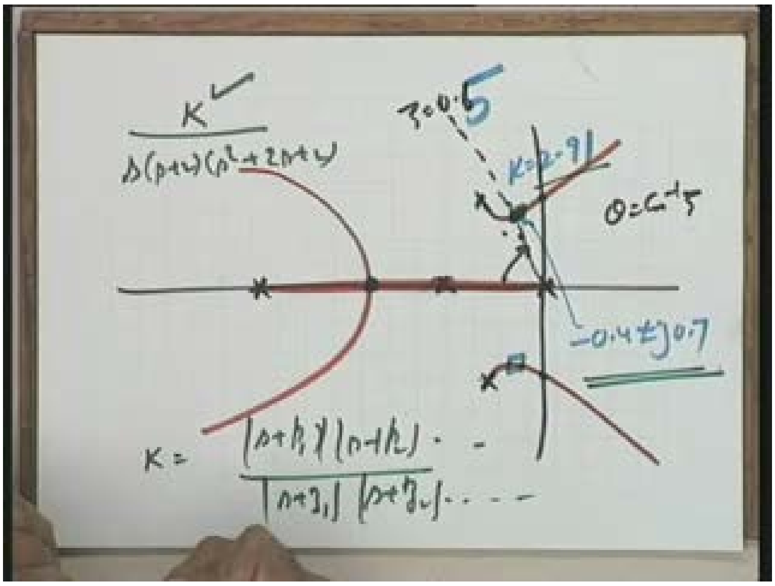

Now, under consideration as I told you it is a comprehensive example. Well, it has all the ingredients which are needed in a root locus design. So do not worry about the complexity. What are the other things? The other things are finding the other two closed-loop poles; after all it is a fourth-order system. You have found the two poles and you know that the zeta is equal to 0.5 has a meaning only when the other two closed-loop poles are insignificant that is these two poles are dominant in nature and for that you require the location of the other two closed-loop poles.

How to determine the other two closed-loop poles?
Please see, one of the poles is going to start this way, one of the root locus branches, the other root locus branch is going to start this way. So as K increases one root locus branch is going this way and going this way and as K increases the fourth root locus branch is going this way.

So it means you want two points on these two root locus branches which correspond to K is equal to 2.91 because K is equal to 2.91 is the value of interest to you. So it means now you think of the procedure; you want two points on these two branches which correspond to K is equal to 2.91.

Using a computer you will immediately do it but by graphical methods it is trial and error. You see that you can take a point over here, apply magnitude criterion on this particular point and calculate the value of K and by trial and error find a point which roughly gives you the value of K is equal to 2.91. So it has to come graphically through magnitude criterion only. But you see, since very rough values are acceptable in the design so this is not going to take lot of time and you can hopefully make these calculations and the value available with me is minus 1.4 and minus 2.85, these are the two values please see. So it means the two roots are the real roots corresponding to K is equal to 2.91. It does not go to these branches corresponding to these roots. This was your 1 value, this is minus 1.4 a pole over here.

How much was the cutoff point please? I do not remember, should I take it this way? Naturally I have to take it this way so here it is minus 2.85. So you see that for the closed-loop system these are the four poles and you find that your dominance condition is not satisfied because the real pole the real portion of the complex conjugate poles is 0.4 and the other poles is 1.4 it is roughly three times away and you want roughly four to five times away, so naturally this total analysis will come from simulation. But you have a rough idea that whatever peak over showed you expect corresponding to zeta is equal to 0.5 the actual over shoot will be slightly more than that because of the presence of this. This will be different than that not more you see.

What will be effect of the pole please?
It depends on what is the effect, it depends upon the relative residues; as far as the 0 is concerned it gives a pronounced peak but a pole does not necessarily give a pronounced peak it depends upon the relative residues so I make a statement that the peak overshoot corresponding to zeta is equal to 0.5 may not come up because of the presence of these nearby poles and hence it will be clearly shown by the simulation exercise only and you have to enter into the design cycle suitably so that the complete acceptance of the response of the system is guaranteed as far as user requirements are concerned.

So n this particular case I say that for K is equal to 2.95 the overall closed-loop transfer function has the poles and zeros like this 2.95 over (s plus 1.4) (s plus 2.85) (s plus 0.4 plus j 0.7) (s plus 0.4 minus j 0.7) these are the closed-loop poles of the system corresponding to K is equal to 2.95 which you have taken corresponding to zeta is equal to 0.5 value.

[Conversation between Student and Professor – Not audible ((00:39:48 min))] this is what I said, probably you cannot, yes, you can provide it you have that much of patience and it is difficult to have that much of patience and therefore it is the rough calculation. After all you want to check the dominance condition. Even the dominance condition is not quantitatively known so it is a rough estimate that where does the pole lie. That is why I am making a statement that at all these exercise you may laugh at thinking that when computer can do all these for you why you should do it graphically or analytically. But really it is useful as a learning module.

Once you learn all these things through this particular process then you can appreciate, analyze the computer graphs very quickly and easily and the chances of errors are reduced there and that is the purpose. So I do not say that even, that is why I made statement, do not worry, from the examination point of view also you just outline the procedure very clearly. Now 2.91 you should not end up with for example, 0.9 that will be too approximate; 2.91, if you end up with 2.5 or 3 well, it is fine, it is fine 2.53 even 3.2 does not matter but you should appreciate, you should understand and you should say that because for want of time you are giving the procedure.

I think with this example all the rules probably have been given to you very clearly and let me take couple of more examples and then the next lecture will be completely on design. The other example that I take is the following: K A s(s plus 2) it is a tachometric feedback, you can imagine this to be a position control system with tachometric feedback. There are two variables I am taking K A the amplifier again and K t the tachometric constant, r here, y here. I want to design this system for a specified value of zeta for example.

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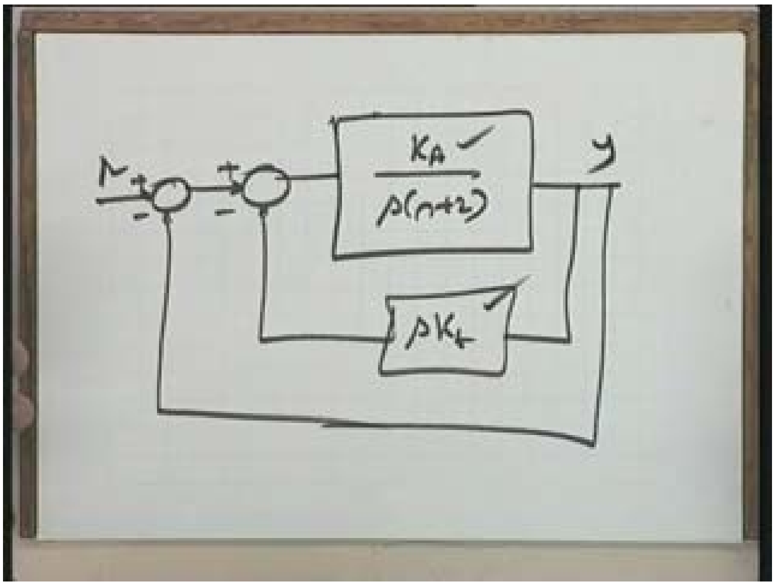

Resolving this into a single loop configuration you see I can easily get this diagram; please help me whether this is okay, well, let me write it separately; it will be K A over s(s plus 2) divided by 1 plus s K A K t over s(s plus 2) so this gives me G(s) over 1 plus G(s) H(s) this gives me K A here divided by s squared plus (2 plus K A K t) s I hope this is okay; in case of error please do help me; y r I have resolved this into a single loop configuration; inner loop has been resolved and resolving the inner loop I get this as the equation K A over s squared plus 2s this s the damping term I have added to this (Refer Slide Time: 42:57) so that this becomes the overall configuration of the system.

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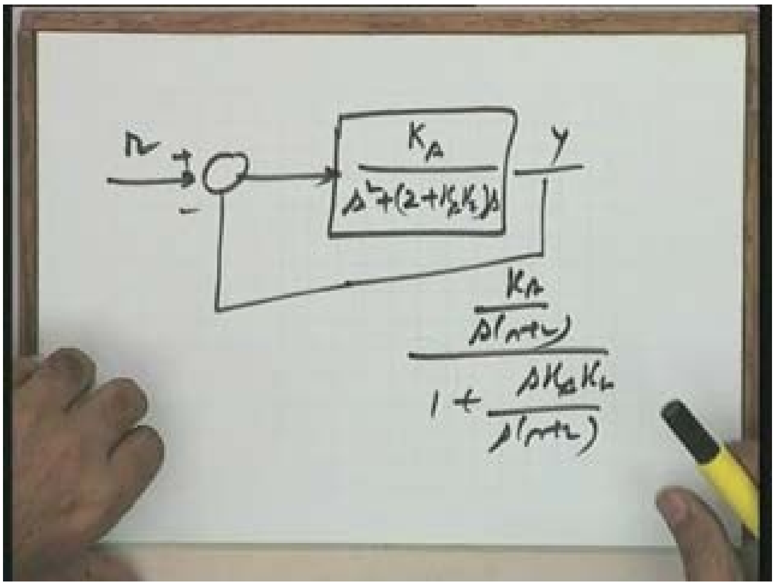

What is the characteristic equation?
The characteristics equation in this particular case turns out to be: s squared plus (2 plus K A K t) s plus K A equal to 0. Please note this point now. I have given this idea to you earlier that poles and zeros of the system may be different than the poles and zeros of the open-loop configuration given to you.

In this particular case I put it in a different way s squared plus 3s plus K A plus K A K t s equal to 0. Putting it in this form 1 plus K A K t s divided by s squared plus 2s plus K A is equal to 0. If I am interested in studying the variation of K t I will like to put K t as a multiplier as far as the function is concerned. So you will see that this function has been formulated as 1 plus KF(s) equal to 0 where your K is the root locus gain which is equal to K A K t. Assuming that K A the amplifier gain is known to me in that particular case variation of K t can be studied with respect to this particular equation.

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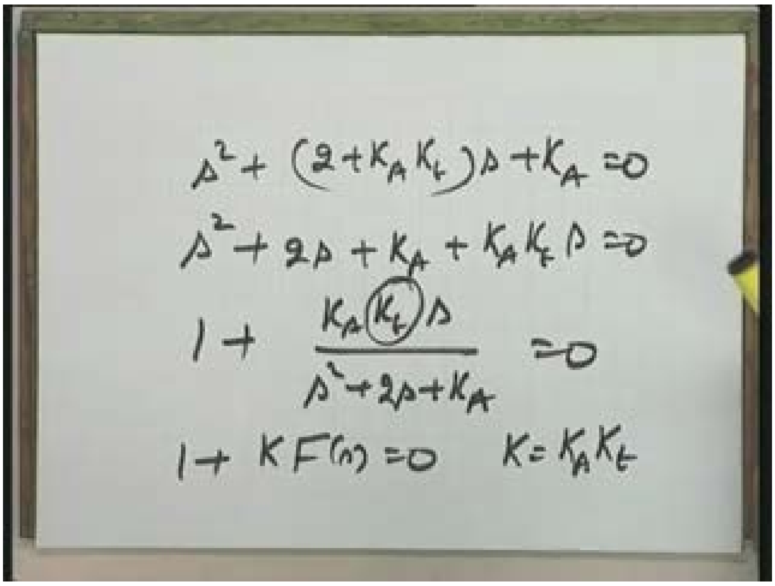

So you please note that 1 plus KF(s) is equal to 0 is an equation where K is the root locus gain and this root locus gain you have to bring out suitably as a multiplier and this is that value which you want to vary to study the effect of that particular parameter.

A question to you: K A is equal to 60.give me the value of K t so that damping is equal to 0.5. Come on; make an attempt please, quick. I write the characteristic equation again: 1 plus 60 K t s divided by s squared plus 2s plus 60 equal to 0. K A is equal to 60 the amplifier, I am taking one parameter at a time so the amplifier gain K A is fixed. In that particular case I get this as the suitable characteristic equation suitable for making the root locus sketch. I want you to give me the value of K t so that damping of the overall system is 0.5 so I need the root locus sketch for this. Now, poles and zeros are not necessarily the poles and zeros of the original system they are the poles and zeros of the root locus function. So, in this particular case you find a 0 at this particular point and there are two poles which are complex conjugate poles.

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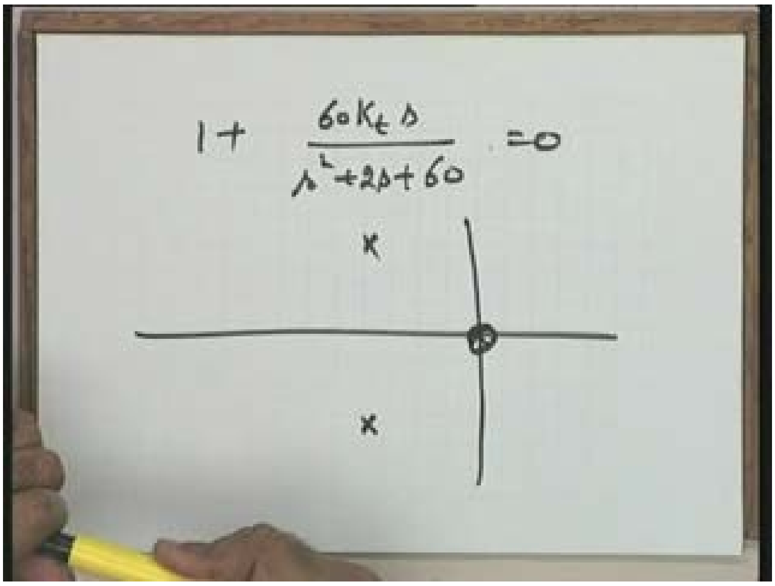

Now I leave this as a simple exercise to you. If I have the values in my notes I will give you otherwise the general sketch, I think let me give you the general sketch. This is on the root locus (Refer Slide Time: 45:51), the root locus in this particular case is going like this; here is the break end point these are two root locus branches. A simple exercise for you to prove that in this particular case the root locus branches complex root locus branches are in fact part of a circle. This I will leave it to you. But suppose this is drawn in this particular case what is the net exercise? The net exercise is to draw this particular line at zeta equal to 0.5.

Let me see if I have the result. Yes I have the result available zeta equal to 0.5. Now, corresponding value of K can be calculated and from that K the K t can be calculated, K t is equal to K A by 60, the result is K t is equal to 0.0958, for those who may try this example I am giving this result otherwise the procedure is good enough 0.0958 is the value of K t at this particular point. Remember that the K parameter is 60 K t and not K t you have to calculate K t, the root locus gain in this particular case is 60 K t the value of K A is 60 and K t can be calculated from this so corresponding to zeta is equal to 0.5 you can take this angle as cos inverse zeta you can take this point and from this particular point you can calculate the value of K and hence K t. So this design has been made.

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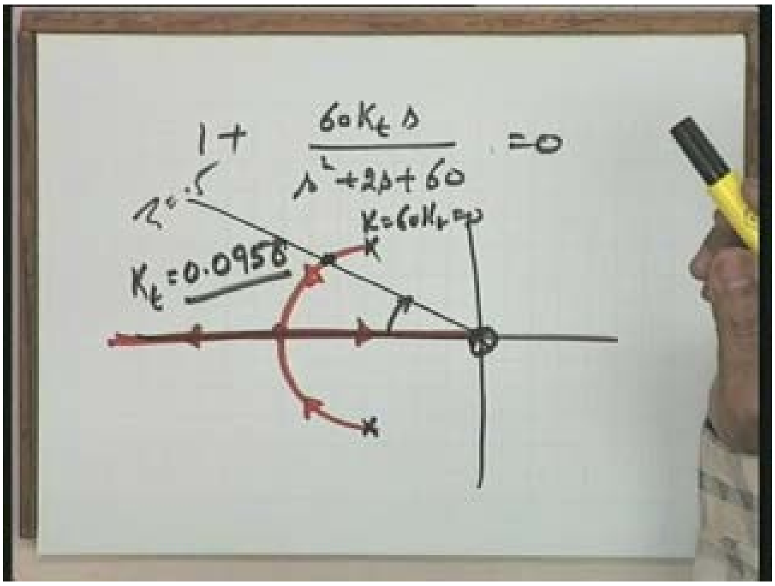

Now help me please; if I now change, any question please? I think it is analogous to the previous diagram rather it is simpler. Now my point is that if I may change K A to 100 after all I assume that K A is also a variable parameter if I make K A 100 can you tell me what type of root locus sketch you are going to get. I retain the earlier sketch also. Corresponding to K is equal to 60 what are the roots? The roots are here.

Corresponding to K is equal to 100 can you help me the real part of the roots?
Just change K is equal to 100; real part of the roots is going to be the same it is minus 1. Real parts of the roots is minus 1 so corresponding to K is equal to 100 the open-loop poles of the function F(s) are the following and therefore if the original root was like this corresponding to K is equal to 60 now the root locus diagram will appear like this corresponding to K is equal to 100 and hence the two branches now will appear like this: one is going this way and the other is going this way (Refer Slide Time: 48:43).

Now, if you want to investigate what is the value of K t when K A is equal to 100? Procedure is same, write the zeta line. the only thing is that now, instead of investigating this point you investigate the value of K A the root locus gain at this particular point and the value of K which you get at this particular point divided by 100 is the value of K t. So, if you have visualized, slowly I have given you a procedure of using the root locus for studying the effect of two parameters instead of only one parameter. So you see that initially it was only one parameter varying.

Now I have two parameters K A the amplifier gain and K t the tachometric constant varying. I find that for K A is equal to 1 value the two roots are here, for K A another value the two roots are here and the root locus branches are changing this way. So I can call these roots as root contours; instead of a root locus plot this I say root contours. You see that this each portion of this root contour is a root locus plot and for a particular value of one of the parameters the other parameter varies. So you can see that actually if you consider the denominator s squared plus 2s plus K A equal to 0 this particular line is actually nothing but the root locus plot of this particular equation you can check this please. So it means the open-loop poles of the function 1 plus F(s) is equal to 0 are in effect to the closed-loop root poles of this particular function or the roots of this particular equation s squared plus 2s plus K A equal to 0. Corresponding to K A is equal to 60 your two roots are here, corresponding to K A is equal to 100 your two roots are here and so on.

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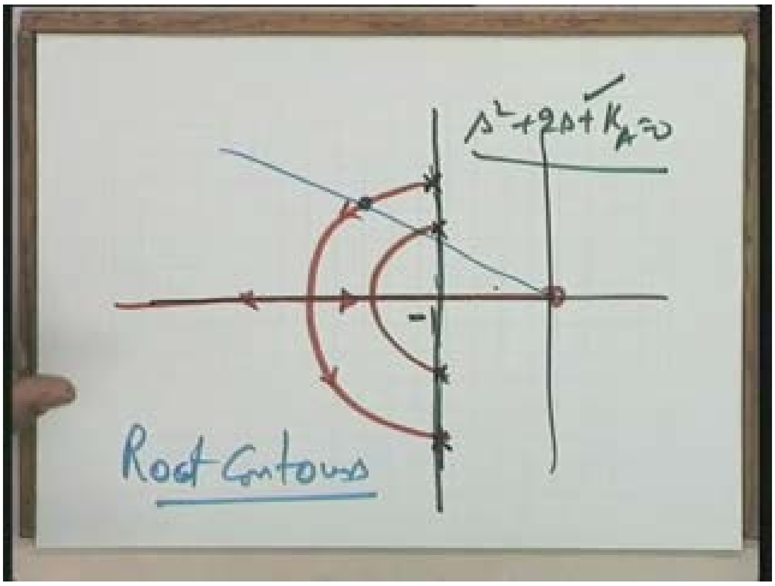

So what you have done in effect or let me take the next slide. If you have two parameters K A and K t 1 plus K A K t s divided by s squared plus 2s plus K A equal to 0. If you have these two parameters K A and K t the procedure now is the following. First of all you concentrate only on this denominator which is a function of only one parameter and you will note that the roots of this particular equation are the open-loop poles of the total function open-loop poles. So it means, first of all study, s squared plus 2s plus K A equal to 0 this is a simple function, it is a simple quadratic function so you could easily get the roots but if it is a complex function you can really put this also in a root locus format. This is 1 plus K A divided by s (s plus 2).

So, if I consider this s is equal to 0 this is minus 2 you know that the root locus is this as far as variation of K A is concerned though in this particular case I did not need this because this was a simple quadratic and a simple quadratic we could get the roots of this particular equation directly by solving a quadratic equation. But suppose this denominator itself is a complex function and there is only one parameter so first of all you can get the roots of this complex function with respect to the change of that parameter by studying this equation and transforming this equation in the format of a root locus equation.

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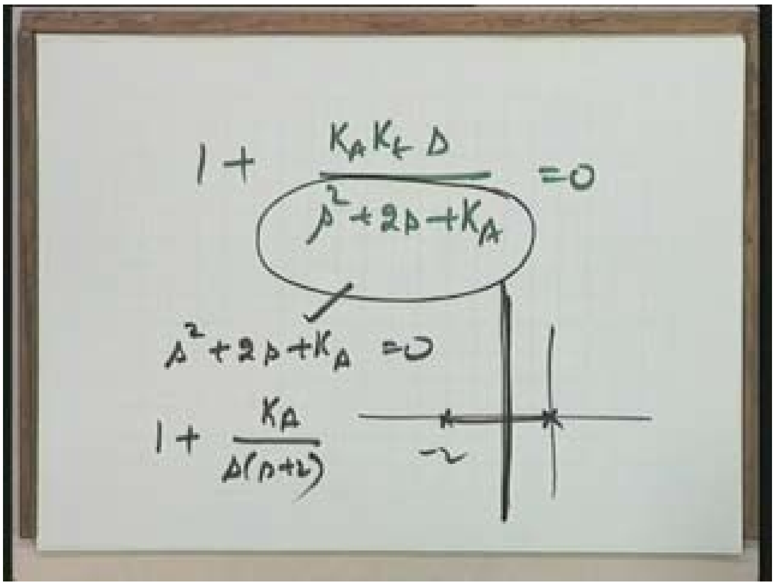

This is the root locus branch. Now once you have got this, this particular root locus gives you actually the open-loop poles for the complete function and then you go back to this, you take any two points of this, taking these as the open-loop poles you draw the total root locus branch, take these two points, taking these as the open-loop poles you draw the two root locus poles. You could take these also but normally you do not use you K A value and that is why coming to the practical values I am taking the open-loop poles on these lines and making the total root locus and therefore for various values of K A which give you the open-loop poles on this line your root locus your root contours are going to be like this.

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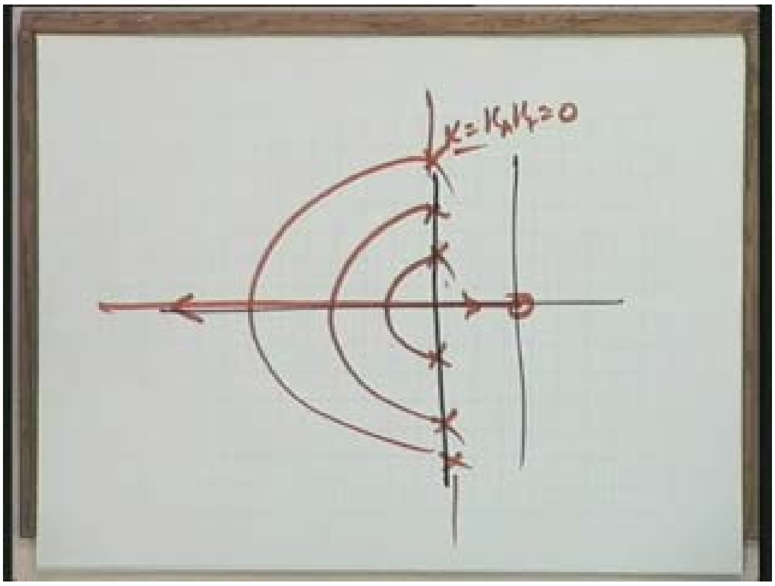

These are the root contours for various values of K A. For every point here K is equal to K A K t is equal to 0, for every point here K is the root locus gain which is equal to K A into K t but each point here or each pair here corresponds to one particular value of K A because this total vertical line is the root locus of the denominator equation as K A is varied so that way we can make the root locus contours of a system and this gives us the method of studying the effect of more than one root locus branch or one parameter on the root locus plot of the system.

The last point I have a minute and a simple point. Let me take G(s) is equal to K e to the power of minus s tau D divided by s. It is the case of a dead time equation. Now you see that whatever method you have seen the dead time has not come anywhere. So what you can do is the dead time can be approximated by the Pade's approximation so this can be written as (tau D by 2 s) divided by (1 plus tau D by 2 s) this is the approximation.

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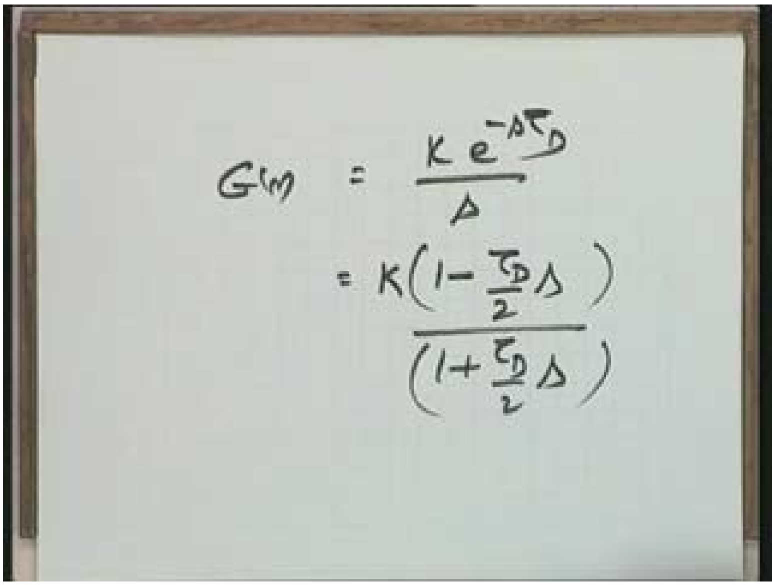

Now, if you see your pole-zero pair, please see, this can be written as minus K [Conversation between Student and Professor – Not audible ((00:54:48 min))] by s, yes naturally yes the pole is there, yes this is the function now. Now you see that bringing it in the standard pole-zero form I can write this as this is equal to (s minus 2 by tau D) divided by s (s plus 2 by tau D).

You will please note the interesting feature in this particular case is the following. Your G(s) is equal to minus K(s minus 2 by tau D) divided by s(s plus 2 by tau D). So you see that if you write your function it is 1 minus G(s) equal to 0 because of this minus sign over here. So what is your angle and magnitude criterion if the minus sign appears outside? The magnitude criterion remains the same G(s) magnitude is equal to 1 but the angle criterion becomes angle G(s) is equal to plus minus 2q into 180 degrees. Now you see it will become an even multiple of 180 degrees because G(s) is equal to 1 is your equation, please see. Your equation now becomes G(s) equal to 1 earlier it was F(s) equal to minus 1. Because it was minus 1 the requirement was odd multiple of 180 degrees; with this minus sign coming outside it is an even multiple of 180 degrees and if you take even multiple of 180 degrees the rules will change now 2q so wherever (2q plus 1) appears in the rules it will become 2q because now it is an even multiple of 180 degrees.

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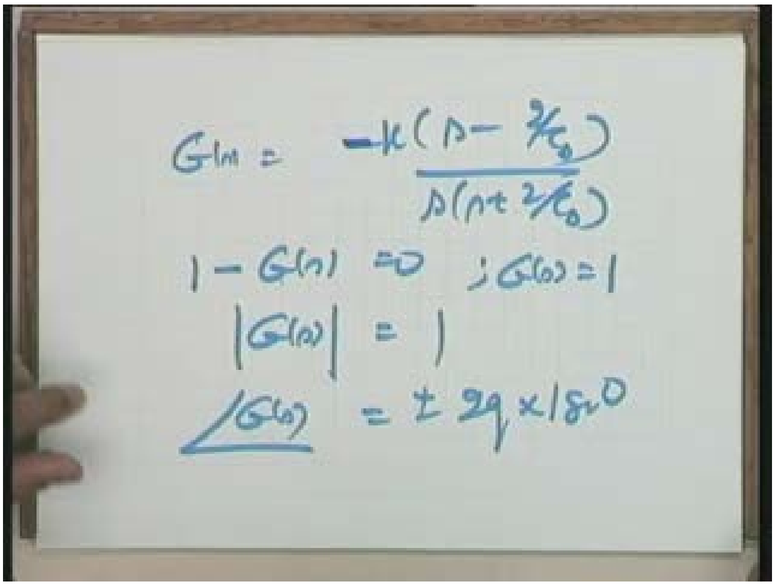

Give me the root locus sketch for this example please. I am sorry for delaying you a little. s is equal to 0 over here, pole over here, this is 2 by tau D here (Refer Slide Time: 56:38), this is minus 2 by tau D over here. Help me please how do I make the root locus sketch in this particular case. Tell me whether this branch is there on the root locus or not? Apply the rule. no as per the previous formula, please see; (2q plus 1) changes to 2q but now because of the negative sign you apply the angle criterion you will find that all those segments which are there on this with even number of poles and zeros to the right are the branches on the root locus. These are the only changes you are going to make. One: this real axis segment search and the other, in all the rules (2q plus 1) is replaced by 2q there is no other change, all the rules will exactly remain the same but for these two changes the real axis segment search and (2q plus 1) being replaced by 2q.

So, if real axis segments search is changed in this particular case you find that this segment is there on the root locus, this segment is there on the root locus (Refer Slide Time: 57:43) and therefore your root locus branches are going to look like this. This is going this way, this is going this way, one is terminating at infinity, the other is terminating at infinity on this side, and I hope this is clear.

Just see; one is going this way going to 0, the other is going this way going to infinity; I again repeat the only change is this in terms of real axis segment search and replacing (2q plus 1) by 2q whenever there is a minus sign associated with it and this minus sign associated with it is an example that of a dead time which always gives you this particular value when you approximate it by a rational function.

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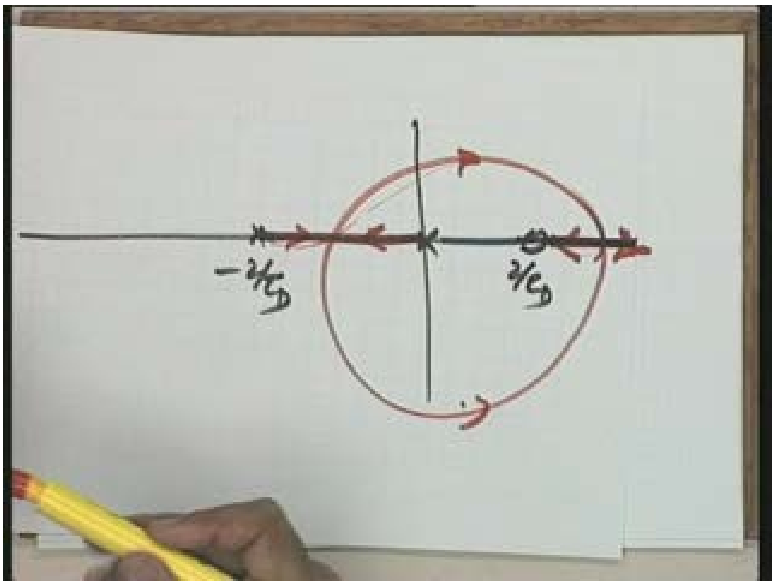

Thank you very much.