Control Engineering
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Lecture - 31
Compensator Design using Root Locus Plots (Contd….)

Let us revisit the equation first we had taken for the root locus plot. Today's discussion will primarily be concerned with the guidelines for sketching the root locus, its interpretation we will see in the next class when we take up the design. If you recall, in the last lecture, I said that any problem root solving problem wherein the roots of the characteristic equation are to be seen or to be viewed as a parameter is varied from 0 to infinity could be represented this way: 1 plus F(s) equal to 0 where F(s) can be put in the form k pi i equal to 1 to m (s plus z i) these are the zeros and pi j equal to 1 to n (s plus p j). Any problem could be put it this form and just let me say that this k is nothing but the root locus gain I will call it, root locus gain.

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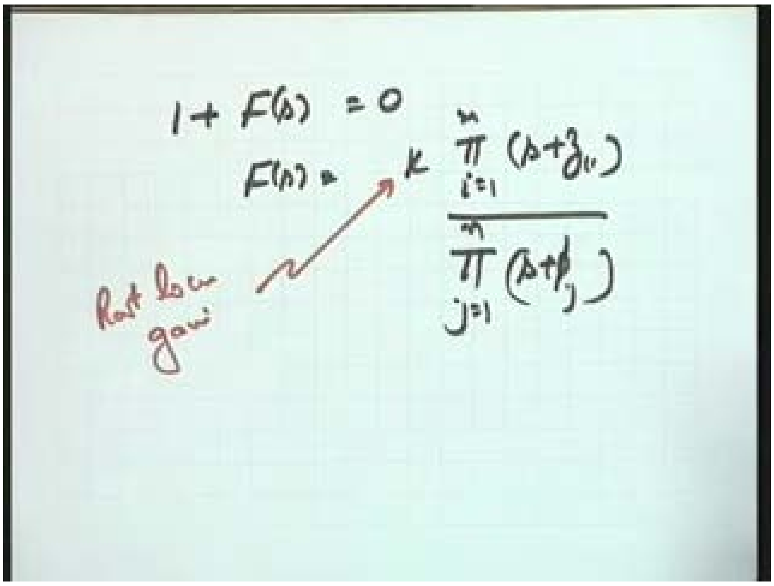

To give you typical examples of applications let us see how the real life problem will be transformed into this particular form. A simple example we have already discussed could be that of a plant, let us say plant is 1 over s plus 1 into s plus 2 this is your plant model and the parameter under consideration is amplifier gain K. In that particular case you see that the closed-loop system this closed-loop system has a characteristic equation which is given by 1 plus K over (s plus 1) (s plus 2) equal to 0. So naturally as far as this system is concerned your F(s) can be identified as this quantity which consists of an amplifier gain K and the plant transfer function 1 over (s plus 1) into (s plus 2). So naturally you find that in this particular case the root locus gain is nothing but the amplifier gain; I need your attention please at this point, the root locus gain K is nothing but the amplifier gain and the open-loop poles of the function F(s) are same as the open-loop poles of the closed-loop system. These open-loop poles are at s is equal to minus 1 and s is equal to minus 2. So you find that this particular equation which is the characteristic equation which gives you the closed-loop poles of the system has been put in this particular format.

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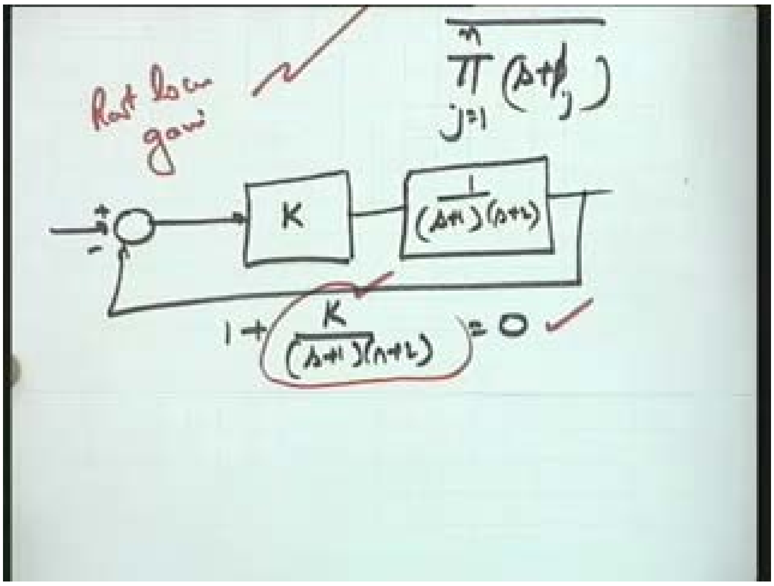

Take another example; other example I am taking is that of a plant with this as the model 25 over (s plus 1) (s plus 2). Let us say that there is a feedback alpha into s, you know that in a position control system it is a tachometric feedback so it consists of the minor loop here (Refer Slide Time: 4:03), the major loop being a unity-feedback.

Please see, in this particular case help me please what if F(s) function. First let me resolve the minor feedback loop, if I resolve the minor feedback loop I have 25 over s squared plus 3s plus 2 divided by 1 plus 25 alpha s over s squared plus 3 s plus 2 this is equal to 25 divided by s squared plus 3s plus 2 plus 25 alpha s. This is once I resolve.

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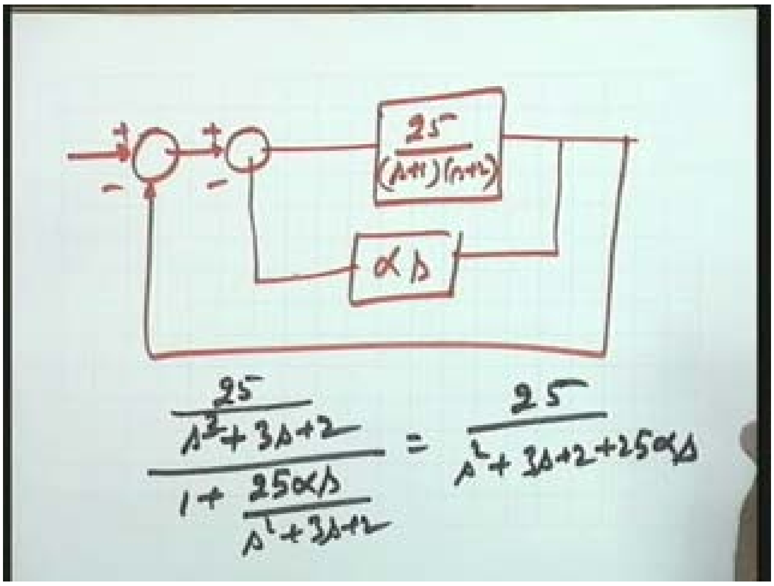

I need your attention on this point please; that k, what is the root locus gain I am trying to explain to you. In the earlier example it was simply purely an amplifier gain and the poles and zeros of F(s) were same as the poles and zeros of the open-loop transfer function of the given system. I need your attention at this point now where I have taken a tachometric feedback and with this tachometric feedback resolving the minor feedback loop this I get as the forward path transfer function. Help me please; what will be the characteristic equation of the system? The characteristic equation of the system you can easily see is s squared plus 3s plus 2 plus 25 alpha s plus 25 equal to 0 look at your notes please. You will find that this is the characteristic equation of the system.

Now, what is the parameter which is under variation? It is the alpha, it is the tachometric constant, it is the damping factor which is under variation. Though you see that if you are studying the effect of alpha using a computer you will simply find the roots of this equation as alpha is varied from 0 to infinity or whatever range of alpha you have decided depending upon the practical hardware. Depending upon the hardware you are going to use you will vary the value of alpha and look at the roots of this particular characteristic equation which are nothing but the poles of the system closed-loop poles. But if I am using the root locus method please see that the root locus method envisages that the equation be written in this form: 1 plus F(s) equal to 0 where F(s) is equal to K pi i (s plus z i) over pi j (s plus p j).

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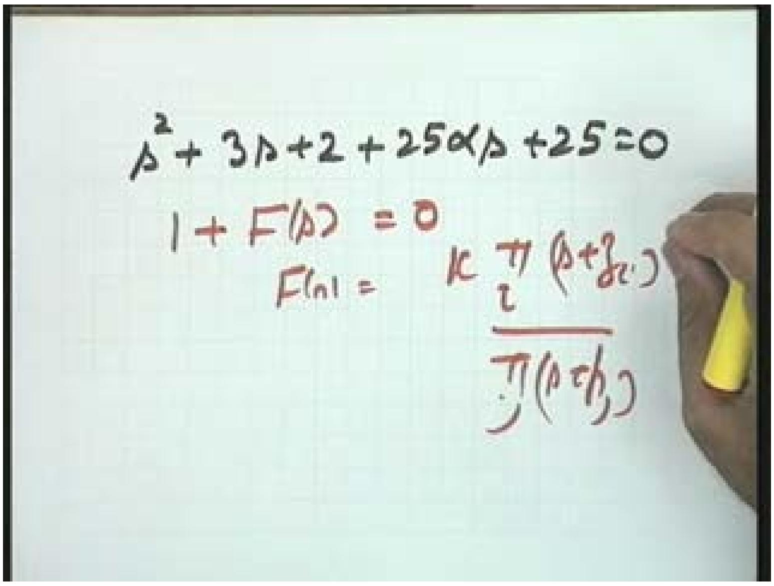

You will see that the root locus method wants that your characteristic equation be interpreted in this way though you have to ensure that the roots obtained from this equation are same as the roots obtained from the characteristic equation. That is the value of s which satisfies the characteristic equation must satisfy the equation 1 plus F(s) is equal to 0. So I really want that the equation I am studying should be put in the form 1 plus F(s) equal to 0. If you do it for this equation please see that 1 plus 25 alpha s divided by s squared plus 3 s plus 27 equal to 0 becomes your equation which could be written in the form 1 plus K (s plus z 1) where z 1 is 0. In this particular case z 1 is 0 divided by (s plus p 1) into (s plus p 2) where there are two poles that divides the second-order factor.

So you see that in this particular……. that is why I want to make it very clear that when I am taking F(s) the poles and zeros of F(s) are not necessarily the poles and zeros of the open-loop system. Now the 0 at s is equal to the…… 0 at s is equal to minus z 1 which is 0 that is at the origin and the two poles at s is equal to minus p 1 and s is equal to minus p 2 have no interpretation in terms of the open-loop poles and zeros of the system under consideration. you will please note; though these are open-loop poles and zeros of the F(s) function under consideration and the F(s) function we have formulated in such a way that it takes this particular point this particular value and now this K (Refer Slide Time: 8:23) which I have referred to as the root locus gain is nothing but 25 alpha. if you vary if you make a root locus sketch as K is varied from 0 to infinity accordingly by a scale factor of alpha you can change the scale to alpha s by a scale factor of 25 you can get the numerical values of alpha from the root locus plot.

You please see that the root locus plot envisages that your characteristic equation is written in this form (Refer Slide Time: 8:49) and in this form whatever k comes over here I will call it a root locus gain which is not necessary because the gain word immediately it comes to one's mind that it is like an amplifier gain in a system. Amplifier gain is true but it is a very specific case of the total application of 1 plus F(s) is equal to 0. It could be any parameter you want to study but the only thing is this that your characteristic equation should be reformulated in such a way that the parameter of interest comes as a multiplier, this point is to be noted and that parameter of interest which comes as a multiplier will be referred to as the root locus gain.

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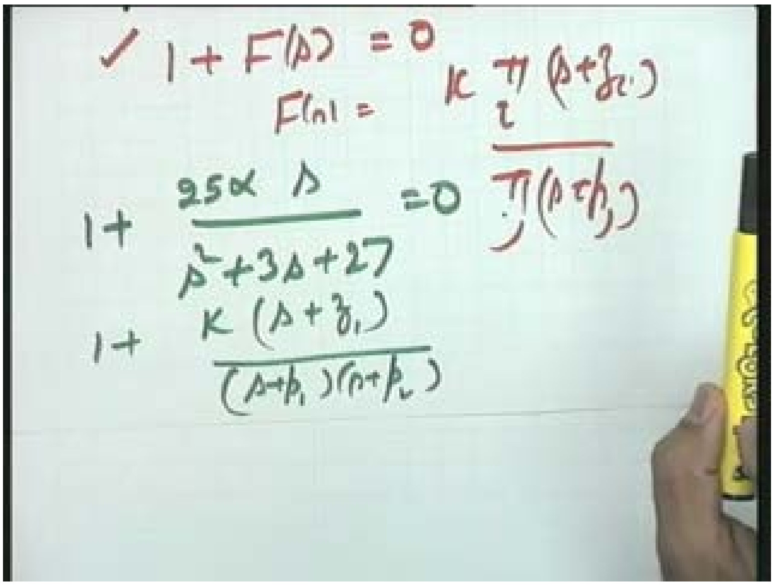

Examples we will follow; the design examples we will follow do not worry but when I am taking poles and zeros and the gain K you should keep in mind that it really does not mean that these poles and zeros are necessarily the poles and zeros of the open-loop transfer function though that is the case we very frequently come across. But there may be a situation of this type where these poles and zeros have been created so that the original characteristic equation can be written in this particular format. I hope my point is clear and the design examples we will surely follow on tachometric constant and all that.

I have a specific objective today and to complete that objective I immediately rush to this equation: 1 plus F(s) equal to 0 or 1 plus K pi i equal to 1 to m (s plus z i) over pi j equal to 1 to n (s plus p j) whatever may be the source of this equation I call K as the root locus gain not necessarily as the system gain, I do not call it as system gain and z is are the zeros of the function F(s) and p js are the poles of the function F(s) and I restrict my discussion to K greater than 0 which is the most commonly encountered situation. If the other situations come for K less than 0, well, the root locus methods can be identically applied on those situations as well. But let us restrict our system to a through K greater than 0 and secondly m of course will be less than equal to n because of realizability requirements.

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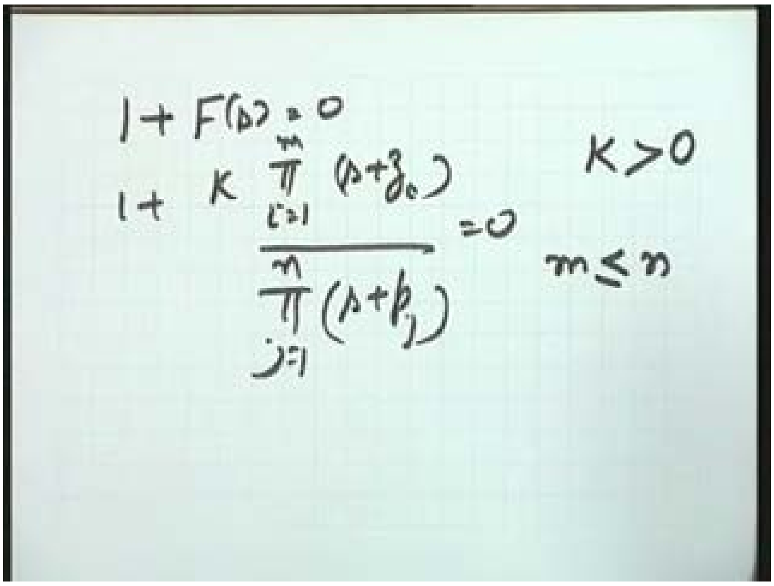

So this is the situation; I want the root locus plot of this particular system as K varies from 0 to infinity and you recall that equations available with me are pi i equal to 1 to m (s plus z i) magnitude divided by pi j equal to 1 to n (s plus p j) magnitude into k equal to 1. This is your magnitude criterion or magnitude condition as we have given.

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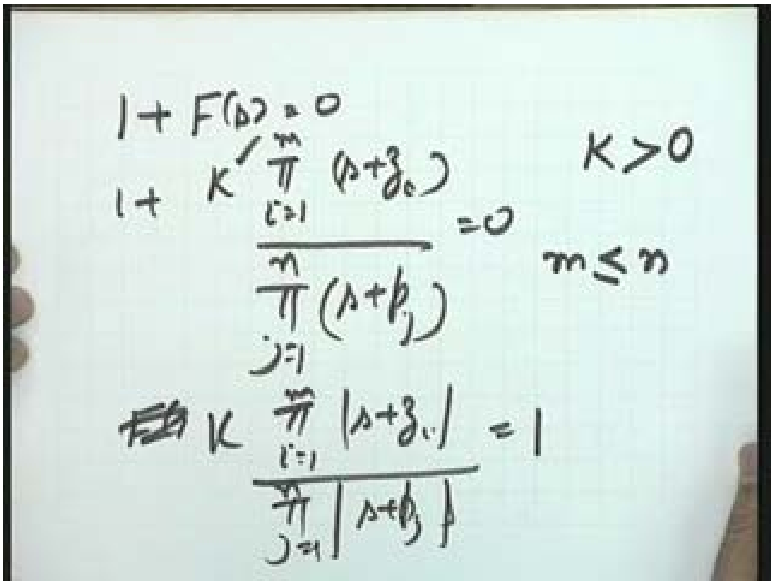

Now the angle condition is going to be sigma i is equal to 1 to m (s plus z i) angle minus angle sigma j equal to 1 to n (s plus p j) angle is equal to what…. is equal to an odd multiple of minus 180 degrees. So I have put it in this form plus minus (2q plus 1) into 180 degrees where q is equal to 0, 1, 2 and so on. This is your angle condition.

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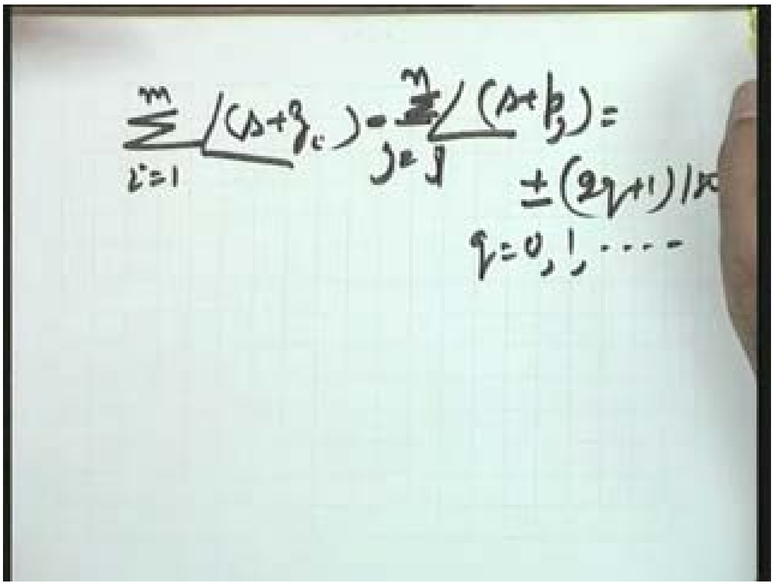

You have told me last time that magnitude condition is satisfied by any point in the s plane.
So our modus operandi is going to be like this: number 1: scan the total s plane where the angle condition is satisfied. Look at this point please. Scan the total s plane where the angle condition is satisfied. Join all those points with suitable graph suitable lines those are the root locus branches. If you scan the total s plane you get root locus branches because every point on these lines which you will make is a point on the root locus for some value of K. Because last time I hope you agreed to that every point on the s plane satisfies the magnitude condition. So you scan the total s plane for the angle condition and every point on the

branches so obtained will become the root locus point for some specific value of K so that gives you the total root locus plot as K varies from 0 to infinity, so one point.

The second point…. for example, at a particular point [Conversation between Student and Professor – Not audible ((00:13:22 min))] the magnitude condition you see, let me say that angle condition is this let me put the magnitude condition here K (s plus z i) magnitude pi i equal to 1 to m divided by (s plus p j) magnitude pi j equal to 1 to n equal to 1 this is my magnitude condition.

You consider any point in the s plane you see you substitute that value you calculate this magnitude, if you take K as inverse of this magnitude your condition is always satisfied. So it means there is a value of K for every point in the s plane on which you can calculate this magnitude there is a value of K for which this condition is always satisfied. But a root locus point will mean that both conditions 1 and 2 must be satisfied so I proceed like this: first I look at the angle conditions. You see, every point in the s plane does not satisfy the angle condition that you can check. So I scan through all the entire s plane and locate the points which satisfy the angle condition join all those points and that gives me the root locus plot because the magnitude condition will definitely be satisfied for any value of K for some value of K so that root locus plot which I will get by application of condition 1 can suitably be scaled with respect to the parameter K. So you can put the values of K all along the branches so that at every point for that value of K your magnitude as well as angle condition is satisfied.

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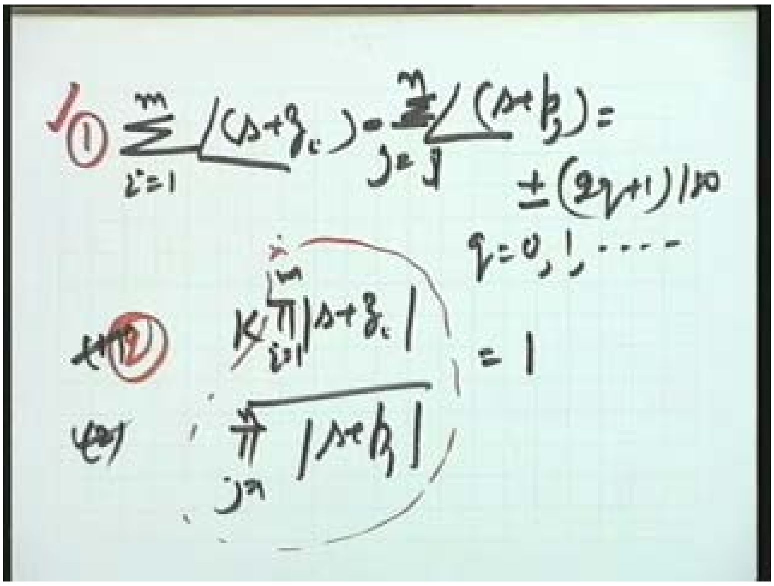

I hope you get my point I have to go a little faster because I have the objective of completing a particular portion today and therefore what I will do is I will miss the proofs of the guidelines of sketching the root locus. There are simple proofs you see given in your text book, in case of any doubt please bring the questions in the tutorial class. I will give you the rules without the proofs and we will use these rules through examples to make a sketch of the root locus that is how to satisfy the angle condition.

The question now is this that you scan the entire s plane so that angle condition is satisfied.

A brute force method, you can say, will be like this: (Refer Slide Time: 15:38). Let us say this is your minus p 1, this is minus p 1, a value of minus z 1 I am taking, another value of minus z 2 I am taking, I could take two poles here also minus p 3 minus p 4 these are the open-loop poles and open-loop zeros of the function F(s). I am making it very clear that I am making reference to the function F(s) and I have to scan the total s plane.

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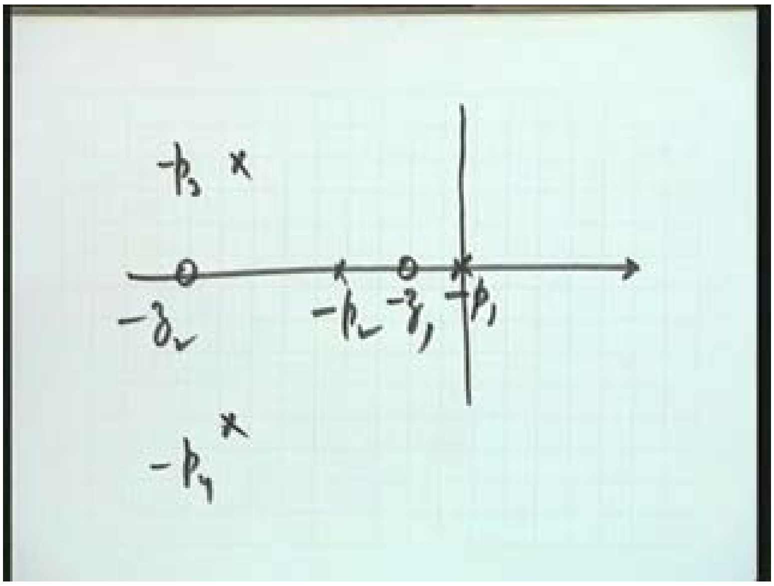

Now let me take this as the trial point. The trial point is s 0. I want to see whether this point satisfies the angle criterion, if yes, yes it is a root locus point because there will be an associated value of K and to do that please see this point I draw the vector from all the open-loop poles and zeros to this trial point that is why the graphical tools will be required, I draw all these vectors. You see that the angle contributed at the point s 0 is s plus p 1 this angle, this angle is s plus z 1, this angle is s plus p 2 and so on. How about this angle please? This angle you take this as the horizontal axis with respect to this you measure, this angle is s plus p 4 (Refer Slide Time: 16:57) and here is an angle s plus p 3.

So you see that you take all the poles and zeros of the function F(s) and draw the vectors to the point s 0 the trial point and with respect to these angles now you check, you just see that you sum up all the angles contributed by zeros, sum up all the angles contributed by poles the total angles contributed by zeros minus the total angle contributed by poles should be an odd multiple of minus 180 degrees. If it satisfies then you are fortunate that you have got a trial point at the first trial itself which satisfies the angle criterion. If it does not then depending upon the difference, depending upon the deviation you are getting from 180 degrees you can take your trial points here and there and satisfy this. Now you can definitely curse me if I say that this is the only method of root locus plot I will give you.

Instead of going for this method naturally you will like to go to the computer and make the computer give you all the roots this is the computer-aided graphing and all the computer-aided design tools are equipped with computer-aided graphing of the root locus today. But fortunately there are some guidelines which quickly give you an idea as to what should be the points in the entire s plane which should be tried for the angle criterion, which should be tried I say, does not mean that the guideline will give you the root locus plot straight away but the guideline will take you very quickly close to the actual points. Even if you do not get the actual points you see, even if you make a very rough sketch after all a basic idea of design you have got with paper and pencil you can definitely go to the computer for the complete and accurate design before the design is implemented.

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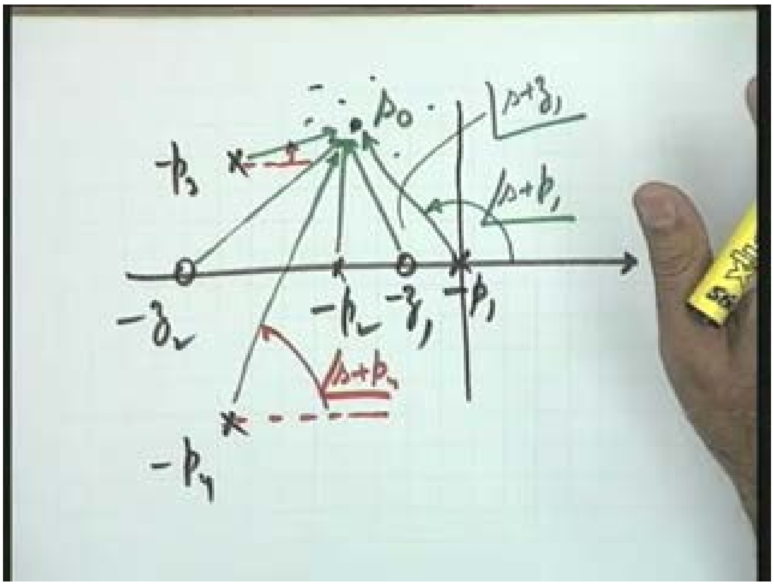

I will like to mention today that with the computer software available in the computer design control design direction it appears that all this paper and pencil exercises are useless because just you sit at the terminals for different values of alphas or K or whatever value you want to have it immediately gives you the root locus plot and you can really visualize the root locus plot and on the basis of the root locus plot available at the terminal you can definitely make a decision on the design. But still you see that to do that exercise this basic knowledge is required that is why we say that this classroom debate this tutorial debate is useful. The only thing is this that we need not worry too much as to how will we numerically get these values computer will help us, you have to qualitatively see as to how the root locus plot is interpreted that is more important in our discussion. If there is any problem, well, this calculation is very big, this calculation I cannot do in the examination, this calculation is difficult do not worry about that after all, the computer is available to do the calculations for you. The purpose here is this that the qualitative aspects of the design methods should be very clearly understood.

So what I am saying is this that the basic guidelines which I am going to give you will give you a very rough root locus sketch and the basic decisions about the design you should be able to make on the basic of the rough root locus sketch. For example, whether you will like to go for a PI controller or a PD controller or a PID controller or any other alternative maybe you can initially decide before you sit on the computer terminal and that initial decision is possible through the rough root locus sketch without making any calculations.

So I am giving some guidelines which normally do not require any calculation or very trivial sort of calculations and these are the guidelines in terms of some seven eight rules I am going to give you and using these rules you will be able to make a rough root locus plot.

As I mentioned that the proofs of these rules I am not going to give you because of want of time.

Rule 1 let me put it this way. Can I say that the rule 1 is a symmetry rule; any root locus you are making you write in your own language please, I will not write the total language over here. Symmetry rule says that any root locus plot you are making has to be symmetrical with respect to real axis because any complex roots whenever they occur they have to occur in complex conjugates in any real system then and only then your characteristic equation will have real coefficients.

So one rule is this that if you are making an upper part of the root locus part very accurately do not worry about the lower part you simply take the mirror image of that because this is going to be symmetrical with respect to real axis, rule 1.

Rule 2: the root locus gain you say varies between 0 and infinity. So I say that the K is equal to 0 is at open-loop poles of F(s). You see, it means all the root locus branches start at the open-loop poles of F(s); I hope this is fine. So it means this again should be clear that the number of root locus branches is equal to number of open-loop poles of F(s) that should also be clear. And secondly, K is equal to infinity so it means [Conversation between Student and Professor – Not audible ((00:22:53 min))] yes please? This step you see that how many if I have taken F(s), please recall we have taken m zeros and n poles, it is alright. So K is equal to 0 at open-loop poles and in addition I say that there will be n root locus branches because every root locus branch will start at an open-loop pole at K is equal to 0.

Since there are n root locus branches each starting from an open-loop pole, you have to give me a terminal point for every root locus branch please. The terminal point will naturally occur at K tends to infinity so the terminal points are number 1: m zeros of F(s), how about the remaining n minus m please? n minus m braches will terminate at infinity, in which direction we will see but they will definitely terminate at infinity. You have to observe these rules you see.

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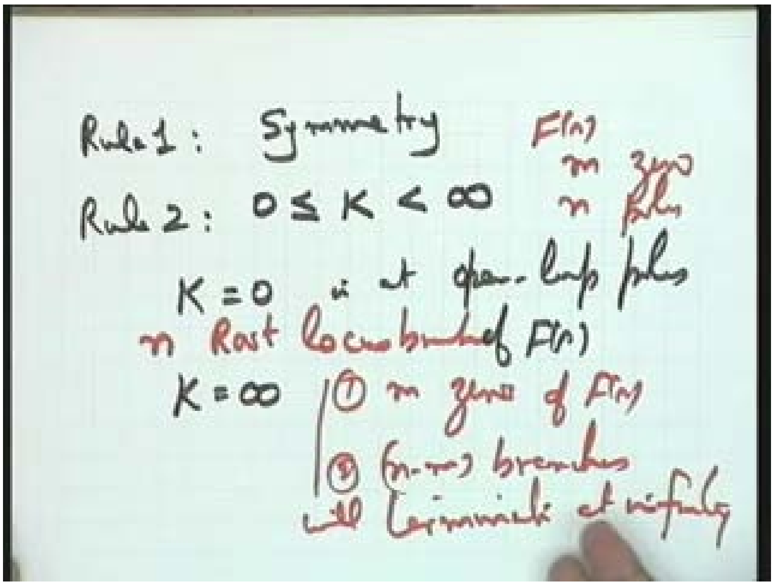

The proofs are very simple but I am just saving time. Look at your text or bring your questions in the tutorial class. I repeat here; there are total n root locus branches. The starting point of the n root locus branches are the open-loop poles of the function F(s) and the starting value of gain K is equal to 0 and the terminal point of the n root locus branches are m zeros of F(s) and n minus m infinity points in the s plane. Where are those infinity points, what are those directions of those infinity points we will see. But these are n minus m n minus m infinity points which we are going to consider, this is rule 2.

Let me go to rule 3. Any point you see if it is not clear you have to please do ask me. Rule 3 is the real axis segments. You are scanning the total s plane. So let me scan the real axis first out of the total s plane, the real axis. You see, any point on the real axis you may take, see this rule number 3 please, any point. If the number of…… I make a statement: if the number of real axis poles and zeros open-loop poles and zeros; if the number of real axis open-loop poles and zeros to the right of the plane is odd in that particular case this point lies on the root locus. I repeat it again or let me repeat it through an example.

Let me say that I have an example: there is a pole here (Refer Slide Time: 25:45), there is a 0 here, there is a complex sphere here, there is another 0 here let me take this particular sketch. Now, if I consider this point please see that and I consider the number of open-loop poles and zeros, number of real axis open-loop poles and zeros to the right of this point it is odd and therefore this particular point is a point on the root locus and hence this total segment is a root locus segment. Whether it is a complete branch or not we do not know but this particular segment will…. every point on this red segment will be a point on the root locus because it satisfies the requirement that to the right of every point of this red line the number of open-loop poles and zeros to the right is odd.

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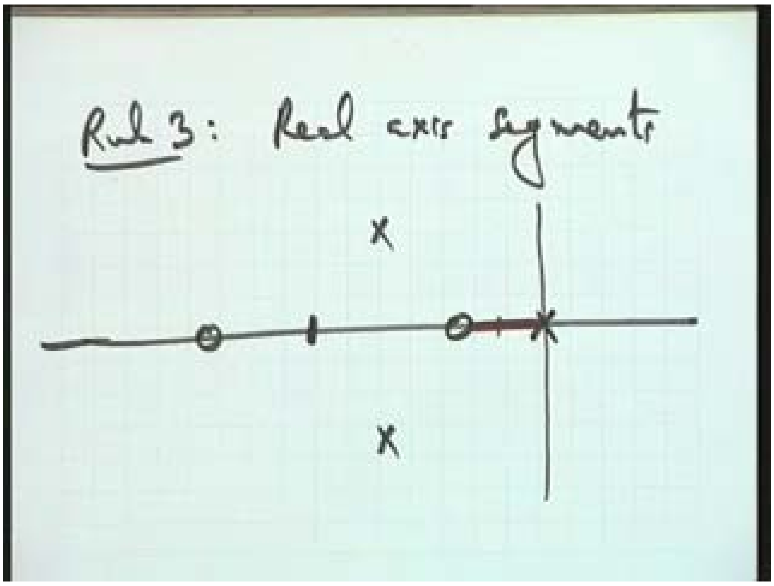

Look at this segment this total segment the green one. The green one if you find the number of open-loop poles and zeros is not odd and therefore this particular green segment does not lie on the root locus. Look at this particular segment again. It is again odd and therefore this lies on the root locus, this lies on the root locus. So it means and how about this? Naturally 0 is even so this also does not lie on the root locus as far as the right hand side is concerned. So it means out of the entire s plane the real axis has been appropriately scanned. It says that, you simply look at the count of number of open-loop poles and zeros to the right of that particular point and that segment will give you you can make a decision about the total segment by application of this rule.

Now, since you see this sketch is in front of me and I know rules all the rules let me just as a passing reference really give you the root locus plot of this also. But do not mind, no, I have not given you the complete rules but let me give you the root locus plot in this particular case how will it look like. You see that K is equal to 0 is my starting point. At all the three you put K is equal to 0, K is equal to 0 here, K is equal to 0 here there are going to be three root locus branches and there are going to be three terminal points: one terminal point is here K is equal to infinity, other terminal point is here K is equal to infinity. Do not mind this completion without giving me the rules.

You see that it appears as if one of the root locus branch is just complete because you are starting from K is equal to 0 and you are going to K is equal to infinity, well you have to prove it but by other rules it will become very clear. But at this juncture I may tell you that this gives you complete root locus branch because a starting point and a terminal point has been obtained and this entire thing lies on the root locus it cannot reach this point through any other root as you will see by other rules so this is one branch. Now you can see that this particular branch total thing lies on the root locus not necessarily it is a single branch, my statement is this total segment lies on a root locus plot and I did not make a statement that this constitutes a single branch of the root locus.

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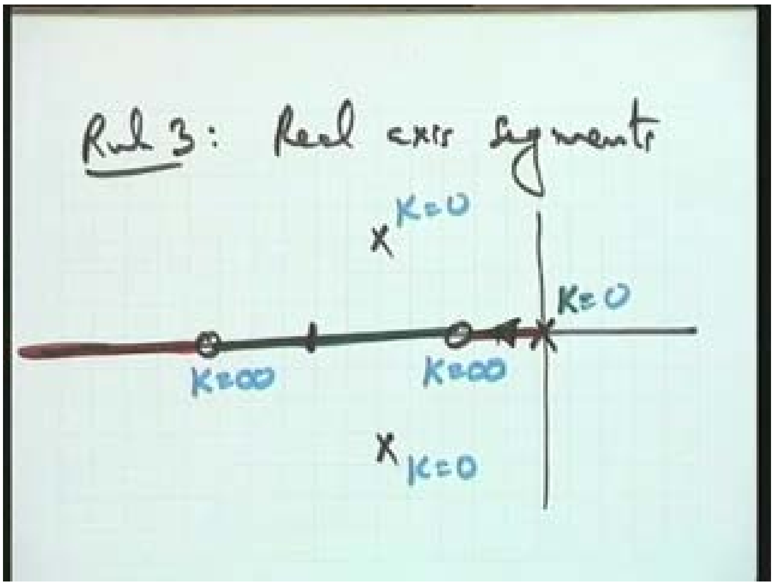

Any idea please, how to take up?
In this particular case you see that the root locus plot will appear like this: the branches will start from here, will go this way and will break off at this particular point. You see, the rules are yet to come but you see that K is equal to 0 this is the root…. the direction is going this way, another branch because every branch starts at K is equal to 0, it goes this way, when the two branches meet you see it corresponds to multiple root as you know so these are the two complex conjugate roots corresponding to these two branches, these are the complex

conjugates as K is increased further, complex conjugate roots as K is further increased (Refer Slide Time: 30:00) these become the repeated roots as K is further increased and this point we will refer to as the breakaway point. At this particular point as K is further increased the two branches break away into different directions: one is going this way and the other is going this way. The one which goes this way has a terminal point to infinity and hence the statement that there will be two terminal points given by zeros and one terminal point in this particular case is given by infinity and this becomes the root locus sketch.

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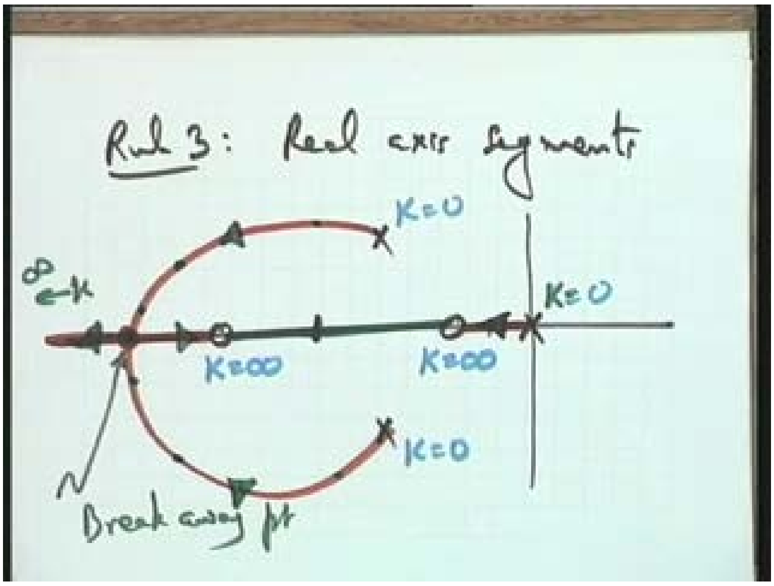

This is the breakaway point at which the two branches which were coming they meet and breakaway and the breakaway means after this particular point as you know the two roots become real and the system becomes over damped. This root is naturally a rare; it is a first-order term. So you can just see what is a closed-loop system. Till this particular value of gain at this particular K there is some value of gain K dash, till the value of gain K dash you have got two complex conjugate poles and one real pole and after this K dash all the three poles are real poles, the closed-loop poles I am referring to because the root locus plot gives you the closed-loop poles only. You have your starting point as the open-loop poles of the function F(s). The more complete rules when they come the diagram will become more clear.

Rule number 4 I go to now. Rule 4 says that, you see that, please see, you have made a decision, I can say the directions of asymptotes, asymptotes (directions) directions first of all. You see that you said that n minus m branches go to infinity. in this particular case you see (Refer Slide Time: 31:58) I gave you this as the infinity and it was obvious also but otherwise there are infinite ways of going to infinity and hence let us really come out to this point that what are the ways and in which directions it can go to infinity.

Please see, the rule number 4 says that it will go to infinity only in the directions given by this formula. It says that (2q plus 1) into 180 degrees divided by n minus m where q can vary from 0, 1…. how many please? It can vary to what extent, please tell me, what is the value of….. up to what value of q you have to take, that is how many angles do you require? [Conversation between Student and Professor – Not audible ((00:32:40 min))] yes please, you have started with 0 so angle is going to be n minus m minus 1 so you need n minus m directions and this particular formula is going to give you n minus m directions. So I will apply it on some formula some problem.

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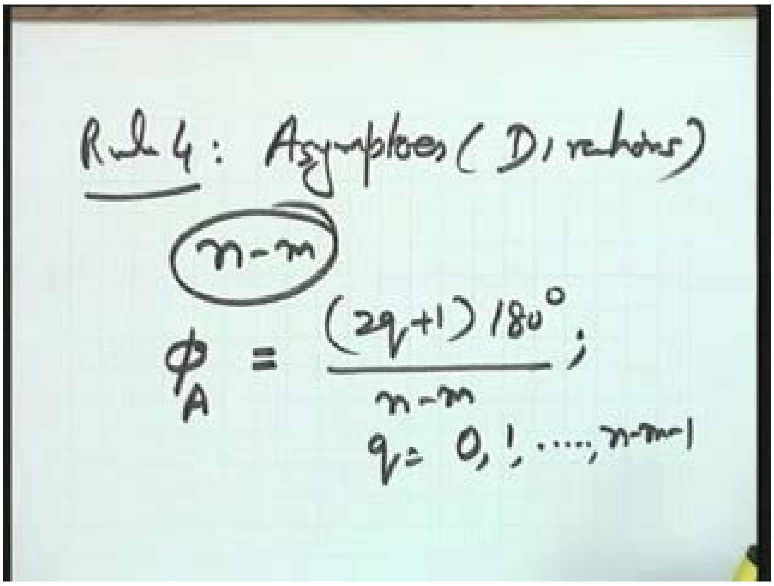

In this particular case for example if you apply you are going to get 180 obviously. One: you will get only one angle and that angle you are going to get as 180 degrees you can apply this formula.

Rule 5 let me say, you see that asymptotes you have already taken but what is the centroid of the asymptotes, from which point on the real axis you should draw these asymptotes? Directions? We have taken angles. But what is the point with respect to which the angle should be measured that particular point in the… has been referred to as a centroid. Centroid is a point on the real axis. From the centroid you are going to measure the angles. So this centroid is minus sigma A this is equal to pi of real parts of poles minus sigma of real parts of zeros divided by n minus m. This gives you the directions along which you are going to get the asymptotes, the direction are given by minus sigma A, the centroid is given by this particular point (Refer Slide Time: 34:18) and the angles are given by this.

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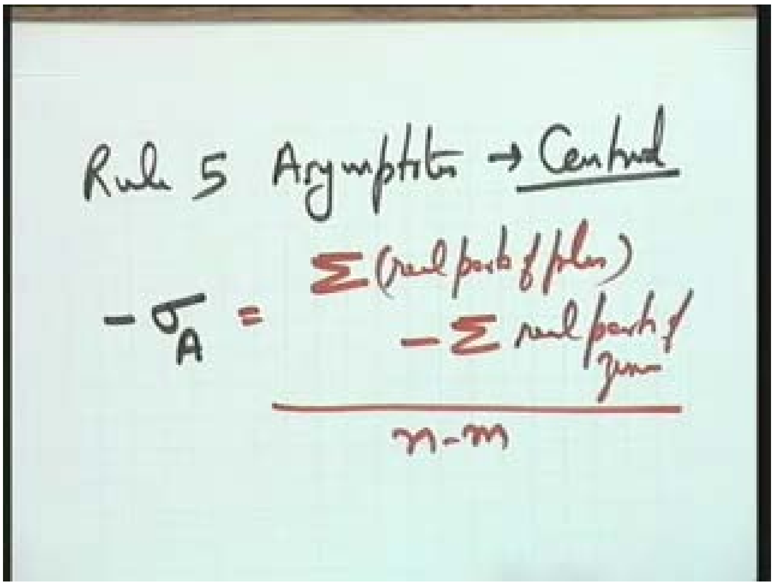

Now I take a numerical example. Well, quickly if you can make the calculations, fine otherwise I have a solution with me, I will take the calculation from my solution to really take care of the speed at which we go.

I take F(s) function is equal to K divided by s into s plus 1 into s plus 2. Set the value of n, n you find is equal to 3 and m is equal to 0 so naturally all the three root locus branches will go to infinity there is no zero terminal point available. Now what are the angles? The angles, please apply the formula and check, the angles in this particular case will turn out to be 60, 180, 240 degrees. Those of you who can quickly check you make your calculations. The minus sigma A you will get as minus 1 this if you need an elaboration I can give you this here (Refer Slide Time: 35:27) this is 0, this is minus 1, this is minus 2 so sum of the real parts of the poles it is minus 1, minus 2, minus 3 there are no zeros. So you have minus 3 divided by three which turns out to be minus 1 so sigma A is equal to 1 or the point at which the angles 60 degrees, 180 degrees, 240 degrees are to go is given by the minus 1 point.

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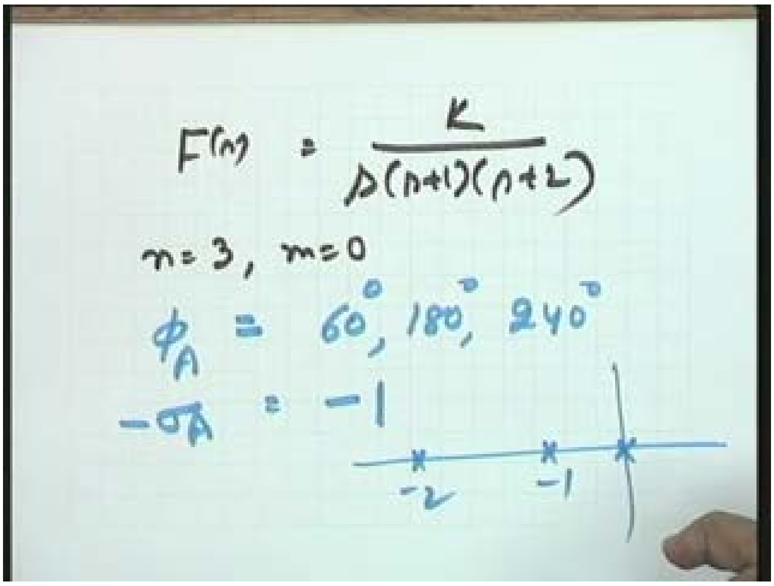

So let us make it, though the rules are not complete and I do not claim that on the basis of the available roots you can make a sketch but at least you will appreciate the sketch I make. This is minus 1 here, minus 2 here or you give me some help also if you can really intuitively feel as to what will be the sketch (Refer Slide Time: 36:12). K is equal to 0 at this point, K is equal to 0, K is equal to 0 the three axis minus 60 here and these are the three axis. So I think you might have got this feel that since this is the starting point maybe one of the branches is going this way and it is true K going to infinity, there will be three branches, since there are three open-loop poles there will be three branches so one of the branches…… you see, every branch gives you a closed-loop pole for some value of K that should be clear so one of the branch is going this way means one root is always a real root lying on this particular segment for any value of K from 0 to infinity.

Now the other two branches; you see K is equal to 0 naturally when you start off with larger value of K they have to go this way because if you go this way it becomes unstable and secondly this particular segment is not a segment on the root locus. Look at the segments please. This total is a segment on the root locus, well; this is not a segment on the root locus (Refer Slide Time: 37:25) so there is no other way actually that the two branches can start of. These two branches have to start of this way because this is a segment on the root locus and the green line is not a segment on the root locus because the total number is odd on to the right of these points.

So you see that, I think an intuitive feeling that there will be a point at which the two roots will be coming repeatedly because after all if the roots are moving this way as K is increasing the two roots are real but a point comes when the two roots get repeated and after that point if K is increased further the two roots are expected to become complex conjugate and therefore, intuitive I use here and hence with using intuitive i I can say maybe this is my root locus diagram because after all I know that they have to meet at point to give me the repeated roots and they have to go along these asymptotes also. So if I apply all these things on this particular case {pro} ((00:38:29: min)) probably this could be the root locus diagram available with me.

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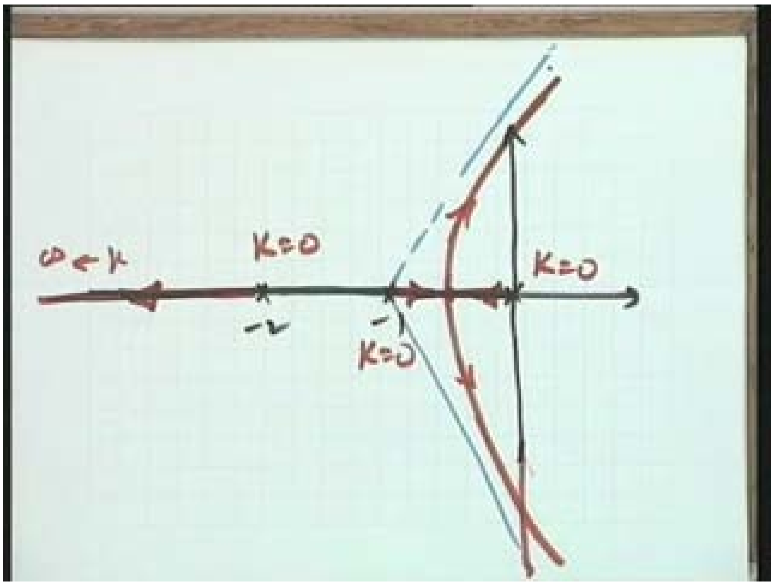


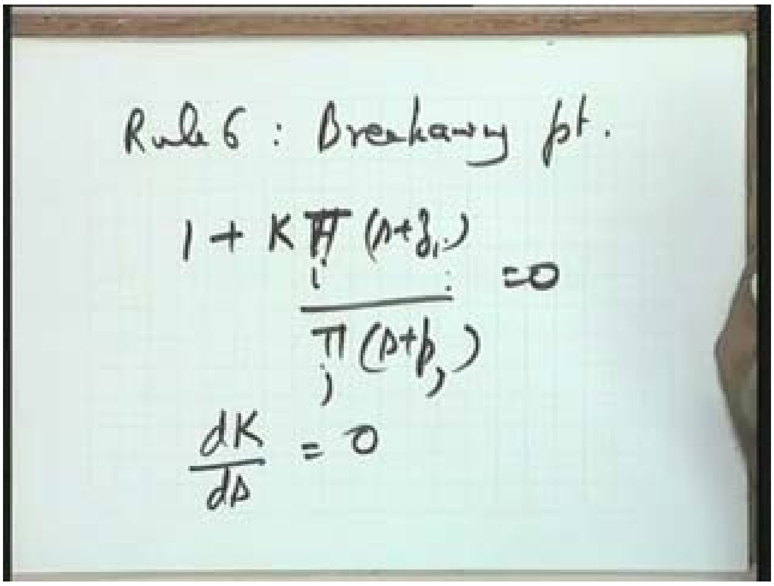
Now you see that one point: if I had this particular point, let me call this point as minus B, the point at which the two root locus branches breakaway I would had been in a more comfortable situation to make a root locus sketch, there cannot be a breakaway point here because the root cannot breakaway, here only the two branches are meeting and they will breakaway. You see that this can easily be, intuitively be felt or otherwise you can check by taking points here and there that these points are going to breakaway at an angle of 90 degrees. That is when you take them away from this you take them at an angle of 90 degrees and then take it this way. This you can check by an angle criterion, taking a point close to this particular point you will find that the angle criterion requirements says that this particular angle will be 90 degrees for the angle criterion to be satisfied.

So it means if I know the point minus B which is the breakaway point you see the situation becomes a very happy situation because i can take this point take an angle of 90 degrees and then an asymptote to this I will be more closer to the actual sketch. So it means my next rule, please give me the tally, what was the rule number, I think 5 we have already reached let me go to rule number 6 and rule 6 I say what is the breakaway point, the breakaway point please see 1 plus KFs 1 plus K pi i (s plus z i) over pi j (s plus p j) equal to 0.

You will note this particular point, please see (Refer Slide Time: 40:18) what is the situation at breakaway? On the real axis the values of K are increasing and at this particular point the value is maximum as far as the real axis segment is concerned. Do you agree with me please? At this particular point where the roots are repeated the value of K is maximum with respect to the real axis segment. Now it will increase only, as the value of K increases further you are going to get complex conjugate roots. Actually this is not the proof; it is just an argument to give an intuitive feeling.

The value at which the breakaway point occurs will satisfy the equation dK by ds is equal to 0 that is extremization of K with respect to s. I am not giving this as the proof but this will be the rule we are going to take up for getting the breakaway points. That is you get K as a function of s from this particular equation you can easily get taking this to the numerator (Refer Slide Time: 41:18) this to the denominator you have got the value of K as a function of s, take the derivative of K with respect to s, you are going to get an equation, solve that particular equation to get the breakaway point.

(Refer Slide Time: 41:31)



Now apply these particular rules on the problem under consideration; 1 plus K over s (s plus 1) (s plus 2) gives me K equal to minus….. yes please, (s cube plus 3s square plus 2s) or else s plus 1 also no, that is right [Conversation between Student and Professor – Not audible ((00:42:01 min)) yes that is right s square 3s cube that is fine. So this is your K please, take the derivative of this dk by ds is equal to minus, negative of course does not matter but still let me continue with this (3s squared plus 6s plus 2) a quadratic equal to 0.

A quadratic here and I am going to refer to my notes to give you the value otherwise it is a quadratic you can quickly calculate this. The values I get s 1, 2 s minus 0.423 minus 1.577 it gives you two roots. See this point please; it gives you two roots.

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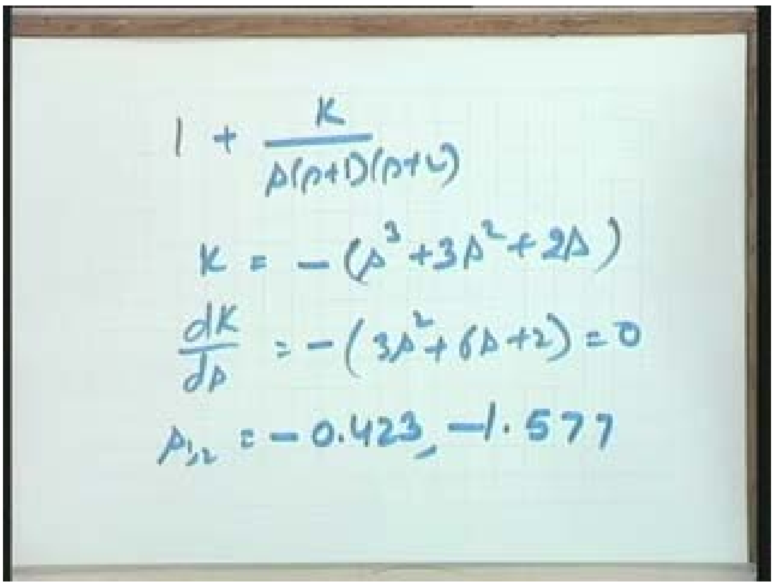

Now look at the diagram. I can take back the same slide.

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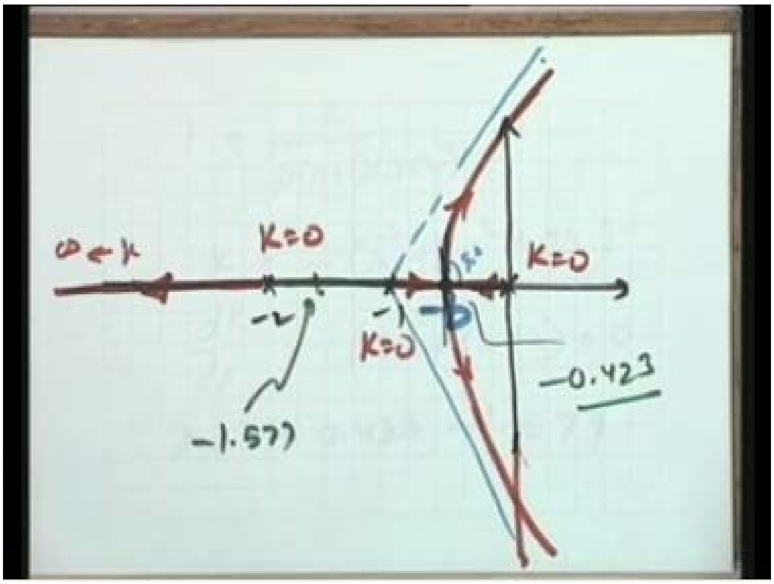

This is minus 1 and this is minus 2 (Refer Slide Time: 42:49) and your points are minus 1.577 and the other point is minus 0.423. You see at this point please; all the roots of the equation dK by ds are not necessarily the breakaway point they are the candidates for the breakaway point because they give all the points where the extrema could occur. You are interested in the extrema in this region, you are interested in the extrema in this particular region so it means you have to evaluate all the roots which you have got and those roots which otherwise satisfy the angle criterion becomes the breakaway point.

Since this point does not satisfy the angle criterion (Refer Slide Time: 43:47) the green line is not a part of the root locus at all, it means it does not satisfy the angle criterion and hence this cannot be a breakaway point. Therefore the only breakaway point is minus 0.423 as far as this case is concerned.

Now, as far as the… now the breakaway points necessarily a little complex example I may take, the breakaway points necessarily are not the points on the real axis. Let me take another example; the example is: 1 plus K over s (s plus 4) (s squared plus 4s plus 20) just to give you a passing comment you see I am not solving this example fully but let us apply the rules of this example; s is equal to 0 is my starting, this pole at minus 4 I have pole and these are the poles at the complex conjugate poles I am going to get at these locations.

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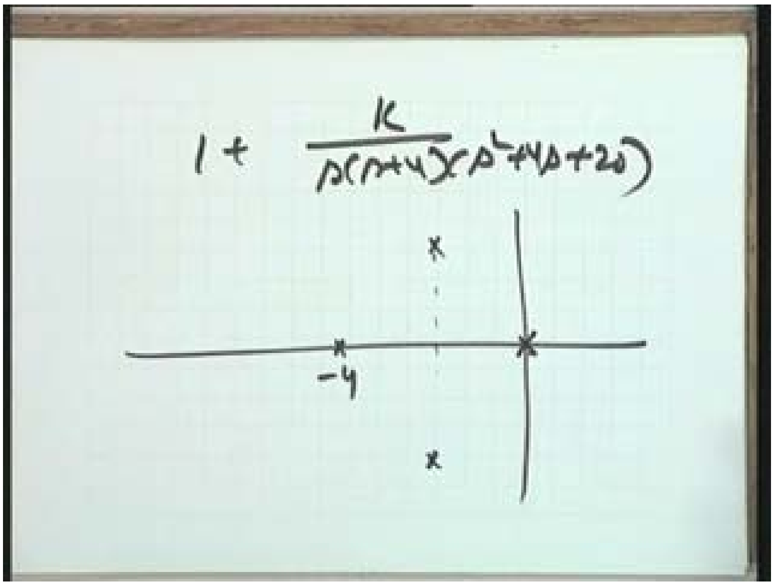
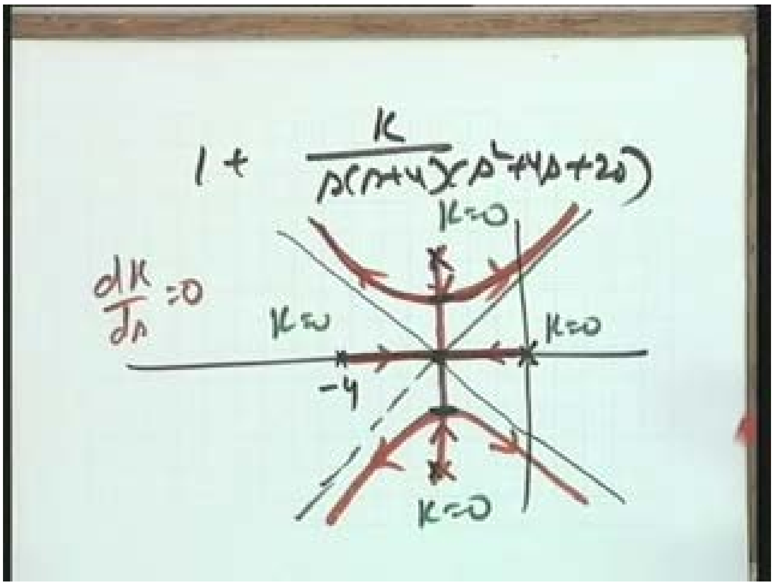


Please see that I have not equipped you with the total rules but whatever rules we have applied you apply those rules and you will find that this is your centroid and these are the asymptotic angles whatever rules we have applied, we have already given these are the points asymptotes. You know that this segment will be on the root locus (Refer Slide Time: 45:06). So, since these two points are coming this way they are meeting at this particular point they have to breakaway.

In this particular case it is more or less clear because of the symmetry that this point minus 2 will be the breakaway point but yes you can test it by solving the equation dK by ds equal to 0. If the breakaway they go like this. Help me please; is this the complete root locus diagram? No it is not as you have rightly said because these are the four segments: K is equal to 0, K is equal to 0, K is equal to 0, K is equal to 0. In fact that is why I wanted to give you this example; if you find the breakaway point you will find three points: one is this and the two other points you are going to get here which are the complex conjugate points, since these points satisfy the angle criterion they are valid breakaway points and therefore in this particular case now again you have to apply your intuitive eye on this (Refer Slide Time: 46:14) this is the complete root locus plot. More rules are required to get this particular plot.

(Refer Slide Time: 46:24)

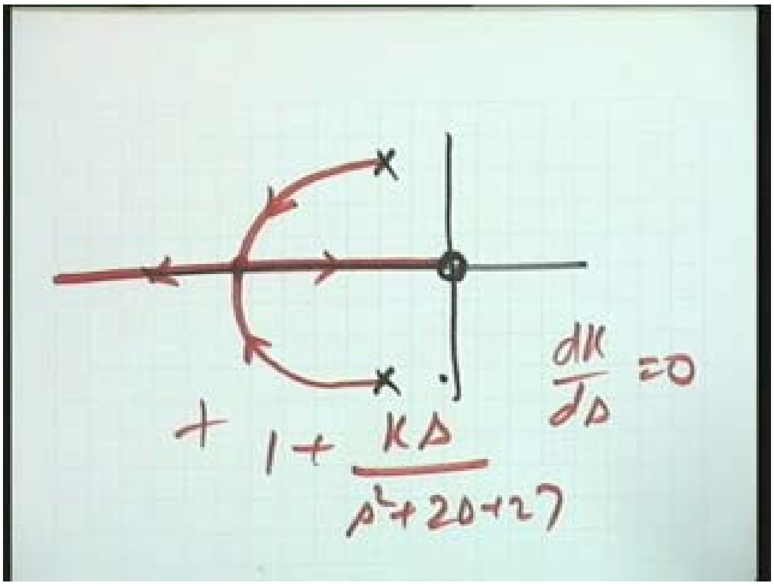


This is the complete root locus plot, you see there are no zeros so all the four branches are terminating at infinity along the asymptotes; all the four branches are terminating and that way you are getting a root locus plot where there are three breakaway points.

One more again to exemplify this rule. Let me take this particular sketch, the sketch I have just now taken today itself right in the beginning. Let us take this situation. Help me please; corresponding to this situation what type of root locus will you suggest. In this particular case there are two poles: one will terminate at 0 the other at infinity and this you know is the total root locus branch. So is it not that the two branches will come and breakaway? Of course I have to give you the further rules. So it is going to be, breakaway could be this also in the books, sometimes it is said break in but I think breakaway is quite a general term, these two are approaching this way and they also breakaway so we will be rather using the term breakaway for this situation also, they breakaway and after this particular point the two move in this particular direction; one going towards, yes please, Sir how do we get the breakaway point in this case? Same rule; dK by ds is equal to 0 will give you the point.

You see that in this particular case for example your characteristic equation is 1 plus Ks divided by s squared plus some value we had taken let us say 2s plus 27 hopefully this was the value, you have to apply the same rule that is dK by…. that is why I said that he breakaway comes this way or the other way. For example, in this particular case also this is a local extrema which is given by the rule dK by ds is equal to 0. There may be more than one points but this particular point which you will get on this axis satisfies the angle criterion and hence your breakaway requirement is satisfied.

(Refer Slide Time: 48:23)



Any question on this please?
Sir, breakaway is always 180 degrees…….. yes you see, you have another point please. As far as this example is concerned you see there are two branches in this case the two branches are approaching at 90 degrees so breakaway the angle at which the two branches which approach is 90 degrees or the two branches after approaching breakaway is 90 degrees as in the earlier case.

For example, this angle (Refer Slide Time: 48:53) is 90 degrees two branches are approaching and breakaway at 90 degrees. In this particular case two branches will approach at 90 degrees and then breakaway along 0 or 180. But you see that, well, it very interesting, the example I can give you with respect to his question.

You take this structure. You see, in this particular case you can now see in your book, particular numerical values, this also is a possible root locus plot that depends upon the numerical values. You see that the two meet and then breakaway along the asymptotes, if the four are symmetrical you are getting this root locus plot. So in this particular case please see, his question was whether the roots always breakaway at 90 degrees. I say that the roots breakaway at an angle of 180 divided by r where r is the number of branches which are meeting in this particular case one two three four four branches are meeting at this particular point and if you solve your equation dk by ds is equal to 0 for this particular example you will get only one valid breakaway point all other points will not satisfy the angle criterion. So it means at this point there is a fourth order root of the characteristic equation and now 180 divided by r is equal to 45 degrees and therefore all these branches will divert will leave this particular point at an angle of 45 degrees with respect to real axis.

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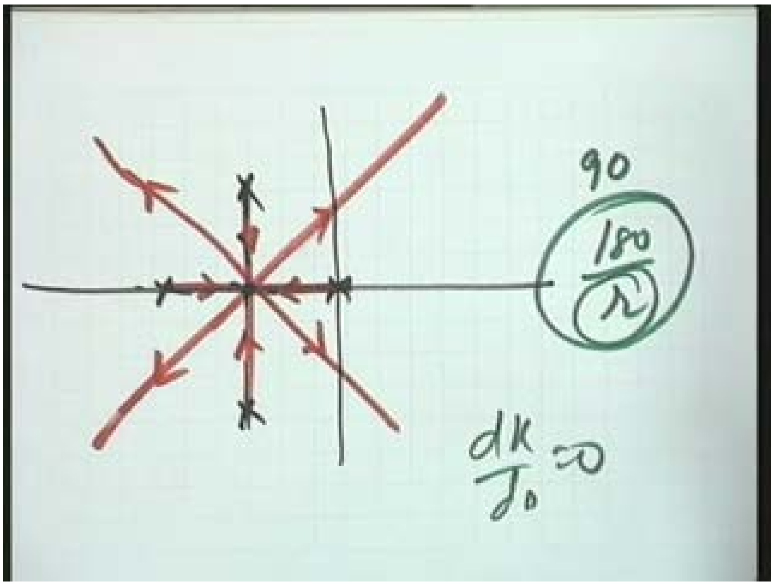

[Conversation between student and Professor: 50:35] Sir r is always even, r is equal….
yes it has to be….. come on think of an odd situation, you probably cannot think of an odd situation because of the symmetry involved. So this is as far as the breakaway point is concerned. You see, I have two more rules and it is 1057, I think two more rules and a comprehensive example will be taken on Friday, thank you.