Control Engineering
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Lecture - 30
Compensator Design using Root Locus Plots

Well friends, from today onwards we take up an altogether new phase of discussion the control system design wherein we cash on the background we have established; the background on the various type of controllers, the background on the commonly used hardware and all other aspects of the subject we have discussed. Now, when I say control system design please see that it is a very big umbrella. Under the design the selection of the plant is also a very important attribute, the selection of sensors for the system is also an important factor and so many other decisions you have to take as far as the total design of the system is concerned the decisions which are based on many a times in addition to performances requirements decisions are based on a economic and other considerations as well.

But when I say that what we are going to discusses in design I think let me make it very clear; I assume that all other decision have all the being taken because the considerations are so widely different that in a classroom discussion we cannot capture those considerations; the considerations about the plant, the considerations about the sensor, the considerations about the controller the type of controller the structure of the controller is also decided.

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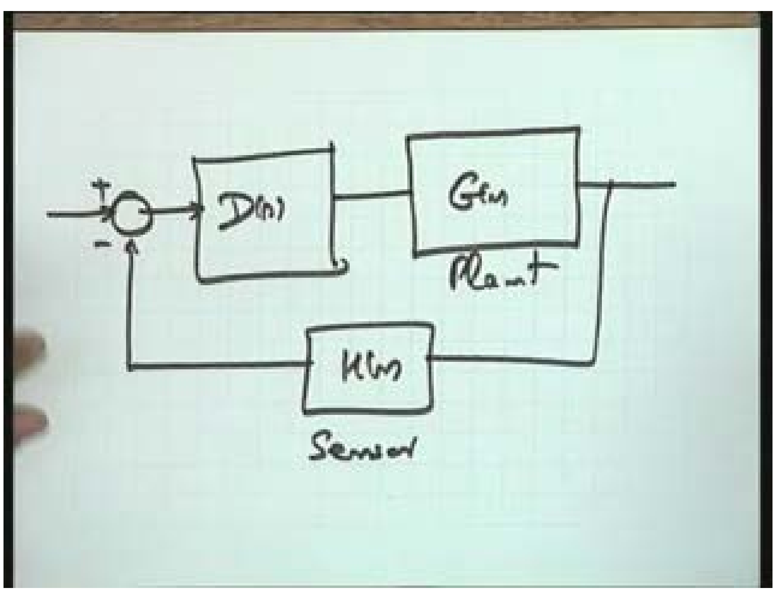

A design will simply will mean for me actually a compensator design. What is a compensator? Compensator is that component of the total control system which is going to compensate for the deficiency in the plant. If this particular D(s) was not there in the picture then also it is a complete control system with the plant and the sensor and a feedback loop. But that complete control system if it satisfies your requirements naturally you do not need any compensation but if it does not satisfy your requirements then you need compensation and you will add a suitable compensator a hardware device which is going to compensate for the deficiency in the performance. So the design in this classroom is primarily focused on the design of the compensator and not the design of the overall system.

I am sure I have given enough background on the problems associated with sensors, the problems associated with plant, the error detectors and all other factors which make up a control system. The background which we have established I am sure will help you in taking a decision when you take up a true industrial application for design. So, in the limited time the discussion may primarily be focused on the compensator design. The word compensation, the word compensator now onwards should be clear that it is a piece of hardware which you are interfacing to the plant which already has been decided. you see that, I assume that the change in G(s) the change in H(s) is now not possible once you have taken a decision on certain considerations, the only change you can do is that in D(s) because that change first of all is easy and secondly is not very costly. After all what will be this D(s) what is the change required? A little change in the Op-Amp circuit; D(s) will consist of Op-Amp circuit or it will consist of software in a digital computer. So a change in D(s) requirement will mean that changing the software or changing the Op-Amp which constitutes the controller. So the only change required is in this aspect D(s) only.

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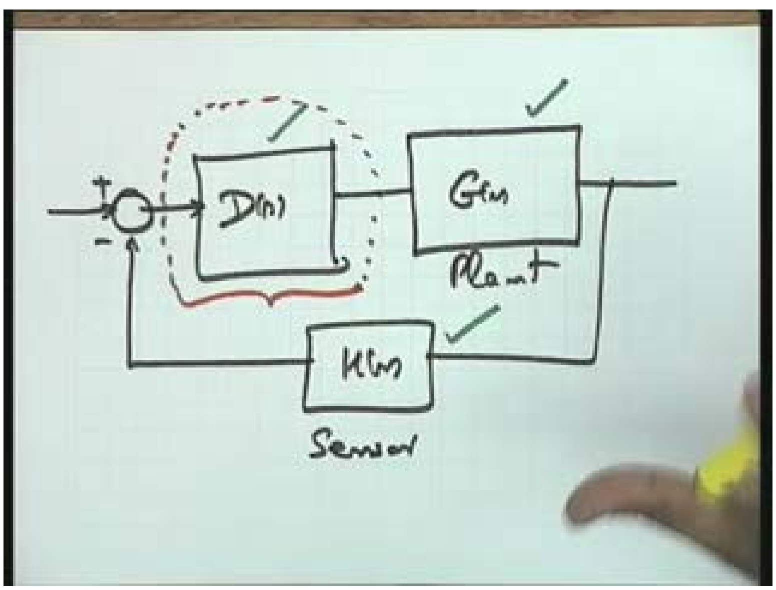

We are going to take up as to how a suitable D(s) can be selected so that all our performance requirements can be satisfied. And I first of all take up a technique called the root locus method. Qualitatively we know what performance do we want and now quantitatively we have to design a D(s) so that the particular performance is achieved. You recall the performance, our performances is in terms of t r, t p, zeta, omega n, M p, settling time, steady-state error and so on. These are the performance measures and we have seen that many of performance are measures are conflicting. but there are two important points; two important parameters: zeta and omega n which give you completely the transient performance of the system and these zeta and omega n you know that can be translated into the desired locations for closed-loop poles.

So can I say that actually the total performance which I have given you in terms of all these measures (Refer Slide Time: 5:53) as far as the transient part is concerned steady-state part is separate, as far as the transient part is concerned this transient part can appropriately be translated into a pair of dominant closed-loop poles. So if you have a large order system, if you have a system higher than second-order in that particular case you have to see that all other poles should become insignificant so that these zeta and omega n have a got the meaning in terms of peek over shoot and settling time. So this aspect we will see in our design examples, given any of these attributes you can take in to account these attributes suitably so that the performance can be translated into a pair of complex conjugate poles. And now I say that whether your performance is satisfied are not it is more appropriate to see in terms of a root locus plot than any other method.

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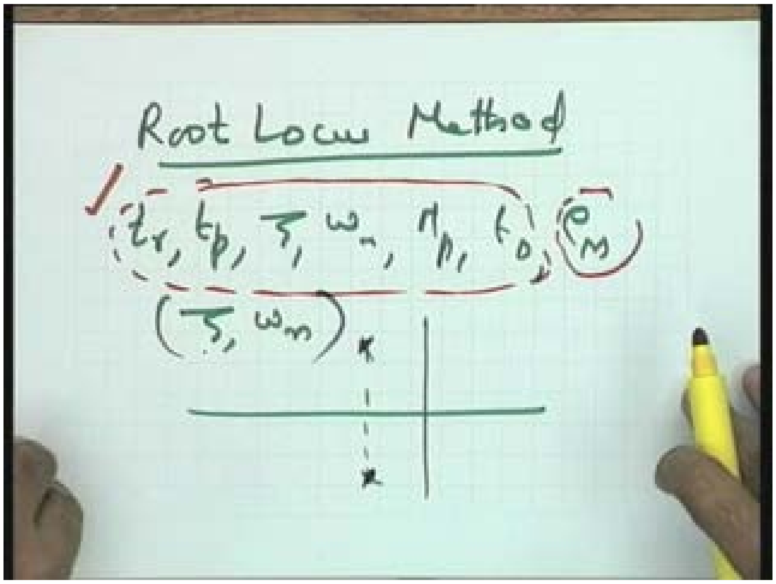

Now, what is a root locus plot; that I like to exemplify first through a simple example. Let me take a very simple example K over s plus 1 into s plus 2 is my plant and K is one of the parameters which is subject to change may be an amplifier gain. Now, as far as the closed-loop poles of the system are concerned you know that closed-loop poles of the system are the roots of the characteristic of the equation. Let me write the closed-loop transfer function as K over s square plus 3 s plus 2 plus K, yes, this is the closed-loop transfer function and therefore closed-loop poles are s 1, 2 minus 3 by 2 plus minus 1 by 2 1 minus K under root please check whether this is okay; 1 minus 4 K under root probably; so 9 minus that is right please check if there is an error you tell me immediately.

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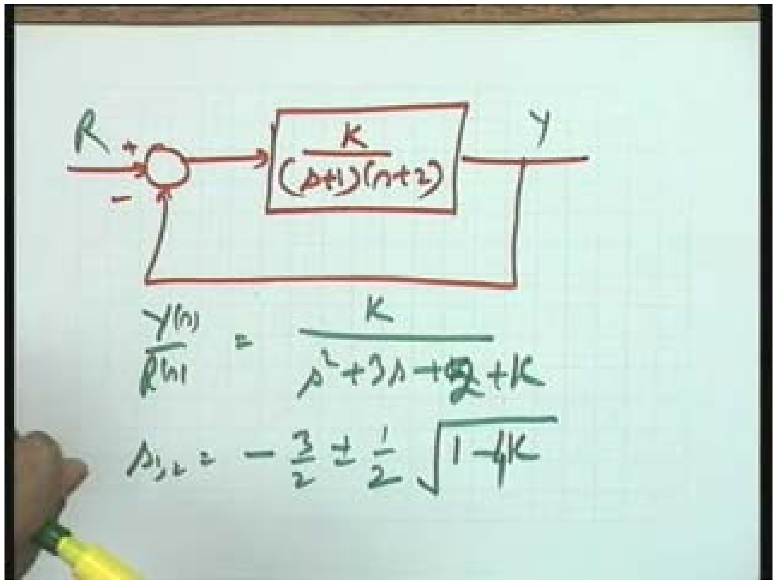

So these are the closed-loop poles of the system: minus 3 by 2 plus minus 1/2 1 minus 4 K under root. Now let us see how does the system behave as the value of K changes from 0 to infinity; after all this is the total change which you can make.

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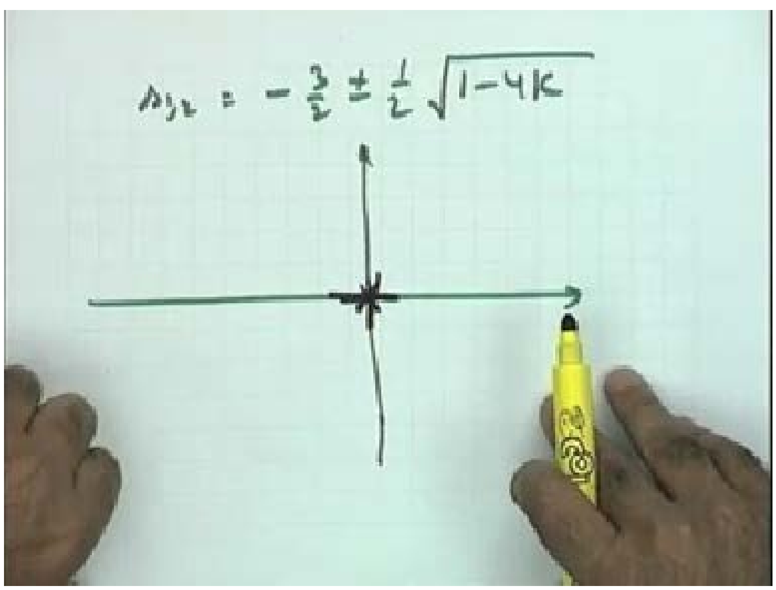

SO, s 1, 2 let me rewrite this equation as minus 3 by 2 plus minus 1/2 1 minus 4 K under root; please help me here give me the locations of the poles as K varies. What happens when K is equal to 0 please? What happens when K is equal to 0? In that particular case you find that the closed-loop are same as the open-loop poles of the system.

I am sorry let me make a fresh sheet. s 1, 2 minus 3 by 2 plus minus 1/2 1 minus 4 K and your open-loop transfer function is K over s plus 1 into s plus 2. Help me get started with K is equal to 0. So K is equal to 0 means you find one of the poles is at minus 1 and the other is at minus 2 so I make a point here as a minus 1 and another point here is minus 2 and let me write K as a running parameter K is equal to 0 at these two points so these are the closed-loop poles. Now increase the value of K increase. Increasing the value of K you can easily check please the poles will be at this particular location (Refer Slide Time: 9:25) for a slight increase in the value of K.

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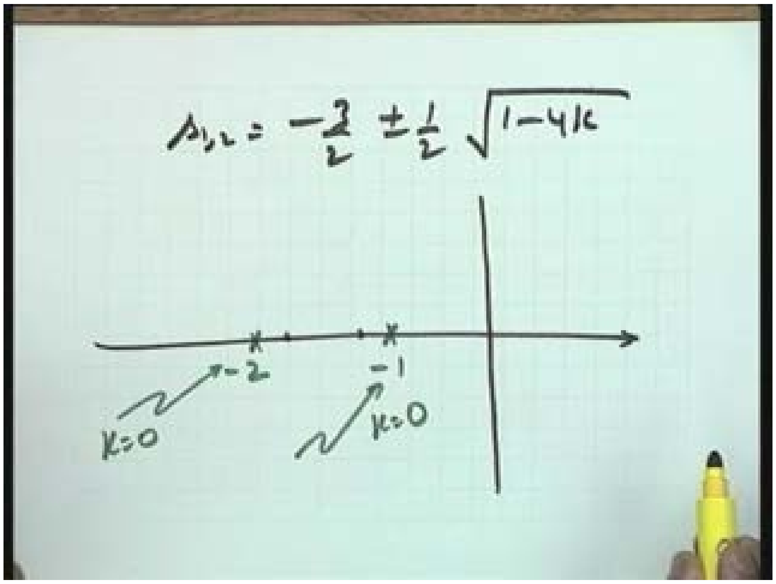

Help me what will happen if I make K is equal to 1 by 4? If I make K is equal to 1 by 4 I get the poles at minus 1.5 repeated and real. At K is equal to 1 by 4 the two poles are repeated and real and they are located at this particular point. So you please note what is happening in terms of a locus. If K is increasing one of the roots is moving this way I can make an arrow here, you can check this by simple calculation this being a simple quadratic. I need you attention at this particular point please. One of the closed-loop poles is moving on this line as K is increasing from 0 onwards and I reach the value 1 by 4. Another root is moving on this line as K is increasing from 0 and reaches the value 1 by 4 and these two roots meet at a common point where the root is minus 1.5 and the value of K is 1 by 4.

Help me please, if I increase K further what will happen? What will happen to the roots if I increase K further?
The roots become complex conjugate and you can easily establish this this being a quadratic; the roots always lie on this line because the real part is always minus 1.5, the roots have to lie on the vertical line the real part being always minus 1.5, only this part changes and this part now becomes a complex part for K becoming greater than 1.4 (Refer Slide Time: 11:03). So it means, I can visualize it this way, please see that as if one of the roots is moving from this point at which the root was minus 2 one of the K is increasing, one of the roots is going this way and may be it goes this way and the other root you start from this point it moves this way and may be it then goes this way. It does not matter it all whether you trace your root locus this way that this root

goes this way the other this way or the opposite because it really does not matter because after all you are interested in two closed-loop poles; whatever way you track you see the two closed-loop poles will remain the same. So it is really not significant whether you call this as one branch and this as the other or you make it the opposite way.

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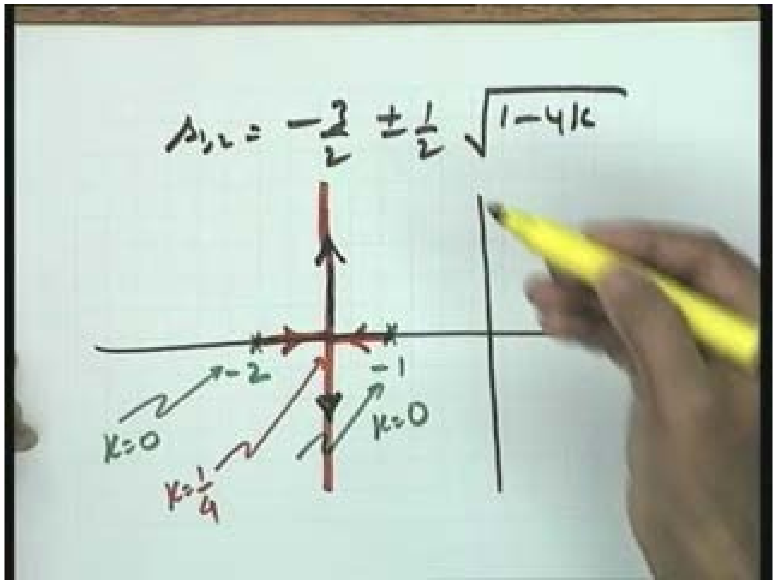

So what I want to say is that there are two branches of the root locus, which root, the root of the characteristic equation or the locus of the closed-loop pole of the system. One of the closed-loop poles will always lie on this particular locus from this point to this point (Refer Slide Time: 12:16) and the other closed-loop pole will always lie on this particular locus as K is varied from 0 to infinity. So what I want to say this that whatever information you wanted to have about a system in terms of rise time, settling time, peak overshoot, zeta, omega n I want to say that this is clearly visible in this form in the root locus plot.

First of all you see, from the root locus plot it is very clear that for any value of K from 0 to infinity the system will never become unstable because the closed-loop pole will never cross the J omega access. See the conclusions; this is a very simple situation, in complex situation also we will be able to make conclusions about the system very quickly and easily from the root locus plot. What I say is that a root locus plot is a complete description of the system in terms of variation of one parameter of the system which is your design parameter. So once the complete description of the system with respect to the design parameter is known to you you simply translate your requirement performance requirement and get the value of the design parameter that is going to be root locus method.

So you see that there will be two steps in the root locus method. You consider what is your design parameter it may be the derivative constant of a PD controller, it may be integral constant of a PI controller, it may be an amplifier gain or it may be any other thing, at this juncture you assume as if there is only one parameter but there may be more than one parameters as well but start with a only one parameter. So what you do is you plot the root locus diagram with respect to that parameter as it varies from 0 to infinity. So it means the total behavior of the system with respect to that parameter is visible to you in terms of the root locus sketch and then you can translate your requirements and look at the root locus sketch to find out what should be the appropriate value of that parameter. These points will become very clear later but at this juncture it should be very clear to you that a root locus plot is a complete description of the system with respect to a specific parameter of design.

So in this case K, you find, for example, for example if you take this point you known that the roots are complex conjugate so if you take larger value of K what will happen you know that the system damping will increase or decrease tell me? If I increase the value….. from this point to this point if I go what will happen to system damping please? It will….. [Conversation between student and Professor – not audible ((00:15:01 min))] decrease and increase both terms are coming. Please see that it will decrease. The system damping is given by…. this is theta cos zeta is equal to cos theta; you can check that this point also as the roots are going this way, you are coming from the situation of over damp to critical damp and to under damp and as you go this way as K increases the system damping will decrease this you can easily check. So you see that the effect of K system will never become unstable, system damping will decrease with the value K, and up to K is equal to 1 by 4 the system is critically damp over damp all these conclusions can easily be made from the root locus sketch.

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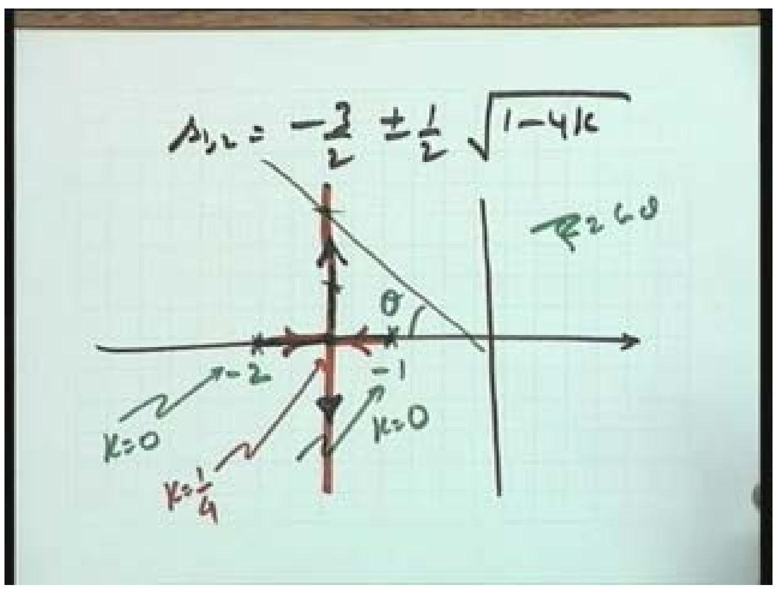

Questions welcome please at any stage. Come on; make little change in your diagram. I now say that your system becomes K over s into s plus 1 into s plus 2 this is your open-loop system. What is the difference now with respect to the earlier diagram? You will note that the difference is this that the plant now has an integrator the plant has an integrator. What is the characteristic equation of the system? It is a third-order characteristic equation as a function of K. Though I do not claim that you can immediately get the roots analytically; assuming that for a different values of K from 0 to infinity you have got the roots.

How will be root locus diagram look like and suppose you have got the roots whether the complete information is available or not let us see. The open-loop poles are at s is equal to 0 at minus 1 at minus 2. I leave this as an exercise to you. Of course you need not to do it on the computer, I am going to give you the methods how to make a root locus sketch. But the root locus as I know will be of this nature (Refer Slide Time: 16:51). Please see, these are the root locus branches. This is the total root locus diagram. You can imagine that you will write the characteristic equation of the system, vary K from 0 to infinity, get the roots and plot them and join them by line. If you do this exercise the red lines will come. At this point I hope you will not mind accepting this statement.

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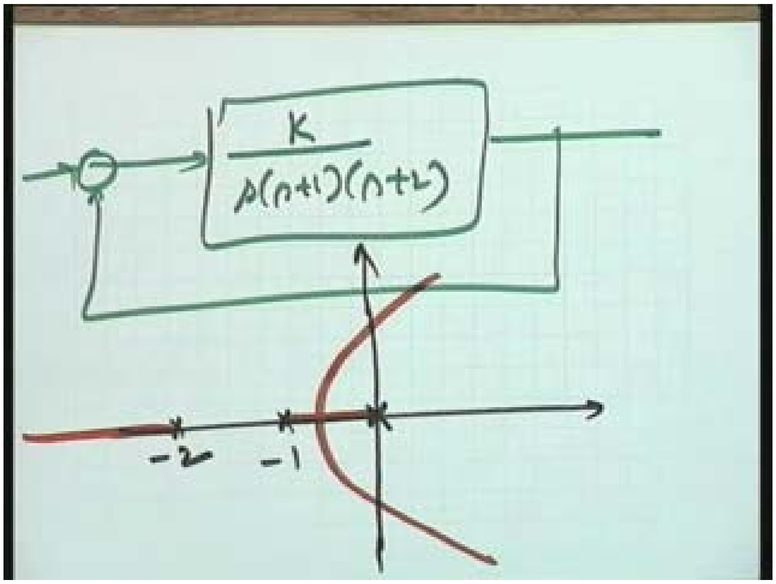

Now let us see the conclusion. The conclusion is very visible from here. Look, this is the point at which K is equal 0 (Refer Slide Time: 17:25), this is the point at which K is equal to 0 and here is the point corresponding to K is equal to 0 you are at the open-loop pole points at K is equal to 0. Increase the value of K. If you increase one root will this way, other root will go this way, third root will go this way these are the three branches. One branch going like this, this way, this way, the other branch going like this, this way, this way and the third is here so for any value of K you are going to get three roots from the root locus plot. Increase K further, you are reaching this particular point where the two roots are repeated and real and let us say the third root is here. Further increase in K gives you complex conjugate roots.

Now you see that this system clearly shows, as you are increasing the value of K you are drifting towards the imaginary axis and your system is becoming more and more to the system is going more and more towards oscillations and instability. And the third root tells you, please see, the third root also gives you the location, you can test the dominance condition and see whether the contribution of the third root on the dynamics of the system is going to be significant or not. If this particular third root is about four to five times away compared to the real part of these complex conjugate roots in that particular case the effect of this particular third root on the dynamics of the system is going to be negligible.

Increase your K further, you are coming at this particular point and you are going closer to be imaginary axis, at this particular point there is some value of K which makes the system oscillatory. You further go to and increase the value of K as system become unstable. So you will find in this particular case that this point you are getting as K tends to infinity, this point is going to infinity as K tends to infinity and (Refer Slide Time: 19:21) here is a point which goes to infinity as K tends to infinity. The only point I have left to you is that you compute it on computer and make this as the sketch.

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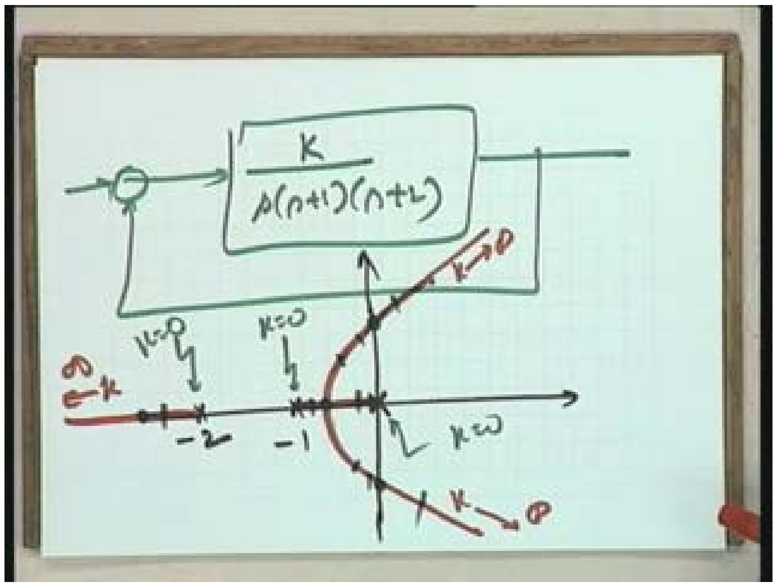

So can we say that this particular diagram has got three branches: for one branch an arrow is shown like this from K is equal to 0 to infinity K is a running parameter, the other branch K is equal to 0 to infinity K is a running parameter, the third branch K is equal to 0 to infinity with this as the third branch as I told you that really scanning this way or this way is immaterial because the two roots are going to be complex conjugate and symmetrical.

So you just see that if you have a root locus diagram the effect of parameter K is totally visible from the root locus diagram this is what we are going do with the help of the so-called root locus technique. Everything about the system is visible. Routh stability criterion did give you the range, you see that the information that from K is equal to 0 to K is equal to this point the system is stable and K is equal to this point to K is equal to infinity the system is unstable this information is available to you from Routh stability criterion as well. But it really does not give you the system poles in that particular region while the root locus plot gives you the system poles in that particular region that is important.

You can at this point say that what is a root locus method after all? If I have a characteristic equation I can sit on the computer terminal and get the poles for any value of K I am interested in. my argument is same as I gave you in connection with the Routh stability criterion. There, only numerical values will be available to you, you will definitely need this type of sketch to see how the parameter is going to affect the closed-loop poles so those numerical values are also required to be plotted. So I do not say that the computer is not useful in root locus method; the use of the computer is only to the extent of getting this plot. Once this plot is available you have to apply your own judgment to see what should be the value of the parameter K which satisfies you performance requirements.

Yes please?

[Conversation between student and professor……((the third root does not affect the transient performance the one which starts from minus 2)) ((00:21:37 min)) why?] ((Does it affect the transient when you are deciding the designing the system)) ((00:21:43 min)) yes, this we have discussed that the third root of a system if it lies in the insignificant range then it does not affect the transient. In this particular case, for example, if the numerical values…… I have not given you numerical values because at this particular point what is the value when the roots are there? I take complex conjugate roots, this is the real part, let us say it is minus 0.5 and let us say this is minus 3 yes it will not affect so it means the root locus plot it will give you that information as well; you cannot say you cannot conclude that the third root does not affect the transient performance.

Let me take a situation where this 0 is close to this (Refer Slide Time: 22:27) Sir, in this particular case…….in this particular case it depends….. what I want to say is that in general you have to determine the third root and check the dominance condition if it does not in that particular case your performance translated into these two poles is the acceptable performance and if it really affects then the performance you have translated in terms of these two poles is not the true performance and you have to suitably account for these effects as well. I hope this is clear that the third root is also given by, third or any number; in a higher order system it will be more than 3 so those roots other than the dominant roots also have to be investigated and the dominance conditions to be examined. So if it is satisfied you are happy if not then you have to suitably re-enter the design cycle so that the affect of the other roots other than the dominant roots is accounted for.

Fine, let me take one more example, say K over s plus let me take it at 5 divided by s plus 1 into s plus 2 you know that what this, this now has a 0 which is equivalent to adding a PD controller to the system. You see with the background, now you see mostly I will be working with transfer function to save time hardware I cannot bring in. But any transfer function should immediately give you a corresponding system in your mind so that you can visualize that system in your mind. This is a type-0 system with a PD controller. For example, a typical system could be like this: it could represent other situation as well. But one typical situation could be a type-0 system with a PD controller with types like your system could be a temperature system or a liquid level system or any other system you can think of.

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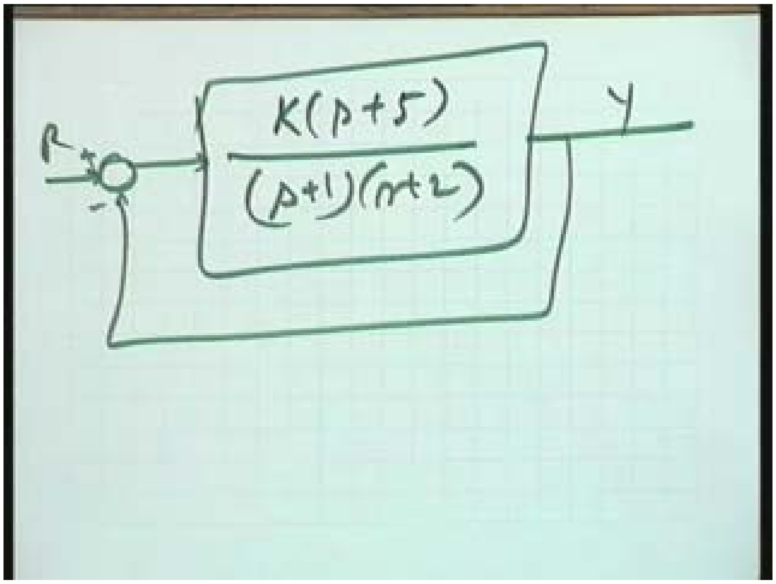

Now let us see what is the root locus sketch of this. Let me make it here itself though there is an overlap. But since these poles in zeros will be available to me here so I make a sketch here itself. s is equal to minus 1 minus 2 and a 0 at minus 5. Please see, zeros I will represent by a circle. It is not too scale or let me make it little more appropriate; if not exactly at two scale it should make some sense minus 1 minus 2 minus 5 0 by a circle so this is the situation. Now I again tell you that get me the characteristic equation, K is a running parameter and with respect to K the system should be…. the characteristic equation should be investigated. So in this particular case if you do that what you get is the following:

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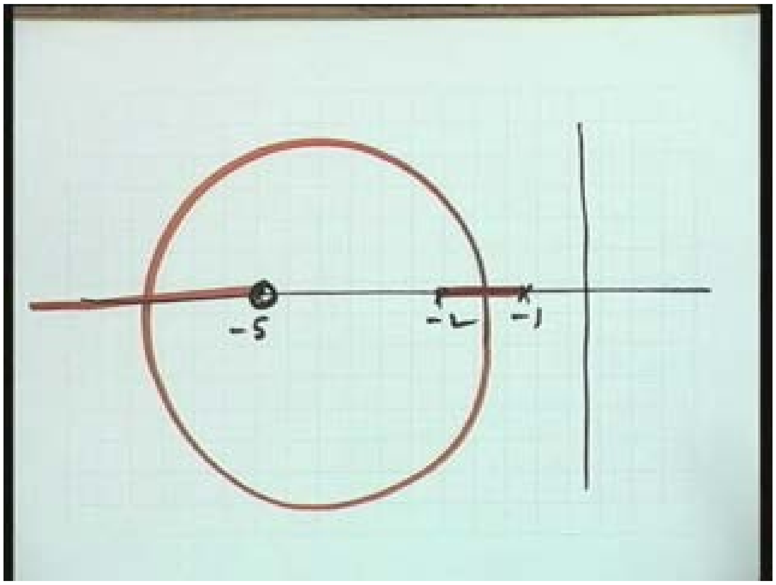

Please see, this what you will get as the root locus sketch when you vary K from 0 to infinity. Let us see how does it come up for various values of K. K is equal to 0 here K is equal to 0 here two poles are there open-loop poles you start with it. There are two branches you see, only two branches because for a second-order system there cannot be more than two closed-loop poles and hence there cannot be more than two root locus branches. By now you must have understood every root locus branch corresponds to one closed-loop pole or one root of the characteristic equation.

Come on help me please what you visualize? You start from K is equal to 0 so naturally you are going this way (Refer Slide Time: 25:57) you increase your value of K you are going this way; at some value of K your roots are repeated. Increase the value of K further you are going this way that the roots are becoming complex conjugate. [Conversation between Student and Professor – Not audible ((00:26:09 min))] that we will see. At his juncture do not worry, please see about the general…… I am going to give this information also. But at this point you just see the general shape assuming as if may not be a circle even. Though it is a circle no doubt but let us say that it is not even a circle it is any shape.

Concentrate on the aspect of how this particular system is going to convey the information about the transient performance of the system. These are the branches. Now you see that the two roots meet again on the real axis. It means there is again a value of K for which the roots are real and repeated. Increase the value of K further one of the roots goes this way (Refer Slide Time: 26:47) the other roots goes this way, at this particular point K is equal to infinity and at infinity K is equal to infinity. These arguments why at this particular point K is equal to infinity….. please at this moment do not worry about this, this is coming in this sequel, I am going to prove all these things to you. But at this point you just see that if this is the root locus sketch available to you it nicely tells you what is the effect of the parameter K on the system performance.

You find that because of the 0 you have added to the system you see that not only the system is stable the system is relatively more stable because your vertical line has been pulled on to the left hand side, you see this point please. Without the 0 you recall the root locus diagram it was a vertical line, when you added an integrator the vertical line was pulled towards the imaginary axis so it means the integrator was making the system less stable and when you have added a 0 into the system say a PD controller then the vertical line has been pulled towards the left hand side which means may be the system has become now more stable, one observation which we have made.

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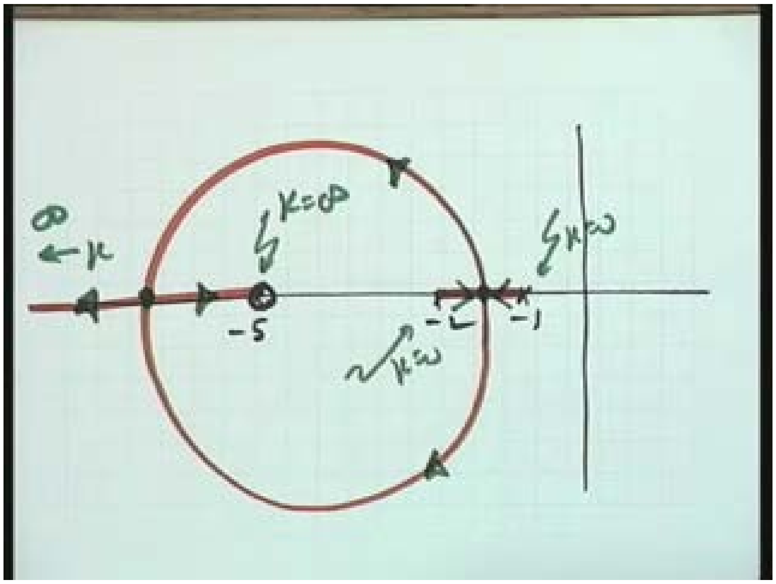

The second observation is this that you see that after this value of K the roots are complex conjugate and the real part of the roots is increasing. You know that if the real part of the…. the imaginary part is not very appreciably increasing, please see that, imaginary part from this point to this point the effect is not that appreciable but real part is increasing drastically and you know that real part is zeta omega n and you know that 4 by zeta omega n represents settling time. So it means larger the value of K faster is your system as far as the settling time is concerned. Zeta omega n is the settling time and zeta omega n is increasing as you see when you increase the value of K from here. So it means the system becomes faster it settles down faster as the value of K increases from the value from the from these repeated root values (Refer Slide Time: 28:55) where they are critically damped to the values where it becomes under damped. So this way the root locus plot gives me the picture about the total behavior of the system.

It means now if I have this sketch and all those t r, t s, t p, M p etc have been translated into two closed-loop pole locations, in that particular case what is after all the design? The design simply will be this that find out where is that closed-loop pole locations if these are the locations find out the corresponding value of the parameter K your parameter has been designed. In this particular case there is no third root so there is no problem of dominance condition, yes, one thing you will have to see in this parameter, you see, the system zero at s is equal to minus 5 is same as the closed-loop zero, you can see this or you can even see G over 1 plus G type of equation.

The open-loop zero at s is equal to minus 5 is same as the closed-loop zero at s is equal to minus 5 so you have checked this but you should see where is this 0 lying; if this 0 lies close to your pole location you have to account for the effect of the 0 appropriately and you know that how to account for the effect of 0, this 0 is going to you peaking effect early so in that particular case what you will do? Please tell me whether you will pull your roots this way or this way to account for the effect of the 0? These are the closed-loop poles which way you will like to pull? You see that you will like pull this to this way so that zeta decreases (Refer Slide Time: 30:29) zeta should increase. You want to see the effect of this s is equal to minus 5, the s is equal to minus 5 if it is close to this particular point it is going to give you more peaking phenomena that is it is given to pronounced peak so what is your design; you will like to design for a lesser value of zeta; for a larger value of zeta I am sorry I made a confusing statement; I repeat my statement again; you know that the effect of 0 is to give you early peak and larger overshoot; I repeat my statement: it is going to give you an early peak and larger overshoot so it means you should design for a little larger value of zeta so that the effect of the 0 makes it what you practically want makes it what is acceptable to you and larger value of zeta the appropriate zeta line you have to take account of and larger of zeta is given by this line compared to this line. So it means if earlier if your root location are here you will prefer these two root locations if the effect of 0 is significant. I mean did I take care of the confusion I made you see? Is it all right please? If this is the location you will most appropriately design for these locations (Refer Slide Time: 31:55) so that the effect of 0 is taken care of.

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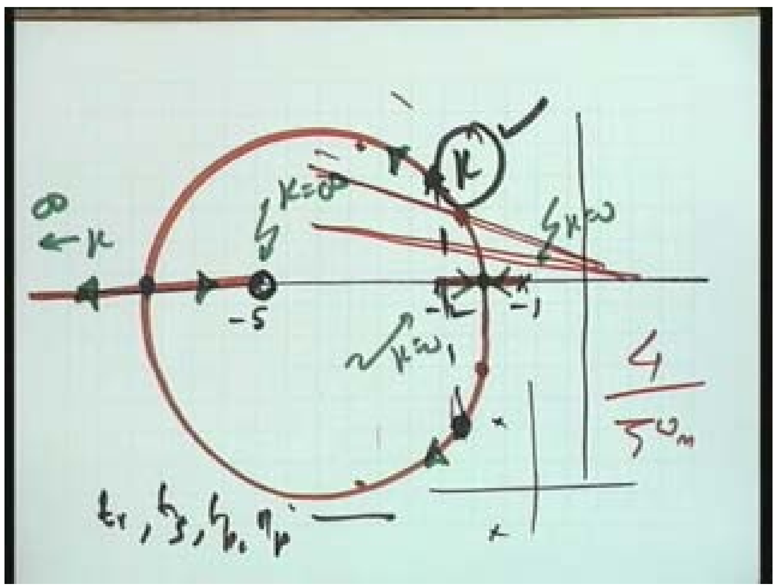

Yes, if there is any question on this or doubt on this please do tell me. You can find that compared to these two you just compare these two locations: this and this (Refer Slide Time: 32:10) this zeta is larger compared to this zeta. The zeta is becoming smaller and smaller when you go this way. Of course in this particular design you can never get a zeta lower than this, the line which is tangential to this; you cannot get a value of zeta lower than this that is the upper limit that is the limit you get.

Now I am comparing this point to this point if the effect of 0 is significant then I will try to pull the desired poles a little this way so that the peaking phenomena pulls the poles on the other side and the value of peak overshoot becomes acceptable to me. Is it okay please? Say it clearly. Yes, it is not coming it means there is some doubt. If there is doubt please do raise a question. [Foreign language…..33:02] some green signal at least has come for me to go ahead.

Now you see that I take the complete situation G(s) plus minus H(s). This is my R(s) Y(s) a complete general situation let me now take. Let me say how the root locus diagram is going to help me, how do I construct the root locus and how the root locus method is going to be applied in the general case.

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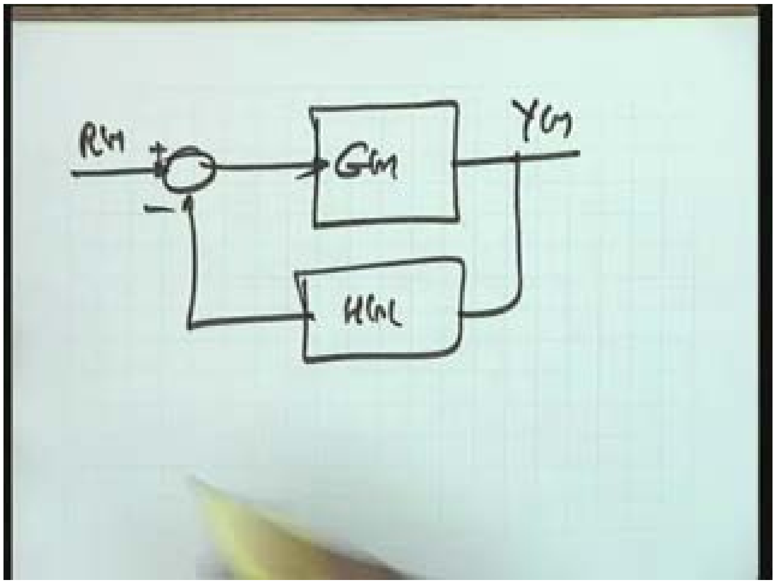

You know that in this particular case the closed-loop transfer function is Y(s) over R(s) is equal to G(s) over 1 plus G(s) H(s). See that if I break my loop here the transfer function between this output you can say that is B the feedback signal, the transfer function between B and R is G(h) so in the terminology….. various terminologies have been used, a loop transfer function or an open-loop transfer function it should not confuse you please that, I may be using the these words loop transfer function or open-loop transfer function for G(s) H(s) so it means I say that for this particular system G(s) into H(s) is an open-loop transfer function. The definition is coming from this particular reasoning that if I break the loop over here this is the output processed by the sensor. So the transfer function between the output processed by the sensor and the input is G(s) into H(s) when the loop is broken. So, in that particular case this transfer function may be refer to as the open-loop transfer function.

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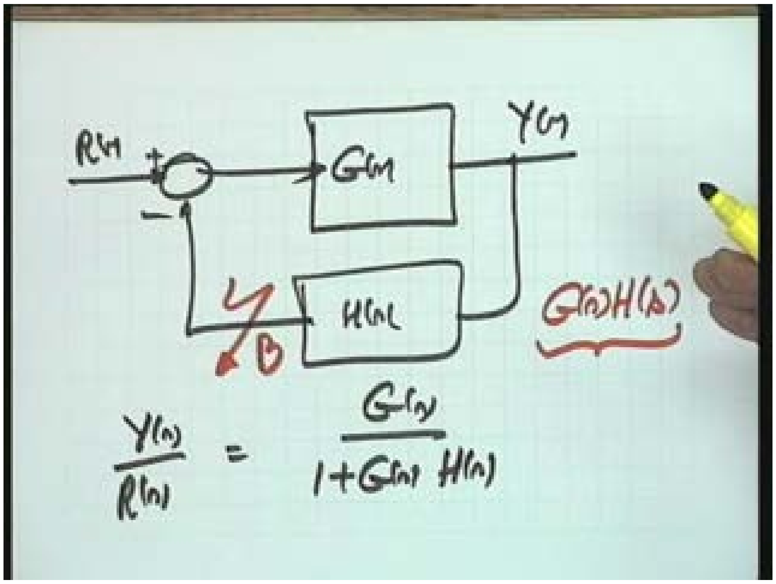

For a unity-feedback system you can very easily see that the open-loop transfer function is same as the forward path transfer function because G(s) is the forward past transfer function that will become open-loop transfer function as well.

So I see that the characteristic equation of the system becomes 1 plus G(s) H(s) equal to 0 where G(s) into H(s) is the open-loop transfer function and I am interested in the roots of this equation because the roots of this equation I know are nothing but the closed-loop poles of the system and they are going to give me the transient behavior of the system. Now, in whatever form the open-loop transfer function may be given to you it is always possible to bring it in this form: K pi i equal to 1 to m (s plus z i) over pi j equal to 1 to n (s plus p j) always please see. After all this is a polynomial divided by a polynomial, factorize the two polynomials one constant will come out because these are s and s so if you are taking s here and not a coefficient here a factor will definitely come out so that factor is K and therefore any form of G(s) H(s) may be given to you it is always to put that in this particular form and in this form K is the gain of the transfer function z is are the open-loop zeros and p js are the open-loop poles of the transfer function G(s) H(s).

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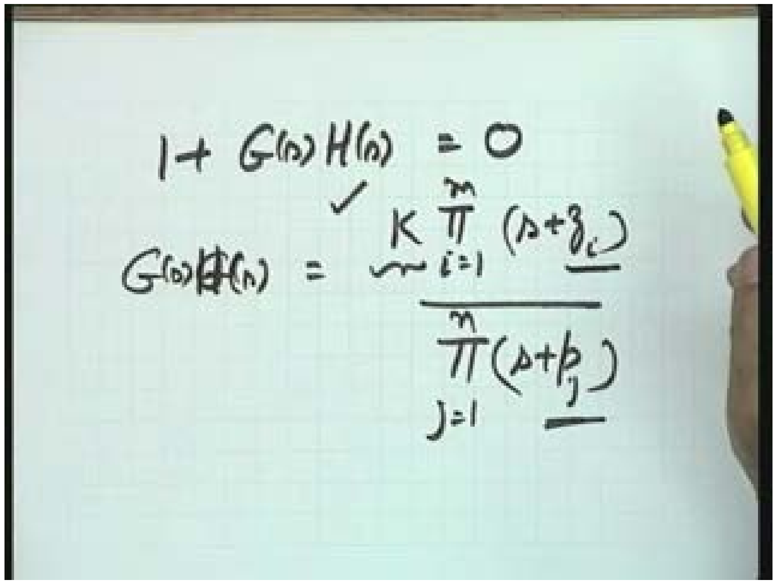

This form is more convenient as far as the root locus sketch is concerned and therefore I make a sketch with respect to this particular form. z is and p js will be positive values if these poles and zeros lie in the left-half plane and normally they lie in the left-half plane though right-half plane possibility is not ruled out.

As I said the zeros could be in the right-half plane and the poles here also could be in the right-half plane because after all your open-loop system can be unstable and the closed-loop will make it stable. The point I want to mention over here is that your closed-loop poles cannot be in the right-half plane but your open-loop poles can definitely be in the right-half plane, who prevents it? After all your open-loop system can be an unstable system, the purpose of designing the system is to design it to be a stable system. The closed-loop poles cannot be in the right-half plane but the open-loop poles can be. But mostly in most of the situations we will come across the poles and zeros of the open-loop transfer function will lie in the left-half plane. So this total function (Refer Slide Time: 37:39) if you do not mind I just name it by a single variable single polynomial function F(s) so in that particular case I can say that the characteristic equation of the system is given by 1 plus F(s) equal to 0; 1 plus F(s) equal to 0 is the characteristic equation of the system.

Now the root locus method says that you look for, you see that, you scan for the entire state space sigma j omega entire state space you scan through and find out those points for which 1 plus F(s) is equal to 0. If you join those points that becomes the root locus plot. This is the definition of the root locus plot. You have to scan through the entire s plane and join all those points appropriately so that for a particular parameter which is varying from let us say 0 to infinity if you find the roots of this particular equation then that particular plot becomes the root locus plot. So equivalently can I say that, any point you take, let me take a typical point over here this typical point will be a point on the root locus plot if at this particular point 1 plus F(s) is equal to 0.

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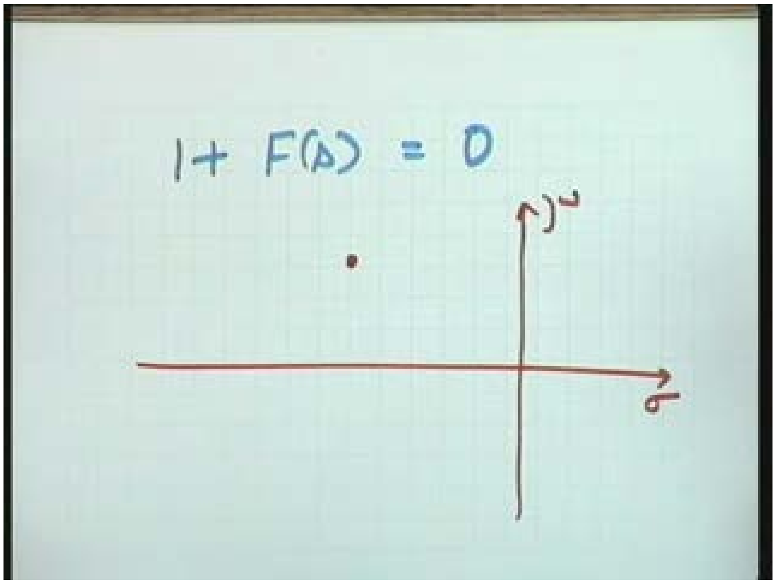

Or equivalently can I say, if at this particular point F(s) is equal to minus 1 or equivalently I can say F(s) magnitude equal to 1 and what is angle F(s) equal to please help me? 180 degrees or a multiple of 180 degrees the only thing is that it has to be an odd multiples for minus 1. So I am now putting it plus minus 2 q plus 1 into 180 degrees where q can be 0 1 2 3 and anything. I am just putting it in this form to take care of all possibilities that is all. It has to be…. it is minus 1 so it has to be an odd multiple of 180 degrees that is all, that is the only point and that is why I am putting it in this form angle F(s) is equal to plus minus 2 cube plus 180 degrees.

Thus, it means any point here in the s plane is a point on the root locus plot if this satisfies these two considerations: one is the magnitude of F(s) is equal to 1 and the other angle of F(s) at this particular point is equal to an odd multiple of 180 degrees.

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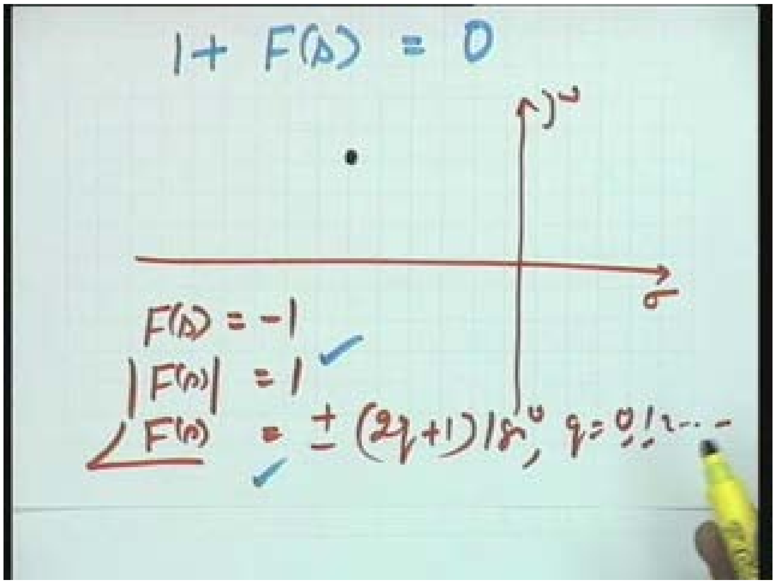

So let me put this in a more appropriate form: F(s) now has been taken as a parameter K pi i equal to 1 to m (s plus z i) divided by pi j equal to 1 to n (s plus p j). Here I need your help please; what is the magnitude criterion? The magnitude criterion will demand that K of course is a scalar constant. I am assuming K is greater than equal to 0 less than infinity. This is a positive parameter I am assuming so it is a scalar constant. So K pi i equal to 1 to m (s plus z i) magnitude. After all, the modulus can be taken inside divided by pi j equal to 1 to m (s plus p j) magnitude should be equal to 1. This is one condition and I call this as magnitude criterion or magnitude condition you may call it.

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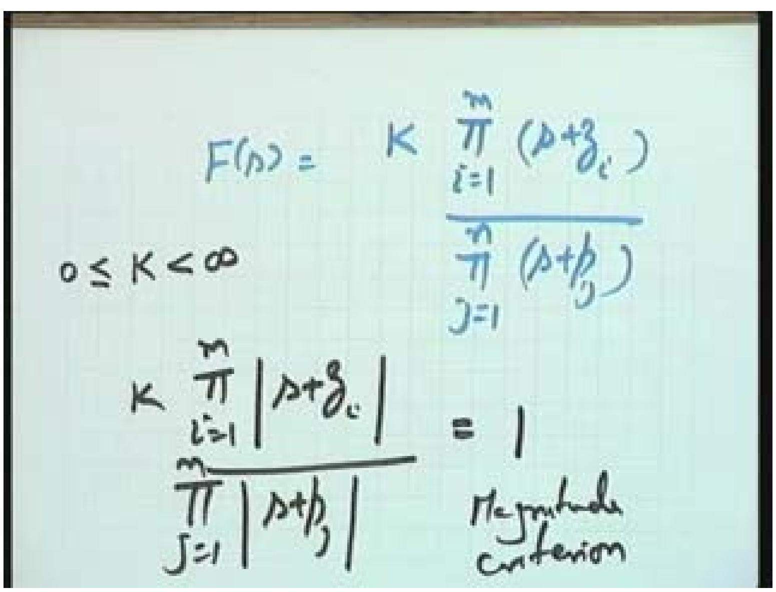

What is the angle condition please tell me? From here the angle condition will be: angle of s plus z i summation i is equal to 1 to m minus angle summation j is equal to 1 to n s plus p j is equal to plus minus (2 cube plus 1) 180 this is your angle condition please. You can see that there I think the summation point should be clear because the angles are added and this negative sign should also be clear. So these are the two conditions: one is called magnitude condition and the other is called the angle condition. So, any point which satisfies these two conditions is a point on the root locus plot. If this is clear to you then you have to answer my question please. You just keep the two conditions in mind; the magnitude condition and the angle condition.

My question to you is this that you scan through the entire s plane and tell me for what points on the s plane is the magnitude condition satisfied? If at all this question has an answer, do answer this question or you say it is a vague question. My question is this that at what points on the s plane is the magnitude condition satisfied? It is a general condition; circle was a specific structure of poles and zeros; at what points in the s plane the magnitude condition is satisfied?

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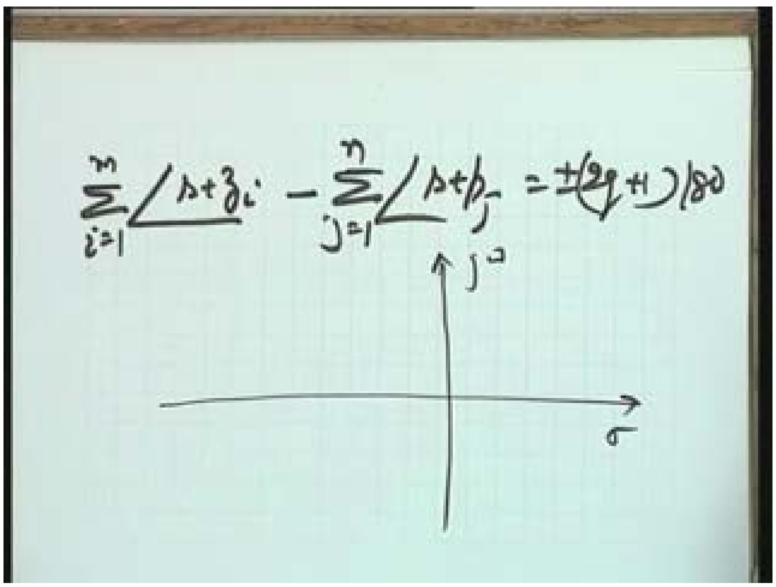

I give you some feeling about this. Let me take up the same example I had taken so that the magnitude and angle conditions become clear. I take the example of K over s into s plus 1 into s plus 2. What is this function? This is your F(s) function. I will come back to my question please. I definitely need the answer to my question from you only. Just little hint I am giving by taking this particular example: F(s) is equal to K over s into s plus 1 into s plus 2 we have already taken. Now you see that the magnitude condition is K divided by s magnitude s plus 1 magnitude s plus 2 magnitude equal to 1.

Help me, what is the angle condition?
The angle condition in this particular case is minus angle s minus angle s plus 1 minus angle s plus 2 equal to plus minus 2 q plus 1 into 180 degrees, is it all right please? These are the magnitude and angle conditions.

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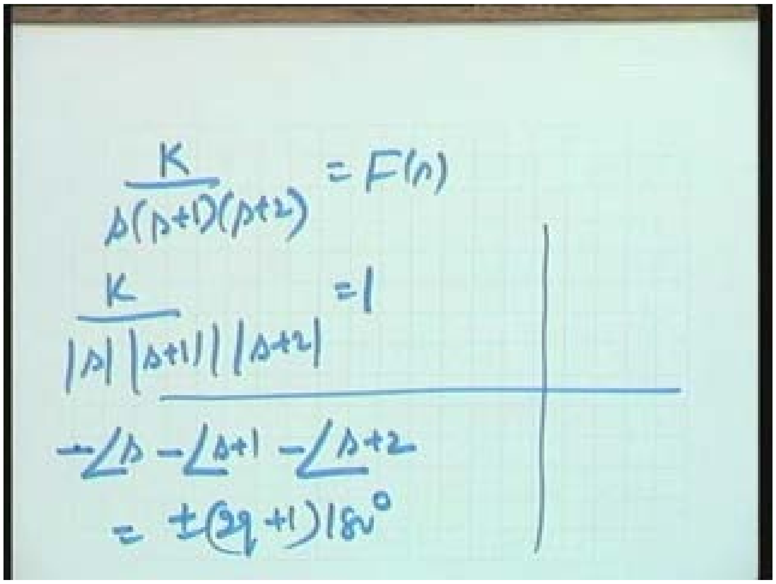

You know the root locus diagram. Do not concentrate on that. Let me take a point and let me call this point as s 0. Help me, how do I check whether the magnitude condition is satisfied or not? This s 0 has got a real part sigma and an imaginary part j omega. So what you do is in this particular case you take this particular situation:

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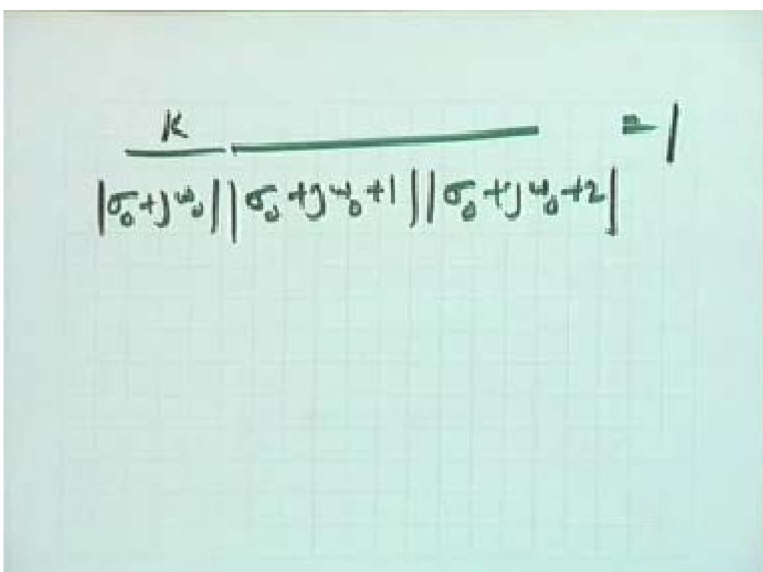

K over sigma knout plus j omega 0 magnitude sigma 0 plus j omega 0 plus 1 magnitude sigma 0 plus j omega 0 plus 2 magnitude is equal to 1. If this condition is satisfied for some K you say that the magnitude condition is okay. This is because this point s 0 equal to sigma 0 plus j omega 0 (Refer Slide Time: 45:07). So, in the magnitude condition you substitute your s is equal to s 0 plus j omega 0 calculate the magnitude of this, calculate the magnitude of this, magnitude of this, K divided by the multiplications of these magnitudes should be equal to 1.

Please see equivalently what is s? s is equal to 0 is this point. Please see sigma 0 plus j omega 0 magnitude if graphically visualized is nothing but this 0 distance on this particular plot. See the point let us say minus 1. If you look at the point minus 1 then the point s plus 1 or s 0 plus 1 is nothing but this distance visualized on the graph. And similarly, the third point at minus 2 is this particular distance visualized on the graph. So it means actually a magnitude condition will really not require testing these or calculating these magnitudes all the time. If you have a graph paper and you can read these red distances to scale to scale by which you have taken this minus 1 minus 2 in that particular case this K divided by this distance into this distance into this distance should be equal to 1 for the magnitude condition to be satisfied.

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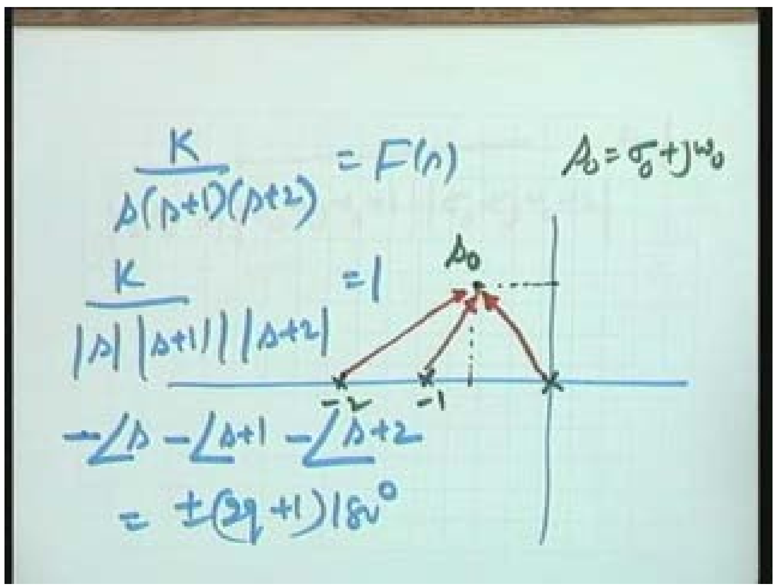

How about the angle condition? Look at the angle condition. Angle at s 0. You see again you can do your calculations using the calculator. You substitute your s is equal to sigma 0 plus j omega 0. What is the angle? Angle is tangent inwards omega 0 over sigma 0 in general. But in that particular case what is this angle? This angle tangent inverse omega 0 over sigma 0 is actually nothing but this particular angle and hence if you have a graph paper available you need not do the calculations. You substitute your s is equal to sigma 0 plus j omega 0. Angle s 0 is equal to tangent inverse omega 0 over sigma 0 is same as this angle drawn by this blue line this can be easily be examined. Is it alright please, this angle or I making an error?

Please see that the angle contributed at this particular point is going to be equal to this. What is the angle? omega 0 and omega 0 is this and sigma 0 is this (Refer Slide Time: 47:49) so at this particular point if you are testing what is the net angle contribution as far as this particular pole is concerned? [Conversation between student and professor –not audible ((00:47:58 min))] yes, this angle you calculate s plus 1; this angle you calculate. So these are angles which contributed this particular thing these particular factors. So if you take these angles some of these angles with a negative sign because these angels are in the denominator because these angles are in the denominator so when they come in the numerator they will appear in the negative sign so you have to see this particular angle and the total angle that is the contribution at this particular point gives you the angle criterion. If this is satisfied, if both these conditions are satisfied you will say that this point meets the magnitude and angle criterion and is therefore a point on the root locus plot. This is definitely a point on the root locus plot in that particular case.

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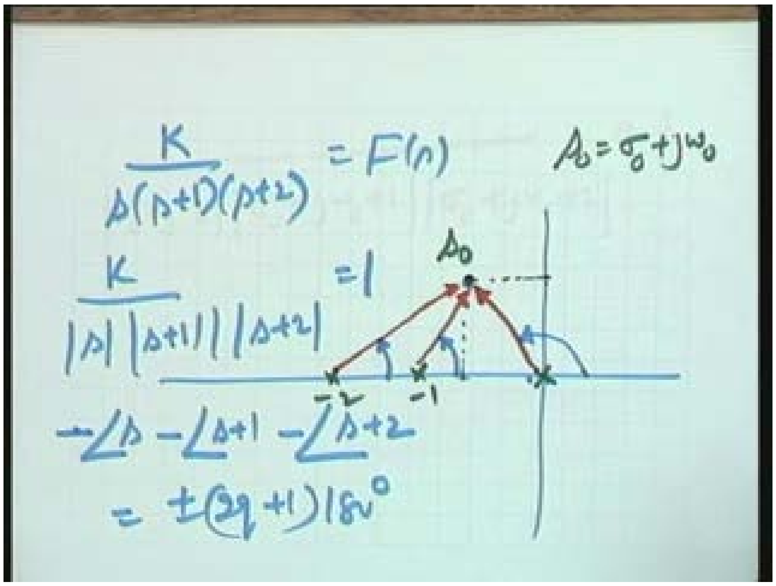

Now can I ask you my question? This was a very specific case. My question was general. You have F(s) equal to K pi i equal to 1 to m (s plus z i) over pi j equal to 1 to n (s plus p j) what are the points at which the magnitude condition is satisfied? [Conversation between student and professor –not audible…. the distance from the distance from the open-loop pole equals three times K by the distance from the open-loop pole ((00:49:39 min))] yes, K is the variable from 0 to infinity so my question is still not answered. What are the points on the s plane for which the magnitude criterion satisfied? Yes…… [Conversation between student and Professor-not audible ((00:49:39 min))] well, say it loudly….. yes, all points in the s plane satisfies this requirement. You can just think, imagine this, this is the very right answer.

Please see, you consider any…. after all your K is a variable from 0 to infinity. Now it may look like a trivial question to you. you see that you take any pole-zero combination, any pole-zero combination you take, so if you take any pole-zero combination and take any point in the s plane you take the magnitude of K equal to inverse of that magnitude, well, the magnitude criterion is satisfied. So you see that the magnitude criterion (Refer Slide Time: 50:23) is always satisfied for any point on the s plane but it does not mean that any point on the s plane is a point on the root locus because it is both the magnitude and angle criterion which are to be satisfied.

Can we make a similar statement on the angle criterion that all the points satisfy the angle criterion; please see, the answer will be no. So it means now, my procedure of making the sketch the root locus sketch is going to the following that you only concentrate on the angle criterion and scan the entire root locus plot scan the entire s plane and make all those points or connect all those points at which the angle criterion is satisfied so all those points are definitely points on the root locus plot because they are going to satisfy the magnitude criterion for some value of K. So it means making a root locus sketch will mean scanning the entire s plane for the points at which the angle criterion is satisfied because once that particular sketch is available to you in that particular case that particular sketch become the root locus plot because the magnitude criterion will definitely be satisfied for some value of K.

So now, [yes please……… 51:52 ((all points satisfy the magnitude condition)) ((00:51:47 min))] yes, root locus plot simply means that K is a variable from 0 to infinity, you see this K, otherwise the plot will not come otherwise it is a point. If you take K it is a point where is the question of plot the word plot becomes meaningless because for K a particular value then these will give you only number of roots equal to the order of the system.

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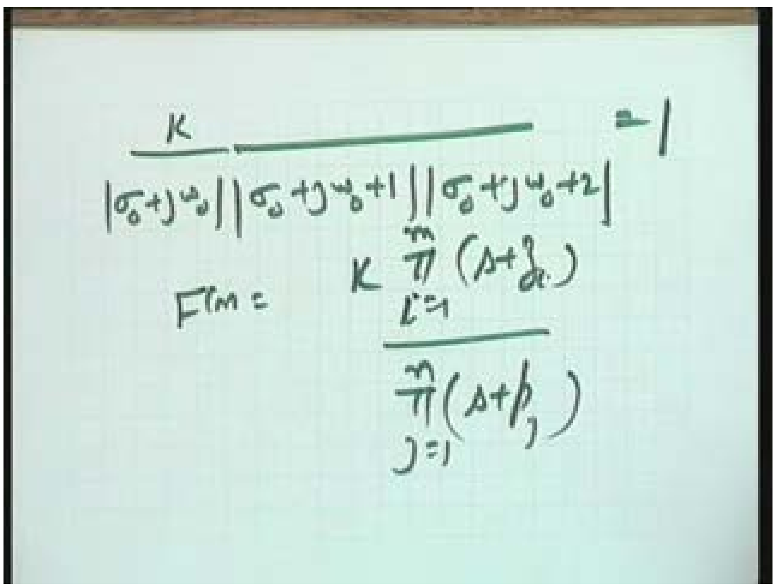

So, when we are talking of root locus branches, when you are talking of root locus plot so naturally it means that the K is a variable from 0 to infinity because I have restricted it to a positive value and for any positive value from 0 to infinity all the points for which the angle criterion is satisfied will become the points on the root locus plot for some value of K.

Look at the sketch we had made for the pole-zero system. minus 1 minus 2 minus 5 so now you see that if you apply the angle criterion I did not give you the method, if I apply the angle criterion on this (Refer Slide Time: 52:58) all the points on these things satisfy the angle criterion on the red lines and all other points do not satisfy the angle criterion that is also important. The points on the red lines only satisfy the angle criterion and all other points do not satisfy the angle criterion. Therefore I can say that all the points on these red lines are the root locus branches for some value of K and the K as…. I will give you the methods, K can be scanned this way K is equal to 0 is here and K is equal to infinity at this zero branch.

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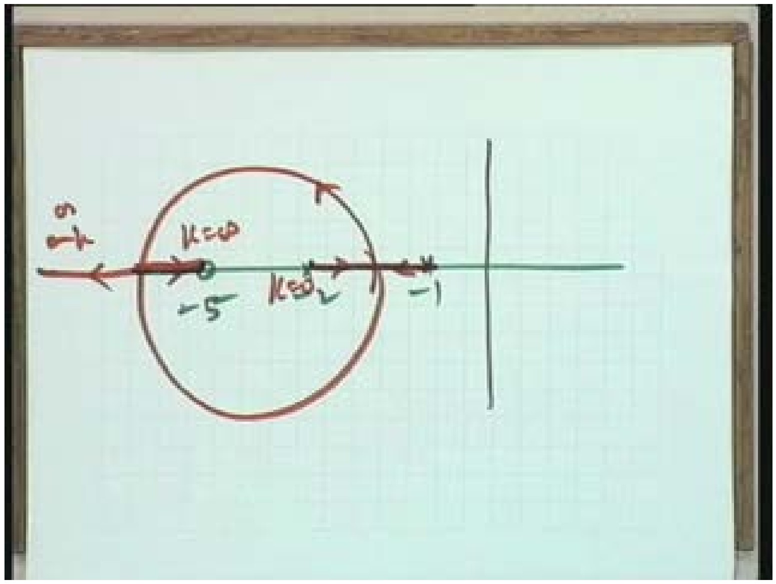

Now there was a question over here from him as to what is the center and the circle of this particular radius; this particular root locus plot. I leave this as an exercise to you. If G(s) H(s) this is my last point is equal to K s plus b over (s plus a 1) (s + a 2) please see that it is a simple exercise for you. This particular circle has a radius equal to (minus b, 0) for you to prove this and center the center in this particular case is going to be b minus a 1 into b minus a 2 under the root this is your center circle, I leave this as an exercise for you. (minus b, 0) is the center and this is the radius. Since this is a simple one we could obtain an analytical expression but in general analytical expressions are not possible and I am going to give you the root locus sketch in method how to sketch the root locus and I conclude my discussion with the last statement that I start with the angle criterion; given an open-loop transfer function I scan the total s plane that satisfies the angle criterion, join all those points to get the root locus plot, thank you.