**Control Engineering**
**Prof. M. Gopal**
**Department of Electrical Engineering**
**Indian Institute of Technology, Delhi**
**Lecture - 3**
**Introduction to Control Problem (Contd….)**

Well friends, I have been giving you various examples to introduce control system terminology and the basic feedback structure of control systems. Before I conclude this topic I will like to give couple of more examples.

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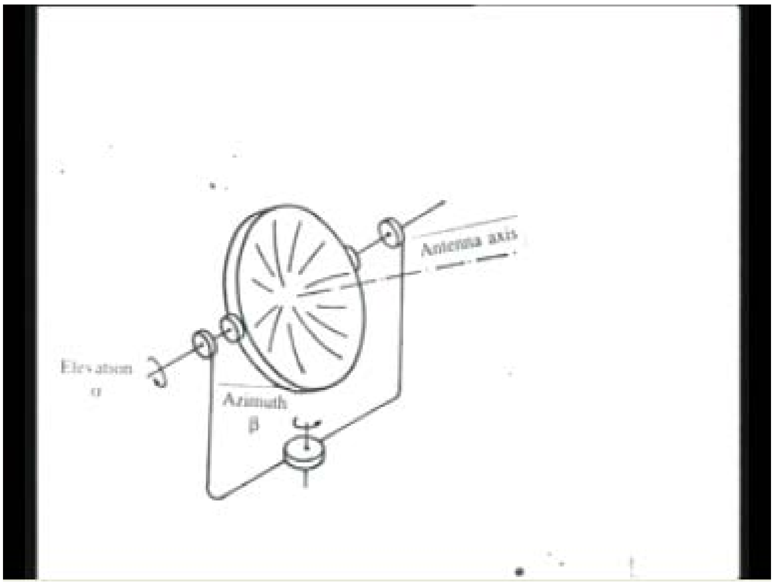


The example I take here is that of a servo system or a servo mechanism for steering of an antenna. It is a typical radar application where the requirement is this; that the antenna is aligned to the target the target plane and it is required to follow the target. So it means, in this particular case the command which is given by the radar sensor the radar sensor detects the deviation between the antenna axis and the target's position and the error between the antenna axis and the target's position is the command signal for the servo system. Now this command signal is given to the servo mechanism which is required to steer the antenna and the problem is that the antenna should monitor or should follow the command so as to reduce the error to zero. So let me take this typical antenna configuration.

As you see over here (Refer Slide Time: 2:28) that there are two degrees of freedom: The elevation angle around the horizontal axis, the azimuth angle beta which is around the vertical axis. So, in this particular case naturally this becomes a multivariable system as per the terminology given to you because it is a two input case. However, as was the case in the earlier example of automobile driving system fortunately in this particular case the interaction is small and we can neglect the interaction. You recall, I told you that if the coupling or the interaction can be neglected in that particular case from the point of view of design a multivariable system can be treated as a subset of single input single output systems so as the case here.

We can design a servo system for driving the antenna as far as the elevation angle is concerned and other servo system for driving the azimuth angle or for controlling the azimuth angle of the antenna. So now let me take a typical system which will be required for the azimuth control of the antenna assuming that interaction between the two subsystems is negligibly small and we can treat the system as a single input single output system. Here is a simple block diagram description which I am going to give to you for driving the azimuth angle of the antenna.

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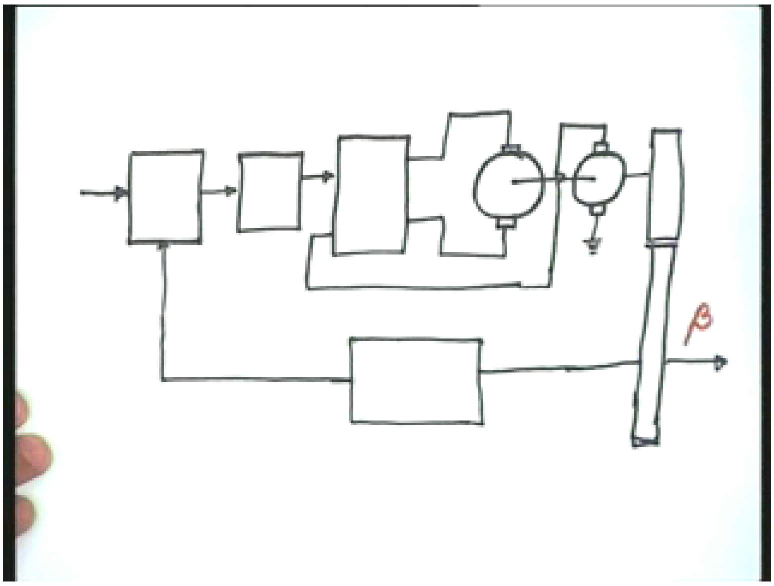


What I require is this; this is my beta the controlled variable the azimuth angle. Beta r will become the command signal as given by the radar sensor. So it means this particular variable has to reach this block which is the error detector and hence I require a suitable sensor here (Refer Slide Time: 4:21) for sensing the azimuth position of the antenna. Here I can use a shaft angle encoder. See the nature of the signals. We will discuss all these systems in details later. Here only the basic principle you have to concentrate on.

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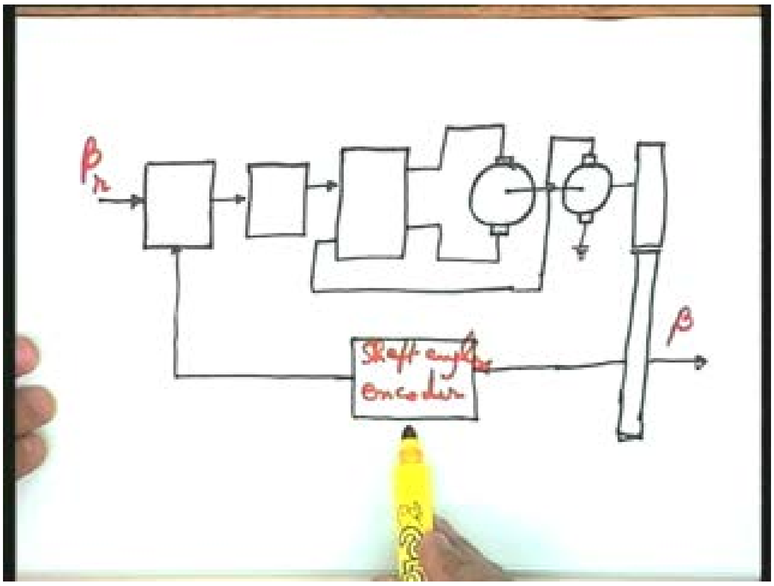


The shaft encoder is a device as you will see later. The output of this device is a digital input which is in proportion to the analog signal given. So it means here it is a digital signal, the feedback signal which is proportional to the azimuth angle beta. so here I can have a computer which accepts two signals: one the command signal; the command signal could also be or will also be in this particular case a digital command proportional to the required beta r and feedback signal is a digital signal proportional to the actual beta the actual angle. So this computer (Refer Slide Time: 5:27) which is a controller in this particular case generates an error between the two beta r minus beta and this particular error is used to generate a suitable manipulated variable a suitable manipulated signal which drives the motor so as to force this beta to follow beta r.

In this configuration, which is a typical configuration used practically what I do is, here I use A to D converter so that this particular digital signal which is a control signal, recall the nomenclature, here I can put this as u it is a controlled signal this signal is converted into an analog signal because my actuating device is an analog device. I have here a power amplifier. So this becomes the signal power given to the power amplifier, this particular signal controls the power supply to the motor (Refer Slide Time: 6:28) so that this particular motor which generates the torque is able to drive the angle beta.

You will please note between the motor shaft and this beta that is the antenna I have used a gear train. As you will see that the antenna the torque requirement for the antenna is much larger than the torque produced by a typical motor. In this particular case a DC motor has been used, a DC armature controlled motor has been used. The torque produced by the motor is lesser than the requirement of the antenna so for the torque magnification a suitable gear system is designed so that this particular torque the motor torque is able to drive the antenna axis. So in this case where is the feedback loop. You will find that the feedback loop in this particular case is through this particular channel that is the output is fed back and it is compared with the reference signal.

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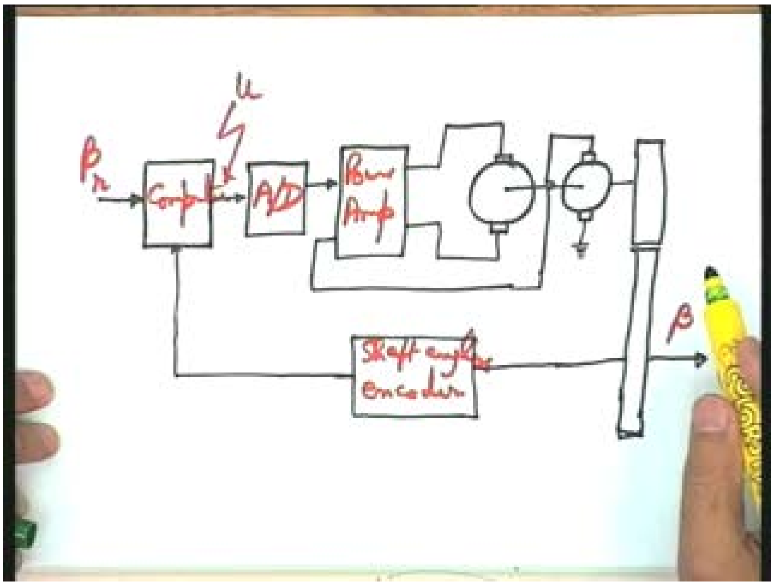


In comparison to the earlier block diagrams I have given to you, you will find one change here and that change is the following that in addition to this feedback signal one additional feedback signal is being given and that signal is through the tachogenerator. This is the tachogenerator (Refer Slide Time: 7:54) which is coupled to the motor shaft. So it means the output here is going to be a voltage which is proportional to the shaft velocity. So, in addition to the position you are feeding back the velocity also.

What is velocity after all?
It is nothing but the derivative of the controlled variable. I need your attention here though the details will come later. So in this particular configuration, in addition to feeding back the controlled variable a signal proportional to the controlled variable I am feeding back a signal proportional to the derivative of the controlled variable also and this type of control configuration I may say that it is proportional plus derivative control. You will see at a later stage various control schemes are possible and in this system a practical scheme is proportional plus derivative controlled where a signal proportional to the controlled variable and another signal proportional to the derivative of the controlled variable is fed back so as to control the motor to realize the objective of the command tracking.

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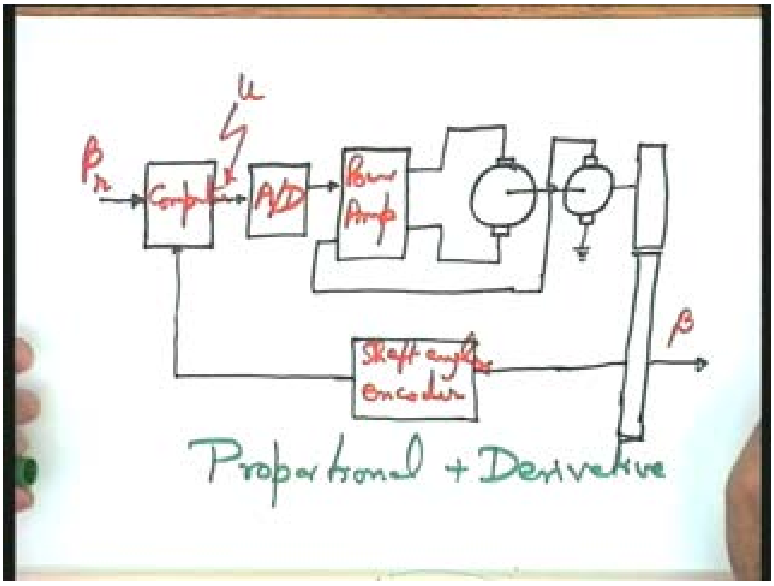


The others control variables, the others controlled schemes as you will see later are proportional plus integral control scheme or proportional plus derivative plus integral control scheme. These are some of the controlled schemes which have which are being proved to be quite effective. We will study these schemes and in addition to these schemes all other controlled schemes will also be investigated. So, for this configuration let me put it in a standard……. for this particular system the standard feedback structure will look like this. This (Refer Slide Time: 9:49) is your command signal beta r the command for the azimuth angle given by the radar system. This is your feedback signal beta coming from the shaft angle encoder in suitable physical form may be a digital number voltage or any other, depends upon the hardware we are going to use. These two are compared and therefore this becomes the error detector block.

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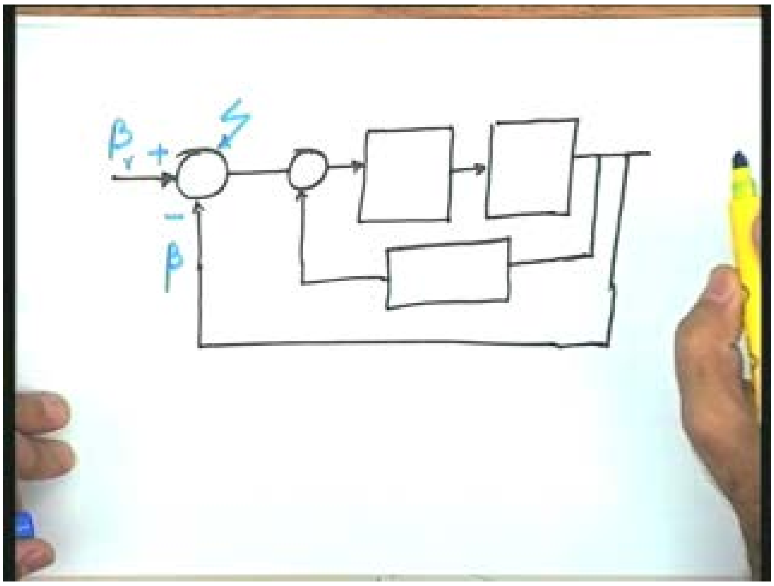

After comparison I have a signal which is e cap the actuating error signal. This particular error signal compares is the actuating signal for this particular system. Now this signal e cap……. now we have another signal coming from this and here is your tacho; a tachogenerator over here which gives you a signal which is proportional to the derivative of the controlled variable. Now this signal is the signal given to the power amplifier. The function of the power amplifier is to change the power level so as to meet the requirements of the motor. So here I can say in a typical configuration I can put here motor plus load as the plant. So in this particular case what is load; load is the antenna, the antenna is being driven through a gear train and the gear is coupled to the motor shaft. So please see that when I want to get a suitable mathematical model for this system, for model based design the antenna, the gears and the motor shaft all will be taken up as the plant of the system and the typical parameters of the plant will become moment of inertia J and the viscous friction B.

As you will recall in one of the earlier systems the mass, the damper and the spring were taken as the elements of physical model of a mechanical system. Similarly, for rotational systems the typical parameters of physical model are moment of inertia J and the friction B.

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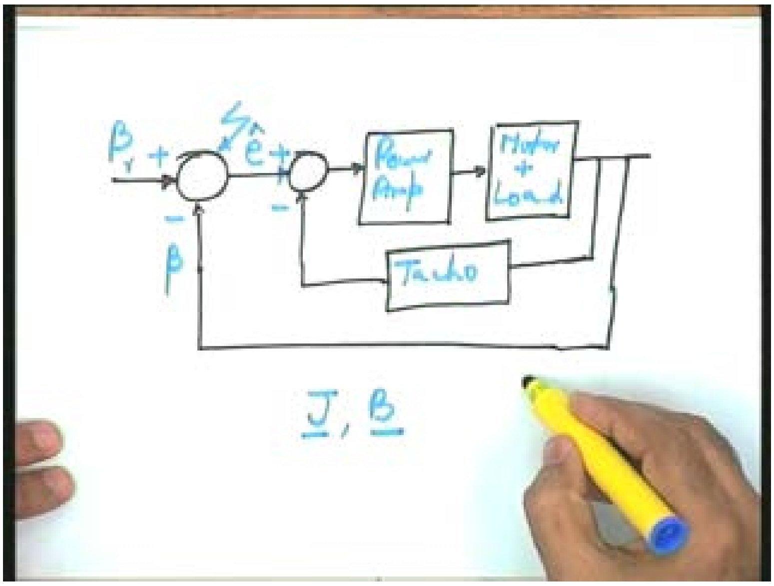


So you can say that I can put this as the load J and B on the shaft which is being driven. This will become my schematic for the load (Refer Slide Time: 12:09). This is the shaft, moment of inertia J and viscous friction B become the load parameters and these are the parameters which I will be using when I go for mathematical modeling for the system.

So one more example. Again a very useful example in industry I like to take and that is of the speed control system. Think of applications of speed control system.

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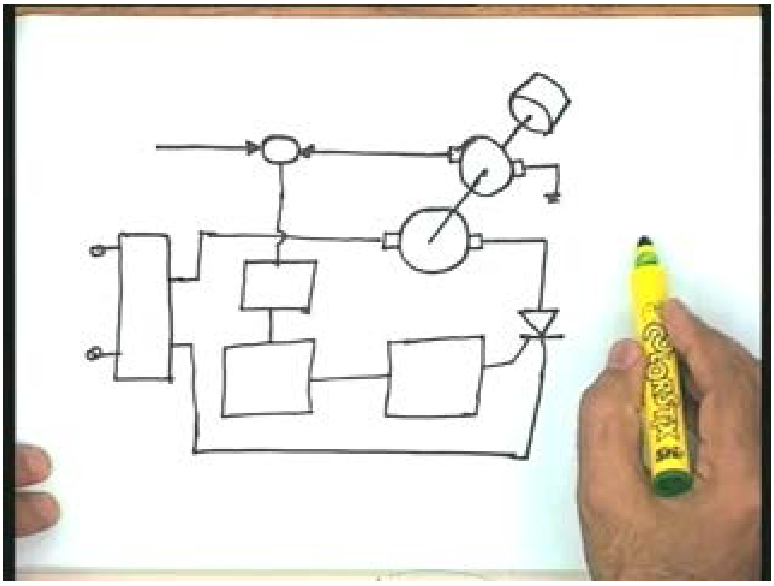

You see that take steel mills, paper mills, you see you will find that the constant speed rollers are required so as to move the product. So this is the roller these are the two rollers (Refer Slide Time: 12:50) and they are to be driven at constant speed so to have a constant linear velocity for that. So you require a control system for driving these particular rollers at constant speed. So these rollers become the load for your system. Please see there are so many disturbances.

For example, if this is a steel roll the variations in the thickness of this particular roll which are beyond your control sometimes will become will constitute a disturbance, in that particular case it is going to affect the speed of the rollers. And many other disturbances may act on the system; the problem is the control requirement is this that in spite of disturbances acting on the system these rollers should be driven at constant preset speed. So take one of the rollers for example and I write a system I give you a block diagram description as to how a suitable control system can be designed to achieve this objective of a speed control with respect to the commanded speed position. So here I will say that the commanded speed position is omega r. In this particular diagram this is the commanded speed position. So this is the reference signal which is to be compared with the actual feedback signal.

Where is the actual signal?
Here is the tachogenerator (Refer Slide Time: 14:18) which is coupled on the motor shaft. See this particular sub-portion subsystem; this is the DC motor again in the system configuration as well, this is a tachogenerator coupled on the motor shaft and here is the load. I can say that this particular load has the parameters J and B the moment of inertia and the viscous friction. This tachogenerator generates a signal omega. Actually it is not the speed, it is the voltage which is proportional to the speed, this is the sensor and the suitable form of the signal is a voltage proportional to this particular speed. So your reference signal also is not going to be the speed it is going to be a voltage proportional to speed. Your error detector may be an operational amplifier an op amp circuit which accepts this particular signal a voltage proportional to speed and another feedback signal which is a voltage proportional to speed. So this op amp circuit which is an error detector in this block diagram is modeled by

the schematic. So these two signals are compared and a signal which is proportional or which is equal to the air between the two is generated.

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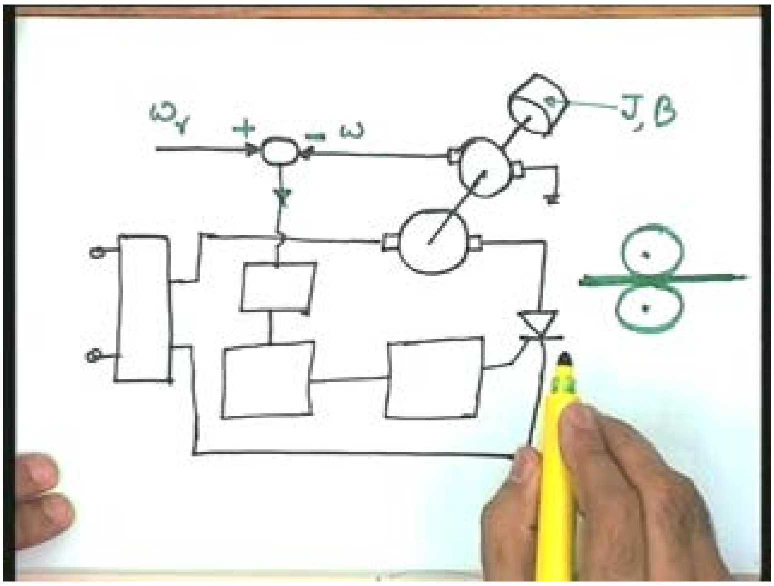


This is an analog signal as per this configuration. If I want to go for a digital control scheme so I can put over here an A to D converter block. So this A to D converter block gives me a digital signal here which is fed to the computer. So here is the DC motor and here I have the rectifier circuit. Rectifier circuit AC supply here is giving the power to the DC motor.

What is the objective of the control scheme?
The objective of this control scheme is to control the delivery of the power to the motor depending upon the error between the commanded position and the actual position. In this scheme this is being realized by a silicon control rectifier. Don't worry about the details of the hardware please. The details of the hardware are going to come later. See the feedback structure inherent structure in the control scheme. So you will find that this I can say is SCR trigger control; a suitable circuit for trigger control.

Now you can see, depending upon the error I can control the triggering of the silicon control rectifier which in turn is going to control the power supply to the motor and this control signal or this manipulated signal the power to the motor is going to suitably control the torque generated by the motor so as to force omega equal to omega r. If omega r is a time varying signal, in some application you may require the speed as a function of time the controlled system over here is a tracking system and if omega r is a preset value a set point in that particular case this control scheme is a regulator. So both the types of applications and speed control, a regulator system and a tracking system will be taken up at a later stage.

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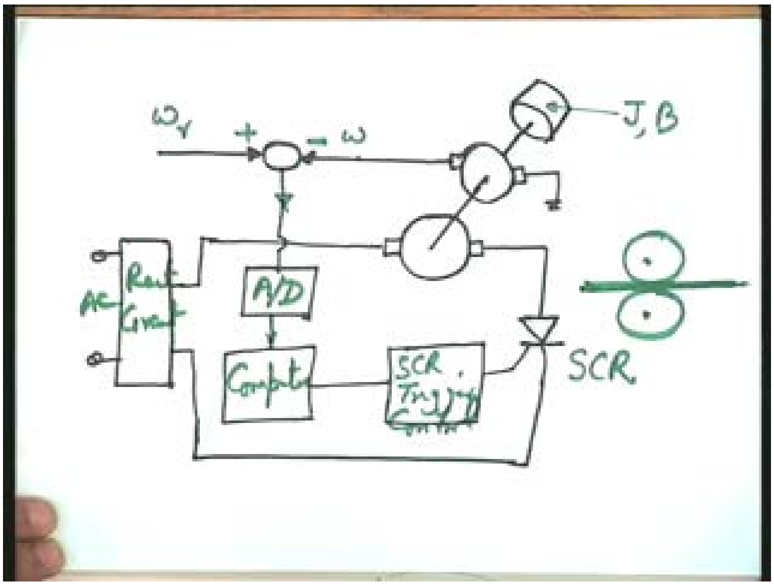


I think with these applications I hope you might have you would have got the feel of a total feedback structure and how it is used in automatic control systems. We have taken examples from the domestic sector; we have taken examples from the industrial sector. Now I think it is time that we settle down for the details of these systems. To get you the det……… to give you the details let us look at the basic subsystems we will come across again and again.

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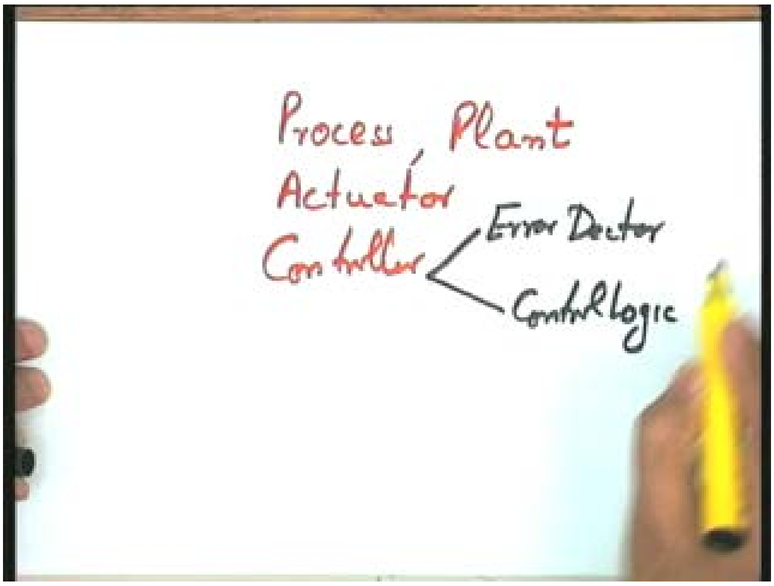


The process or the plant equivalent terms; I hope you understand this term very well now, the actuator; these are the terms should be very well understood because they will be repeated quite often; the actuator; recall the function of the actuator please the function of the actuator is simply to change the power level of the control signal to make it compatible to the motor or to the plant.

What is the controller?

The controller as you have seen consists of basically two subsystems: One: the error detector which compares the command signal or the reference signal with the actual controlled signal and it is going to have a sub-block the control logic. i have given you couple of examples. In some systems the control logic for simple proportional logic and in one of the systems particularly that of radar tracking the logic was a proportional plus derivative logic. So you will require a suitable hardware to realize the control logic the proportional logic, proportional plus derivative logic or proportional plus derivative plus integral logic.

So, in addition to this the other components are: the sensor which is going to generate a feedback signal so as to make it compatible with the command signal or the reference signal and the error detector has to compare this feedback signal with the reference signal.

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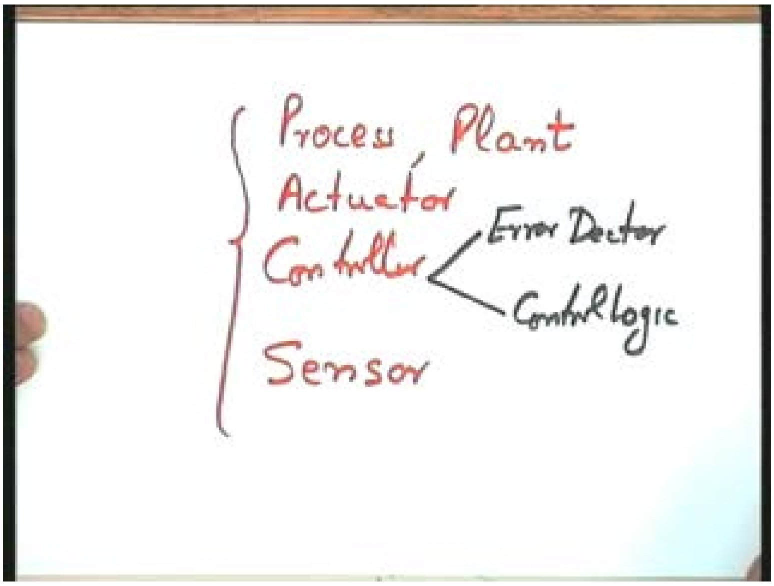


So now what are these systems?
These are the basic systems we will come across. And in a model based design I will require the mathematical model of these subsystems. I will use these mathematical models in the overall configuration to generate a mathematical model of the total feedback control system. So this is the next stage of our discussion. That is, before I take up the mathematical model of the entire control system I will like to concentrate on the mathematical models of these basic subsystems. You will see a common point that all the systems which will come up in our discussion are linear time invariant systems. We will limit our discussion to linear time in variant systems only. So it means I should concentrate on modeling of linear time invariant systems and this is the next phase of our discussion wherein the basic principles, the basic laws of physics for modeling of linear time in variant systems will be taken up.

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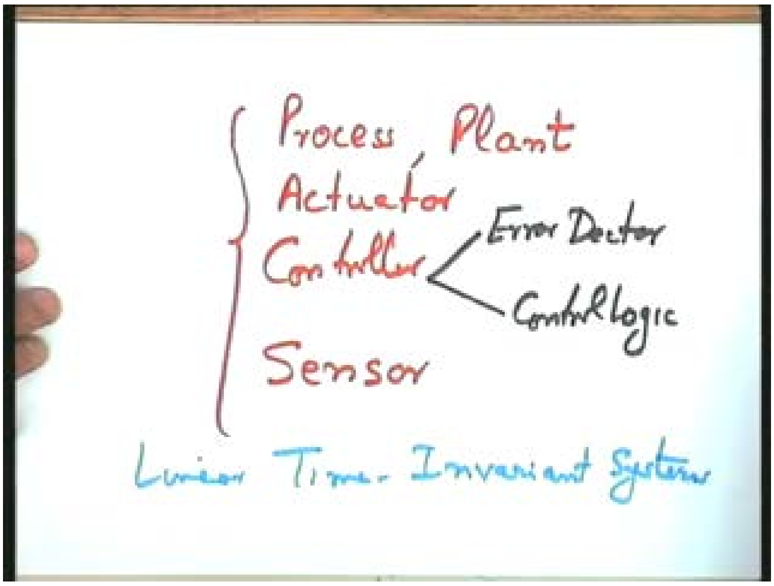


Well, I would like to mention here before I take up this that in some course or the other you definitely have come across the modeling of electrical systems, mechanical systems that is the spring mass damper systems or moment of inertia, viscous friction systems, the fluidic systems, the thermal systems. So the basic laws of physics to model these systems are known to you. I can assume this and go ahead with my discussion. But I think it will be appropriate if a quick revision of the basic laws is taken up so that the nomenclature is very well understood the way we are going to take in this particular course. So don't mind the repetition if it is a repetition. So I like to give a quick review of the basic laws of physics as applicable to linear time invariant systems.

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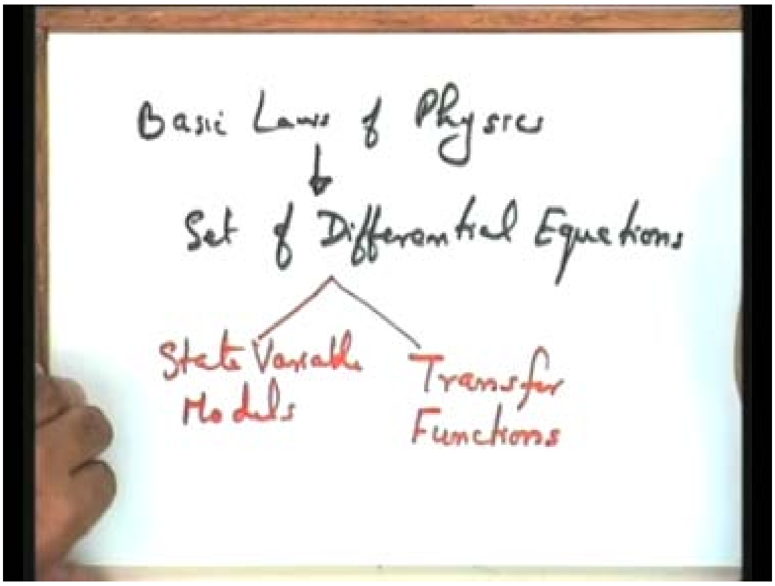


I like to classify my discussion this way that the basic laws of physics applied to the systems under consideration will give you a set of differential equations as you know. The differential equations obtained this way may not be directly usable for the purpose of analysis and design.

We will like to manipulate, we will like to play with these equations so as to put them in a suitable form in a convenient form and the two convenient forms in control systems are the state variable models and the transfer functions. So let me quickly see what is the concept behind the state variable models and then we can take up the transformation of these models to transfer functions. To illustrate the concept of state variable models I will like to take a very simple example, a simple electrical circuit.

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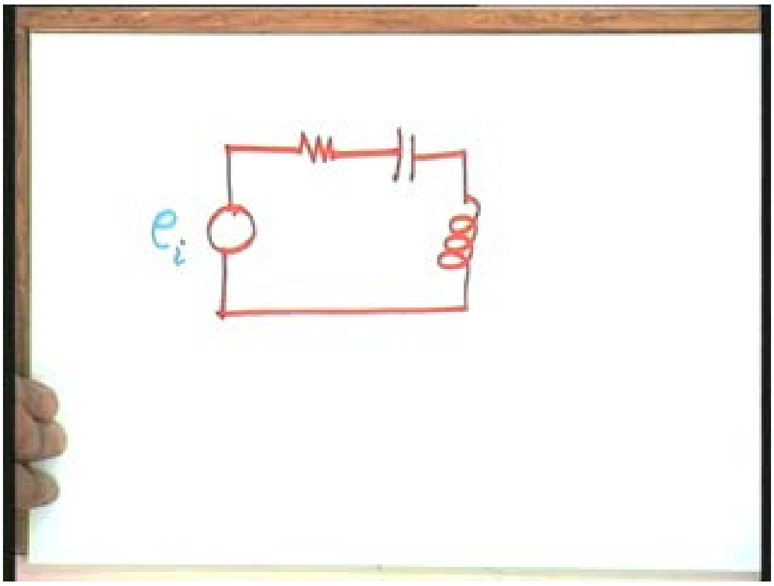


In this particular circuit the input variable of this voltage is e i this is the input variable. What is the output variable in the system? Or, in general, in any system please see output variable is an attribute you are interested in. In this particular circuit for example the current through any of these elements or the voltage across any of these elements could be output variable of interest. So, output variable in a system is an attribute of interest to me, the attribute I want to study. However, you will please note in this system, if I give you these two variables the voltage across the capacitor and the current through the inductor (Refer Slide Time: 24:49) in that particular case in terms of these two variables e(t) and i(t) you can get any variable of interest to you. So it means any output variable you may define it is uniquely available to you algebraically if the variables e(t) and i(t) are known to you.

So I can say that, in that particular case, these two variables e(t) and i(t) completely characterized the system which is a simple electrical network. So e(t) and i(t) for this become the characterizing variables and any output information you may be interested in can be obtained in terms of these characterizing variables.

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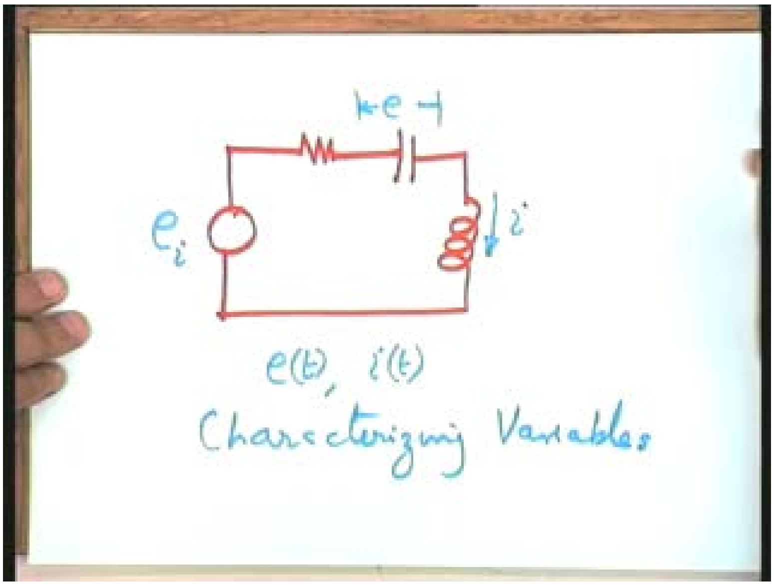


You will see an interesting link between the energy and the characterizing variables. So the energy stored in the capacitor as you see is given by 1 over 2 Ce squared and that in inductor is 1 over 2 Li square. So the dynamics of these characterized variables please see is nothing but the redistribution of energy within the system which is the cause of the dynamics in this particular system or in any system the redistribution of energy.

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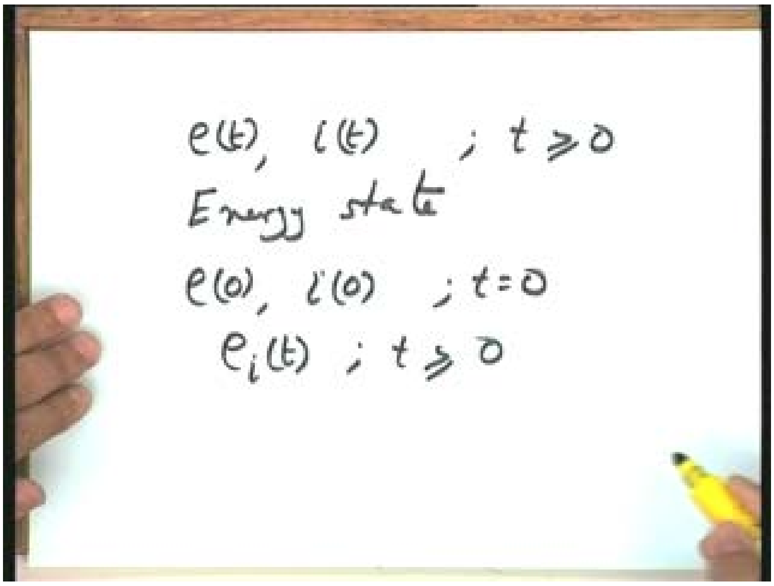


So now let me say that if I have these characterizing variables e(t) and i(t) which characterize the energy state, you see the word energy state of the system at any time t I have the total information about the system in that case. And how do I get e(t) and i(t). You will see that e(t) and i(t) at any time t greater than equal to zero is available to me if e(0) i(0) at t is equal to 0 are known to me and in addition the external input e i(t) is known for t greater than equal to 0. This is the excitation.

If this total excitation to the system (Refer Slide Time: 27:11) which is the initial energy state of the system and the external input for all time is available in that particular case these energy variables, the characterizing variables are known to me for all time and these variables therefore can be defined as the state variables of the system. I need not go to the formal definition. I hope the term is very clear. The state variables are a set of characterizing variable which give you the total information about the system at any time provided the initial state and the external input is known to you. I hope this is okay.

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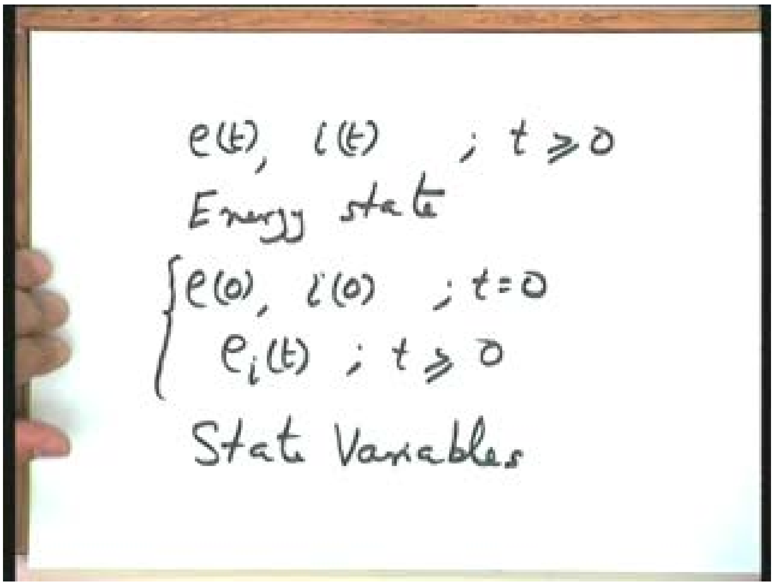


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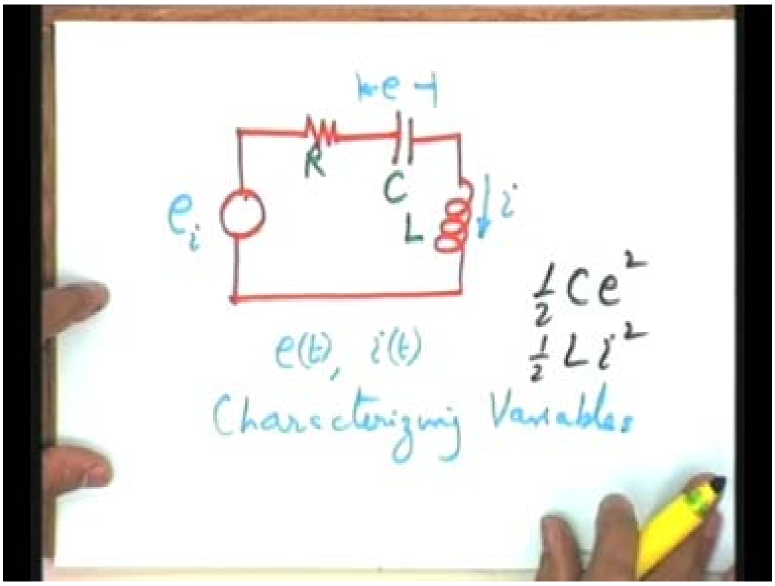

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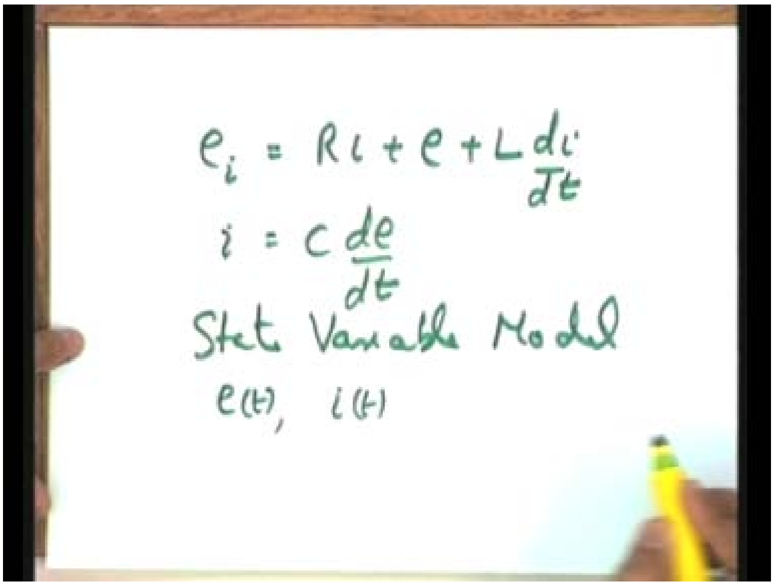

Coming back to the original circuit the resistance where parameter is R, the capacitance is C and the inductor is L and the inductance is L and the current in this particular circuit if I so applying the basic laws I can write the equation as; e i equal to Ri plus e plus Ldi by dt. This is your loop equation please see. And another equation I can write about this system is this that: i equal to C de by dt. These are the two equations and this set of two equations actually constitutes the mathematical model of the system.

The definition of the state variable model which is nothing but reorganization of this mathematical model is that I should be able to get the characterizing variables from that particular model. The characterizing variables being e(t) and i(t). The standard practice is to write the state variable model in terms of the first derivatives of the characterizing variables.

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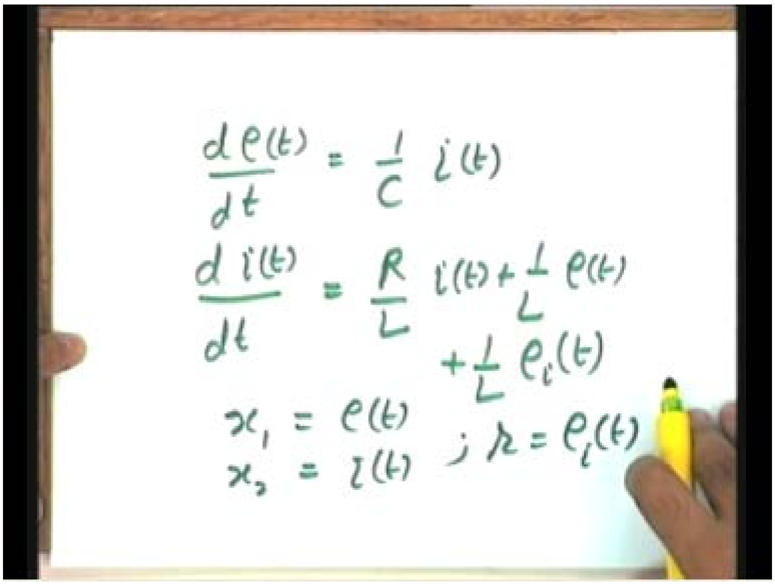

So I need my equations in the form; de(t) by dt which in this particular case you find is nothing but is equal to 1 over C i(t). The other equation in this case will become; di(t) by dt equal to please help me this will become R by L i(t) plus 1 by L e(t) plus 1 by L e i(t). This becomes my equation. So this is the first derivative of one of the state variables e(t) and this is written in terms of all the sets of state variables in this case only I take and this is again the first derivative of the other state variable and on the right hand side I have got the state variables and the input variable.

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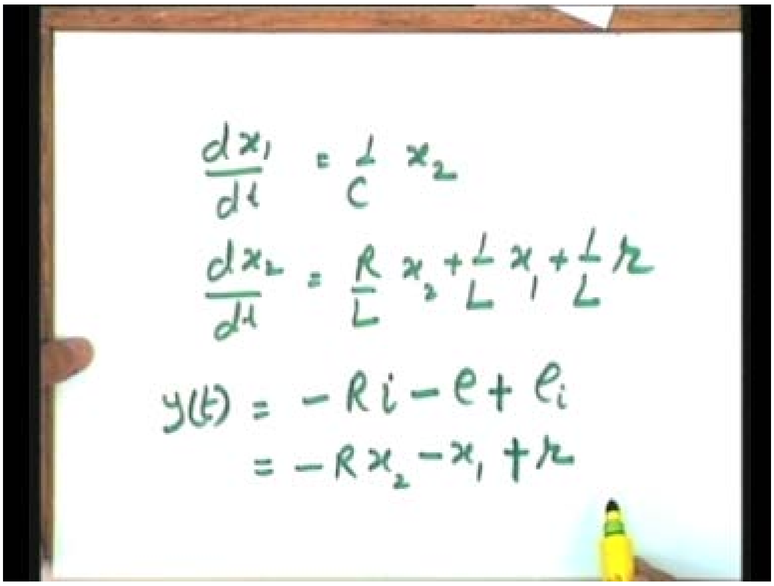


If I use the standard nomenclature, if I say x 1 is the variable e(t), x 2 is the variable i(t) and r is the input variable e i(t) you will please see that your state variable model for this system becomes dx 1 by dt equal to 1 over C x 2 dx 2 by dt equal to R by L x 2 plus 1 by L x 1 plus one by L R. these are the two equations which is a set of state equations.

Now look at the output information you are interested in. By definition any output you should be able to get in terms of the state variables. Let us say that the output you require is the voltage across the inductor.

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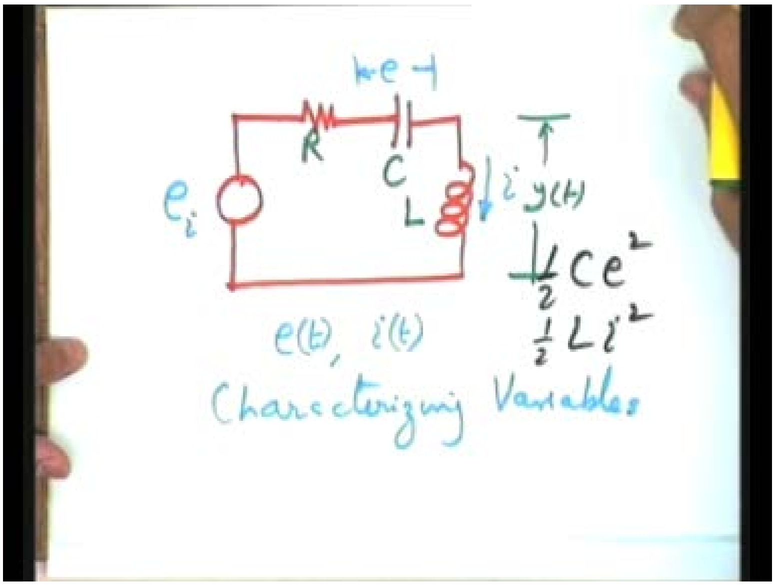


Recall the circuit, the voltage across the inductor. So, if y(t) is the output which is the voltage across the inductor, I can take this particular diagram again and define this as the output y(t). So you will see that this y(t) is given by minus Ri minus e minus e i in terms of your state variables and the input variables. If I put it in terms of x 1 and x 2 this becomes minus Rx 2 minus x 1 minus r sorry plus r (Refer Slide Time: 32:06) is it okay please? Minus Ri minus e plus e(i) and I have made the correction here in this equation as well. So you see that the output is written as an algebraic equation as a readout function in terms of the state variables and the input R.

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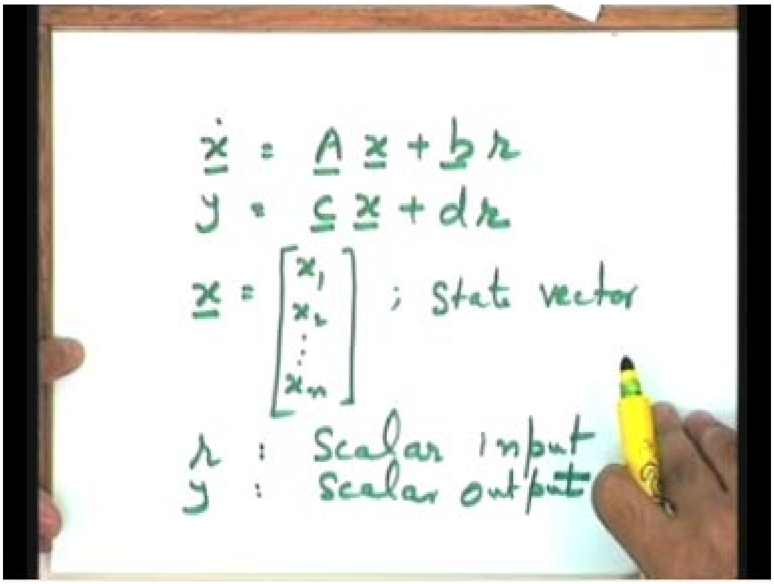


Now let me put it in a standard format. In the standard format this model can be written as: x dot is equal to Ax plus br see this something which is known to you. As I said that it is a sort of review. y the output is equal to cx plus dr. See in terms of our original system whether it fits into this standard model or not. x is a vector, this underline representing it to be a vector which is a set of n state variables x 1 x 2 x n.

Now, in this case this is the state vector (Refer Slide Time: 33:22); r is a scalar input because I am considering a single input single output system.

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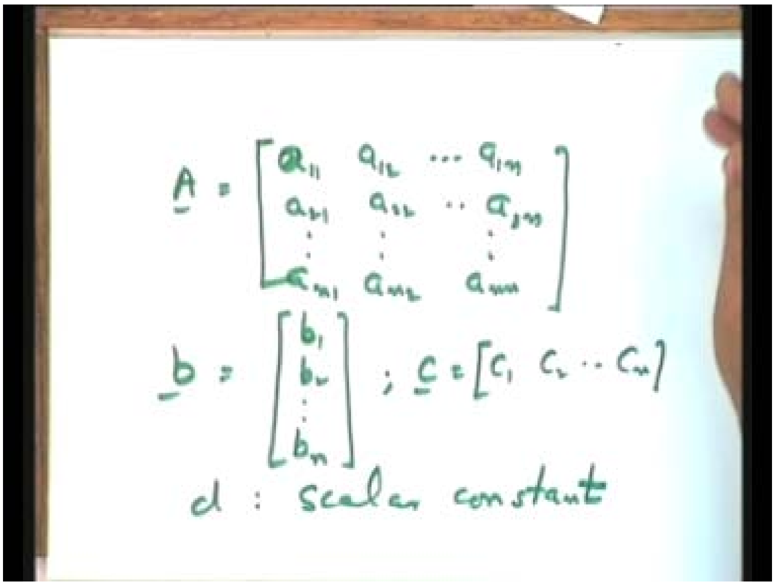


Why in this particular case is a scalar output and what are ABC and D matrices?
Let us see what is A please tell me what should be the dimension of A. Obviously in this particular case if x is an n into one state vector consisting of n state variables A is going to be: a 11 a 12 a 1n a 21 a 22 a 2n and let me take the last row as a n1 a n2 a nn. This is the matrix. How about b? b is going to be a vector and n into 1 vector b 1 b 2 b n a column vector and the C is going to be a 1 into n row vector, let me say it is c 1 c 2 c n, d is a scalar constant. This is the standard format: x dot is equal to ax plus bu.

You will please note that in my nomenclature which will be followed throughout the course the lower case letters with an underline are reserved for vectors, the upper case with the underline is for a matrix and whenever there is no underline it is a scalar. This is the convention, this is the nomenclature I am going to follow throughout and this is the standard single input single output state variable model.

A simple exercise for you and a quick revision of the basic laws of physics for the mechanical systems as well.

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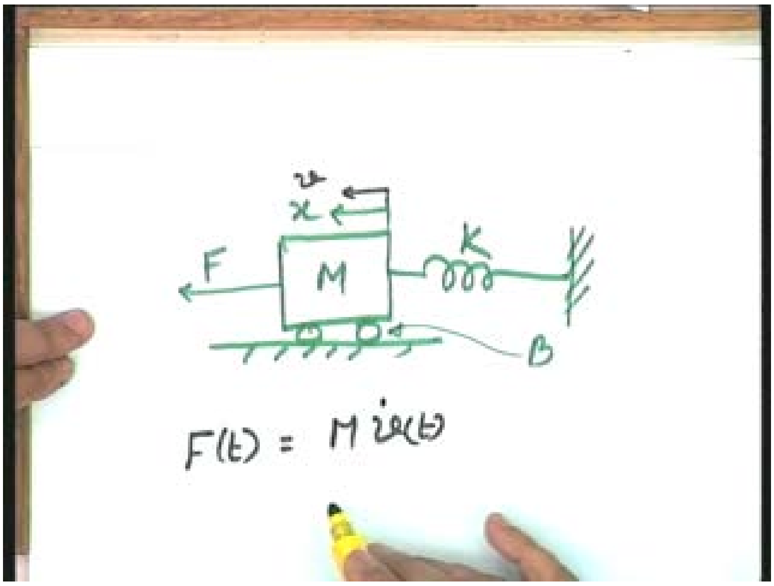


I consider mass spring damper system M K frictional coefficient B let me say and the applied force F, the displacement is x. This is the system let me take and I want to first of all apply the basic laws of physics to get the differential equation for the system and then those differential equations will be manipulated to put them in the standard state variable format. So the basic equations I did not do any revision the basic equation in this particular case is quite obvious that the input F(t) is equal to Mv dot (t) dot stands for the derivative and here v is the velocity variable. v is the velocity variable and therefore Mv dot is one of the terms over here, the other will become Bv plus Kx please see this is your basic equation. This is a differential equation model of the system. Ft is the applied input, v is the velocity n of the mass and x is the displacement of the mass. I want to rearrange this differential equation in a state variable format.

As I pointed out the rearrangement in the state variable format I am doing because the analysis and design of control systems in terms of state variable format will be more convenient. Help me please. In this case I have to suitably define the characterizing variables as is visible from this equation or in general you please note that the displacements and velocities of masses in translational mechanical system give you a convenient set of characterizing variables. So in this particular case I say x(t) and v(t) are the characterizing variables for my system so I can define the state variables as x 1 is equal to x and x 2 is equal to v as the state variable model of state variables for the system and I want to get the matrices ABC and D which naturally should be giving me the second order model.

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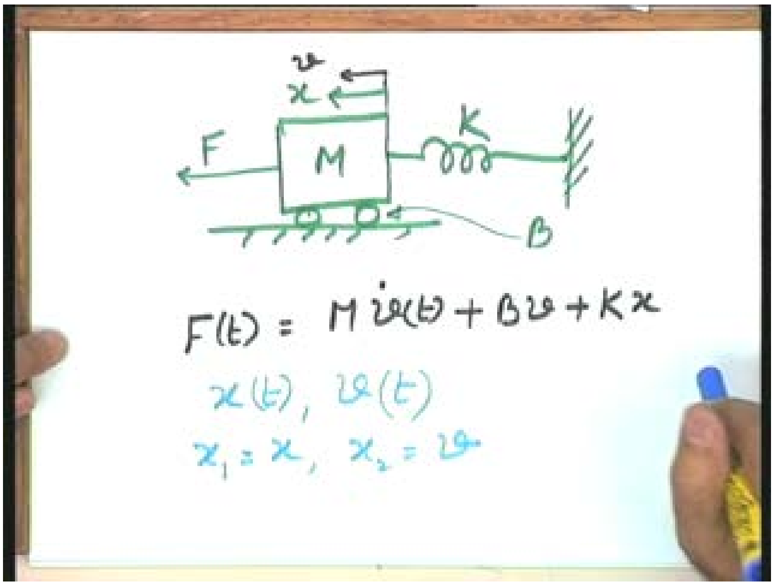

So, since in this particular case the point to be noted is, there are two characterizing variables, the system under consideration is a second order system, the state vector x will be a second order vector and the A matrix will be a 2 into 2 matrix B will be 2 into 1 vector C will be 1 into 2 vector and D if required will be a scalar constant. Very simple you see, please make an attempt and follow me.

In this particular case the requirement of the state variable model is this that I should express the first derivative of the state variables in terms of state variables and input variables. So please note that.

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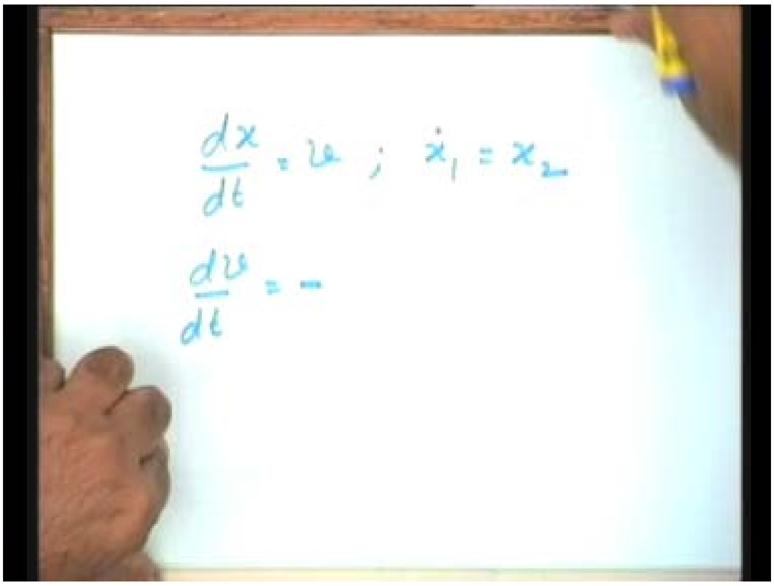

As far as the first state variable is concerned that is the displacement I get a simple equation; dx by dt is equal to v; in terms of x 1 and x 2 it becomes x 1 dot equal to x 2. I hope this equation is okay. x is the displacement and v is the velocity and I want to express the first derivative of the state variables in terms of state variables and the input variables. So this is the first equation I get as far as my state variable formulation is concerned.

How about the second equation?
My requirement is this that I should get this first derivative of the state variable. So I really want to write in terms of dv by dt equal to……. and on the right hand side I should have only the state variables and the input variable. Look at your equation which we have derived. We can write dv by dt is equal to……….. Well I can write keep the original equation with me; the original differential equation obtained using the basic physical laws.

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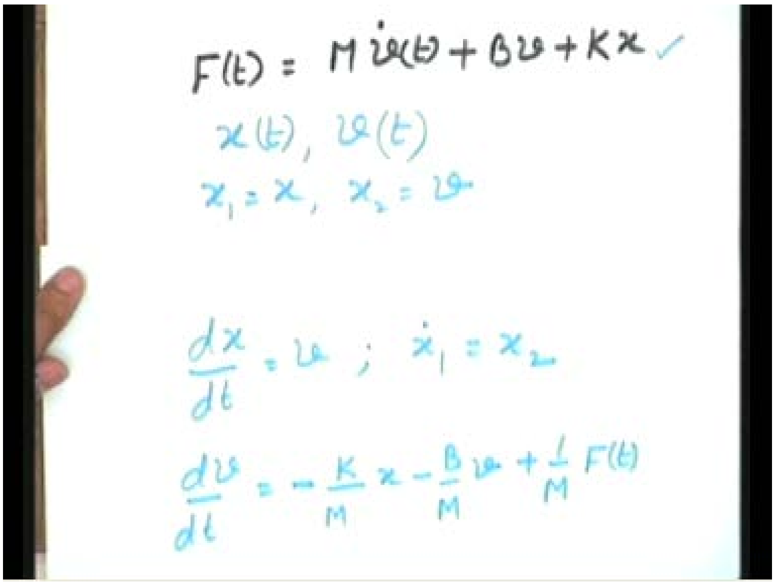


I rearrange this equation to write dv by dt which can be written as minus K by M x minus B by M v plus 1 by M Ft. I hope this is okay.

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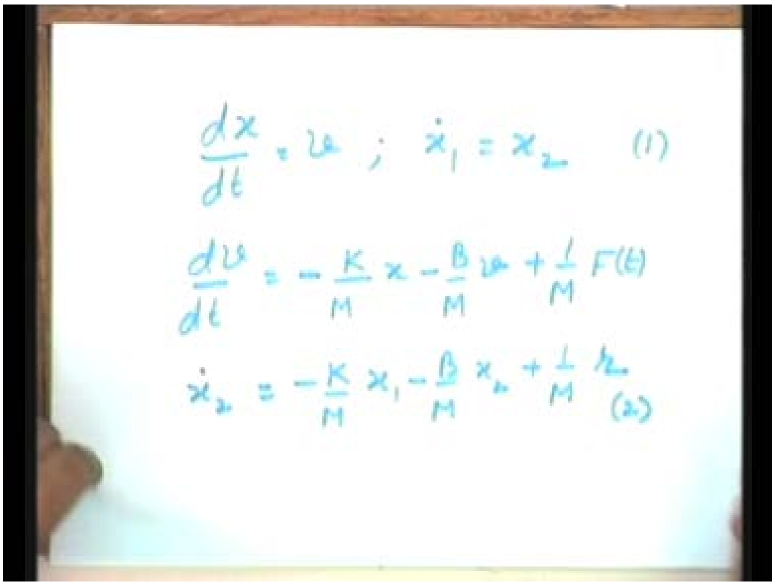

Now, in terms of x 1 and x 2 variables this equation becomes x 2 dot is equal to minus K by M x 1 minus B by M x 2 plus 1 by M r. Now look at these two equations; equation 1 and equation 2. Let me put it in a standard format, the format of the state variable model. Please see whether this is okay.

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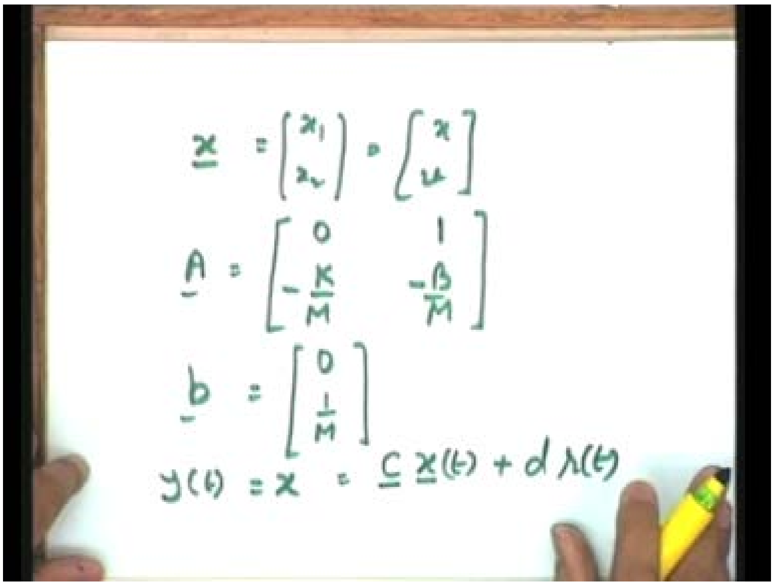

My state vector x is equal to x 1 x 2 is equal to x v the displacement and velocity. The A matrix is 0 1 minus K by M minus B by M. Please see whether this okay? This is your A matrix in our standard state variable model. How about the b vector? Look at the b vector please. The b vector in this particular case becomes a 0 here and 1 by M. this is the b vector, I am getting it in the form x dot is equal to ax plus bu.

How about c and d?

Please see the c and d matrices depend upon the output variable you define. An output is any attribute of the physical system. So if I say that the displacement of the mass is the attribute of interest to me in that particular case, please see, your output variable y(t) becomes equal to x itself and writing it in the standard format cx plus du please see this becomes c x(t) plus d r(t) please see I am sorry I am using r as the input variable and not u so this becomes d r(t).

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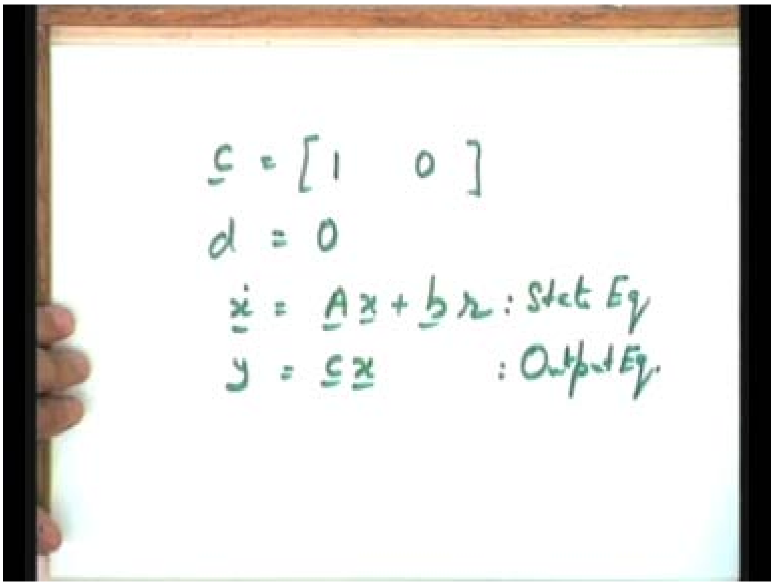


So in this case your c will become equal to a 1 into 2 vector and I am sure you are able to get it this is going to be 1 0 because x is the displacement that is the x 1 variable and your d that is the scalar constant is going to be 0 in this particular case and therefore I have got a model x dot is equal to Ax plus br and y is equal to cx. This let me say is a state equation and this is the output equation (Refer Slide Time: 43:31) state equation and output equation.

Student conversation: can you explain the last line?
Yes, the last line I said about the output you want to say that is y(t). The state variables are x 1 and x 2 the displacement and velocity variables and by definition if x 1 x and v that is the displacement and velocity are known to you at all time in that particular case any attribute of the physical system must be known to you. So it means any attribute of the physical system should be an algebraic sum of the state variables x and v in addition to the input variable which is the force. So I say recall the physical system it is the mass spring damper system; if I say the displacement of the mass is my output variable, I am defining, the user's requirement is that he wants to control the displacement of the mass in that particular case that becomes the output variable of interest. So the output variable I want to express in terms of the state variables. So in this particular case, fortunately, one of the state variables is directly the output variable is it not and therefore your equation becomes y(t) the output variable is equal to x the displacement which is the state variable x 1 itself which in the standard format can be written as c into x plus d into r (Refer Slide Time: 00:45:04).

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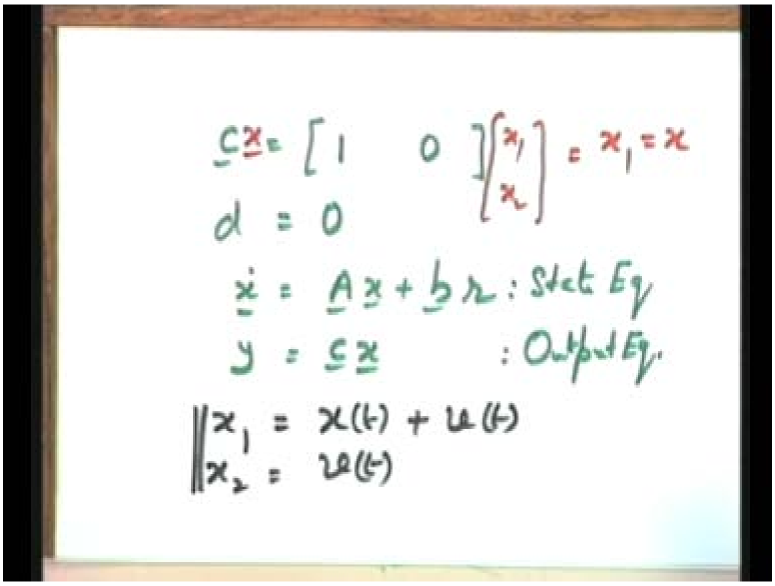


Now c if I take at 1 0 and multiply it by x 1 x 2 because I want to get c into x. I am going to get this as x 1 only which is equal to x the displacement of the mass. So writing it, actually it is so simple that y is equal to x this is the equation. We are writing it in this form and trying to realize the c matrix because A B C and D matrices I want to identify as the basic units of the state variable model which I will be using later in system simulation or for designing purposes. Is it okay please? So this is the state variable formulation we are going to take up (Refer Slide Time: 45:50).

Now before I conclude today's discussion I like to point out one thing. In these two examples I have given you physical variables as the state variables of the system. You will please note the state variables are defined only as a mathematical convenience. I am making a point which is quite important please I need your attention. The physical variables are input to the system and the output of the system, they have necessarily to be physical variables.

Consider for example the mass spring damper system. You have defined the displacement and the velocity as the state variables. Help me, what is the problem if I take x 1 is equal to x(t) plus v(t) as one of the state variables and x 2 is equal to let us say v(t) as another state variable? So, as far as x 2 is concerned it is velocity but how about x 1? Sum of displacement or velocity these two put together is simply a numerical value it has no physical meaning it is not a single physical variable of the system. So as such x 1 and x 2 are not independent physical variables of the system but you can definitely get state equations in terms of x 1 and x 2 and your output y can be defined in terms of x 1 and x 2; your output equation will change in this particular case but for given input the output will be uniquely defined. I hope you are getting my point.

The x 1 and x 2 variables in between are not unique; you can define in all most infinite ways. The only thing is this that your state variable formulation will always give you a unique input for a given unique output for a given input. The state variables are a matter of convenience and depending upon the analysis and design conveniences we many times define different types of state variables.

Take the example of the electrical circuit we have taken. We have taken voltage across the capacitor as one of the state variables. Well, take charge stored in the capacitor as the state variable there is no problem. In that particular case instead of voltage e the charge q will become the state variable. However, whether q is the state variable or e is the state variable the output of interest to you will be uniquely defined for a given input; you will get the same value for all definitions of the state variables. I hope this point is well taken and the discussion on the other forms of state models that is the impulse and the transfer function model will be taken up in the next lecture. Thank you.