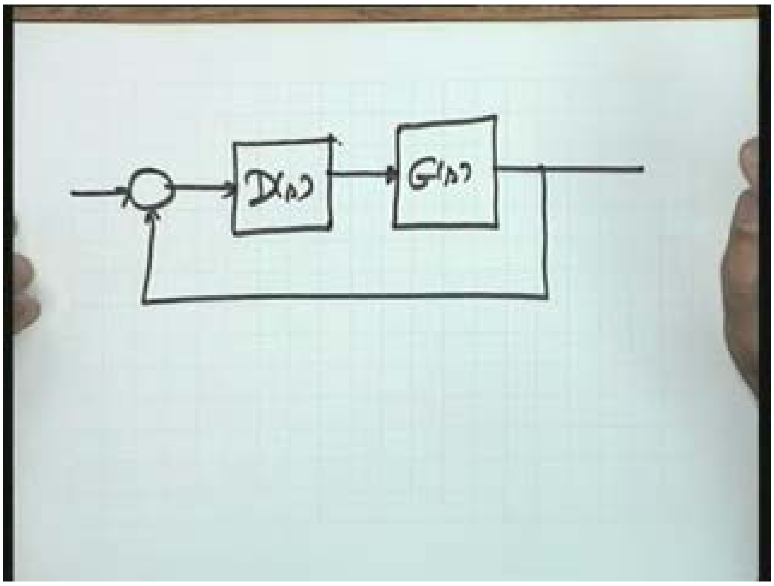
Control Engineering
Prof. Madan Gopal
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture - 29
The Performance of Feedback Systems (Contd...)

Well friends, in the last lecture I told you that, as far as this topic of the performance of feedback systems is concerned, we will be covering two more things: one: the discussion on steady-state accuracy and then a complete design example. I hope we will conclude our discussion today on this subject.

Let me take up the aspect of steady-state accuracy. I have taken it earlier, now I just give it a different quantitative color. The word steady-state accuracy is not new to you; you know that by final value theorem you can determine the steady-state error of a system. So let me give some quantitative information about it as to how to handle it when we enter into a design cycle. For this quantitative description I take this block diagram G(s) is the plant model and let me say D(s) is the controller in cascade and I consider the steady-state accuracy to standard reference input or command signal.

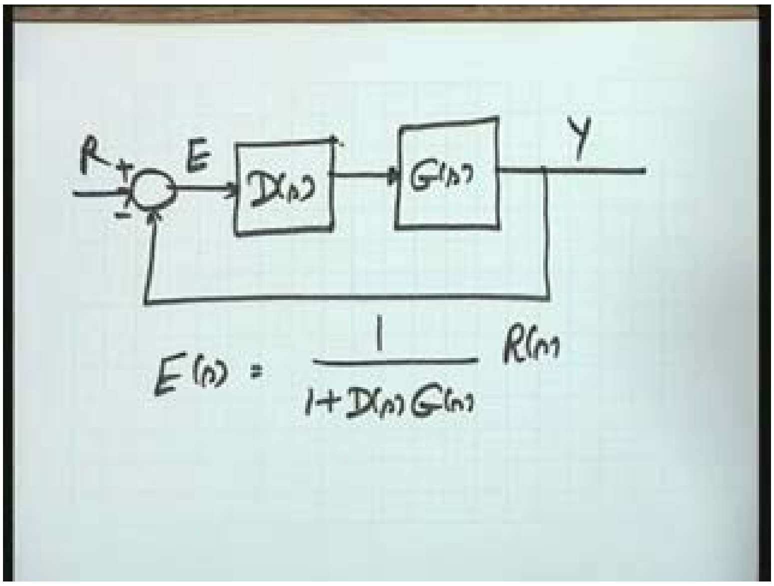
(Refer Slide Time: 1:58)



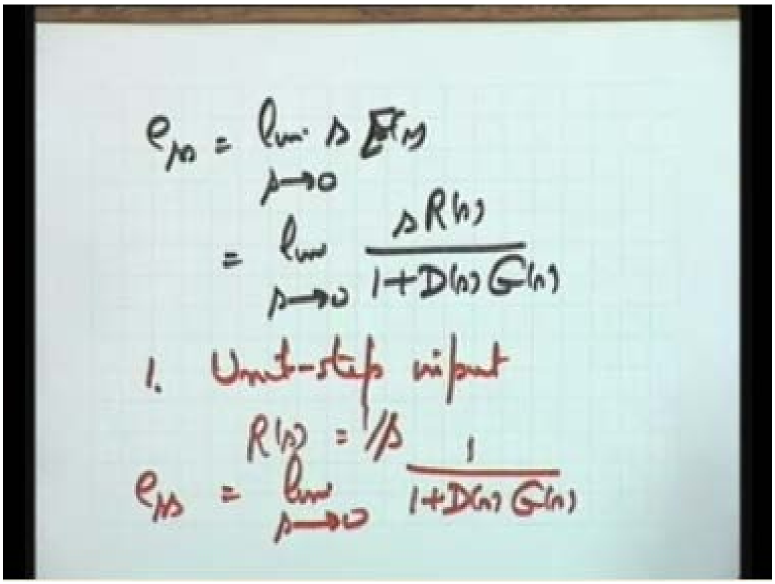
Of course if you have a disturbance signal you can establish a block diagram between disturbance in the output and apply the basic steady-state error relationship between the disturbance in the output there is no problem. But the standard results I am going to give you between this input the command input R, the output Y and the error E it is your standard feedback system. You know that E(s) is equal to 1 over 1 plus D(s) G(s) into R(s). You will note that now the G(s) in the relation we have used earlier is being replaced by D(s) G(s) because the total forward path transfer function or the total open-loop transfer function of the unity-feedback system is D(s) G(s).

I am splitting it into these two parts because studying the total system with respect to this configuration will turn out to be easier when I take a PD, PI or a PID control scheme. So this is that is known to me very well and I know that by application of the final value theorem I can get the value of the steady-state error.

(Refer Slide Time: 2:49)



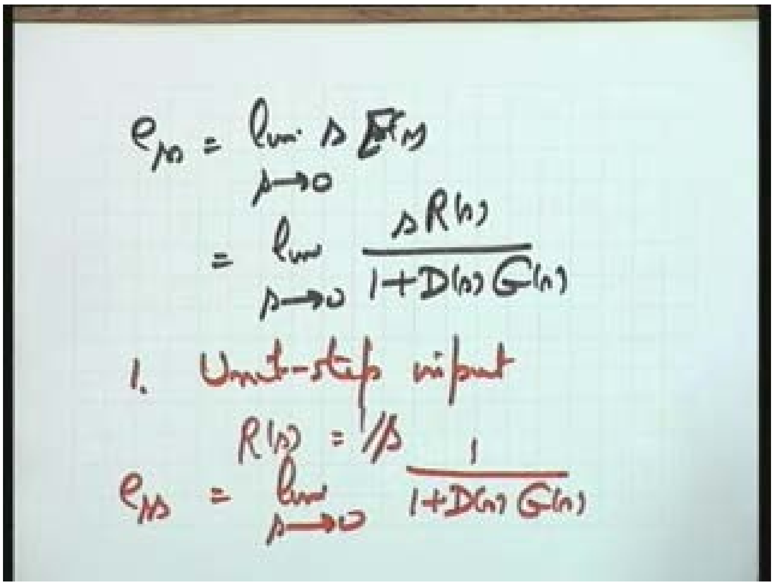
(Refer Slide Time: 00:03:10 min)



So e ss equal to limit s tends to 0 it is E(s) sE(s) this becomes the steady-state error. So let me substitute for E(s): it is limit s tends to 0 sR(s) divided by 1 plus D(s) G(s). Now, please consider the situations one by one. We said that we will handle three situations only. One: a step input so let me say a unit-step input because that really does not matter because the steady-state error will be scaled up or down depending upon the magnitude of input I am taking.

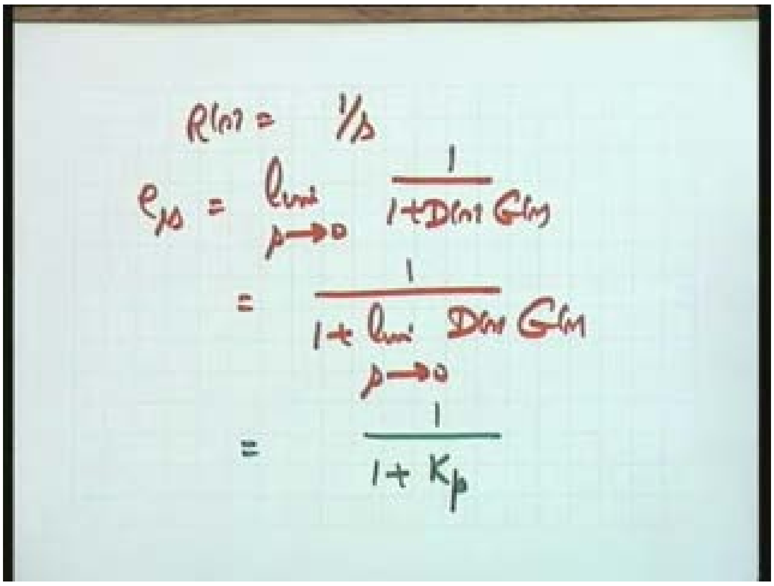
For a unit-step input you know Rs equal to 1 by s and therefore your e ss becomes equal to limit s tends to 0, look it is 1 over 1 plus D(s) G(s) with s gone removed with s cancelled the limit is given by limit s tends to 0 1 over 1 plus D(s) G(s).

(Refer Slide Time: 4:20)



So let me put it again here: R(s) is equal to unit-step 1 over s and steady-state error is equal to limit s tends to 0 1 over 1 plus D(s) G(s) which can be written as 1 over 1 plus limit s tends to 0 D(s) G(s). Why did I write in this particular form; the reason being that I will recognize this as equal to 1 over 1 plus K p where K p in the literature is referred to as position error constant. It is a very important term in the literature because it is extensively used to describe the steady-state error. So it means, instead of saying, instead of specifying the steady-state error of a system to a unit-step input equivalently you can specify that the K p of the system is this much where K p is called the position error constant. The word position is coming because the step input signal in the so-called motion controlled systems is the position input, you please see. I think at this point itself let me explain this particular point.

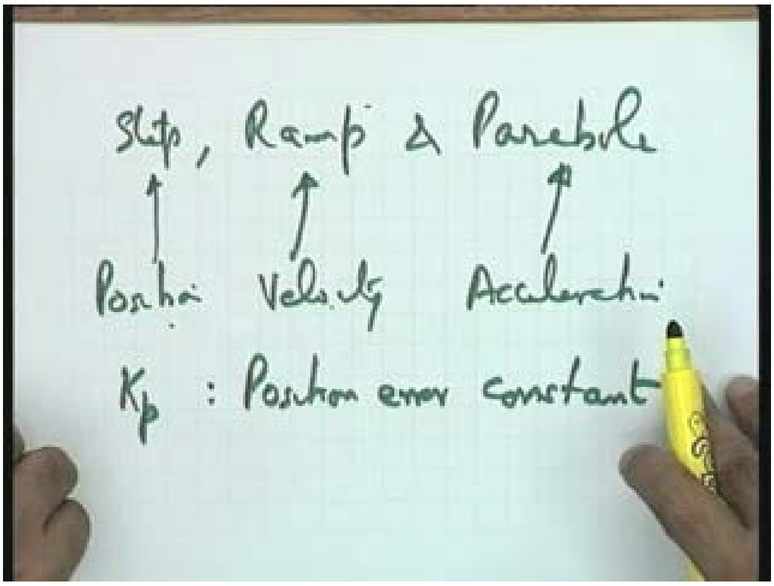
(Refer Slide Time: 5:05)



You have step, ramp and parabola; these are very general terms these are the mathematical terms you see because the function is described mathematically by step, ramp or parabolic function. But in control system diagram it is quite commonly used, this particular input as a position input, this particular input as velocity input and this particular input as acceleration input (Refer Slide Time: 6:05). The reason is simply that in motion control systems the step really corresponds to position, the ramp corresponds to velocity and the parabola corresponds to acceleration. But the meaning of the words position, velocity and acceleration will change if your system is let us say a temperature controlled system or a liquid level controlled system.

Still you see, if you do not confuse yourself then in that particular case the words position and step; velocity and ramp; acceleration and parabolic could be interchanged you see. You keep in mind that these also are functions and these functions mathematically are given by step, ramp and parabolic. So we will go ahead with the terms used in the literature. In the literature K p has been referred to as position error constant but this you will definitely keep in mind that this position error constant really does not mean that it is applicable only to position control systems.

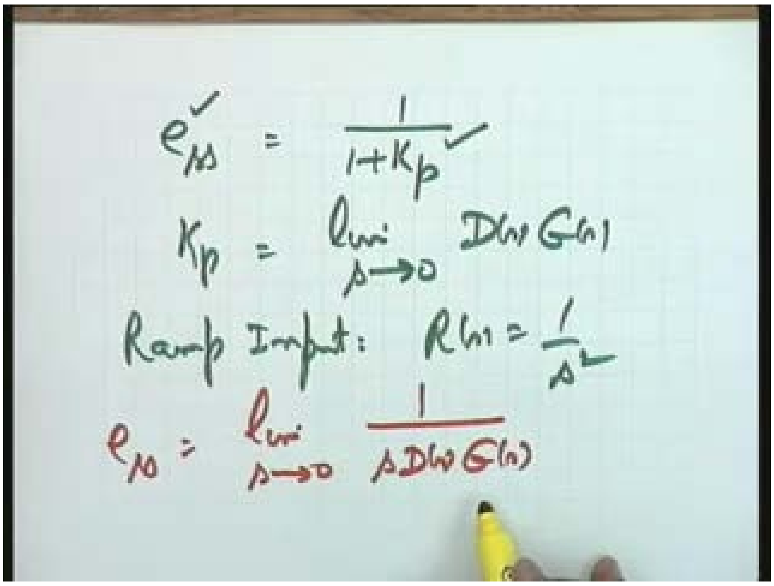
(Refer Slide Time: 7:13)



It is a very general term maybe applicable to any system wherein the step input is being given and for that system I can say now that the steady-state error is equal to 1 over 1 plus K p and this particular steady-state error cannot be given in terms of e ss or K p only where K p let me give you as a final formula as limit s tends to 0 D(s) G(s).

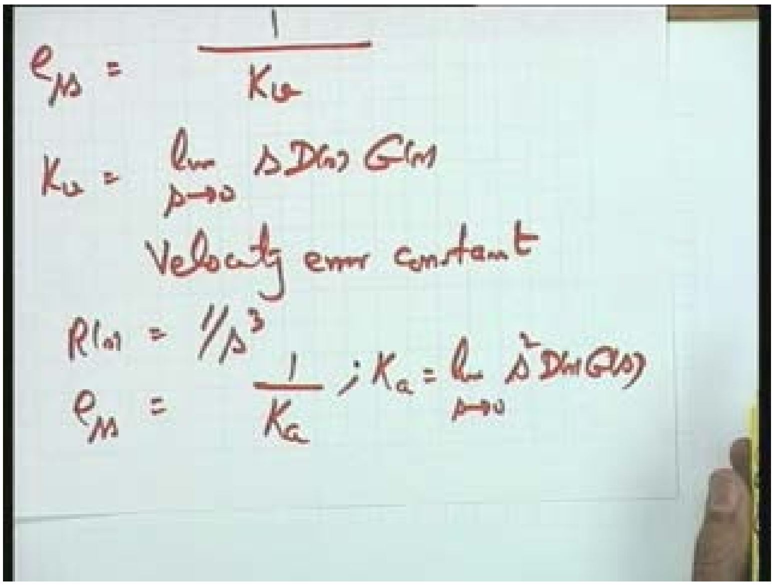
Go to the next input, I go to the ramp input now. The ramp input is as you know R(s) equal to 1 over s squared. Please help me, give me the value of steady-state error for the ramp input. e ss as you can see from the formula you have I am directly putting this result limit s tends to 0 it is 1 over s D(s) G(s) please check this whether this is okay. I have substituted R(s) is equal to 1 over s squared in the standard result and let s go to 0. This is for you to verify, I remember the final result that is why I have put it in this form.

(Refer Slide Time: 8:21)



If that is okay with you in that particular case I rearrange this result in the form e ss is equal to 1 over K v where K v is equal to limit s tends to 0 sD(s) G(s). You see, this K v in the literature is referred to as velocity error constant. You see, I am giving you now equivalent ways of specifying the steady-state accuracy. The only point is this that if I give you system K v is equal to this much it should automatically be clear that the input to the system is a ramp input I need not specify that. While if I give you steady-state error I will have to specify the input along with. So this is an equivalent way K v is the velocity error constant. And lastly and identically of course R(s) is equal to 1 by s square if you take please do it for yourself e ss will turn out to be equal to 1 over K a acceleration error constant where K a is equal to limit s tends to 0 s squared D(s) G(s). This is your K a expression please and that is all you need not go to higher order inputs because we have decided that we will go for ramp, step and parabolic inputs.

(Refer Slide Time: 9:54)

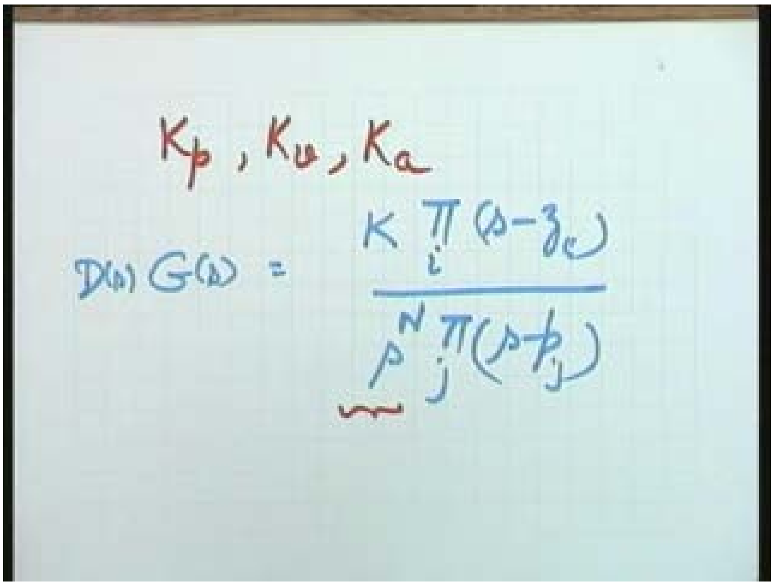


Therefore the conclusion of this discussion is this that the steady-state accuracy can be described either in terms of steady-state error with a specified input or equivalently I may give you K p K v or K a of the system these are the error constants of the system. However, you will have to keep in mind that these error constants are defined for step signals, for unity signals, unit-step, unit ramp or unit parabola but if your steady-state if your input is other than this in that particular case appropriate scaling you will have to do, this you have to keep in mind and the scaling will become straightforward. This is the point I wanted to introduce.

One more point I want to introduce is the following: I have the relationship for these three; let me look at G(s) as a general expression G(s) or D(s) G(s) why G(s); the total forward path transfer function now is D(s) G(s) with respect to the block diagram I have taken. I hope you will not mind if I put it in this form (Refer Slide Time: 11:07): I (s minus z i) divided by s to the power of n j (s minus p j) where z i and p j are zeros and poles respectively and s is also a pole at s is equal to 0 of order N.

Now you can just think in your mind as to whatever type of transfer functions we have so far discussed can always be put in this particular form, it is a very general form, it is going to be convenient in the analysis I am going to make that is why I have written in this form. But this is a very general form, all the transfer functions we have handled so far can definitely be put in this particular form a constant, zeros, the only thing is that specifically the poles which are at s is equal to 0 I have taken out that is all and my attention is going to be focused on this particular point that is why the poles which are on the origin of the s plane have been specifically pointed out otherwise all others are poles, these poles maybe complex conjugate poles, these poles may be real poles and whatever maybe the general nature there is no problem at all.

(Refer Slide Time: 12:15)



[Conversation between Student and Professor – Not audible ((00:12:16 min))]
Anyone who answers his question, can N be negative? Yes, it can be, there is no problem, but I have not taken the zeros on this specifically because that does not disturb me let it be 0. This, as you will see has a specific role to play that is why the poles at s is equal to 0 with N greater than 0 I am taking; he is very right, N can be negative but I want to focus attention on the poles on the s plane. The negative N will mean that there is a 0 on the origin at the s plane, well, in that particular case I am capturing it by one of the z i is equal to 0, I am capturing it that way because it is not disturbing my analysis, is it okay please? I hope you have got this point. So, for me now N is positive, I think yes let me in that particular case make it very clear that N is greater than equal to zero it could be equal to 0 of course so in that particular case there is no pole on the origin.

Now you see that in all the formulas which we have seen for K p K v and K a your limit s goes to 0 in D(s) G(s) or s multiplied by D(s) G(s) or s squared multiplied by D(s) G(s). Now, if you look at this expression if s goes to 0 please see that this is not going to create any problem and same will be the situation he pointed out whether if N is negative and s in the numerator also is not going to create any problem for me because the corresponding magnitude will be 0 and 0 is a defined number, does not create any problem for me. And this also, let s is equal to 0, this does not create any problem for me but if I put s is equal to 0 here this number becomes infinity. Therefore as far as the steady-state accuracy is concerned note this point please this factor s to the power of N is going to play the primary role there; the major contributory factor to affect the steady-state behavior of the system is the factor s to the power of N.

Yes please, suppose [Conversation between Student and Professor – Not audible ((00:14:31 min)) from the numerator, because suppose we have S2 in the denominator S2 as the numerator also then this will not go to infinity… so….. should be the net)) ((00:14:43 min)) right, I get your point; it does not mean that s to the power of N here will go to infinity for every input we are taking. You see, at least this is there that for some inputs for example depending upon the value of N so if there is a cancellation naturally it will not be there but I am making a general statement and after the general statement I am going to substitute specifically the values of R(s) and see which power of N creates problem for us.

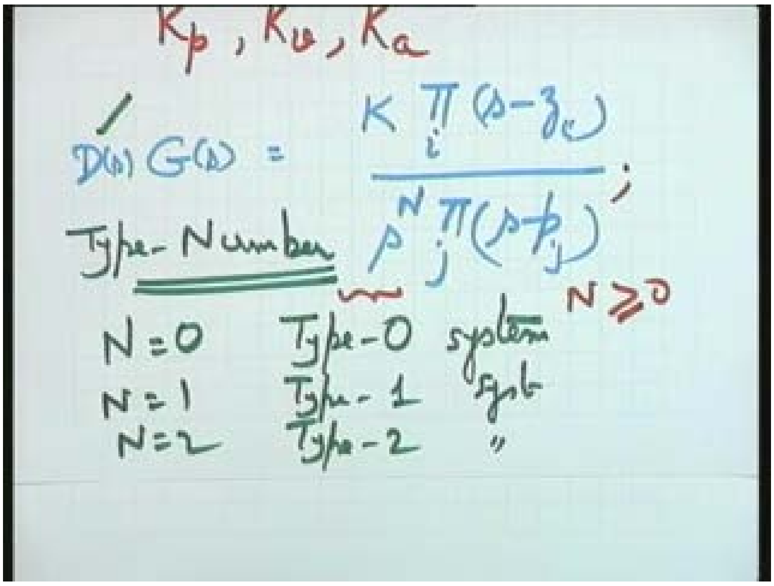
His point should please be noted that for every value of N it is not going to create problem because after all this is going to get multiplied with s squared depending upon the type of input we are going to handle. So I am making a general statement at this point that this s to the power of N may create problem, will create problem let us at this juncture not use the statement so this may create problem and that will depend upon the nature of the input and if it does in that particular case you feel that it is because of the infinite contribution it is giving to the magnitude of D(s) G(s) at s is equal to 0 and specific cases in response to his question will automatically come when I take the different values of R(s) here.

So it is because of this only you see a specific name has been given to the system with respect to s to the power of N. Please see that if N is equal to 0 the system is referred to as a type-0 system because it is a major role to play as you will see. His question also will get answered where I take the values of inputs but this s to the power of N, this s to the power of N which corresponds to poles on the origin or which corresponds to number of….. see this statement please number of integrators in the forward path of the system is it okay this 1 over s to the power of N is N integrators, number of integrators, why do I make this mention because you can suitably with the help of your controller D(s) control the number of integrations in the forward path.

PI controller is an example by which you are introducing an integrator in the forward path by design; you are intentionally introducing integrators and it will become clear by this analysis as to why the steady-state error to step input will always be 0 when an integrator is introduced, this will become clear. We have always….. all the examples we have taken so far we have seen that the steady-state error was zero whenever there was an integrator this will become clear but at this point this may please be noted that the number of poles at the origin is same as number of integrators in the forward path and number of integrators you can control by suitable control on D(s). Therefore if you require a specific type of steady-state behavior maybe you can realize it by controlling number of integrators in the forward path.

This question which I am posing to you will get answered automatically when I take the different values of N with different values of input combinations. So I made a mention N is equal to 0 is referred to as a type-0 system, N is equal to one obviously is a type-1 system and N is equal to 2 is a type-2 system; not that the type number stops here it will be type-N system in general but we are stopping here with the observation that normally we do not come across systems which are more than type-2 so this is known as type-number of the system. Type-number is primarily related to the steady-state error behavior, the type-number.

(Refer Slide Time: 18:29)



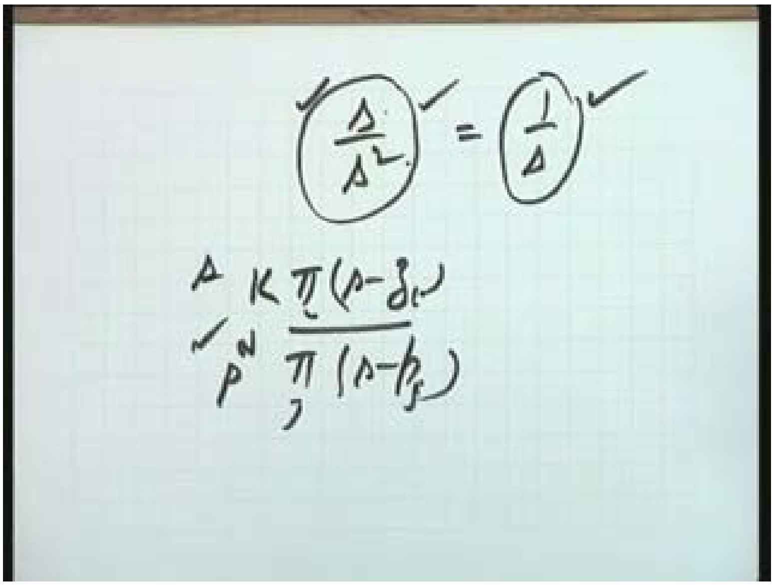
Now let us see how the type number is going to affect, yes please. What would be the type number of the system say s by s squared? s by s squared is a type-1 system obviously. That means ((you are trying)) ((00:18:42 min)) you are cancelling and 0 surely but you said that in practical cases you can never…… you see that when you say s by s squared it means you are giving me a model so it means this is 1 by s; now if you say the practical questions it means practically this system with this particular transfer function will approximate an integrator but will not be an exact integrator.

After all when you say s by s squared it means the 0 is at least close to the pole at the origin. So it means the real system… one thing is the 1 by s, you have a perfect integrator so this particular system with this type of behavior will approximate an integrator because of imperfect pole-zero cancellation. But it does not mean that there will not be equivalent effect the only thing is that the perfection which his represented by this equation will not be realized in an actual system, this is what I made a statement.

Since type number is defined with respect to mathematical model I will call this as a type-1 system. [Conversation between student and professor: Then can we add negative values of N2 Refer Slide Time: 19:44] negative values of it…. you see that if there is a negative value of N2 yes it means there is a numerator in the s, there is a pole in the s. So what I want is this that this particular transfer function which you have written you have written after all possible pole-zero cancellations have taken place. I am not retaining any pole over here. So it means this final form this N number has come after all cancellations have taken place again. By pole-zero cancellation I mean as far as the mathematical model is concerned it is the perfect pole-zero cancellation. The statement about robustness which I made that the pole-zero cancellation which you show up in your mathematical model is not exact when you go the actual system but yes it approximates to that value in the actual system as well if the parameters do not wander too much around the nominal value.

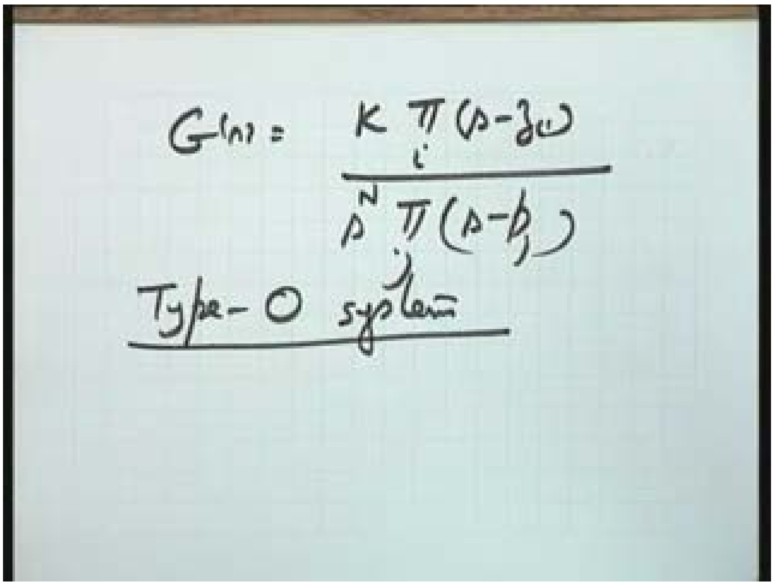
So his point may please be noted. When I am defining the type number I am really not assuming that there will be an s in the numerator; I am assuming them this particular transfer function which you have written has been written after all possible pole-zero exact pole-zero cancellations if any in the model. I hope this point is also well taken.

(Refer Slide Time: 20:59)



If that is okay then let us take one by one the situations; keep G(s) in mind, G(s) is equal to K sigma i pi i (s minus z i) s to the power of N j s minus j this is the transfer function. Now help me please; consider a type-0 system. First of all can you give me a physical example of a type-0 system, we have really put in lot of effort on modeling, lot of time was spent on modeling so in between some questions if I give you you must answer it. Type-0 system I want some physical example.

(Refer Slide Time: 21:36)



[Conversation between Student and Professor – Not audible ((00:21:48 min))] Proportional control, a constant amplifier, a constant amplifier. You see, you give, this is a device you have given which can be modeled as a type; constant amplifier is a zero-order system rather.
A type-0 system will have dynamics as well; you give me a control system. His answer is right I do not challenge that but an amplifier is not a control system by itself, I want you to give me an example of a feedback control system with dynamics but type-0 system. Sir, any second-order system… equation…… that I could answer that is very visible here, I want physical example; mathematically it is visible here you see, why second-order, you simply take s here in….. [Conversation between Student and Professor – Not audible ((00:22:39 min))] temperature control system, yes, you recall the models.

In a temperature control system the model was obtained as some k over s plus 1 that was the model, liquid level control system the model was obtained as k over s plus 1 and I think in one model it was a second-order term s plus 1 into s plus 2 but still it is a type-0 system. Now the general statement motor control is wrong because the motor transfer function was k over s into s plus 1. It is a type-1 system s into s plus 1 but I do not say that your answer motor control is totally wrong. But if you use your motor in speed control application then it is a type-0 system because your transfer function with respect to speed was k over s plus 1 please see that, you have to equip yourself with all these things. When I am asking a question I am asking with respect to physical interpretation.

Take a motor control system but use motor and speed application it is a type-0 system. But if you use your motor in a position control application it is a type-1 system. You see the order; somebody said that take any second-order system. In defining the type of a system order of the system is not important. It could be an nth order system, it could be 100th order system but what is important is the number of integrators in the forward path. The order is not important you could take any order, how many integrators are there in the forward path that is why I wanted you to recall the type of models we have derived.

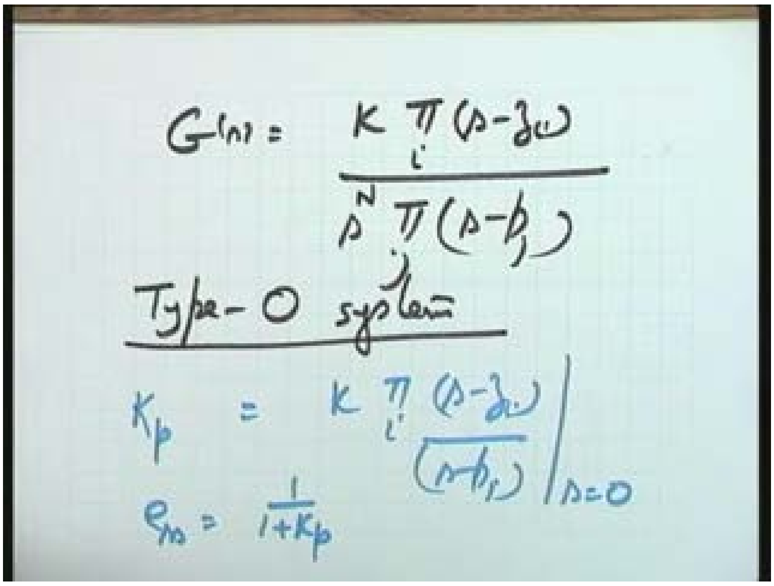
You go back and look at your notes and you will find that these statements are right: the temperature control examples, liquid level control examples we have discussed in the class, we have modeled them as type-0 system, speed control as type-0 while position control as type-1, any example of type-2 please since we are discussing it? We have discussed some example, either in the class or in the tutorial that I do not remember at present but any example of type-2 system you could give me; at least one example comes to my head which I have discussed maybe in the tutorial [Conversation between Student and Professor – Not audible ((00:24:51 min))] attitude control, yes, attitude control of a satellite was an example whose model we obtained as k over s squared a simplified model and s squared means, well it is a second-order system but this second-order system has got two integrators in the forward path and hence it is a type-2 system.

So please see that I have not given you any example to my mind which is a type-3 system
I have not given you any example. It does not mean that system example cannot exist but I simply say that mostly it does not exist. However, in a very specific design, you may really….. because of very high demand on ……. you see, I may give you an example; some of the radar systems could be type-3 system could you tell me why; very interesting discussion is going on that is why you say I am making this statement. Some of these radar systems, antenna control systems could be type-3 systems even.

When I said that most of the systems will be either type-0 type-1 type-2 except in very specific situation can you imagine that an antenna control system will be a type-3 system, why? If it is yes then the reason should be given…….[foreign language………. ((00:26:07 min))] [Conversation between Student and Professor – Not audible ((00:26:08 min))] you see that it depends, you should answer it this way; if a type-3 system comes it means it is very much dependent upon the nature of the input. You take a numerical control, a machine tool is not going to revolve and give you a job like this it will be a ramp, a parabolic shape or some systematic shape but in a case of antenna control system what is the input the input is maneuvering of the target aircraft. You see that you are after all targeting your antenna on the aircraft where your missile is to land, if there is no static accuracy with respect to the enemy's aircraft in that particular case the missile cannot land at the appropriate place. So you want, if you find that it is the military aircraft you are pointing, so military aircraft means the maneuvering will be so fast that you want the capability on static accuracy to a very high standard, so in that particular case you may require that the acceleration error should also be 0 as you will see that with a type-2 system error to acceleration inputs cannot be 0, the input acceleration input should also give you a 0 error that may be a demand on the system in that particular case you may design a type-3 system in that particular case. But please note that stability of the type-3 system is a very risky proposition so it means you will have to make a very careful design so that you can really not pay a heavy price in terms of instability of the system.

So the example I am giving you that type-3 system possibilities is also not ruled out but such applications are rare and mostly we go for type-0 type-1 and type-2 systems. This point on type-3 system will become further clear when I give the acceleration error constants. I will comment on this again. Type-0 system since you have referred to this point, help me please what is the value of K p for a type-0 system? This is equal to…. look at this point, it is K pi i (s minus z i) over (s minus p j) s is equal to 0 so what is the steady-state error to step inputs; the steady-state error to step inputs is 1 over 1 plus K p which is finite.

(Refer Slide Time: 28:52)

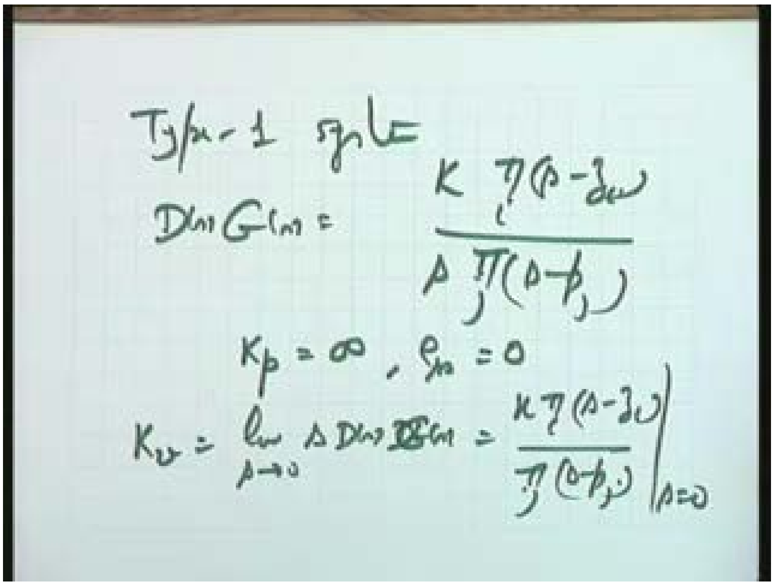


How about the steady-state error to ramp input; that is what is K v. I think let me not write it you can keep on answering quickly. What is K v in this case please? A type-0 system. I want you to give me the value of K v, I am sorry take D(s) G(s) total including the controller; is K v 0? Is the answer given by him is correct K v 0? So what is steady-state error to ramp input? Infinity, same will be the situation with K a please, K a is 0 and the steady-state error to acceleration input is infinity so it means please see that your type-0 system cannot meet high demands on steady-state accuracy it can at the most give you reasonable steady-state behavior if the inputs are only the step inputs or the constant inputs this should be very clear now. A type-0 system cannot meet high accuracy demands. So, if high accuracy demand is there as you will see you will have to intentionally introduce integrators in the forward path.

Is the table complete please? K v is equal to 0 and K a is equal to 0 and hence steady-state errors is equal to infinity. Anyone who could not get this point please? I hope this is with all of you. In that case I go to type-1 system please. I think you can immediately do the necessary calculations; I will be interested more in the conclusions based on this s only j (s minus p j) look at this type-1 system please. A type-1 system will always have K p equal to infinity and therefore e ss is equal to 0 as far as step inputs are concerned. Therefore whenever a type-1 system is involved you should not calculate the steady-state error.

In one of the tutorial classes one of you observed that that the steady-state error is always 0 it means it has got reasonably good steady-state accuracy steady-state behavior but if you see K v K v in this particular case is limit s tends to 0 sD(s) G(s) and you can see that this is equal to K pi i (s minus z i) over pi j (s minus p j) s is equal to 0 a finite value. So it means a type-1 system will give you a finite value of K v and hence steady-state error for ramp inputs.

(Refer Slide Time: 31:21)

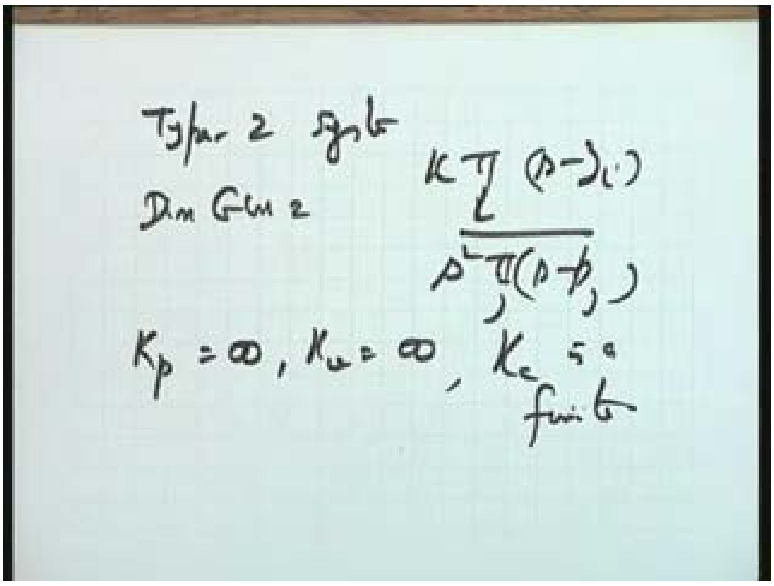


But now I leave it to you, you calculate K a. K a in this particular case will turn out to be 0 and hence the steady-state error to acceleration input is infinity. So it means, by introducing one integrator in the forward path you have improved the steady-state behavior of the system, you have improved this.

Now, lastly the type-2 system. type-2 system if I take well D(s) G(s) is equal to K pi i (s minus z i) over s squared p i j (s minus p j). Well, the calculations will immediately come K p is equal to infinity, K v is equal to infinity and K a is a finite value. So it means a type-2 system will have zero steady-state errors to both step and ramp inputs; a good system from the point of view of steady-state accuracy. But you please note that the price is being heavily paid and the price is in terms of stability requirements, two integrators in the forward path are going to create problems for you as far as stability design is concerned. So the situation is not that rosy that is why we have to strike a compromise a tradeoff between the accuracy and stability. But this I see, a type-2 system will have zero steady-state error to both step and ramp and a finite steady-state error to acceleration inputs.

Now you fit in the argument I gave for the position control system. If you want the error to be zero for acceleration types of inputs also in that particular case you will like to go for one more integrator and hence it will become a type-3 system and such applications are rare very few wherein you require three integrators in the forward path.

(Refer Slide Time: 33:22)



In typical industrial control applications I will say you are mostly satisfied. General statements I am making; very general application where the user has not given a very specific demand to you in that case I say that you are mostly satisfied with zero steady-state error to step inputs and finite steady-state error to ramp inputs. If the user has not specified anything else in that case mostly what you do is in that particular case, can I make a statement that if the system has an integrator inbuilt into it you may like to go for a position or a PD controller. But if this system has not an integrator inbuilt into it you may like to settle for a PI controller is it okay? The statement please?

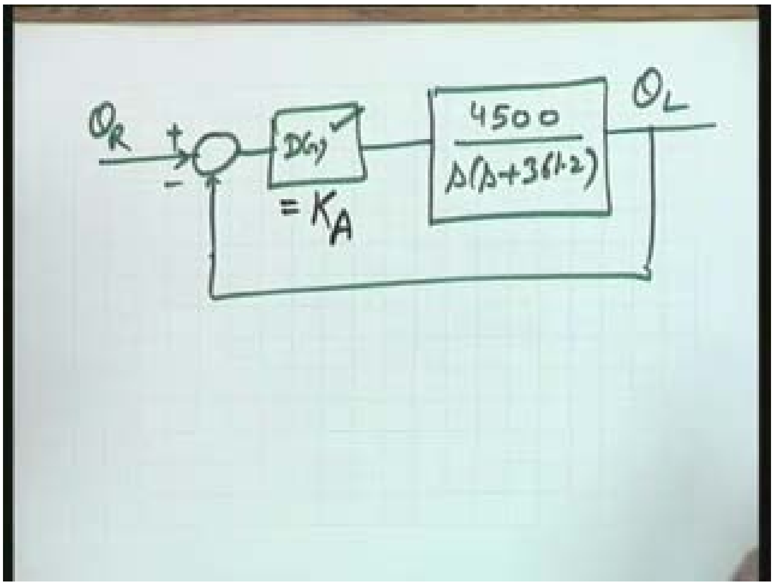
I am making a statement that in general most of the control systems we come across, general applications where there is no specific demand from that user in that particular case a zero steady-state error to step and a finite state error to ramp is more or less okay. So as a general guideline if you have to make a quick decision as to what type of controller you want in that particular case you look at the plant if the plant is a type-0 system in that particular case, well, if you go for an integrator in the loop that will be better unless the user has given you a specific demand because the steady-state accuracy will be improved. But if the plant itself has got an integrator for example, the motor example you gave, in the position control application in that particular case you may try at least try whether the performance is satisfactory for only position control or with a PD controller because a PD controller will not increase the type number of the system while an integrator increases the type number of the system. Is it okay please?

[Conversation between Student and Professor only a decontrolled… the time number of a system ((00:35:32 min))] only a decontrol, yes, a question please, a decontrol only if you take in the forward path again it is a question where the cancellation comes. So if it is a type-0 system it does not…. otherwise mathematically speaking yes but since the decontrol the derivative control does not affect the steady-state error of the system, normally only decontrol is never used in practice it is a PD control: proportional, proportional derivative, proportional integral, proportional integral derivative these are the combinations we use.

Mathematically yes, on the basis of whatever cancellations you get your answer will come on that because you are working on a model. So, by cancellations whatever answer you get it is a valid answer but practically only decontrol is not used because it does not affect the steady-state behavior and you go for PD type of control in practice where the type number will not be affected.

Any other question please?
If this is okay then look at the time and the requirement of a design example. So let me make an attempt otherwise I will probably leave it in between and complete it in the tutorial because next time definitely I want to go for a new topic. I take a complete design example as a position control example you have just now referred to. This is your theta R, this let me say is a D(s) a controller, this I take as the plant, I have a typical value in mind I will not give you the numerical values here (Refer Slide Time: 37:18) s plus 361.2 it comes from some specific values so I am using those values over here theta L the load and I want to design a controller D(s) for this particular system.

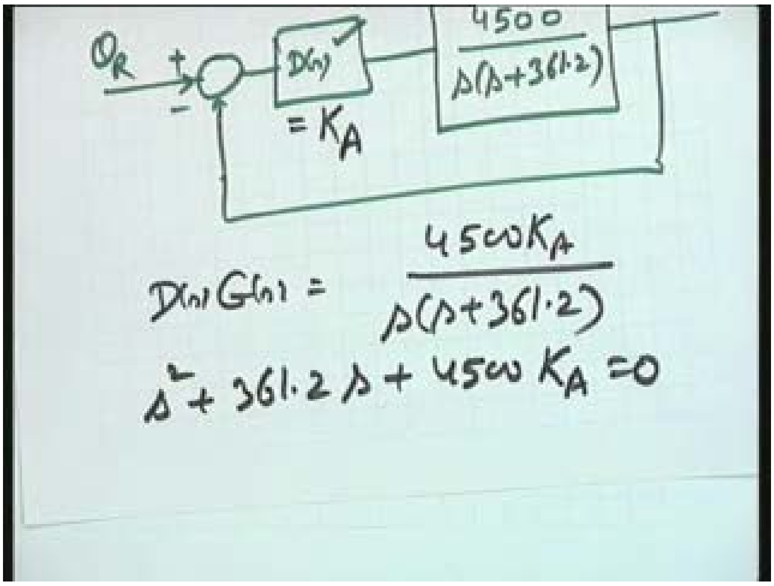
(Refer Slide Time: 38:03)



Keep the physical system in mind. The physical system is a position control system it is an amplifier it has a motor, it has a power amplifier, it has a gear train it has everything which a position control system has. So I am modeling it this way so that I can go ahead. Suitable numerical values of all the parameters have given me this value of G(s) and naturally in this particular case I will first like to try only a position controller because the system is a type-1 system. So in that case I try only an amplifier K A and see whether my performance requirements are satisfied or not. So with this K A I find D(s) G(s) is equal to 4500 K A divided by s into s plus 361.2. If my performance requirements are not satisfied with only K A then I may go for a PD controller or a PI controller or even a PID controller.

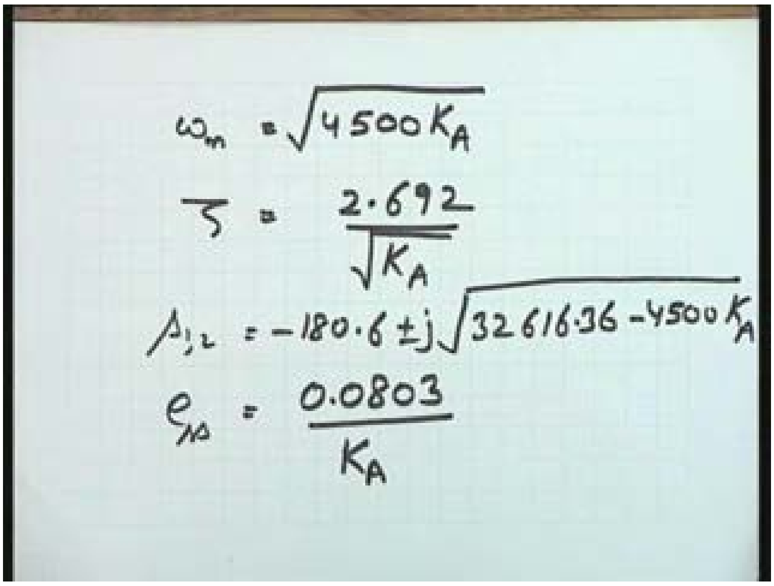
So in this particular case you find the characteristic equation I want to determine the zeta and omega N. Fortunately please note, the characteristic equation for this system is coming out to be in the standard second-order form, you can just see: s squared plus 361.2 s plus 4500 K A equal to 0 is the characteristic equation and this is a function of K A only.

(Refer Slide Time: 38:57)



Now look at these terms. These, you are very well equipped to make the calculations from the characteristic equation given to you.

(Refer Slide Time: 00:39:10 min)

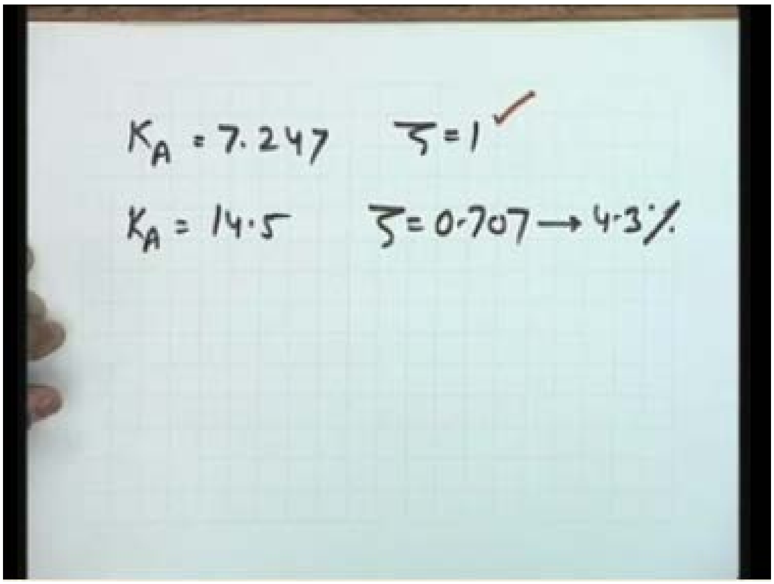


I get these values of different parameters. You can just take down these values and satisfy yourself that these values are correct just by applying the basic relations we have discussed so many types. The natural frequency omega n is 4500 K A under the root a function of K A, the damping is a function of K A these are the closed-loop poles of the system again a function of K A, steady-state error in this particular case turns out to be this. Tell me please whether the steady-state error is an error to step input or to a ramp input I have not written here; should it be to step input or to ramp input please?

[Conversation between Student and Professor – Not audible ((00:39:53 min))] step input, step input no. for step input it is decidedly 0, how can I give it in this expression. I have not written for you….. you have to note it that this has to be ramp input neither the step nor the acceleration because I know that it is a type-1 system. For step input it is 0 and for acceleration input it is infinity, it has to be finite only for ramp input and therefore for ramp input the steady-state error turns out to be this value. So these are the basic relationships I have.

Now let me take some numerical values please. I hope you have noted down, all of you; just verify and if there is any error you do please inform me about that otherwise hopefully the calculations are right.

(Refer Slide Time: 00:40:37 min)



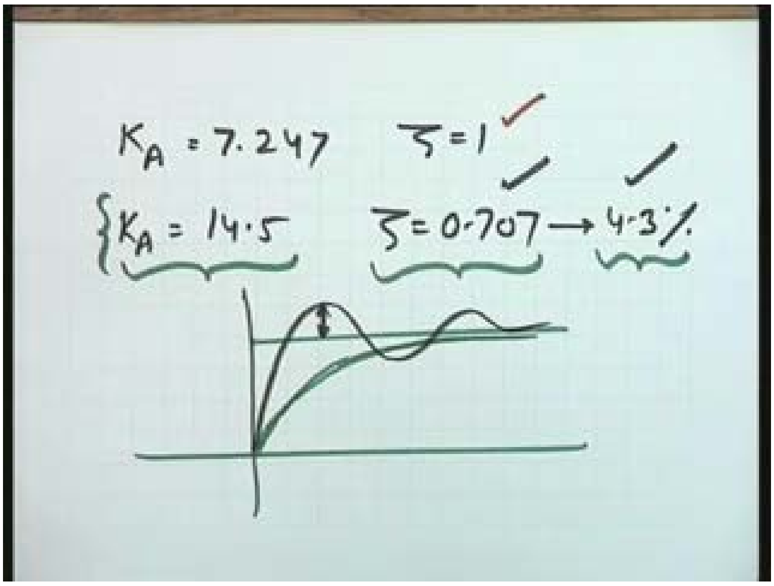
Now let me take the value of KA equal 7.247. Why did I take this value? Just for the sake of comparison I have taken a value which gives me zeta equal to 1. So you see that I have used this expression only (Refer Slide Time: 40:53) to get the value K A I am giving you. Do not worry about the numerical calculations at this stage, look at the concept. I have taken a value of K A so that zeta equal to 1 a critically damped case. Corresponding to this zeta do not worry about these statements (Refer Slide Time: 41:10) corresponding to this zeta equal to 1 look at the response I am going to get the response for the standard second-order system I am going to get is this.

Now I know that for a general application this value of K A is equal to 7.247 may not be acceptable to you the reason being that the settling time for zeta is equal to 1 is reasonably high. Unless the user tells you that I want zeta is equal to 1 you will not like to design for a zeta equal to 1 because the settling time is reasonably high. Of course the rise time will also….. I mean rise time as such is not defined because….. what is rise time in this case please? Just you apply, literally if you apply your definition it is infinity, rise time because it will cut that axis only at infinity so rise time actually is defined for under damped system.

Let me not use the word rise time for this application but settling time I can of course define and the settling time in this particular case is reasonably large as we have discussed from the settling time or normalized settling time versus zeta curve you can recall. So in this particular case it means I will really like to go for a moderate value of zeta and now using the same formula i get K A is equal to 14.5 which gives me zeta is equal to 0.707 which gives me peak overshoot is equal to 4.3 percent, all the calculations you can verify easily.

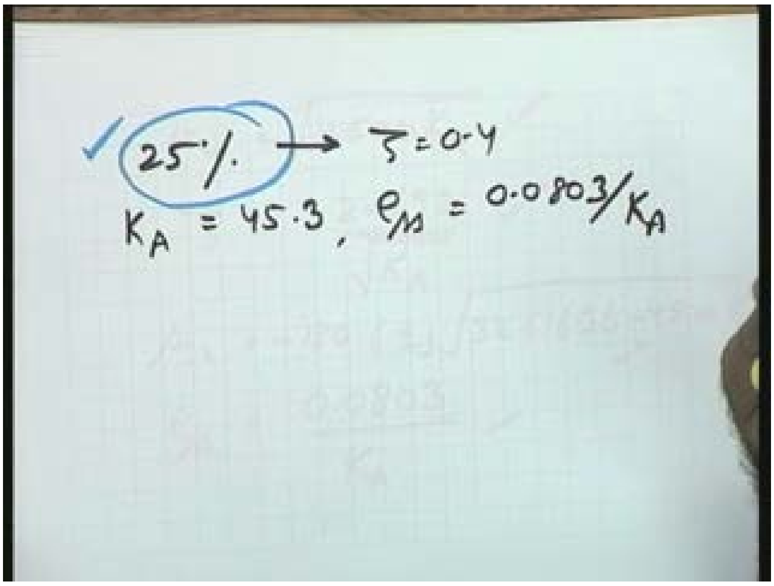
Now you can see that as far as peak overshoot is concerned maybe it is acceptable 4.3 percent is not large peak overshoot. Well, zeta is equal to 0.707 is a good value because the settling time in this particular case is going to be low, probably the lowest settling time, probably the lowest you see and correspondingly the rise time in this particular case will be low the peak time will be low everything is quite good and 4.3 is the only price I am paying. What is the price I am paying? Overshoot of 4.3 which looks quite reasonable. Hence, it means in this particular case this value is alright.

(Refer Slide Time: 43:18)



Now you see that what is the corresponding steady-state error. (Refer Slide Time: 43:29) let me go back to the original slide. The steady-state error in this particular case is 0.0803 divided by K A the value of K A you have taken as 14.5 so this is the steady-state error. So I can simply say that in this particular case the steady-state error is fixed at this particular value 0.0803 divided by KA where KA is equal to 14.5. But if you want to improve on the steady-state error what you will do is you will have to increase the value of K A. So it means if the steady-state accuracy demand is high you will have to take the large values of K A in that particular case you will have to allow lower values of zeta though zeta is equal to 0.7 is moderate. So, if you allow lower values of zeta what will happen? The price being paid is that the peak overshoot will increase. So it means the situation of no further tradeoff will come that you will say that no more values of K A if the peak overshoot is exceeding the limit you have imposed on the system. After all, otherwise the system may become unstable.

(Refer Slide Time: 00:44:41 min)

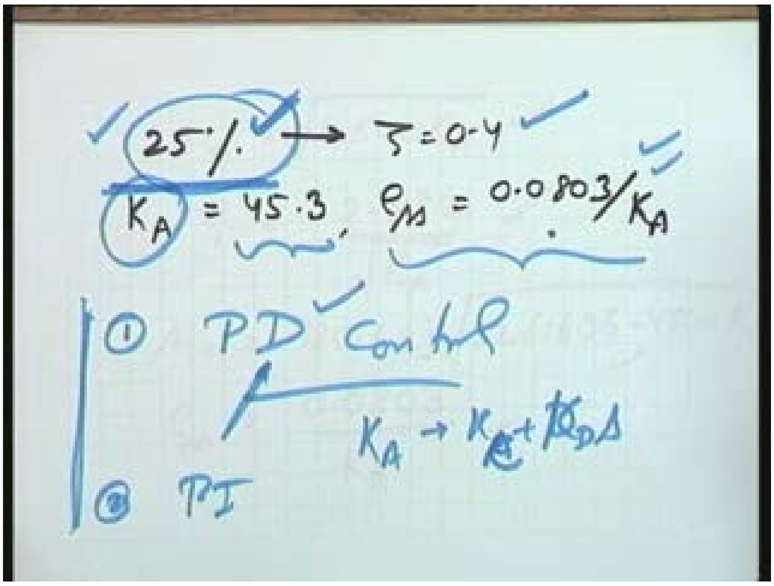


Let me say that the limit you have imposed on the system is that, look, you see, your peak overshoot should not be higher than 25 percent. Now you see, though the settling time will be larger but still you are allowing the lower values of zeta because you want to get better steady-state accuracy, you want to get better steady-state accuracy please so in that particular case you have allowed 25 percent over here and this corresponds to zeta is equal to 0.5 and this zeta is equal to 0.5 is 45.3 in this particular case and the steady-state error correspondingly is 0.0803 divided by K A.

Now you will say to the user to the person for whom you are designing the system hands off because you will not like to decrease the steady-state error further the reason being that this will be dangerously close to instability. So, as far as proportional control is concerned you have reached the limits. Now, since I cannot take PD and PI controller so in just one minute you give me the clue. So, if you want to improve the steady-state accuracy further what will you do please? Still you do not want this particular 25 percent requirement to be violated.

One thing is there that yes, first thing is that PD control is also possible you can try this please. though PD control as such does not allow, the derivative control as such directly does not affect the steady-state error but what it will do; you see that the derivative action will provide for the damping so what you do is it is going to allow you to take larger values of K A without violating the requirements of 25 percent of overshoot, a simple exercise for you, you replace your K A by K c plus K D by s and satisfy yourself that it will allow you to increase the value of K c the proportional term because the damping is being provided additional damping is being provided by the derivative term. So a PD controller can also improve the steady-state error please see; the reason being that, well, the increase in K A the risk was the damping was being reduced. Well, now you are adding a derivative term, an additional friction into the system is being put in so that the damping is restrained at the required value and hence it is going to permit a higher value of K A a higher value of K A means a lower steady-state error.

(Refer Slide Time: 47:28)



And second, of course, you have answered, PI control is definitely going to reduce the steady-state error to 0 the only thing is this that you have to be cautious in the design that your transient accuracy requirements are also satisfied. And these things cannot come analytically, graphical methods, the root locus method and the frequency domain methods very well established methods of control system design will help you design the values of PD, PI or PID controllers so that the steady-state and transient accuracy requirements are satisfied and this is the subject of discussion next time, the root locus method we will start with, thank you.