Control Engineering
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Lecture - 27
The Performance of Feedback Systems (Contd.)

Well friends, let us resume our discussion from the point where we left last time that is transient performance specifications were under discussion. Recall, we have decided that a step response may be a unit step response is good enough to specify the transient response of a standard second-order system. I will see to it that I convince you, the same specifications will hold good for higher order systems as well.

So the specifications specifically we had taken were the rise time, here is t r, this is your peak overshoot (Refer Slide Time: 1:36) we have named it as M p and this is the time to peak it is t p and the fourth specification we had taken was the settling time, let me say this is band which normally I said is taken as 2 percent or 5 percent, if the response goes into this band and never comes out again then the point at which it enters this particular band is called the settling time because any value within this band is acceptable to you and you say, practically, this system has settled.

For a standard second-order system we had taken up the transfer function Y(s) was equal to omega n squared s (s squared plus 2 zeta omega ns plus omega n square) this is your response transform in which r(s) is equal to 1 by s has been taken.

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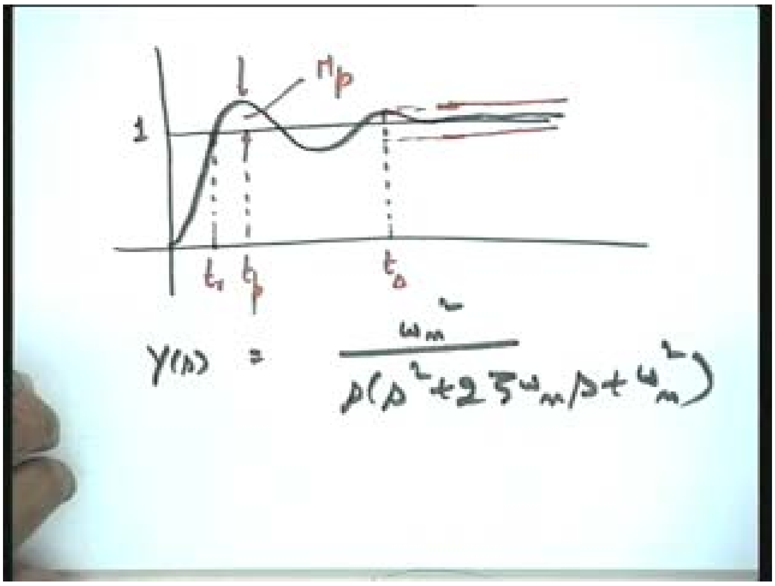

Now the inverse Laplace transform of this gives me y(t) this is equal 1 minus e to the power of minus zeta omega nt over 1 minus zeta squared sin of omega dt plus theta where theta is your damping angle which has been taken as equal to cos inverse zeta. Recall the relationships we have derived: t r is equal to pi minus cos inverse zeta divided by omega n 1 minus zeta squared is the rise time.

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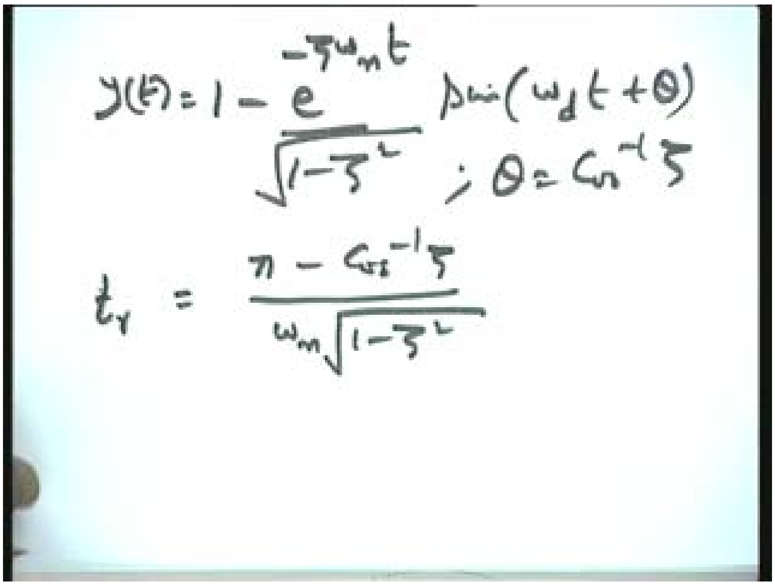

Now let me make a mention here, a passing comment here as to how the rise time varies when zeta end omega n change. You find from this particular expression that rise time is a function of both zeta and omega n. You see that practically the effect of zeta on rise time is small. I can tell you this with the help of a simple sketch. Let us say, on this particular axis I take zeta and here let me take normalised rise time omega n t r. If I take this value in that particular case the effect is the curve is almost like this very rough curve I am giving you so that you get an idea as to how the parameters zeta and omega n affect the rise time and hence the speed of response.

M p we had taken, M p the peak overshoot is equal to e to the power of minus pi zeta over 1 minus zeta squared under the root. You find that this M p is a function of zeta only the damping ratio. This is also very clear that the rise time is a measure of speed of response. You will like that the system should rise to the 100 percent value as quickly as possible but in addition you will like that the peak overshoot may not be too high because a peak overshoot a high value of peak overshoot means it is dangerously close to the instability point, a little variation of parameters here and there may make you system unstable.

So what is normally expected is this that a peak overshoot in the range of let us say 5 percent to 40 percent is considered acceptable. Well, 0 percent is perfectly all right but as you will find that the 0 percent value is going to give you the higher rise time that is the 0 percent value is going to take more time to raise to the desired value. So the normal range of 5 to 40 percent is taken and this range as you can easily calculate lies between zeta 0.4 to 0.7 practically.

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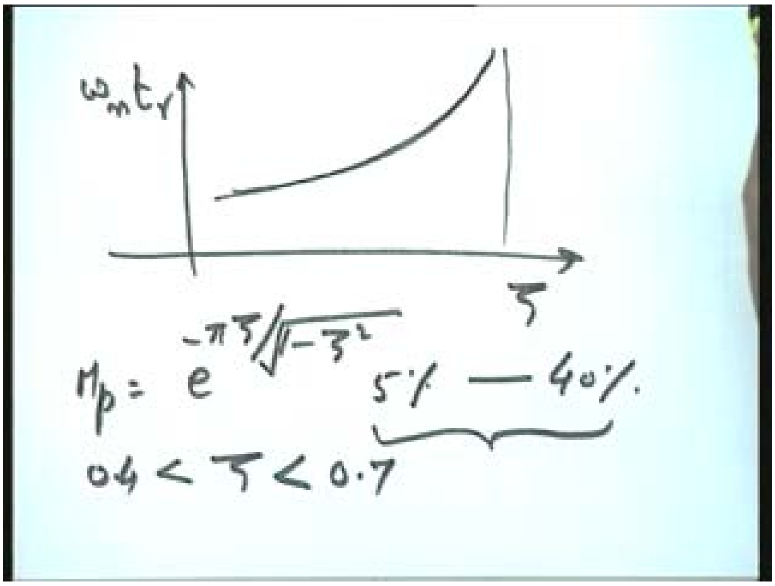

In this particular range zeta is between 0.4 and 0.7. So if you consider this range between 0.4 and 0.7 you find omega n t r is approximately constant. You see that the variation in omega n t r is very small. I need your attention at this particular……… this is a very important point I am going to make as to how do we calculate the parameters zeta and omega n.

Let me see if I have got a specific numerical value. Fine, this should be alright I think in this particular range if we take. So zeta is equal to 0.4 to zeta is equal to 0.7 the numerical value I was checking was this. Please do verify the corresponding values of peak overshoot from this relationship, you can easily calculate. The normal value of….. I am not remembering the exact numerical value that is why I have encircled in a red circle for you to verify that.

So, if I take that 0.4 to 0.7 is the normal range of damping ratios which are taken in industrial applications then with respect to this normal range you find that the change in the rise time is very small. So it means, as far as normalised rise time is concerned it is not very much affected by the damping ratio, an important point, it is a qualitative statement, quantitatively yes, as you decrease the damping the normalised rise time is reducing but that reduction over the range we are interested in is so small that we can practically say that normalised rise time is not affected by the change in the damping ratio over the range between 0.4 and 0.7. So if that is the case then omega n t r is approximately constant and please note that in that case we can say that rise time t r is an inverse function of omega n the undapmed natural frequency.

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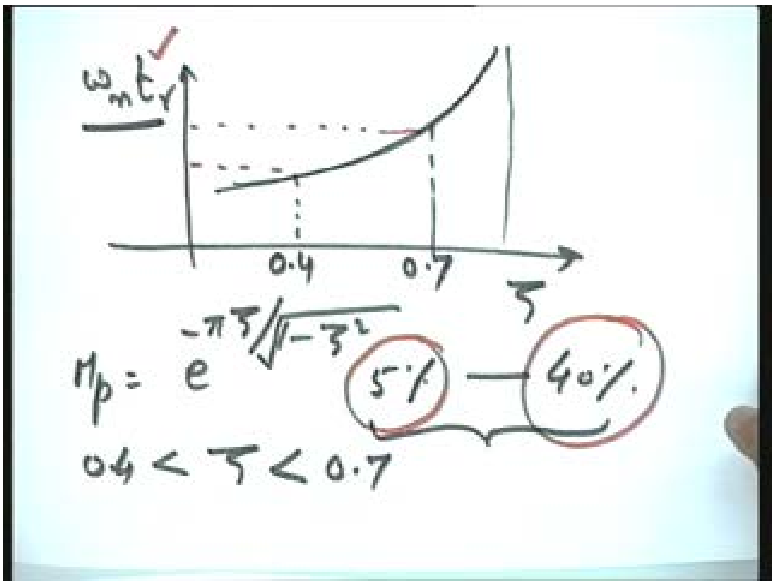

Very important conclusion will be used all through even in frequency domain discussion; you say that t r is equal to 1 over omega n into a certain constant. So it means t r is a function inverse function of omega n though zeta affects it but not to a great extent. So it means what is the design tip; the design tip will be you make the parameters, you change your parameters, you design your parameters in such a way that omega n becomes as large as possible. So it means a good design will always call for, please see, it is always set in your mind as to what are the design values you are looking for; a good design will always call for a largest value of omega n because you want, if possible, the rise should be instantaneous, if the rise should be as quick as possible you make your omega n as large as possible.

So you see that if that were the situation in that particular case probably we could have easily taken omega n is equal to infinity or close to infinity but unfortunately this is not possible this is not allowed, the reason being, though the frequency domain performance specifications I am going to discuss, the main important reason is this that, if you make your omega n larger the bandwidth of the system is larger; the précised definition of the bandwidth I will give later but at this juncture I want you to exercise your intuitive information of bandwidth. For a very large value of omega n the bandwidth of the system becomes very large, so what will happen; there is no problem if this relationship is to be seen so let it be a large bandwidth but what will happen if the large bandwidth is there then high frequency signals, normally the high frequency signals are noise signals, the high frequency signals will also enter the loop and will affect the system performance.

You see that normally the useful signals the controlled signals in the process in a loop are low frequency signals in all typical industrial control applications. All high frequency signals are noise signals. A larger bandwidth means the noise signals will affect the loop performance and therefore the output which you are getting may be a result of the noise signals and not the actual signal and hence the performance of the system is at risk. This is one of the most important considerations of limiting your rise time. You see, you cannot take your rise time as small as is theoretically possible. Because there are practical considerations you want to limit the bandwidth so that the noise signals are not acted upon by the process acted upon by the loop, the noise signals are filtered out and for that reason omega n is limited and hence rise time is limited.

So keep that general notion into your mind that the speed of response and the bandwidth in the frequency domain have got one-to-one relationship since an infinite bandwidth is not allowed so zero rise time is not allowed and there has to be a suitable compromise and that compromise will depend upon the specific hardware you are going to use.

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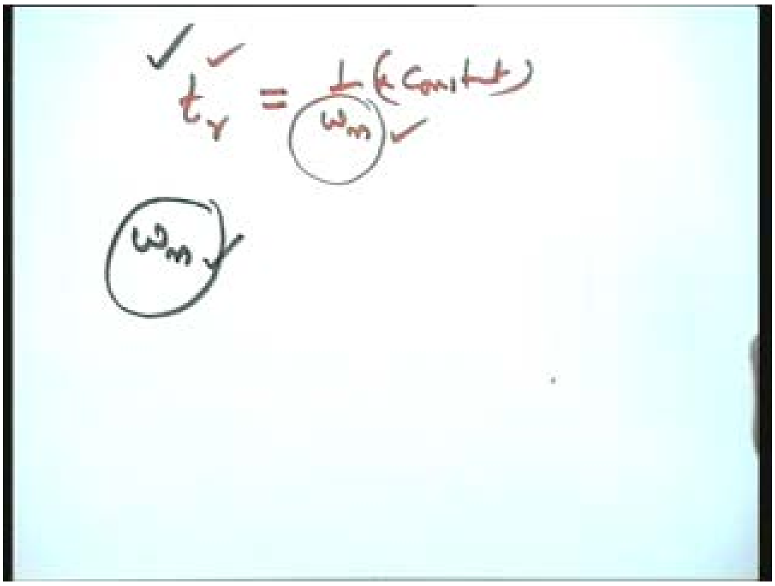

For example, if your sensor is very precise, if your sensor does not create high frequency noise signals may be a high bandwidth is allowed, may be lower rise time can be obtained but if your sensor is of poor quality in that particular case you will see to it that high bandwidth should not be allowed and therefore there will be a corresponding limit on the rise time. So in general, yes please [Conversation between Student and Professor ((there does a high omega and also a text stability)) ((00:11:32 min)) high omega and affecting the stability let us see.

You see that at this juncture we are taking up the second-order system and if you see this second-order system response, second-order system poles minus zeta omega n these are the poles (Refer Slide Time: 11:53). So, high omega n is this particular part. So the stability will be affected when you take up both zeta and omega n in such a way that these poles are close to j omega axis. so the poles as you will see are primarily affected by zeta the stability is primarily affected by zeta it gives you higher amplitudes because you go closer to the j omega axis, it gives you higher amplitudes that is higher peak overshoot and hence the risk of stability. That is why you will see that stability is guided by this M p is equal to e to the power of minus pi zeta over 1 minus zeta squared under the root.

A 100 percent overshoot means your poles are on the j omega axis and if you exceed the values it means you are risking the stability. So it means, since you posed this question please zeta you can say is an indicative of relative stability and roughly speaking though we are taking a standard second-order system we have to suitably extend this logic to higher order system but as a rough measure I can say that zeta is a an indicative of stability relative stability and omega n is an indicative of rise time that is the speed of response.

You will like that the omega n should be larger limited by bandwidth considerations and you will like that zeta should lie between 0.4 and 0.7 for stability considerations, larger than 0.7 peak overshoot is reduced but the rise time all increases in the process those with a value larger than 0.7 but with values less than 0.4 there is a likelihood that the peak overshoot will become high and the dangerous risk of going closer to instability will be there.

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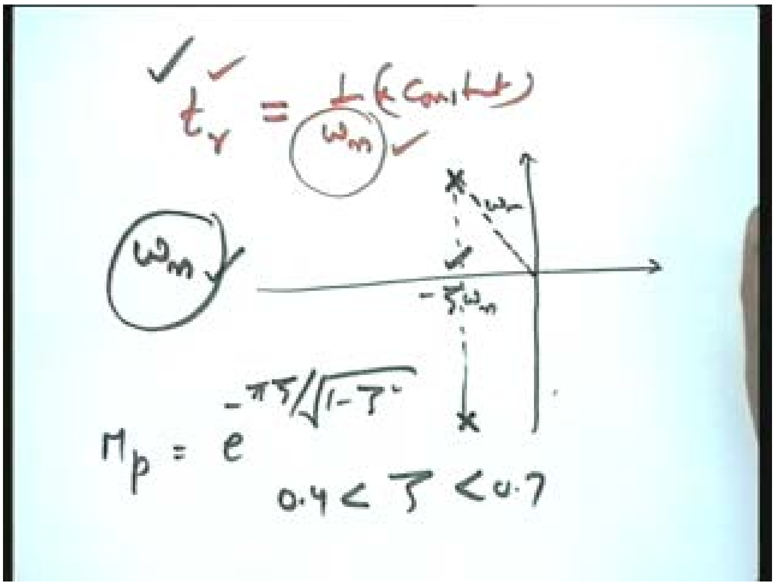

These are two important points we have discussed; t p is identical to rise time as far as qualitative behaviour of the system is concerned. You can see that t p is equal to pi over omega n 1 minus zeta squared under the root. A larger value of omega n gives you a smaller value of t p the peak over the time to peak and it is obvious smaller t p means quick is the rise of the system to the desired value and hence you are in a better situation because the system rise will be quicker. With this now I go to settling time, a very important concept in settling time is involved and i will really need your attention and your discussion on this because the settling time, as you will see has been presented in different ways in texts.

My presentation is as follows:
I put it this way please, this is 1 and here is the typical response (Refer Slide Time: 14:42). Let me take the band and typically let me take this to be a 5 percent band. Please see that let me take a particular value of zeta is equal to 1. [Conversation between Student and Professor ((every 5 percent from the 1)) ((00:15:04 min)) 1 that is this is 1.05 0.95 percent with respect to the final desired value. let us say that this is the critical damping corresponding to zeta is equal to 1. Now, on this particular axis I am taking omega nt normalised time and on this particular axis I have y.

Please see now, keep this curve in mind because I may not be having this slide all the time in front of me. You must keep this curve in mind as to what is the corresponding effect when zeta keeps on changing that is when more and more oscillations come up.

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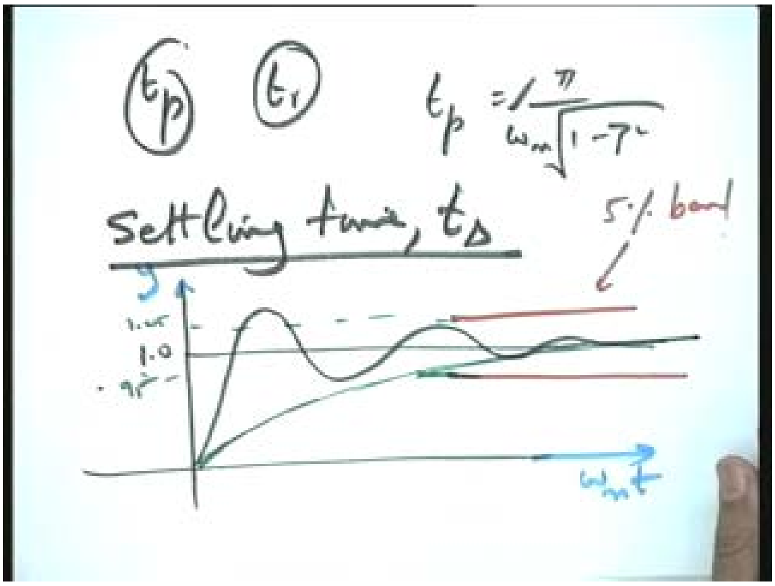

Just see, here I am taking zeta verses omega n t s settling time, zeta verses omega n t s. Let me take the step response curve here also at this particular point, let me put the band here as 0.95 to 1.05 (Refer Slide Time: 16:07). Let us say that this is the curve, you see I am exaggerating this scale to make it clear, I will definitely request your attention on this point please. Let us say that this is the curve corresponding to zeta is equal to 1 and I assume that at corresponding to zeta is equal to 1 at this particular point the corresponding value time here I am taking as 5. So it means zeta is equal to 1, this is omega nt, this is your y corresponding to zeta is equal to 1, the normalised settling time is 5 I am taking this as the value.

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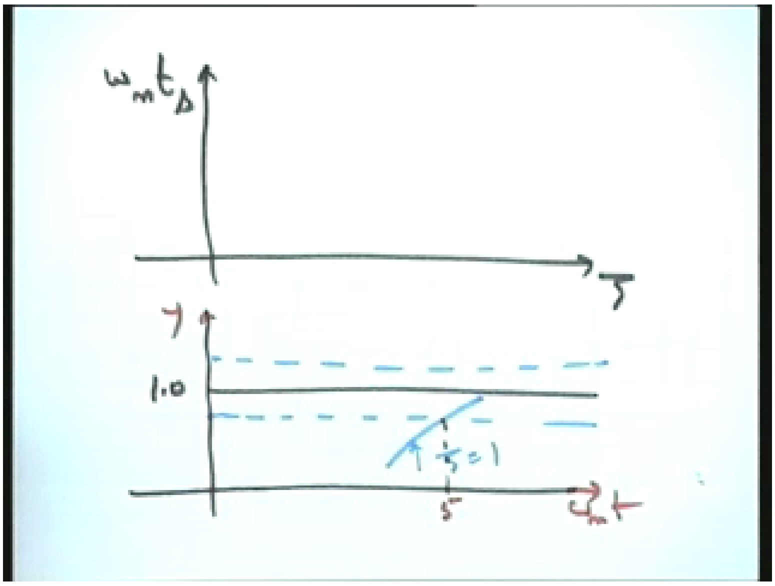

Help me please, what will happen and let me take this point here as 5, I am at this point (Refer Slide Time: 16:59). What will happen if zeta becomes equal to 0.9, I need your help please. Keep that curve the total normalised curves in your mind, zeta is equal to 1 is the critical damping case and I say that normalised settling time as observed from the curve is 5. Now I am taking zeta is equal to 0.9, help me, what will happen to normalised settling time will it decrease or increase? Decrease, let us say that it decreases to 0.4. These values 5 and decreases to 4 these values are more or less correct values because I know the numerical values over here. I am taking it still further, I take the value to zeta is equal to 0.7 help me again, I think you will say it will again decrease, is it okay please? Will it decrease please at zeta is equal to 0.7? At zeta is equal to 0.7 I am taking this value. But please note, here is a point of caution, at zeta is equal to 0.7 when you take this curve it touches the upper bound it just touches the upper bound please at zeta is equal to 0.7.

Now help me, up to zeta is equal to 1 to zeta is equal to 0.7 I agree to what you said that the normalised settling time is decreasing and I have made this as the plot and it is obvious that it is decreasing because it is entering into this particular band early as the zeta value is decreasing but at zeta is equal to 0.7 as you can examine from the generalized normal curves you will find that the overshoot is 5 percent. It means this is going to touch this particular line, well, there is no problem, still this particular curve lies within the 5 percent band.

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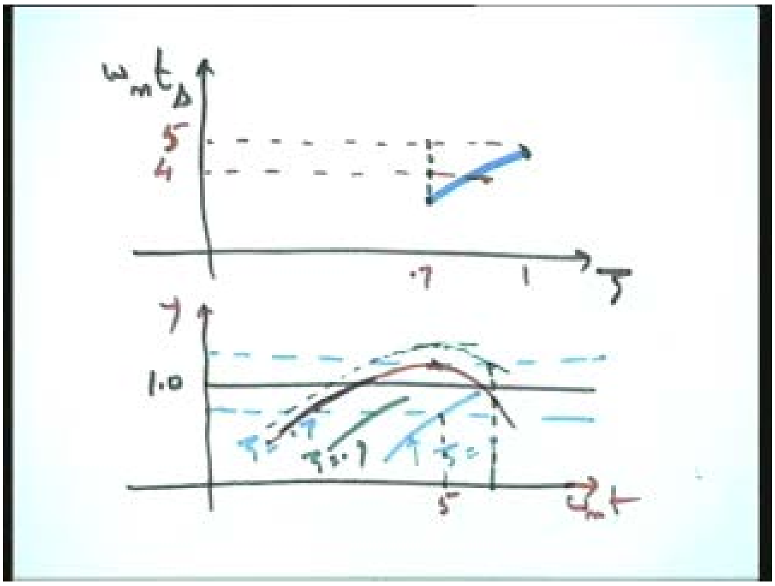

Now you have to help me, what will happen if zeta is slightly less than 0.7? [Conversation between Student and Professor – Not audible ((00:19:05 min))] in that particular case please see this particular band is going to be crossed and hence there is no decrease; you see that the new settling time will be the time at which this particular point enters this particular band again. So it means, at zeta is equal to 0.7, I can say that there is a sudden rise in the settling time. Look at the definition; settling time is not a linear curve, there is a sudden rise in the settling time at zeta is equal to 0.7 because of the 5 percent overshoot. Now if there is a slight decrease in 0.7 it is going to cross the upper boundary and therefore it will re-enter at this particular point. Let me say that it re-enters….. well, since the numerical value is known to me I will redraw the curve which gives you roughly the same numerical value which comes from the normalised curves.

This is your 5 percent band (Refer Slide Time: 20:10), this is zeta equal to 0.1 case zeta is equal to 1 case at 5 please zeta equal to 0.9 case at 4 zeta is equal to 0.7 case it is at 3 zeta is equal to 0.7 case it is at 3 but this 0.7 crosses this particular band at 4.4 please just because I remember the value this is obtained from the normalised curves, this can however be examined either by computer simulation or by looking at the normalised curve given in any standard reference. So at 4.4 it is going to touch this particular point so it means for zeta less than 4.4 the settling time is going to be more than 4.4.

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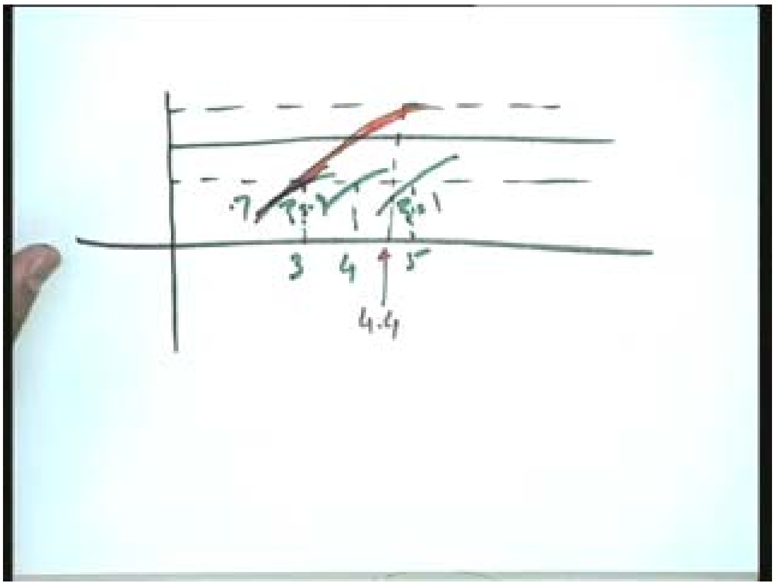

Look at zeta less than 0.7 the settling time is going to be more than 4.4 so look at this particular point, you are here at 4.4. Now you see, this you can examine very easily, these settling time is increasing now, the settling time is increasing as zeta is decreasing. Come on check this particular point 0.7 go to let us say 0.55, 0.55 is going to cross here so naturally the normalised settling time is more than the time corresponding to zeta is equal to 0.7.

In case of any question I like to answer it. If zeta is decreasing from the value 0.7 you have gone to 0.55, it crosses this particular point so now it crosses this 5 percent band at this particular point and let me say that this point is 5.1. The scale is exaggerated it has no meaning 5 to 5.1 is shown like this, do not worry about this, this is at 5.1, decrease your zeta further, if you decrease your zeta further let me take zeta is equal to 0.43. In that case it crosses this particular point at 5.2 but an interesting feature at 0.43 is this that it has an undershoot which just touches the lower boundary at the value 7. At zeta is equal to 0.43 it enters this particular band (Refer Slide Time: 22:41) at 5.2.

So it means, if you see this particular curve (Refer Slide Time: 00:22:46 min) it means up to zeta is equal to 0.43 I can easily go like this, it is an increase in the value of the normalised settling time but at zeta is equal to 0.43 what you find is this that suddenly there is a rise and the rise and the rise is to a value of 7 because there is an undershoot over here of 5 percent so it means for any value of zeta slightly less than 0.43 the value of settling time is going to be more than 7. Extending this logic you see that (Refer Slide Time: 00:23:23 min) from this point onwards the curves are going to be like this.

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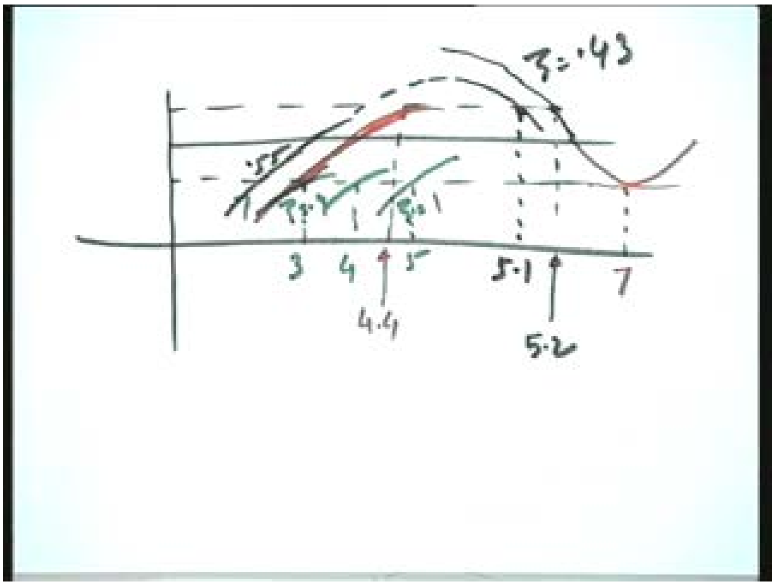

So let me draw a neat curve now and you find that the settling time equation is very difficult to capture in terms of an analytical expression. Let me take this way zeta is equal to 1 zeta is equal to 0 let me take this point and so on, this is going to be the curve the crucial point of interest to us is roughly zeta is equal to 0.7 and between zeta is equal to now you see the difference; between zeta is equal to 0 to 0.7 the normalised settling time, it is the normalised settling time please, omega n is still in the picture; the normalised settling time is increasing and in this particular case from 0.7 to 1 the behaviour is different as you will see 0.7 to 1; as zeta is increasing your normalised settling time is increasing and on this side a zeta is decreasing your normalised settling time is increasing and hence this particular curve has to be kept in mind before you take up this particular performance specification.

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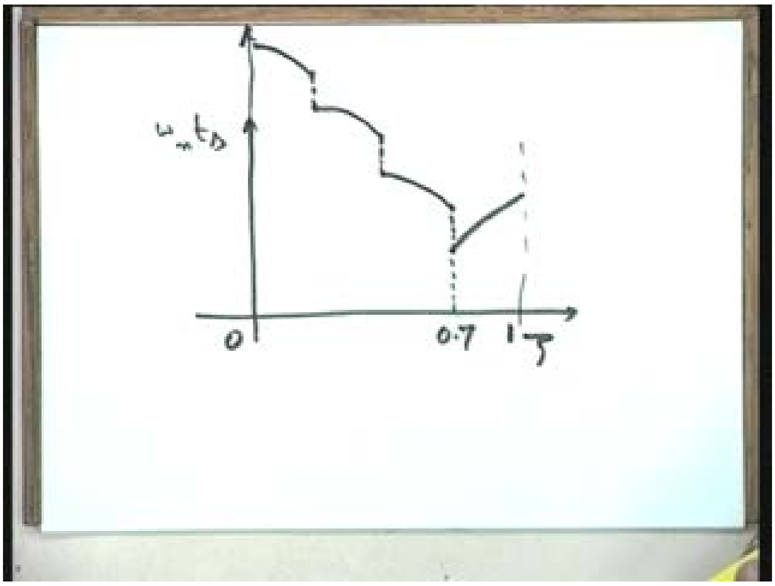

Now, since the behaviour in this particular range is more or less identical except these particular bumps (Refer Slide Time: 24:54) at least this linear this relationship is valid that as zeta decreases the normalised settling time increases. Let us try to capture this in the form of an analytical expression. You see that your response is like this (Refer Slide Time: 25:09) these are your envelopes. Let me take the envelope and take a point at which it enters 0.95 or it enters 1.05 let me take this as the point because at this particular point I want to determine as to what is going to be the settling time t s. So it means 1 plus e to the power of minus pi minus zeta omega nt divided by 1 minus zeta squared is equal to 1.05 you can roughly take this as the t s value.

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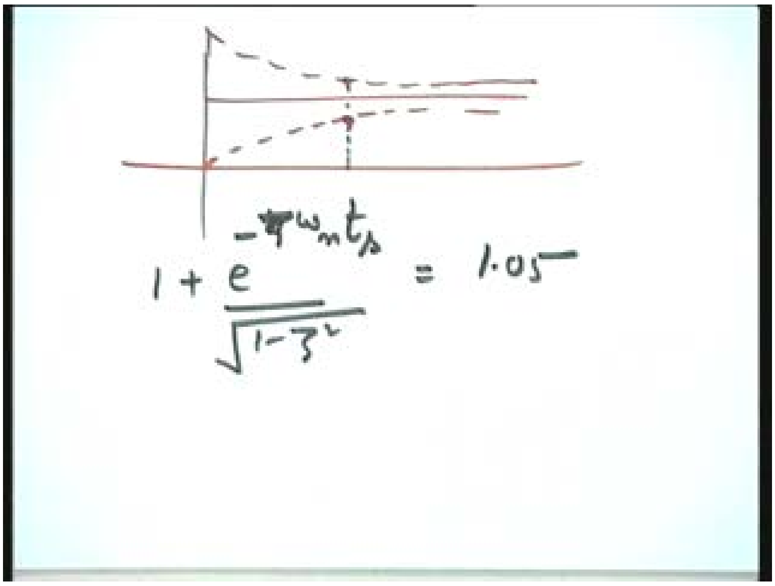

Please see, I am taking roughly this…… instead of going to this curve in between I am concentrating on the envelope. I said that let us capture this particular part (Refer Slide Time: 26:01) in the form of an analytical expression an approximate analytical expression. getting an analytical expression for this part is going to be difficult so I am going to capture it in an approximate analytical expression and I go this way please: 1 plus e to the power of minus zeta omega n t s divided by this value is equal to 1.05 that is this is the point and this point I consider is the point corresponding to the settling time an approximation because I know all these curves will always remain within this particular envelope and hence this seems to be a reasonable choice for the settling time if I am looking for an analytical expression. If that is the case in this particular case please see your zeta omega n t s is equal to log of 0.05 1 minus zeta squared under the root.

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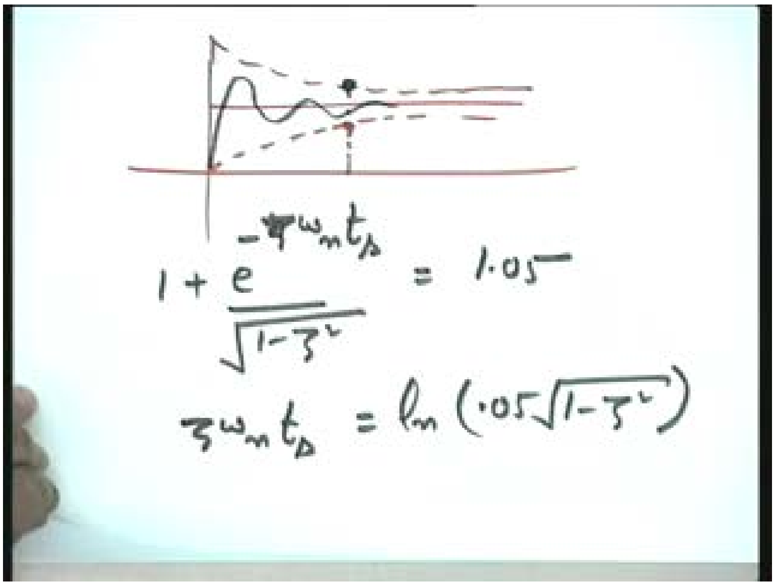

Therefore from here I get normalised settling time omega n t s is equal to 1 over zeta log of 0.05 1 minus zeta squared under the root this is your normalised settling time and analytical expression on the approximation of envelope guiding the dynamics of this system.

Now if I look at this particular part please see (Refer Slide Time: 27:30) this particular part it varies from minus 3 to minus 3.3 for zeta lying between 0 and 0.7. I am looking for that part only between zeta is equal to 0 to 0.7 because behaviour is identical in that particular part. In the other part behaviour is going to be different. And since the part which is of prime interest to me is only between 0 and 0.7 because mostly damping ratios are between 0.4 and 0.7 and therefore I am interested in an analytical expression in that range only and you find that the analytical expression is like this omega n t s is equal to 1 over zeta log of this value. Keep this in mind that the variation is only between 3 to 3.3 over the entire range of zeta. So it means t s is equal to 1 over zeta omega n and here it is log of 0.05 1 minus zeta squared under the root where this value within the bracket changes only from 3 to 3.5 when zeta varies from 0 to 0.7.

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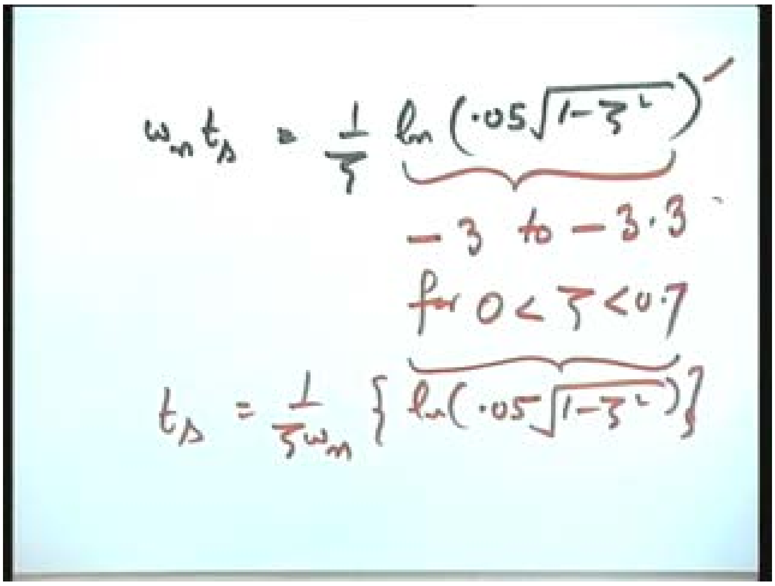

Now, before I make a general conclusion I will say that let us take a 2 percent tolerance band because this is more often used as a specification 2 percent tolerance band. Now I will leave this as an exercise to you but that exercise has to be done only through computer simulation because otherwise this exercise is not that simple that the minimum value of zeta at which this occurs is 0.76. from 1 to 0.76 the value of normalised settling time is decreasing and from 0.76 to 0 the value of the normalised settling time is increasing for a 2 percent tolerance band and therefore for a 2 percent tolerance band please see zeta omega n t s becomes equal to minus 3 minus why minus 3……. L n 0.02 1 minus zeta squared under the root; please see this is for 2 percent tolerance band identically. And if I take the range zeta 0 to 0.76 this value changes from minus 3.9 to about minus 4.34; this is the range from minus 3.9 to about minus 4.34 as zeta changes in the range from 0 to 0.76 I hope you are getting me.

The minimum value occurs at 0.76 and therefore I want to capture the relation from 0 to 0.76 the behaviour of the settling time for this particular range because in this particular range the zeta verses normalised settling time behaviour is quite well known in a monotonic expression but the only thing is in between there are certain bumps. So if I consider this you will find that t s in this particular case is minus 1 over zeta omega n log of 0.02 1 minus zeta squared under the root. While the value within the brackets is between 3.9 to 4.34 for all the values of zeta lying between 0 and 0.76.

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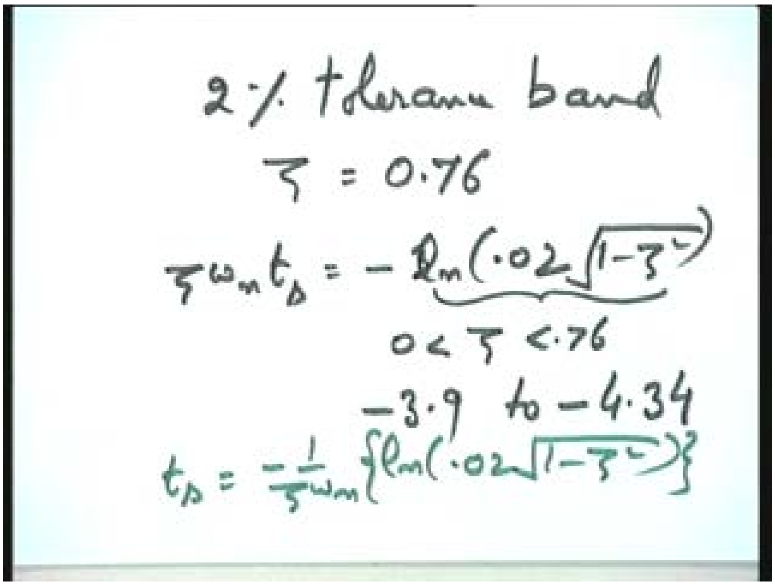

At this point please any question you see, any question, I can repeat this; this is a very important point you see because the settling time this always remains a question as to how the settling time how the zeta verses settling time curve comes. You see that it really is not a linear relationship; we are trying to capture it into a simple analytical expression only for a limited range of zeta. Fortunately this range of zeta is of prime interest to us and therefore we will be able to utilise this particular expression. Now, all the time using this expression is quite difficult. Can you give me any specific significance of this product zeta omega n. Yes please, I want to get an answer from you, any specific significance of the product zeta omega n? [Conversation between Student and Professor – Not audible ((00:31:57 min))]

It is the time constant of the envelope curve. Please see, zeta omega n is the time constant of the envelope curve; tau we have taken, it is not zeta it is reciprocal of the time constant. Time constant tau is equal to 1 over zeta omega n. the envelope curve you recall…. I mean the expression is before you; it is: e to the power of minus zeta omega nt over 1 minus zeta squared under the root 1 plus this is the envelope curve so naturally the e to the power of minus t by tau in this form if you visualize your tau becomes equal to 1 over zeta omega n. this is the curve this is 1 this is your envelope curve (Refer Slide Time: 32:45).

Now if you look at this expression (Refer Slide Time: 32:49) I hope you will not mind after all it is a design cycle. Please see, when we are taking this approximation in a design cycle it is always valid to make the approximation because after all your final confirmation of design is going to be by simulation. The approximation made is this that for a 2 percent tolerance band the settling time is approximately equal to four time constants of the envelope curve. I need your attention please.

Look at this particular range; this range is 3.9 to 4.34; average value is not being taken because figure four will turn out to be easier in the design iterations. we are taking four into time constant of the envelope curve and we are going to take up the settling time is equal to 4 into tau or 4 divided by zeta into omega n for all values of zeta between zeta is equal to 0 to 0.76 because after all if the exact value does not turn out to be this value it will be a small variation from this value and this small variation will clearly show up in simulation. Your design is finally going to be confirmed by simulation so let us make the design iterations simpler.

This is what is done in normal practice (Refer Slide Time: 34:20) and I take that t s is equal to 4 by zeta omega n as the standard relationship for my discussion throughout the course. I may make a mention here; different textbooks may give different formula for this different relationship for this at least four five different forms are in usage. But you see practically speaking it really does not make a difference, it is only a qualitative help, after all quantitatively you are going to simulate your design and if the design is not acceptable it will clearly show up in simulation so that is why we have to stick to one value and to my mind this value is more appropriate because at least it is easier to work with.

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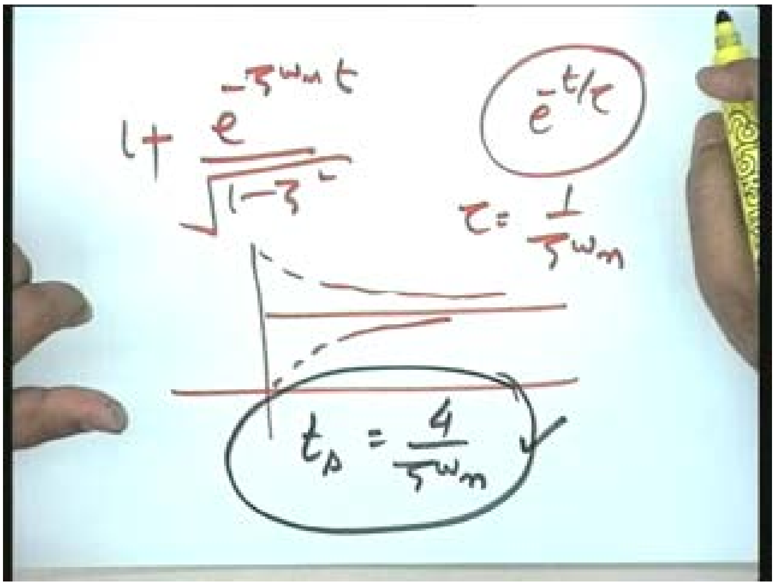

Therefore I consider the fourth formula now: t r I have given to you let me not repeat it here, t p I have given to you the peak time, the M p value has been given to you and the settling time now is obtained as 4 over zeta omega n as the relationship for our design exercise.

[Conversation between Student and Professor – Not audible ((00:35:28 min))] for 2 percent and now you see that….. okay since you have raised a point look at the 5 percent tolerance band, what should be the approximation? Can give me some crude approximation for 5 percent tolerance band? 3 is probably a very important very useful figure 3 into the time constant of the envelope. So, for a 5 percent tolerance band you will take your settling time is equal to 3 tau where tau is the time constant of the envelope curve is equal to 3 over zeta omega n when you take a 5 percent tolerance band. But again please see, if I do not specify in my design exercises the tolerance band by default we are going to take this to be a 2 percent tolerance band and therefore t s is equal to 4 by zeta omega n is going to be our formula. By default we will always assume that well 2 percent tolerance band is specified because this is more often used otherwise as you have very rightly said t s is equal to 3 by zeta omega n could be a reasonable choice as far as the value of settling time of 5 percent tolerance band is concerned.

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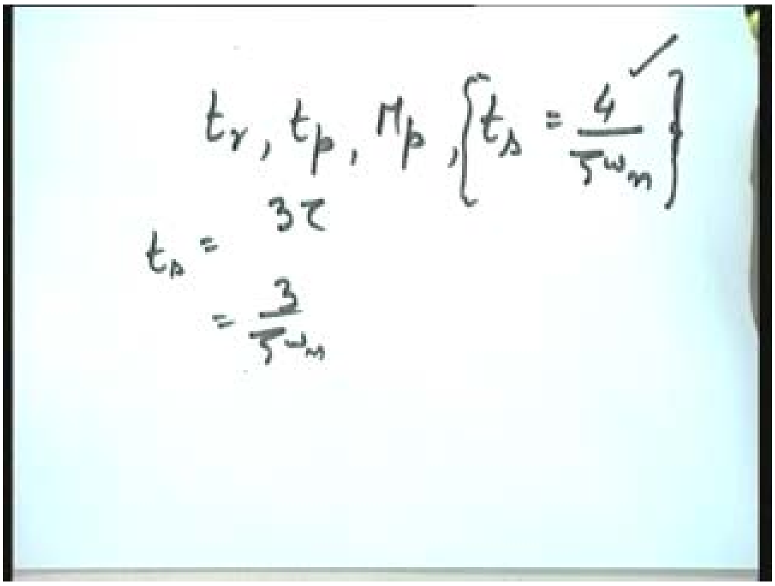

Now one more specification is left you see before I conclude the discussion on second-order system and that specification is on steady-state error. The steady-state, yes please…..
[Conversation between Student and Professor …… 37:05]…((omega n is going to help t s t r t p everything)) but will it lead to saturation of pressure or something because it concerns numerator)) yes, now you please see that, well, I thought that can I just delay answer to your question after the steady-state is reached because all the four I am going to take together as to what is the effect of zeta and omega n and how the conflict is there; I am going to answer this please, so let me take this relationship also and then I will answer your point. There is definitely a conflict as you will see.

Now, 1.0 I am taking, now you can see that for the step response the steady-state error is 0
this is clear from the curve or it will be clear from the expression as well: y(t) is equal to 1 minus e to the power of minus zeta omega nt 1 minus zeta squared under the root sin (omega dt plus theta) you can see let t go to infinity, as t goes to infinity y goes to 1 and therefore at in the limit t tends to infinity y tends to 1 and hence the steady-state error is 0.

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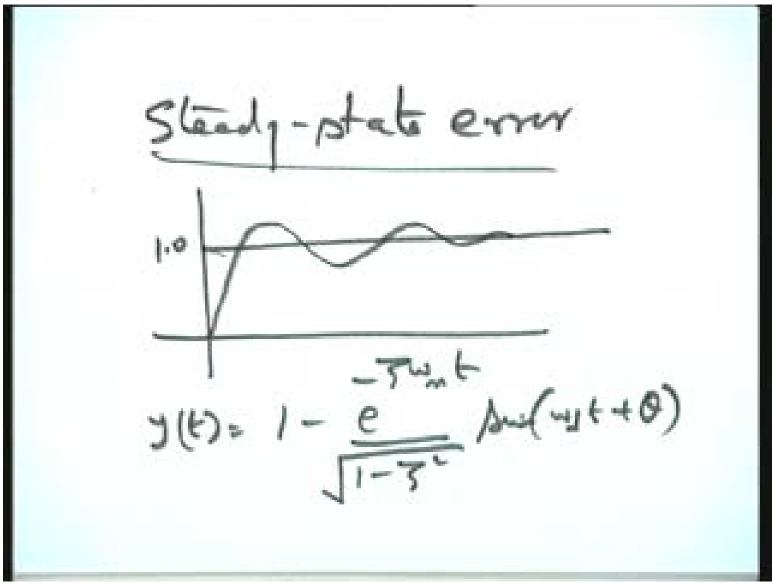

Now, as I said that if you have steady-state error 0 to a step input you cannot be very happy, it is not a happy situation that your design is acceptable to the user, no, design is acceptable from transient performance point of view as far as step input response in concerned. If the steady-state performance is to be evaluated you have to consider all the inputs at least the standard inputs step, ramp and parabola. So step we have taken, now let us take the ramp.
For the ramp R(s) is equal to 1 by s squared this is for the ramp and Y(s) becomes equal to omega n squared over s squared (s squared plus 2 zeta omega ns plus omega n square).

Let us take the inverse of this: y(t) equal to… well, I remember it because of quite often usage t minus 2 zeta omega n in this particular case minus e to the power of minus zeta omega nt over 1 minus zeta squared sin of omega dt plus theta, this is going to be the value of y(t) as t tends to infinity. Let me check this expression please. The value of, yes please, well, this is the value of y(t).

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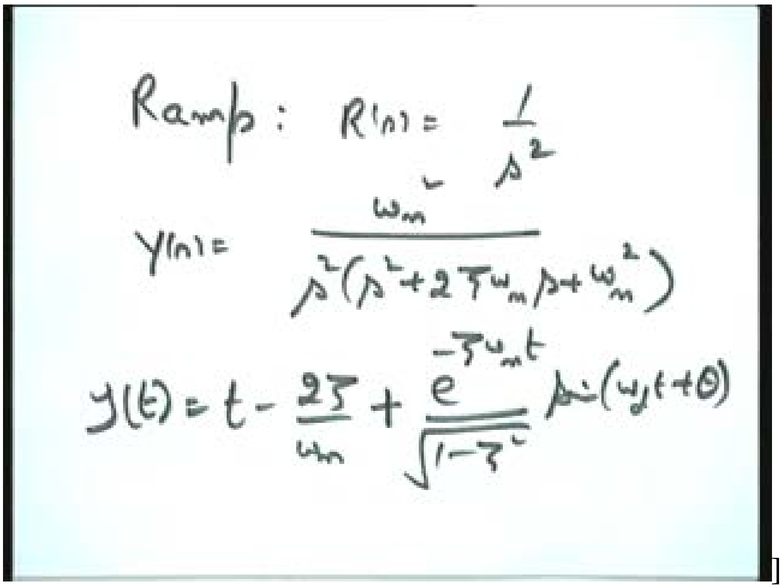

]

Now help me please, how will the system behave as far as the steady-state behaviour is concerned. Let us make a sketch of this. If I make a sketch of this response: t verses y(t) the input is r(t) is equal to t r(t) is equal to t please see let me make a total sketch. Once I have got the expression available with me what is the harm in making the total sketch. The total sketch will turn out to be like this (Refer Slide Time: 40:29): you find, as far as transient is concerned it is an oscillation so no new behaviour, the oscillation was clear from the step response as well. So it means, as far as the transient response is concerned the step input has not added to the information and that is why normally we study the system the study the transient response of the system to a step input.

But look at the steady-state behaviour. What is the steady-state value?
Please see that as t tends to infinity your y steady-state is equal to t minus 2 zeta by omega n, y steady-state equal to t minus 2 zeta by omega n. So it means this steady-state value is going to follow r(t) is equal to t with an error of two zeta by omega n and hence this I can say is the steady-state error 2 zeta by omega n. Once the transient are over this is the steady-state response. If you plot the steady-state response this curve is going to be the steady-state response and it is going to follow the ramp input with an error of 2 zeta by omega n. So therefore I can say or you can say e ss is equal to input r minus y ss is equal to t minus t plus 2 zeta by omega n and therefore the steady-state error is equal to 2 zeta by omega n.

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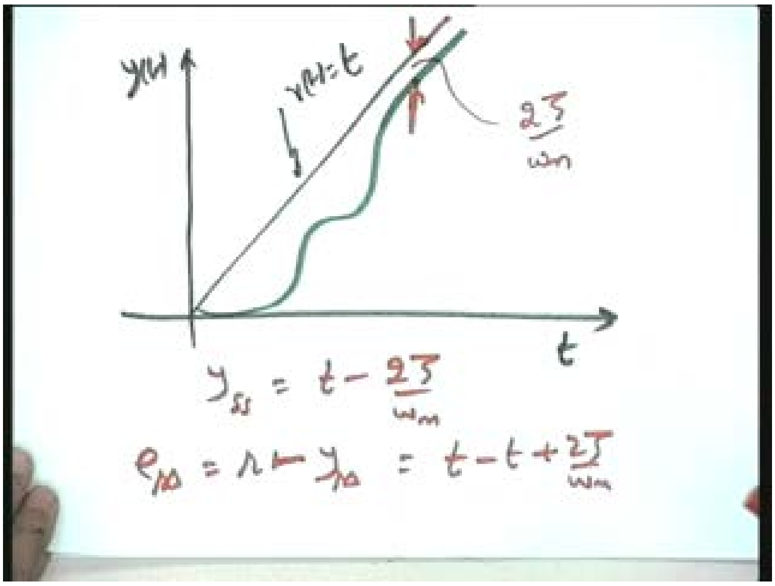

But once I have made a statement that we are not interested in the transient behaviour. In that particular case there was really no necessity of taking the inverse transform of this (Refer Slide Time: 42:04) because we can definitely determine the steady-state behaviour using final value theorem. So it means the steady-state behaviour of the systems will be investigated using final value theorem wherever applicable and the total transient response need not be evaluated for any input other than the step input please.

Now help me apply the final value theorem on this result please: omega n squared over s into s plus 2 zeta omega n is your G(s) plus minus this is your R here, this is E and this is Y. What is E(s) by R(s) equal to? 1 over 1 plus G(s) this we have used number of times, this expression is G(s) please (Refer Slide Time: 42:53), this is E(s) by R(s). You are interested in applying the final value theorem so E(s) is going to be R(s) divided by 1 plus G(s).

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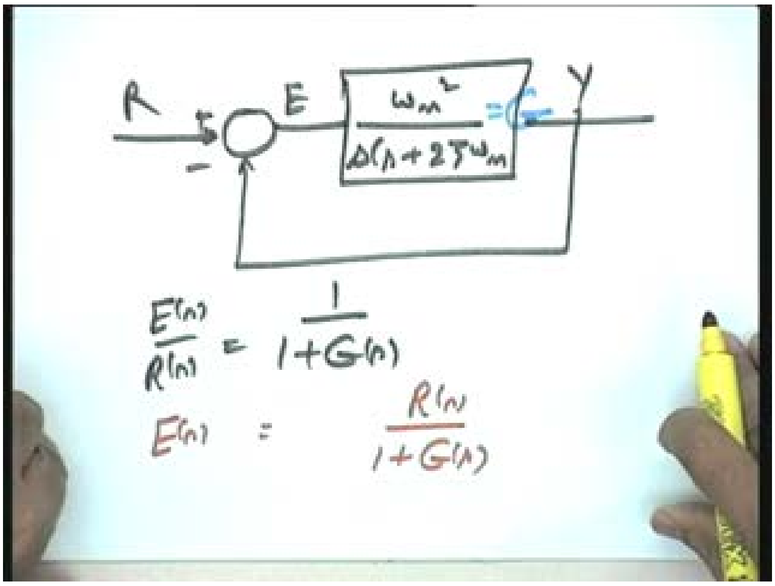

Help me please or you independently do it and see that whether we get the same result 2 zeta by omega n or not. e ss is going to be equal to sR(s) over 1 plus G(s) limit s tends to 0 this is the final value theorem limit s tends to 0 sE(s). I hope this is okay E(s). Now you please keep this thing in mind while applying the final value theorem do not make the error. Many a times this error is made. The final value theorem is applicable if and only if sE(s) has no poles on the imaginary axis or the right-half plane that is the function is analytic that is necessary otherwise the final value theorem is going to give you the wrong results because the system is not stable in that particular case, E(s) function is not stable in that particular case.

So sE(s) should not have poles on the imaginary axis or in the right-half plane this is the requirement for the final value theorem. So in this particular case for this specific case under consideration what is E(s) equal to? E(s) equal to 1 over 1 plus G(s) it is omega n over s into plus 2 zeta omega n. Yes come on, help me please, quickly give me the value of E(s). I do it, you also independently do it and verify that this is a squared plus 2 zeta omega ns plus omega n square, this is your E(s) expression, s into E(s) that is right.

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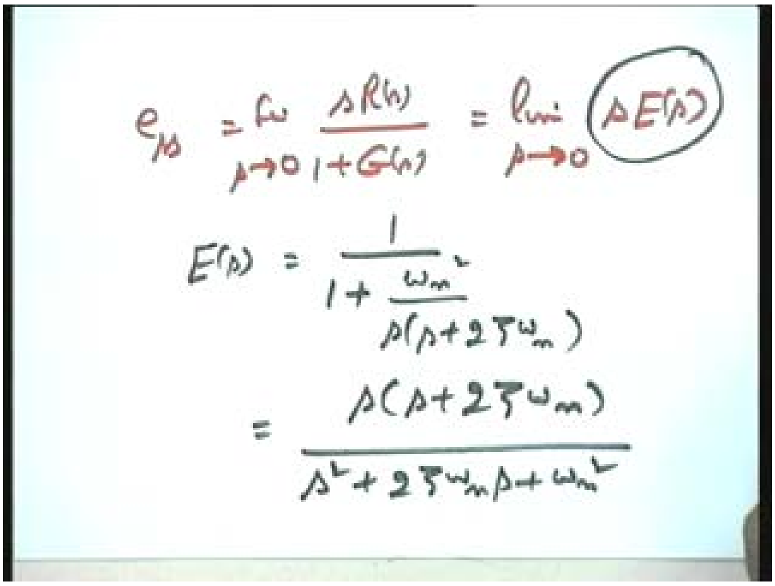

So now what is R(s)? R(s) in this particular case is 1 by s squared. This is not E(s) I am sorry that is why I was not getting the result (Refer Slide Time: 45:01). It is E(s) by R(s). I am yet to multiply it by R(s) please, it is E(s) by R(s) expression please.

Now what is E(s) equal to now?
E(s) equal to s plus 2 zeta omega n divided by (s squared plus 2 zeta omega ns plus omega n squared) into s, is it okay please? R(s) is equal to 1 by s squared has been substituted. What is sE(s)? sE(s) is equal to s plus 2 zeta omega n over s squared plus 2 zeta omega ns plus omega n square. Please see that this particular function has no pole on the imaginary axis or in the right-half plane and hence final value theorem is applicable. I want you to check this point at every point. In the case of sE(s) we are able to check this point and we are getting this particular result.

What is the steady-state error now?
e ss is equal to….. are you getting the same result, I think yes, you are getting the same result 2 zeta by omega n and therefore the steady-state error can be evaluated using the final value theorem. Yes I know that I have only two minutes and in two minutes you answer my question and then leave the hall.

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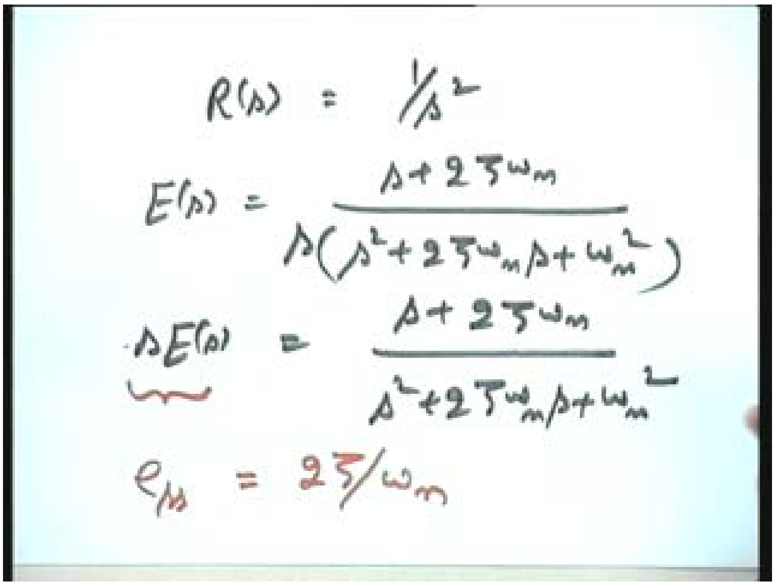

My question is this that what is the steady-state error to unit parabolic input? This is my question to you. His question about t r t p t s and the conflicting nature of that I am going to answer next time. You answer this question for me; R(s) is equal to 1 by s cube [Conversation between Student and Professor – Not audible ((00:46:43 min))] or, no it is not defined, sE(s) becomes equal to s plus 2 zeta omega n over s squared plus 2 zeta omega ns plus omega n square into what please? Into s.

Please see that the final value theorem in this particular case is not applicable because sE(s) does not satisfy the requirements of final value theorem. But still can you answer from this very expression what will be the steady-state error? Do not apply the final value theorem I do not say that but still I think the answer to my question is visible from this expression. [Conversation between Student and Professor – Not audible ((00:47:23 min))] Look at E(s) E(s) is equal to s plus 2 zeta omega n over s squared s squared plus 2 zeta omega ns plus omega n squared. After all you are going to invert this to get et and then let t go to infinity this is what you will do.

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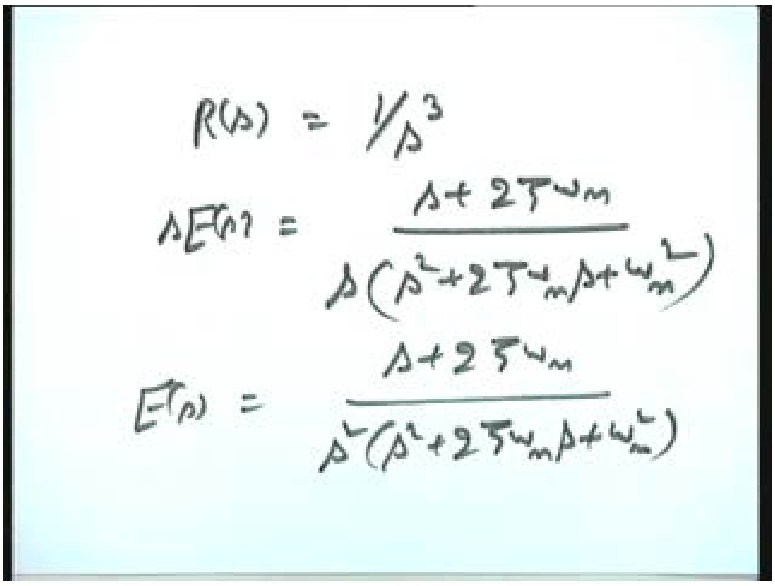

Now please see that inverting this means getting the impulse response of this. Is it not okay? Now, if this has a double pole at the origin in that particular case this function is an unstable function. So it means if you invert this, et is going to grow with the time and surely as t tends to infinity the value of E will go to infinity and hence the steady-state error for a parabolic input is infinity. So you see that the simple standard second-order system did not behave is not able to behave properly when you go to parabolic inputs. for a parabolic input as you see from over here the steady-state error becomes infinity but it really does not mean that our design is useless, it simply means that hopefully this particular system will be put to use wherever the inputs which are going to come which are going to be applied to the system can be appropriately represented by steps and ramps only the inputs will not be faster than the ramps.

Thus, if you really require the steady-state error to parabolic input also to be a finite value in that particular case the system in the present form is not acceptable to you, you will have to carry out a suitable design and the design exercise we will do later for sure, thank you.