Control Engineering
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Lecture - 26
The Performance of Feedback Systems

Yes friends, let us start with our let us start with new phase of discussion today the performance of feedback control systems. Though this is not a new discussion altogether, in some form or the other what do we need from a control system, how do we specify the performance of a control system, all these has been discussed in many cases, qualitatively. So I think, before we go to the design problem it will be necessary to specify the performance quantitatively. The discussion today is primarily concerned with the quantitative specifications of performance of a control system.

Recall the qualitative discussion for example to start with. The qualitative discussion if you recall, the performance called for the stability of the system, the system has to be stable. So an unstable system is not going to perform so the primary requirement on the performance of a system is that the system must be a stable system. The second aspect we discussed qualitatively was on transient response. We want that the transient response of the system should be acceptable by acceptability I mean that the system should rise to the desired value quickly and should not have large overshoots, should not oscillate around its equilibrium state before it settles down to the equilibrium state. The third point raised was steady-state accuracy. The steady-state accuracy is concerned with the steady-state response; after the transients are over whether at the steady-state the response of the system the output of the system is equal to the commanded value or not, if not then there is an error steady-state and most of the systems will require a very good static accuracy so that after the transients are over the output follows the command accurately.

Next we had taken was the sensitivity and robustness issues. Sensitivity and robustness issues called for the effect of variations in the system parameters due to errors in modeling or due to changes of the parameters with usage in time. You see that we wanted that the system must be robust, that is, the system must perform acceptably even under the situation that the parameters of the model are different than those used in your control system design. The other point we discussed was disturbance rejection. In this particular case again the requirement was this that when the environment affect the system when the disturbances uncontrolled disturbances affect the system then the output should reject their effects that is the output should not be affected by this at least at steady-state, during transient state the effect of the disturbance on the system should die down quickly. So this is what we qualitatively found is the requirement on a control system.

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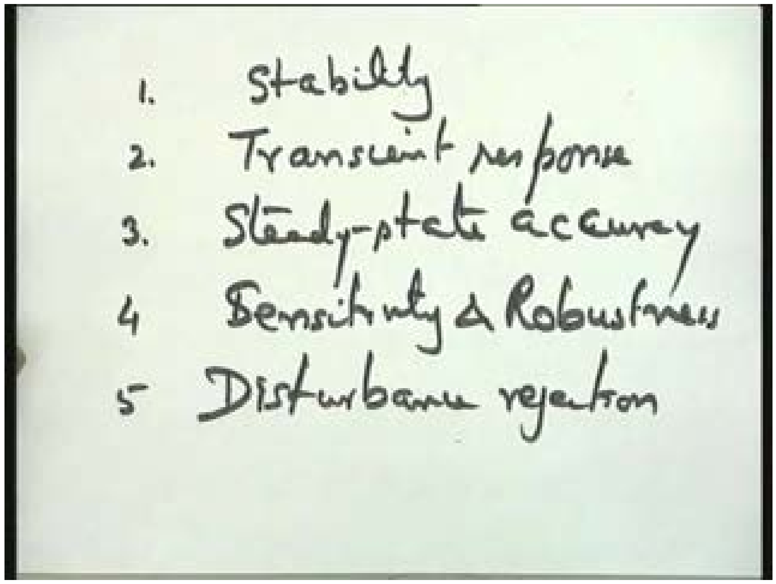

Now, various quantitative relationships for these measures were also discussed. But now coming to the design what we are going do is we are going to specify the performance in terms of transient and steady-state accuracy. Once the performance is specified in terms of transient and steady-state accuracy and a control system is designed after that we will analyze that system to see whether other requirements on robustness and disturbance rejection are satisfied or not.

I think, at this juncture, this point must be very clear. I told you this point earlier also that as far as the current research in control system design is concerned methods are being developed wherein the sensitivity and robustness can be specified quantitatively right in the beginning and the design is carried out to meet that requirement on robustness. But since this subject is yet to mature, you see the research in this area started somewhere in early 80s, since this subject is yet to mature this subject still has not been brought to the classroom. So we discuss the classical way which definitely is going to provide the background for the discussion on robustness also later.

The classical way of discussing control system design is to specify transient and steady-state accuracy and then analyze the control system design whether it meets your requirements on disturbance rejection and robustness or not. If not then you re-enter this cycle; after all design is an iterative process, you re-enter the cycle till you find that your robustness and disturbance rejections requirements are satisfied to a reasonable extent. So, as far as quantitative specification is concerned it will be on transient response and steady-state accuracy. Of course, the stability of the system is the prime requirement because without stability without that condition being satisfied all these specifications do not carry any meaning.

So what we normally do in design, first we find out the domains of various parameters which result in stability. As we have done using Routh stability criterion, couple of problems we have solved for simple cases of one or two parameters. So we find the domains of parameters which result in system stability. Once the system stability is guaranteed that is within those domains a suitable search is carried out so that the transient and steady-state accuracy requirements are satisfied. Once those requirements are satisfied you go back to the original system, simulate the system to find out whether your robustness and disturbance rejection requirements are satisfied or not. So, with this background I take you now to the specifications on the transient and steady-state performance.

Let me take the transient performance. You see that if I have a system a typical system I could take this way to simplify the situation. Though the discussion can easily be carried out to a more general case I take a unity-feedback system. There is no loss in generality in this you see, the discussion can easily be extended to a non-unity-feedback system. So now you know that in this particular case Y(s) over R(s) equal to G(s) over 1 plus G(s) is the function of the system.

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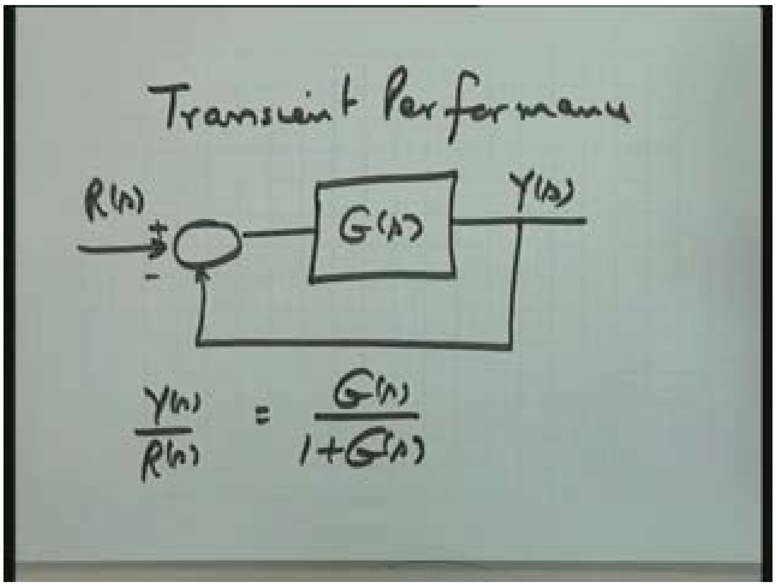

You are interested in the transient response of Y(t) in response to the input signal R(t). Now look at a practical situation please; in a control system you can never decide beforehand as to what type of input signal will be coming to the system. After all the control system has been designed to take up any situation it comes across and therefore the input signal to the control system really cannot be decided beforehand. So we should use some standard signals, some test signals so that if the system is designed for those test signals it should perform satisfactorily for any input signal which it takes up. And you know that the transient performance of a system is dictated by the poles of the system and is really not affected by the nature of the input. you see that the poles, if all the poles (Refer Slide Time: 8:57) are in the left-half plane it has been discussed it has been proved by now that the system transients will die down will die out if all the poles are in the left-half plane.

So it means, as far as the transient performance is concerned it is based on the system characteristics and not on the input characteristics and therefore if I excite the system by any input in that particular case all the modes of the system will be excited and we will be able to look at the transient performance of the system and hence it is really not necessary to take up more than one inputs and you know this has already been given to you that the test inputs which we have already discussed are the step, the ramp and the parabola. There is no limit to these you see, the step, ramp and the parabola; you could extend higher order polynomials as well. But we find that these test inputs are standard inputs are good enough to study the performance of a control system. After all, these are not the actual inputs, these are the inputs, if a system is satisfactory with respect to these inputs it is hoped that it will perform satisfactorily with respect to any input it comes across and we have to satisfy ourselves that this statement is true.

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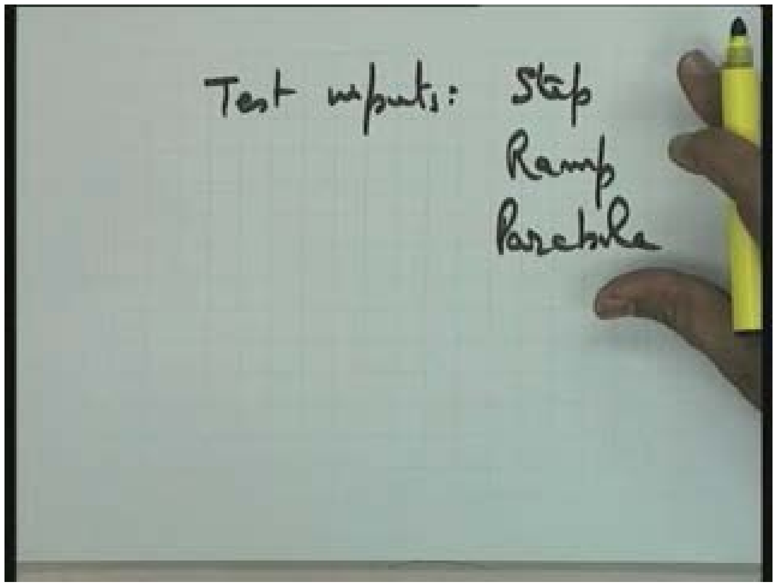

Let me talk of the transient performance first. Now you see, what is the difference if I take a step input, ramp input, parabola input or any other input when I know that the transient modes are dictated by the system poles and therefore one of these inputs will suffice and obviously the simplest one will be better and a step input is taken for the purpose. So it means from now onwards almost exclusively throughout our course we will be taking step input response of a system to quantitatively specify the transient characteristics of the system. The reason should be very clear because I am interested to see the behavior of various modes of the system; the input nature is not going to affect the transient modes of the system. So step input will suffice for me and therefore I consider that r(t) the input is equal to mu(t) which is a step signal and let me say that this is the input 1 I could take. After all if I take a step of larger magnitude it is not going to change the nature of the response it is again going to change only the magnitude of the response and the therefore unit-step input is the slandered signal we use to study the transient phenomena. For the transient characteristics unit-step input is the standard input we take.

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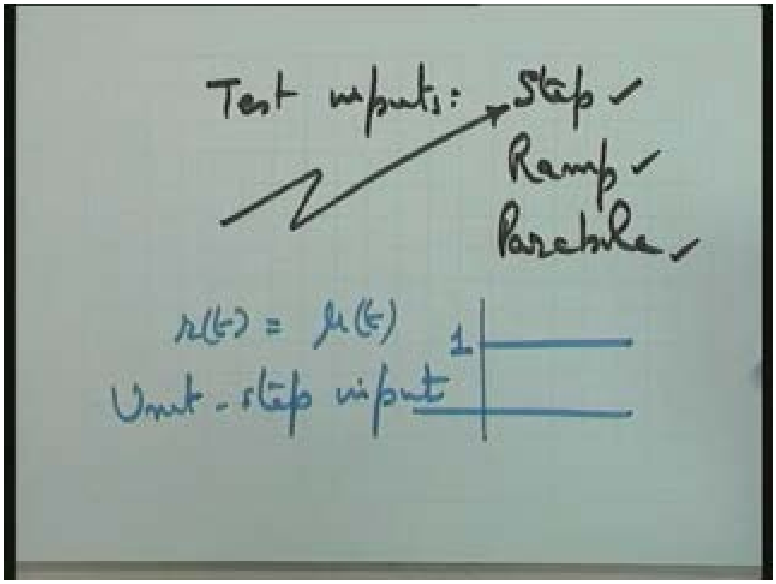

Come to the study state response now. Now, if I consider the steady-state response, now recall, what is the steady-state response; steady-state response depends upon the system characteristics as well as the input. You consider any situation, take the transfer function, and determine the steady-state and transient response components of the system you will find that the steady-state component of the system depends upon the characteristics of the system as well as the input function. So it means I cannot take a single input to specify the steady-state performance of a system. Effectively speaking I should really have the actual input which the system is going to take up so as to clearly and rightly specify the steady-state performance. But since in a control situation the input is not known to me.

for example, take a tracking radar, tracking radar is going to track the aircraft motion. now how do I know beforehand as to what will be the profile of the aircraft motion, it is not known to me. take a numerical control of machine tool, I cannot say beforehand as to my machine which type of jobs it will do; will it taper, will it do it a circling job or will it cut a parabolic profile or any other profile; after all if I have designed a control system for a machine tool for a cutting machine it should be able to cut any profile it is called upon to work with.

So naturally I cannot decide the input function for my control system it could be anything because it depends upon the users' specification of the job. Consider residential heating system. Well, you see the effect of the environmental temperature on my control system is going to change from the summer season to the winter season because the environmental temperature is going to change drastically so the disturbance signal also is going to change and hence I cannot specify the disturbance signal. So you see that the inputs cannot be specified and the steady-state performance is definitely a function of the input.

So how do we come out of this dilemma?
You see that a way of coming out of this dilemma is to consider the actual input as a suitable summation of polynomials. In general, let me take a polynomial function r(t) is equal to 1 over k

factorial t k mu(t) it is a polynomial function of time. You see that this is sure I hope you will agree that you give me any function, well; it will be possible to decompose that function into a suitable set of polynomials. So it means, if my system is going to behaves satisfactorily for all these polynomials so it means I can really rest assured that the system will behave satisfactorily for the actual input. So a set of polynomials I am going to take as the standard inputs as far as specification of the steady-state performance is concerned.

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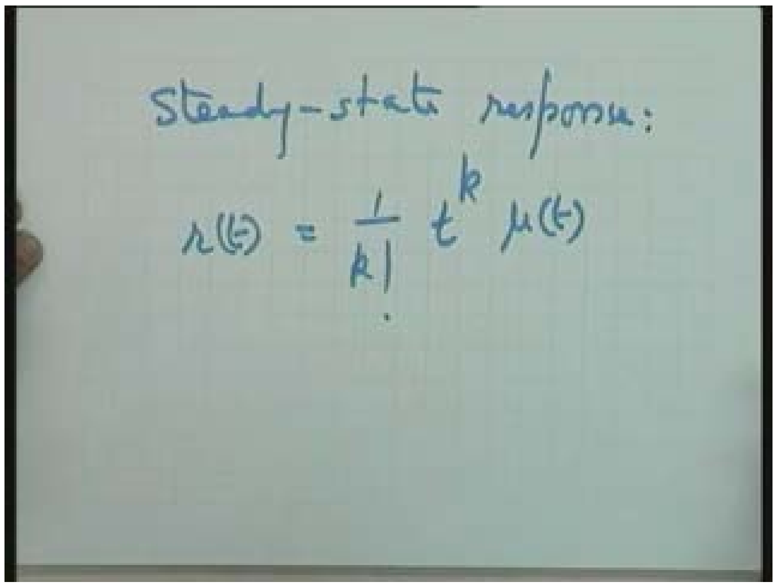

Now let me take up k is equal to 0. If you take up k is equal to 0 you find that your r(t) is nothing but the unit step function, it is a zero-order polynomial. For k is equal to 1 r(t) is equal to t mu(t) it is a ramp function, it is a first-order polynomial. Now this point may please be noted that r(t) is equal to t mu(t) is a more difficult signal for the system as far as the steady-state performance is concerned. Take k is equal to 2 r(t) equal to 1 by 2 t square mu(t) it is a parabolic function, it is a second-order polynomial. You see that, as you keep on increasing the values of k the polynomial is becoming or the input is becoming faster and faster.

So, normally what is seen is that the actual input applied to the system is not so fast as to require k more than 3, please see, it is only an experimental evidence; it is only the evidence seen in the actual seen as far as the actual usage of the control systems. Mathematically you require a large value of k to specify any input function as a set of polynomial functions. But actually you do not require more than k is equal to 2 because the system inputs are not that fast because these inputs the second-order polynomial or a parabolic function is fast enough and if a system is able to perform well for the second-order polynomial it is going to perform satisfactorily for any input it is hoped.

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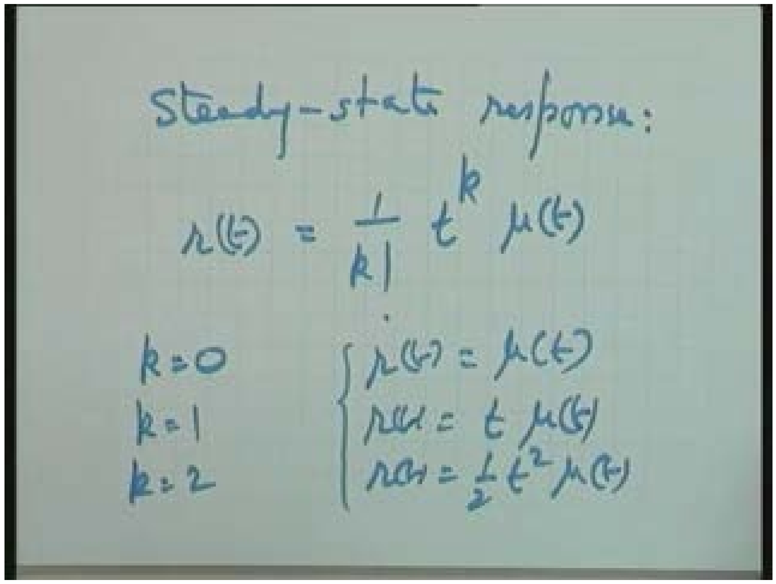

Now, as you will see when we come to practical systems, you see, normally we do not take even k is equal 2, for the sake of completeness okay some difficult control problems do require k is equal to 2 but normally k is equal to 2 also is not taken up, k is equal to 1 is found good enough. You will find that k is equal to 2 raises or gives raise to difficult control design problems. control design becomes most difficult with increasing values of k. So it means there will be some value of k may be k is equal to…….. I think if I can quote, may be after k is equal to 4 it cannot be stabilized at all. So it means the stabilization problems keep on increasing as the odd as the k keeps on increasing. So it means, even for k is equal to 2 after all, your requirement is this that all these steady-state performance or transient performance has to be satisfied within the umbrella of stability requirements. So, as the order k increases the stability becomes difficult to maintain and therefore larger values of k though mathematically quite satisfying but practically will become very difficult. So, k is equal to 2 is almost ruled out unless there is a very specific situation and as I am telling you mostly we will satisfy ourselves with k is equal to 1 because it satisfies the practical requirements, it satisfies the industrial control requirements in many applications.

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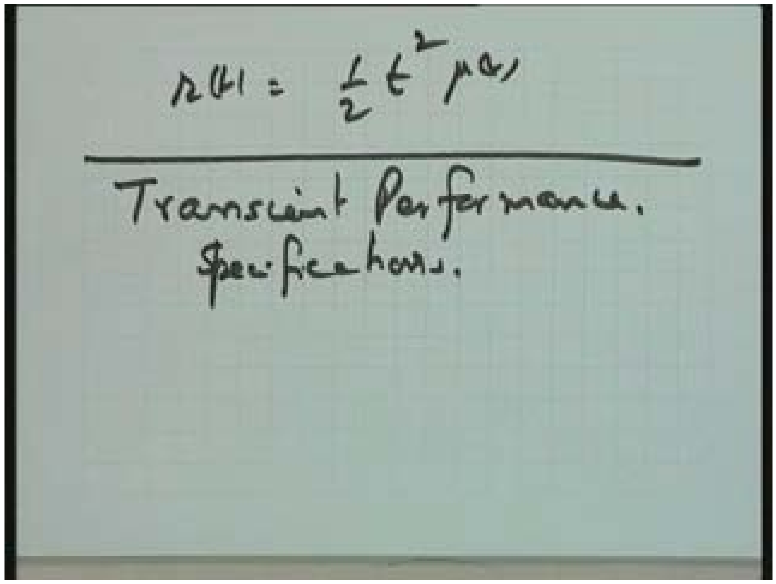

So I hope this point is now clear that as far the steady-state performance is concerned I will be referring to the unit-step, the unit-ramp and the unit-parabola. Unit-parabola the word unit is coming because its derivative is unity; it gives you the unit value. so in this particular case unit slope value it gives you once you take the derivative of this: r(t) is equal to 1 over t squared mu(t) take the derivative it is a unit-slope so let me call this as a unit parabolic function on account of that. So these are the three types of inputs I will be referring to when I come to this steady-state performances specification. But please note that the difficulty of design increases with increasing value of k. Your design is going to more difficult because you will not be able to maintain stability of the system. So with this now I think I can straightaway go to the transient performance specifications.

How do we specify the transient performance?
I am not taking any mathematical model. I simply say that you go to industry, take any typical industrial control system working there, excite that system by a unit-step input or a step input. You see, unit step is only the scaling of the response it should not make any difference, you just excite the system, the industrial system by a step input, let me call this is a unit-step input and measure the response, the response in all likelihood in more than 70 percent cases or even you can take a larger figure you see will turn out to be of this nature (Refer Slide Time: 20:31). This is what I am saying is the industrial scene today. So it means I am going specify the performance as such because all these systems are working in industry and they give this type of performance naturally this performance is adequate, the reasons will become very clear as to why this performance is better, why the industrial control systems will perform this way if excite the system by a step input.

Now you can see that it is a damped oscillation. So you will note that damped oscillation is accepted you are not taking a system with no oscillations because it is not going to be an acceptable system to you as you will see. So it means some of the overshoot in the performance is acceptable to you because most of the control systems are working this way. So the specification of transient response effectively will mean specifying this curve for a particular system. But if I specify this quantitatively like a graph in that particular case you see the control design will become difficult therefore this particular graph I translate into a set of indices. What are those indices or what are those measures I take? Please see this. First measure I take is this time (Refer Slide Time: 21:51) the time taken for the response to come to 100 percent of the desired value for the first time. It will come later also at this value but this time which I am referring to as t(r) the raised time of this system is the time for the response to rise from 0 percent to 100 percent of the final value for the first time. This is one index I take.

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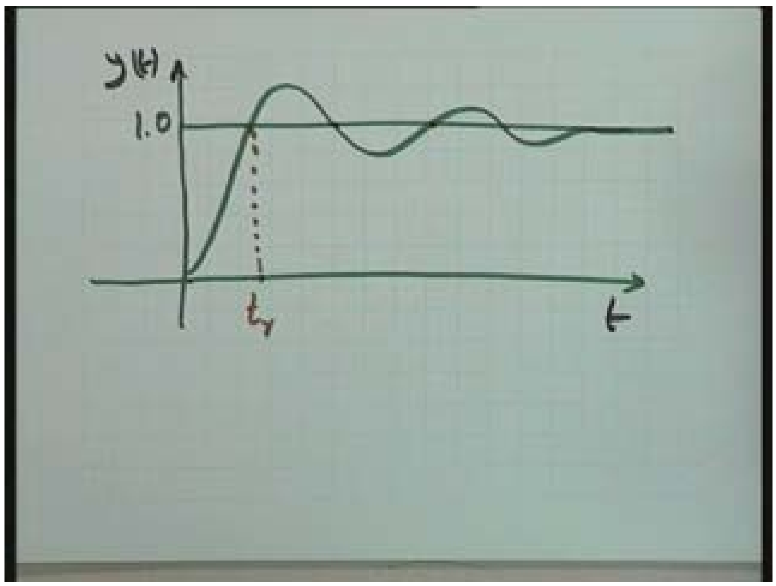

The other index I take is the following: I take this particular magnitude that is how much it has over what is the overshoot and it is the peak overshoot as you will see because the next undershoot and second overshoot etc these magnitudes are going to be lower and lower in a damped oscillation. Since these magnitudes are going to be lower so it means if the peak is acceptable to you the other undershoots and overshoots also will become acceptable to you. So M p the peak overshoot is the second index which I am going to use. And of course the time taken to reach this peak overshoot t(p) I will call it peak time is another index, I will write the words here the rise time t(r) I need not write the definition I hope this is clear now, the peak time t(p) the peak overshoot M p and what else, the next and last I take is this settling time.

The settling time you see that if I give you an exponential function e to the power of let us say exponential function I give you e to the power of minus t by tau (Refer Slide Time: 23:44) this system this particular function settles to the zero value only as t tends to infinity. So mathematically speaking, the settling time of this function is infinity but practically if I consider this particular function can be settled down when this particular value is negligibly small and the word negligibly small depends upon the application. You may say that if it has settled down within 2 percent of the desired value, okay, you can find that the system function is said to have settled down completely.

So what I want to say now, this is a higher order system not necessarily a first-order function. So if you take the mathematical mathematics mathematical response, in this particular case exact settling time will may will may will turn out be infinity, t tends to infinity will be the time required for this particular response to exactly come to steady-state. But what we are going to do is to specify a perturbation band, a tolerance band so that if the system settles within this particular band practically you will consider as if the system has reached the steady-state though exact value of 100 percent or exact value of 180 is yet to go to, but with in this tolerance band if the system has reached in that particular case you can say that the system has settled completely and this tolerance band (Refer Slide Time: 25:16) is normally taken as 2 percent of the final value or 5 percent of the final value depends upon the application.

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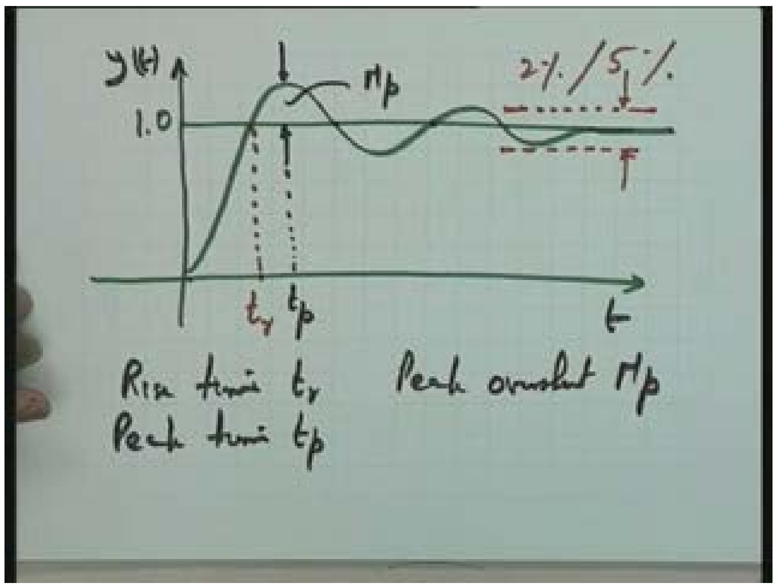

If the application requires a larger static accuracy in that particular case 2 percent of the tolerance value is taken otherwise 5 percent of the tolerance value may turn out to be acceptable. So, as per the graph you find that……. yes, please help me, what should be the settling time, could I take this as the settling time (Refer Slide Time: 25:53) because once the system has entered this particular band as per this graph it is not coming out and if the response having entered this particular band is not coming out you can say that the system has settled within the accuracy of this specified tolerance band. So I can say that this is the settling time of the system. And lastly, this let me say settling time; t s let me call it.

Lastly let me say the steady-state accuracy. Steady-state accuracy I will take separately because steady-state accuracy as we have seen is not specified only to a step input it is specified to step ramp and parabolic inputs. So, only if I am considering the step input response I can consider only the transient response specifications. Now you can see that if I give you these four parameters you can almost reconstruct this step response and hence the complete graph of the system is more or less specified if I give you these four values t r t p M p and t s. Now what is the requirement? The requirement is this, you see that the t r should be as small as possible because if t r is not small if it takes larger time then your system is a sluggish system; it takes lot of time to rise to the value you require. The t p should also be small so that whatever is the peak it should come quickly and then the system should settled down; t s naturally should be small, M p should also be small. So I want all these values to be small but unfortunately these things are conflicting as you will see. If you tend to make one value small the other becomes larger and that is because of the design problem otherwise the design would have been a set of mathematical equations to solve.

You see, the design does not turn out to be a set of mathematical equations for solution; the design is an iterative process because these requirements cannot satisfied simultaneously; these are conflicting requirements as will become clear shortly.

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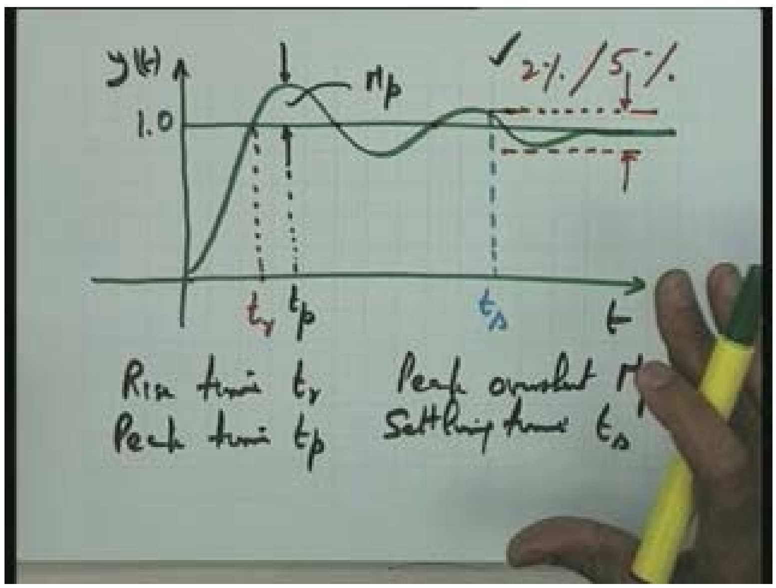

Let me go to the steady-state performance specifications. Steady-state performance specifications e double s is equal to limit t tends to infinity [r(t) minus y(t)]; recall the original block diagram I had given you. the original block diagram I may redraw it here because it is a simple one: plus minus r, e and y, this is a unity-feedback system I have taken therefore this e is nothing but the system error it is r minus y and value of this e as t tends to infinity because the system will completely settle down as t tends to infinity mathematically so using this mathematical relation gives you the steady-state value and this r t now (Refer Slide Time: 29:02) could be a step input, a ramp input or a parabolic input. So it means, as far the specification on steady-state performance is concerned the user may give you the specifications like this: e ss to unit step and or e ss to unit ramp and or e ss to unit parabola. This point may please noted, this and or because if the requirements on steady-state accuracy are not very stringent in that particular case a very simple situation is that only steady-state error to unit step is specified.

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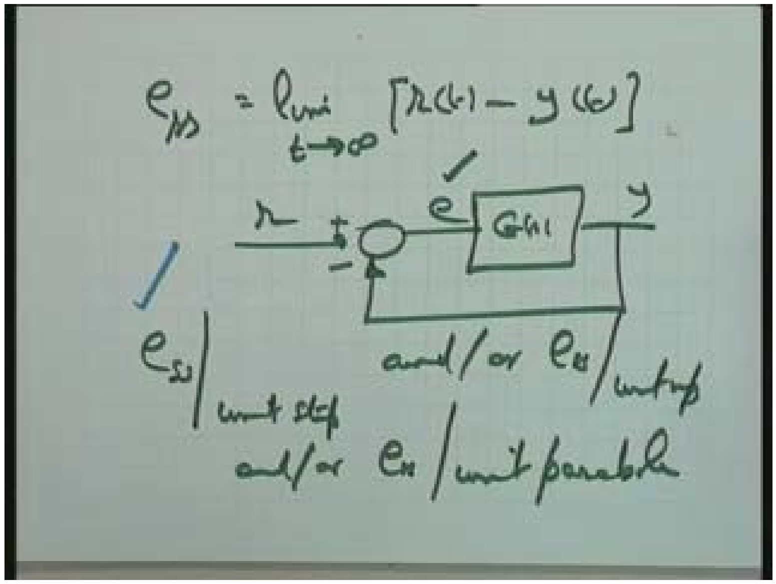

You will find that your design problem becomes very easy. As a designer you will be very comfortable if such a situation is there. The design problem will become more and more complex if more and more specifications are given. For example, if unit ramp steady-state error is also added, well, the design problem become difficult and it still becomes more difficult if I specify these steady-state errors to unit parabola as well. So these are the specifications.

And in any particular control design problem user tells you that these are my specifications on transient and steady-state accuracy and now you give a design for me and once you give the design of course the user will definitely want that the actual system, when your design is implemented on an actual system it should set it should operate satisfactorily otherwise what will happen when the user takes your design from your table and implements it in the actual situation it will not work, the reason being, the model you have used on your table is definitely different than the actual process he is going to work with so it means your design has to be robust. So it means every design has to be declared complete only after extensive simulation study.

So what you will do is whatever the design you have carried out you will go to the computer and vary the parameters of the model, varying the parameters of the model means going closer to the actual physical system because you really do not know how the actual physical system is going to behave. The simulation you can do is the best possible tool available in your hand is that you vary the parameters of the system and implement the design you have made for the nominal parameters.

If your design gives satisfactory performance for all the variations all the reasonable variations in the parameters you have made in that particular case you are sure that when the user takes your design to the actual application it will give him satisfactory performance and that is the final stage of design. So with this background I think now we can enter into the design cycle straightaway without wasting time unless there is a question. If there is a question I like to answer otherwise I enter into design cycle straightaway. Fine, I take it positively; no question means you have understood me, fine.

The design cycle I start with a standard second-order system. I do not mean that the actual systems in industry will be second-order systems I do not mean that. But I definitely can guarantee you that if you have understood the design for a second-order system you will be able to extend this design cycle to any system you come across and that is why the design of a standard second-order system is extremely important. This is my starting point; once I give you all the tips for this Standard second-order system the extensions will be very obvious and be it a tenth order system the tips we are going to finalize for a standard second-order system will be applicable there as well. So do not think that it is a fictitious situation; though it is mathematical situation not a real life situation so to capture this mathematical situation I take G(s) as s plus 2 zeta omega n this is the R input R(s) (Refer Slide Time: 33:31), this is the error E(s) and here is the output Y(s) this I consider as a unity-feedback standard second-order system.

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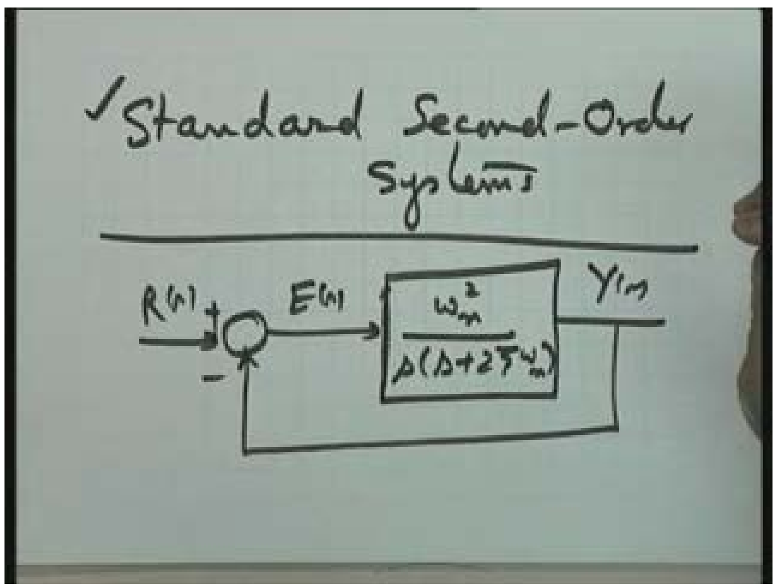

Now let me carry out everything I have talked of; the transient response specifications, the steady-state response specifications as far as the standard second-order system is concerned. I first analyze the system, design means what; design means I am going to introduce a controller here, let me say that this D is the controller D(s) so that this particular standard second-order system meet the specification you impose on the system that is the design exercise but before I take up this D(s) into the loop I simply analyze my system first that is I determine those measures, those indices for the given system without any design, let me call that this is the analysis problem I am going to carry out first and after the analysis I will incorporate this D(s) the controller into the forward loop and then see how to get the value of D(s) so that the specifications on transient as well steady-state accuracy are met.

So, for me now it is only G(s) in the forward loop I do not take D(s) so G(s) is equal to omega n squared over s into s plus two zeta omega n. So what is closed-loop transfer function please?

Y(s) over R(s) equal to help me please omega n squared over s squared plus twice zeta omega ns plus omega n squared. This is your standard second-order system you are already conversant with; zeta and omega and you now their physical meaning, zeta is the damping ratio, omega n is the undamped natural frequency.

You now zeta is equal to 0 is an undamped system, zeta between 0 and 1 is an under-damped system, zeta is equal to 1 is a critically damped system and zeta greater than 1 is an over damped system you now this and you also know that the characteristic equation of the system is del s equal to s squared plus 2 zeta omega ns plus omega n squared.

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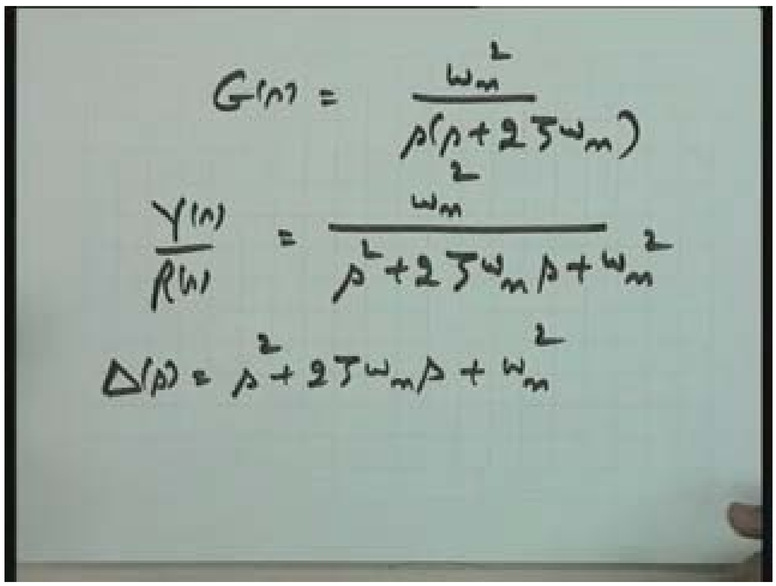

Now, from this point onwards please see, any equation I give you any statement I make I tell you, you will be using that statement till the end of this particular course. These are very important quantitative relations I am going to give you and therefore I want you to absorb these relations permanently in your memory in addition to your note book please. The characteristic equation is known to you. Help me please, what are the characteristic roots. These are the characteristic roots: minus zeta omega n plus minus j omega n 1 minus zeta squared under root are the characteristic roots (Refer Slide Time: 36:33) please.

Give me the sketch of these roots as zeta is varied please. Help me please, real s, imaginary s, well, zeta is equal to 0 please see is going to give me the roots as plus minus j omega n, j omega n here minus j omega n here these are the roots please j omega n minus j omega n are the two roots corresponding to zeta is equal to 0. Now how about zeta is equal to 1 please? Zeta is equal to 1 is going to give me the double root, please see zeta is equal to 1 you substitute it is going to give me the double root at minus omega n. So now it means for the critically damped system the two roots are at this particular point and for an undamped systems the two roots are on the j omega axis and if I draw this semicircle you will find that this is actually the locus of the roots as zeta is varied between zero and one.

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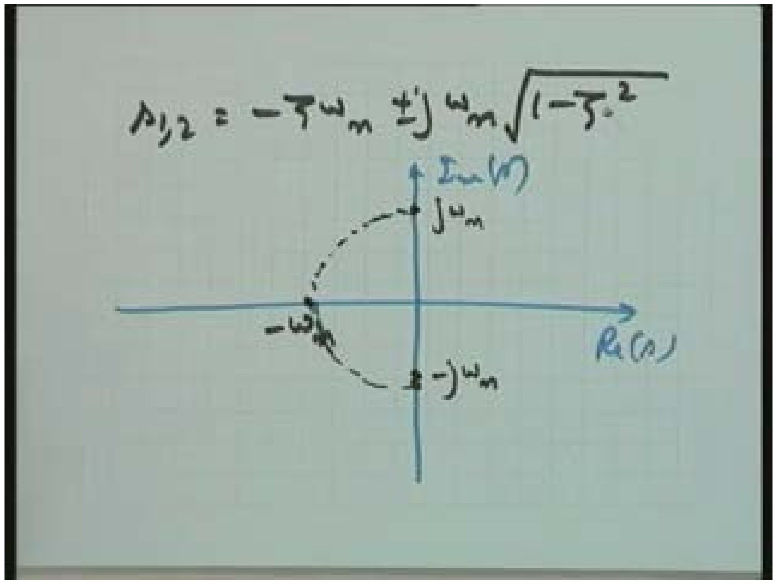

Take a typical value, come on, take this value please, and at this particular point give me the real part the real part is going to be equal to minus zeta omega n and the imaginary part is equal to j omega d is equal to j omega n 1 minus zeta squared under the root. This is your j omega d. The omega d as you know is the damped frequency. So minus zeta omega n is the real part of the root. Please see that this zeta omega n in my discussion is going to have a very important meaning and you can see that this zeta omega n magnitude corresponds to the real part of the complex conjugate root pair.

Help me please, if I draw a line here and take this angle as theta (Refer Slide Time: 38:49) could you give me the value of theta please; I want you to give me the value of theta. This value theta is going to be obtained from this particular triangle, yes, what is cos theta is equal to? Cos theta is going to be equal to this value, this is omega n this value divided by this value, what is the radius? The radius of the semicircle is omega n, is it okay? The radius of the semicircle, you can see that the radius of the semicircle……… you can see this point also that this is minus zeta omega n, this is omega n 1 minus zeta squared under the root (Refer Slide Time: 39:30) take the magnitude, the magnitude of this is going to be omega n and therefore cos theta is going to be equal to zeta. If you need explanation to this well tell me please; cos theta this theta I am going to call it as the damping angle because this theta is not a function of omega n it is a function of zeta only.

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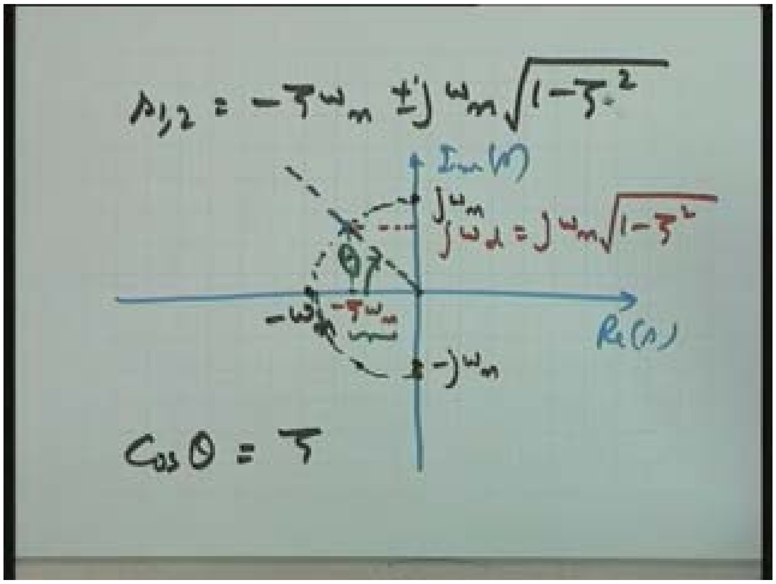

I repeat this diagram; you see that this is corresponding to zeta is equal to 0, the locus is a semicircle with the radius equal to omega n, the locus is a semicircle with the radius equal to omega n, this particular root (Refer Slide Time: 40:14) is minus zeta omega n this particular part and this is j omega d is equal to j omega n 1 minus zeta squared and this I have taken as theta. So I hope this is very clear from this particular diagram that this line is a zeta line as it is called or damping line because this angle is a function of damping only so the damping angle theta is equal to cos inverse zeta. So it means, if I give you the value of zeta the root the corresponding root is definitely going to lie on this angle so how to draw it.

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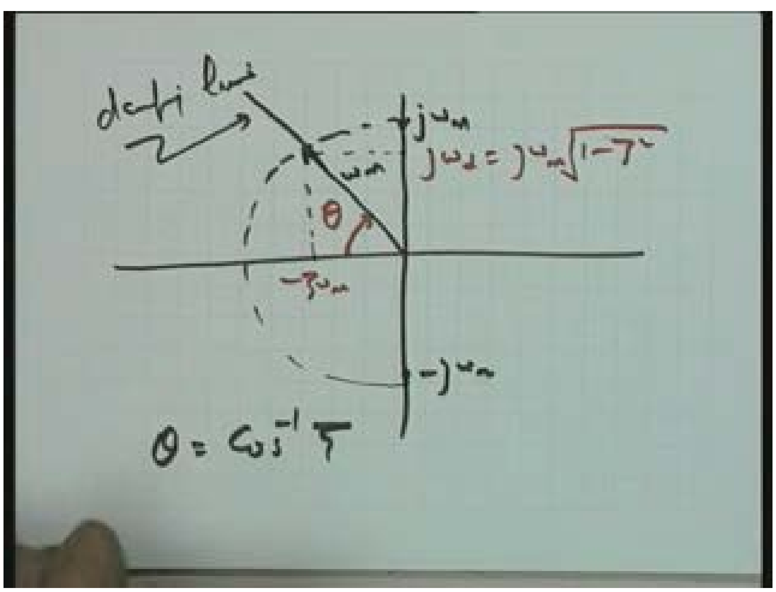

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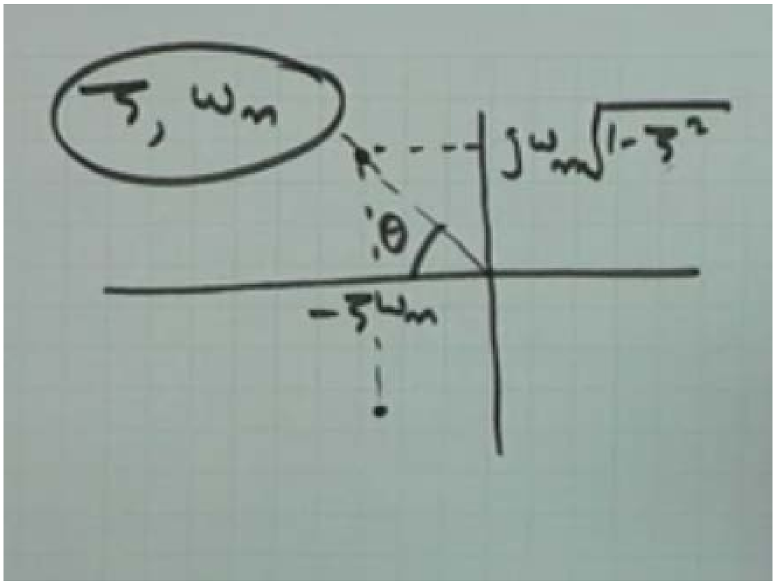

Suppose I give you zeta and omega n, say zeta and omega n are the two parameters which describe the personality of a standard second-order system. How do I get the roots? The roots graphically I can get this way: draw a line here at an angle theta is equal to cos inverse zeta, this is your zeta line, cut a magnitude equal to omega n these are the roots of the corresponding system, these are the closed-loop poles of the system or the roots of the characteristic equation.

The real part is obviously minus zeta omega n and the imaginary part is j omega n 1 minus zeta squared 1 minus zeta squared under the root; the damped frequency is coming over here. So this way we have the relationship, please see. What I want……. this point is very important here as I said, what I have told you that, if I give you the values of zeta and omega n you can translate these values into closed-loop poles, so is it not equivalent that the zeta and omega n of a standard second-order system are specified or the closed-loop poles of the system are specified.

The statement, though at this particular point does not look to be very important is going to be the basis of root locus design. The zeta and omega n are the characteristics of the transient response of the system. If I specify zeta and omega n to you I can translate this into the desired locations of the closed-loop pole and the design procedure then will be to force the closed-loop poles of the system on the desired location, equivalently will mean satisfying the transient performance specifications. So it means the required zeta and omega n and have been translated into these locations for the closed-loop poles.

As I said, primarily we are interested in an under-damped system. So look at the under-damped system please, under-damped second-order system omega n squared over s (s squared plus 2 zeta omega ns plus omega n squared) this is the response function. You will note that I have taken r(s) is equal to 1 by s a unit-step function. So this is the response function for me you see and I want to analyze this particular response function. We have already done in this particular course. You can otherwise independently try it again, the inverse Laplace transform of this. This is such a standard function that almost all control engineers will remember it you see, so I do also remember. this is (Refer Slide Time: 43:44) sin omega n 1 minus zeta squared t plus theta where theta equal to cos inverse zeta, this is the function I get please; y(t) the response of an under-damped system between zeta is equal to 0 to 1 that is your closed-loop poles are complex conjugate poles. This is the response function. This is, as you see now, is the damped natural frequency and theta is the damping angle coming over here.

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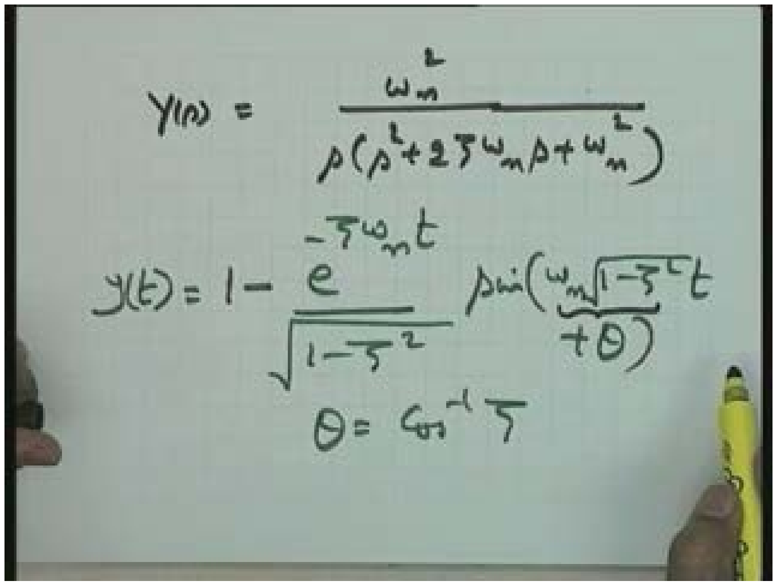

Now if I make a sketch of this function a typical sketch something we have already done, this is your unit-step function, a typical sketch is like this: now the two curves the envelop curves we have already discussed you see if you look at your notes, these are two envelop curves and let me write the functions for these envelop curves it is 1 minus e to the power of minus zeta omega nt divided by 1 minus zeta square under the root and this curve is equal to 1 plus e to the power of minus zeta omega and t 1 minus zeta squared under the root, these are the two envelop curves please.

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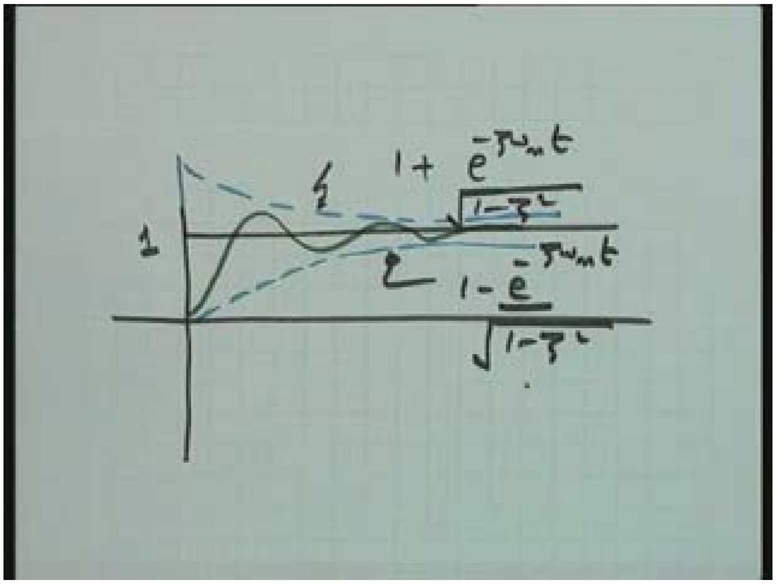

Now you see that these envelop curves has an important role to play as far as the transient decay of this system is concerned. you can note that the faster these envelops decay faster is the decay of the actual response because the response always stays within these two envelops and the time constant of this envelop, help me please, what is the time constant of the envelop which is an exponential curve the time constant tau I want to get answer from you [Conversation between Student and Professor – Not audible ((00:45:41 min))] 1 over zeta omega n that is why I said that zeta omega n product plays a very important role; zeta omega n, inverse of zeta omega n gives you the time constant of the envelop of the standard response and you know that the lower the value of the time constant the faster is the response, that is, faster is the decay of the system. So the decay of the envelop curves is a guideline to the decay of the actual response of the system, this will come in our discussion later.

Come on, couple of equations, I have some 10 minutes available and with your help I hope I will be able to derive these equations quickly. I write y(t) equal to 1 minus e to the power of minus zeta omega nt divided by 1 minus zeta squared under the root sin omega n 1 minus zeta squared t plus theta this is the response please. You note one point you see; the t is appearing with omega n in this particular expression, this is omega nt here, this is omega nt here so it is more convenient for me to make a plot with respect to normalized time omega nt rather than time t. I can make a plot you see for various value is of y, I am taking on horizontal axis the normalized time omega nt.

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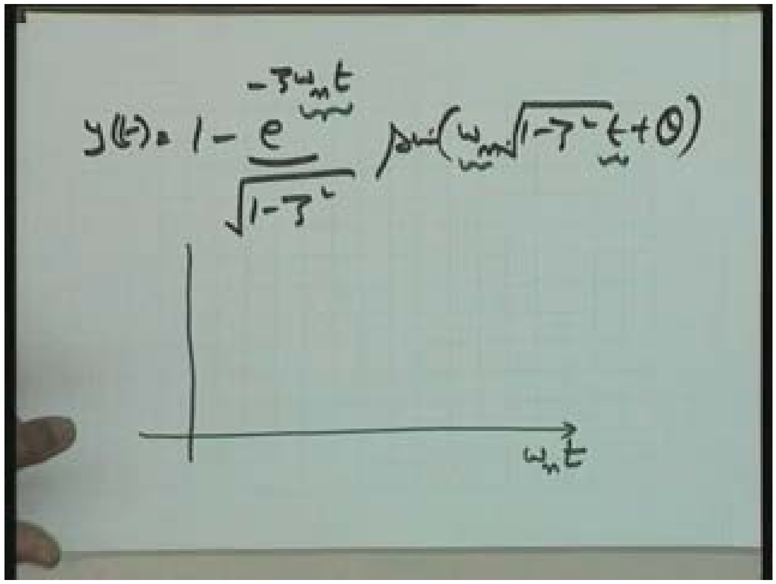

So if I take this normalized time omega nt zeta is the only parameter left you see. So y is the response and this 1 is the input, I can take now the response with respect to zeta. As you will visualize, for lower values of zeta the response is going to be more oscillatory. So let us say that this is the typical response for zeta equal to 0.1. Now you increase the value of zeta, increasing the value of zeta your response may look like this (Refer Slide Time: 47:53). Let us say that this is for zeta equal to 0.5, these are the rough sketches I am making. If you increase it further you can go to zeta equal to 1 and zeta equal to 1 you know that it is a critically damped system, this is zeta is equal to 1 is the response curve where the oscillations just vanish. For a value of for the value of zeta just lower than this there will be an oscillation.

Now, if you increase the value of zeta it is very well known to you that the system becomes sluggish though there is no overshoot but the system responds to settle to the value will also be very large and normally this response for zeta greater than 1 is not acceptable unless there is a very specific application.

Normally zeta greater than 1 in a control system situation is not acceptable to us because the response become very sluggish. Now you find, in this particular case you see, observations, qualitative observations, lower the value of zeta lower the value of t r please. So it means your rise time is lower for lower values of zeta but lower the value of zeta larger is the peak overshoot. You can see that, at this juncture, qualitatively, without even going to the quantitative expression, you find that the peak overshoot and the raised to time conflicting each other. Well, a lower peaked overshoot and a lower rise time are the requirements of design.

Rise time will mean the speed of response and peak overshoot corresponds to relative stability.
Say, large peak overshoot means there is a tendency to go towards oscillation and the system becoming unstable. After all this is your nominal system keep in mind, the actual system parameters will be different than the nominal system and therefore there is a possibility that the actual system parameters may drive the system to instability. So it means a very large peak overshoot is not acceptable to you and similarly a large rise time is not acceptable to you and

these two are conflicting requirements as you say and similar situations you are going to get as far as settling time and the peak time is concerned.

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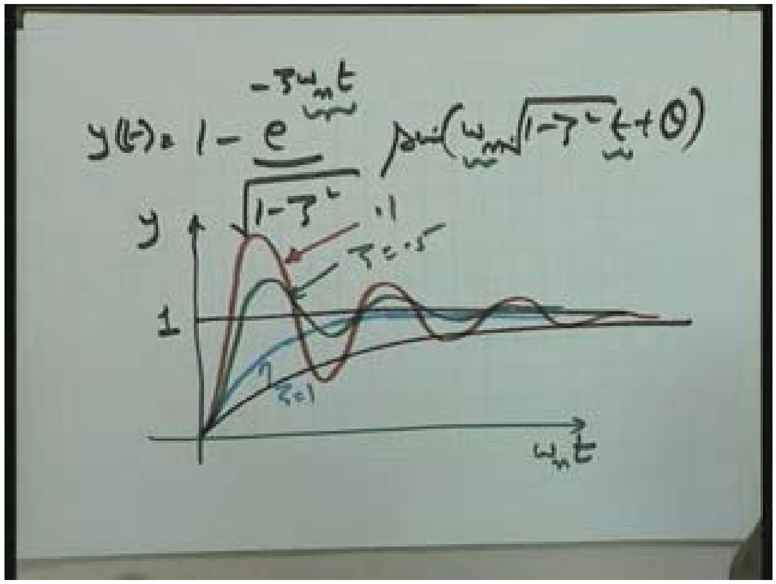

Let me go to the quantitative situation please, help me please. From the response y(t) equal to this expression which we have written zeta omega nt give me the value of t r please, come on independently do it and then compare with the result I am going to write. I want the value of all the four indices for this particular expression please. The first index is t r. So it means the equation to get the t r is going to be y(t r) equal to 1 for the first time that is going to be the value of t r y(t r) equal to 1. So in this particular case please see; e to the power of minus zeta omega nt r divided by 1 minus zeta squared sin of omega dt plus theta should be equal to 0 this is what I want for the first time. This will become zero only when this sin term become 0 and sin term will become 0 only when this is equal to pi (Refer Slide Time: 51:24). The other values will also give you but for the first time if you are considering so I will take it pi.

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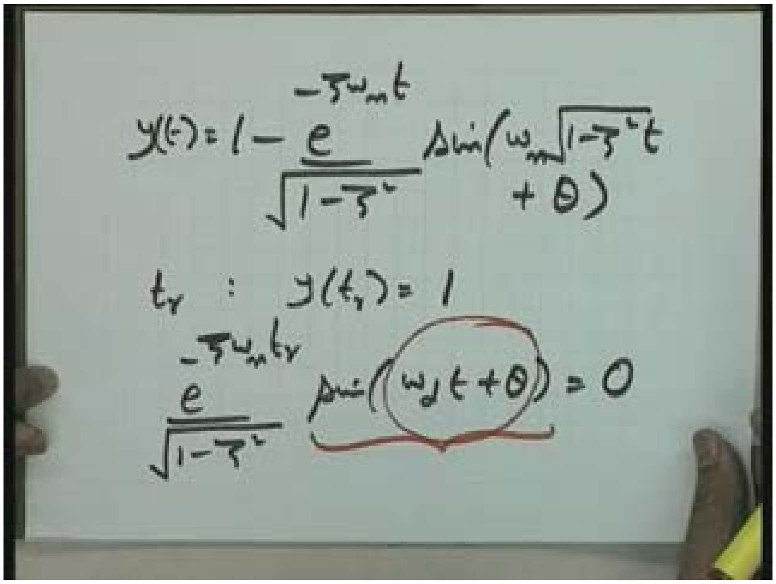

So, in that particular case please see, I am getting t r equal to pi minus theta divided by omega ds the expression; t r equal phi minus cos inverse zeta over omega n 1 minus zeta squared under the root is the expression for the rise time. You have to keep that in mind because I will be using this quite often in my discussion throughout. You can see that rise time is a function of both zeta and omega n and I will resolve this conflict of conflicting specifications, conflicting design requirements this will be resolved later. At this particular point you can just see that t r is a function of both zeta and omega n.

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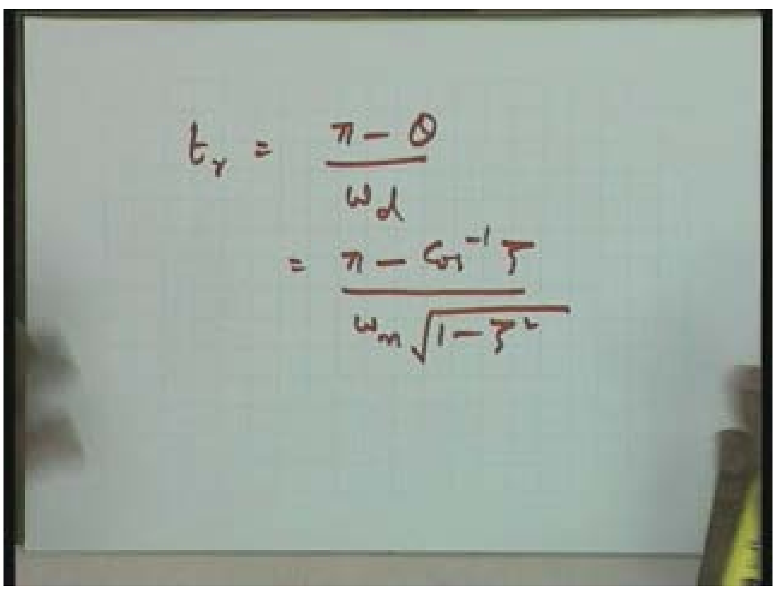

Come on, help me please for t p. t p the time to peak how do I get that please?

You see the response, the response is of this nature and t p means this time so it means actually it is nothing but a point of extrema of the response. It is a point of extrema and I want to get the value of t p where the derivative of y is equal to 0. So set the derivate of y equal to 0 please you see that y again 1 minus e to the power of omega n 1minus zeta squared t plus theta this is your y please, yes give me the derivative please dy by dt equal to and then I manipulate that derivate and give you the expression. This is zeta omega n e to the power minus zeta omega nt (Refer Slide Time: 53:20) the derivative of this divided by 1 minus zeta squared sin of omega dt plus theta, what is next please minus taking the derivate of this now e to the power of minus zeta omega and t divided by 1 minus zeta squared cos of omega dt plus theta into what into omega d.

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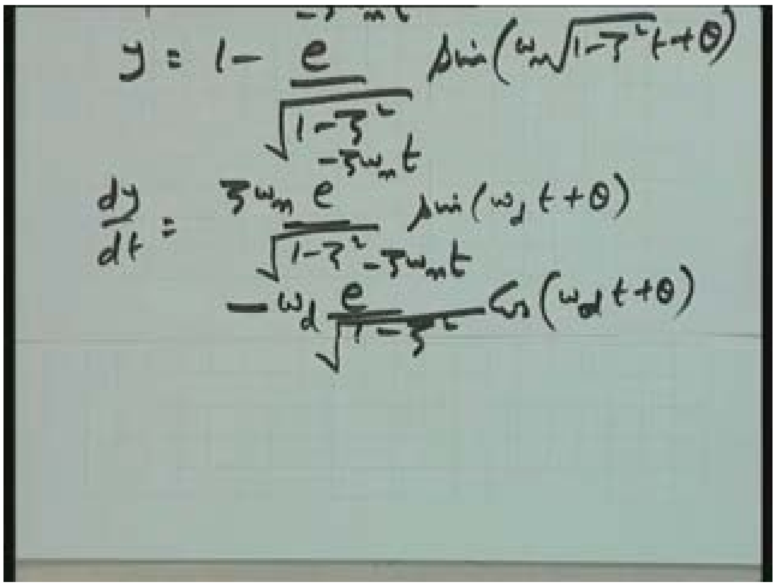

Please see this expression; all of you please satisfy yourselves that this expression is okay. This is the derivative I have taken of the value of y and naturally this equal to 0 at the extrema so Conversation between Student and Professor – Not audible ((00:54:07 min))] the negative of….. dy by dt expression at this particular this I am taking equal to 0 why negative of this please help me? So now in this case you see I have to take this value equal to 0 and from there I get the value of time t which I call as the t p time.

Help me please, the manipulation of this. What I am doing is this way: omega n e to the power of minus zeta omega nt divided by 1 minus zeta squared under the root out zeta sin omega dt plus theta minus…. help me please what shall I write here? What shall I write here with me? 1 minus zeta under root cos omega dt plus theta this should be equal to 0.

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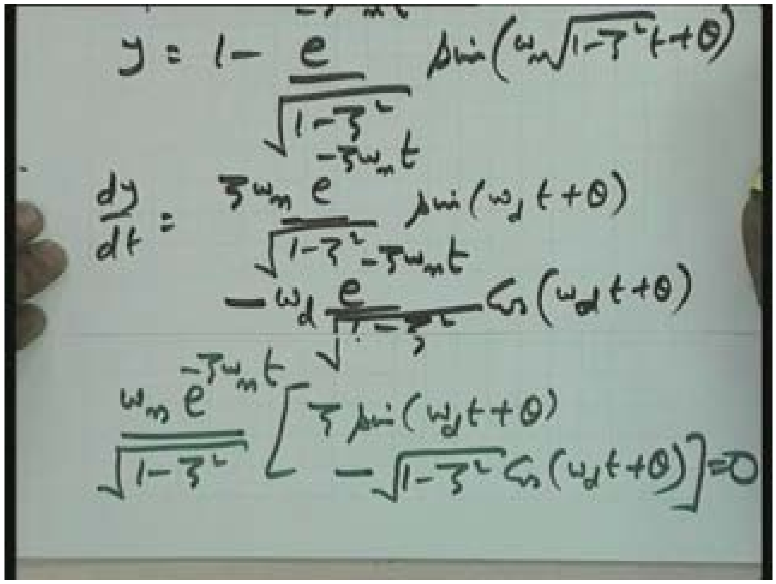

Now give me the manipulation of this equation please. I make you recall that diagram I had given you regarding the damping angle. You see that this was your damping angle theta and here is a typical point which corresponds to complex conjugate roots. This is minus zeta omega n and here I have omega n 1 minus zeta squared under the root j of course the magnitude is this (Refer Slide Time: 55:33). Help me what is zeta equal to? Zeta equal to sin theta and 1 minus zeta squared under the root is equal to cos theta. You can see it very clearly from this particular triangle that zeta is equal to zeta is equal to cos theta and 1 minus zeta square under the root equal to sin theta therefore the expression cos theta sin omega dt plus theta minus sin theta cos omega dt plus theta equal 0.

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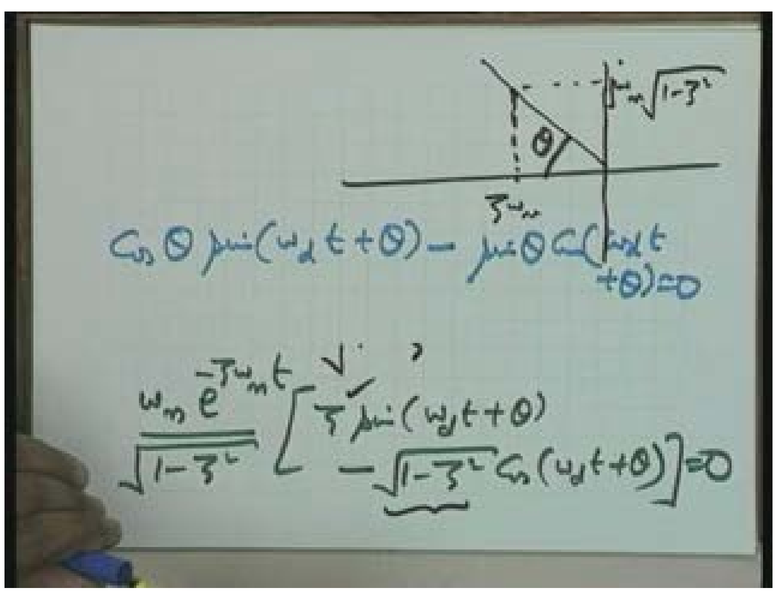

Now simplify this; this is going to be sin a minus b; sin a minus b it means sin of omega dt equal to 0, theta goes away, sin of omega dt is equal to 0 and this will occur, you see that, now see the expression sin omega dt equal to 0 will occur at many points but you are interested in the peak overshoot, the peak will occur at the point corresponding to phi it cannot accurate 0 because that cannot be the peak it occur at phi. So t p is going to be equal to pi over omega d equal to pi over omega n 1minus zeta squared under the root; another important expression I want you to remember.

Now if I want you to find the time to first undershoot please you take here 2 pi. The time taken to first undershoot will become 2 pi by omega d, the time taken to second overshoot will become 3 pi by omega d and so on. However, in my design cycle I will really not be interested in that I will be interested in peak overshoot and the peak overshoot is given by this expression t p is equal to pi over omega d.

Once you have taken this value since it is just…. yes please, just a minute oh just a minute please just this expression I wanted to complete; what is M p? I think you can go quickly once you derive M p for me. What is M p? M p is going to be equal to the overshoot over and above 1. So can I write; this is y(t p) minus 1 so y(t p) minus 1, since the time is already over so I give this result to you a simple manipulation. in the expression for y you substitute t p and manipulate, M p you are going to get this as e to the power of minus pi zeta over 1 minus zeta squared under the root it is a simple manipulation. In the y expression which we have derived substitute t p is equal to pi by omega d and simplify, your peak overshoot turns out be e to the minus pi zeta over 1 minus zeta squared under the root.

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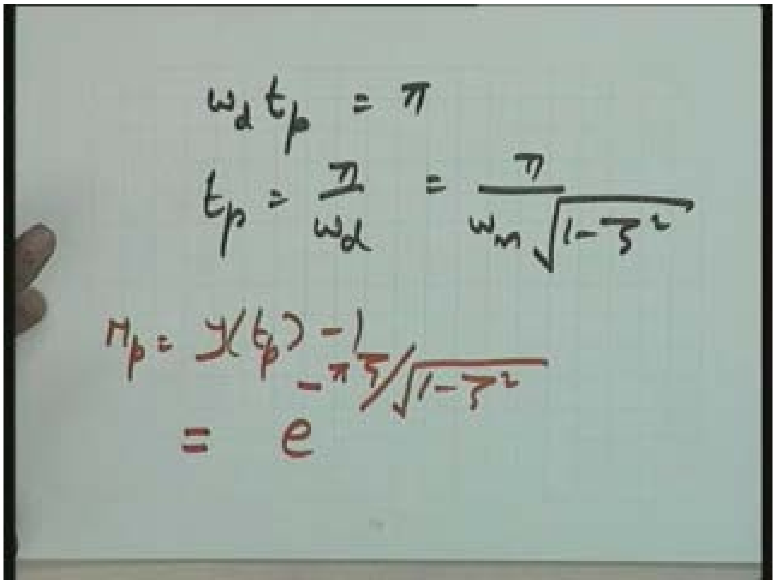

Thank you, I will continue with discussion on the transient performance specifications.