Control Engineering
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Lecture - 25
Concepts of Stability and Routh Stability Criterion (Contd..)

Friends, let us take up the discussion on stability, continuation of our yesterday's discussion. As far as the Routh stability criterion is concern I made a mention that it does not give you the root locations but it does answer the question of absolute stability. That is, it will tell you whether there is a root in the right-half plane or not and we have seen that it can also tell you about the roots on the j omega axis. So let me say that let me summarize our discussion: del s is equal to 0 is the characteristic equation of the feedback system under consideration and we are interested in the stability of the closed-loop system. So we said that first of all you form the Routh array using the coefficients of the characteristic equation. Once the Routh array formulation is completed we have to look at the first column. The first column elements give us the answer on stability.

We are not interested in the magnitudes of the elements of the first column, we simply look at the sign changes and our conclusions were, as per the Routh stability criterion, number one: if all the signs are positive in that particular case the system or the characteristic equation does not have any root in the right-half plane, this was one point. Second point: if there are sign changes in that particular case there are roots in the right-half plane and the number of roots in the right-half plane is equal to the number of sign changes in the first column. The third point: the third point was referred to the observation that there may be a situation that you cannot continue with the Routh array formulation. An example was given to you in which, under the third situation let me say, let me club, let me bring it under two points, two classes: one class we had discussed that there is an all zero row.

If a complete row of a Routh array turns out be an all zero row as you have noted last time then you cannot continue with the array formulation because in the next row you will get the infinite elements. In that particular case I told you that the indication is that there is an existence of there is an existence of symmetrical roots; two real roots in the left-half and the right-half plane, two roots on the imaginary axis, symmetrical or two pairs symmetrical with respect to the imaginary axis and there could even be repeated roots. And we discussed that looking at the auxiliary equation maybe we can sort out whether the roots lie on the imaginary axis or whether the roots lie in the left-half and the right-half plane symmetrical about the imaginary axis.

Another possibility is this that, look, you have only the pivot element as zero only the pivot element that is the first element of a row that is the element which is in the first column of the Routh array is zero and other elements or not zero, at least one of them. So I put this (Refer Slide Time: 4:23) at least one element in the row is non-zero. Look at this situation; I am referring to this situation as a case wherein you consider a row the first element a zero but in that complete row at least one non-zero element is there. So this situation is different than this situation where the first element is a zero but in addition all other elements are also zero.

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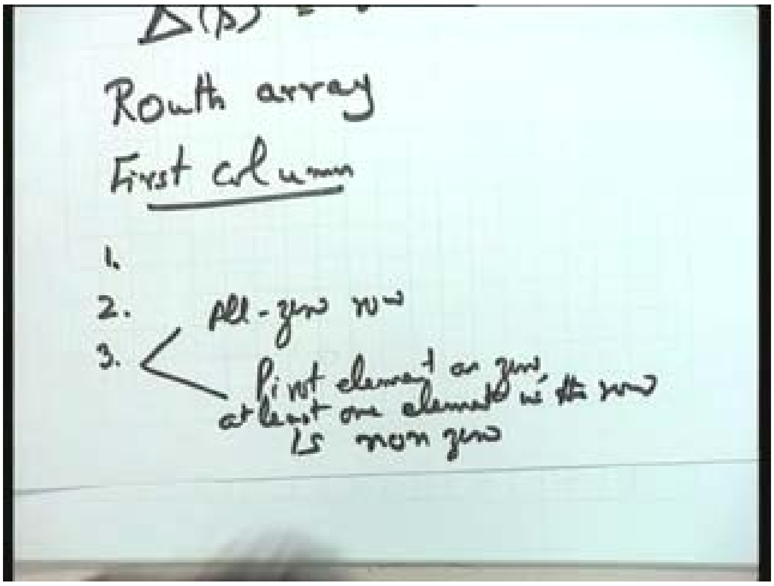

So in this particular case also you can visualize, since the pivot element is zero and the array formulation needs a division by the pivot element so naturally the array formulation in this case will also not be possible. And how to sort out this situation that also I think will come up very well through examples I am going to discuss. These are the cases, other than this case probably we do not come across any case and therefore all the problems we come across can be classified into one of these cases I have given to you.

Now what I do is I take an example and let us say this example lies in which of these cases and how to handle that situation………..,

[Conversation between Student and Professor – Not audible ((5:38))]….yes you have a question please? There is the once again what are the cases under which all zero row all zero row….. let it come through this example if you do not mind. This example is an example of an example of an all zero row and I will explain your point also while explaining this example, discussing this example. This is the example please see, let us go for the array formulation s 6 s 5 s 4 s 3 and so on these are the various rows, the count of the rows. Now the first element being an even power of s so let me take all the even coefficients here 1 8 20 16 s 5 2 12 16. Well, yesterday I was making lot of errors. Hopefully I will be care full today but you have to help me if I make an error please.

These are the two rows I have generated (Refer Slide Time: 6:37) from the given characteristic equation. Let me simplify this 1 6 and 8…. help me please, create the other row, the s 4 row the elements which I have for the other row are 2 12 and 16 s 4 row 2 12 and 16 please check whether this is okay. If this is okay I simplify this also and this is 1 6 and 8. The generation of the next row s 3 row gives me….. please see now here it is 0 0 so I have come to an all zero row situation.

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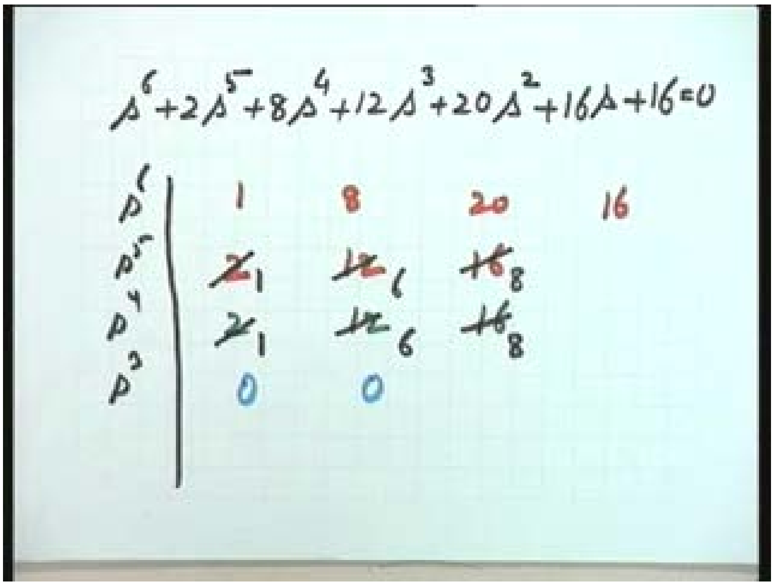

Therefore I cannot continue with the array formulation now because s 2 row will give me an infinity element. Now his point as to what are the situations which cause an all zero row, before continuing with this let me answer his point please. The situations which cause an all zero row are the following: one as I said there could be a pole pair on the imaginary axis symmetrical. Naturally they have to be complex conjugate pair only but there could be double poles also there could be multiple poles also I am exemplifying it through a pole pair.

There could be symmetrical complex conjugate pole pairs symmetry with respect to imaginary axis, there could be a situation like this two real poles is symmetrical about the imaginary axis. I have exemplified all single poles you could take the situation of multiple poles as well but symmetry is the cause of giving you an all zero row that point is to be noted, symmetry. That is why I said that an all zero row will always appear against an odd power of s because the auxiliary equation will turn out to be within even power of s. Just see the situation: an all zero row will always appear against an odd power of s in the array formulation.

As you have seen in this particular case (Refer Slide Time: 9:07) the all zero row has appeared against s 3 but at this juncture I can only say that the system is not stable. Whether it is marginally stable or unstable we cannot say, please note, marginally stable because we cannot say which of these situations do exist which we have listed over here
So for that I have to continue with this and continuation is done with the help of auxiliary polynomial. So in this particular case the auxiliary….. let me reserve this for use later, so I construct the auxiliary polynomial for this particular problem here A(s) equal to…. look at your notes you will find that it is constructed from s 4 row so I am writing this as s 4 plus 6 s squared plus 8 as the auxiliary polynomial; 1 6 and 8 are the coefficients corresponding to an s 4 row, s 4 being an even number, 4 being an even number I am taking all even numbers as the powers of s in my auxiliary polynomial. Any question please, you must raise a question if you have any.

Now the claim is this that this auxiliary polynomial (Refer Slide Time: 10:22) is exactly a factor of the given characteristic equation. So as I said last time one way is this that you factor it out and try the Routh stability criterion on the reminder polynomial to see whether the roots lie in the right-half plane or not. But you see, an alternative is this that you take the derivative of the auxiliary polynomial and continue with the array formulation then whatever conclusion you have from the Routh array they hold on the reminder polynomial, this point may please be noted. Whatever conclusion you will have the resulting Routh array constructed by replacing the all zero row by the elements of the derivative of a well, that conclusion holds only on reminder polynomial. So you will have to have your own conclusions own decisions on the auxiliary polynomial separately.

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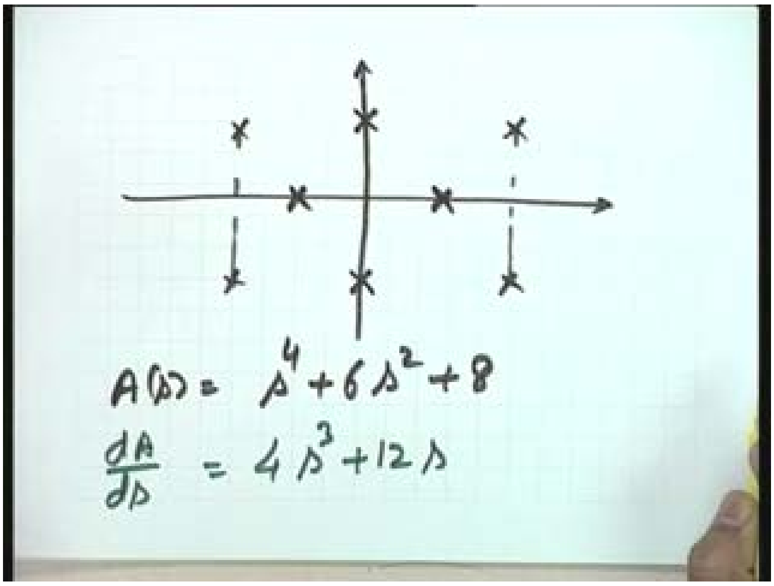

So let me first look at the derivative. So if I take the derivative of this: dA by ds I get 4 s cube plus 12 s. So it means the coefficients are s 3 row, the next row after s 4 is naturally s 3 and the coefficients of s 3 row are 4 and 12. Now let me pick up the original slide. Let me take this 0 and 0 let me replace by 4 and 12 the coefficients of the derivative of the auxiliary polynomial. Now I can further divide this row by 4 to make the calculations simple 1 and 3. Come on please, make a quick calculation now and tell me what are the coefficients of s 2 s 1 and s 0.

I look at my notes and give you the values 3 8 1 by 3 and 8 please see if there is any error (Refer Slide Time: 12:20). You can do a quick calculation and tell me if there is any error here. I hope this is okay. So, in that particular case let me look at the first column please. if you look at the first column you find that all the coefficients are positive meaning thereby that the del(s) over A(s) polynomial has no roots in the right-half plane; del(s) is the original characteristic polynomial and A(s) is the auxiliary polynomial, it has no roots in the right-half plane; this is the conclusion as far as the Routh array formulation using the derivative of the auxiliary polynomial is concerned.

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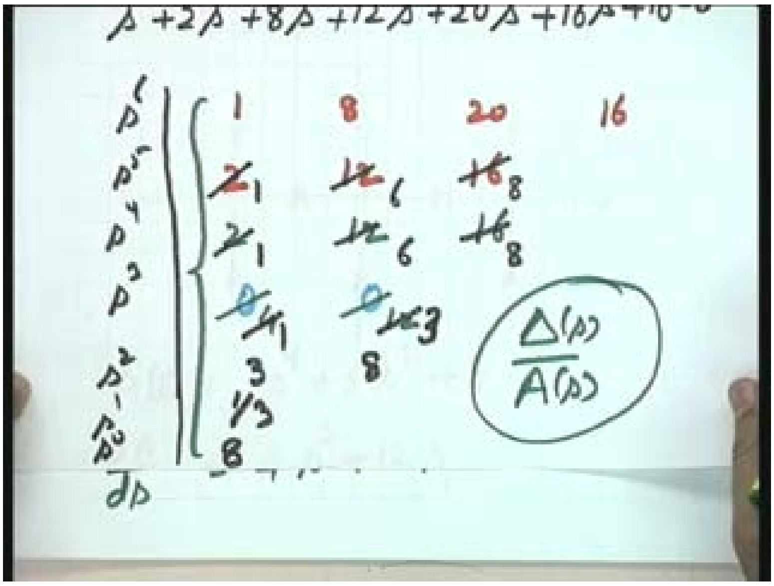

Now look at this particular auxiliary polynomial because that is a factor now. Now you find that A(s) equal to s 4 plus 6 s squared plus 8. Though it is a fourth-order equation, please see you should not be worried about this because it is a specific fourth-order equation wherein s square you can put equal to z and it becomes a quadratic in z z squared plus 6 z plus 8 so it is a quadratic in z and therefore solving this equation is easy. And the solution which I have got is the following that is you first solve for z and take the square roots s turns out to the plus minus J under the root and plus minus J 2; plus minus J square root 2 and plus minus J 2 meaning thereby, please see the symmetry now, the symmetry is this that this pair at J under root 2 and another pair at J 2 has caused this all zero row and this you have got directly from the auxiliary polynomial.

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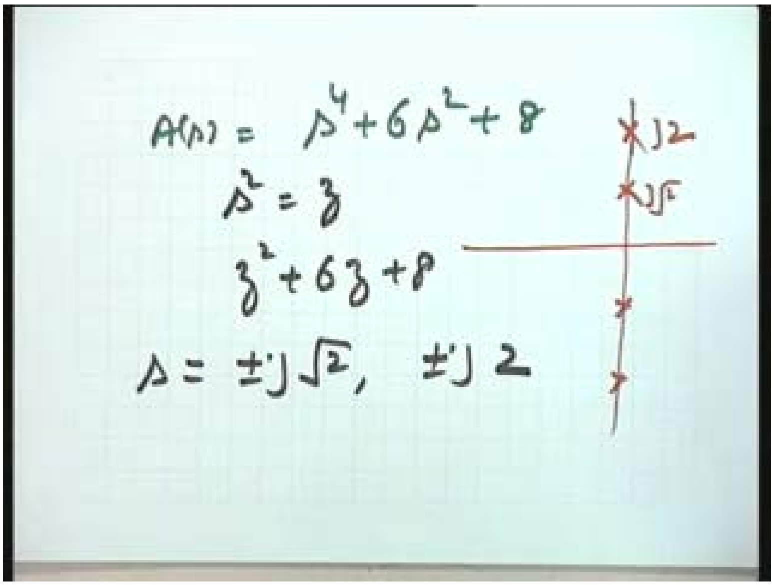

Now look at the fourth-order polynomial factorization was not difficult for you because in effect it is a quadratic in z. So now you see that the conclusion is that this particular system is a marginally stable system. In addition, you can say that the system will oscillate at a frequency under root 2 radiance per second and at a frequency 2 radiance per second. So the marginal stability can come because of a constant magnitude in the output, marginal stability means it is bounded you see, a constant magnitude could occur but in this particular case since the poles are complex conjugate so it means marginal stability is coming because of the sustained oscillations and the oscillation frequency as you see is given by under root 2 radiance per second and 2 radiance per second. I want you to note this point very carefully because I am going to use it shortly in the design exercise. These are the oscillation frequencies. I hope this makes the point clear as to when do we get an all zero row and how to sort out that how to sort out that particular situation, it is a standard procedure which you can follow.

The next case I said there may be a situation that the row is not an all zero row but still you cannot continue with the array formulation. Let us see this example please. in this particular example, help me, in array formulation; s 5 s 4 s 3 s 2 s 1 and s 0 this is a row count (Refer Slide Time: 16:06) and let us see the coefficients now: 1 2 and 6 and here I have 3 6 and 1 I hope this is okay, yes. In this case now you give me the values please 1 2 s 5 plus 3 s 4 plus 2 s cube plus 6 s squared plus 6 s plus 9. Though I promise that I do not make an error but still to get that value of zero is requiring a change. Yes, come on please, let me just divide this total thing by 3 it is 1 2 and 3 help me what is the next row it is s 3 row, the calculation with me is 0 and 3 is it okay I hope this is okay.

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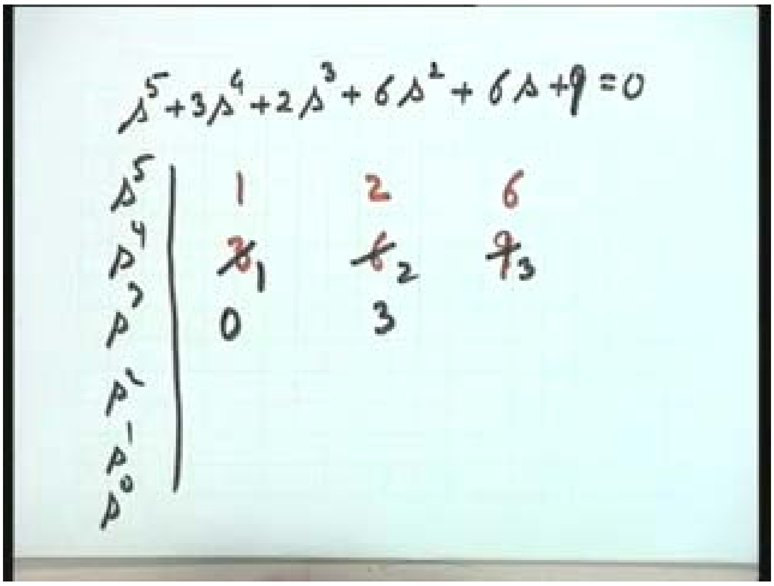

Now zero and three please see it is not an all zero row and since it is not an all zero row you cannot make a conclusion of this sort that there are symmetrical rows. In effect you see this does not indicate anything specifically, my point may please be noted. This is simply merely because the coefficients of the characteristic polynomial or of this nature that the manipulation of these coefficients has resulted into a zero. At least at this juncture no conclusion can be made regarding stability, instability or marginal stability. It is simply a play of the coefficients you see, after all the algebraic manipulations of the coefficients is going on, you can simply say that the coefficients have turned out to be in this particular case of the value that there is a zero here so you cannot make any conclusion. but help me please, if I perturb the coefficients of the characteristic polynomial a little, by the word a little I mean let us say 1 percent in that case what will happen, you can just see, I hope you will agree, if there are roots in the left-half plane and the roots in the right-half plane their physical locations cannot change to an extent that the left-half plane will jump to the right-half plane or vice versa.

You see that after all I am not claiming that Routh stability criterion is going to give me the exact location, I am interested one in absolute stability or instability; so if I perturb the coefficients of the characteristic polynomial by let us say 1 percent so that this zero problem is taken care of in that particular case the left-half plane roots hopefully will remain in the left-half, the right-half plane roots will remain in the right-half and the conclusion about stability or instability will not be disturbed. However, there is a little risk and that risk is this that if your original characteristic equation by chance had roots exactly on the imaginary axis in that case by making this 1 percent or ½ percent change you will not know from your resulting polynomial whether the roots have drifted to the left-half side or the right-half side. So it means there is a risk of making in error if there were roots on the j omega axis. But if you know beforehand that there are no roots on the j omega axis then probably making a little shift in the roots of the in the coefficients of the characteristic polynomial will not change the answer on absolute stability.

So let me first assume that such a situation exists. [Conversation between Student and Professor…… 19:51] ((what is the roots j omega axis give you all zero row so we not an all zero row)) ((00:19:56 min)) very rightly said, the point is very very rightly said. Since it is coming in my explanation I think I will wait for a minute and go to the next example and explain his point or in this very example.

This is the point I was going to make that if there is in all zero row it will be indicated, if there are poles on the imaginary axis it will be indicated by an all zero row and you can be cautious about perturbation. If there is no all zero row so in that particular case there are no root on the j omega axis it cannot exist and hence your perturbation exercise does not involve any risk. I will explain this point further.

So at this juncture, let me assume, as if there are no roots on the imaginary axis in that case the perturbation is possible and instead of perturbing all the coefficients here what we normally do is we only change this zero by an epsilon where epsilon is a positive number. Positive I am taking for convenience that is all because the coefficients here have been….. the first coefficient has been taken as a positive coefficient. So I am replacing this zero by in epsilon a very small positive number epsilon tending to 0 this actually is the result of perturbing all the coefficients of the characteristic polynomial, I need not do this exercise because effectively what do I expect is I expect that all those perturbations will change the magnitudes in such way that this zero will be replaced by a small number that is all. So let me replace it by a small number, continue with the array formulation and let epsilon tend to zero look at the signs of the first column coefficients and then I can comment on the stability of the system.

Yes, [Conversation between Student and Professor……((minus 1 in the first column)) ((00:21:43 min)) yes, ((we will be taking epsilon as positive or negative)) ((00:21:48 min)).

Now, if the previous row had a minus 1 in the first column there also does not matter because epsilon can be positive or negative depending upon the perturbation you give. Let it be positive, epsilon will finally tend to zero. You see that the only point is this; you make an array formulation with epsilon positive and epsilon negative if the perturbation exercise is right, if there are no roots on the imaginary axis both of these exercises will give you the same result on stability. The result on stability will not change if whether you take epsilon positive or negative if the original characteristic equation does not have poles on the j omega axis. So I am taking epsilon positive only for convenience. You can make an attempt you can do the same example by taking epsilon negative and you will find that your final result remains the same.

[Conversation between Student and Professor – Not audible ((00:22:43 min))]
Let us see because finally you see epsilon tending to 0, here in this particular case if you take it this way……. in this okay I think we will do the exercise both ways and it will come out very easily. Let me take epsilon, what is the next thing please? It is 2 epsilon minus 3 upon epsilon is the value which approximately can be taken as minus 3 upon epsilon because epsilon equal to 0 I can take, without creating any infinity problem I can create this, this is minus 3 upon epsilon.

What is the next value please?
The next value I am taking is s 3, give me the s 1 row please, the s 1 row in this particular case, it is, put the epsilon limit, in this particular case it turns out to be 3, 3 I say that, you just please do the calculation yourself and let epsilon go to 0, it is 3 with me is it okay please and the next obviously is going to be 3. Now you make your conclusions both ways taking epsilon positive epsilon negative and satisfy yourself that the conclusion remains the same. The conclusion remains the same in this particular case, is it agreeable to all?

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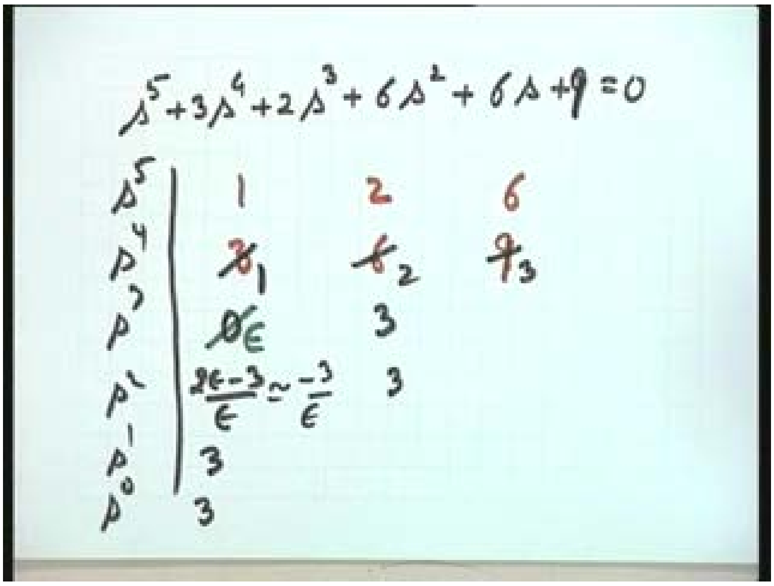

[Conversation between Student and Professor – 24:06…….((Sir but single specific suppose be at s one and s were row)) ((00:24:08 min))] ((then the conclusions will be different epsilon positive or negative, what is your question please? and the s 2 row what is the value expected?.......... plus you see that you see do not create any fictitious situation this way, the claim, you see the logic, the logic is this: in th**e** characteristic equation if the roots are in the left-half or in the right-half plane away from the imaginary axis whether you perturb the coefficients by minus 0.01 percent or plus 1 percent it is not going to make a change because it will not cross the j omega axis. You take an example and let epsilon go both ways, you generate an example in which the answer is different and bring it to me that is better. I leave the problem to you, you generate an example in which the answer will be different and then bring that example to me.

The basic perturbation analysis says that if the roots are not on the imaginary axis the answer will remain the same. I leave this as a home exercise for you. and again if there are roots on the imaginary axis, please see; now epsilon tend to 0 I make and the row becomes an all zero row if the row becomes an all zero row in that particular case it indicates that there are roots on the imaginary axis and any conclusions now if you make by epsilon tending to zero is a risky conclusion.

Please see, in this very example you imagine that there was a situation that when you put epsilon tend to 0 and an all zero row is coming still you can make conclusions when you let epsilon tend to 0 but those conclusions are risky. If all zero row is not coming I assure you that the conclusions are not risky whether the epsilon is taken as positive tending to word zero from the positive side or from the negative side. But if when you let the limit epsilon go to zero if it turns out be an all zero row then there is a risk that the roots may jump from one side to the other and your conclusion when epsilon goes from negative to zero or from positive to zero can be different in that situation only and therefore we do not rely on our conclusions in that particular case.

What you should do is this that since an all zero row is available you can create in auxiliary polynomial, divide the original polynomial by the auxiliary polynomial, extract does roots and then re-apply the Routh–Hurwitz criterion on the remainder polynomial. [Conversation between Student and Professor – Not audible ((00:26:49 min))] no, if epsilon tend to 0 is coming out to be an all zero row in that particular case the previous row is going to give you auxiliary polynomial and epsilon tending to 0 will give you the finite values including zeros, the finite values and that particular situation will give you a polynomial which will have roots because you know that an all zero row can come only when there is a symmetry of this type, this, this (Refer Slide Time: 27:25) these types of symmetries and these types of symmetries will not give you the epsilon values as such, these type of symmetries will give you the finite values when the epsilon tends to 0.

You come and make an attempt on couple of examples and you will find that it works. Why I am not going to this, actually I have already spent two hours on this. Though I very much know that this is only a theoretical exercise and I am not going to use this in the design exercises as you will see shortly. So I really do not want to spend more time on it now. I am going to give you the conclusions and you generate examples, you look at the text book.

For example, the text book, your text book has an example in which epsilon tends to 0 gives you an all zero row and from that all zero row using that all zero row you go to the previous row, you extract, you make an auxiliary polynomial, you divide the original polynomial by the auxiliary polynomial and re-apply the Routh–Hurwitz criterion on the reminder polynomial please.

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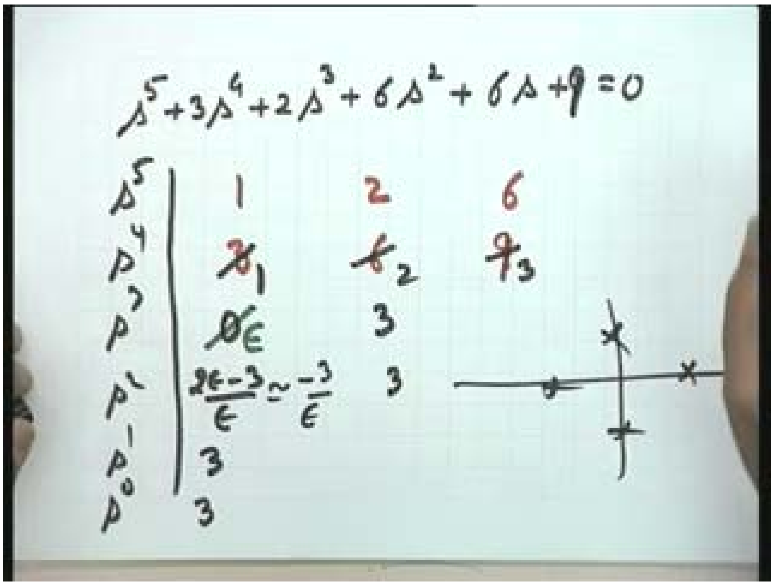

[Conversation between Student and Professor – Not audible ((00:28:23 min))] You can, you see, it is when you make epsilon tend to 0 in this particular case it gives you a finite value. Look at the example please, look at that example; I really want to save because i want to finish it today. Just look at the example given in your text book that there is an all zero row over here so that particular situation will take you to a polynomial and that particular polynomial will be a well-defined polynomial when epsilon tends to 0 and you divide the original characteristic polynomial by that polynomial to get the remainder polynomial and apply the analysis on that.

I leave this as a numerical example for you to complete it and the example of this nature is given in your text book. Today I have to conclude my discussion on stability. Yes, yes there was a question somewhere [Conversation between Student and Professor ((first of all you are placing zeros by epsilon and then you were saying that we put a limit like epsilon tends to 0 and then we get an all zero row)) ((00:29:13 min))] you see that I am not replacing all the zeros by epsilon, if there is an all zero row you have not to replace it by epsilon you see, in that particular case this indicates that there are roots on the imaginary axis or there are roots symmetrical with respect to imaginary axis.

I am [can……29:48] a situation in which there is only one element which has a zero which is the Pivot element. This Pivot element I have taken as an epsilon and using this epsilon as a positive number small positive number I construct this. So once I construct this I get this as a zero now let epsilon go to zero from the positive side. So if it goes to zero I find positive, positive, positive, negative, positive, positive looking at the sign only, in that particular case, as far as this formulation is concerned there are two sign changes and hence there are two roots in the right-half plane.

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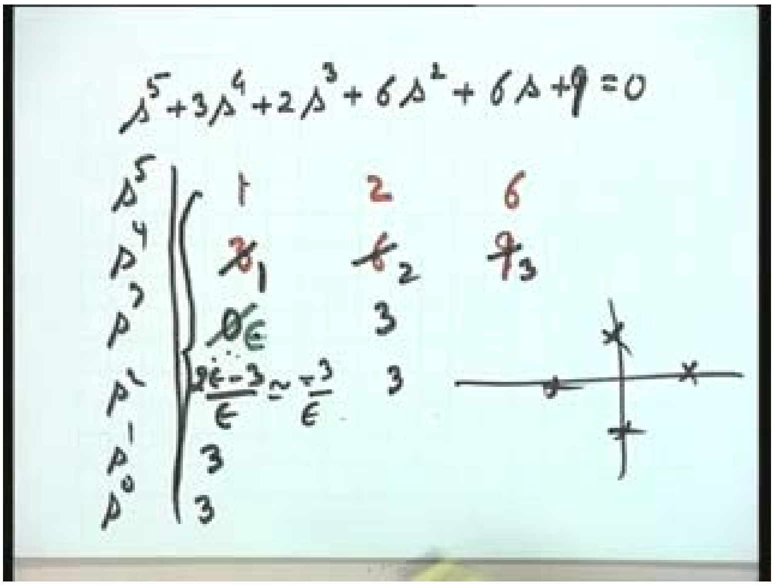

Now I said that if you come across a characteristic equation wherein when you let epsilon go to zero you have an all zero row. Now in this particular case you do not get an all zero row when you get epsilon tend to 0 because if you get epsilon go to zero you make it zero it is not an all zero row, you make it zero it becomes infinity. So, like you are getting a zero here for example you may get two coefficients here so that when you make epsilon equal to zero both of them become zero. In this particular case you are not getting zero, you are getting this as infinity, this as zero but this as zero is not giving you an all zero row situation. So limit should exist when you put epsilon equal to zero you see and that limit should give you zero zero as an all zero row, in that particular case I am saying that there is a situation wherein the particular given characteristic equation has got an auxiliary polynomial with roots on the imaginary axis.

Well, again, not necessarily with roots on imaginary axis, there may be roots on imaginary axis, you have to examine they auxiliary polynomial, if it has roots on the imaginary axis divide out the total polynomial by the auxiliary polynomial and apply the Routh stability criterion on the remainder polynomial. Is it okay please, does it answer your point? You see, in this particular case for example let epsilon go to 0 epsilon equal to 0 this does not give you all zero row, this does not give you defined elements. [Conversation between Student and Professor – Not audible 32:07] ((polynomial will be in terms of epsilon)) the auxiliary polynomial even you put epsilon equal to 0 in that particular case the auxiliary polynomial will have definite coefficients.

Well, in that case, well, unfortunately that particular example, anyone with the text book? If you have a book I will see the example right away that numerical coefficients I have not brought, if you are so keen okay let me reserve it for next time, next time I will bring that. You see, of hand it is difficult to create an example of this type; I will have to bring the characteristic equation which results into this particular situation.

I will be happy if I leave this as an exercise to you so that I complete my discussion today. You will find that when epsilon goes to 0 you will get the characteristic polynomial auxiliary polynomial, divide the original characteristic equation by the auxiliary polynomial, get the reminder polynomial an apply the Routh–Hurwitz criterion on the reminder polynomial.

What is the problem please?
You can take this as an exercise to be completed through the text book and let me continue with my discussion, is it okay? But still after looking at the text book if you will find that this particular problem needs a discussion I will bring it here because I want to complete the stability portion before the minors so let me continue with my discussion please.

Yes please, I take this as the next example: s cube plus 7 s squared plus 25 s plus 39 equal to 0 is the example and in this particular case I am interested in studying the relative stability of the system, please; extension of the Routh stability criterion for relative stability analysis. By relative stability I mean, look, you see, if you had known that the root is here in that particular case this is the root let us say the location is minus p what is the corresponding transient please; the corresponding transient because of this root as you know is e to the power of minus pt because it is a pole 1 over s plus p, if you inverted the transient corresponding to this root is e to the power of minus pt, is it okay please.

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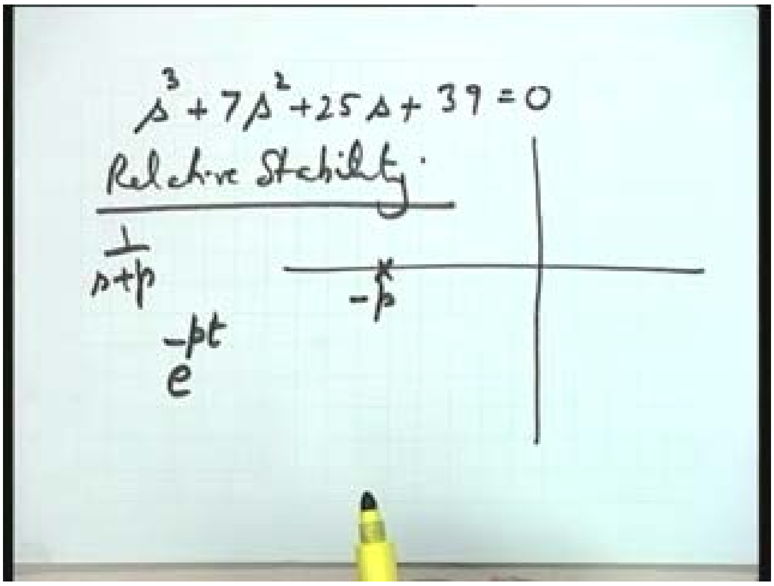

So, if there was in information available to you that not only that the root lies in the left-half plane rather the location of the root is given by this it means naturally you know more about the system than merely the absolute stability. So it means exact root location gives you more information about the system then the Routh stability criterion. Though Routh stability criterion cannot answer the exact location but it can help you one thing. Suppose it tells you whether all the roots lie to the left-half of this line I think to certain extent you have some information about the dynamic nature of the system also. In that particular case you will know that at least the corresponding roots or the corresponding transient is faster than the root at this particular location. Please see that e to the power of minus pt could be written as e to the power of minus p by tau where tau is the time constant of this particular mode. Help me, smaller the value of the time constant how about the response; faster will be the response.

Smaller the value of the time constant faster will be the response. It means larger the value of p faster will be the response. It means deeper is the root in the left-half plane faster is the response, deeper is the root in the left-half plane faster is the response. So your answer so far has been whether all the roots lie in the left-half plane or not. But if your Routh stability criterion can answer this point whether the roots lie to the left of this particular line let us say you have defined this line to be equal to s equal to minus sigma.

If you have defined this line to be equal to s equal to minus sigma in that particular case you will note that all the transients are at least faster than e to the power of minus sigma t. You see, some information more than the earlier information is available though the exact location of the roots will give you more information than this but a compromise between the two situations is possible that; one: only information about the root locations in the left-half plane is available and secondly the exact root locations are available. This, I say that, well, Routh stability criterion can be utilized to give you the relative stability analysis.

The word relative stability is being used in terms of the dynamic response of the system. That is, though the system……. there are two systems both of them are stable which is relatively more stable; the system whose transients die out faster is relatively more stable, the system which has poles deeper into the left-half plane is relatively more stable, I hope you are getting my point. The system whose poles are relatively deeper into the left-half plane will give rise to transient modes which die faster. It means naturally that system is relatively more stable it comes to the steady state quickly.

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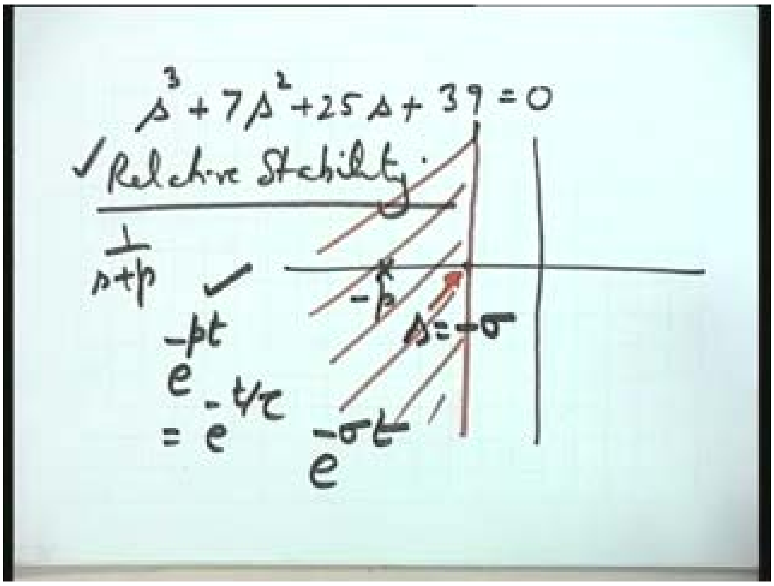

Now you see that I want to extend the Routh stability criterion to this situation that I want this answer to this question to be answered whether the roots of this particular equation lie to the left of this vertical line defined as s is equal to minus sigma meaning thereby whether the transients which are the given by this particular characteristic equation are faster than e to the power of minus sigma t or not. So help me, anyone who could suggest a method of extending the Routh stability criterion for that situation.

Let us say sigma is equal to 1. I want to check whether all the roots lie to the left of the vertical line s is equal to minus 1[Conversation between Student and Professor – Not audible ((00:38:35 min))]. If you apply the Routh condition on the original polynomial it will answer whether the roots lie in the total left-half plane or not. Stability is answered, you can check. Let us assume that it is available. You could check this polynomial the first column of the Routh array has all positive signs. This you could check please, the first column of the Routh, it means it is absolutely stable but that does not answer whether all the roots will lie to the left of this line.

[Conversation between Student and Professor – Not audible ((00:39:12 min))] Yes, if your point is taken, if you put s is equal to minus 1 it will only answer whether there is a root at s is equal to minus 1 or not. If you put s is equal to minus 1 in the original characteristic equation the only thing can be answered is this that whether the root is at s equal to minus 1 or not. It does not answer whether the……. and if it is not, well, the characteristic equation will not be satisfied but it does not answer whether all the roots lie in the left-half plane then as the suggestion as come from the other side is that you put s is equal to s cap minus 1 new variable s cap in the original characteristic polynomial.

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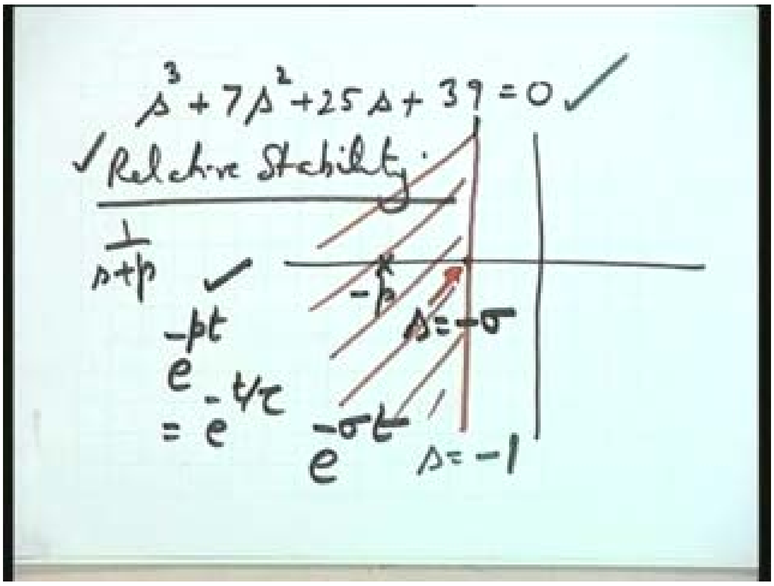

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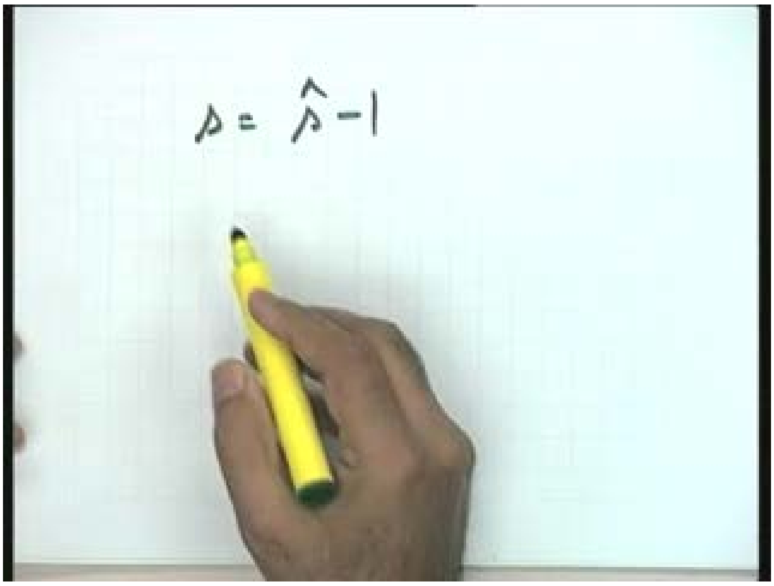

Now you please see, apply your conclusions, let me put it, let me take this slide. If I put s is equal to s cap minus 1 in the original characteristic equation I give you the resulting characteristic equation is this. This is the resulting characteristic equation. now you apply your Routh stability criterion in the s cap plane and I leave this to you to satisfy that the first column elements of the Routh array corresponding to this characteristic equation has all positive signs meaning thereby all the roots of this characteristic equation are to the left of this particular line which is the imaginary axis of the s cap plane. But s cap equal to zero is equivalent to s is equal to minus 1. So, if you translate this information to the original plane s plane it means all the roots are guaranteed to lie to the left of the line s is equal to minus 1. So it means the characteristic equation under consideration has got all the poles to the left of the line s is equal to minus 1 and hence it partially answers relative stability.

[Conversation between Student and Professor – Not audible ((minus 1 or plus 1)) ((00:41:19 min)) s cap minus 1 because you put you see your conclusion is on s cap so when s cap equal to 0 is the imaginary axis of s cap, s cap equal to….. you have applied your Routh stability criterion on s cap plane so this is the imaginary axis which corresponds to s cap is equal to 0. So s cap is equal to 0 is the imaginary axis of the s cap plane, s cap equal to 0 is the minus 1 axis of the original s plane and hence whatever is the conclusion with respect to the imaginary axis of the s cap plane can be translated to the minus 1 axis of the s plane and hence the answer on relative stability.

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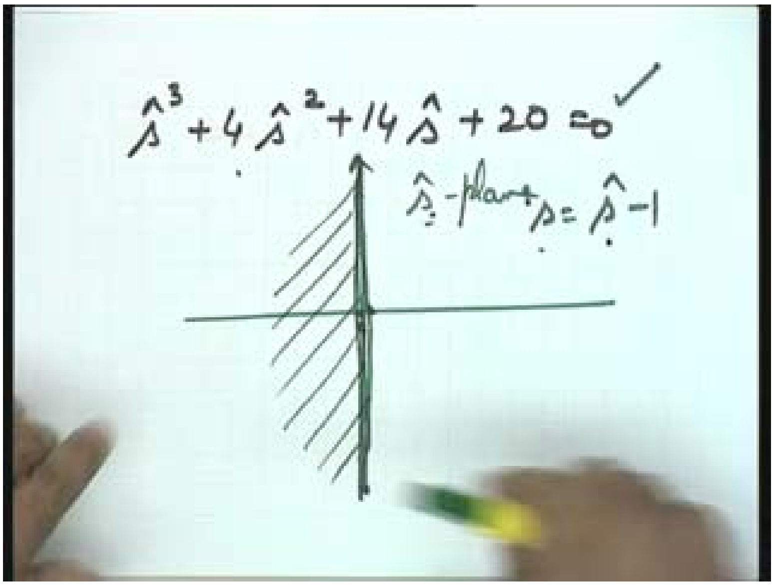

The next point which I have to complete before I let you go because this is a part of the question paper which I have set: G(s) is equal to K this now I take as the feedback system in which there is a parameter K. the parameter K is a design parameter I do not know the value of K and I want to calculate the value of K so that the system remains stable may be a range of the parameters K.

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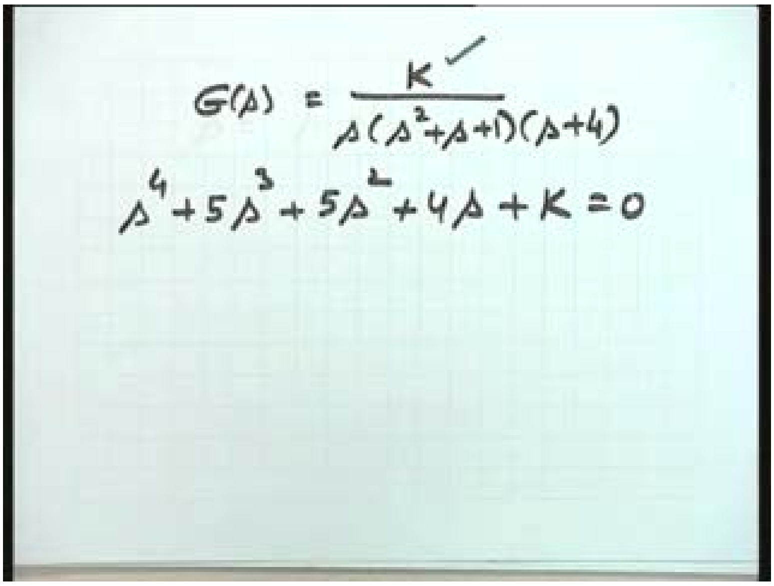

Now this is G(s) please see, your characteristic equation will become 1 plus G(s) equal to 0 which I have written over here 1 plus G(s) equal to 0 is the characteristic equation and I have written the result this way. Naturally your characteristic equation will turn out to be a function of K. So now let us make a Routh array. Obviously the Routh array will be a function of K s cap 4 s cube s squared s 1 s 0. So, if I write the elements over here 1 5 K 5 and 4 here come on please, give me the s 2 row s 2 row turns out to be 21 by 5 it will be K here and now I am writing here from my notes please check up 84 by 5 minus 5 K divided by 21 by 5 this is the element at this particular position (Refer Slide Time: 43:43) do check up please I have been making lot of errors and s 0 element is going to be K.

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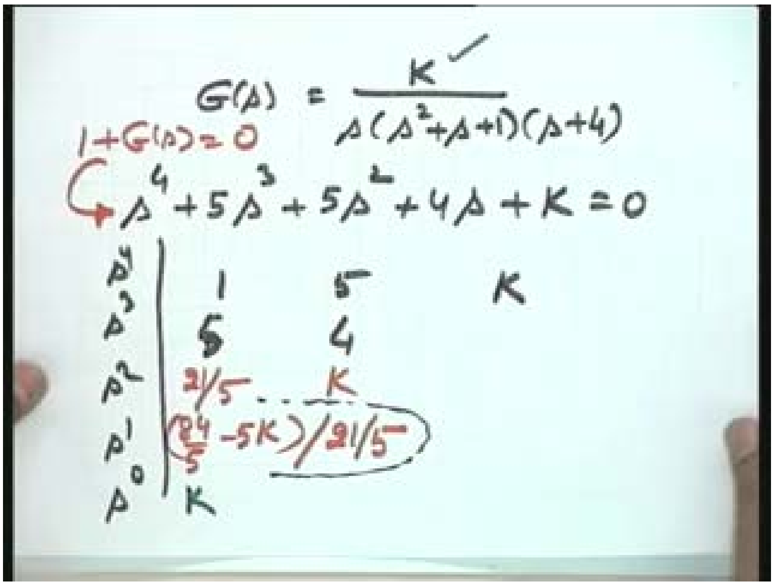

So, as far as the Routh array is concerned the Routh array is a function of K. So it means naturally your stability analysis is going to be a function of K. Is it okay please, this element, the element on this? So come on, what should be the restriction or a set of restrictions on the parameter K for the system to be stable. One restriction you obviously see is that your K should be greater than 0 which otherwise is also true if a physical system will have K greater than 0 an amplifier gain for example.

Now look at the other restrictions. The other restriction is that the element s 1 should be the element of the first column here corresponding to s 1 should be greater than 0 which gives me K should be less than 84 by 25 please check up, K should be less than 84 by 25. So it means unless K is greater than 0 less than 84 by 25 the system is going to be an unstable system. This is the stability restriction that is why I said that in any control system design the Routh stability criterion gives an important contribution because it tells you that what is the range in which you can vary your parameters. Whatever may be your performance specifications after all you cannot make your K go outside this range because there is no question of satisfying performance specifications when the system turns out to be an unstable system and this is the subgroup of our discussion, this is the range of our discussion which we are going to use in the later stages you see, all other was only a theoretical exercise and that is why that epsilon problem also I left in between.

We are going to use Routh stability criterion as a design tool because it is going to tell you as to what is the range of the parameters K for which the system is stable. So I know that this is the range. Help me please, you start with K is equal to 0 and go towards 84 by 25 and you find that your system is a stable system. What will happen, you tell me now, if K equal to 84 by 25 or K is greater than 84 by 25. I think greater than 84 by 25 is a clear situation the system turns unstable. So it means actually if you visualize as K is increasing the closed-loop poles of the system are drifting towards the right-half plane and at K is equal to 84 by 25 you get an all zero row and hence the roots are on the j omega axis. You have rightly said, corresponding to K is equal to 84 by 25 the roots are on the j omega axis.

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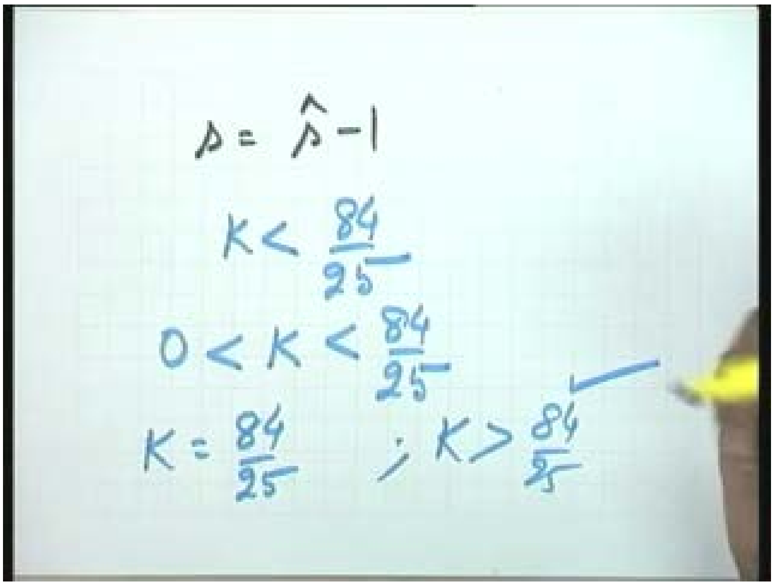

Come on, give me the roots please? Plus minus okay j 2 fine, I take this as the roots as given by him. So please see that if the calculation is right which I am sure you people are giving a right answer to whatever the calculation requirements are coming. So in this particular case as K is increased from 0 the roots are drifting towards the j omega axis and then into the right-half plane and at K is equal to 84 by 25 the roots are on the j omega axis and the system oscillates with the frequency to radians per second. So it means you are turning your stable system to an oscillatory system, the osculation frequency has been determined to be K is equal to 2 radian per second and once K exceeds this value…………… […….…47:46…….foreign language words] whatever is the value let me take it as plus minus j omega 0 please see, you are also behaving like me in that case you see; it is plus minus j omega 0 which you can determine from the characteristic equation, is it okay, which you can determine from the characteristic equation and hence this is the oscillation frequency and the system oscillates at this particular frequency.

The last example, I have a margin of two minutes as the watch shows. I take up one more example. G(s) is equal to K over…. okay let me take this slide I have written this (Refer Slide Time: 48:22) G(s) is equal to K over s into s plus 1 and H(s) is equal to e to the power of minus s tau D and I have a feedback system of this nature G(s) H(s). Come on please, now in this particular case s tau D is the delay and your characteristic equation becomes 1 plus Gs Hs equal to 1 plus K e to the power of minus s tau D over s into (s plus 1) equal to 0. Please see, it is not a polynomial equation because of the term e to the power of minus s tau D and the Routh–Hurwitz criterion is applicable only to polynomial equations. So it means, as such, when you have delay element your Routh–Hurwitz criterion is not applicable.

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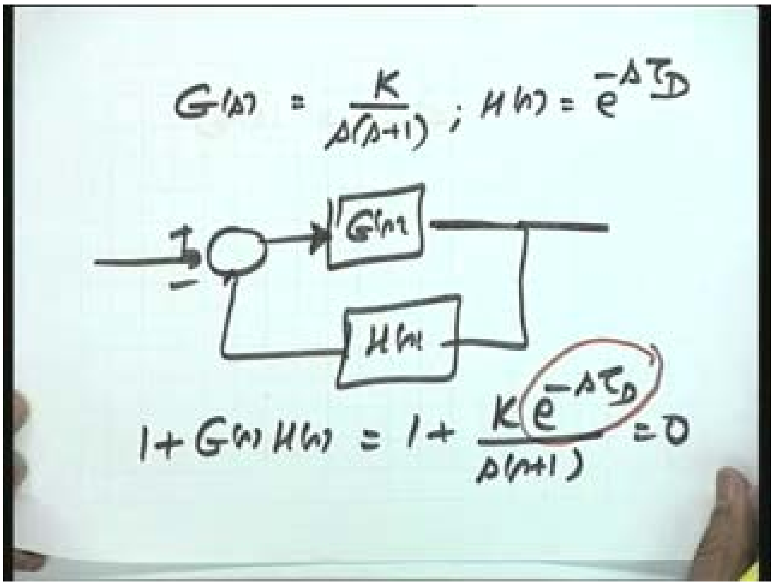

However, if you approximate your delay, let us see e to the power of minus s tau D can be approximated as 1 minus e to the power of minus s tau D. It is a very gross approximation. actually a better approximation is which I have given in my discussion to you earlier; it is 1 minus tau D by 2 s divided by 1 plus tau D by 2 s it is a better approximation with better frequency domain characteristics. And in the literature it is refer to as Pade's first-order approximation. If you take this approximation then also your characteristic polynomial gets converted into a characteristic equation which is a polynomial characteristic equation: e to the power of minus s tau D being replaced by this.

Now what I am doing is just to make the analysis look better I am writing this in the form 1 minus T(s) divided by 1 plus T(s) where now you can see that capital T has been taken as tau D by 2.

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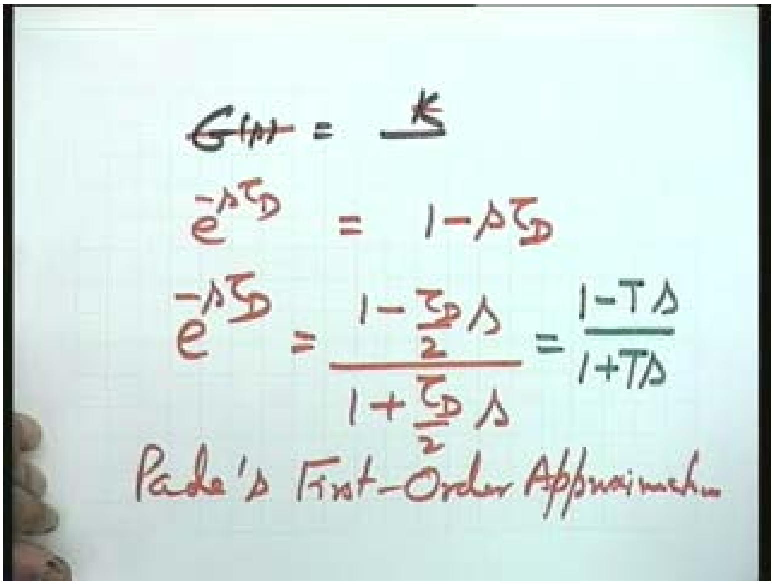

Come on please, you just replace e to the power of minus s tau D by this value and get a characteristic polynomial which naturally will be in terms of two coefficients now K and T, there are two unknown parameters. Now, my design is concern with finding the limits of these two unknown parameters so that the system remains stable; earlier case was only with respect to K and now there are two unknown parameters.

Now, since it is only a third-order polynomial so naturally the time taken in a row formulation will be small and we can quickly write this particular result and find out the ranges of the parameters K and T for stability. Come on please, give me the Routh array for this: T 1 minus KT 1 plus T and K. next let me write this as, from my notes, 1 plus T minus 2 KT minus KT squared by 1 plus T and this is K. You can check it later; you just concentrate on this conclusion. Look at this, you see that for the system to be stable T greater than 0 K greater than 0 which physically also is true no problem but this quantity should be greater than 0 for the system to be stable.

So look at this (Refer Slide Time: 52:07): 1 plus T minus KT into what is this value KT plus 2 should be greater than 0 or KT should the greater than or less than or marginal value you take should be KT plus 2 divided by no no no no no it is T plus 2, it is T plus 2 yes please it is T plus 2 so KT marginal value the border line value of KT is equal to T plus 2 divided by T plus 1.

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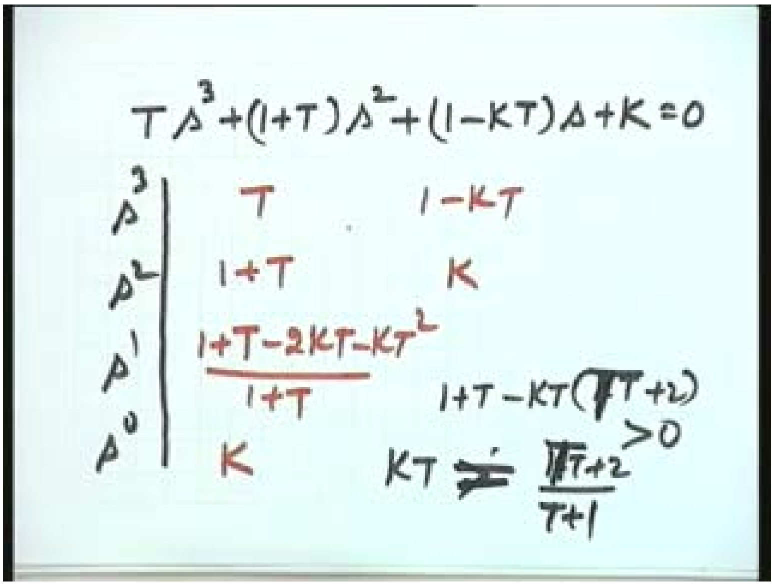

I rewrite this; the border line value of the two parameters which results into instability which results into a border line case a marginal case is KT is equal to T plus 2 divided by T plus 1. Please see, it is a function now. If I make a sketch T versus K it turns out to be like this and this is the stable range and here it is an unstable range; this you can check very easily. Please see that you can see now the conclusion, this last statement I am making. As you increase the value of K value of T what is T please T is the dead time, it is the transportation delay. As you increase the value of T what happens, what is the conclusion on stability; the margin of stability reduces because the system is now stable only for a lower value of K. You just see, I am making the statement: as you increase the value of T, that is, as dead time in the system increases the margin of stability reduces because the system can become unstable now for a smaller value of K. So it means this example very well illustrates that dead time causes instability in the system and therefore as far as possible the dead time in the loop should be reduced because more the dead time less is the margin on the amplifier gain K for the system to remain stable. However, in this particular case the total plane the total range has been divided into two parts. Any design which you make should correspond to this restriction that the values of K and T must lie in this range for the overall system to remain stable.

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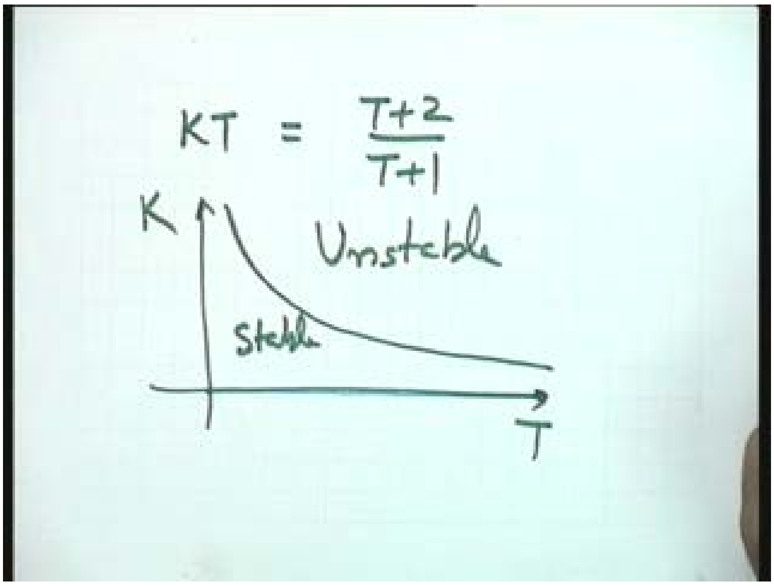

Thank you very much.