Control Engineering
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Lecture - 24
Concepts of Stability and Routh Stability Criterion (Contd..)

Well friends, let us revisit the conclusions we arrived at last time regarding stability. I will put it this way: suppose I have a closed-loop system of this form: G(s) is the open-loop transfer function or forward path transfer function, just let me take it for simplicity or unity-feedback system, R is the input and Y is the output. We know that the closed-loop transfer function is Y(s) by R(s) equal to G(s) over 1 plus G(s). The characteristic equation of the system is 1 plus G(s) is equal to 0. The roots of this characteristic equation are same as the closed-loop poles of the system and these roots dictate the stability of the system.

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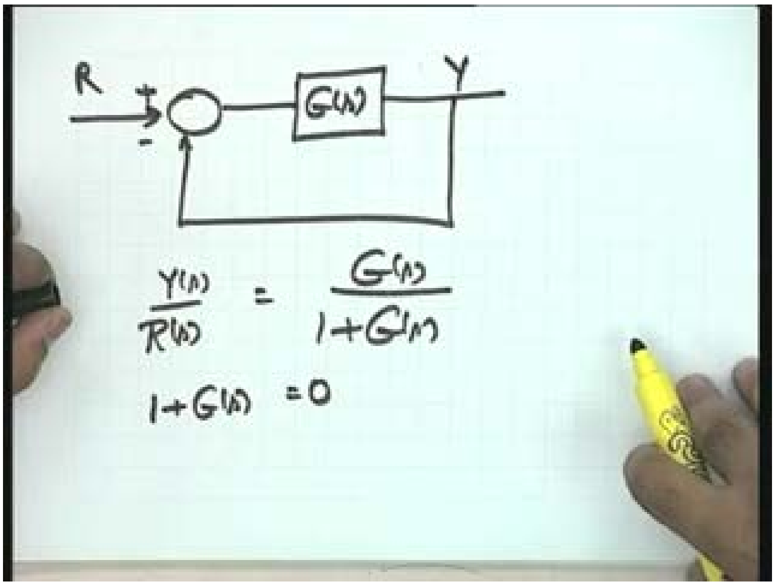

You recall the conclusions regarding stability. The roots of the characteristic equation or the closed-loop poles of the system should have this behavior for the system to be declared stable or unstable. One, case number one let me call it, when do we call this system stable? This is what we established last time. A system is stable when all the roots of the characteristic equation are in the left-half plane strictly in the left-half plane you will call that particular system is stable system. Though we had derived, if you recall, the asymptotic stability and BIBO stability these two definition were given but then I made a statement that in most of the situations, in mostly commonly occurring situations the two are equivalent therefore we will drop the word BIBO or asymptotic and we will consider the system to be a stable or an unstable system.

A system is stable if all the roots of the characteristic equation are in the left-half plane, all the roots I am referring to this point. A system is unstable if either of the two or both of these happens. One: at least to one root in right-half plane, at least one root in right-half plane this could happen or at least one root pair on j omega axis, please see.

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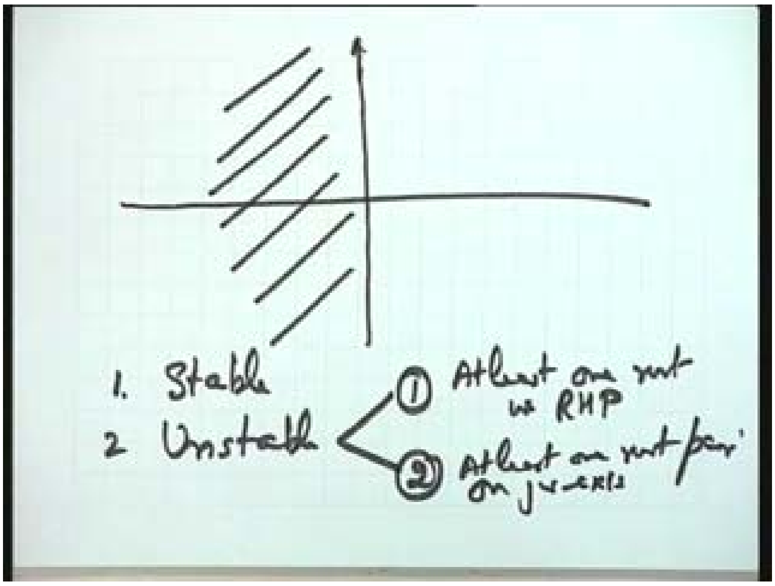

Well, both of these could happen so this is the minimum condition, in that particular case what will happen, if this condition or this condition gives you a growing mood in the response a simple a single growing mode can definitely turn the system to be an unstable system. This is the situation (Refer Slide Time: 3:48) this is the right-half plane at least one root should be there in the right-half plane or a pole pair on the imaginary axis will definitely lead to a growing mode.

The third situation is captured under the definition called marginally stable. A marginally stable system is that system which we have defined this way that, it has all the roots characteristic roots on the left-half plane but for some simple roots on the imaginary axis. The word simple should be very carefully noted. That is, these are the distinct roots, it could be a root on origin, could be a root pair on j omega axis but these are not multiple roots. We have seen that simple roots but for some specific classes of inputs always lead to a bounded output and hence if those specific classes are of no concern to me in that particular case the system may be acceptable to me that is why I bring this particular class of systems under the banner marginally stable system. So it means, for a specific application marginally stable system has to be further investigated to see whether the system will be acceptable to me or not.

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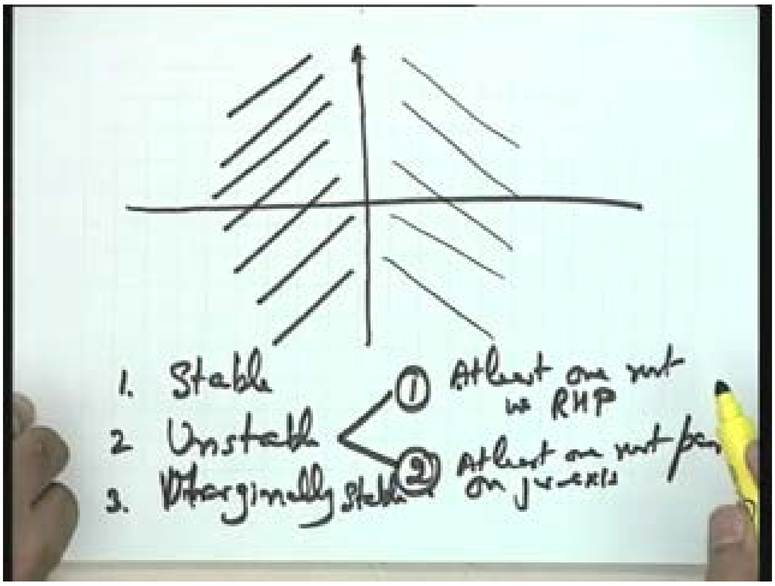

These were the conclusions we derived we arrived at last time. Now let us take a step forward how to determine stability that is my point. And, in general, the characteristic equation can be written as a polynomial an nth order polynomial a 0 s n plus a 1 s n minus 1 plus a n minus 1 s plus a n equal to 0 this is my characteristic equation which has been derived from the equation 1 plus g s equal to 0. So it means the roots of this particular equation or nothing but the poles of the closed-loop system.

Now you are interested to know whether the system with this characteristic equation is stable or not. You will recall, the zeros of the closed-loop system do not contribute stability because they simply change the magnitude of the response they do not give rise to mode of the response. That is why in stability study we are not interested in the zeros at all because they are not going to contribute to the stability analysis.

Yes please, [Conversation between Student and Professor – Not audible ((00:06:14 min))] Yes, that I think I will come to that. yes, if a zero, his point please, zero that way is important, if a zero cancels a pole and if that particular pole unfortunately is in unstable mode in that particular case, in your characteristic equation that mode is not visible and you may declare the system to be a stable system though it may be an unstable system because of that mode which has been canceled by zero; because exact cancellation is never possible, it is only a mathematical cancellation and in an exact physical system that particular mode which is cancelled by a pole by a zero in its mathematical model may lead to instability. However, that particular situation will come later in our discussion.

At this juncture, let me assume that I have a system wherein the zero does not cancel a pole. And I may make a statement that in most of the physical systems it appears like that. That is, the zeros do not cancel a pole unless you design it particularly. Such system, since the question has been raised, let me refer to that are referred to as controllable and observable systems. Though the concept of controllability and observability is not coming in our discussion in any controllable and observable system the zeros cannot cancel a pole.

And since he has raised a point please note that it is only under this assumption that zeros do not cancel a pole that BIBO stability is same as asymptotic otherwise the two concepts are different because if a zero cancels a pole BIBO stability will not show that because the pole has been cancelled. By asymptotic stability that cancellation does not occur because there you are working with a state variable model and hence the effect of that pole will show up in asymptotic stability and the two conclusions regarding stability may become different. So I am referring to that situation, I am basing my discussion on this assumption that there is no cancellation which is true in most of the physical situations.

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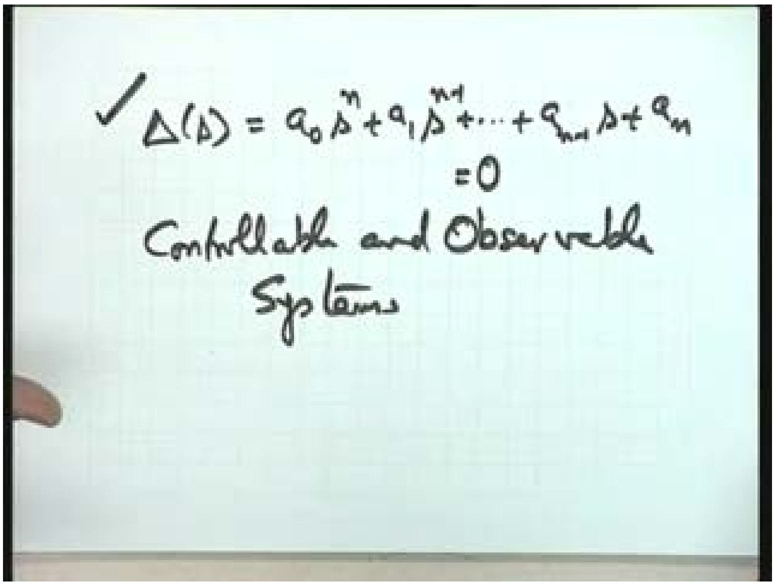

So, coming back to this particular equation the characteristic equation I want to study the roots of this particular equation to see whether the system is stable or not. Well, you will say that why study this problem; you will simply feed this particular polynomial to a CAD package and the CAD package will immediately give you the roots of this particular equation where you have the answer for stability. So, as such, if the answer on absolute stability is concerned probably we should not continue with our discussion because the computer aided software is available today to answer the question, in every lab you are equipped with computer added computer software so that you give this polynomial as an input it will give you the poles it will give you the roots of the equation, you examine the roots vis-à-vis explain and declare and talk about it and give you a conclusion about stability. But the point of further stability analysis is not referring to is not referred to only absolute stability we are going to use the criterion I am going to discuss in the design problem.

For example, these coefficients are a 0 a 1 a n minus 1 a n are functions of some of the physical parameters of the system. Let us say an amplifier gain k, let us say a time constant of a sensor, let us say the torque constant of a motor so these are the coefficients which are the coefficients of the characteristic polynomial they are directly affected by the system constants the physical constants of the system. If I have a question as to what are the ranges of the constants of the system which I can take so that the system always remains stable probably the numerical solution of the equation will not give an answer to this problem or even if it gives an answer it will be a difficult procedure of getting the answer, try to get my point please.

My question is this that give me the ranges of all those parameters which you can take without any risk so that for any parameter value within those ranges the system will remain stable. So in that particular case you see the numerical solution because the computer software will definitely require the numerical values of these coefficients and the numerical values of these coefficients you cannot feed for a design and problem and hence we look forward to a stability analysis technique which is helpful in design problem in addition to giving the answer on absolute stability and the technique we are going to take up here is the Routh stability criterion.

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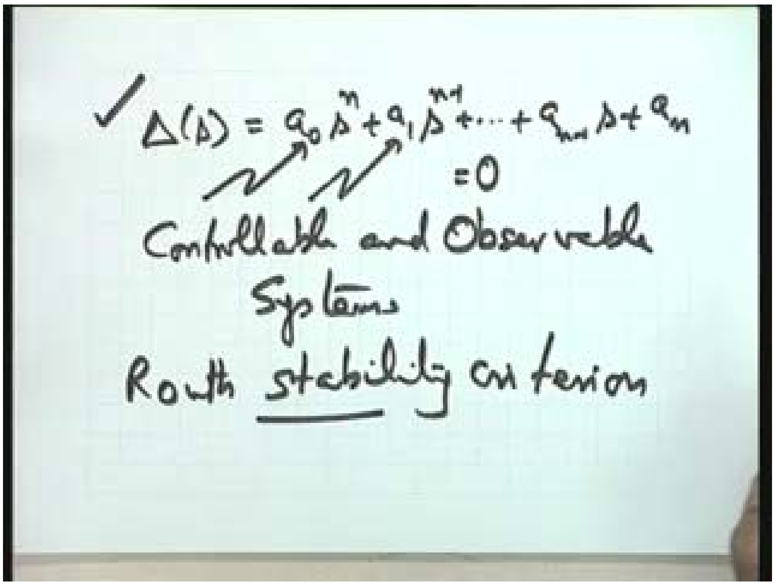

See the objective in mind, the Routh stability criterion, finally if I do not prove to you that Routh stability criterion has got properties better than the numerical solution of the equation in that particular case there is no advantage in taking such a criterion. I hope I will be able to convince you that it has got certain merits which we are going to utilize in our design problem. So, my today's discussion is primarily based on the use of Routh stability criterion on the study of on stability analysis of a system with a given characteristic equation.

So let me rewrite the characteristic equation here: a 0 s to the power of n plus a 1 s to the power of n minus 1 plus a n minus 1 s plus an equal to 0, just see that. One general statement I like to give which does not kill the generalization please note. Can I take always a n is not equal to 0; let us see what happens if an is equal to 0. You see, if a n equal to 0 what happens s is equal to 0 is a factor so it means you now that s is equal to 0 is a root of the characteristic equation, why to determine this using Routh stability criterion when it is so clearly visible. So I am assuming that characteristic equation in this characteristic equation which is in front of me a n is not equal to 0. So, if a n equal to 0 you take out the root at s is equal to 0 and reframe your equation so that the last coefficient of this particular equation which is a coefficient of s to the power of 0 should be non-zero, one point.

The second point I am making is that a 0 is always greater than 0 it is only helping me analyze you see; there is no loss in the generality because what is there after all; if a 0 is negative I will make it positive by multiplication of the entire equation by minus 1 that does not change the analysis that does not change the answer on stability. So these are the two things I will keep in mind and right my criterion on the basis of this.

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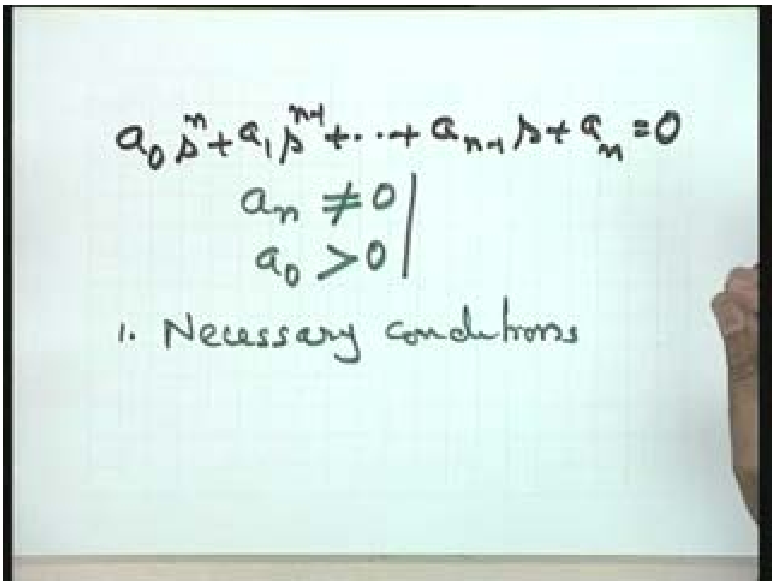

Now, before I take up the criterion, please note that there are necessary conditions which this particular characteristic equation must satisfy for stability. Very simple proof you see, you could even take some examples and satisfy yourself, I am avoiding the proof to save time. The necessary condition is this that all these coefficients in this particular polynomial must be positive.

Now I am making the statement positive under the assumption that a 0 is greater than 0 otherwise I would have said must be of the same sign that is all. So now I can make a statement that all these coefficients of the characteristic equation must be positive for the system to be stable and this can easily established. A zero value also destroys the necessary condition for stability. That is there should be present and positive, a particular coefficient if a particular coefficient is zero that is sufficient to conclude that the system is not stable. So it means all these coefficients in the characteristic equation must be positive and non-zero for the system to be stable, this is the necessary condition to be satisfied. So, if you have a polynomial not satisfying this condition so it means for stability study you need not proceed for the application of the Routh stability criterion.

Now let me go this sufficient condition. Well, even if these necessary conditions are satisfied it does not guarantee that the system is stable; it simply says that there is a possibility that the system is stable and hence the sufficient conditions should be examined and I go forward to the criterion now to establish the sufficient condition. By the way please help me, I do not know whether I asked this question last time or not whether the Routh criterion has been done in some course or not, circuits or something, okay fine…… [Conversation between Student and Professor – Not audible ((00:15:10 min))]

Please see that, again I will not give the proof, involved algebraic proof is there, from the application point of view this particular criterion. I form a Routh array an array formulation and from the array formulation I will be able to make a conclusion regarding the criterion. so the first term is s to the power of n, so let me write is s to the power of n, write here s to the power of minus 1 s to the power of minus 2 and s to the power of 1 s to the power of 0. This is simply a count of the rows you see, well, so many rows are involved and this I am keeping as an index of the number of rows. The first two rows are given to you and the coefficients of these first two rows are given by this characteristic equation, how do you we write these rows please see it very carefully. s to the power n first coefficient is a 0 so write the first coefficient here it is a 0, s to the power of…. then you write leaving one coefficient, you see then after a 1 it is a 2, if n is e 1 so it means all the coefficients here corresponding to even powers of s will appear and if s is odd that is if n is odd then all the coefficients corresponding to odd powers of s will appear in the first row. So in this particular case I am writing a 0 a 2 a 4 and so on.

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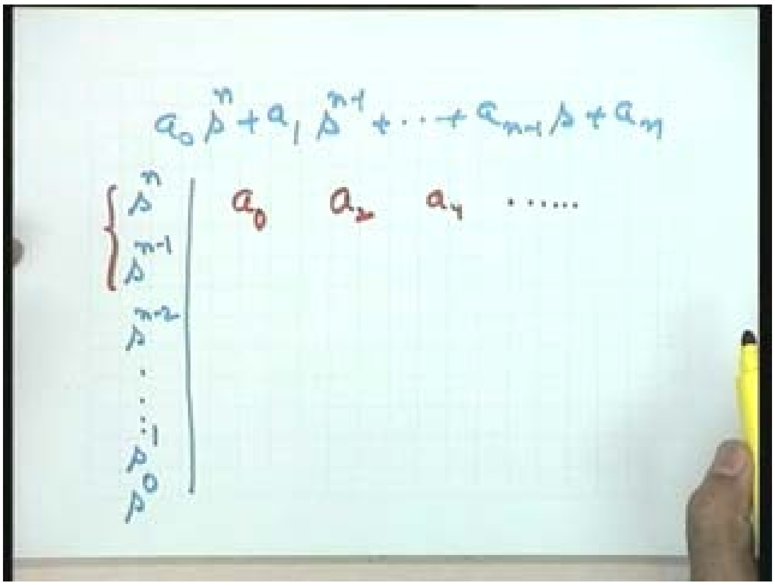

You definitely should raise a question in case there is a doubt. Similarly, s to the power n minus 1, I said coefficients of s to the power of n minus are also given to you. The first coefficient is a 1, again if n minus 1 is even, well all even coefficients will appear, if odd then all the coefficients corresponding to odd powers will appear. So in this particular case it is a 1 a 3 a 5 and so on. So the two rows you have seen are available to you; these are the two rows of the Routh array formulation so the total ere formulation I am going to get over here using these two particular rows. As I said the physical feel you may not be able to get because I am not going to prove the criterion so it is only the algebraic method which you are going to get as to how to apply this particular method for stability determination stability analysis.

Now what I do is I take this pair (Refer Slide Time: 17:50) I need your attention here please; I take this pair, take up this coefficient a 1 multiply a 1 with a 2; a 0 with a 3 and write the coefficient here a 1 a 2 minus a 0 a 3 divided by a 1. This is the first coefficient of the row s to the power of n minus 2. Now please note that s to the power n minus 2 has lost that meaning that this is the coefficient here corresponding to s to the power of n minus 2, it should not confuse, it is only an index of the number of rows; if you start with n the next is n minus 1, n minus 2 and you will go down to 0. You could write nth row and n minus 1th row, n minus 2th row whatever index you would like to take. so please there should not be any confusion that the coefficient here corresponds to the element or corresponds to the term s to the power of n minus 2 in the characteristic equation, no. once you write the these two rows (Refer Slide Time: 18:59) the characteristic equation has lost its significance you need not look at the characteristic equation, all other rows will be constructed looking at these two rows only. So the s n minus 2th row is under consideration, the first element of s minus 2th row is coming and that element I repeat now is constructed like this, you take this pair and this pair cross multiply starting with this a 1 into a 2 minus a 0 into a 3 divided by a 1 is the first element.

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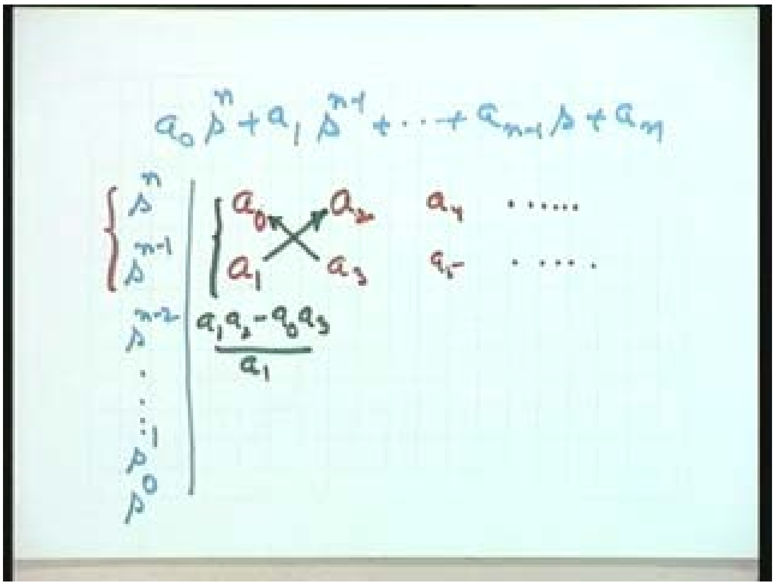

Let us look at the second element of this. Since the space is not there let me put a star here and write a star separately, star equal to what? Star will be equal to a 1 into a 4 minus a 5 into a 0 divided by a 1. So I am writing a 1 into a 4 minus a 0 into a 5 divided by a 1. So I hope the procedure now gets known and the next element double star here; if there are coefficients here it will be a 1 into this coefficient minus a 0 into this coefficient divided by a 1. If there is no coefficient then in the multiplication game you will take the coefficient to be equal to zero. A missing coefficient will be taken as zero coefficient in the process of construction. I am sure when I take up the examples this will become more clear but you just set the rule in your mind. So this way I have constructed the next row s n minus 2th row I call it.

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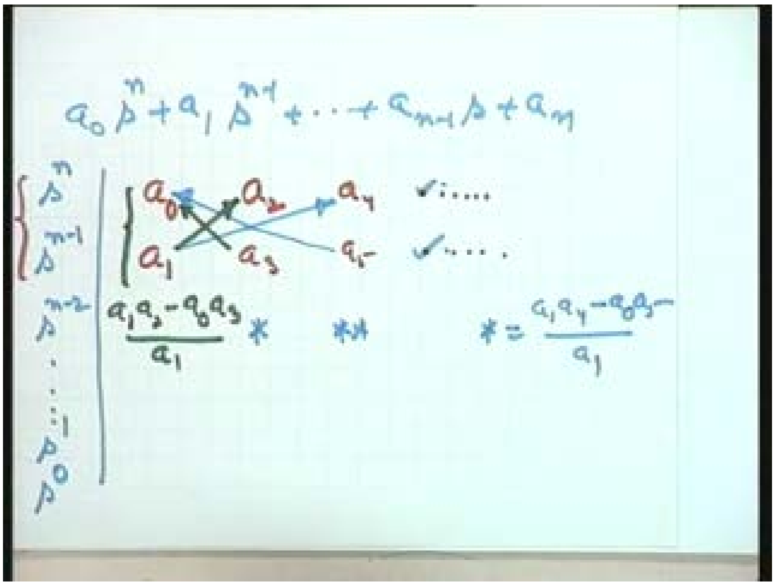

Now go to the next one. The next row after this, the procedure is same, now you take these two elements (Refer Slide Time: 20:53) take these two elements so this particular element now here, let me say, a dot will be constructed like this, this into this minus this into this divided by this, the same procedure goes. Now I think, instead of talking about this general case it will be useful if I immediately go to a numerical example because this will exemplify the technique better. So let me take a numerical example: I take s 4 plus 8 s cube plus 18 s squared plus 16 s plus 5 equal to 0 the Routh area.

I have not yet given the conclusion as to how to determine this stability, I am simply giving a numerical example to give the procedure of construction of Routh area. so what I will do is I take the first two rows corresponding to s 4 and s 3 s 4 and s 3 come on please, you can independently write your coefficients and compare with my result. I am writing this as 1 here 18 here and 5 here (Refer Slide Time: 22:00) because this is an even power so all the coefficients corresponding to even powers are coming over here; s 3 is an odd power so all the coefficients corresponding to odd 8 16 taking 0 as an even please see this is s to the power of 0 I am taking zero power as an even power that is why 1 18 5 and 8 16 the two rows have been constructed.

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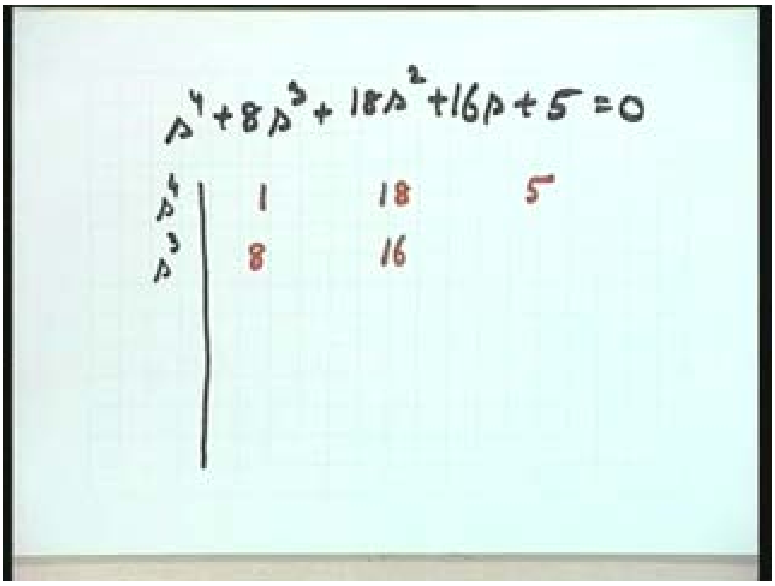

Now I will not look at the original characteristic equation. I will now construct the row s to the power of 2 the next row in the descending order, the first element of the row will become 8 into 18 see this 8 into 18 minus 1 into 16 divided by 8 I have got the answer with me it is 16 you could check please if there is an error; 8 into 18 minus 1 into 16 divided by 8 so I get this as the coefficient, please see. Next coefficient please? Now, next co efficient you see that 8 into 5 let me take a different color here 8 into 5 minus 1 into 0 the missing coefficient I will take as 0 divided by 8 that obviously gives me 5. Actually if one of the coefficients is missing are 0 you can see that this element simply gets transferred here obviously this element gets transferred here because it is 8 into 5 divided by 8.

Next, let me take next as s to the power 1 row. so when I am taking s to the power of 1 row I will now look at 16 into 16 minus 8 into 5 divided by 6 the calculation here is 27 by 2 some of you must check please for an error in my calculations 27 by 2. How about the next help me please, what should I write over here? [Conversation between Student and Professor – Not audible ((00:24:14 min))] No, now you are taking 16 into a missing term, this is the missing…. the first row will not come over here 16 into a missing term minus 8 into a missing term divided by 16 which is obviously 0 or you need not write 0 it is a missing term again. So it means no more coefficient over here, it simply says that in this particular row no other coefficient need to be determined.

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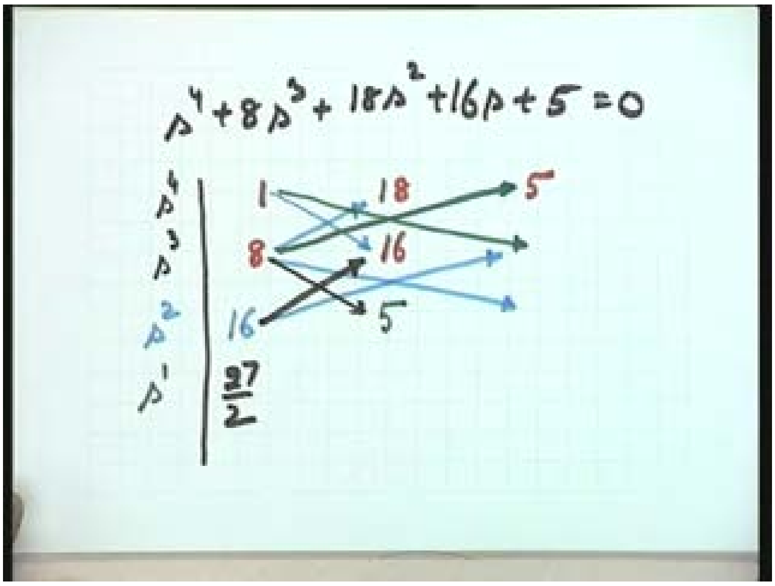

Let us go to the last one the s to the power of 0 coefficients. I think if you have seen the procedure for s to the power of 0 no calculation need be done. Please help me, what is it is 5 because there is a missing term so this coefficient will get transferred to this place and hence once you have reached s to the power of 0 by this particular systematic procedure you will say that your Routh array formulation is complete, this is the complete Routh array.

Now, Routh suggested that in this particular case the answer of stability will be given by examining the coefficients of the first column of the Routh array (Refer Slide Time: 25:25) and when you examine the coefficients we have to examine only their signs and not their magnitudes. The criterion says that if there is no change in sign that is if all of them are positive, because now we can say positive because the first coefficient has to be positive and we have taken a 0 to be greater than 0. So if there is no change in the sign if all the coefficients are positive it is guaranteed that all the roots of the characteristic equation are in the left-half plane and the system is stable, please see.

Though you see in this particular case formulation of the Routh array even on computer is much easier than solving this particular equation, still I do not think the advantage lies in this aspect you see; the advantage lies in the design aspect which will come later. Here the Routh array gives you an answer on absolute stability looking at the first column, if there is no sign change in that particular case the system is absolutely stable there are no poles in the right-half plane.

Now, though in this particular case the system is declared to be stable let us see the other part of the Routh stability criterion. It says that if there are sign changes in that case the system is not stable and the number of roots in the right-half plane is equal to number of sign changes. So, in addition, it gives you an information on the number of roots in the right-half plane. So I say that number of sign changes in this first column is equal to number of roots in right-half plane. So it means the system is definitely unstable and additional information on the number of roots you can determine. However, please note, it does not give you the location of the unstable roots while the numerical analysis will give you the location. It simply tells you that so many roots are in the right-half plane the location cannot be determined using this particular criterion.

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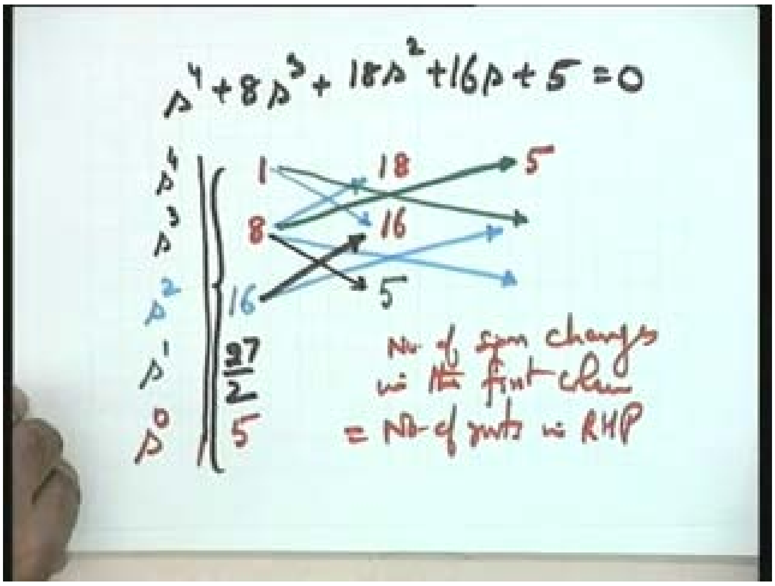

Take up another example: 3 s 4 plus 10 s cube plus 5 s squared plus 5 s plus 2 equal to 0 again just to give you an experience on construction of Routh array and the application of the criterion; s 4 s 3 s 2 s 1 s 0 I can write straight away because I will be coming down to s 0. So the coefficients now 3 5 2 you can see this 10 5 these are the coefficients. Now one thing, one change let me make over here. There is no problem there, you see, since in this particular case a complete row is divisible by 5 I can do that without changing my conclusion on the stability of the systems. So please see that multiplication or division of a complete row by a positive constant will not change the conclusion of stability, you may do that if it helps you in calculations, that is all. So naturally you see lower the value quicker will be the calculation. So let me put it this way. Now s 2 you give me the value of a s 2 please; it is 7 by 2 s 1 is going to be 2 and s 0 yes, did I make some error [Conversation between Student and Professor – Not audible ((00:29:18 min))] yes yes yes yes I am making an error.

Please see, let us not hurry up 10 minus 7 by 2 here will be 2, this will be 2 over here and it is not 2 now here it is a complete term; take up this pair and multiply it 7 by 2 into 1 minus 2 into 2 divided by 7 by 2, as per my calculation it is minus 1 by 7. Now it will be 2 here I made an error there, please see. Do check quickly please minus 1 by 7 is the coefficient in this particular row s 1 row which I have obtained by using this cross multiplication 7 by 2 into 1 minus 2 into 2 divided by 7 by 2.

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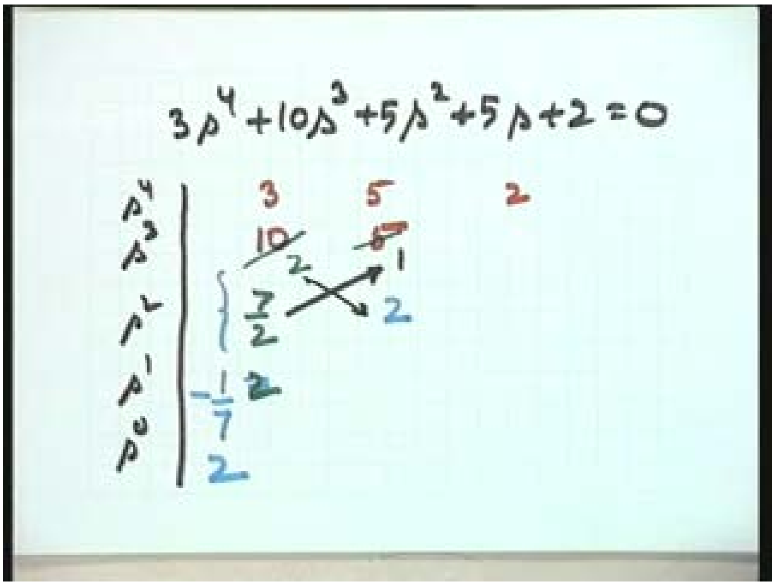

So, looking at the first column you can immediately say that these systems whose characteristic equation is given over here is definitely an unstable system. How many roots are there in the right-half of plane, please note? The number of sign changes it has been referred to. So, positive to positive no sign change, again positive to positive no sign change, positive to negative there is one sign change (Refer Slide Time: 30:32) and negative to positive again a sign change. So there are two sign changes and hence in this particular case there are two roots in the right-half plane as per the Routh stability criterion.

[Conversation between Student and Professor – Not audible ((00:30:49 min))]
Yes, we have not yet come to that…. if this is the situation, yes the roots on the j omega axis do not exist. I am going to come to that situation also. So it means the Routh stability criterion has got an extension to the situation where the roots are. Yes, if all of them are positive then definitely all the roots are in the left-half plane. If there are sign changes the roots are in the left-half and the right-half plane and the number of roots in the right-half plane is equal to the number of sign changes. There are definitely no roots on the imaginary axis if your conclusion or if your Routh array formulation is like this.

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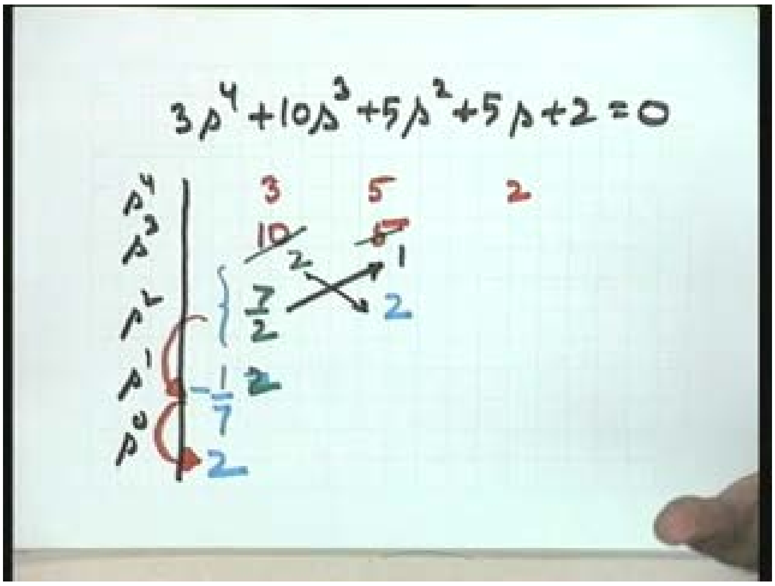

Now let me go to a situation. Naturally such a situation is bound to exist because the roots can be there on the j omega axis. and to come to that particular situation I think I will take up an example first: s 5 plus s 4 plus 4 s cube plus 24 s squared plus 2 s plus 63 equal to 0 this is the system I have taken and let us now apply the criterion on this system s 5 s 4 s 3 s 2 s 1 s 0. Come on please, the first power is an odd power so all odd coefficients 1 4 2 even coefficients 1 24 63 so let us get to the formulation procedure. So the formulation procedure demands that the coefficient here will be given by this into this minus this into this divided by 1, I use the table available with me it is minus 20 please do check and this as per my table is minus 60…. [Conversation between Student and Professor – Not audible ((00:32:51 min))]….2 minus 61 so it means I have an error here, please do take it 62 here what shall I take here 62 because the array formulation should be correct I might have made an error in noting down the characteristic equation. So in this particular case it is minus 20 minus 60 so let me divide the entire thing by 20 positive constant I get minus 1 minus 3.

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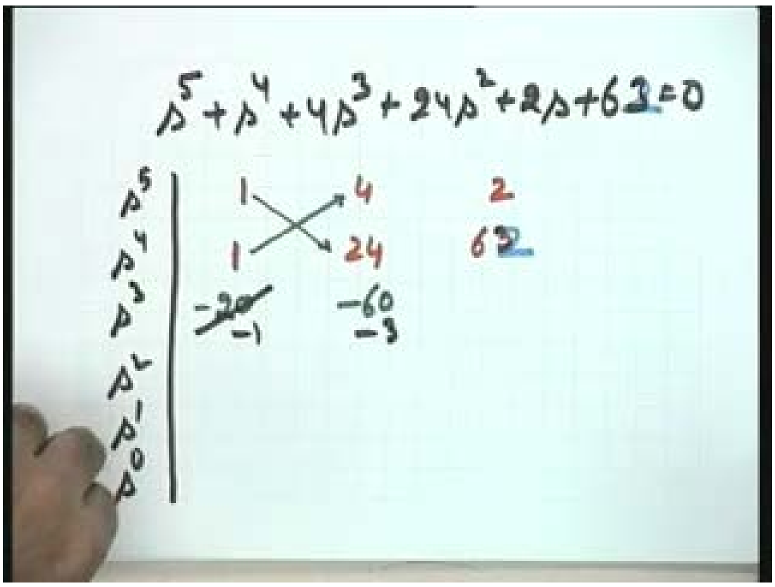

Come on please, next coefficient. Next coefficient in this particular case, no, I next take a 63 what is the problem in this example, I have made some error in this example, please, 1 4 3 okay 1 4 3 yes, I take up the example again, some error has been made over here because the coefficient formulation asks for 63 later so I cannot make this change, probably the error lies over here plus 3 s plus 63 it is no adjustment you see, it is giving you trying to give you an example in which the complex roots in which the roots on the j omega axis lie otherwise let it be that characteristic equation you could have gone ahead with the stability analysis there was no problem, do not think that I am making some manipulation. The manipulation if at all any is to create an example in which there are roots on the imaginary axis that is the only thing.

Come on, let us hope that there is no error now; s 5 s 4 s 3 s 2 s 1 s 0 1 24 63 1 1 4 3 lot of errors I am making I am sorry 1 24 63 (Refer Slide Time: 35:00) please do help me you see I do not know why in this case I am making lot of errors, do help me now. Come on please, now I will take the coefficients which you give me, yes, what is the coefficient here minus 20 is okay now okay, next is minus 60 I hope this is okay now the adjustment has been properly made. Come on, let me now divided this entire thing by 20 a positive constant, I get minus 1 here and a minus 3 here. What are the values at s 2? s 2 is 21 and 63 is it okay? Fine, I am happy you see there is no error here. So in this case I again divide it by 21 and get 1 3. Come on, how about s 1 please? [Conversation between Student and Professor – Not audible ((00:35:50 min))]

s 1 both zeros 0 and next of course missing is 0. How about s 0? No s 0 is not 0.

[Conversation between Student and Professor – Not audible ((00:36:00 min))] Undefined……. so please see that this is the situation. His question was when do we come across the situation, there could be poles on the j omega axis. Please see that in this particular case a complete row in the Routh array is a zero row and the problem is this that you cannot continue with the array formulation, the coefficients of the next row becomes undefined as it has been rightly said. So actually in this particular case when a complete row turns out to be a zero row note this point very carefully, this is an indicator and this is the only indicator that when a complete row turns out be an all zero row in that particular case there is a possibility of roots on the j omega axis. Not necessary the last row, any row in the Routh array, any row in the Routh array. May be in another example a row earlier than this row may appear. So what I am referring to is any row in the Routh array if it turns out to be a zero row in that particular case the possibility of the j omega axis roots exist.

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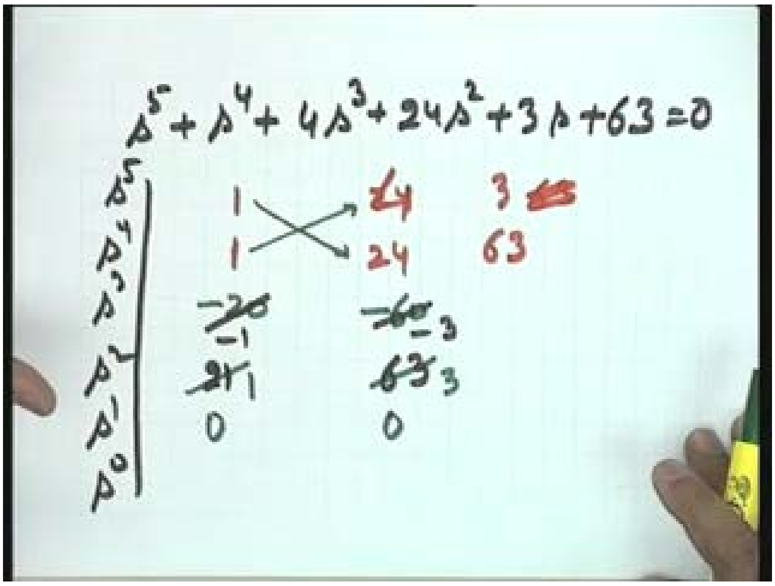

In fact if you come across an all zero row, again you see, without proof you will have to digest this statement, in that case one or more than one of the following will happen if there is an all zero row. See all the possibilities which could be occur, one at a time or more than one at a time there will be a pole pair symmetrical with respect to the imaginary axis. So it means even with all zero row the system may be purely unstable and may not have roots on the j omega axis. This is one possibility.

Another possibility is this that there is a pole pair on the imaginary axis. Yes, this possibility gives rise to the situation of imaginary axis. The third possibility is this that there could be two complex pairs symmetrical with respect to the imaginary axis. So in this particular case also the system is unstable. So you see that if you come across an all zero row nothing can be said, all these possibilities even the multiple roots are possible, there may be a multiple imaginary axis root in that case the system will be unstable. So the only thing is that you have to further study you have to analyze this particular system because when you come across an all zero row you cannot make a statement on stability of the system.

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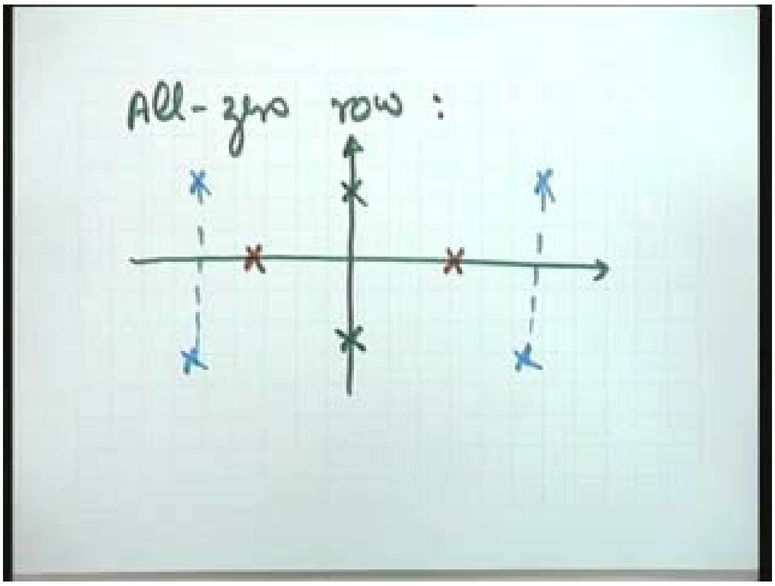

Let us see how do we study that and to study that I come back to the Routh array we have formulated. You see that this is an s 1 row, you go one row above and that one row above is s 2 row, when I get an all zero row I construct an auxiliary polynomial and the auxiliary polynomial for this example is given as s squared plus 3s. Please see how do I get this? This is an all zero row, I go to a row, one row above, the coefficient of this is s 2 so it means this particular coefficient will be tied to s 2 and the next coefficient will be tied to s 0 and therefore the auxiliary polynomial becomes a s is equal to s squared plus 3 so this becomes the auxiliary polynomial of the system and the result is this that the auxiliary polynomial is definitely a factor of your original characteristic equation.

Since the auxiliary polynomial has got the roots at, at what location please?..... On the imaginary axis it means in this particular case the roots are on the imaginary axis and the system definitely is a marginally stable system if not an unstable system because the stability is yet to be determined.

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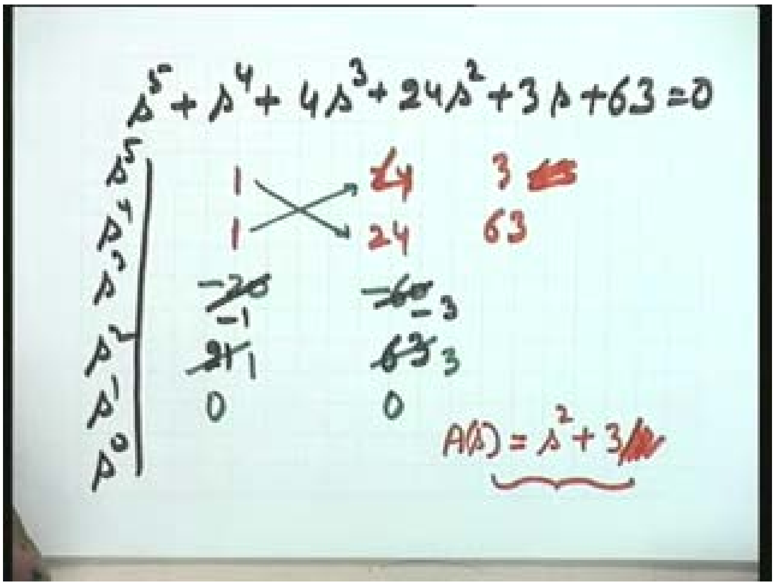

I explain the point again. Please see, when you come across an all zero row in that particular case you cannot proceed further with the array formulation. So what…….. [Conversation between Student and Professor – Not audible ((00:40:28 min))] if s 0 row you will see that one thing is there, see his question please, an s 0 row is an all zero row he says, let us see whether it is true or not. Can anyone answer, probably the answer lies in the discussion I have given. If this is the row in that case one of these possibilities is not possible because this so-called auxiliary polynomial will be first-order which will give rise to a single root and single root does not gives rise to all zero row. So it means s 0 having an all zero row will not occur and if it occurs in that particular case it is not related to this particular situation, you can go for the formulation of the Routh array as per the earlier conclusion.

The auxiliary polynomial formulation will demand that the root will be at the odd power of s always, please see, look at these equations; the root will always……… the all zero row will be an odd power of s always it cannot be an even power of s because the symmetry involved here is going to give rise to an all zero row; though the proof is not being given but from the statement you try to conclude as to what are the situations which will give rise to an all zero row. The all zero row has to be an even power of s because the auxiliary equation has to be an odd power of s because the auxiliary equation has to be an even power of s giving rise to these roots which are symmetrical.

Now you will see, this situation again, if you come across an all zero row the conclusion is this that the auxiliary equation which you form from the row one row above is a factor of the original characteristic polynomial this is the result. And over and above….. now this result does not conclude, does not force you to conclude, does not make you conclude that the system is marginally stable. Can we say that please tell me that this particular pair of roots on the imaginary axis (Refer Slide Time: 42:34) is definitely a factor of this characteristic polynomial, is it sufficient to conclude in this example that the system is marginally stable; probably not because we have not said any thing regarding the remaining roots. The only thing it says is that these two roots are on the imaginary axis the two roots given by the characteristic by this particular auxiliary equation but the remaining roots you have yet to examine and how to examine the remaining roots please; the remaining roots could be examined in two ways: One, you divide the total characteristic polynomial by this auxiliary polynomial so you get the remainder and you apply your Routh array formulation on the remainder polynomial; naturally such a situation will not arise because that polynomial will not now have this type of situation and therefore the conclusion regarding the number of roots of the remainder polynomial in the left-half or the right-half plain has to be taken into consideration before you declare this system to be a marginally stable system or an unstable system. You see, this system at the most can be marginally stable it cannot be a stable system, of course this as an established. You simply want to examine whether this particular system is unstable or not. For that what you can do is that you take up the original characteristic equation divide this characteristic equation by this auxiliary polynomial, go to the remainder polynomial and look at the remainder polynomial that is going to give you the result.

Alternatively as has been suggested please see that you take the derivative of the auxiliary polynomial, note this point please: dA by ds derivative of auxiliary polynomial with respect to s please help me it is 3s plus 0. [Conversation between Student and Professor – Not audible ((00:44:26 min))] sorry lot of errors today I am sorry 2 s plus 0. Now this is s to the power of 1 so what you have to do is this that go to the row corresponding to the s to the power of 1, replace these zero coefficients by the coefficients of the derivative of the auxiliary polynomial. So what are the coefficients? 2 and 0. The coefficients of the derivative of the auxiliary polynomial it is 2 and 0; 2 corresponding to s to the power 1 over here and 0 means it actually missing term. Now it says that you continue with the Routh array formulation and all the conclusions which now you get from the first column they are actually valid on the remainder polynomial.

Come on, please complete it and give me the conclusions. In this case the completion is no problem at all because this will be now 3 (Refer Slide Time: 45:25) this will be 3 with the 2 and 0 replaced with 0 and 0 replaced by 2 and 0 now the problem of infinity is not occurring and therefore you can write this coefficient as 3. Look at the first column please. If you look the first column you find that there are two sign changes. So it means now if you go back to the original polynomial you can conclude that the original polynomial has got two roots in the right-half plane, two roots on the imaginary axis and obviously the balance one root in the left-half plane and the system is decidedly an unstable system. So, in this case you see, when you have to come to this situation the answer will be given like this that is you have to look at the auxiliary polynomial quickly and replace the coefficients by the coefficients of the derivative of the auxiliary polynomial and complete the array formulation okay.

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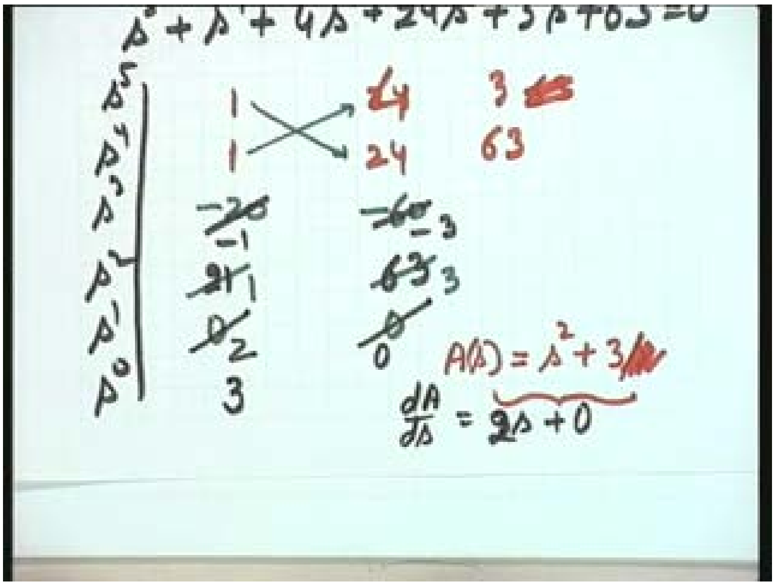

I will continue with it next time, thank you.