Control Engineering
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Lecture - 23
Concepts of Stability and Routh Stability Criterion

Well friends, today we get started with the stability problem, very important problem, the most crucial problem in the design of control systems. In fact any design which you make, any adjustment in the controller parameters which you make are constrained within this stability domain that is the stability conditions have to be satisfied. And to give you an idea of stability I will be cashing on your intuitive knowledge of stability and then I will build on quantitative methods on that knowledge.

What I do is I take a linear system here. Now, you can imagine any physical system in this block having discussed so many systems so many different systems in this class. In this particular linear system now let us see inside and outside of this linear system to see what are the variable of interest to us or what are the variables which make a difference to the system under consideration. So let us first see the environment. The environment I will say; one is the command signal or the reference input which you are giving as per your demand, the other input acting on the system is the disturbance let me call this as w and y the output is that particular variable in which you are interested. So you see, you will recall, mostly we have been confronted with these three variables in all the block diagrams we have been using in the control system analysis and design.

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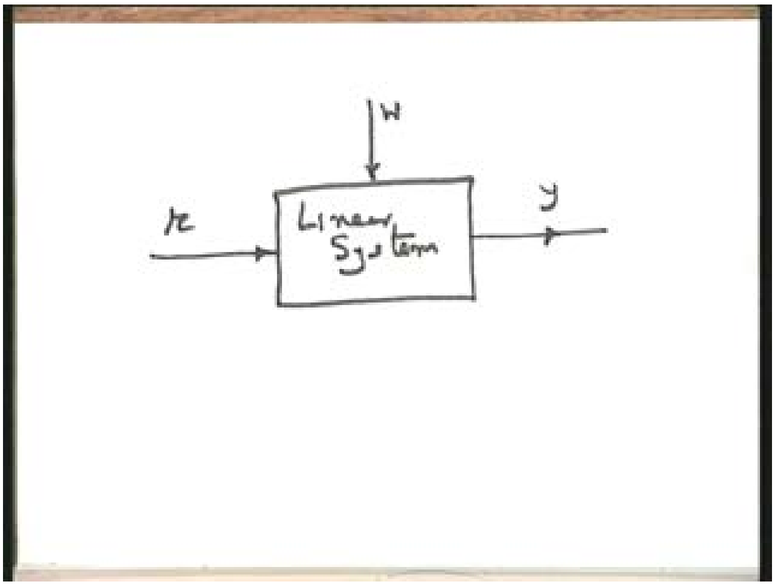

Now, in addition to these three variables I think you will appreciate that there is something more also. Because as far as the output is concerned the output is that attribute of the system in which you are interested in which the user is interested but the output does not describe the dynamical state of the system completely, you know it already. The dynamical state of the system is completely described by all the state variables and let me say in a typical system the state variables are n x 1 x 2 to x n.

Now you see that, well, if the mathematical model, I need your attention please, if the mathematical model of the system which you might have written in the form of a transfer function Y(s) over R(s) is equal to G(s); suppose you subject this system to an input R(s) and you analyse the response of the system Y and you find that the Y response is acceptable to you it is nice, it satisfies all the performance requirements but it does not mean that the actual system when subjected to this particular input also will give you satisfactory response, there could be problems.

What are the problems please?
The problems are the following: Y may be behaving satisfactorily but who knows; in response to this input which you have given or in response to the disturbance w some of the state variables of the system which you are not observing in the output Y may be unstable. So, if they are not converging to any steady state when subjected to an input w or an input r in that particular case when a physical system is subjected to this r what will happen, you will not get Y because some of the states will be going to instability and therefore will be destroying the system anyway. So, if the system will not be working because of some of the states being driven to instability in that particular case though your model was predicting an appropriate Y your physical system will not give you that Y. So it means, all that analysis we have done so far we have being missing this point all through and this is an important point.

You see, what is the use in working in with a model when we actually use the physical system, does not give the response as predicted by the model. But anyhow, if before you go for simulation before you go for actual experimentation if you are sure that all these variables are at least stable they may not behave in a particular transient way that is you may not dictate the transient of these variables because you are not interested with them, the attribute you are interested in is the output y only and therefore you will monitor the transient of y and you are not interested in monitoring from time to time basis the transient of any of these state variables. So, the only precaution you want to take is that none of these state variables becomes unstable or none of the dynamical modes of the system become unstable then and only then whatever analysis and design you make on the model of a given by a transfer function will make sense otherwise experimentation can give widely different results and therefore I make a statement that the stability subject the stability concept is extremely important, you can go and experiment your design on the physical system if and only if you can ensure that the physical system will remain stable and therefore to look at the stability aspect I may when I go to the design, you see, after the stability discussion now again I will be concentrating on G(s) only because my design is based on Gs because I am interested only in the transient attributes of Y and not those of x 1 x 2 x n. But surely this I will do if and only if this guarantee has been given to me that the system is stable that is all the state variables will be converging to a steady state and hence this the uh for in response to w or r or any other parametric change conditions any of the modes of the system will not go unstable. This is what I want to say and the stability concept will guarantee or will give you quantitative methods to study that.

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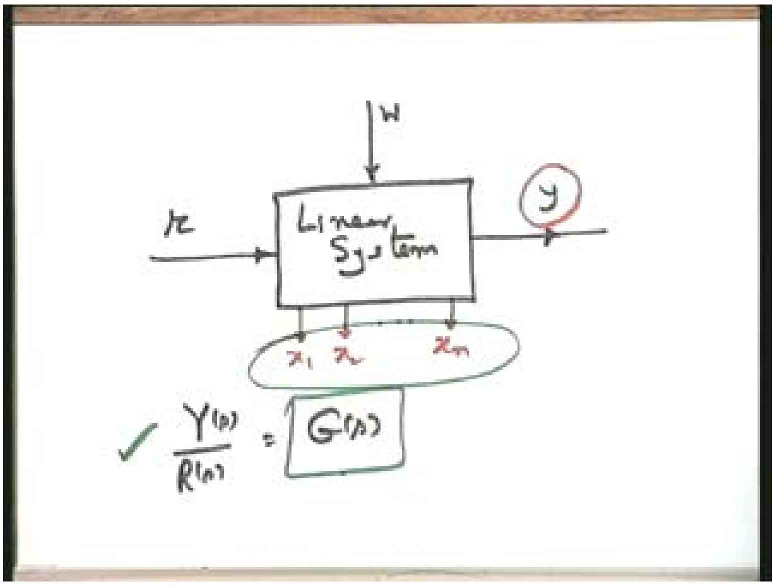

Now you look at this; how do I study that?
Again you see, even in that modelling , even in that study some approximations I will make but those approximations really at least as a pointer to stability will not do any mismanagement there. what I do here is this that suppose the w comes on the system, anyhow w is not known to you, the w is unpredictable, its magnitude is not known to you but I think this you will realise that any change in w really in effect changes the storage of energy within the system. So it is equivalent to changing the initial conditions of the system x 0 and by the basic definition of state variables initial conditions at T is equal to 0 is a representative of energy storage of the physical system. So it means, when you really subject the input to either r or r to w it equivalently means that the energy storage at the point this system is subjected to, that type of input has changed. So it means, if I can vary, you see this particular point now, if I can vary the initial conditions x 0 quite widely in the state space, practically if I scan all these the entire state space and still I find that x 1 x 2 x n remains stable that is they will not diverge it means I am sure about my stability.

See this point please: w I do not know but this much I know that this w is going to affect the initial energy storage and hence initial conditions on the system. And therefore what I do is as a change in the initial conditions I will scan the entire state space all the possible initial conditions which the system can take; if I simulate the conditions with respect to all those conditions and look at the values of x 1 x 2 x n and if I am satisfied with those values I will be satisfied with the stability.

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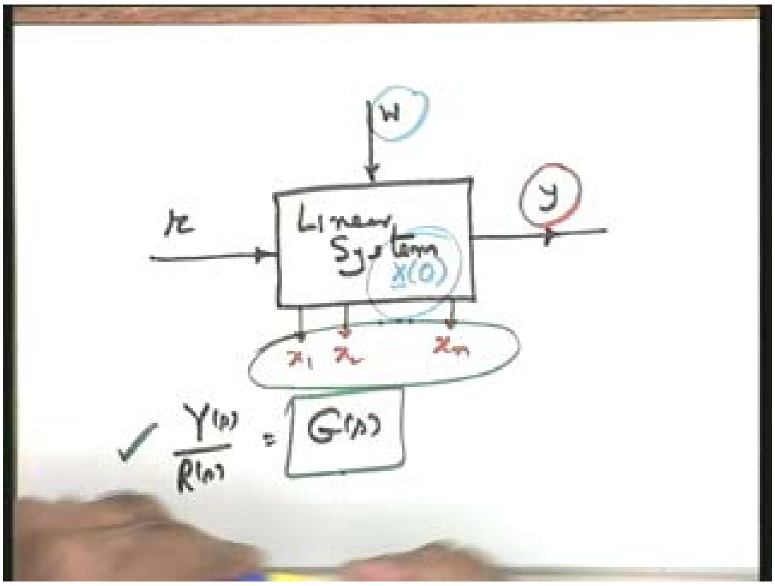

Now, as far as the stability concept is concerned the concept intuitively is known to you, quantitatively it has been studied in the literature under two different classes: One I call it BIBO stability as the term has been used in the literature. BIBO stands for bounded-input and bounded-output stability let me write it, bounded-input bounded-output stability and number two I take zero-input stability. I think in this particular case also the point will be clear as to why these two aspects are there. Again the assumption is that the system under consideration, it is an important assumption; the system under consideration is operating under its linear range.

You see that how complex is the design. Both these concepts will become meaningless as soon as you go beyond that range because your superposition is not valid. This is based on the principle of superposition. There are two sources of excitation: the disturbance or the initial conditions and the external input. I am taking one of these conditions at a time and applying superposition to study stability. So naturally this assumption, the validity of this assumption demands that your operation is strictly in the linear range, as soon as you go to the non-linear range the concept of stability which we are discussing is no more valid and the stability of a non-linear system is quite complex to determine, is quite complex to analyse and I will not take you to that particular region of stability analysis therefore I make an assumption and I will assume that when I experiment my design on a system the linearity range will be adhered to and if the linearity range is preserved in that particular case the concept under discussion will become valid.

So you find that under bounded-input bounded-output BIBO stability I assume that the system is initially relaxed. by relaxed system, I mean that x at (t is equal to 0) is equal to 0 and t is equal to 0 is a reference time and for time in variant system at any time you inject the energy you inject the input becomes the time t is equal to 0. The systems under consideration, again, the subclass is a linear time invariant system both for time varying and for non-linear systems the discussion on stability is quite complex and beyond our scope. So t is equal to 0 is my reference time and at that reference time I assume that the system is relaxed. If the system is relaxed, if the initial energy is equal to zero in that particular case I may not be interested in studying the state variables of the system because all the state variables are unexcited at that particular point and I concentrate on bounded-input bounded-output stability. That is, if r the external input is bounded stability should guarantee that y is always bounded. This is the condition. And since initial conditions are all zero you will realise that stability can be studied in terms of the transfer function because a relaxed system can be fully described by a transfer function model.

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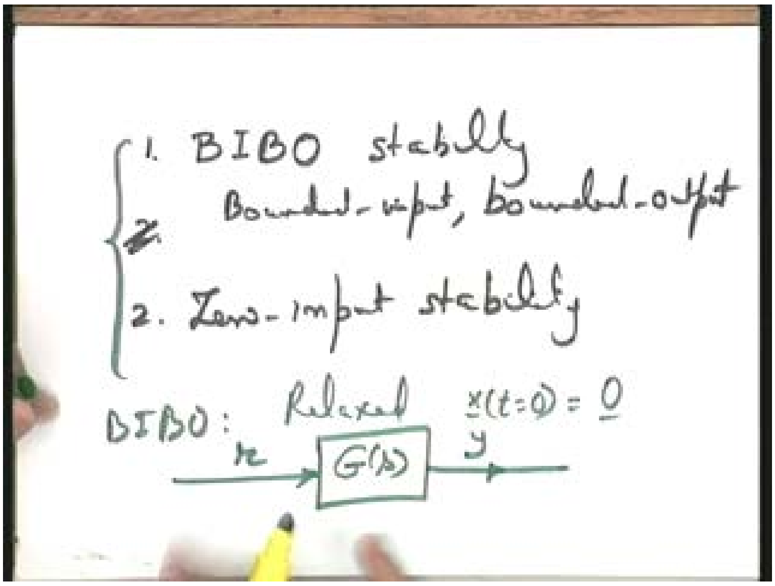

So one concept of stability the bounded-input bounded-output concerns the relaxed system and when you subject the system to an input r the output is y. The quantitative method which you should give which we should establish should guarantee that y will always remain bounded for any bounded r. This is what you keep in mind; this is what I will make you recall when I go to the quantitative methods on bounded-input bounded-output stability.

First I am taking only…. that is, you see that w or this point may please be noted you just take w into consideration. Now, when you are analysing the system you are subjecting it to external inputs. So external inputs whether it is a wanted input or a disturbance input how does it matter? So it simply means that, if the initial condition is zero, if externally on the system any input which is bounded comes in that case the output must be bounded. So the only requirement on disturbance… disturbance is also included in BIBO but the only requirement is that the assumption that the disturbance is bounded and is not unbounded.
Now, though I have used r of course but it does not mean you see after all it is a it's a simulation experiment, it is a laboratory test, it is not the physical system is not been subjected to any input.

Now you take it to be actual situation: let this input (Refer Slide Time: 14:26) be the command input or the disturbance input coming on to the system from the environment. The condition on bounded-input bounded-output stability is this that: if the input coming from the environment is bounded the output of the system must remain bounded.

[Conversation between Student and Professor – Not audible ((00:14:41 min))]

Yes, okay, disturbance, I think in the tutorial this was very clear.

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Whether you take Y(s) over R(s) or you take or you take Y(s) over W(s) as far as the characteristic equation of the system is concerned it is always the same if you recall the tutorial problem. And as you will see when we develop the quantitative methods the stability is governed by the characteristic equation only and not the zeros of the transfer function and therefore it does not make a difference whether you are taking Y(s) over R(s) or Y(s) over W(s) and hence the statement that this R is a general input coming at any point in the system; you see, you take it this way (Refer Slide Time: 15:31) that the transfer function is different, simply it means that the input r is being injected into the system at different points, that is all whether it is disturbance or a command signal. And as you have seen in the tutorial problem and which is a general statement that the characteristic equation will remain the same the poles of the system will remain the same and the quantitative methods will tell you that the stability is governed by the poles of the system.

Any other point please?

Fine, if that is okay then let me go to the second aspect. The second aspect is the zero-input stability. Zero-input means the external input is being assumed to be zero. Now it does not mean that the external input is not coming. I am now scanning x 0; it is the equivalent way of seeing the effect of external input on the state variables. Well, again, now you may say that how can I assume or how is the x 0 changing otherwise. Well, x 0 is changing either because of the initial energy storage at t is equal to 0 or the energy is being injected into the system from the external means. So it is an equivalent way as I told you that our physical requirement is this that any changes may come all the state variables and the output variables should remain bounded should remain within my boundary, this is my requirement and these are equivalent quantitative means of doing that.

So now, in the mathematical model I will assume that external inputs are zero and the only change or the only input which is changing the dynamics of the system is the initial state and I will scan this initial state totally in the entire state space. So I think equivalent meaning should be very clear. This change may be occurring because of the external inputs only but this is an equivalent way of studying the dynamics because it goes nicely as far as mathematics is concerned but if this is assured the corresponding aspect will also be assured that if the initial energy is changing because of the external inputs the states will remain bounded.

So I want to study the effect of variation of x 0 on x t for all time t greater than equal to 0 this is what I am interested in. and the mathematical model I take for this purpose, you already know that model, we also discussed in the class x dot is equal A x this is the model I take for this purpose (Refer Slide Time: 17:56) where x at t is equal to 0 I take by definition equal to x 0 which is subject to change.

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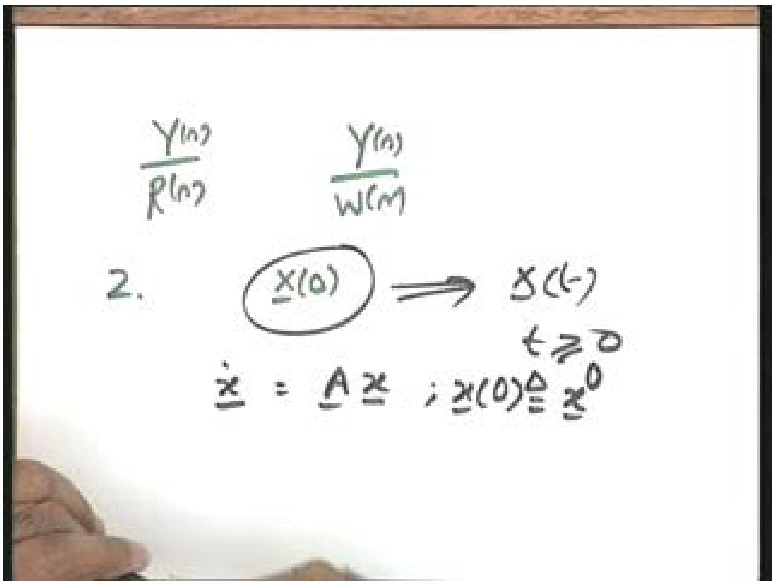

So now what is the problem, the problem with me?
Zero-input stability is the title given to this; under zero-input stability I will vary this x not and calculate x(t) and see whether x(t) remains bounded for all time; if yes I will say that this system preserves zero-input stability. Now put the two together; you see, in that particular case I think intuitively it can come, for a linear system since superposition is valid intuitively it can come that for any r any w which are bounded the system states and hence the system output will remain bounded if these two conditions are satisfied.

Now, one definition at this stage I may give may be at a later stage we may use it: a system where input is zero and the system is time variant is an Autonomous system. So it means we are considering an Autonomous system that model for studying the zero-input stability, fine.

Let me now come one by one, let me first take up BIBO. This aspect is known to you about the poles and zeros but since some couple of points I want to inject into it I will quickly scan through. let me take this as the linear system and here is the input r (Refer Slide Time: 19:31) input r is a general input now please, w r different should be clear; I do not mind whether it is w it is the input which is being given to the system and this is output y. if impulse response of this system is g(t) you know that we have discussed in the class impulse response model is a complete characterisation of a relaxed system. So since the system is a relaxed system the impulse response is a complete characterisation; the output y(t) by convolution is given by 0 to infinity g(tau) r(t minus tau) d tau impulse response being the complete characterisation of the system.

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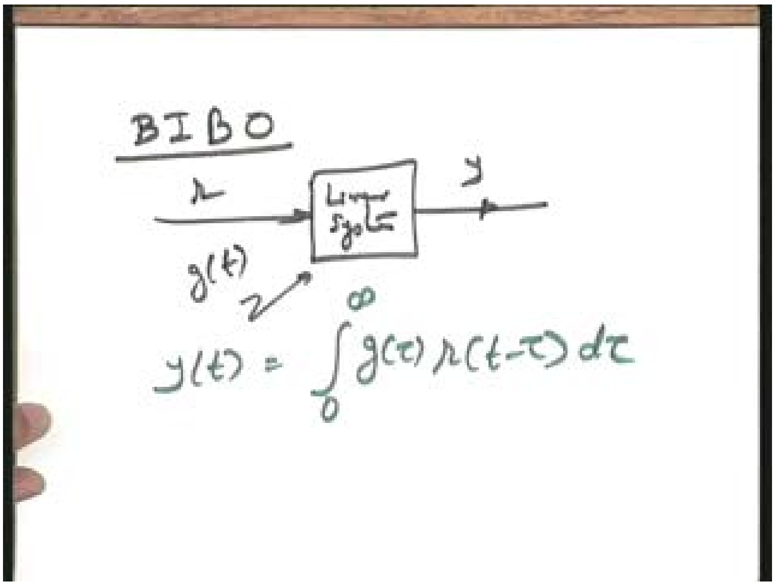

I am interested in the magnitudes so let me take mod on both the sides; this becomes less than equal to the basic…… modulus theorem can be applied and if I take these modes inside this becomes less than equal to this quantity (Refer Slide Time: 20:35). And now just see, if I say that this is bounded so it means modulus of this is less than equal to M where M is a finite value; I can take this as less than equal to 0 to infinity g(tau) d tau and this quantity should be less than equal to N less than infinity where M is also less than infinity a finite quantity.

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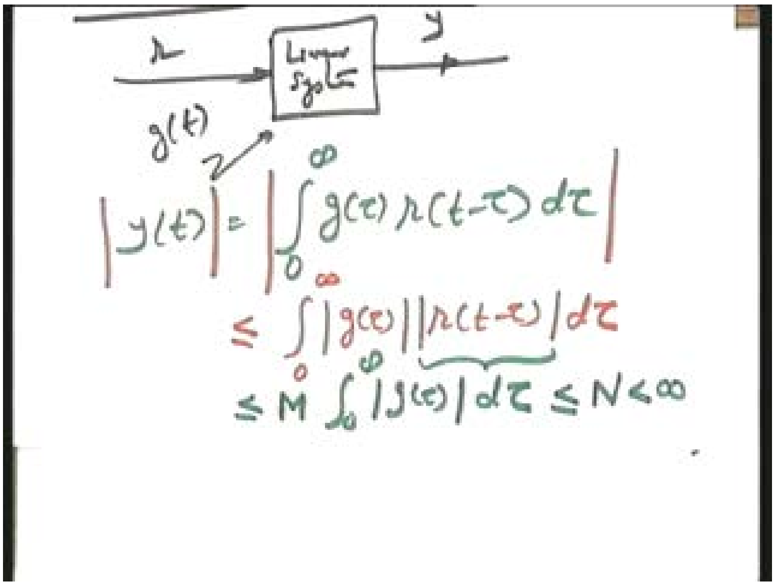

So it means, if I ensure this particular equation the bounded-input bounded-output stability condition is satisfied or equivalently now you see that the integral of the modulus of impulse response should be finite. I have come to this condition: BIBO stability condition has finally come to this point that integral of the modulus of the impulse response should be finite. So let me put it separately, the system is bounded-input bounded-output stable if and only if the necessity and sufficiency can be proved. Let me not go to the deep mathematical proof. It is both necessary and sufficient that 0 to infinity g(t) mod dt is less than infinity.

Equivalently speaking, the area under the absolute value curve of impulse response should be finite because in the mod you are taking you should change the sign, negative sign to positive and then take the area under the curve. So if you have an impulse response the total area under that curve taking all signs to be positive should be finite, this is my requirement and this requirement now I will like to translate to the poles and zeros requirements which you already know. I take the transfer function model now instead of the impulse response model: b 0 s to the power of m plus b 1 s to the power of m minus 1 plus b m by a 0 s to the power of n plus a 1 s to the power of n minus 1 plus a n. You will please note that m is less than equal to n. the realisable system has to be either proper or strictly proper; m cannot be greater than n because it will become an improper transfer function that way.

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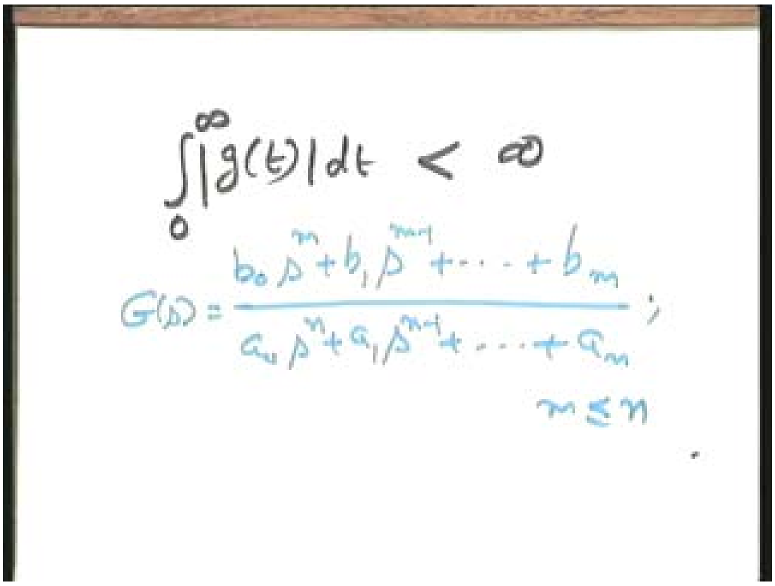

Now look at this: if I take the partial fraction expansion of this, this is what I meant; in this particular case please note, the characteristic equation of the system is delta(s) a 0 s n plus a 1 s n minus 1 plus a n equal to 0. This is the characteristic equation of the system and the roots of this characteristic equation give you the poles of this transfer function.

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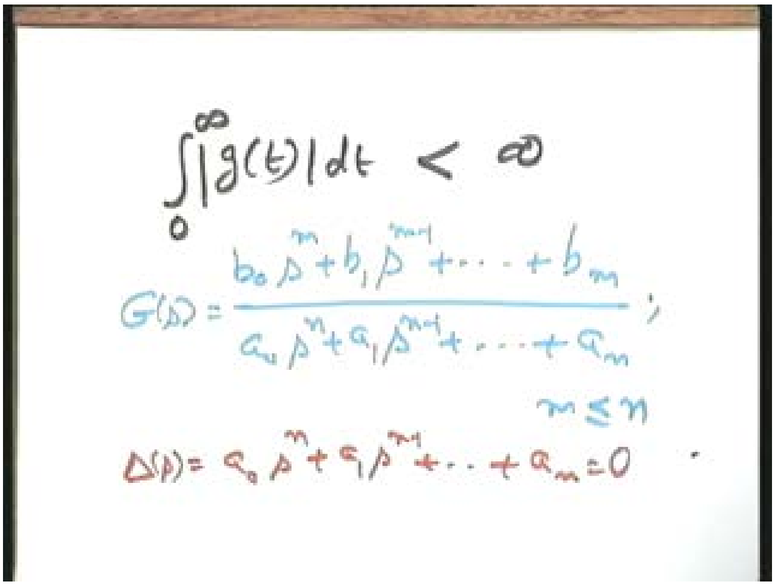

Now if you take the partial fraction expansion of this the zeros of the transfer function will only change the magnitudes of the response and will not change the character of the response. The character of the response whether stable or unstable, whether constant, growing or decaying will be given by the roots of the characteristic equation. You know that a first order factor gives you exponential response; either growing or decaying a second order factor gives you oscillatory response either growing or decaying and so on and therefore the stability is determined by the roots of the characteristic equation (Refer Slide Time: 24:02). The zeros only shape the transient response they do not alter the answer of stability. Therefore let me concentrate on the roots of the characteristic equation or the poles of the system.

Now let us look at…… I will not give the details here because the concepts are too simple to be given here. Let me look at the s plane and scan the entire s plane. If there is a pole here, if there is a pole here you know that the response decays impulse response decays.

What is the transfer function?
R(s) is equal to 1 and therefore Y(s) equal to b 0 s to the power of m plus b 1 s to the power of m minus 1 plus b m divided by a 0 s to the power of n plus a 1 s to the power of n minus 1 plus a n this is the response transform because you are considering the impulse response and for the impulse response your R(s) is 0.

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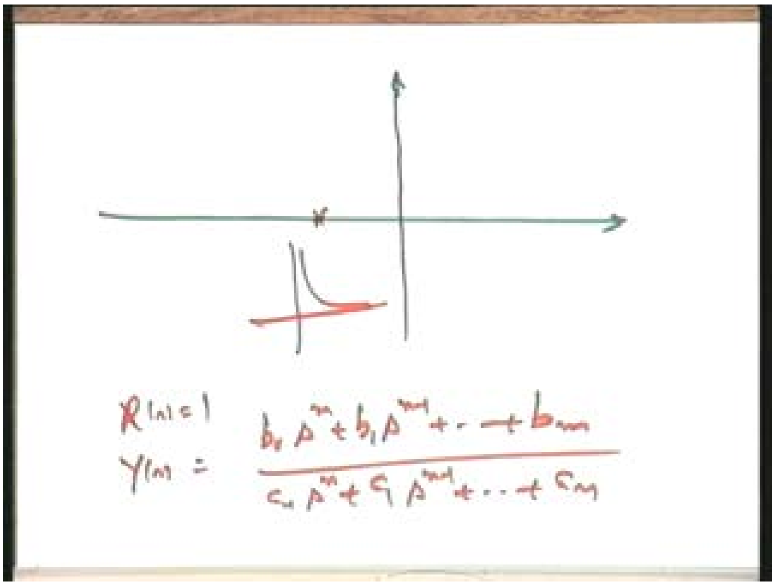

Now imagine that the factor one of the factors of the denominator is at this point and if you invert it is going to give this type of response (Refer Slide Time: 25:16) and you take the some of the area is finite. So this particular pole at this point is not going to give you any problem.

Similarly, a pole pair at this point is not going to give you any problem because the response in this particular case will be oscillatory but a decaying sinusoid. So if you take the area under this, absolute curve, you convert negative to positive and then take the area it is going to be finite. Similarly, instead of a single pole you take a pole pair, the pole pair you see, the response is going to be of this nature, this you can establish very easily. For a pole pair the response is going to be of this nature and again the condition of the area under the absolute curve of the impulse response is finite is also satisfied.

So I find that if the poles whether real, complex, distinct or repeated anyway they should appear if the poles lie in this particular region in that particular case it is guaranteed that the condition, necessary and sufficient condition which we have established for stability is always satisfied and therefore the system will be BIBO stable.

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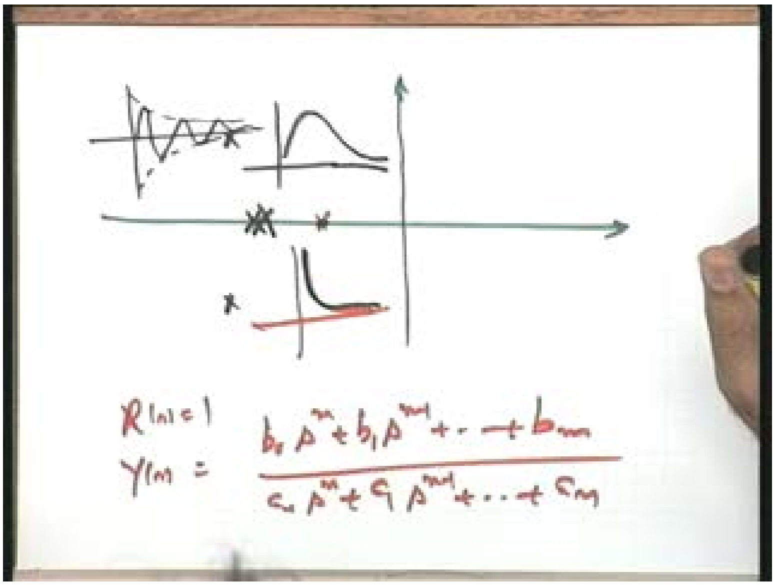

Look at the right hand side. Now, if I look the right hand side take a pole here the pole here gives you response of this nature, the pair here gives you response which is growing exponentially, you can take double pole, double pole will aggravate the situation. In that particular case, I say that, if there is any pole real, complex, repeated, distinct in any form in the right half plane the system is unstable BIBO unstable. This may please be noted. Even one pole in the right half plane will make it unstable, because one mod if it grows to infinity, well, the system is unstable so the condition of stabilities of all the poles in the left half plane and the condition of instabilities of at least one pole in the right half plane is making the system unstable.

[Conversation between Student and Professor – Not audible ((00:27:18 min))]
Yes, next I am coming to this. Now, to scan this completely the part which is left is the j omega axis and here specific attention is required. Let me first take a pole at the origin (Refer Slide Time: 27:33); if you take the pole at the origin….. I think since this being a very special case and I have some comments to make let me scan the j omega axis on a separate slide.

Let me take a pole at the origin, please help me, the corresponding response for a pole at the origin is a constant. Now, if you take the area, well, naturally, the area is infinite because this is constant t is equal to t tends to infinity the area will be infinite. So basically if you apply your definition of BIBO stability a pole at the origin means that the system is unstable, no doubt about it. Similarly, you take a pole pair, you take a pole pair in this particular case you have an oscillator, and in this case also, since you are taking the area under the absolute curve. So, if you take the absolute curve of this and then take the area it is again going to be infinite. It means a pole pair also gives you instability.

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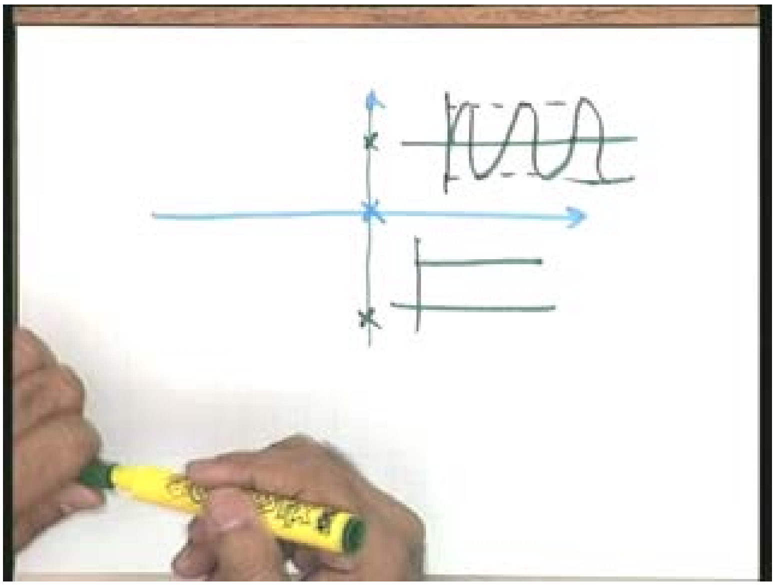

Now, before I comment on this let me take a double pole. I take now a suppurate graph, a double pole on the origin. A double pole on the origin has got this type of curve, please; you can take the inverse response after all, so this is decidedly unstable. Similarly, a pole pair, complex poles that is imaginary poles but a pole pair in that particular case your response is of this nature, please (Refer Slide Time: 29:11); a growing sinusoid, a pole pair so the system is again unstable. So if you apply the basic definition, in that particular case it is seen that whether a single pole or a pole pair on the imaginary axis makes the system always unstable if you apply the necessary and sufficient conditions of stability.

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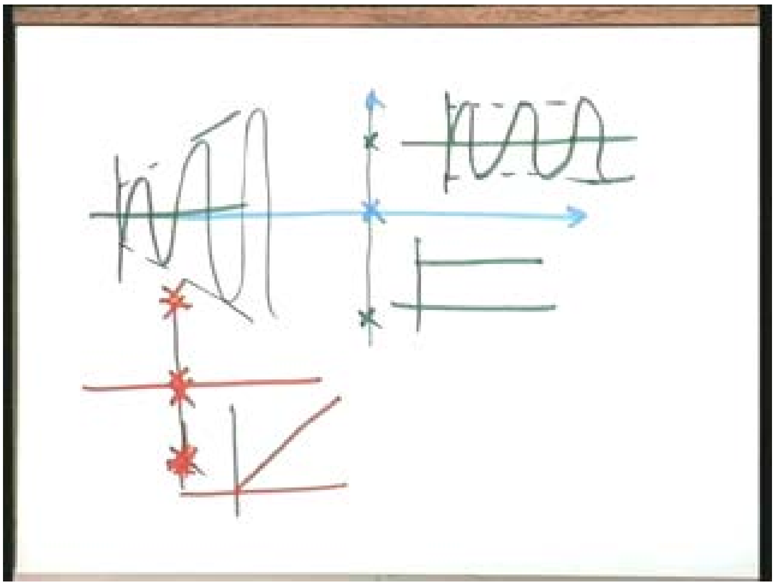

Now, you see that, as far as mathematics is concerned the mathematics has declared all the systems with poles on the imaginary axis to be unstable. But let us see from the users' point of view. I take a typical transfer function G(s) is equal to numerator N(s) divided by s into s plus j omega into s minus j omega equal to Y(s) over R(s); I need really you are attention on this particular point.

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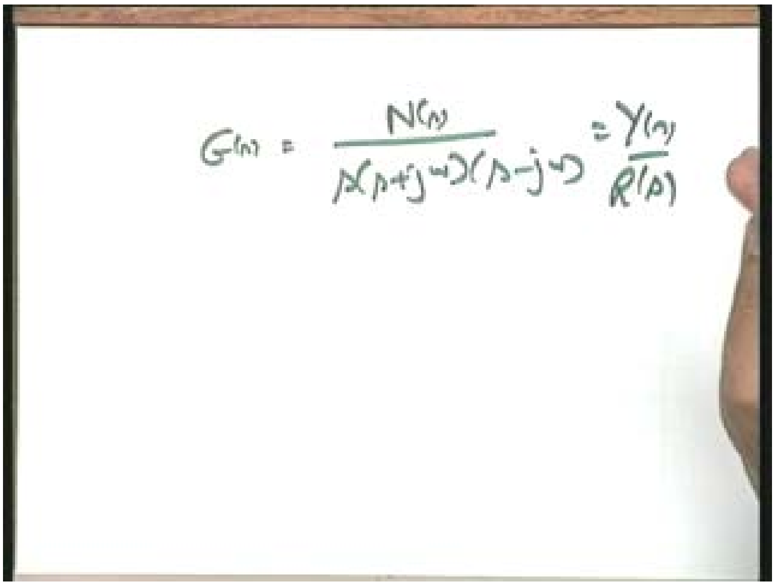

Mathematical condition, necessity and sufficiency you have established and the system has been declared unstable no doubt about it. Now the user may say that, look, even if you are mathematics say it is unstable I may still be interested in it. What are those situations in which the user will be interested in even if the mathematics has rejected the system and those situations could be seen very easily.

Look at this system and forget about the mathematics. You scan all the possible R(s) all the possible inputs which the system can be subjected to. Now it is a little intuitive feeling which you can get from here or from the mathematics you can prove that this particular system will give you bounded response for all the possible inputs except one and what is that except one; that except one is that input for which the input pole matches these ports.

You just imagine that R(s) is equal to 1 over s or R(s) is equal to 1 over s squared plus omega square. For this particular system if you avoid these two inputs, you name any input for which the system has a growing response, for all the other inputs the system is going to have may be an oscillatory response, may be a constant response but it will not be growing. Though the mathematics has rejected the system but we look at this particular point that unless the system pole matches one of the poles on the imaginary axis and makes it a double pole, you see that matching one of the poles at the imaginary axis means converting a simple pole into a double pole into a multiple pole and a multiple pole is decidedly growing; we have seen this particular point, you will please see, if there is a double pole at the origin (Refer Slide Time: 32:03) this is the response, if this is a double pole pair this is the response, this is decidedly growing. But if you have single pole and if the system is not subjected to these two inputs the response is always non-growing.

Now it is up to us whether we reject this system or accept this system because it will depend upon the application. So I need your attention and comments if any; I make this statement that this is a case of poles on the imaginary axis only, that too simple poles, because if there is a multiple pole the system is already unstable; I am considering only as a special case the situation where there are simple poles on the imaginary axis and I make a statement: unless the input matches one of these poles the system output will remain bounded and hence I may like to accept that system. So it means for this particular transfer function these are the two situations. Since it is a boundary situation, mathematics is rejecting it, and the user analysing it so let us try to give it a meaning marginally stable system.

Marginally stable system really will not mean that it is acceptable. The word 'marginal' because for want of any suitable word in the literature such as critical stability, limited stability, marginal stability so many new words are being used so many words because there is no appropriate word which gives the correct message, but the message is this that, well, if that is the situation you have to be cautious, you look at the system more carefully and see whether you can accept that system, if you can accept that system, for you it is a stable system why do you call it a marginally stable system then, but you do not accept that system naturally, that is an unstable system for you. This is the point you have to take note of that now we have under bounded-input bounded-output stability class a new concept where we coin the word 'marginal stability' for that.

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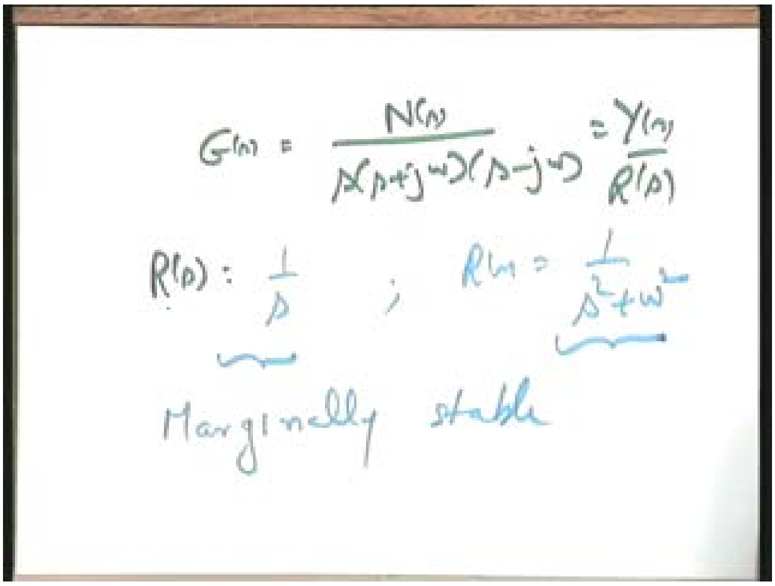

Now, since looking at the time I want to finish at least the stability concept today. So let me now quickly go to zero-input stability unless there is a question from you, I will go to the concept of zero-input stability. Look at this now, requires a little knowledge of state variables but I think that much you already have; I will be able to cash on that knowledge, I take the linear autonomous system: x dot is equal to A x and initial condition is x 0 by definition is equal to an x 0t value. Let me take an autonomous system. I want to study the zero-input stability so input I have taken zero but initial state is the energizing input to the system.

Now let me take the Laplace transform of this. s I no sorry let me take the Laplace transform, the Laplace transform will become sX(s) minus x 0. you will note that the underline means a vector and this means Laplace transform X(s) by definition, X(s) by definition is equal to x 1(s) x 2(s) x n(s) this should be clear in my vector notation. Similar is the case for x 0. So it is s X(s) minus x 0 is equal to A X(s); capital I am using for the Laplace transform Laplace domain and underline for a vector.

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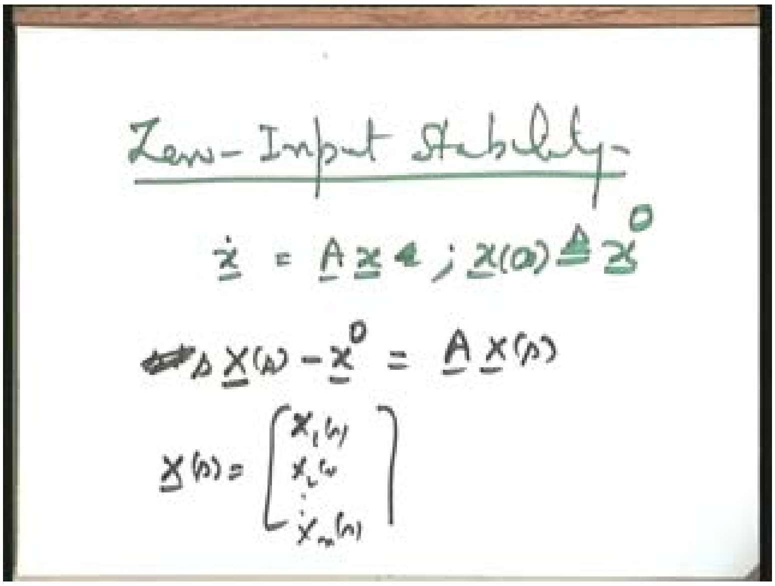

A is a matrix of constant coefficients. So this equation now which I have written I reproduce here s X(s) minus x 0 equal to A X(s) could be represented as (sI minus A) X(s) equal to x 0. You can see this point very easily. Just a rearrangement, this I is in identity matrix. From here I get X(s) is equal to (sI minus A) inverse x 0. If there is any gap you please raise an alarm I will explain that point otherwise I think it is quite simple quite obvious (sI minus A) inverse x 0. This I am interpreting in the form (sI minus A) adjoint divided by sI minus A determinant x 0. The inverse can be represented as an adjoint and a determinant matrix. adjoint you know consists of the elements of the cofactors.

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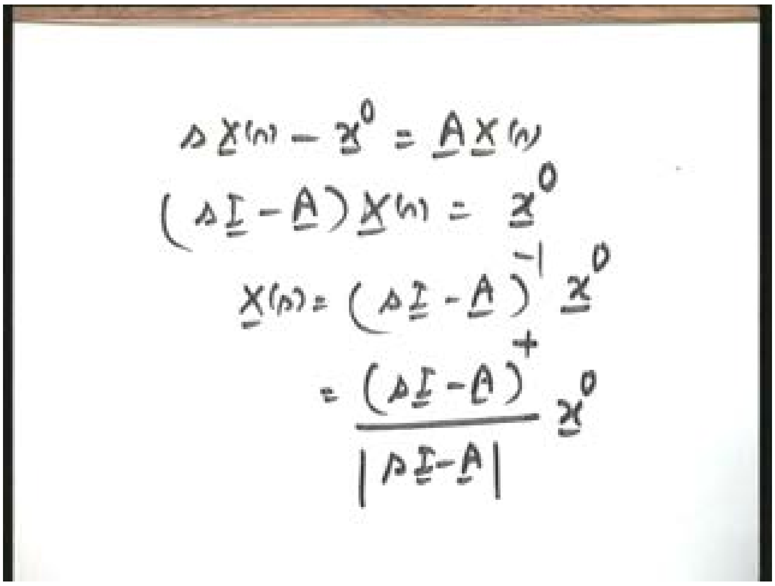

Can you give me the dimension of this particular matrix or I will put the question in a different way but I expect the answer from you. X(s) is equal to (sI minus A) adjoint divided by sI minus A determinant x 0 I want to study this; I want to study the inverse transform of this because for any input x 0 in the entire state space I want to study how X(t) will behave so I am interested in the inverse transform of this. Help me please; what will be (sI minus A)? (sI minus A) will be an nth order polynomial for n into n matrix A and let me write this polynomial as alpha 0 s to the power of n plus alpha 1 s to the power of n minus 1 plus alpha n this could easily be seen that this determinant polynomial is an nth order polynomial for in n into n matrix A.

Since you are interested in x 0 so it means the dynamics again is going to be guided by this particular nth order polynomial (Refer Slide Time: 37:57) this also is called the characteristic equation but it is the characteristic equation obtained from the state variable model of the system. It is a characteristic equation because the dynamics is characterised again by the roots of this equation as you will see.

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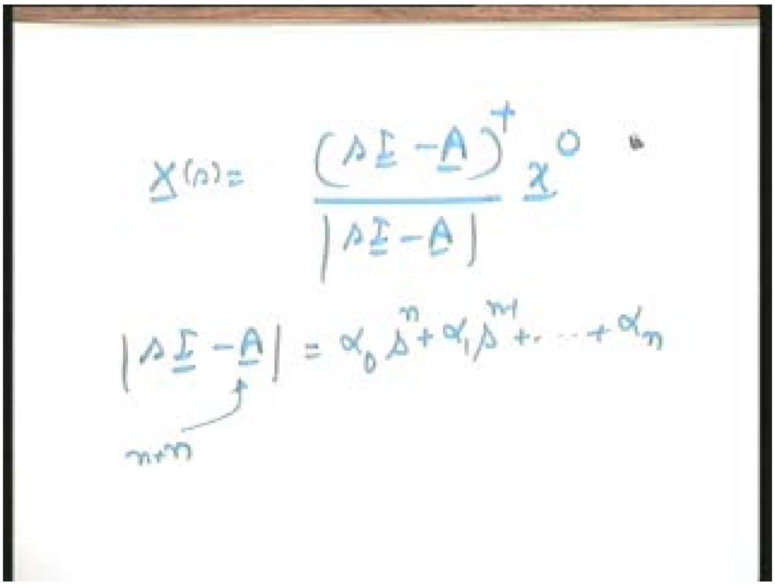

Now you tell me, this is what my question is which is coming to you; can you comment on the individual entries of the adjoint matrix (sI minus A); and if the answer comes I will not explain otherwise I will have to split up this particular matrix. Imagine (sI minus A) and give me the individual….. any statement, any comment on the individual entries of this particular matrix; will it be a polynomial? If yes, of what order?

[Conversation between Student and Professor – Not audible ((00:38:48 min))]
n and n minus 1 of the two answers, please see, it is n minus 1th order. If you want I can explain the reason or it should be intuitively very clear: s 1 0 0 0 1 0 you take this as the matrix; these are the elements a 11 a 12 a 1 and so on. Now individual entries you take. If you take individual entries what is the entry of this adjoint? The entry of this adjoint is you will take the cofactor of one element; if one element goes in that particular case there are n minus 1 factors which will be multiplied and hence the polynomial will be n minus 1th order polynomial. This could easily be established.

let me not write the complete…… what you take is you take a simple A matrix, you take a identity matrix, establish (sI minus A) and take the adjoint .you could take a simple equation also, you will definitely get the feel that every entry of (sI minus A) every element of sI minus a is an n minus 1th order polynomial.

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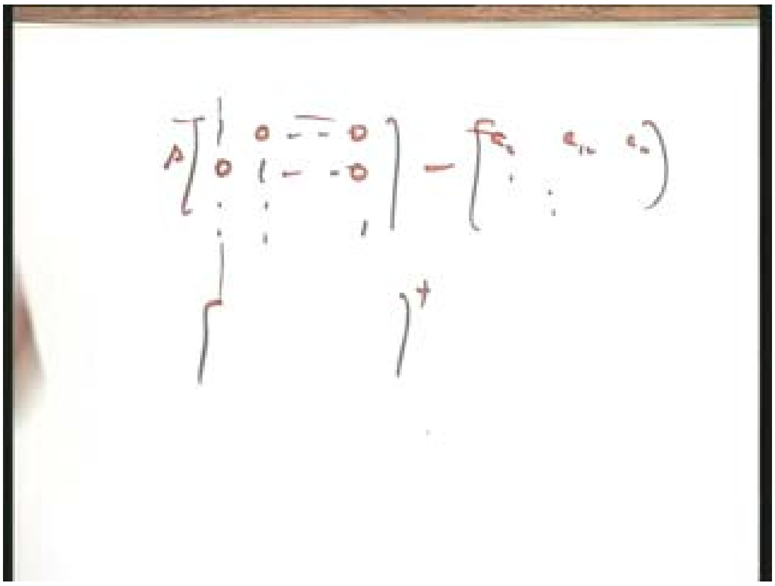

I want to cash on this if you agree with this. What will happen in that particular case the X(s) is equal to….. see all the entries of this particular matrix and n minus 1th order polynomial divided by this polynomial everywhere. So an n minus 1th order polynomial will give you the zeros of individual entries in the overall system while this will give you poles. So it means you can forget about zeros as far as stability is concerned. So it means the stability is given by, the zero-input stability is given by only the roots of this particular equation and you can forget about the zeros which will come because of the adjoint matrix (sI minus A) adjoint.

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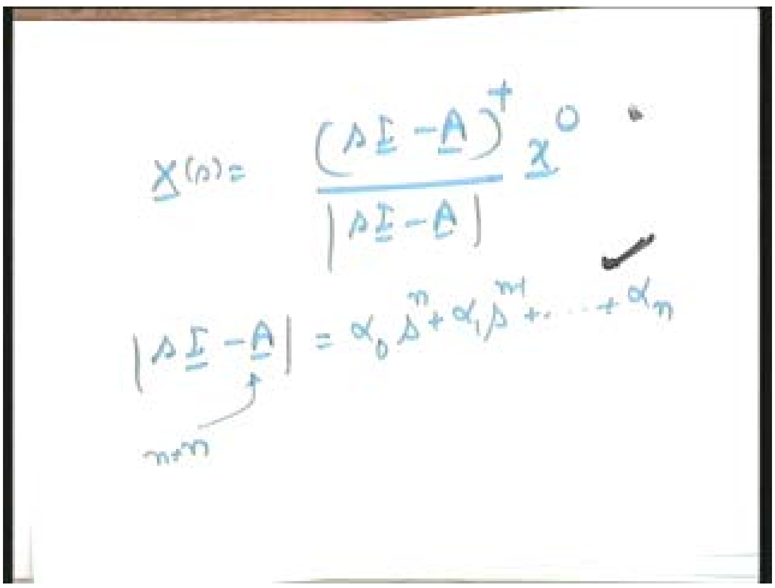

Yes any question please? I have not inverted it because really I want the qualitative feel, I do not want at this stage the qualitative information. If you get this feel in this particular case X(t) will always ….. now what happens X(t) will always remain bounded will always give you the required response if you look at the roads of this equation if they lie in the left half plane because the polynomials of this particular thing the elements of this particular matrix are all of order n minus 1.

Those of you who really could not get the feel of this just take one simple example and split it up and get the adjoint matrix; I am sure it is going to come to your satisfaction. I can proceed further at this particular stage and give you the definitions as given in the literature.

the literature says that the word asymptotic stability has been coined; asymptotic stability, if all the roots of the equation sI minus A determinant equal to 0 are in the left half plane in that particular case what will happen please tell me, all the states will go to zero, it should be very clear; asymptotically means X(t) the definition means that X(t) limit t tends to infinity will definitely go to zero not only bounded, why the word asymptotic is coming should be very clear, you can see the responses, you can see the original equation, your X(t) is equal to Laplace inverse of (sI minus A) adjoint divided by sI minus A determinant into x 0.

Take any x 0; x 0 is simply a constant, it is a scale factor, it is doing the same thing as the zeros of the transfer function you see, it is only a scale factor; the dynamics is going to be guided by sI minus a determinant. So, if all the roots of this equation are in the left half plane in that particular case all the dynamic modes decay to zero and therefore X(t) will decidedly decay to zero as t tends to infinity. So the condition of asymptotic stability is this that the state vector decays to zero as t tends to infinity which happens when all the roots of the characteristic equation are in the left half plane.

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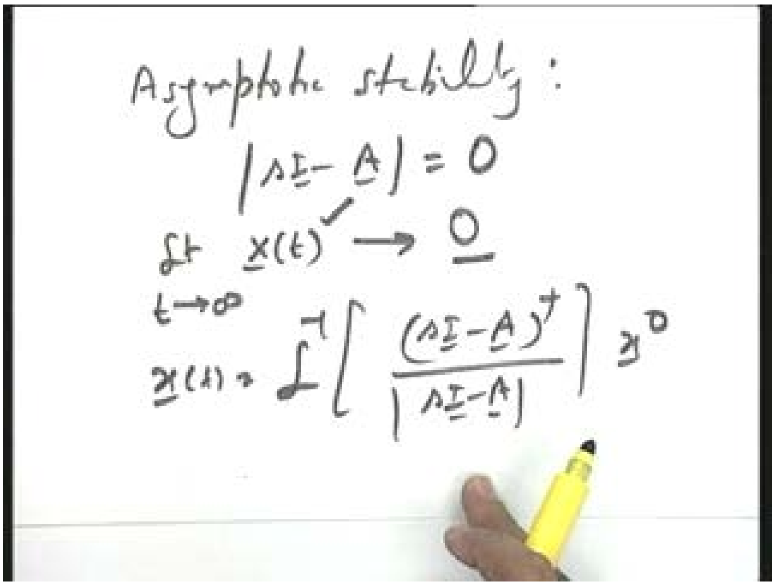

Now the condition of instability should be very clear. You take any root in the right half plane and look at this equation, definitely the state variables will grow to infinity and hence the system is unstable. Let us see the situation of simple poles on the imaginary axis. Similarly, you see multiple poles on the imaginary axis you can see. Multiple poles on the imaginary axis, look at the response, the response will be growing multiplied by x 0 will again give you an unstable situation because the state X(t) will go to infinity. But how about simple poles at the imaginary axis? Now tell me please, is the condition of simple poles on imaginary axis equivalent to getting a finite value for all t or not? You will please see, if there are simple poles on imaginary axis x 0 is simply going to change the magnitude so it means X(t) will remain bounded in certain region if there are simple poles on imaginary axis.

I am not now matching the poles it is a different situation. I say that if all the poles are in the left half plane the X(t) goes to zero; asymptotic stability; if at least one pole is in the right half plane and or multiple poles on the imaginary axis the system is decidedly unstable because X(t) will grow. But if there is a simple pole on the origin for example what is the response contribution; origin is going to give a constant response multiplied by this constant x 0 so X(t) remains bounded in certain region. You take the poles on the imaginary pair; so this will give you an oscillatory response multiplied by constant will only change the magnitude of the oscillation and therefore X(t) will remain bounded. So it means any simple pole on the imaginary axis will give you bounded state response and this again you can club under the marginal stability condition.

Now you can say that asymptotic stability and BIBO stability will become equivalent; very interesting point please; if and only if sI minus A determinant is same as….. this a very crucial statement….. as the denominator of G(s). You see that if sI minus A determinant is same as the denominator of transfer function of the same system in that case whatever we have been saying about BIBO stability and asymptotic stability is just equivalent there is absolutely no difference and fortunately, if not hundred percent, in most of the real life systems available to us this condition is satisfied.

This condition, though in this class we will not be discussing this condition, this condition is referred to as control ability and the observe ability of a system. It means if this condition is satisfied the system in the literature is referred to as controllable and observable the concept which we are not going to discuss in this class; the only statement I want to make is that in most of the real life situation the condition that sI minus A determinant is same as the denominator of the transfer function is satisfied and hence the zero-input stability ensures BIBO stability and vice versa. If that is the case then let us forget about the terms BIBO stability.

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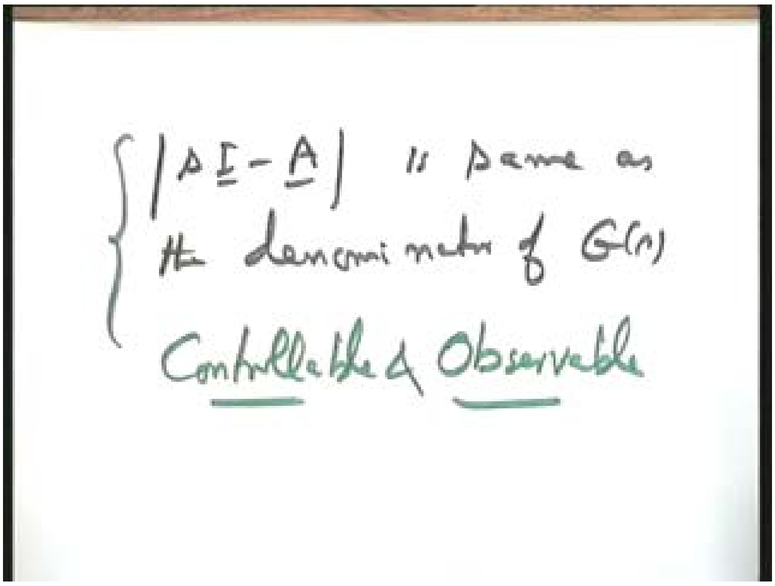

From now onwards I will drop the word BIBO stability. For me, the words only stability may be sufficient, stability. If all the poles are in the left half plane the system is stable; BIBO and asymptotic both of them are……..this will not be true only for a specific subclass of systems where the condition that sI minus A determinant is not same as the transfer function denominator. So if we exclude that subclass of system from our discussion which will be a very minor loss in that particular case we can forget about the two terms and for me stability is one term and instability…… a system is unstable if at least one pole is in the right half plane or a pair of poles on the imaginary axis the system is unstable.

Marginally stable: the system is marginally stable if all the poles are in the left half plane except one single pole or a simple pair on the j omega axis. Or simple pairs you see, you can have more than one; for example this and this well, this is also marginally stable but it should not be pole pair. So, if simple poles on the imaginary axis and all other poles are in the left half plane then the system under consideration is marginally stable.

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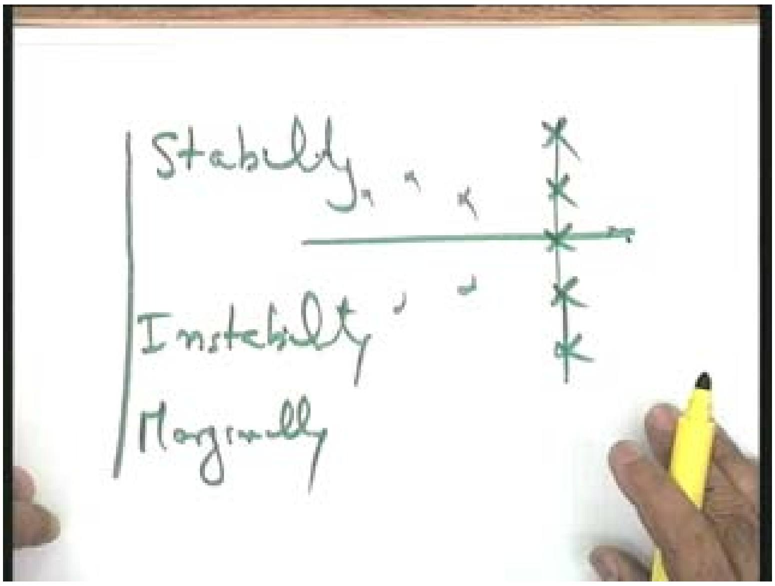

So for me, from today onwards stability, instability and marginal stability will be the three terms I will be using, thank you.