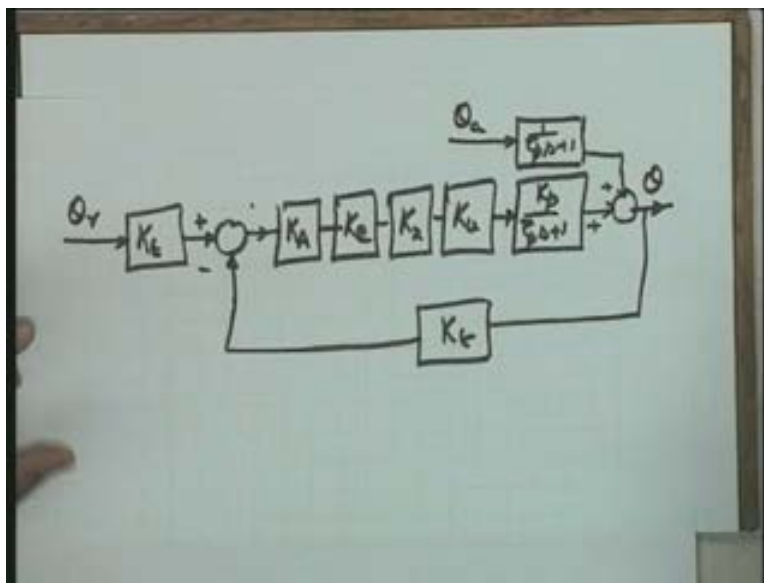


Control Engineering
Prof. Madan Gopal
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Lecture - 22
Basic Principles of Feedback Control (Contd...)

Let us recall the temperature control process we had discussed last time. The reference temperature I take here as θ_r I give you the block diagrammatic description once again this θ_r through, the setting K_t was converted into a voltage, a voltage signal was compared coming through a thermocouple sensor this gave me the error signal here this error signal was amplified using an amplifier K_A (Refer Slide Time: 1:35) this then was going to an electropneumatic transducer with constant K_e , a valve positioner with constant K_x , a valve gain K_v was taken to get the flow rate here, a process transfer function K_p over $\tau_p s + 1$ was taken and here I had a summing point to sum up the disturbance effect let me take this as the disturbance effect model which we took as 1 over $\tau_p s + 1$ and θ_a the ambient temperature the environmental temperature is the disturbance. The output is θ and this is sensed by a thermo couple sensor and is fed back. This is the block diagram of the system we discussed.

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I will try to quickly revise as to what were the conclusions. The conclusions were the following that when the process is a first-order model here (Refer Slide Time: 2:40) and we have neglected the dynamics of all these components in that particular case we found the following: the gain K the loop gain was directly affecting the transient performance as well as steady state. The conclusions were, larger the loop gain better is the transient performance because the feedback system time constant reduces. And secondly, larger the loop gain less is the steady state error. But then I pointed out to you that the gain is limited by certain considerations. One consideration

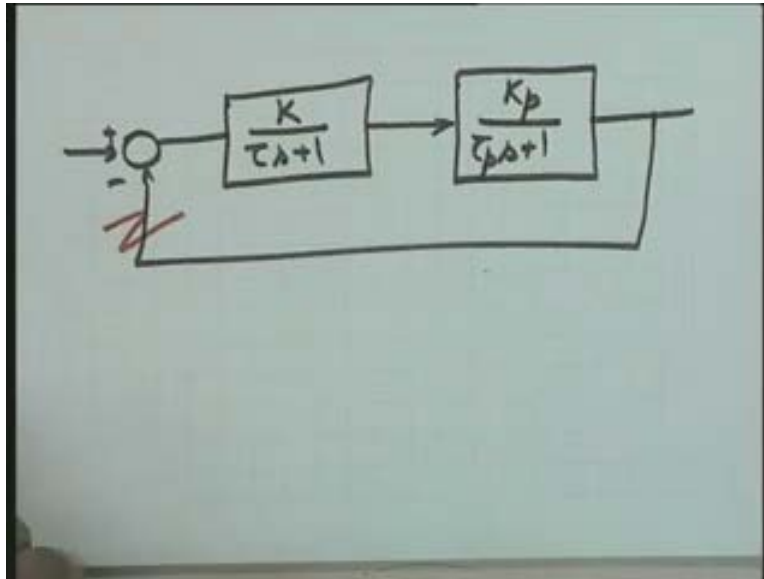
was that all these components will be driven to saturation and therefore your system will no more be a linear system and hence the loop gain choice is limited. The second consideration really led to good amount of discussion also, some questions from you were regarding instability. I said that, as the gain K increases what will happen; the approximation of neglecting this dynamics is no more that accurate; the dynamics becomes more effective and once the **dynamic become** dynamics becomes effective it leads to system instability.

There was a question as to when the time constant of this system is being improved. Why not the time constant of this also changes accordingly and the correlation of the two that is the approximation still remains valid. Well, some qualitative answer was given. For want of the discussion on the stability and the root locus I really could not a full answer which otherwise I would have liked. But now, after reviewing yesterday's lecture, I find that may be your intuitive total information on the stability and a root locus diagram could be utilized to give a more complete answer to that question.

So what I do is the following: **I assume the loop of this** I assume a loop of this form now, a total gain is clubbed into a single gain call it K and let us say τ is the time constant of this component a typical component in the loop you may select and you take the τ as a time constant of this component. Please see, the tools which I use in my answer in details will be coming later the root locus diagram and the stability. But I think I will be satisfied if I satisfy you on that question.

So $K p$ over $\tau s + 1$ is the process and here is let me not consider the disturbance at the moment and let me consider this has the process. Please see, if I break the loop here the open-loop transfer function is K over $\tau s + 1$ into $K p$ over $\tau s + 1$ please note. Both the poles one at s is equal to $-\frac{1}{\tau p}$ and the other at s is equal to $-\frac{1}{\tau}$ are in the left half of the s plane and the system is stable. I mean, in addition to stability other requirement of sensitivity and robustness are there but at least when stability is concerned, well, the system is stable.

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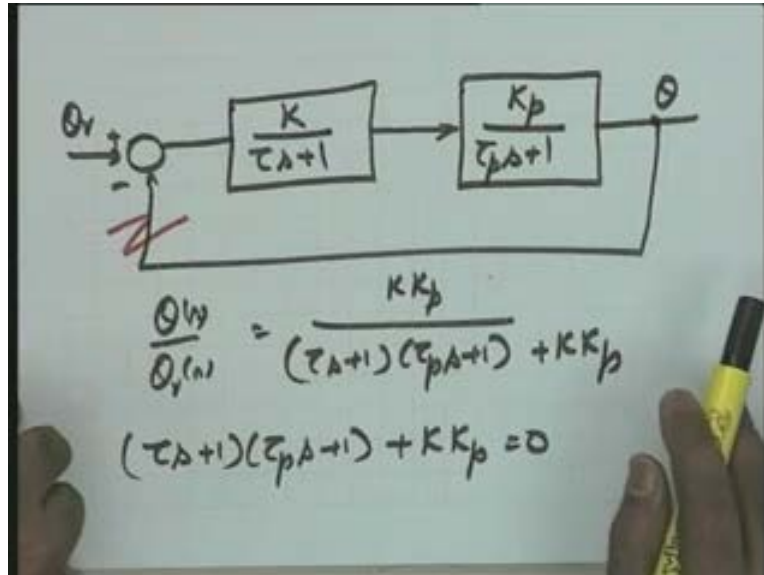


Look at the situation when the loop is closed. If I close the loop $\theta(s)$ this is θ and this is $\theta_r \theta(s)$ over $\theta_r(s)$ the closed-loop transfer function now becomes $\frac{K K_p}{\tau s + 1 \tau_p s + 1 + K K_p}$ this could easily be reduced. This is the overall transfer function of the system (Refer Slide Time: 6:26). And you know that the poles of this system are nothing but the roots of this characteristic equation.

What is the characteristic equation?

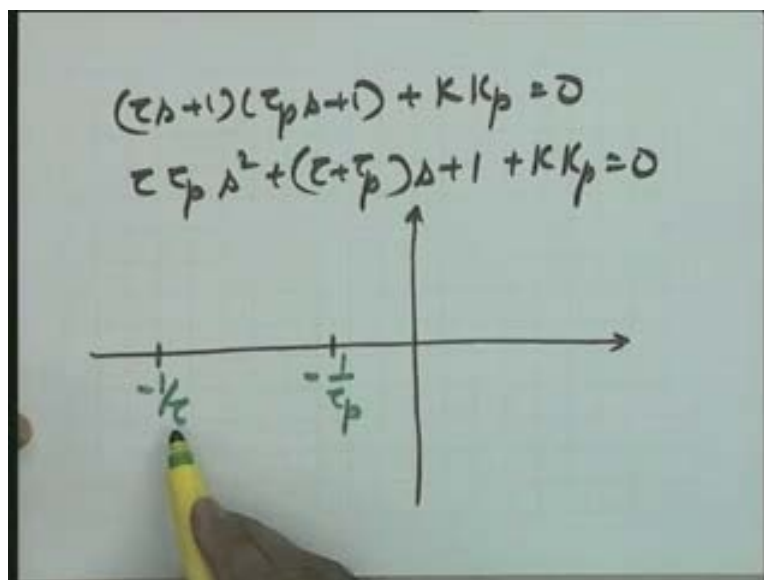
$\tau s + 1 \tau_p s + 1 + K K_p = 0$ is the characteristic equation and the roots of this characteristic equation are nothing but the poles which are going to determine the stability of the system. So I want you to study that naturally through a root locus diagram we will be studying in detail later; but at this juncture, I want to see what happens to these poles when K increases. So I now concentrate on this particular equation the characteristic equation and study the effect of change of K on the roots of this equation and therefore the poles of the closed-loop system.

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Rearranging this gives me $\tau_A \tau_p s^2 + (\tau_A + \tau_p)s + 1 + KK_p = 0$. Now please see; I make a diagram as K changes and **little quantitative aspects of the problem I leave to you for complete calculation**, what happens. If you take K is equal to 0 the two roots which you can easily determine are at these two points. I need your attention here please; $-1/\tau_p$ and $-1/\tau_A$ these are the two roots when K is equal to 0. K is equal to 0 means or equivalently you can say situation of low gain as you will see. So these are the two roots.

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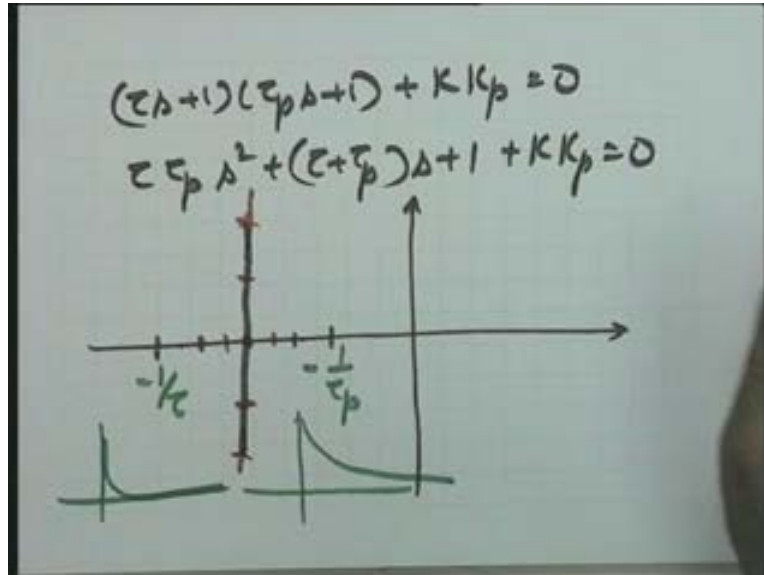


You will find the coming that tau has been neglected with respect to tau p means the tau is much larger than tau p it simply means that, that particular component other than the process is faster than the process then tau is, yes, tau is **tau is** much smaller than tau p I am sorry tau is much smaller than tau p it means the component is faster than the process and tau is smaller than tau p is reflected in the two root locations. You know that if this a root lies deep into the left half plane the decay curve of the corresponding dynamic mode is faster compared to the root close to the ω axis. So, if you see the dynamics of this particular system the dynamics of the system corresponding to this particular tau will quickly decay compared to the dynamics of tau p and therefore I can neglect tau with respect to tau p I can neglect the dynamics of this system other than the process and I get the simple situation.

Now let us see what happens when K increases; please do this calculation. For a particular value of k p you increase the value of the k you will find that the roots come here (Refer Slide Time: 9:28) the two roots of the characteristic equation which are the closed-loop poles of the system. The two roots come here, please see; I mean the situation is now complex because of the interaction under the feedback loop. Now you increase your gain K further the two roots go there still an increase of the gain will bring the two roots at the same point that is you get a double root. So it means you please note that as your gain K is increasing your approximation of neglecting the dynamics of the system is becoming less valid. I hope it is obvious now from this diagram, maybe from our yesterday's answer it was not so obvious because the root locus..... this is a root locus diagram as it is said and we are going to study the root locus at a later date. But from this point you see that as I increase K the two roots are coming closer and therefore the tau cannot be neglected with respected to tau p. That is, a dynamics corresponding to this root is not much faster compared to the dynamics corresponding to the other root and hence the dynamics of the other component of the system also becomes effective, still go on increasing the root K.

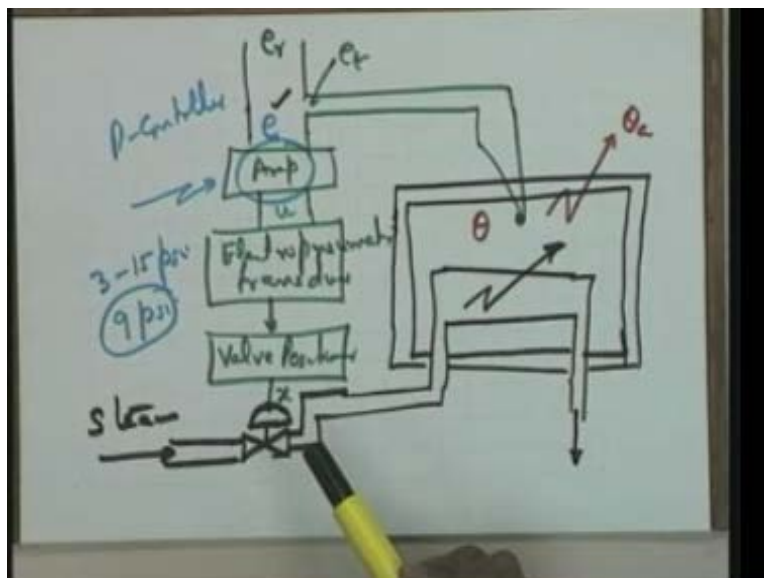
If you increase the K you will see later that the roots now become complex conjugate. that is, for still a larger value of K one root is here and the other root is here (Refer Slide Time: 10:5) so now you know that if the roots are complex conjugate the poles are complex conjugate the system response is oscillatory. So it means, because of the interaction of the dynamics of the Electropneumatic transducer or amplifier with the process the response which was earlier exponential only has now become oscillatory. You keep on increasing the roots the oscillations will keep on increasing. So in this particular case when you have taken only one tau the response that is these roots never go to the right half plane, still you see instability has not been predicted but again now I cannot go to that depth that I give you what happens when one more dynamic element is taken, here at least you take the qualitative aspect that if I increase one more tau the system characteristic equation becomes a third-order system and with a large value of K there is a distinct possibility of these roots going to the right half side and hence the system becoming unstable.

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So I hope, with this explanation the answer to the question as to what happens to the two time constants under the feedback becomes clear. The approximation naturally become less valid as K keeps on increasing and hence the feedback system is prone to instability and this is one of the important factor in design. If you want to improve the accuracy you will have to see side by side that the stability is not threatened. Well, next, I started with the concept of integral control also. So now let me take up the integral control at this point. **You have this particular system in your notes; you may not draw it again.** Look at this system what did we do in this particular system.

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I said that this error e (Refer Slide Time: 12:51) will become zero if initial trimming of the Electropneumatic transducer and the valve positioner is done in such a way that this temperature θ is equal to the equilibrium value. Now help me please, give me the answer; what happens; the trimming has been done, the system is in operation. But as I said the chamber may be used for testing the electronic components you have manufactured.

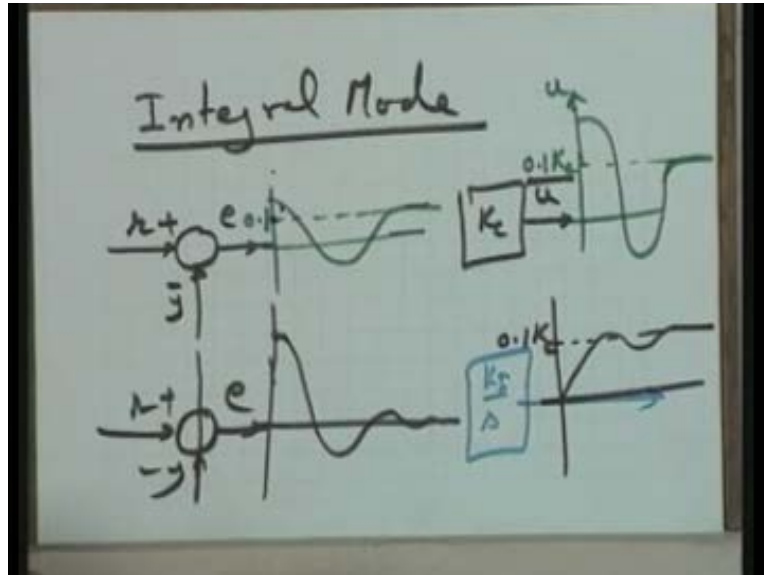
Now, suppose the demand of testing says that you increase this temperature by 10 degrees now how can you do? If you want to increase this temperature by 10 degrees meaning thereby that the command will change by 10 degrees; if the command changes by 10 degrees for the new steady state to occur you see that you require an additional energy that additional energy can come if and only if there is some input to the Electropneumatic transducer. So it means the additional energy will come if and only if there is an error signal. **See this point, my explanation please needs your attention.**

I want to say is that, well the system is operating in steady state there are no disturbances, forget about disturbances but still you see the demand of the output variable may change depending upon this specific applications, your demand says that you increase the temperature but 10 degrees, the 10n degrees increase will come by additional energy. so the additional energy, there are two ways you see: One is this that you shut the system down go back to the basic elements, retrim the components so that the adjustment of the Electropneumatic transducer and the valve positioner is accordingly done with respect to the 10n degrees requirement. But if your requirement keeps on changing on minute basis or second basis you really cannot every time go and retrim this, you want some automatic arrangement and to go for an automatic arrangement the suggestion is an integral control.

The claim is this that the integral control if you incorporate in the system in response to your changing demands the integral control will automatically change the trimming of the components, see this point. You will not require going back to those components to change the energy. The automatic trimming, the automatic readjustment, automatic balance of the requirements will be done by the integral control this what I am going to explain to you. But before that you see that unless you go for the trimming if this is the arrangement done in that particular case it is very necessary that a steady state error occurs because it is the steady state error only which will give an additional input over here and this additional input can excite the electro-pneumatic transducer in such a way that additional energy is available to the system by an additional opening of the control valve over and above the opening which existed before the new demand on θ_r .

Now let us say how the integral control will achieve this objective. Again this was the slide we were discussing last time.

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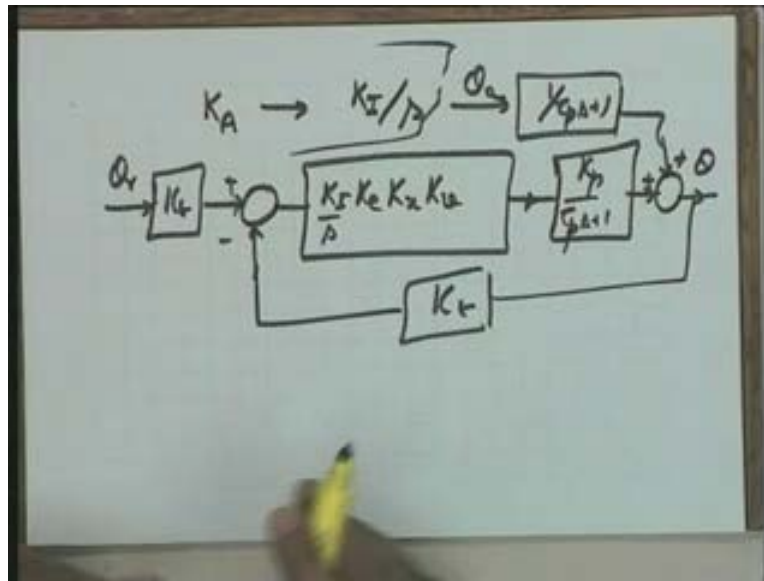
In this particular case let us say r is the input and y is the output. Because of the additional demand there is an additional energy requirement and let us say that the additional energy requirement is met when a steady state error of 0.1 exists. Well, how this 0.1 has been taken, nothing important in it, some numerical value I have taken. So let us say an additional energy of $0.1 K c$ is the output of the amplifier or the proportional controller and this output of the proportional controller is able to meet the requirements for the new steady state and therefore this is a must this particular steady state error (Refer Slide Time: 16:52) because this is a must $0.1 K c$.

Now let us see you have an integral controller installed. If there is an integral controller installed integral controller can integrate this error; you can suitably have these constants $K I$ and other constants in the system so that when the steady state occurs the integral controller can give you the required amount of the energy which is $0.1 K c$ so it means I have to suitably adjust my value $K I$ that is a design concept. At this juncture, you have to only take the qualitative appreciation. How do I get the value of $K I$? You leave it to me to explain to you later. But you see that at least it is clear that an integral control can provide this particular energy automatically if a suitable design is made and if this energy is available it is equivalent as if Electropneumatic transducer and the control valve had been re-adjusted and therefore the error can go to zero and the error does go to zero when we really take up the integral control into the picture.

So the purpose of integral control is actually the automatic re-adjustment; this is one example, anywhere in demand may come automatic re-adjustment for the new equilibrium position and therefore within integral control it is possible to achieve zero steady state error and hence integral control is basically employed for getting better steady state accuracy but there is a price to be paid. You will see through the example I am going to give you that this very integral control is going to create instability problems much more than the problems that existed with only proportional control and for that I take up the system which we have been discussing the temperature control system.

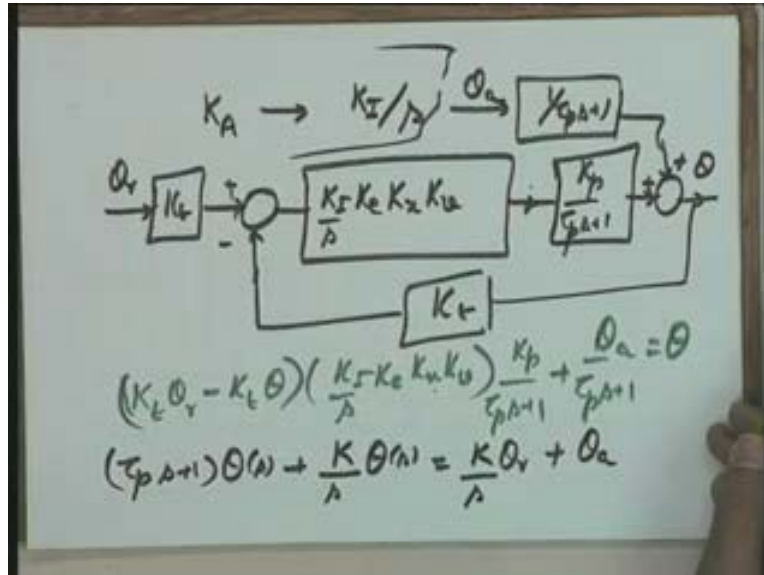
Now what I do is that the K_A amplifier I replace by K_I by s an integral controller and qualitatively study the effects of the integral controller on the system. K_A is being replaced by K_I by s , **I need your help please**. θ_r is here, K_t the two voltages being compared now let me put a single block here K_A was the block let me put K_I by s after that it will be K_e next is your K_x next is your K_v and you are getting a flow here. Now this is a process K_p over $\tau_p s + 1$ and here is a disturbance; I will like to study the effect of the disturbance as well 1 over $\tau_p s + 1$ and the disturbance is θ_a , this is θ_a and through K_t the thermocouple constant it is coming here.

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Let us go the same way which we employed for portion control. I write the system equation, help me please. $K_t \theta_r - K_t \theta = K_I \int \theta_e dt$ $K_x K_v$ have reached this point, after that I write $K_p / (\tau_p s + 1)$ this point plus $1 / (\tau_p s + 1)$ into $\theta_a = \theta$ is my equation. From the block diagram I have written this equation. Please rearrange this equation in the form of transfer functions and other things. So let me do the first arrangement here itself. let me take it on this side the $\tau_p s + 1$ into θ in the Laplace domain plus collect the θ terms $K_t K_I K_e K_x K_v$ let me put all of them into a single constant k and therefore this becomes into K_p also this becomes K over s **is it alright, yes** $\tau_p s + 1$ goes over here K over s into θ that is right, this has gone on this side (Refer Slide Time: 21:15) is equal to..... this now becomes K over..... K_t gets multiplied K over s **[Conversation between Student and Professor – Not audible ((00:21:30 min))]** no K_t term I am multiplying with those this sum $K_t K_I K_e K_x K_v s$ into θ into $\theta_r + \theta_a$. **Please see if there is any error, I hope this is okay.** i hope this okay fine. I have re-adjusted the terms θ on one side θ_r and θ_a on the other side; over s will come here also please see (Refer Slide Time: 21:57) I am multiplying this here $K_t K_I K_e K_x K_v$ over s , K_p is also is getting much into θ_r with $\tau_p s + 1$ going on to this side, **I hope this is okay, fine.**

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Rearrangement of this equation gives me: $(\tau_p s + 1 + K/s) \theta(s) = K/s \theta_r + \theta_a$ or equivalently this can be written as: $(\tau_p s^2 + s + K) \theta(s) = K \theta_r(s) + \theta_a(s)$. yes please, give me the steady state performance of the system for a step input in command and a step input in disturbance.

A step input in command: So what is $\theta(s)$ over $\theta_r(s)$ please? This is $K / (\tau_p s^2 + s + K)$. A step input in command means it is $\theta_r(s) = 1/s$ and you find that your θ ss is equal to 1 for any value of K which was not possible in only proportional control. θ ss is equal to 1, you just apply your final value theorem; $\theta_r(s)$ is equal to $1/s$ and the final value theorem says that $\lim_{s \rightarrow 0} s \theta(s)$; I hope I need not write this step. So it says that θ ss is equal to 1 irrespective of the value of K and hence the steady state error e_{ss} is equal to 0 when the integral control has been employed. But still we have to see the disturbance. Come on, tell me please; in response to disturbance, well, I need not write anything; is it really visible that the output θ at steady state will become equal to zero in response to disturbance? That is θ / θ_a you take which is $s / (\tau_p s^2 + s + 1)$ and apply the final value theorem; is it okay please?

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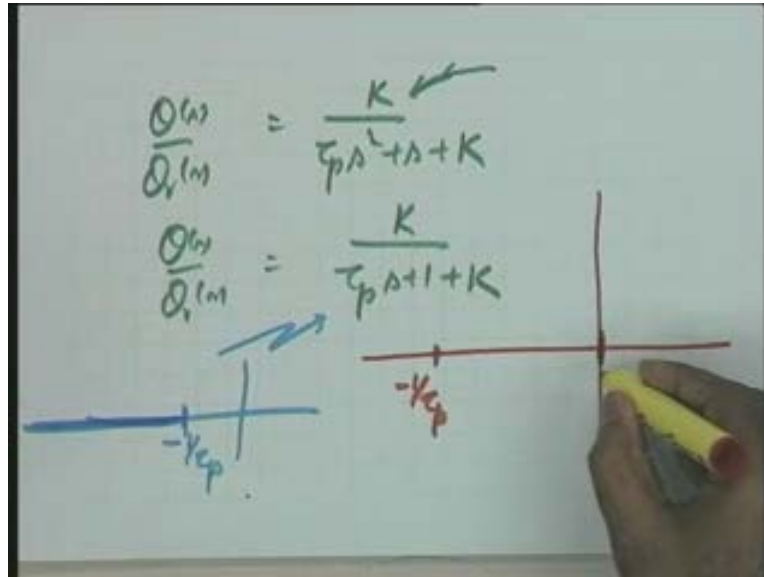
$$\begin{aligned}
 (\tau_p s + 1 + \frac{K}{A}) \Theta(s) &= \frac{K}{A} \Theta_r + \Theta_d \\
 (\tau_p s^2 + s + K) \Theta(s) &= K \Theta_r(s) + \Theta_d(s) \\
 \frac{\Theta(s)}{\Theta_r(s)} &= \frac{K}{\tau_p s^2 + s + K} \\
 \Theta_{ss} &= 1 ;
 \end{aligned}$$

Please see that this is theta ss is equal to 1 in response to command step and theta ss is equal to 0 in response to disturbance step. So it means, surely, if the disturbance and the command inputs are constant signals the system steady state error is going to be zero; it is the perfect steady state performance you will like to achieve and you have achieved it which was not possible with proportional control. Any questions on this please? If there is any gap in these equations that also you could ask me I can write those equations please. If this is well-taken then I go to the transient performance.

Transient performance let me say theta s over theta r(s) let me take is equal to K over tau p s squared plus s plus K. Now you see that originally your system, I need your attention please, originally your system was a first-order system and theta s over theta r(s) was there for that system was K over tau p s plus 1 plus K. Please see, assuming that the model is okay, let us first assume that the model is okay then we will introduce other dynamics. If the model is okay you will find that for this particular system for this particular system the pole of the system initially is at minus 1 over tau p and it goes on to this side as K increases. I again I am exploiting your intuitive knowledge of how to make the root locus diagram without actually defining what is a root locus diagram.

So you see that when I take K is equal to 0 the pole is at minus 1 over tau p and when I increase K the closed-loop pole of the system is going down deep into the left half plane and a pole going deep into the left half plane means that the system is becoming the response is becoming faster and faster. So there is no constrain there is no limit assuming that the model was correct. But now look at this situation; look at this that is K over tau p s squared plus K plus 1; in this particular case if I make a root locus sketch, the two poles are at K is equal to 0 and K is equal to minus 1 over tau p when s is equal to minus 1 over tau p when K is equal to 0 please see. I am at this point and this point.

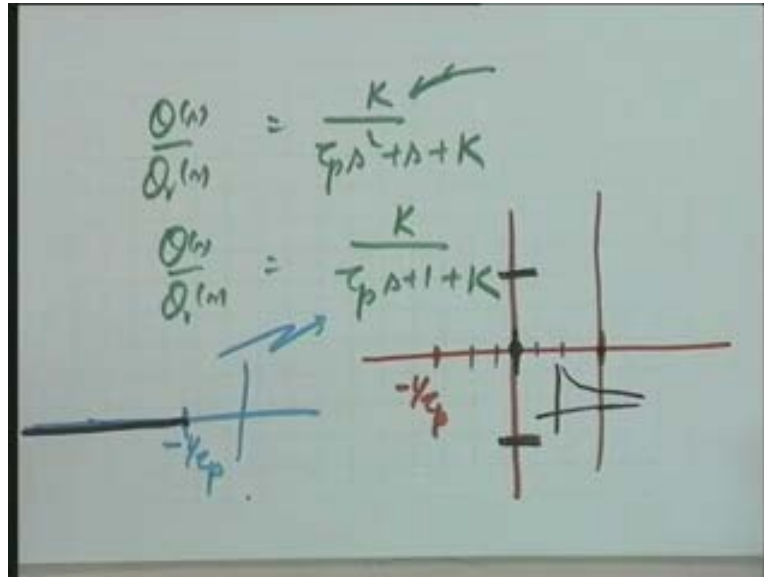
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You will please note that the response due to the pole at the origin will never decay, it is a constant response. So it means if there is a pole here the transient will not decay. Now you increase your K , if you increase your K your poles are moving like this; still an increase your poles are moving like this. So, if your poles are moving like this and there will be a K when the poles will bifurcate and will become complex conjugate, all now is happening because of the integral control. This is all now happening because of the integral control. Now there are two poles and the poles are moving like this (Refer Slide Time: 27:17). If the poles are moving like this one aspect you may please see; in this particular case it was possible to drive the pole to the left half plane wherever you want it so it means there was no limit in improvement in the transient performance. While in this particular case at the most you can drive your poles at this particular point.

You please note that the response is I am making a statement: the response is dictated by the pole closer to the g omega axis because it is the pole which decays which takes more time in decaying; the pole which is farther away from the g omega axis takes less time for decaying. So it means, as far as the transient performance is concerned it is dominated by the pole closer to the g omega axis other pole has got relatively less effect and now you can see this particular pole can be driven anywhere you like while this particular pole that is the dominant pole that is the pole closer to the g omega axis can be driven at the most to this particular point. So it means one thing you are fixing a limit; because of the integral control now there is a limit on the improvement in the transient performance which you can achieve. Not only that, if you increase the K in this particular case the response becomes oscillatory. in this particular case (Refer Slide Time: 28:40) the poles are again not going into the **left half plane** right half plane but you see, an integral control has converted your first-order system into a second-order system.

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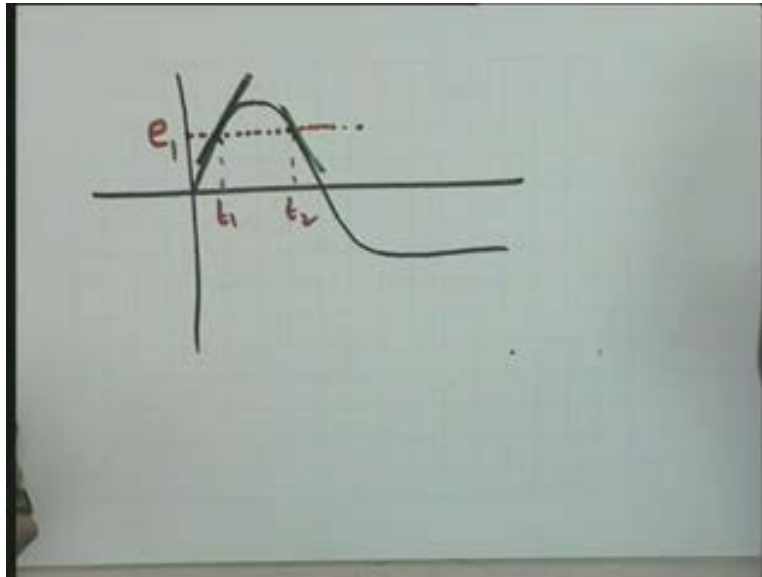
Now, even if single more time constant becomes effective in the system the system can become unstable, even if a single time constant. In the earlier situation a single time constant was not creating that bad a situation but now even if a single time constant becomes effective in the system in that particular case it becomes a third-order system and instability is distinctly introduced into the system. So it means it is visible from here that, though it is a simple example, from this example it is visible that integral control **is having many a times** will have many a times negative effect on the transient response of the system or limiting effect on the transient response on the system and can even lead to instability of the system as is qualitatively explained to you though it is not visible from the example we have taken this being a very simple example. But there is a distinct possibility that the system will be driven to instability; that is it will be driven to unstable region.

The third and the last aspect which I want to take is that of derivative control. Now you see that I take a typical error curve; let us say a system has got this type of error. now please see, if you go for only proportional control so what will happen; at this particular error (Refer Slide Time: 30:25) **I will definitely need your attention at this particular point please**; corresponding to this error e , let me call it e_1 what is happening; the proportional control will have the same action, the same amount of manipulation of energy will be there at e_1 at this time t_1 as well as at time t_2 because the error is the same. In the two situations t_1 and t_2 since the magnitude of the error is same, see the control signal will be same and therefore energy will be the same but you will not like to go for that the reason being because you know that at this particular point the error is rising (Refer Slide Time: 31:07) and at this particular point the error is falling.

So it means if the error is rising you will like to take a more severe corrective action so that the rise is also prevented, not only the magnitude the tendency to rise is also prevented so you will like to take a more severe action at this point compared to this particular point. That is, actually this slope is nothing but telling you or making the controller knowledgeable about the future error. More this particular slope, that is, if you are going vertically up it means the knowledge

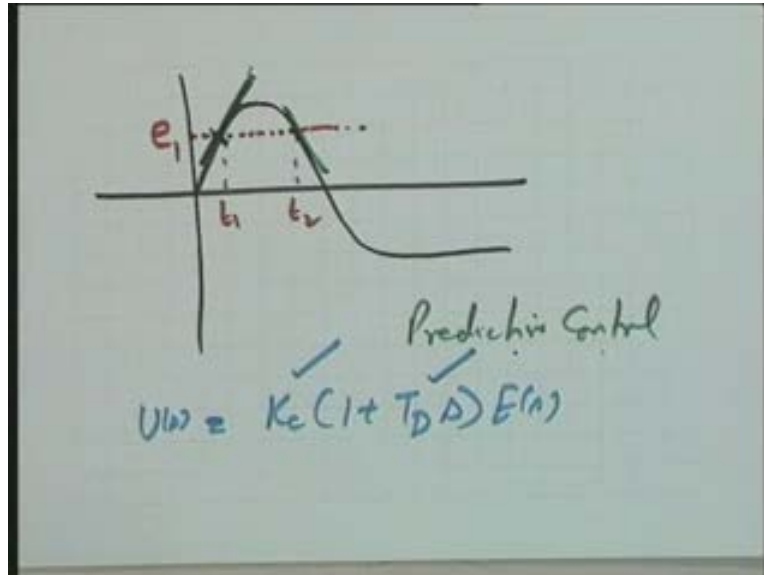
given to the controller is that the errors in future are going to be more severe and if this slope is near zero, horizontal line, the errors are not going to be more severe. So the derivative control means that you just introduce a signal not only proportional to the magnitude of the error but you introduce this information also to the controller that, look, this much is the derivative.

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So can I say that a derivative control is nothing but a predictive type of control? It predicts what is going to happen in future and prediction is coming through the slope of this line qualitatively. The prediction as to what is going happen in the future is coming through this particular slope line. So it is a predictive type of control and what will happen is that the errors in that particular case if you are able to give this particular information also it will have more damping. That is, as if you know you are driving a vehicle and you know what is going to happen, if there is a sharp slope you will actually apply your brakes because you know that heavy oscillations or instability can be caused because of that so it is just taking that predictive action so more damping will be introduced. At this particular juncture, if the errors are more you are going to introduce more damping and hence the transient response of the system can be improved with the help of derivative control. And as you know that the derivative control is going to be $U(s)$, it is going to take the form $U(s)$ equal to $K_c (1 + T D_s) E(s)$ where $E(s)$ is the error. So, in addition to proportional signal there is a signal which is the derivative signal which is also coming into the loop.

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Couple of points I like to mention over here; you see that **if you take** if this steady state error becomes constant that is when you reach a steady state what will happen, in that particular case the effect of derivative on this error is zero, so one point, the derivative control is effective only in the transient mode and has no effect on the steady state error because the derivative of a constant is zero. So when you reach the new steady state the derivative control has lost its role. At the new steady state it is only the proportional control or proportional and integral control which is providing the required amount of energy at the new equilibrium point. The derivative control has played its role only during the transient period by suitably controlling the breaking or damping on the system.

One more point I think I like to give it over here itself. Now let us say that you are introducing this (Refer Slide Time: 34:35) the effect of this qualitatively I am going to see. But one point you must note over here; when you implement this there are problems. So it means this type of equation is for qualitative study and not for implementation.

Come on, please help me, what are the problems this type of control can introduce into the system (Refer Slide Time: 35:00) $E(s)$ this $T_D s$ term. Let us say that at some point of time may be because of the sensor or other disturbances a signal of the form $0.01 \sin 10t$ has appeared into the loop. I just give you an example; 0.01 is too small a magnitude and if you are working only in the proportional mode may be you can forget about it. The system itself is a filter a low pass filter so it means when such a signal passes through your plant the high frequency signal will be blocked automatically. But if there is a derivative control in between what is happening, you take the derivative of the signal $0.01 \sin 10t$ that is what is the value please? $10 \sin 10t$, yes, $10 \sin 10t$ that is what is the value please? $10 \sin 10t$ a signal which you have introduced this magnitude 10 may become even more than this signal the useful signal that is the error signal which is manipulating the energy signal. So it means, because of the noise accentuating properties this type of derivative action is never employed. You will always go with a suitable low pass filter when you implement this. The details of the filter when we come to the

design will be given later. At this point we will like to go for only the qualitative aspects of derivative control assuming that such a signal does not come and if it comes we have a filter which will take care of these signals.

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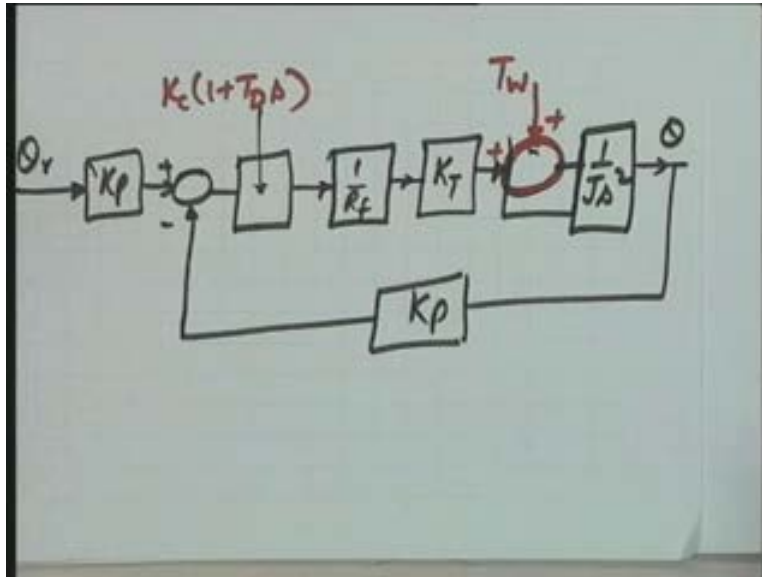
$$U(s) = K_c (1 + T_D s) E(s)$$

\swarrow $0.01 \text{ A} \cdot 10^3 \text{ t}$
 \swarrow $0.01 \text{ A} \cdot 10^3 \text{ t}$
 \swarrow $1 \text{ A} \cdot 10^3 \text{ t}$

Now, to explain the derivative concept to you I take up a position control system. Now I think the schematic need not be given. You have been equipped; the block diagram cannot directly explain the system to you. I take the command θ_r the position command, K_P the potentiometer plus minus, after that let me have here a derivative control (Refer Slide Time: 37:06) so it means I take here $K_C(1 + T_D s)$ as the controller. To make the analysis simple, after this particular controller I take it to let us say, field control motor let me take, $1/R_f$ is the field resistance, I am even neglecting the field inductance only to approximate it. So this is the field current, let me have a constant torque constant here this is the torque.

Note the approximations involved, field inductors normally is not negligible. so this is the torque and here is okay sorry, at this point let me introduce the disturbance (Refer Slide Time: 37:49) plus plus here is a T_W here and at this point now I have the plant and let me assume the plant to be purely inertial plant only to help reduce the size of the equations I am going to write otherwise the conclusion is valid when you take the damping also. So this is θ , this θ is being picked up from here by a potentiometer and is available for feedback.

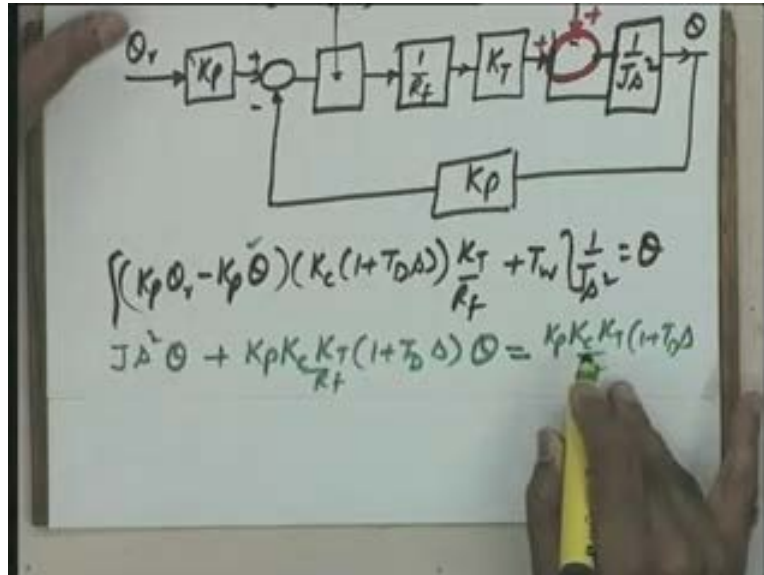
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The approximations should be clear to you that I have just taken only to make the life simple as far as mathematics is concerned. Give me the equations please? $(K_p \theta_r - K_p \theta)$ $(K_c (1 + T_D s))$ $K_T / R_f + T_w$ $1 / J s^2$ equal to θ . **See this point**, I have simply followed the loop and I have written this particular equation following the loop.

Anyone who is able to detect any error in this equation, I hope this is okay; I have not made an error. Now I do the same thing. I manipulate this equation; take all θ terms on one side. so $J s^2 \theta$ has been taken on this side plus this θ term multiplied by K_s so it is $K_p K_c K_T / R_f (1 + T_D s) \theta + K_p K_c T_w \theta$ is here K_T / R_f this is fine. Now you take, this is equal to, θ terms have been taken on this side; this is equal to $K_p K_c K_T / R_f (1 + T_D s) \theta_r + T_w$. I will rewrite this on the next slide but I think it is clear here: $K_p K_c K_T / R_f (1 + T_D s) \theta_r + T_w$.

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What I am doing here now? This total thing I am replacing by a single constant K. So in terms of a single constant K my equation becomes: $(Js^2 + K T Ds + K) \theta(s) = K(1 + T Ds) \theta_r + T W$. This becomes my equation please in terms of the single constant K manipulating the earlier equation. Now you can write in terms of the transfer function. first let me take it with theta by theta r equal to with respect to the step input please $K / (1 + T Ds)$ over $Js^2 + K T Ds + K$. **Help me please, give me the indices.** I told you that, as far as second order factor is concerned the personality of the system is described by zeta and omega n. **Come on, give me the personality parameters for this system.** The omega n becomes equal to K / J by J under root.

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$$\begin{aligned}
 (J s^2 + K T_D s + K) \theta(s) &= K(1 + T_D s) \theta_r + T_w \\
 \frac{\theta}{\theta_r} &= \frac{K(1 + T_D s) + T_w}{J s^2 + K T_D s + K} \\
 \omega_n &= \sqrt{\frac{K}{J}} ; 2\zeta \omega_n = \frac{K T_D}{J} \\
 \zeta &=
 \end{aligned}$$

How about zeta please? $2 \zeta \omega_n$ is equal to K/J and therefore zeta; **come on, give me the expression please to make you active in the game.** Give me the expression; I want T/D also to appear over here which will naturally. Just see, T/D by 2 since I remember the expression, it is for you to verify K/J under the root. So now you find, this you can verify later also you see if you are not able to do immediate adjustments [**Conversation between Student and Professor – Not audible ((00:41:51 min))**] is it okay or is there any error? The 2ξ , yes, $2 \xi \omega_n$ is equal to K/J . **oh yes oh yes oh yes oh yes** since I remembered the final expression so earlier expression even if it was wrong I could write it. **I simply out of memory wrote this expression that is fine.** $2 \zeta \omega_n$ is equal to $K T/D$ by J and hence from this expression the value of zeta can be obtained.

Now you please see that it is visible, the effect of the damping, the effect of derivative control on the transient response is visible over here, you can increase the value of zeta that is you can bring the un-damped or underdamped system towards more damping, critical or over depends upon the specific application and therefore the derivative control is very clear it is going to improve damping of the system.

One point here, I think the last point and I have margined to explain that last point is the following. I told you filtering problem; there was a problem in filtering. **You see that though the denominator is a second-order term your attention please, at least those of you were actively involved in this your attention is needed here;** you see in the numerator $1 + T/Ds$ is there so what will happen, if you give a step change in the command a step change suppose your requirement is a step change, your antenna you are moving from 10° location you suddenly wanted to move by 30° because the enemies plane has suddenly taken a turn so you want to move you want to change it by 20° suddenly, so in that particular case what will happen; you will please note that the step signal is multiplied by T/Ds and therefore a spike that is a signal an impulse type of signal of large magnitude will be injected into the system which is undesirable. Though the feedback action will take care of this finally but you see you are creating a disturbance and then solving it so this particular problem is there in this particular type of control that this θ_r when you give a step change in command $T/Ds \theta_r$ is the signal in the loop which is a signal of large amplitude on short duration and this particular signal is not useful is going to create disturbance in the system.

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$$\theta_r = K(1+T_D A) O_r + T_w$$

$$\frac{\theta_r}{O_r} = \frac{K(1+T_D A)}{J A^2 + K T_D A + K}$$

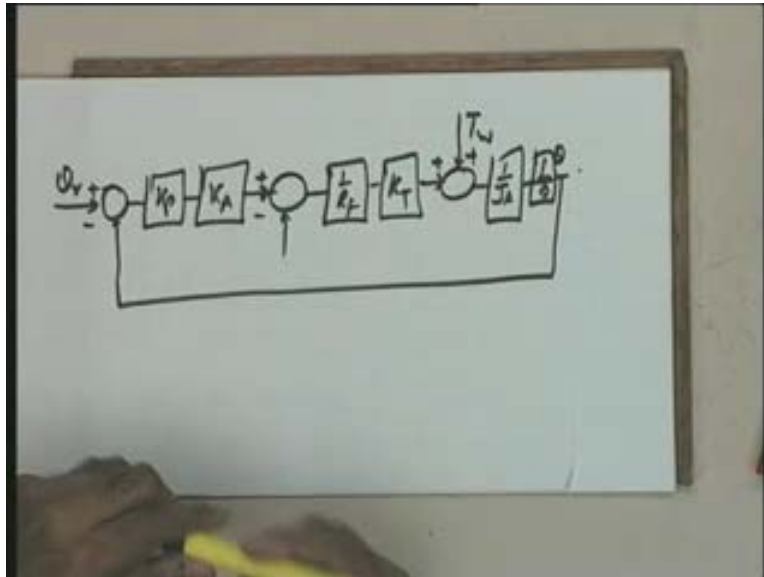
$$\omega_n = \sqrt{\frac{K}{J}} ; 2\zeta\omega_n = \frac{K T_D}{J}$$

$$\zeta = \frac{T_D}{2} \sqrt{\frac{K}{J}}$$

So what is the alternative? A beautiful alternative you already know that is why I said that I like to give this and that alternative is the following: Theta r plus minus..... do not mind the word beautiful it will really turn out to be beautiful you will appreciate this you will find this, K p I am merging it inside, let us say what are the other constants please, the other constants were K A let me use an amplifier only so the so-called beautiful answer uses the amplifier in the feedback forward path and therefore the so-called T Ds does not appear so this is K A and after that I have K T over R f so I have the torque over here and plus plus T W and here please see 1 over Js squared let me split it into 1 over s and 1 over s this is my theta. Now you see that this particular signal in a position control system is physically available to you through a tachogenerator. So, if I take this signal here from here I should have introduced a loop over here. If I take this signal at this particular point actually the loop will not come here at this, this needs splitting, since I have the margin to explain fully I redraw the diagram please because this signal I like to give after the amplifier, K p a potentiometer, this is amplifier (Refer Slide Time: 46:08).

Now let me introduce the signal here so that is why you see redrawing the diagram became necessary. this is the signal I am taking from the feedback loop, this signal now is being given to the field circuit 1 over R f, this is the time constant the torque constant K T and here is my T W the disturbance, 1 over Js I am writing here, 1 over s I am writing here, this is theta and is being fed back.

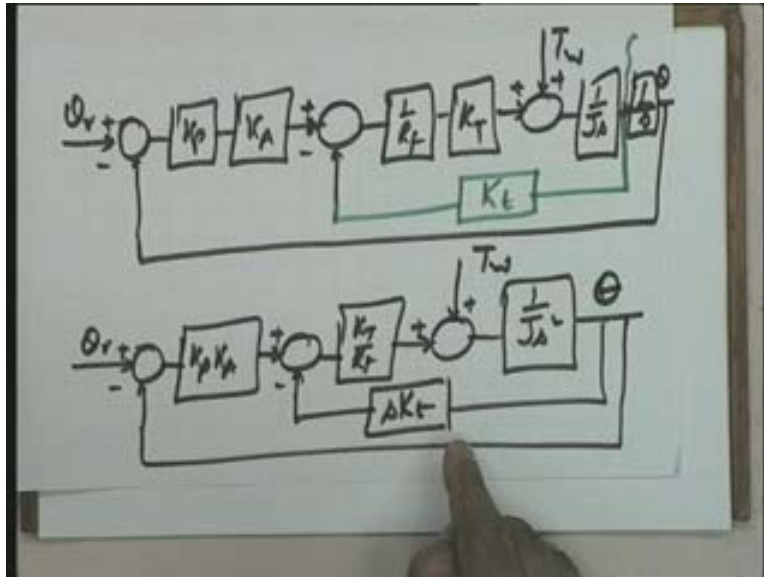
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Now you see what I was saying through the earlier slide is the following that at this particular point that is on the motor if you install a tachogenerator the speed ω a signal a voltage signal proportional to ω is available and that ω can be fed back, this is tachogenerator constant K_T , this ω can be fed back here and now this very diagram which I have drawn over here can be redrawn in the following way: θ_r plus minus here $K_p K_A$ a feedback signal at this point K_T by $R f T W$, 1 over $J s$ squared let me put it now, this is θ , is it not equivalent to $s K_T$ **putting here**? That is a signal which is proportional to the derivative of the output has now been injected.

You have avoided the signal which is the derivative of the command which you do not need really and your control scheme you do not need the information on as to how the derivative of the command signal will change, you only need the information on the derivative of the output so that you can predict what is going to happen in future. So $s K_T$ has been introduced over here (Refer Slide Time: 48:08) now this is equivalent to the derivative control but this $s K_T$ appears in the model, in an actual system no derivative is being taken because tachogenerator is a physical device which is giving you a voltage signal which is proportional to speed so the derivative action is inherent in the operation of the device and the problems of generating a spike because of the derivative or even the noise problems have been taken care of by introducing this type of scheme.

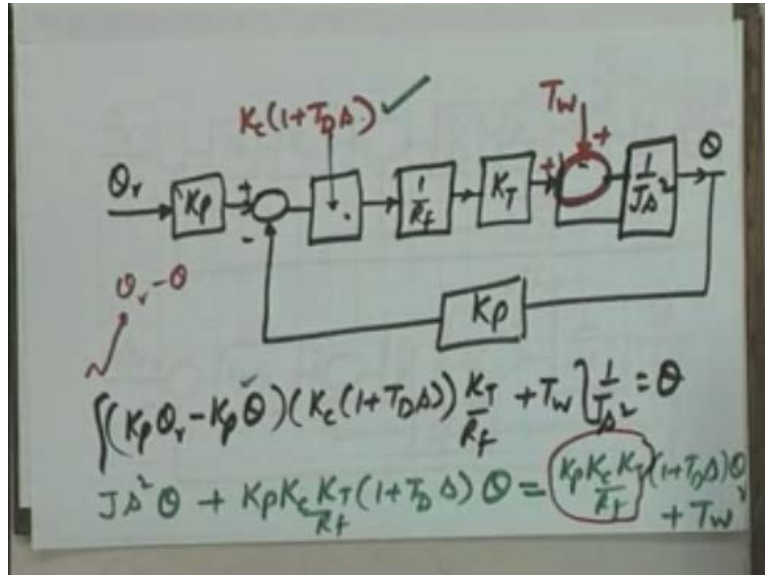
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But please note that this type of scheme is not applicable everywhere. If you have a process control system a temperature control system you do not have any signal which is θ dot inherently available in the system. So I said that if possible you go for this scheme but it does not mean that this scheme is replacing the original scheme. The original scheme cannot be replaced because in every application we may not be in happy situation of this type that a signal which is proportional to the derivative is available to you without actually taking the derivative of the signal through an electrical circuit.

[Conversation between Student and Professor – Not audible ((00:49:16 min))] The spike I said will appear in the earlier scheme, I will go back to the scheme, in this scheme it will not, in the earlier scheme the spike will appear in the following way:

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Suppose you give a step change in command (Refer Slide Time: 49:32) if you give a step change, the system was in steady state and θ_i is equal to θ_r let us say a steady state has appeared. Now θ_r minus θ is equal to 0. Now, if you give a step change in command θ does not change instantaneously but θ_r changes. So since θ_r changes sharply, instantaneously it means the derivative of that signal will be taken through this particular electrical circuit and this is injected into the plant. In this particular case this will never occur. And I am making a claim that the two schemes are important; the only thing is that, yes, wherever this alternative is possible do not go for the other alternative because obviously the noise filtering problems and the problems of the spike are automatically taken care of because the derivative signal is available inherently through the system component (Refer Slide Time: 50:23).

However, as I mentioned to you, you may not be in such a situation always particular in process; in motion control applications yes, but in process control applications we are not as lucky and we have to go for that derivative however we take a suitable filter even to filter out the effect of that spike or the change in the command signal we change..... Or one thing you see, why this thing, you see, this particular signal as I will hopefully explain in my discussion, even this particular circuit you can keep in the feedback loop so that at least this spike problem is taken care of; noise filter you will have to introduce in that situation as well, but at least the problem of the sudden change in the command can be taken care of if this particular controller is put in the feedback loop. These are some of the alternatives and these alternatives really will be discussed when we come to the design stage. And I think all what I had to say on the basic principles is over and my discussion next time is going to be on stability, thank you.