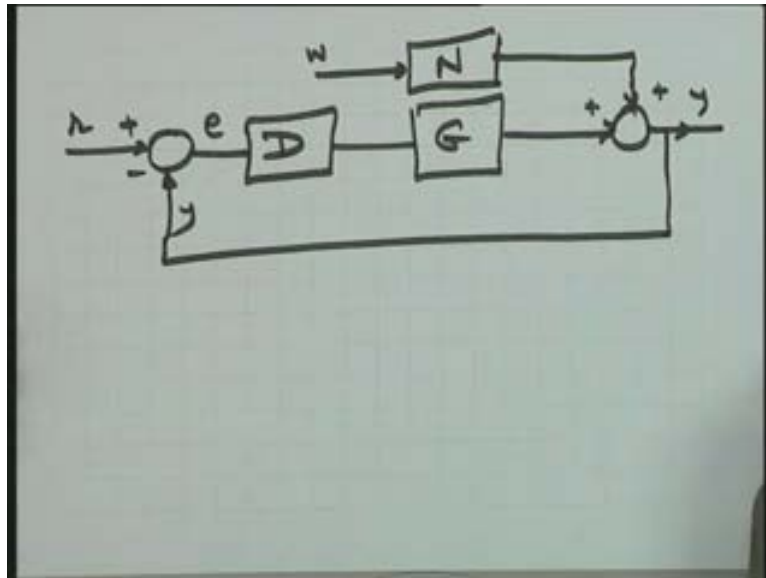


Control Engineering
Prof. Madan Gopal
Department of Electrical Engineering
Indian Institute of Technology, Delhi
Lecture - 21
Basic Principles of Feedback Control (Contd..)

Friends, let me get started with a basic feed back structure once again; r over here is the reference input, D is the controller, G is the plant here and the disturbance enters the plant through the transfer function N the disturbance is w ; the output is y and well, let me take this to be a unity-feedback system so that this is e the system error directly. If this (Refer Slide Time: 1:38) were through a sensor I would have taken this as e cap the actuating error.

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So here to explain the point, I have in mind today, this structure will be more helpful and therefore I am taking this y directly so that e is a system error. I need your attention at this particular point that I am using e here and not e cap. Now this d is a controller, this simplest controller we have been using so far is an amplifier. Various gains of the amplifiers we have used, in various industrial applications we have considered in this particular class. But last time, if you recall, we have seen that the amplifier may not realize all the objectives, the performance objectives are quite conflicting you see, you find that if I increase the amplifier again certain attributes of the system performance are improved while it affects negatively certain other important attributes of the system.

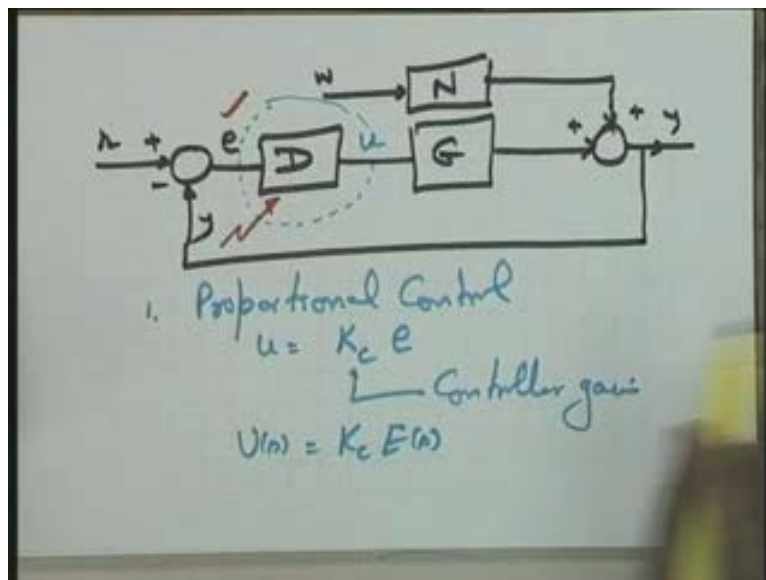
Since I have discussed that in details already I may not revise those concepts. But this is sure, this was our conclusion last time that only amplifier or only proportional control may not realize all the objectives because of the conflicts among them and I hinted at the possibilities that look in that case let us not constrain ourselves to the amplifier only, let us not restrict ourselves to the proportional control only. Though there are absolutely no limitations on what you design as far as D is concerned why do I say that there are no limitations? **You see**, in earlier days there were limitations of implementation because the implementation was done

through hardware primarily pneumatic and hydraulic and electrical in nature and realization of any function D through hardware was not possible and certain classes of functions were restricted from the point of view of realization through hardware available at that time. But now that condition does not exist that restriction does not exist the reason being that you give me any function D I can realize it through a digital computer. So now, it means, as far as your imagination on the controller D there are no restrictions, you can realize any to sort out the conflict, to make a trade-off between the conflicting specifications given on a specific control system.

However, what has been done in the past, if I give you that information, if I give you the details of what is being done today in industry then probably visualization in terms of better control as D will be possible will be more appropriate. So what I am going to do today is giving you the information about what has been successfully done in the past and what is mostly the present practice also but the future is in your hand and let us what type of new controllers you come up with.

So the types of thing which have been done in the past are the following: Only proportional control, well, this is the simplest possible. So naturally we will like to see the possibility whether only proportional control is possible or not. So it means, in the case of only proportional control what will happen, let us say that this is my control signal u , u will turn out to be equal to $K_c e$ where K_c I call as the controller gain. Instead of using as the amplifier gain let me use the general term, now, I call it the controller gain. So in terms of Laplace domain $U(s)$ becomes equal to $K_c E(s)$.

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Now, since this slide is going and I am using the other slide please keep in my mind that I will be giving you the inter-relationships between u the control signal and e the error signal. So, proportional control is one possibility. Look at the other possibilities. The other possibilities used in industry, the other alternatives used in industry are proportional-integral control called the PI control. What is PI control? In this particular case u the control signal is equal to $K_c e$ plus $1/T$ integral of $e dt$. You will find that the control signal consists of two components: one component is proportional to **the error** the system error; since I have

taken a unity-feedback structure one component is proportional to the error and the other component is proportional to the integral of the error. So in terms of Laplace domain I can write this as $U(s)$ is equal to $K_c (1 + \frac{1}{T_I s}) E(s)$ this becomes the transfer function of a PI controller which many a times depending upon the convenience I may be writing this as $(K_c + K_I \frac{1}{s}) E(s)$ I may be writing in this form where K_I you can call as integral gain, K_c in general is called as the controller gain, T_I is the integral time or reset time. These things will become very clear when we take up a particular dynamical situation integral or reset time is T_I . so we will see as to why this is needed and how do you know that we have to go for an integral control. **This, I think will become clear today itself.**

(Refer Slide Time: 7:30)

2 Proportional-integral control
 PI
 $u = K_c (e + \frac{1}{T_I} \int e dt)$
 $U(s) = K_c (1 + \frac{1}{T_I s}) E(s)$
 $= (K_c + \frac{K_I}{s}) E(s)$

Handwritten annotations: "integral" with an arrow pointing to $\frac{1}{T_I s}$, and "reset time" with an arrow pointing to $\frac{K_I}{s}$.

The third possibility is a PD control proportional derivative and I think you can now write the type of controller we are going to have. I am only going to introduce the variables we are going to use. K_c into let us say $(e + T_D \dot{e})$ let me take as the control signal. In terms of Laplace domain $U(s)$ is equal to $K_c + T_D s$ sorry $(1 + T_D s) E(s)$ will become the transfer function. This T_D is referred to as the derivative time or the rate time. **time or the rate time** Again depending upon the convenience of manipulation, no other reason, I may be writing this in the form $(K_c + K_D s) E(s)$ where K_D could be referred to as the derivative gain.

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PD Control
 $u = K_c (e + T_D \dot{e})$
 $U(s) = K_c (1 + T_D s) E(s)$
↳ derivation term
rate or derivative
 $= (K_c + K_D s) E(s)$

Now, if I club all the three modes I have the so-called PID controller very well known in industry off-the-shelf controller; if you do not know anything else about control you will at least try PID controller this is what is said in the industry. So PID controller is just the first trial if you get the success fine otherwise you think of better functions D so that your control objectives are realized. And in many situations your requirement is really met by the PID control and the function, let me write directly in the Laplace domain $U(s)$ is equal to $K_c I$ can write this as $(1 + 1 \text{ over } T_I s + T_D s) E(s)$ as the transfer function which sometimes I may be writing in the form $(K_c + K_I \text{ by } s + K_D s) E(s)$ as the transfer function of the PID control.

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PID Controller
 $U(s) = K_c (1 + \frac{1}{T_I s} + T_D s) E(s)$
 $= (K_c + \frac{K_I}{s} + K_D s) E(s)$

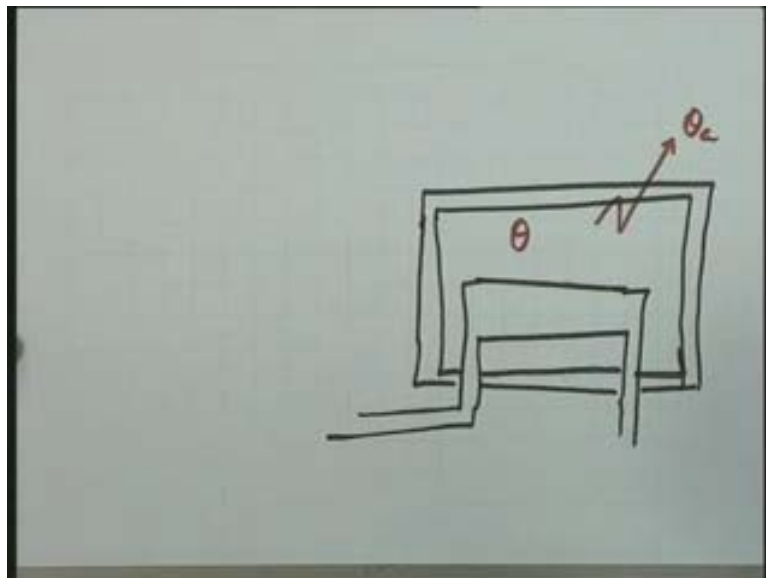
So keep in mind that why do I need these functions and many others which I am not listing here because the only amplifier gain is not able to satisfy all the performance requirements and therefore I have to deal with this situation of conflicting requirements through other

functions. Now one by one quickly through examples we will take up the attributes of these controls: Proportional, integral and derivative modes of control let me call it the attributes of these modes of control will be understood and from the basis on the basis of this understanding for a particular application we will be able to take a decision as to which modes are important to realize or to meet the performance specifications.

To first take up the proportional control I take an example which we have already discussed in the class, if not the same at least the similar example we have discussed in the class. it is a temperature control system, you will please note that; it is a chamber whose temperature is to be maintained at a certain prescribed level and θ_a is the disturbance variable, let me take θ_a as the temperature of the chamber, air is there in the chamber, I mean you can consider the application.

For example, some of the electronic equipment you test for failures under certain temperature condition you can say that it is your testing chamber. Depending upon the temperature you want to use for testing a particular product you have manufactured you will regulate the temperature of this particular testing chamber, put your components in the testing chamber for certain amount of time and see the failure and mode of that particular component. Maybe one of the applications I am refereeing to. But let us not worry about that. Let me see at present the control requirements. I want to control the temperature of this particular chamber at regulated value θ_r .

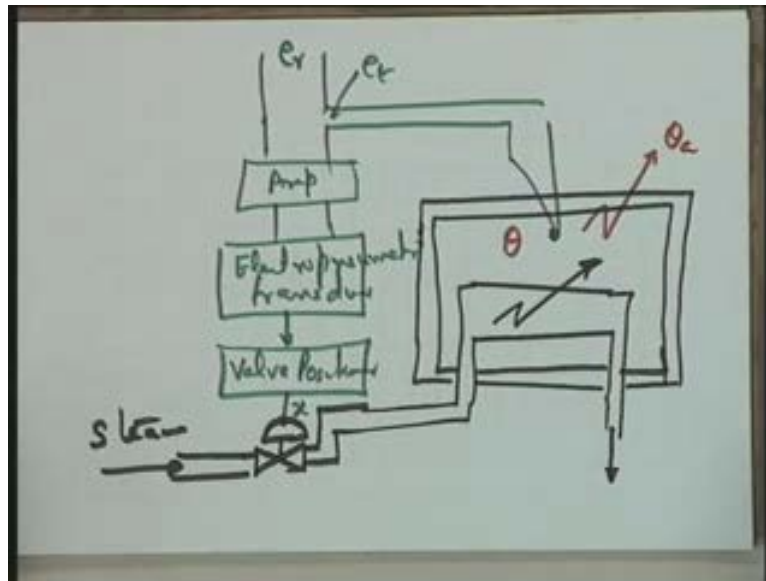
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So, it means θ_r is to be compared with θ_a I need a sensor, so a thermocouple, a known sensor I am taking, let me take that this is the output of the thermocouple I pick up from here, the reference e_r depending upon the type of θ_r you give, the type of command signal is given, e_r is set and this particular signal (Refer Slide Time: 12:29) will be subtracted, this e_t will be subtracted from e_r and let me say I have an amplifier here, the output of the amplifier I give, now all these things are known to you, I give to Electropneumatic transducer. You know that the Electropneumatic transducer will give me a pressure signal. this pressure signal now I give to a valve positioner which is going to position the stem of the valve if it is a translational valve, this valve positioner it means is going to give me x this x is going to

control the valve and let me say that let me take up the situation here this way this is the valve and here is the steam and therefore this is the heat exchanger and the heat is being given to the air of the chamber through this particular heat exchanger the condensate this way (Refer Slide Time: 13:39).

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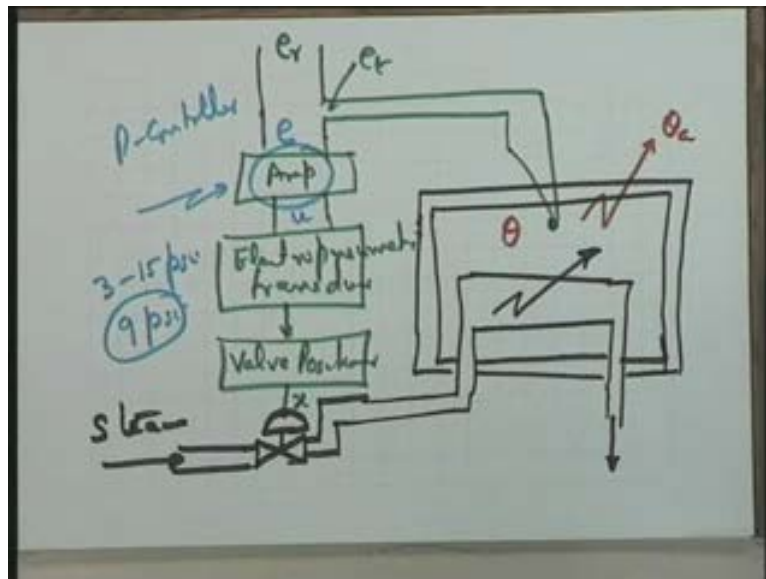
I hope the diagram is clear because we have dealt with many of such systems and now the components and the nature of their model should be immediately clear to you. The theta r e r proportional to theta r the thermocouple here, the voltage the error voltage, the amplifier voltage (Refer Slide Time: 13:56) is going to Electropneumatic transducer to the valve positioner positioning the valve to regulate the steam and hence to regulate temperature theta.

Now I want to see the effect of only proportional control. That is I want only an amplifier here which can control the variable K_c . I want to see how the proportional control is going to affect the various performance specifications I have already given to you. You will help me now please; theta theta a let me take as the disturbance variables. Tell me what do you want? When theta is equal to theta r, I **need your attention and your help now**, what is the value of this error e please at that particular point. When e is equal to theta r proportionally e r will be equal to e t in that case this error will be equal to zero so it means the amplifier output now you can call it U the control signal because now I am calling this as a proportional controller, P controller I am calling this amplifier so it means U is the control signal and naturally this U will be also equal to 0.

Now please note, to maintain theta at theta r you will require the heat exchange so when this U is equal to 0 how do you get this heat exchange, please? This heat exchange you get by steady state or equilibrium setting of the valve positioner and the Electropneumatic transducer. Recall that the Electropneumatic transducer has a variation of 3 to 5 psi. Suppose the hardware adjustment in the electropneumatic transducer is done to let us say a value 9 psi just a tentative value I am taking and this pressure opens this particular valve the just amount maintains theta at theta r when the equilibrium state is there. So it means, under the equilibrium state the error signal is zero, the control signal is zero and the prior setting of the Electropneumatic transducer in the valve positioner maintains the heat balance and that

setting also takes care of the heat loss to the environment but at steady state, assuming that the environment temperature were constant.

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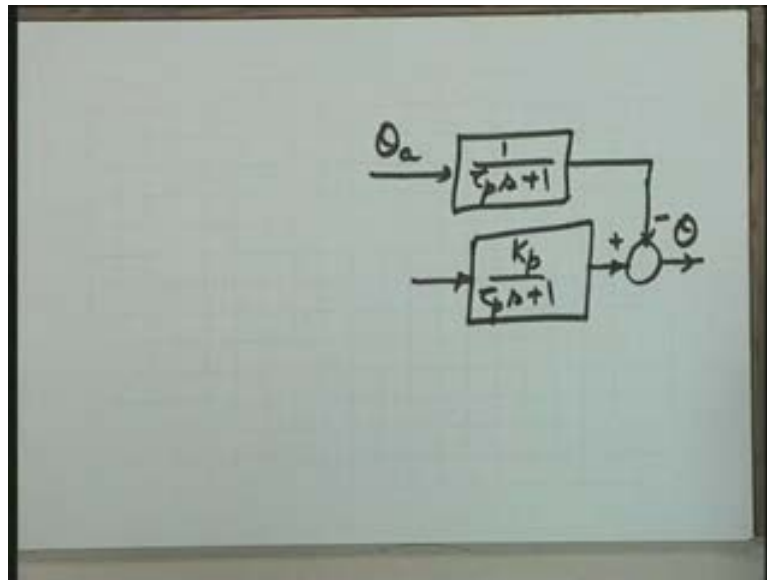
Now the question is this that when the disturbances occur? How can the disturbances occur? Please see that the disturbances primarily can occur in this particular case through the change in the environmental temperature or if you call it, it is not a disturbance or **your command** change in your command signal. These are the two things which can create perturbation from the equilibrium position. Perturbation from the equilibrium position can be caused by these two things: Either we change in the command signal or the change in the environmental temperature.

So, accordingly an error will be created, a control signal will be created and that control signal has to readjust the steam flow rate and hence readjust the heat flow rate so that the equilibrium is restored. This is what I need. And yes please **[Conversation between Student and Professor – Not audible ((00:17:20 min))]** θ_a is the environment ambient temperature. I am saying, θ_a is the ambient temperature and this is actually θ_a bar plus θ_a , I am taking this θ_a as the deviation of the ambient value from the value which was taken when the setting of the transducer and valve positioner was done. After all when you were doing this setting there was a heat loss then also but then that heat loss has been taken care of by that setting but these setting now takes care of the heat loss which was there only with respect to the ambient temperature and then only. So, any variation in the ambient temperature will create imbalance at this particular heat interface and therefore it will require either reduction or increase of the steam flow rate and hence the heat. **Is it okay please? Fine** then.

Now I am not going to the physical equations that is why I have spent so much of time on modeling you see. Now I want you to appreciate the modeling immediately. The chamber; can I take the chamber as a first-order system. So I take the chamber over here as a first-order system let me say $K_p / (\tau_p s + 1)$ is the mathematical model; heat steam flow rate over here and θ is here, this K_p and τ_p you can determine from physical equations or you can determine experimentally. Let us say these valves are determined by me either way.

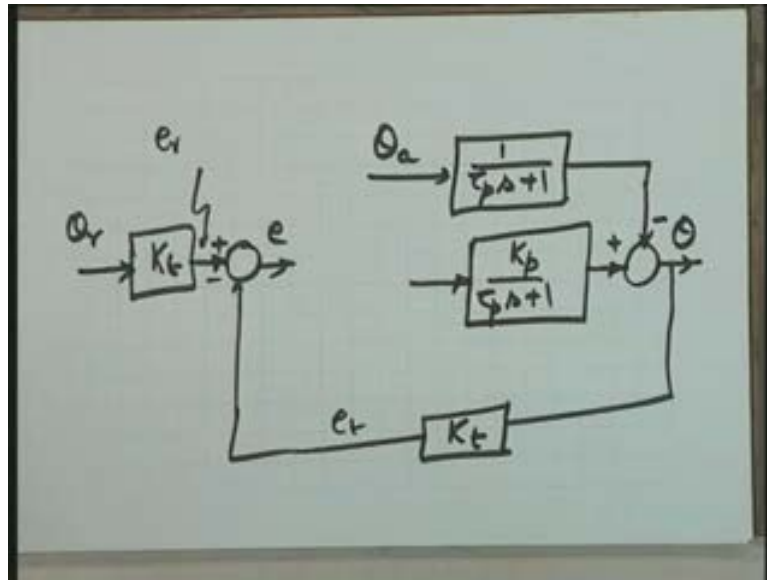
Disturbance affecting the output theta: Let me say this is plus (Refer Slide Time: 18:59) this is minus I am taking, the disturbance is effecting this theta a is the..... now this theta I am taking the perturbation, please note that. Whenever I take a dynamical model it is always with respect to the steady state. The total ambient temperature is not theta a; theta a is the perturbation of the ambient temperature from the point at which the equilibrium point was set. So this is this and let me say that K I am taking as 1, tau of course will remain the same 1 over tau p s plus 1 will mean thereby, you see, this is clear as to why did I take this K is equal to 1 it simply means that theta a I am assuming is directly affecting theta at steady state at the equal ratio. You can determine experimentally if there is a difference then suitable constant will come there.

(Refer Slide Time: 19:51)



Now this theta is coming from the sensor and let me say K t is the thermocouple constant so e t is this particular voltage, a comparison theta r is your command, as we have discussed many times I will be taking K t as the scale factor it is not a physical system at all, it is a scale factor so that I get the reference value e r here; the two are compared and an error signal e is generated.

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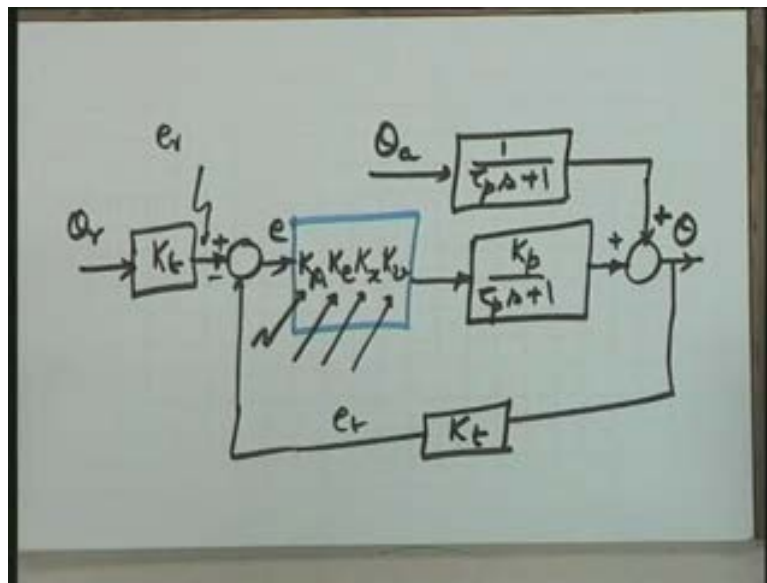


Now, for want of space on my slide I put this particular block in a single block but to show the components you can really separate them out into separate blocks (Refer Slide Time: 20:40). My single block consists of a constant K_A the amplifier gain, a constant K_e the gain of the Electropneumatic transducer, a constant K_x a gain of the valve positioner, a constant K_v the gain of the control valve. So you put four different blocks in your diagram and you get the complete functional block diagram of the system.

You will please note K_A has got the units voltage on one side and voltage on the other, K_e - voltage on one side pressure on the other, K_x - pressure on one side displacement on the other, K_v - displacement on one side and the steam flow rate on the other side. I **hope this is clear**. These are the four basic units; I am taking all these basic units to be zero-order systems. I am neglecting the dynamics. I need your attention at this particular point because I will **like you I will** make you recall this statement. I am making this statement which I will make.

You recall later that all these components are actually dynamical components. A zero-order system effectively means that the system will respond instantaneously and you know that no physical system in nature can respond instantaneously so it means all of them are dynamical systems but the time constants of these systems are negligibly small compared to the plant time constant τ_p and that is why that is being neglected and all these constants or these models are being taken as a zero order model.

(Refer Slide Time: 22:23)



Under this assumption only this is the block diagram of the temperature control system under consideration. And my objective now is to study the effect of the gain K_A on this particular system performance; the steady state, the transient, the sensitivity, the disturbance reduction and everything we have talked about.

Any question on the basic system and its block diagram please?

[Conversation between Student and Professor – Not audible ((00:22:50 min))] Well, you see that, look, it depends, you were right, I am assume I am in this particular taking minus sign over here, θ_a increase increase in θ_a over here will result in increase or decrease what is intuitively, intuitive feeling? Suppose this temperature (Refer Slide Time: 23:18) increases on this side it means the heat outflow will decrease and therefore this temperature will increase, fine. In that particular case let me take here; this to be a plus sign the effect of θ_a on the disturbance θ_a .

Now let me take the describing equation. I am writing this way: $K_t \theta_r s - K_t \theta s$ this is the error this gets multiplied with $K_A K_e K_x K_v$ this becomes the manipulated variable now this I am multiplying by K_p this total thing (Refer Slide Time: 24:05) multiplied by K_p over $\tau_p s + 1$ plus I am taking 1 over $\tau_p s + 1$ this is θ_a , this quantity is here plus $\theta_a(s)$, please help me, I am making an error I fear. $\theta_r K_t K_t$ into θ and error this is the control signal here, this control signal multiplied by this plus θ_a into this will give me θ that is right equal to θ that is right please, please see this point the flow: $K_r \theta_a K_t \theta_r - K_t \theta$ into $K_A K_e K_x K_v$ into $K_p \tau_p s + 1$ you have reached this point (Refer Slide Time: 25:04).

Now you add this signal to this: $\theta_a (1 / (\tau_p s + 1))$ is equal to θ . I hope this equation is alright. Now let me manipulate this equation. Manipulating this equation I get $\tau_p s + 1$ into θs plus $K_t K_A K_e K_x K_v K_p$ into θs is equal to..... yes it is equal to what? This term goes, this will become $K_t K_A K_e K_x K_v K_p$ into θ_r plus θ_a ; please you have to just exercise little effort and see whether..... if I make a mistake you have to really correct me otherwise maybe our conclusions also will be affected from there.

(Refer Slide Time: 26:06)

$$\{K_t \Theta_r \omega - K_t \Theta(s)\} K_A K_e K_x K_v K_p \frac{K_p}{\tau_{ps} + 1} + \frac{1}{\tau_{ps} + 1} \Theta_c(s) = \Theta$$

$$(\tau_{ps} + 1) \Theta(s) + K_t K_A K_e K_x K_v K_p \Theta(s) = \{K_t K_A K_e K_x K_v K_p\} \Theta_r + \Theta_a$$

From $\tau_{ps} + 1$ I have multiplied this side so $K_t \Theta_r \omega - K_t \Theta(s)$ into Θ_r looks alright. Now this term I have taken on this side $K_t K_A K_e K_x K_v K_p$ into $\Theta(s)$ on this side plus Θ_a **fine I think it is okay, please**. Now this total thing let me club into a single constant and call it K . You please note that the amplifier gain K_A will directly change here assuming other constants are fixed. Assuming other parameters of the system are fixed the amplifier gain K_A is directly going to affect K so K I call as the loop gain. So in terms of K I can now write the equation as $\tau_{ps} + 1$ plus $K \Theta(s)$ is equal to $K \Theta_r(s)$ plus $\Theta_a(s)$ where your K_A the so-called proportional control is directly changing the value of K . You can call it if you do not mind as if K is a proportional control constant; it is an equivalent statement.

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$$(\tau_{ps} + 1 + K) \Theta(s) = K \Theta_r \omega + \Theta_a(s)$$

So in this particular case $\Theta(s)$ is equal to K over $\tau_{ps} + 1 + K$ plus $K \Theta_r(s)$ plus 1 over $\tau_{ps} + 1 + K$ plus $\Theta_a(s)$. Please see, if there has been little gap in the derivation, you

have not got my derivation; you leave that gap there because you can fill it up later. Please concentrate on this final equation. From the block diagram I have got an expression between the output theta and the command signal theta r, between the output theta and the disturbance signal theta a. you concentrate on this expression please.

One immediate thing which is noticeable from this point is the following that when I rewrite this expression in the form: K over $1 + K$ 1 over $\tau s + 1$ into $\theta_r(s)$ plus 1 over $1 + K$ into 1 over $\tau s + 1$ **let me take this slide little this side so that it is visible to you** into $\theta_a(s)$ that is τ_p now please see τ is equal to τ_p over $1 + K$. You will please note that τ_p was your plant time constant. That is the time constant of the system without feedback. What is the effect of the feedback? The obvious effect of the feedback in this particular case is to reduce the time constant because K is greater than 0. So you will note that by feedback your system has become faster. The lesser the time constant the better is the speed of response. That is quicker will be the decay of the transients and the system **will soon** will quickly reach to the steady state after the disturbances have died down. So it means I see that the time constant of the feedback system I call it τ has been improved as been reduced so that the system has become faster. One aspect I have noted.

(Refer Slide Time: 29:30)

The image shows a hand holding a yellow marker pointing to a whiteboard with the following handwritten equations:

$$(\tau_p s + 1 + K) \Theta(s) = K \Theta_r(s) + \Theta_a(s)$$

$$\Theta(s) = \frac{K}{\tau_p s + 1 + K} \Theta_r(s) + \frac{1}{\tau_p s + 1 + K} \Theta_a(s)$$

$$= \frac{K}{1 + K} \frac{1}{\tau_p s + 1} \Theta_r(s) + \frac{1}{1 + K} \frac{1}{\tau_p s + 1} \Theta_a(s)$$

$$\tau = \frac{\tau_p}{1 + K}$$

The other aspect, I want you to give me the steady state error. Please help me. First let us consider theta r only. I can apply superposition is being linear system. Let me take theta r equal to 1 over s a unit step input, in that particular case theta s becomes equal to K over $1 + K$ 1 over $\tau s + 1$ into 1 over s . What is the steady state value? Theta ss limit s tends to 0 s theta(s) please help me, what is the steady state value; limit s tends to 0 s theta(s) K over $1 + K$. Let me write the other expression also so that then I can discuss fully whatever is happening.

Let me take now the disturbance input theta a is equal to 1 over s . now what is theta ss is equal to? I leave this as a simple exercise for you minus 1 over $1 + K$ y minus (Refer Slide Time: 30:36), well, it will be plus 1 over $1 + K$. You can just see 1 over $1 + K$ is going to be your theta ss in this particular case.

(Refer Slide Time: 30:43)

$$\begin{aligned}
 1. \quad \theta_r &= \frac{1}{s} \\
 \theta(s) &= \frac{K}{1+K} \left(\frac{1}{s} \right) \frac{1}{s} \\
 \theta_M &= \lim_{s \rightarrow 0} s \theta(s) = \frac{K}{1+K} \\
 2. \quad \theta_c &= \frac{1}{s} \\
 \theta_M &= \frac{1}{1+K}
 \end{aligned}$$

Now, what is the effect of this on the steady state error?

You see, θ_r is the command signal so it means you want θ_{ss} to be equal to 1. You will please note, as the gain K increases θ_{ss} approaches 1 and therefore e_{ss} is equal to 0. As far as the command signal is concerned the steady state error to a command signal which is of step nature which is a constant command signal reduces as the gain of the system increases, one point. And second, what do you want in terms of disturbance. In terms of disturbance you want θ_{ss} to go to 0 so that the effect of the disturbance is filtered out and you find that as the value of K increases your θ_{ss} in response to disturbance goes to zero which is a desirable feature.

(Refer Slide Time: 31:40)

$$\begin{aligned}
 \theta(s) &= \frac{K}{1+K} \left(\frac{1}{s} \right) \frac{1}{s} \\
 \theta_M &= \lim_{s \rightarrow 0} s \theta(s) = \frac{K}{1+K} \\
 2. \quad \theta_c &= \frac{1}{s} \\
 \theta_M &= \frac{1}{1+K}
 \end{aligned}$$

$e_M = 0$ ↗
 $\theta_M \xrightarrow{\Delta t} 0$

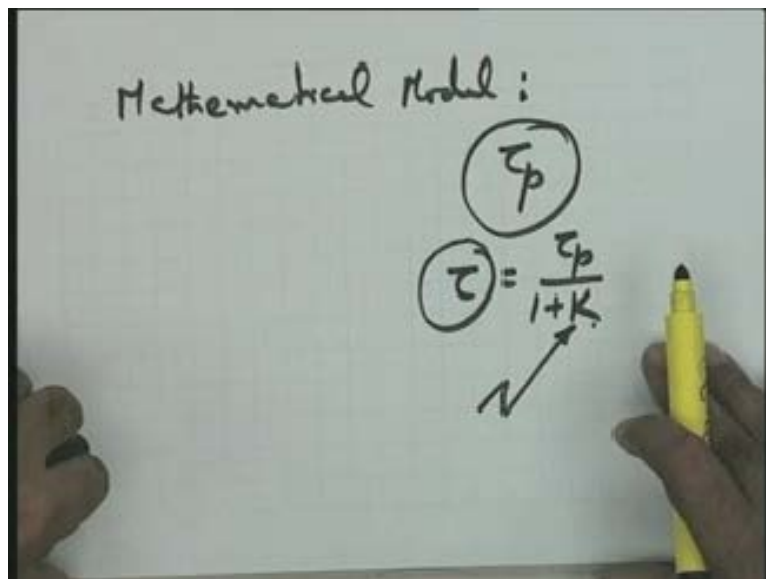
Now you put all the three things together: the steady state performance of this system improves with the high value of K and the transient performance of the system also improves with high value of K . And if you theoretically make K is equal to infinity you have an

instantaneously operating system because τ becomes equal to 0 and you have zero-steady state error because steady state error becomes e_{ss} becomes equal to 0. So it means you have to strike a suitable balance if there is any constrain if there is no constrain you make K is equal to infinity and realize your objectives but if there are constrains then you have to strike a suitable balance.

Now let us see what are the constrains which do not allow us to take large values of K . this I hope is clear that larger the value of K better is the transient performance as seen by the mathematical model better is the transient performance as seen by the mathematical model we have derived and better is the steady state accuracy.

Now the problem.... that is why I am using the word. The problem actually is with the mathematical model we have derived. It needs attention. You will please recall my statement and now I make you recall. I have taken zero-order models for the amplifier, the Electropneumatic transducer and the valve positioner. Why did I take zero-order model is because they assume that the time constants of all these components are negligible compared to the time constant τ_p the time constant of the plant. but when you are operating in the feedback mode, please see, your time constant now is not τ_p , your time constant in the feedback mode becomes τ is equal to τ_p over $1 + K$ so it means what happens; as you increase the value of K the value of τ reduces and hence your assumption which you have made about neglecting the time constants of various component on the system is no more valid. Your assumption becomes invalid **you see** because in that particular case time constants of the transducer, the valve positioner and the amplifier will no more be negligible with respect to the time constant of the feedback system.

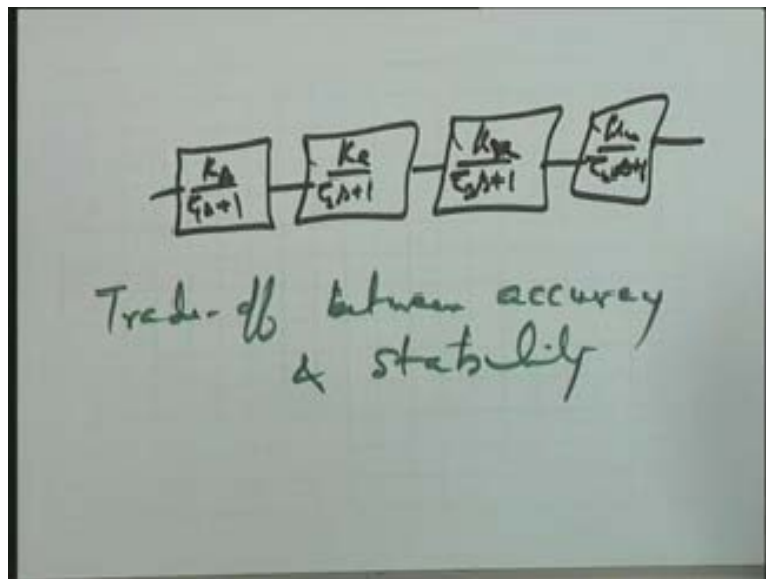
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What will happen is under that situation when you have a large value of K your system may look like this: $K A$ over $\tau_1 s$ plus $1 K e$ over $\tau_2 s$ plus $1 K x$ over $\tau_3 s$ plus $1 K v$ over $\tau_4 s$ plus 1 and these will be the four components now which will be connected in cascade to the plant when K is very large. So what is happening now? Your first-order system has become a fifth order system. At this juncture, probably I have not equipped you with the tools of stability analysis, **maybe within a couple of lectures you will be equipped with the tools**

and you will find that this particular system which is of fifth order; rather, let me say that any system which is of third or higher order will become unstable for large values of K; this statement you consume today without proof but you will appreciate it soon; any system of order higher than 3 will become unstable for large values of K. So it means actually, when we are studying the time response of the system we are so happy that the time constant is being reduced and the speed of response is being improved but you see, that is our illusion because of the approximation in the model when you go to the physical system what to talk of reducing the time constant as soon as you inject a higher gain into the system the system goes unstable and the system naturally will not perform the way you like. So it means there is a conflict between the improvement in the performance of the system and the stability requirements. So it means, actually the word I had used earlier, there is a trade-off required between accuracy requirement that is the steady state accuracy you have seen and the stability. As you improve the accuracy this stability is lost and suitable trade-off is to be settled to see that both the requirements are met with.

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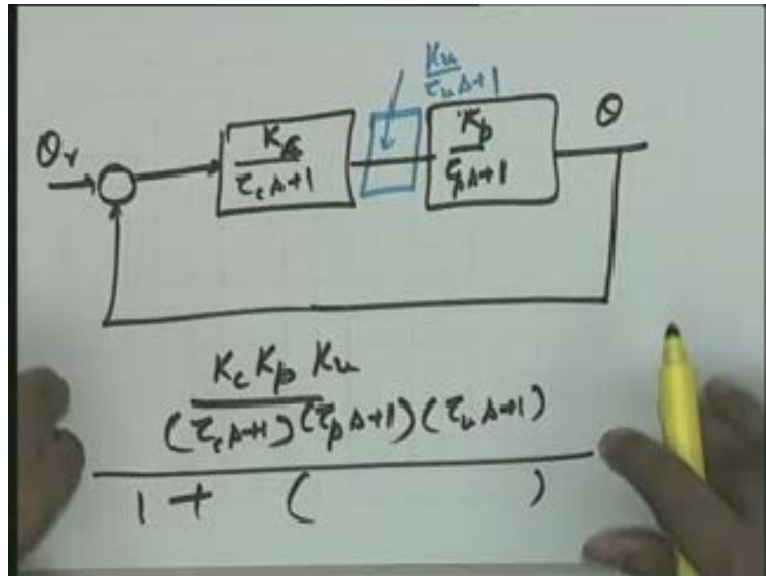


And one more point which I again I think you will recall. you see, larger value of K what will do; with the larger value of K large voltage will be generated by the amplifier, large signal will be generated by the transducer by the positioner and so on so all these devices may be driven to saturation if it exceeds their normal operation region. So it means that again your model is not able to predict but in effect your system has become a non-linear system once you have taken a very large value of K and hence all predictions become invalid. in non-linear system, though we may not be discuss the non-linear system in this class, a non-linear system can enter into an oscillatory mode, can enter into instability and various phenomena are possible and therefore the K large value will really create a conflict and suitable design which is required.

[[Conversation between student and professor]] [37:23....reduced by the same patterns since they are also a part of the loop] [they are also the part of the loop.....let us study this, let me take one time constant.]

Let me say that I have taken this to be equal to K_c over $\tau_c s + 1$ where K_c is the gain and here I have taken K_p over $\tau_p s + 1$. Let me take this situation, this is my output θ_r and this θ_r (Refer Slide Time: 38:02). So in this particular case you find $K_c K_p$ over $\tau_c s + 1$ into $\tau_p s + 1$. **Now let me take actually** Since I have given a statement about a third-order system you will not mind if I introduce one more component K_v over $\tau_v s + 1$ so one more component I am introducing I am making it a third-order system into K_v over $\tau_v s + 1$.

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Now this divided by $1 +$ this entire expression, please see this point gives me the closed-loop transfer function as $K_c K_p K_v$ over $\tau_c s + 1 \tau_p s + 1 \tau_v s + 1$ plus what plus $K_c K_p K_v$ please see, this is your overall expression $\theta_r(s)$ over $\theta_r(s)$ you can see this point. Now the overall expression if you recall for the system without this particular dynamics was $\theta(s)$ over $\theta_r(s)$ equal to some constant K over $\tau s + 1$ plus K . This was the system without the dynamics taken into consideration.

What is characteristic equation in this case please?

The characteristic equation in this case is $\tau_p s + 1 + K = 0$. So what is the root of the characteristic equation s is equal to $-1 + K$ over $K \tau_p$. The root of the characteristic equation as you see is the pole of the closed-loop system and from your earlier knowledge, I said that stability we have not discussed but from your earlier knowledge of system stability you know that if the poles of a system are in the left half plane the system will be stable, use your intuitive knowledge. And in this particular case you see (Refer Slide Time: 40:11), for all values of K s will be in the left half plane and hence if the time constants are really negligible the system is always stable.

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$$\frac{O(s)}{I(s)} = \frac{K_e K_p K_u}{(\tau_e s + 1)(\tau_p s + 1)(\tau_u s + 1) + K_e K_p K_u}$$
$$\frac{O(s)}{I(s)} = \frac{K}{\tau_p s + 1 + K}$$
$$\tau_p s + 1 + K = 0 ; s = -\frac{1+K}{\tau_p}$$

Now look at the consideration of the time constants. by considerations of the time constant you will find that your characteristic equation becomes a 1 s 3 plus a 2 s 2 plus a 3 s plus a 4 equal to 0 this is your characteristic equation it is a fourth-order system (Refer Slide Time: 40:40). It is a fourth-order system now and this is the characteristic equation, the parameters of function of the gain K. Now since it is a **fourth** sorry third-order system, this is a third-order system (Refer Slide Time: 40:52) and hence I have four terms over here.

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$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

Now this has three poles in the s plane and this is what I left as an exercise to you: if you take large values of K some of the poles of this particular equation are driven to the right hand side which was not possible in the earlier case. Large values of K will drive some of the poles to the left hand side and hence this system will become unstable. Now what is the effect on the time constants? Let us see the effect on the time constants. Yes, if I divide this total expression by this accordingly the time constants of this will get reduced, will change but the

system with **these three additional** the two additional dynamic elements have converted the first-order characteristic equation into a third-order characteristic equation and hence have introduced the possibility of instability. So with larger dynamics the possibility of instability has been introduced, and as I said, in general, for large values of K this particular characteristic equation will definitely drive some of the poles to the right of the s plane and hence make the system unstable.

So it means, what is the effect on the time constant when the total dynamics is being taken becomes irrelevant, becomes less important because the system has gone unstable so why to study those basic sector when the higher order system has gone unstable. And once the system has gone unstable there is no point in studying the time constants of various systems because the interaction of various components is leading to instability which is not acceptable to us.

[Conversation between Student and Professor – Not audible ((00:42:51 min))]

Sir, constants also reduce..... no, no well, if the time constants get reduced, it means as far as the basic component is concerned it has the time constant which is different when the feedback was not available. But all these components now are being used under the feedback. So it means the feedback which otherwise..... So you remove the feedback, there are time constants it is a third-order system even then but the system is stable. You remove the feedback link; the system in which the various components have got the dynamics included is a stable system. You can see this from here (Refer Slide Time: 43:33). If these are the time constants; and if you see the poles of these individual factors these poles are in the left half plane. So it means including the dynamics of the time constants also the open-loop system is stable. But what I am saying is that when you close the loop then because of the interaction of these various components the very feedback itself which is helping you improve the robustness which is helping you improve the sensitivity is creating stability problems. So it means a feedback system is always prone to instability while without the feedback link the system was a stable system.

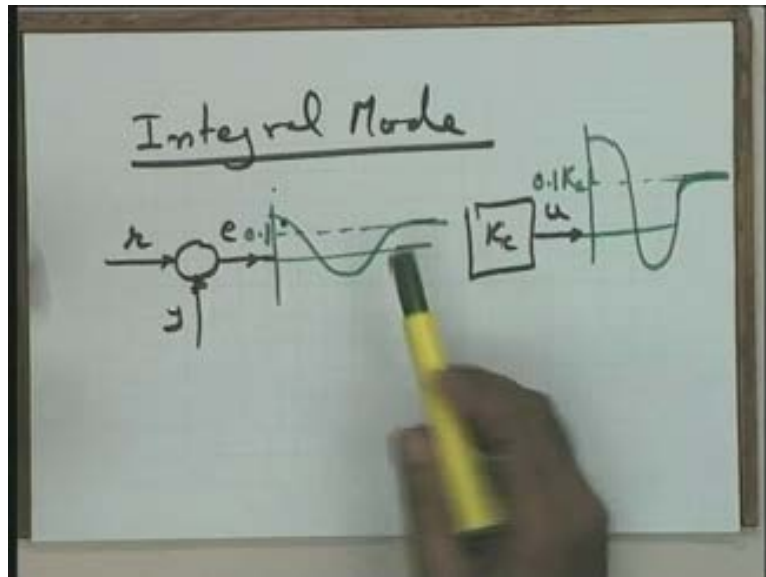
I understand his question. His question is about the dynamical modes. So, if you take those dynamical modes individually in the open-loop mode there is no problem as far as the stability is concerned because all those dynamical modes are giving you open-loop poles in the left half plane. What I am worried about is that when the dynamics of these modes get included in the feedback loop the system becomes unstable which otherwise from your earlier model was not visible, the instability phenomena was not there. So it means feedback which is helping us improve performance is the very cause of problems because it leads us to instability situations.

I have not calculated the time constants because the important point was that when the dynamics gets included the interaction of various dynamical elements is introducing the instability.

The next I take, at least let me introduce the concept looking at the availability of time and then I will take up this mode next time. I intend to complete the discussion next time on the PID controls, Integral Mode. Now you see that, very interesting feature will come out, I am sure you are going to tell me as to how the integral mode will help us. Let us say this is r and this is y the output and here is an error e , **I use an let me take** let me take an error signal typically, let us say that this is the value (Refer Slide Time: 45:57) let me fictitiously take as 0.1 is the value of this error signal at steady state. Now this error signal is multiplied by K_c

to give you u and this is the control signal now and it is an amplification so let me say that this is the control signal and at steady state it is $0.1 K c$. **I need your attention at this point please.**

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By very basic nature of proportional control the error has to exist in the system because if the error becomes equal to zero what will happen; if the error becomes equal to zero in that particular system in that case the energy is not being supplied to the system and $0.1 K c$ is the energy required to sustain the system at the new equilibrium; $0.1 K c$ let me say is the energy required to sustain the system at the new equilibrium and that energy is coming from the error e and let us say that the steady state value of the error is 0.1 . **so it means from** Your original equilibrium was obtained by adjusting the Electropneumatic transducer.

Now the new equilibrium has come. The new equilibrium now requires a different amount of energy and you are not going back to the original system to retune the Electropneumatic transducer to get the new equilibrium value of the heat, I hope you getting my point so how will the new equilibrium come. The new equilibrium can come only from additional energy coming and that additional energy will come only if the error signal is present. So it means the error signal is must. if the error signal is equal to zero it means you have come back to the original equilibrium position which you are not able to because the system is driven to the new situation.

Now what is happening in the integral control?

Please see, in the integral control I achieve this objective this way: e , let us say that the error is truly zero but if this error is going through an integral controller K_I by c in that particular case please see, please see in this particular case what is going to happen the integral of this, the area under this curve and let us say this is steady state value, the area under this curve and finally it is going to **steady**. And let me say that I adjust the K_I value in such a way that at steady state I have the value $0.1 K c$ which is just the energy required to maintain the system at the equilibrium value. So it means, in this particular case just the energy required to maintain the value at the equilibrium point was coming through the error signal. In this case that particular signal that particular energy has come through integration. So it means when

the error was reducing you have used that knowledge to create the signal which will stay for all time in spite of the error going to zero. So it means the integral control gives you the possibility of reducing the error to zero and you will see next time that by an appropriate integral control it is always possible to reduce the steady state error to zero for the type of system we have considered today, thank you.