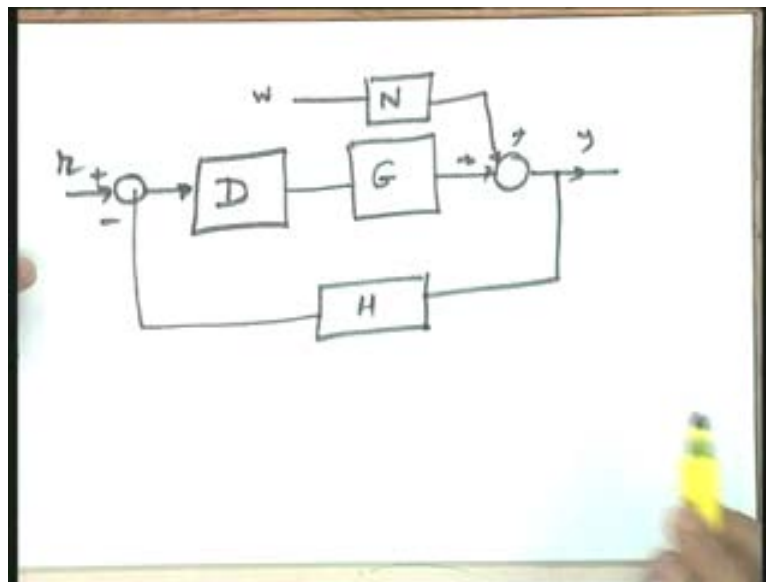


**Control Engineering**  
**Prof. Madan Gopal**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Delhi**  
**Lecture - 20**  
**Basic Principles of Feedback Control (Contd...)**

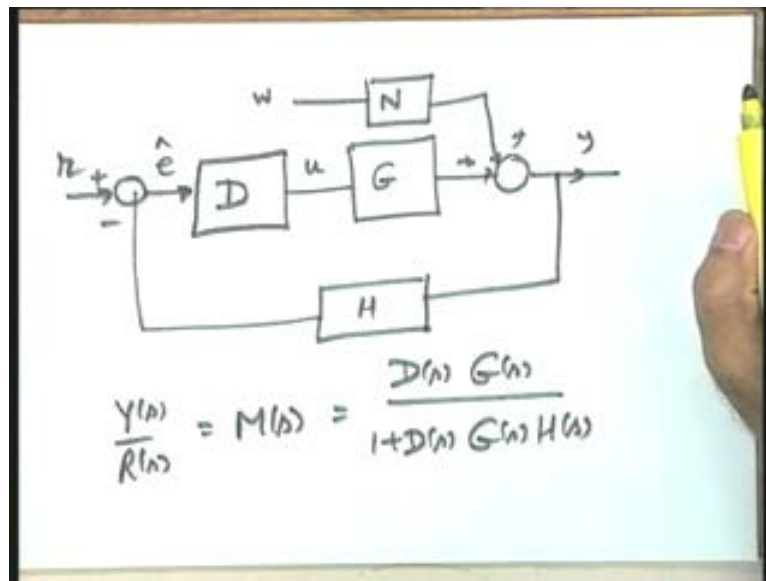
Now friends, basic principles of feedback control were under discussion. We had been discussing last time about the sensitivity and robustness issues. Let me reconsider the basic block diagram. this is the controller (Refer Slide Time: 1:13) with transfer function  $D$ , here I have the plant with transfer function  $G$ , the disturbance to the plant has been modeled through a transfer function  $N$  and here is a disturbance input  $w$ . The output is  $y$ , this output  $y$  is fed back through the sensor with transfer function  $H$ , here is an error detector and my reference input is  $r$  over here.

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This, we have taken as the error signal  $e$  and this has been taken as the control signal  $u$ . This is the basic block diagram with respect to which the basic characteristics of feedback control systems are being discussed. the transfer function between  $y$  and  $r$  is given as  $Y(s)$  over  $R(s)$ , we have designated it by  $M(s)$  is equal to as you know  $D(s) G(s)$  over  $1 + D(s) G(s) H(s)$ . This is your overall transfer function of the closed-loop system.

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The basic result we derived last time was that the sensitivity of the overall transfer function  $M$  with respect to the plant transfer function  $G$ . Please note that I am assuming that there are certain parameters of the plant which are subject to variations and therefore I am studying the sensitivity of the overall transfer function with respect to plant which is equivalent to studying the sensitivity of the system response to variations in the parameters of the plant, it is just equivalent. Instead of the response here I am taking the overall closed-loop transfer function and instead of the specific parameters  $\theta$  I am taking the plant transfer function to which the parameters  $\theta$  belong. This, if you recall was derived to be equal to  $1$  over  $1$  plus  $D(s)G(s)H(s)$ ; to make it very clear let me put it here that this (Refer Slide Time: 3:34) is actually  $G(\theta, s)$  meaning thereby that  $\theta$  the parameter under variation belongs to the plant  $G$ .

One more transfer function was sensitivity function we derived that was  $S$  of  $M$  with respect to  $H$  that was equal to minus  $D(s)G(s)H(s)$  divided by  $1$  plus  $D(s)G(s)H(s)$ . This is the sensitivity of the overall function with respect to the  $H$  that is the sensor transfer function.

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The image shows a whiteboard with two equations written in black and blue ink. The first equation is  $S_G^M = \frac{1}{1 + D(s)G(s)H(s)}$ . The second equation is  $S_H^M = \frac{-D(s)G(s)H(s)}{1 + D(s)G(s)H(s)}$ . A blue arrow points from the term  $G(s)$  in the denominator of the second equation to the label  $G(s)$  written in blue ink above it. A hand holding a yellow marker is visible on the right side of the whiteboard.

Now please see what were the conclusions. Last time we came to the following conclusions: number one, if I increase the loop gain here is the loop transfer function (Refer Slide Time: 4:17) please see loop transfer function; come back to this particular slide (Refer Slide Time: 4:23) you will find that this is the loop and if I multiply all these blocks D G and H I get the loop transfer function D G H and if I increase this loop gain in that particular case I find that sensitivity of M with respect to G reduces.

Now as a comparative index it will be interesting if you keep the sensitivity of M with respect to G is equal to 1 also into focus and this is the sensitivity of the open-loop system. When you break the loop then the sensitivity of the open-loop system is unity. That is, the open-loop system is highly sensitive to parameter changes in the plant. So if I increase the loop gain you please note that sensitivity reduces and with respect to the sensitivity this is one of the important attributes of feedback control. Look at the sensor, please see, if I increase the loop gain (Refer Slide Time: 5:28) the sensitivity of M with respect to H tends to 1 which indicates that the closed-loop system is highly sensitive to parameter changes of the sensor. Therefore at the design stage, as a control engineer we have to be extra careful in selecting the sensors. The sensor should really be a robust and stable sensor, that is the components or the parameters of the sensor should not change with environmental and other conditions; this should be our selection as far as sensor choice is concerned.

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Handwritten mathematical derivations for sensitivity functions:

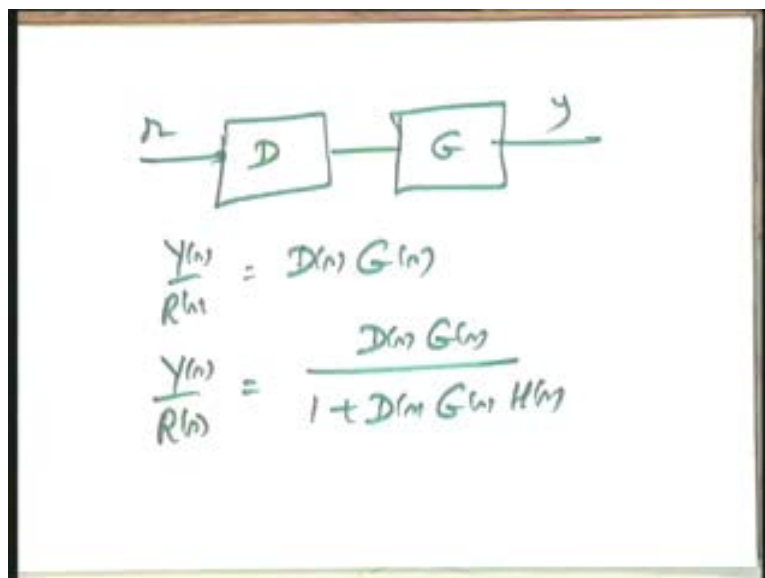
$$S_G^M = \frac{1}{1 + D(s)G(s)H(s)}$$

$$S_H^M = \frac{-D(s)G(s)H(s)}{1 + D(s)G(s)H(s)}$$

$$S_G^H = 1 \quad (\text{Open loop system})$$

One point here to be noted; what is the price we are paying in terms of improving the sensitivity. You will please note that if I consider the open-loop system, here is the open-loop system D and G **I need your attention here please**; this is your input r, this is your output y (Refer Slide Time: 6:30) you find that the gain of the system is given by D into G. Y(s) over R(s) is equal to D(s) into G(s) so it means Y(s) is equal to this much of system gain multiplied by R(s). But what happens in the case of a closed-loop system? Y(s) over R(s) is equal to as you see D(s) G(s) over 1 plus D(s) G(s) H(s).

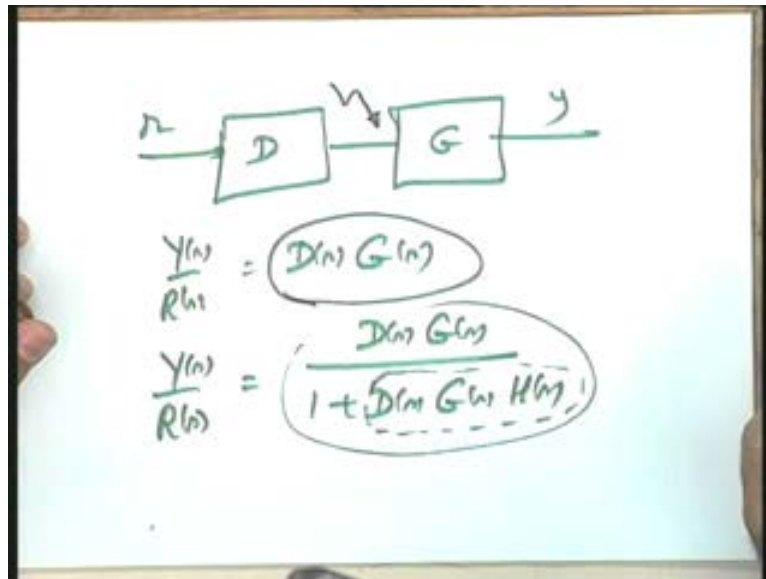
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You will please note that we are immediately paying a price for sensitivity improvement and the price is in terms of loss of gain. The output in this particular case is equal to this particular function multiplied by input. Well, the output in this case is this much of function multiplied by input. However, the loop gain better is the system sensitivity that is lesser is the effect of parameter changes on the system but in addition you will note that there is a larger price paid

in terms of loss of gain. So you see that, well, though it is definitely a disadvantage **but the advantage is how to waive the disadvantage** and this disadvantage can easily be taken care of by..... you can just see, having an appropriate open-loop gain in the system. You see that suppose this is the motor and if the open-loop gain of the motor is made high that is the selection of the motor is such that the gain of the motor is high, I will keep it high because I know that I am going to use it in the feedback loop and in the feedback loop the overall gain of the system is going to be reduced. So it means the selection of the plant or tuning or the tweaking of the plant should be done in such a way that the open-loop gain of the plant is high to account for the loss of gain which will occur because of the feedback action.

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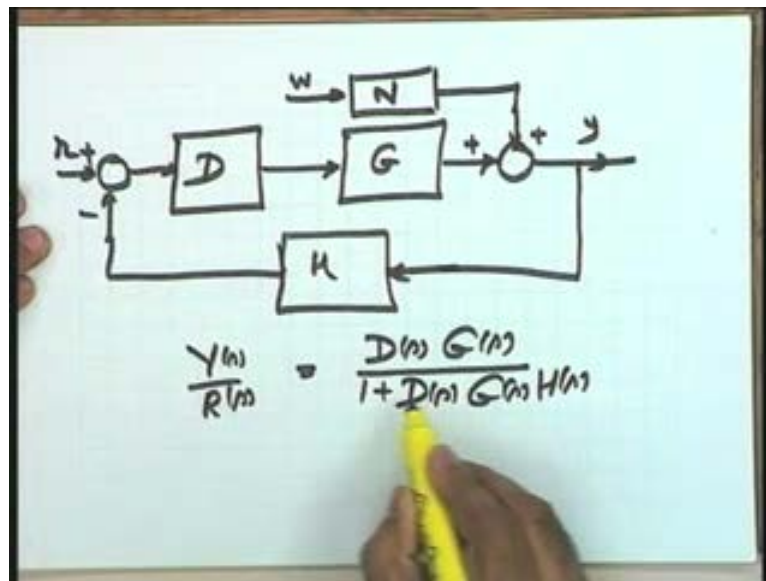
So it means because of the feedback action there is going to be a loss of gain. suppose in the open-loop plant there was certain gain for  $G$  required, for the feedback action the only precaution I can take is that the gain of  $G$  somehow this much of adjustment I will make in the plant that the gain of  $G$  is higher because I know that when I connect a feedback loop the gain between  $y$  and  $r$  will be reduced and to keep it or to bring it to the acceptable limits I can take certain precautions I can do certain adjustments on the plant model  $G$  and this is the way I can accommodate for this loss. But, however, you see that the sensitivity reduction is such an important parameter that the loss of gain is not considered to be a very significant disadvantage.

**Any question on this point please?**

**Why we need a high gain you can always connect an amplifier at the end ....00:09:31.... okay that is one point.**

Please see that he says that the controller transfer function and here I take  $G$  the plant transfer function, well, the disturbance here is..... if disturbance transfer function is  $N$  disturbance is  $w$ . let me take the output  $y$ , reference is  $r$ , the sensor transfer function is  $H$ . Now  $Y(s)$  over  $R(s)$  is equal to  $D(s)G(s)$  divided by  $1$  plus  $D(s)G(s)H(s)$ .

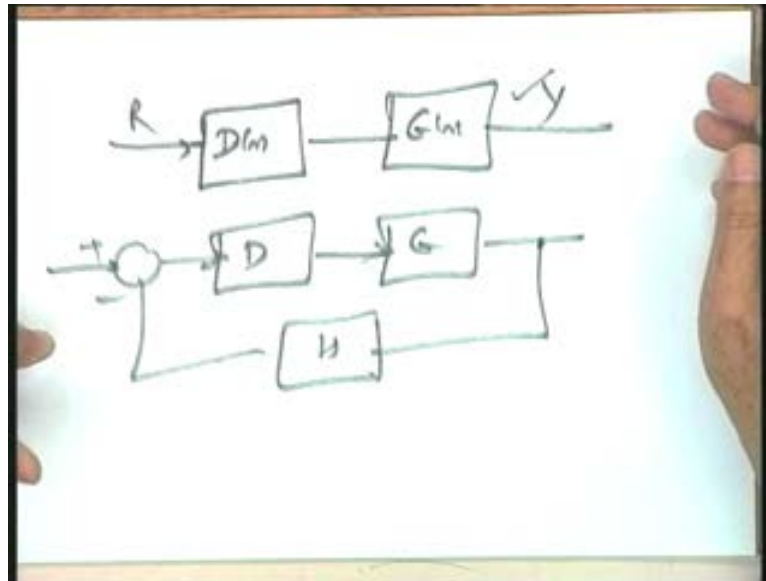
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Now, because of this high loop gain as we find there is a loss of gain Y, the ratio between Y and R gets reduced and he says that if I take an amplifier here in that particular case this loss of gain can be compensated for. **Good suggestion**; but what will happen if the parameters of this particular unit outside the loop change? For example, let us say this is temperature and you want to increase the temperature you want to magnify the temperature. The device for that particular purpose will have its own problems, the parameter variations of that particular device will affect the system output directly because this particular component is not in the feedback loop the feedback strategy is not taking care of the parameter variations of this component. So it means any changes occurring within this component or any environmental effect on this particular component is going to directly affect the system output and therefore we try to compensate for the loss by putting sufficient margin or by increasing the open-loop gain of the plant so that after the loss of gain the output level is acceptable to us.

So now we have seen that, compared to the open-loop transfer function which is  $D(s)G(s)$  Y and R, compared to this open-loop the closed-loop transfer function here.... okay let me forget about the disturbance for the time being H gives me better sensitivity properties, sensitivity reduction which I may say that **please see** probably I think I have made a statement here; the robustness or sensitivity reduction is the primary goal of feedback action. Otherwise probably all those other requirements of let us say the steady state accuracy or shaping the dynamic response or other requirements on the control system can suitably be met with an open-loop control as well.

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You will see shortly, we are going to discuss today itself that all those things which you have in mind that the transient response should behave in a particular way, the required steady state accuracy should be this much, probably these can be met by open-loop control also or if you do not mind I can make a statement that they may be better satisfied with an open-loop control compared to even feedback action. I may get better performance as far as dynamic shape is concerned or as far as steady state accuracy is concerned if there is no feedback action.

However, you see, the real life problem is that of robustness or sensitivity you cannot eliminate this problem and hence you have to go for feedback action and the feedback action is creating its own problems and that is why the design becomes complex. It is really a suitable trade-off between the accuracy and stability and instability is caused because of the feedback action only as you will see. So the feedback action is primarily coming for the robustness or sensitivity reduction and it is going to create problems and we have to suitably strike a compromise between the dynamic response, between the steady state accuracy and between the Robustness which you are going to achieve through feedback.

Next let me take the disturbance rejection. Sensitivity will be coming in detailed discussion later. Now it is only the qualitative appreciation I am going to give you please. All these aspects of design naturally will come in-depth when we take up the design aspects of control systems but the qualitative appreciation must come today itself. This is the objective of today's lecture. Disturbance rejection let me take this aspect.

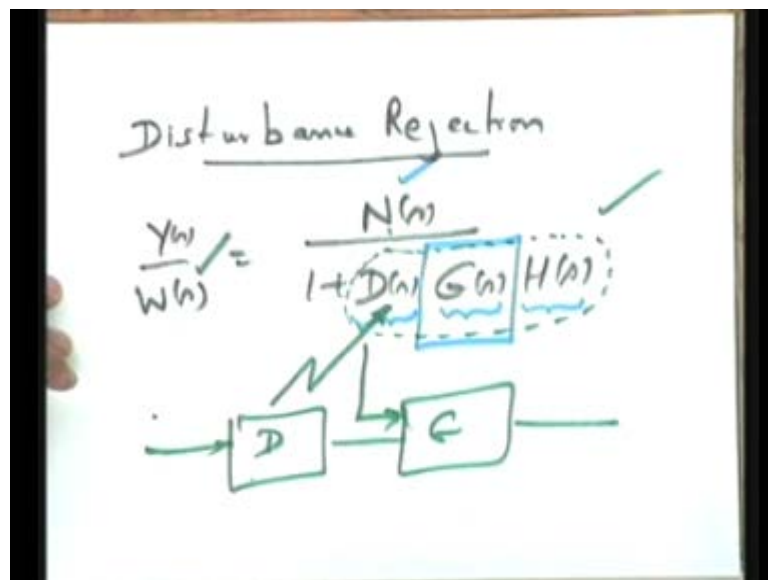
Now if you take.... let me not redraw it, let me project the slide. now if you take  $w$  versus  $y$  the output in that particular case you know that the transfer function is going to be  $Y(s)$  over  $W(s)$  equal to we have already called it  $M$   $w(s)$  is equal to  $N(s)$  divided by  $1 + D(s) G(s) H(s)$ . Please see, either you know it or you can derive it. I leave it to you that  $Y(s)$  over  $W(s)$  the transfer function between the output and disturbance is given by this function  $N(s)$  divided by  $1 + D(s) G(s) H(s)$  (Refer Slide Time: 15:29).

Now let me concentrate on this:  $Y(s)$  over  $W(s)$  equal to  $N(s)$  over  $1 + D(s) G(s) H(s)$ . Now please see, the effect of the loop gain or the feedback action on the disturbance rejection. Again I think it is very clear that if this loop gain is made high in that particular case the disturbance effect is filtered out or is reduced. So it means the rejection of disturbance is also directly related to the feedback property. If you make  $H$  is equal to 0 the disturbance rejection is not possible. All the disturbances which enter the plant will definitely show up their effect in the output because you see that in the process of feedback your controller becomes intelligent because it has the information.

You see that qualitative aspect of control action now because of the feedback action  $D(s)$  is your controller it has got the information about the effect of disturbance on the output. Since it has got information about the effect of the disturbance on the output it can take appropriate action to reduce the effect of the disturbance. But as far as the open-loop scheme is concerned, you just recall this scheme  $D$  and  $G$  let us say disturbance acts over here, the  $D$  controller has no information about the disturbance at all. So if the controller has no information about the disturbance it naturally cannot take any effect to compensate for disturbance it cannot filter out the effects of disturbances and hence in an open-loop scheme if the disturbances come they naturally will affect the output straight away and disturbances as we have seen through our practical case studies, through our hardware examples we have seen the disturbance effects do come.

In almost every case we have come across we have seen that there was an effect of disturbance. So it means absolutely there is no alternative to feedback, you can never go for an open-loop scheme because in the open-loop scheme the disturbances are going to directly affect the controlled variable.

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Well, in the case of feedback action, you find over here that by suitable control of the loop gain, I am talking of the loop gain again and again and through this particular thing I can affect I can control the effect of  $W(s)$  on output  $Y$ . Now at this point, one specific information I would like to give that as far as the loop gain is concerned there are three parameters please:  $H(s)$   $G(s)$  and  $D(s)$ . I assume that  $H(s)$  the sensor you cannot play with



that sensor, you cannot increase the loop gain by increasing the gain of the sensor, the sensor itself has got certain constraints, the parameters of the sensors cannot be allowed to vary because the system is highly sensitive to sensor parameters. So sensor is fixed and is given by the considerations of robustness the design is fixed for the sensor.

How about  $G(s)$ ?

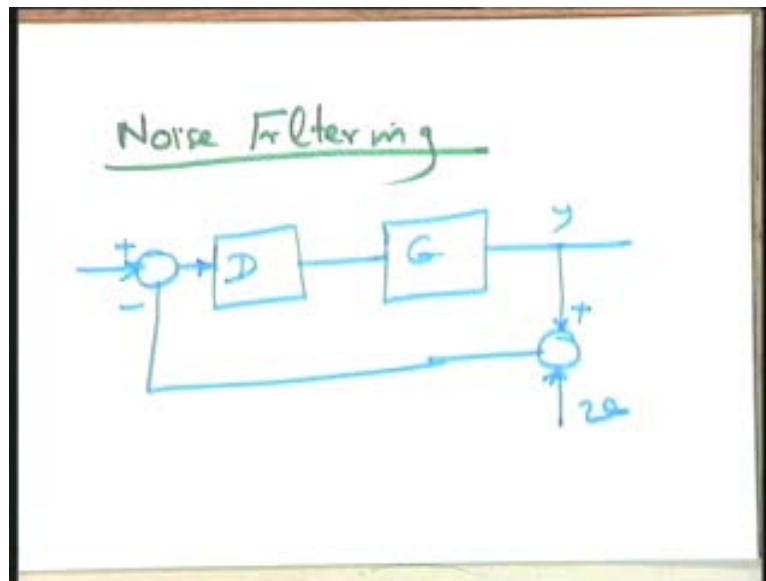
The plant gain also in all probability in most of the cases cannot be varied the plant is a fixed plant; the only parameter which is subject to change which is subject to design is the controller. You will actually change your  $D(s)$  so that the loop gain  $D(s) G(s) H(s)$  is increased and hence the disturbance effects can be filtered out. Now one thing to be noted: Suppose somehow you can change the gain of  $G(s)$  also the loop gain through gain change in  $G(s)$  can be increased no doubt but one point may please be noted you have to be extra cautious about it, what is  $N(s)$  after all  $N(s)$  is the transfer function between the disturbance and the plant output so naturally  $N(s)$  is a function of  $G(s)$ .

You recall so many models we have derived for the disturbance models, in many systems we have derived and we have seen that  $N(s)$  the disturbance link between the plant output and the disturbance  $W$  is a function of  $G(s)$  so it may happen that if you change the loop gain through  $G$  in that particular case it may increase the disturbance effect on the plant because this  $G$  is affecting  $N(s)$  also and this  $N(s)$  is appearing in the numerator. So it means you see that just do not go by this blind rule that increase of the loop gain will improve the disturbance or will reject the disturbance; how is the loop gain being increased that is also important.

If you increase the loop gain through  $G$  there is a possibility that you are increasing the gain of  $N$  and if you are increasing the gain of  $N$  you are causing problems for the system rather than taking away the problems or finding the solutions to the problems. So this is all a complex situation you see; it does not go the sequential way that you increase the loop gain it will reject the disturbances you have to account for the total system in totality and see what are the parameters which are getting affected by change in gain of any of the components. But as a thumb rule I may tell you that we will not touch  $G(s)$  and  $H(s)$  this will may become our fixed components. once you have taken care of may be by adjusting some open-loop gain you might have taken care of the loss of gain after that the loop gain is primarily being increased through the controller design  $D(s)$ .

Next I take noise filtering and I need now I think your participation in setting up the characteristics of the system. Come on please; let us see what is the effect of feedback on the noise effect. We know that, from where this noise is coming. The major source the most important source of noise is the sensor itself as we have seen in our system examples. So let me take a situation like this:  $D$  is my controller transfer function,  $G$  is the plant, this is output  $y$  so as soon as I try to measure it I introduce this noise signal  $v$  and let me say that this is being fed back I am taking unity feedback case to simplify the situation.

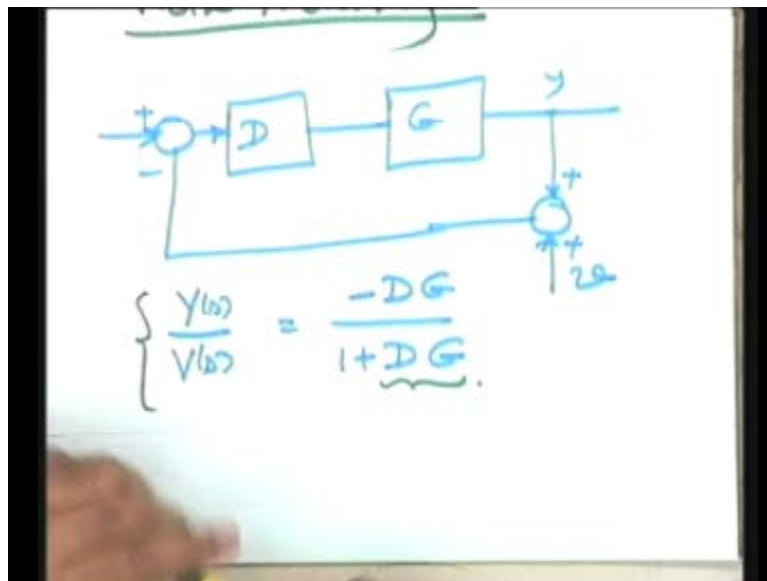
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Take this case only to make the discussion simple otherwise it could be the more complete block diagram there is absolutely no problem in that. Now you have to help me please, this  $v$  is a high frequency noise, high frequency signal which is coming because of the sensor. Recall the temperature control system; thermocouple being close to the tank which is being continuously stirred. Well, you see you cannot avoid high frequency signals entering the loop because after all temperature measurement is a must and to measure temperature you have to install a measuring device.

Suppose you install a thermocouple there so you see that thermocouple is introducing noise in the loop which otherwise was missing in an open-loop scheme. So it means a sensor is the major source of high frequency noise in the loop. And let us see how is the loop gain going to affect this noise effect and I want you to give me the transfer function between  $v$  and  $y$   $v$  as the input and  $y$  as the output. Help me please, I think you can just look at this block diagram quickly and give me the transfer function; I need the answer from you please.

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$Y(s)$  over  $V(s)$  is equal to what? Yes, I wait for a few seconds just.....minus  $DG$  upon  $1$  plus  $DG$   $1$  plus  $DG$  that is right please. You can just view it as a basic feedback loop; this as a reference input, this as an output this will become forward path transfer function minus  $DH$  so it is minus  $DG$  over  $1$  plus  $DG$ . Now you see this effect please.

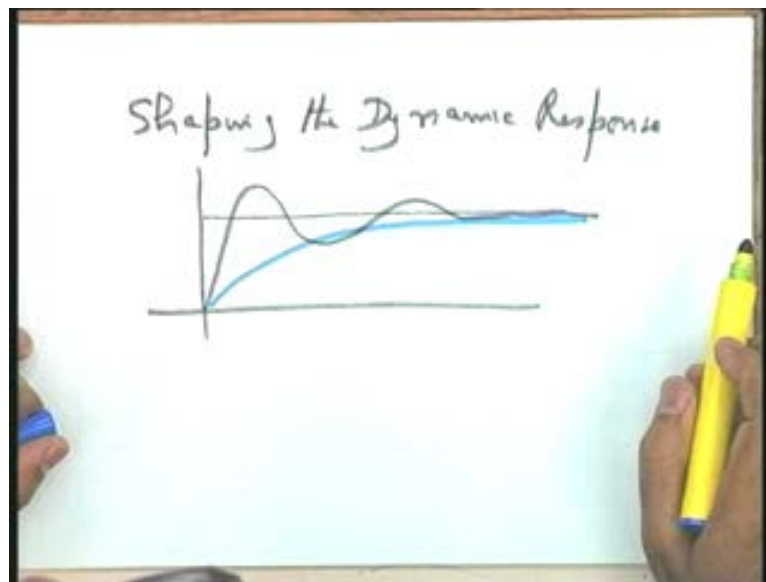
Now, in this particular case, the loop gain is  $DG$ , you increase the loop gain. To increase the loop gain for sensitivity reduction or disturbance rejection is increasing your noise problems. You can just see that when you increase the loop gain  $Y(s)$  by  $V(s)$  is nearly equal to minus  $1$  and hence whatever is the noise signal over here it is directly getting injected into the loop. So it means when you try to improve the performance in terms of disturbance rejection and sensitivity reduction you are increasing the problems in term of noise effect.

Well, as I said there is always a trade-off in the control system design. You cannot meet all the requirements by increasing the loop gain so what we normally do is the following that to take care of the noise problem we do not compromise on the loop gain rather we introduce an appropriate high frequency noise filter. So it means we take care of the noise problem by introducing an additional component and appropriate high frequency noise filter so that this particular noise the high frequency noise is blocked by that particular filter and it is not allowed to enter into the loop gain. This will permit us to go for high loop gains for disturbance rejection and sensitivity reduction.

This point may please be noted down that the increase in the loop gain is creating problems as far as noise filtering characteristics are concerned so it means again, at the sensor point you have to be extra careful. a suitable high frequency is, you will have to study the type of frequencies entering into the system, you will have to study the environment in which your sensor has been installed, the effect of the environment on the sensor and what type of frequencies will enter through the sensor has to be studied and then a suitable sensor filter a suitable high frequency filter should be designed so that the high frequency signals are not allowed to enter into the loop and hence this will not come on your way as far as taking high loop gains is concerned.

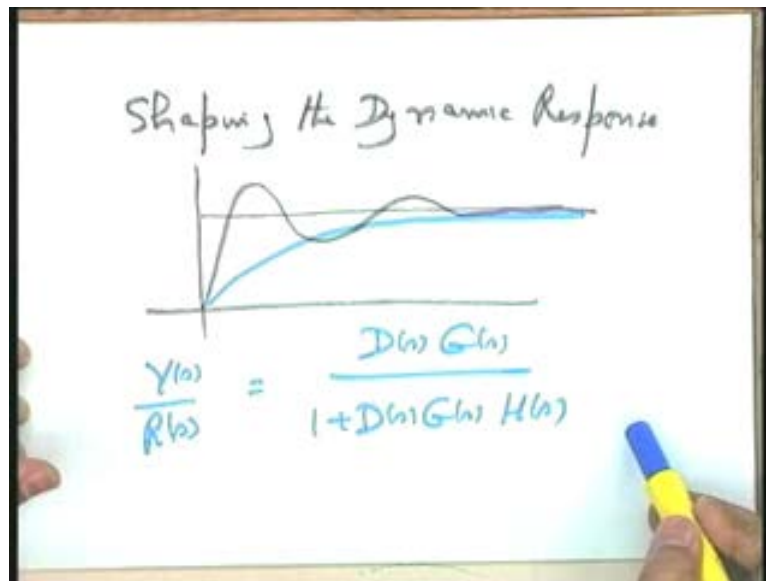
Next feature I take is shaping the dynamic response, **shaping the dynamic response**. You recall what do I mean by shaping the dynamic response. I gave you an example that if this is the value I want I may be interested in this type of response (Refer Slide Time: 26:58); this is one typical example and not in this type, this may be my requirement. It means I am giving the shape of the transient also which the output is required to follow, not only that where the output should settled down.

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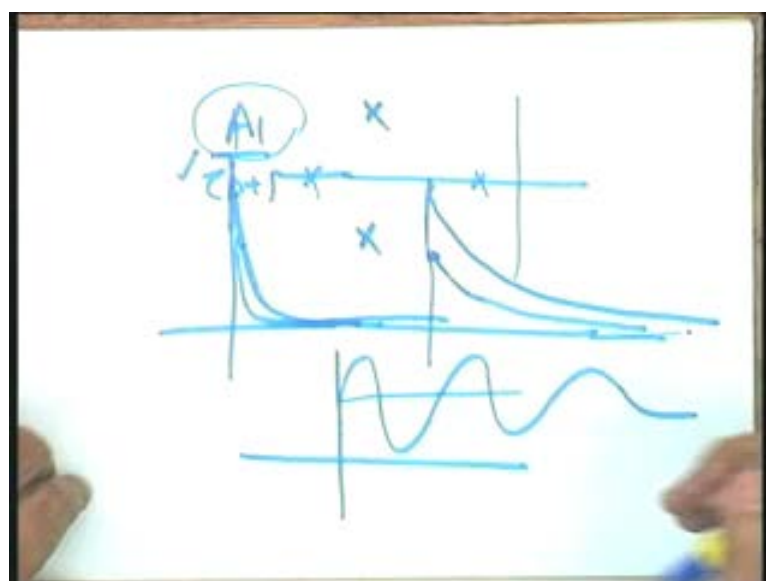
Now you see that if I give you a function  $Y(s)$  over  $R(s)$  is equal to  $D(s) G(s)$  over  $1 + D(s) G(s) H(s)$ ; please recall how do you shape the dynamic response, you will remember I am sure that the dynamic output  $Y$  is a function of the system poles. As far as the zeros are concerned they simply affect the magnitude of the various modes of the response whether the response is oscillatory, whether the response is non-oscillatory, whether it is a first-order factor, whether it is a second-order response all this is going to be guided by the poles of the system and not by the zero. **I hope you remember it very well;** you can appreciate this particular point. As far as the zeros are concerned, when you take the partial fraction expansion for example the zeros will affect only the residues, magnitude and those residues will affect the magnitude of the response the nature of the response will be guided by the pole.

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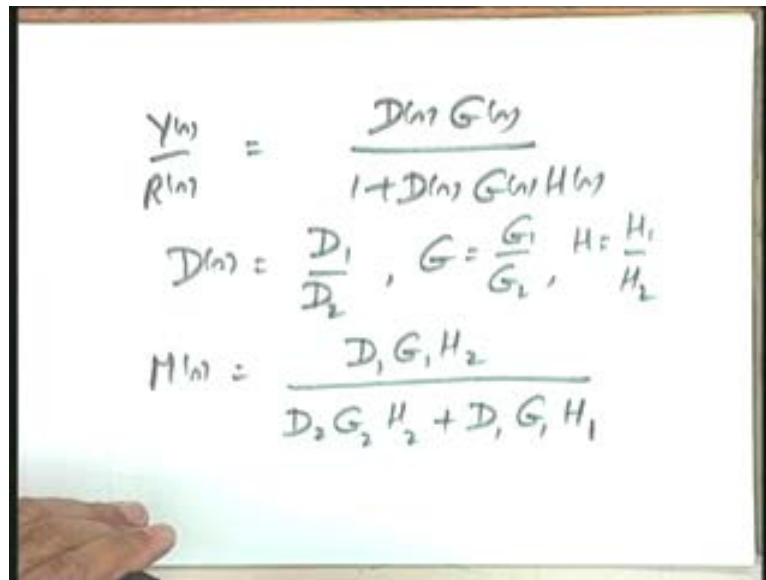
For example, if the pole is a first-order factor  $1$  over  $\tau s + 1$  and in the numerator I have  $A$  the zeros only affect the magnitude of  $A$  and the poles affect this  $\tau$  and you know that if my  $\tau$  is here the response may look like this an impulse response let me take impulse response may look like this and if the  $\tau$  is here the impulse response may look like this (Refer Slide Time: 29:00). So in this particular case you find that the settling is quicker while in this particular case the settling takes time. So it means, what I want to say is that the shape of the dynamic response is dictated by the poles of the transfer function and the zeros of the transfer function only change the magnitude of the response that is your response may become like this depending upon the zeros effect, the response may become like this depending upon the zeros you may change. But as far as the shape is concerned, whether the response is this way, now your poles may be this way; if your poles are this way you know that your response will be oscillatory in nature, damped oscillations or un-damped oscillations. So it means the nature of the poles is going to guide the nature of the response.

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Now let us see how will the feedback action affect this. This is what I want you to appreciate:  $Y(s)$  over  $R(s)$  is equal to  $D(s) G(s)$  over  $1 + D(s) G(s) H(s)$ . Let me take  $D(s)$  as a ratio of two polynomials  $D_1$  by  $D_2$  a qualitative appreciation at this particular point is really needed. I take just a ratio of two polynomials,  $G$  as a ratio of two polynomials  $G_1$  and  $G_2$ ,  $H$  as a ratio of two polynomials  $H_1$  and  $H_2$  and therefore your  $M(s)$  becomes equal to, please see,  $D_1 G_1 H_2$  divided by  $D_2 G_2 H_2 + D_1 G_1 H_1$ .

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$$\frac{Y(s)}{R(s)} = \frac{D(s) G(s)}{1 + D(s) G(s) H(s)}$$

$$D(s) = \frac{D_1}{D_2}, \quad G = \frac{G_1}{G_2}, \quad H = \frac{H_1}{H_2}$$

$$M(s) = \frac{D_1 G_1 H_2}{D_2 G_2 H_2 + D_1 G_1 H_1}$$

Just see that this is your  $M(s)$ . **I hope this is okay**. Just I have substituted these polynomial functions and I have rearranged I get this as the rearrangement. Now please see, if you want to shape the dynamic response it means you want to make it different from the open-loop response and the open-loop dynamics is guided by  $G_2$  because  $G_2$  is going to give you the poles of  $G$ . so it means the pole locations of  $G_2$  are not acceptable to you that is why you want to go and shape the dynamic response. So just see that the pole locations of  $G_2$  are not acceptable to you, by suitable design of  $D_1$  and  $D_2$  you can change the factors of this polynomial and hence you can change the pole locations of the closed-loop system and therefore you can shape the dynamic response of the system.

So, through feedback action I just want to say this point. you see that in the open-loop case the response  $G_2$  was not acceptable to you but through feedback action you can appropriately choose  $D_1$  and  $D_2$  so that **the pole** the roots of this particular equation, roots of this particular polynomial which will become the poles of the transfer function  $M(s)$  can be suitably placed in the  $S$  plane and hence the dynamic shape of the response can be suitably given. Or what we will do is the following that given the dynamic shape you will translate that dynamic shape into desired locations of poles and then you will select  $D_1$  and  $D_2$  so that those locations of poles are realized.

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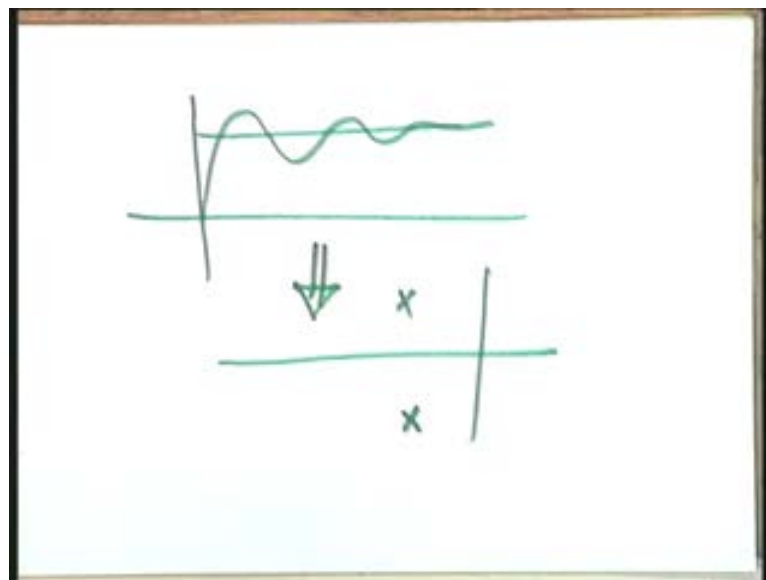
$$\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)H(s)}$$

$$D(s) = \frac{D_1}{D_2}, \quad G = \frac{G_1}{G_2}, \quad H = \frac{H_1}{H_2}$$

$$H(s) = \frac{D_1 G_1 H_2}{D_2 G_2 H_1 + D_1 G_1 H_1}$$

See the design problem please. I really will like conformation from you that my point is followed. My point I repeat; suppose I want this as the dynamic response, of course the design methods are going to come but my strategy will be the following: This dynamic response let us say can be realized by these pole locations, these two pole locations.

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Let us say I have given you the methods, you are equipped with the methods to establish this fact that the dynamic response of this shape can be realized if the pole locations are these. Now let us assume that your open-loop transfer function does not have these pole locations. So in this particular case I want to claim that by a suitable design of  $D_1$  and  $D_2$  that is by suitable design of the controller  $D(s)$  you will be able to force the pole locations of the closed-loop system onto these points so that the dynamic response of the closed-loop systems follows the prescribed dynamic shape. **I hope you are getting my point.**

The prescribed dynamic shape can easily be realized by this way by a suitable design of  $D_1$  and  $D_2$  so that the closed-loop poles are same as these two pole locations. Please do not worry about the quantitative aspects as I am telling you again and again; you try to appreciate the qualitative nature of the result. So this concludes that well shaping the dynamic response through the feedback action is possible if I appropriately design the controller and hence numerator and denominator polynomials of  $D(s)$  so that the closed-loop poles of  $M(s)$  are same as the closed-loop poles which result in the required dynamic shape. But one thing is there my friends; if really this is the objective I can realize this objective of the statement I reemphasized and if this is the objective probably this objective can be realized in a very very simple way in the open-loop control mode itself.

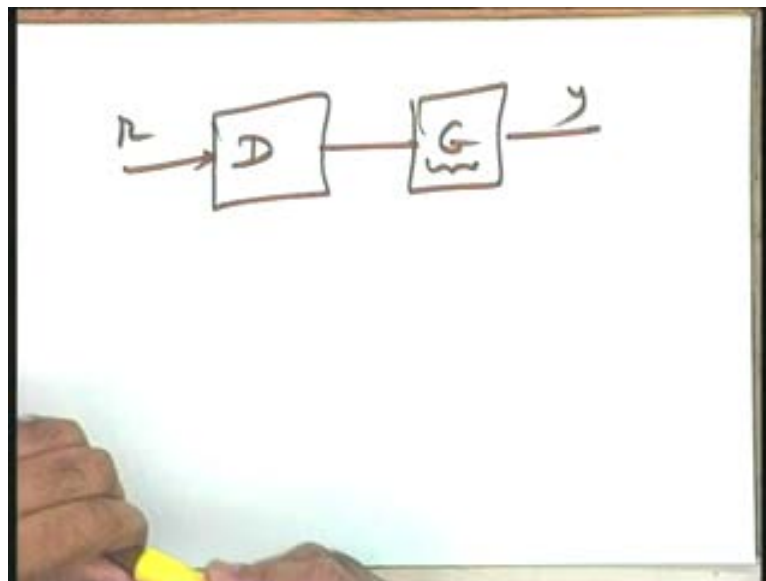
Can you suggest how can I realize this objective?

I will make my objective clear once again; I want to change the poles of the system so that they become equal to the prescribed poles which correspond to the desired dynamic shape of the response. My question to you is this:

Can I realize this objective by open-loop control?

Recall the open-loop control:  $D$  is here  $G$  is here there is no feedback action this is  $r$  this is  $y$ . the poles of  $G$  are not acceptable to you, the poles of  $G$  are not giving you the desired dynamic response. I want to design a  $D$  so that the poles of these overall systems correspond to the locations which are a result of the prescribed shape of the response. Come on give me a suggestion please.

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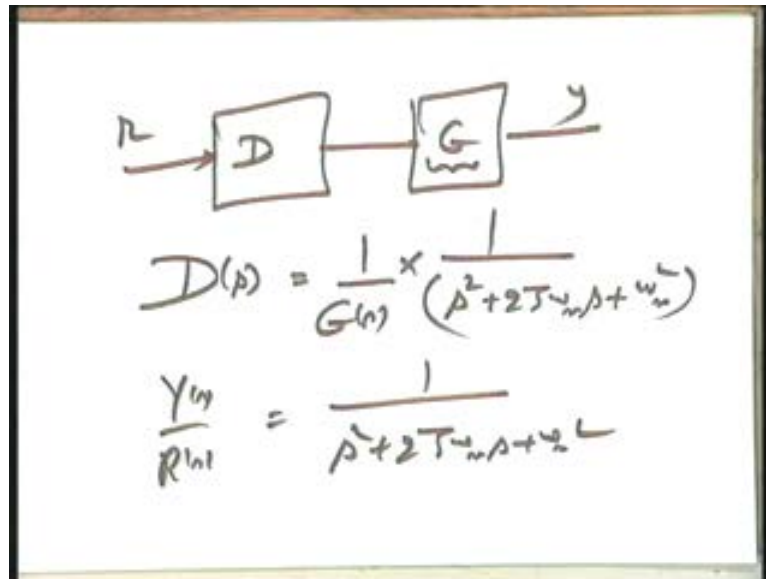
[Conversation between student and professor ... $D_1$  should cancel the pole of  $D_2$  that is fine and tends up to.....35:55.....it is wonderful please, this is what I had in mind]

You see, the most obvious solution which dictates as a feedback action is not required at all; is this first you cancel the poles of  $G(s)$  that is you make  $1$  over  $G(s)$  so poles of  $G(s)$  are canceled out so they are not going to effect the dynamic mode at all.... into you keep the poles which you want, the prescribed poles, let us say your prescribed poles are  $s^2 + 2\zeta\omega_n s + \omega_n^2$ . Let us say this is the prescribed pole. Now if I take this  $G(s)$  what is the overall  $y$  by  $r$ . You will please note that  $Y(s)$  by  $R(s)$  is equal to  $1$  over  $s$



squared plus 2 zeta omega ns plus omega n squared which you claimed is the situation corresponding to the given dynamic shape. So naturally if such a D(s) is realizable in that particular case this is the most obvious and exact solution which is possible. But we do not go for the solution; any answer to this point that we do not go we never go for this solution rather, why; though it is a most obvious solution we never go for this solution please. Come on, I need again some idea from you, okay fine does not matter. As soon as I give you the idea you will definitely appreciate.

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You see that this D(s), please your attention, this D(s) will cancel the poles of G(s) (Refer Slide Time: 37:32) and not that of the actual physical plant because the actual physical plant and G(s) which you are using over here there is always a gap. We have said that you can never model a physical plant exactly. Since you can never model a physical plant exactly it means when you use a controller D(s) realized from G(s) it is not going to nullify the dynamics of the plant though it is nullifying the dynamics of the model.

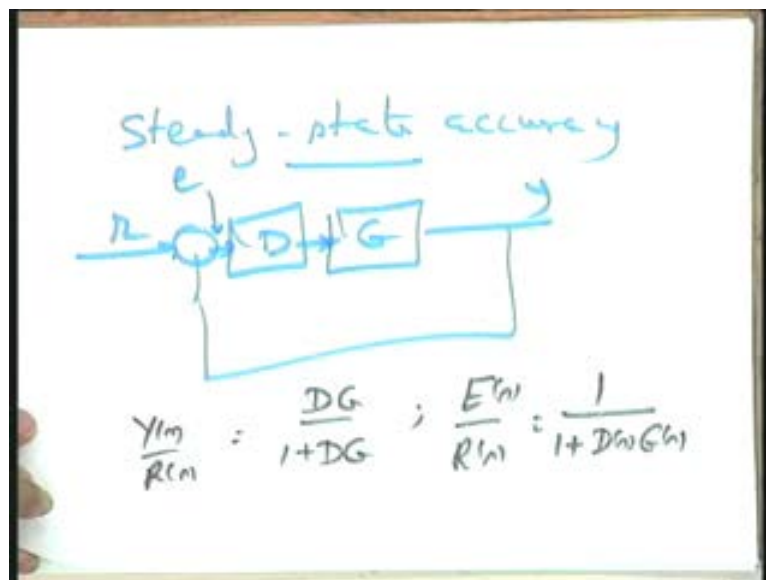
So since there is always a gap between the model and the plant this cancellation as was suggested by one of you is possible with respect to model only and not with respect to plant and therefore there will be an error between y and r because of this imperfect cancellation. And since the controller is not having any knowledge about the error there is no feedback action going over here so the controller is not an intelligent controller it has no information about the errors those errors will keep on building up and your system may become unstable and therefore such a performance or such a controller in which an open-loop dynamics cancelling attribute is used is never utilized.

Not only the modeling errors, this G you have used once for all, D you are not tuning online because the parameters of G may keep on changing with time, the G parameters the actual process, the G parameters will not change because G is only a model but the actual system parameters will keep on changing with time and this change in time is not reflected in D, this D is not an adoptive controller which adopts to the changes in the G parameter and therefore with time the performance of this D (Refer Slide Time: 39:24) will deteriorate. It is because of this sensitivity or robustness requirements that you would never go for this open-loop

solution though this open-loop solution seems to be the most obvious solution possible. So, though feedback action is more difficult you will please recall that transfer function I gave you adjusting the closed-loop poles of the feedback system is more difficult than realizing the poles this way but we go for that difficult design procedure but we do use our feedback principle to take account of robustness and disturbance rejection requirements. I hope this point is well taken, fine.

Now this way let us go to steady state accuracy also. Similar thing, steady state accuracy: Take the basic feedback loop  $G$ , let me take it unity feedback to make it little simple. Now you know that this is your error  $e$ , at this particular you have got the error  $e$ . Now please see, in this particular case  $Y(s)$  over  $R(s)$  is equal to  $DG$  over  $1 + DG$  and  $E(s)$  over  $R(s)$  I leave it to you, a simple exercise, you can look at the block diagram or you can even give me please; what is  $E(s)$  over  $R(s)$ ?  $1$  over  $1 + D(s) G(s)$  please check this point. This is your  $E(s)$  over  $R(s)$ ,  $E(s)$  is the error (Refer Slide Time: 41:00) this is  $1$  over  $1 + D(s) G(s)$ . This can easily be examined.

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Now, what is the steady state error?

The steady state error we can use the final value theorem limit  $s$  tends to  $0$   $sE(s)$  is going to be become your steady state error. This is equal to limit  $s$  tends to  $0$   $sR(s)$  over  $1 + D(s) G(s)$ . This is equal to as you know limit  $t$  tends to infinity  $e(t)$ , I have the basic relationship for the steady state error using the final value theorem.

Now take a specific case please; again a very interesting result will come. Let me take  $R(s)$  is equal to  $1$  over  $s$ ; what is steady state error please? Let me call this as  $e_{ss}$   $e_{ss}$  is equal to, come on please,  $1$  over  $1 + D(0) G(0)$  this is your steady state error; what is  $D(0) G(0)$  it is the D c gain of the loop that is the loop gain at low frequencies. Substituting  $s$  is equal to  $0$  is equivalent to getting the D c gain of the loop that is when the frequency signal is low. When the frequency of the signal is low at that particular point the gain of the loop is the D c gain. So it means  $1$  over  $1 + D(0) G(0)$  is the steady state error for the specific situation under consideration and this is directly dictated by the D c gain. Yes, surely I can reduce the steady state error by appropriate design of the controller.

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$$e_{ss} = \lim_{s \rightarrow 0} \Delta E(s) = \lim_{s \rightarrow 0} \frac{\Delta R(s)}{1 + D(s)G(s)}$$

$$= \lim_{t \rightarrow \infty} e(t)$$

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \frac{1}{1 + D(0)G(0)}$$

Ess is equal to 1 over 1 plus D(0) G(0); make an appropriate design of the controller suitable control of the value D(0) will give me the reduction in the steady state error and hence will improve the steady state accuracy. **So keep this point in mind now** that steady state accuracy gets improved with increase in the loop gain. This is your loop gain after all and I am improving the steady state accuracy with the increase in the loop gain. Come on please; now just let us compare the situation with an open-loop situation. Help me here also, r and y what is the steady state error? e is equal to R(s) minus Y(s) input minus output is the error is equal to E(s) let me take it (Refer Slide Time: 43:36) is equal to R(s) minus **help me please** D(s) G(s) R(s) is equal to R(s) into 1 minus D(s) G(s). So it means, just please look at this point, steady state error becomes equal to, for a step input you can see I will take R(s) is equal to 1 by s and then apply the final value theorem; give me the result for steady state error please 1 minus D(0) G(0) see whether it is okay. I have substituted R(s) is equal to 1 over s and then applied the final value theorem.

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$$e_{ss} = \frac{1}{1 + D(0)G(0)}$$

Block diagram: Input  $R$  enters block  $D$ , followed by block  $G$ , with output  $Y$ .

$$E(s) = R(s) - Y(s) = R(s) - D(s)G(s)R(s)$$

$$= R(s) [1 - D(s)G(s)]$$

$$e_{ss} = [1 - D(0)G(0)]$$

Now, from this expression should I do it or it is okay; **this expression** this expression from here please. Yes I need..... okay fine. In this particular case now you see that you can reduce the steady state error to zero by suitably designing the zero. You make your  $D(0)G(0)$  is equal to 1 so again please see, probably the steady state accuracy requirements can be better met with open-loop control than with feedback if there were no problems as far as robustness and disturbance rejection requirements are concerned.

You see that in this particular case probably you cannot reduce the steady state error to zero, you will have to make  $D(0)G(0)$  infinite to make the steady state error zero. While in this particular case (Refer Slide Time: 45:13) you can make the steady state error zero by an appropriate design of  $D(0)$ . So it means **I want to say is that** what I want to say is that the open-loop control has a solution as far as the dynamic response or steady state response is concerned.

We are going for the feedback action primarily because of the sensitivity and disturbance rejection requirements and those requirements are primary and it is because of those requirements that we are first creating problems because of feedback and then solving those problems as a compromise between robustness and accuracy requirements or stability and accuracy requirements. We have to make a suitable compromise between the two solutions.

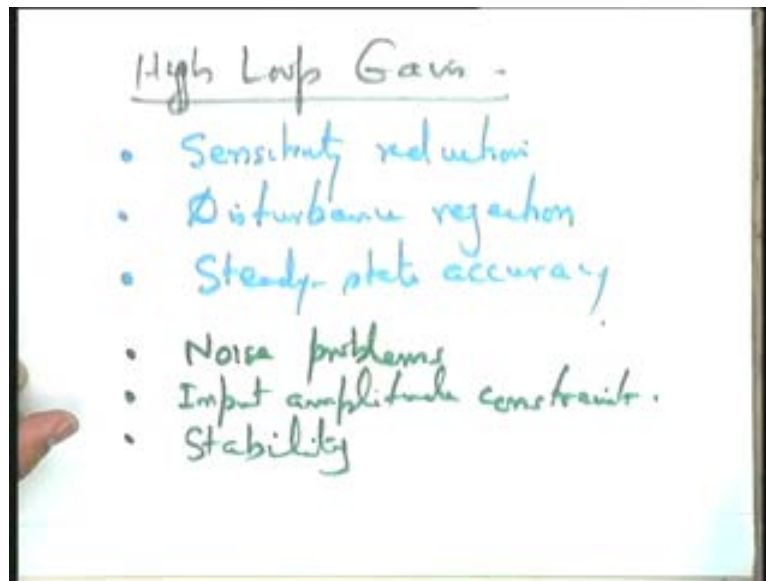
This is what I had to say as far as the qualitative aspects of feedback control are concerned. I will really like to conclude this with certain summery points coming from you. We have seen all through that high loop gain is one of the design tools available with us. May be by high loop gain we are going to take care of many of the design requirements. Let us now list one by one, what the high loop gain can do. Can it satisfy all the requirements or it can create problem for us.

High loop gain as you have seen sensitivity reduction is possible that we have seen. 2) By high loop gain we have seen disturbance rejection is possible and what else please you will tell me by high loop gain? You have clearly seen steady state accuracy can be improved so it means if you go for a high loop gain you find that higher the loop gain just see a thumb rule, higher the loop gain better is the performance of the system in terms of these three requirements. Let us see the problems. The high loop gain can raise certain problem at least three of them I want you to list come on please, the high loop gain problems. One; we have seen the noise problems. That is the high loop gain will allow the high frequency noise to enter the loop and hence the signal to noise ratio may become poor and the performance of the system may become poor.

What else please?

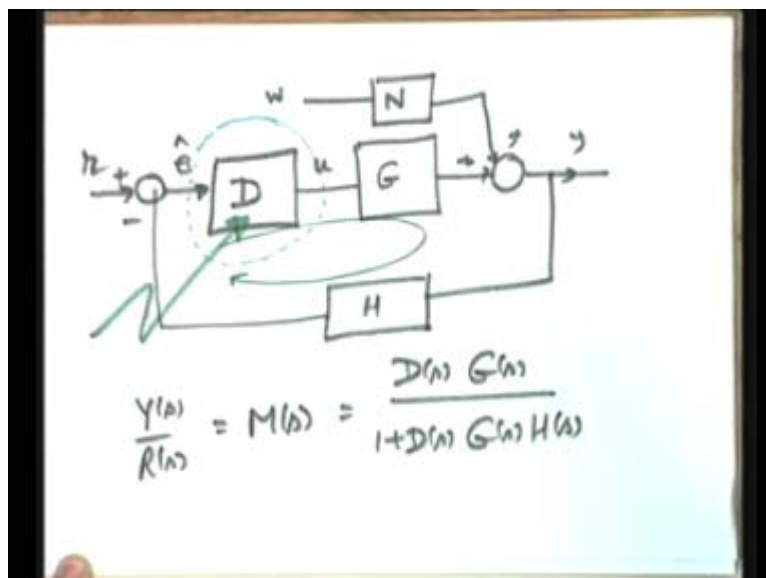
A high loop gain may create very high values of the input signals please see and your system under consideration or some of the components of the system may saturate. So the situation is not that simple that you will immediately improve the performance of the system by high loop gain. the system may saturate, some of the components there so those considerations will limit the high loop gain, the noise considerations will limit the high loop gain, you cannot take infinite gain because of the noise problems, you cannot take the infinite gain because of the input amplitude constrains and the stability problems you will see that is the transient response or the dynamics problem that we will see later in details that is higher the loop gain more prone is the system to instability.

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You will see there higher the loop gain more oscillatory the system becomes and a more oscillatory system can become unstable and therefore high loop gain is limited from the stability requirements also. So it means the design is a trade-off between the various design requirements we have seen. The various design requirements have to be simultaneously met to see that the overall control objectives are satisfied.

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I will conclude my discussion with this statement that: as far as this block diagram is concerned the loop gain can be increased by appropriate design of  $D$ . The concluding statement but very important statement and the discussion on this statement we will follow in the next lecture.  $D$  increasing the gain is actually increasing the amplification gain so here  $D$  may be an amplifier for example, please see, purely an amplifier. The gain of this particular loop can easily be increased by increasing the gain of the amplifier it is so simple a situation. We have always been talking about the high loop gain. If you do not change the gain through

G and H it means you can change the gain through G and most simple solution is changing the gain of D when D could be a simple amplifier. But we have seen that high loop gain is not the optimum solution or may not give you the control solution in many situations. It means you will like to change this D you will like to make it different from the amplifier that is why the discussion of derivative and integral control has been coming.

Please see, when only the gain that is only the amplifier cannot satisfy all the six requirements you will like to change this D so that all the requirements are satisfied and two of the things which we will be referring to which we will be taking up are the derivative action and the integral action. So you see now the need of derivative action and integral action. your amplifier gain here only amplifier gain may not satisfy all the requirements you have to look for alternative  $D(s)$  and couple of suggestions are that in addition to the amplification you take the derivative of  $e$  or and you take the integral of  $e$  and inject that particular signal in the loop. So it means in the loop in addition to injecting a signal which is proportional to the error you may inject the signal which is proportional to its derivative which is given by the integral of this particular signal.

Now, in the next lecture we have to see how this injection of proportional derivative and integral actions is going to satisfy all the six requirements which we have imposed on our control system. Thank you.