Control Engineering
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Lecture - 19
Basic Principles of Feedback Control

Well friends, having taken up the control hardware in depth, you recall, we have taken up control hardware for motion control systems, specific applications of position control and speed control, their elector mechanical servos the DC as well as AC servos were taken. In addition, the process control hardware for applications like temperature control, composition control, liquid level control have been taken. And now you just store that in your memory, keep that in mind. Today onwards I start with feedback control theory and to illustrate the theory as an application example I will be really recalling those hardware examples from your memory so that the control theory applications to real life industrial control systems could be given.

To get started with the principles of feedback control theory I recall the basic feedback diagram we have discussed. Let me say that this is G of course a function of S, let me write it in short SG, this is the plant model (Refer Slide Time: 2:12) here I have a controller and D has been designated to represent the transfer function of the controller, here is a controller. Onto the plant I have a disturbance signal and this disturbance was captured into this type of model, this is N and here is a disturbance W so this is y my controlled variable. This controlled variable y will be measured by a suitable sensor with the transfer function H and this is the error detector here which is comparing the reference signal with the feedback signal to give you the actuating signal e cap.

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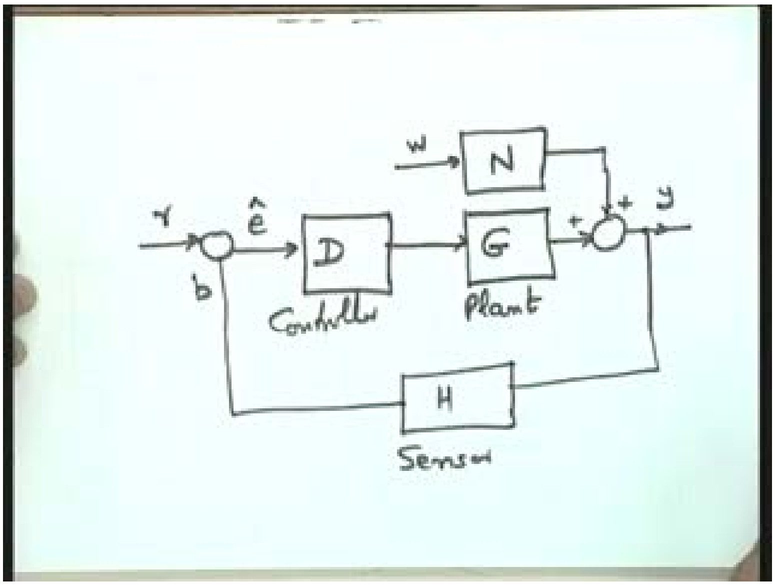

Here is the controlled signal u and here I have taken the reference elements; let me say A as the transfer function of the reference elements and here is y r the command signal or the reference signal which is given to the system. Look at this; this broadly describes the features of any control system you will come across; y is the controlled variable, y r is the command signal, r is a signal derived from y r for the purpose of comparison with the feedback signal, e cap is the actuating error signal and the system error e is given by y r minus y or y minus y r whatever way you take it; y r the reference signal minus the actual signal y, this is e the system error, here I am taking e cap to differentiate it from e because in this particular case the transfer function of the sensor and that of the reference signal generator is also incorporated. So here is a controller which generates the signal u acts on the plant the disturbance acts on the plant and this is the total feedback diagram.

Now we have to decide today as to what is the objective of design, what do we want and what way we are going to achieve the objective. So, our today's discussion basically is going to be the objectives of design.

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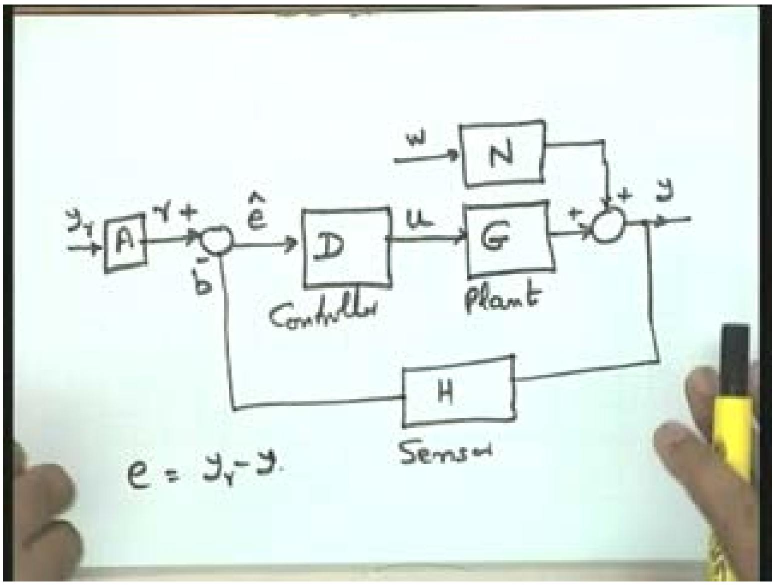

You will keep this block diagram in your mind when I discus the objectives and various ways to implement or to realize those objectives. So, under the objectives I will like to say that actually the output y that is the controlled variable y I want this to be nearly equal to y r(t) for all time t greater than equal to t 0 where t 0 is my starting point. t 0 is the point is the time at which the control is applied and I want that the signal y that is the controlled variable should equal the reference signal for all time t greater than equal to t 0. This I think broadly, as a layman, the user will tell you this only, that look, you give me the value of y equal to y r for all time t; this the control objective in luminance form which the user is going to specify to us.

Now what are the problems?

You see that one of the problems I can take, most frequently found problem in industry that your y r is constant over a long period of time and the control objective is this that the y should approach this value y r as soon as possible and should stay at that value after approaching y is equal to y r see the problem please. This problem is referred to as set-point control problem or equivalently the regulator problem also can be framed in this form. So, in the set-point control problem what you have; the reference signal is your set point which as I said in many control applications in many industrial applications is constant over a period of time, you have to change this value only after a long period, you are not changing it continuously with time in many applications.

Let us concentrate on this set of applications. Then what is the requirement? The requirement will be, as soon as this change y r comes on this system I will like that y should approach this y r as soon as possible. So now let me define the word over here. I define the word the performance measure as the settling time. Settling time for me will be the time taken for y to approach y r. So naturally lower the settling time better is the speed of response. You want the speed of response of the system to be better it should respond quickly theoretically instantaneously. Since instantaneous response is not possible it is going to take certain finite amount of time and that finite amount of time is the settling time, you will like to reduce this particular time so that the speed of response becomes better.

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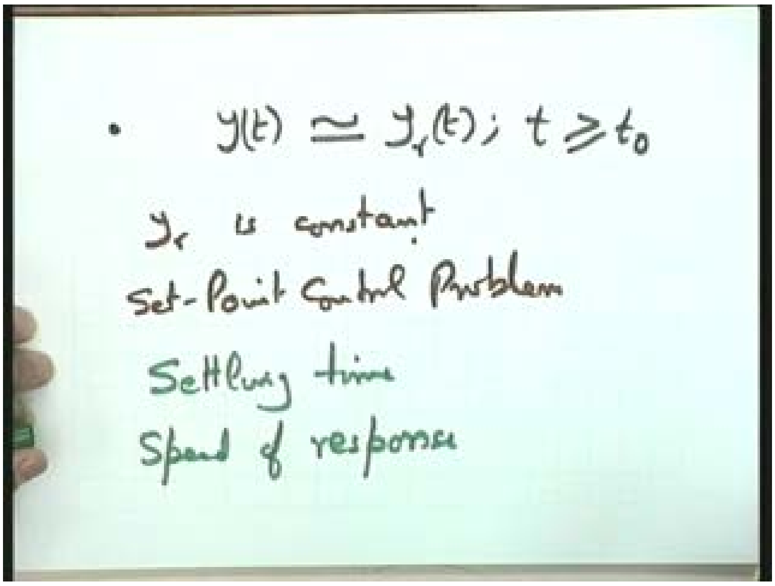

So, in addition to the settling time requirement that is settling time should be rest should be less, what you want; you want the steady state accuracy. That is, once it has reached the steady state it should stay there at the value commanded, that is y r value. so it means the difference between y r and y once the system has reached the steady state is the steady state error and you will like the steady state error to be zero for all inputs coming on to the system. So basically there are two requirements I will say that the system should have lower settling time or higher speed of response and in addition it should have better steady state accuracy that is the steady state error should be equal to zero. So this actually in the nutshell is the controlled requirement but there are problems which are not going to help us which are going to come on our way as for as realization of this control requirement is concerned and those problems have to be taken care of before you say that these control requirements can be met with or your design strategy your design algorithm in addition to keeping these control requirements in mind should take care of the constrains which come on the way of realization of these control requirements. And let me now spell out those details and one by one in a block diagrammatic form we will discuss today and in the next lecture I intend to give the case studies that is the application examples for all the discussions we are going to carry out today.

The very first point which comes when you design a system is the stability. At this juncture, you use your intuitive knowledge of stability; quantitative definition of stability will come later, how we analyze whether a system is stable or not we are going to discuss in depth later. the intuitive information, the intuitive knowledge of stability will be, that well, if there are small changes in the command signal, in the disturbance signal or in the system parameters there should not be large changes in the system response; if that happens then I can say that, well, the system in general is a stable system. And please see that stability is the prime requirement of a system. all those performance specifications, the speed of response or the steady state accuracy you have to meet within the umbrella of stability that is within the variations in the controller which are permitted by stability considerations within those variations you have to achieve the steady state accuracy and the speed of response. So this is going to be one of the requirements which I think I will postpone my discussion for a later date because it will be an in-depth discussion later.

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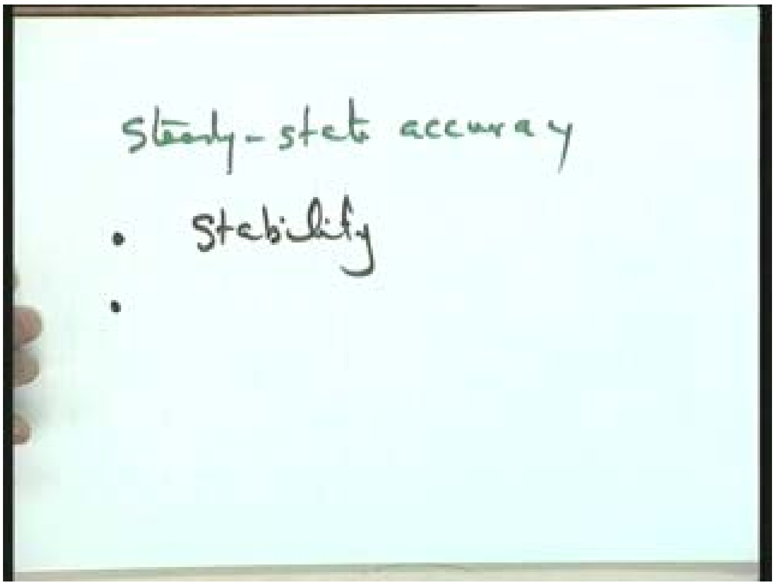

After stability I may say that, well, input amplitude constraints. Well, I assume that the transfer function model adequately represents the plant, the controller, the sensor or any other component of the system that is why the type of block diagram I have displayed we have accepted that block diagram as our basic block diagram for design. But please keep in mind that everything is not that rosy; what will happen if the amplitudes of various signals go beyond certain valve the linearity assumption will be invalid and the basic block diagram on which your design is based will be wrong and hence when you apply your design to a real life situation it will not give you correct results. So it means there are always certain constraints on the amplitudes of the signal in the system, those constraints are defined so that the linear model of the system is valid and in your design whatever design you may carry out; after all before you implement the design in an actual industrial control situation you will like to simulate the environment in the lab and see whether the system works properly or not and in that simulation experience in that simulation experiment you must take care of this fact that there are certain constraints on the signals and your design should meet those constraints. You are not free to take any control signal, you will have to meet those constraints before you give that design to the actual user because your model has been taken as a linear model and linear model is not valid under all situations. We have really had an in-depth discussion on the modeling; you recall, at every stage I told you, well, look it is only a perturbation model; it is only a small variation around the steady state point which is allowed for this model to be useable so hence at the design stage we must take care of those variations, we must see that our design does not go beyond those variations.

The third point I take is the disturbance rejection. I want the speed of response, I want the steady state accuracy; speed of response to an input signal and steady state accuracy to a command signal. But what will happen if there are disturbance signals. So it means, in addition to those requirements I should see to it that the effects of disturbances are properly filtered out; they do not affect the system; that is the accuracy and speed of response should be obtained under the condition that the disturbances are coming on the system. And one more point to be noted, that unfortunately these disturbances are unknown to you; anything known to us is not a problem please see that. Even if the disturbances were known to us deterministically there was no problem, we could have taken those disturbances very easily into consideration as far as our design algorithm is concerned. But the major problem for you for the control designer is this that, the disturbance when do they occur, what are their magnitudes, what are their wave shapes all these things are unknown to you and you want a control system which takes care of even this nature of the disturbance also that it is random in nature that it is unknown to you.

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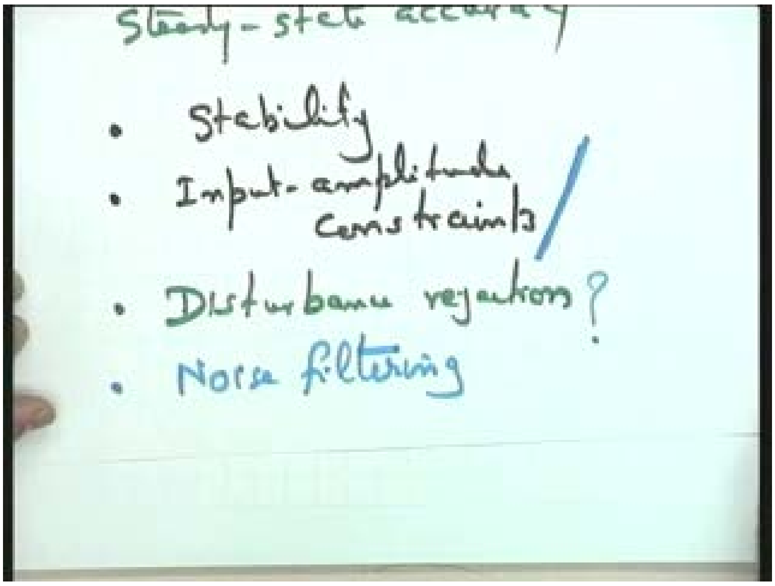

The next point is noise filtering. The very important curse of the feedback control is the use of a sensor. If this were an open-loop control no sensor was required because no feedback information was required and this sensor is a problem because this sensor is going to introduce high frequency noise into the system which otherwise would not have been present. So this high frequency noise through this sensor will enter the loop and it is possible that this signal to noise ratio may be very poor and the response which you are getting is as a result of the noise and not as result of the signal you are injecting into the system and therefore this particular problem of the feedback control must be accounted for and this normally is accounted for by introducing into the feedback loop a suitable high frequency filter. However, in taking in declaring the design to be completed this is very necessary that you take care of the noise filtering characteristics also.

Well, next point: Sensitivity and robustness. Sensitivity and robustness, what is this, the word sensitivity is very well known to us. I will put it this way: The robustness and sensitivity both are related and the difference between the two I am going to explain. We have taken a plant and we have said that the plant model is G(s), I need your attention, specifically, this is a subject of research even today; the word robustness need very clear understanding and in what way we are going to use the word robustness in our design.

Plant is modeled by G(s) and this plant consists of various physical components as you have seen. And I think, in the process of our modeling exercise it became very clear that it is almost impossible to capture all the physical attributes of the plant into a model. Consider for example a temperature control system, you have nicely assumed that the temperature of the entire tank is uniform and theta is the variable which is going to represent that temperature, but no, this will never occur. The temperature control system or that particular plant is actually a distributed parameter model system because the temperature in one corner of the CSTR may be different than the middle point and we have not accounted for that; we have assumed that the temperature of the entire tank is uniform and it is taken as equal to theta.

Take for example a mechanical system, the motor example. We have taken the twisting effect or the spring effect of the shaft to be zero because this resulted into a very simple model for us so we have accounted for only the moment of inertia and the friction. The spring effect has been neglected so it means the model is an approximation. Now you can say that, well, we could have included that, we could have generated a distributed parameter model for a temperature control system, I could have included the torsional effect of the shaft into my model; but please note more complex the model more complex will be the design algorithm and today's literature is not tuned to taking care of a complex model as far as availability of the design algorithms are concerned.

We have design algorithms which assume simple linear model. So, if the complex model you are able to generate then the design algorithm also you will have to you will have to research because the present design algorithms do not meet the requirement. Not that we are not capable of designing or we are not capable of generating the design algorithms but what happens you see the design algorithm become so complex that you see the accuracy achieved in the process of modeling becomes counter productive. So it is seen that is simple design algorithm with a simple model is a better proposition than generating or researching a very complex design algorithm for a very complex system model. So it means at least I should be aware of that that the design which I have carried on a particular plant model G is not accurate because G has got the modeling errors. so you cannot assume a situation where modeling errors will not be there; there are going to be modeling errors and this is one particular point; at least you should test your design; you see may be you have not taken these considerations into your design may be these considerations not embedded in our design but at least we should test our design for various possibilities of modeling errors which can occur.

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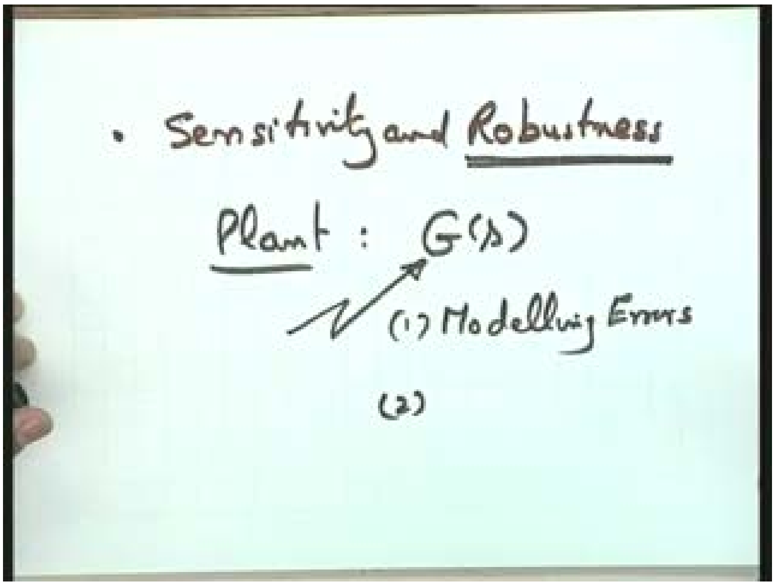

One more point and that point is this that once your design is complete you have implemented that design on a real life situation. But the design was carried out under certain assumptions of environmental conditions. With the working of this system the environmental conditions may change the wear and tear and the aging of the system may affect the parameters of the system. So it means, the model on which our earlier design was based will no more be the same after the system has been on operation for some time. That is, I can say that there will be parameter changes with time. As I said, that I really, first of all cannot include this exactly in my model; secondly, even if I am able to include it exactly the design algorithm will become too complex to be of much use to me and therefore what normally is done is that the design is carried out on the simple model and then we test the system under these conditions and this is known as robustness. If my design is working satisfactorily for finite amount of changes in models whether they are due to the errors or due to parameter variations the system is referred to as a robust control system.

So what is a robust control system?
Design has been carried out, a testing is being done by allowing, now by simulation you can test it for example, allow the changes in the parameters on the system and look at the performance of the system, if it works satisfactorily over certain finite amount of changes the control system which you have designed is a robust control system. And the word sensitivity is related to this particular concept of robustness. If these changes I am referring to are differentially small in that particular case the system is referred to have a good sensitivity, that is, it is insensitive to parameter variations. So you can say that a sensitivity based design or robust control design they have something in common. A sensitivity based design takes care of only differentially small changes in parameters either due to environmental conditions or due to modeling errors while a robust control system takes finite amount of changes in the modeling errors and parameter variations. So this particular aspect the robust control system is actually the subject of intensive research today. However, our discussion in this particular course will be limited to sensitivity based design.

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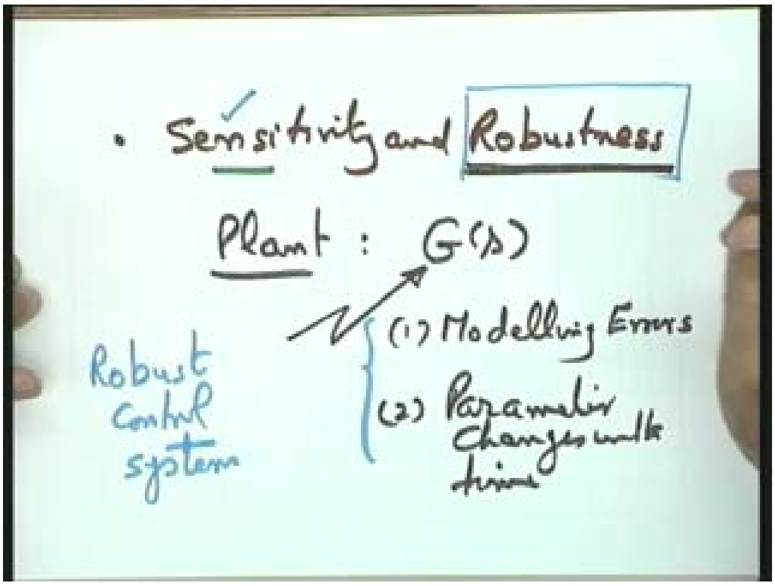

I may be using the word robustness; well, my design is robust, so it means I will be using this word in a loose sense, in a qualitative sense because the analytical methods I am going to use in my course over here will be based on sensitivity that is it will be reducing this sensitivity, so robust word when I use in my discussion should be very clear that it is really being used in a loose terminology and it is only a qualitative form of sensitivity based design. However, this is a very very important concept of the design and it is a subject of research even today.

The next point I like to raise is shaping the dynamic response, shaping the dynamic response, this is also a requirement let us see how is it a requirement. Let me say, let me tentatively give you this picture: This is time and this is my controlled variable (Refer Slide Time: 24:07) and you are at this particular point at t is equal to 0. Please get my point over here; what do I want what do I what do I mean by shaping the dynamic response or how do I improve the transient response of the system.

Now let us say that the command signal is a constant signal which is given to the system at t is equal to 0; t is equal to 0 I can set at any point on my scale and let me represent that signal to be a constant signal y r. So what do you want now? You want this particular y to follow this y r. Theoretically speaking you will like y r, should be like this; well this is my y that is it gives you an instantaneous response. But this is not possible because every component of the system has got certain lag. It takes certain time to react, instantaneous reaction is not possible and hence it is going to take finite amount of time before it settles down to the value y r. So you see that I want two things: One, it should take minimum amount of time to settle down. Let us say that it settles down this way, it settles down this way so it has taken practically this much amount of time to settle down because after this it is at the steady state and the value is let us say equal to y r. This particular point or this much time I will refer to as the settling time of the system; lower the settling time better is the speed of response of the system. This is the concept of speed of response.

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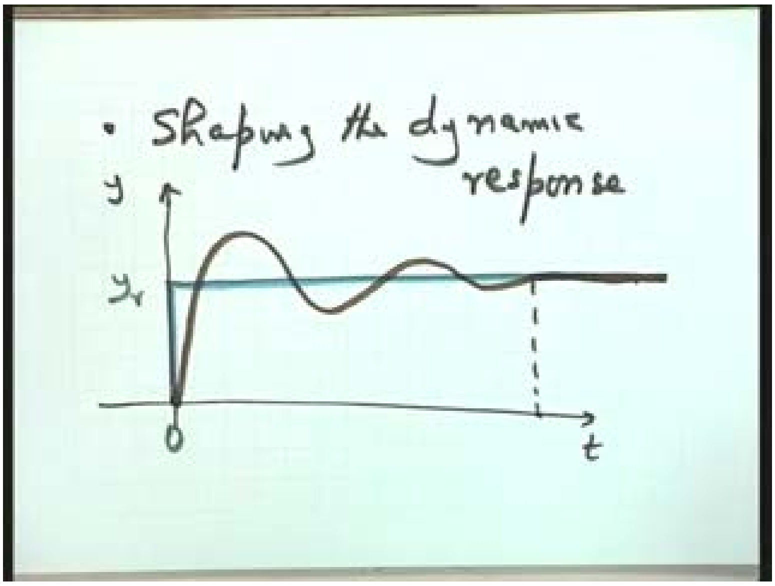

Now what is the steady state accuracy?
I really will want that the system should finally settle down at the value y r; in that particular case steady state error between y and y r is equal to 0. But if it does not go to y r if it stays at some constant value different than y r in that particular case that value you will refer to as the steady state error of the system and you will minimize this particular error this is your steady state accuracy.

Yes please…..
[Conversation between the Student and the Professor – Not audible ((00:26:27 min))] That is right, I will explain this. You said it should or should not oscillate this is what you want to say. Yes, look at this point also, next I was referring to this point only. Now you see that there are various ways for the response to reach this particular point and that is what I mean by shaping the dynamic response. One of the ways I have given like this, that is it oscillates around this particular point and then settles down this value.

Alternatively as he pointed out I may take this situation, let us say it goes this way; I do not allow it to oscillate I let it go to this way it never oscillates, so what happens; here is the shape, that is there is actually a conflict between the two requirements as you will see when you come to the design. As you damp out the oscillations, as you permit the system to follow this particular curve what will happen this may not be visible very clearly from this response so what will happen is the speed of response decreases, this should be evident also. This actually amounts to heavier breaking or damping of the system. you do not allow the oscillations it means you break the response, you damp the response and if you damp the response, naturally, intuitively at this stage you see, I mean quantitatively it will come later, intuitively get the feel if you are damping the response so as to suppress the oscillations naturally the speed of response is going to be killed. So, in that process the system will become a slow responding system which also you do not want. So it means you will like to settle down for a compromise.

On the other hand, suppose you allow the oscillations, so in the process of allowing the oscillations the oscillations may become too large. A too large an oscillation means what? So you see that, it means, recall that maximum input amplitude constrains. If there is a too large an oscillation, at some point of time such a large signal is being applied to some system which may drive that system to saturation or which may even destroy that system. So you see that too large an oscillation may be improving your speed of response but still you cannot allow that improvement because too large an oscillation may drive your system to saturation or may turn the system to other situations which are not acceptable to us. Too small an oscillation or no oscillation situation may damp the oscillations but may reduce the speed of response. So it means actually, as a designer, user will only tell you that user the maximum amplitudes are like this, my speed of response is like this.

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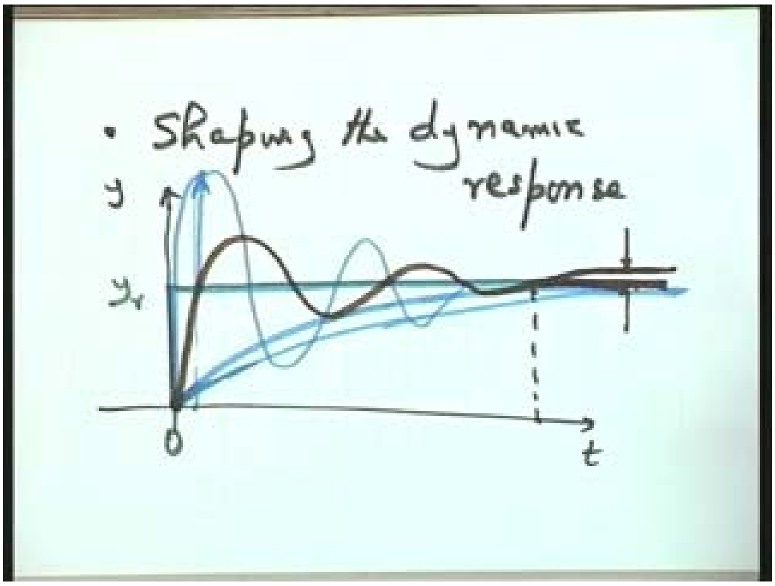

As a designer it will be your responsibility to shape the curve. This is what I want as this that not only this steady state accuracy and the speed of response or the specifications, as a designer you will have to set a particular curve also which the system must follow so that it meets the requirements of input amplitude constraints, it meets the requirements of speed of response and it meets any other requirements which may be imposed on the system. [Conversation between the student and the Professor-Not audible ((00:29:48 min))]

As I told you that, the quantitative definition of settling time will come later. I am going to redefine it later so let me save time. At this point I will say settling time is the time taken for the system to reach the new steady state. And I said that the time taken to reach the new steady state will increase if the system is heavily damped and therefore you will not like to follow this particular curve.

Since you have raised a question, I think I will have to answer it to certain extent. I was avoiding the answer because the detailed discussion is coming later. I say that, well, this is the over damped response and the system has settled down and settling time now in response to your question I may say may be the time for the system to settle down within certain small range of the final steady state value, let me say. The settling time is the time for the system to settle down, it should not come outside this particular range in that particular case as far as this curve is concerned I will say this is the settling time because once the system enters this particular band it never comes out.

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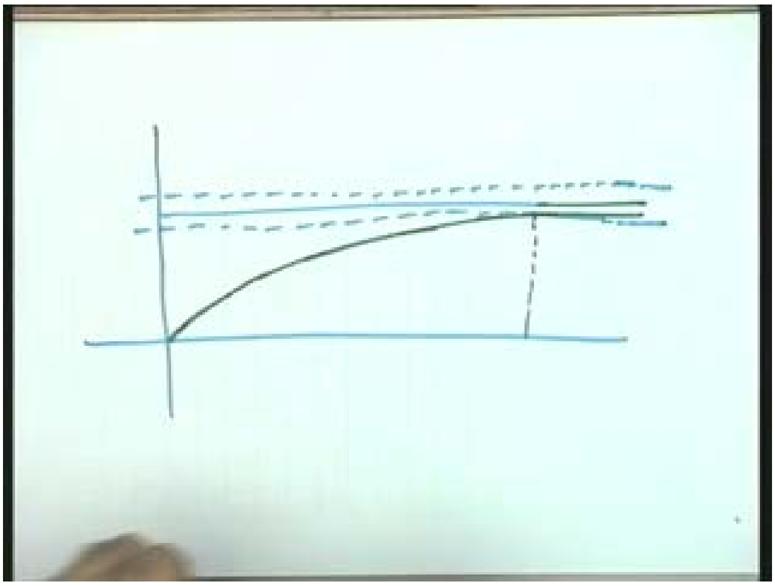

Now you take the other response. The other response is the response in which I allow oscillations and you will see that a typical response may look like this: (Refer Slide Time: 31:24). if a typical response is like this where do you put the line for the settling time, I will like to put the line here because this system has entered this particular band a 2 percent tolerance band or a 5 percent tolerance band depends upon the user requirement; I say that the system has settled if the system has almost reached the final steady state and the word almost am quantifying into a band which is let us say 2 percent band around the steady state value or 5 percent band around the steady state value depends upon our design.

So now you see that, if the system was over damped you see it has taken this much of time to satisfy the definition of settling time and now if I take this particular response (Refer Slide Time: 32:05) it has taken this much of time to satisfy the definition of settling time though in this particular case oscillations have occurred and theoretically speaking I should have avoid oscillations but you see I have to strike a suitable balance. I should allow oscillations to increase the speed of response or to reduce settling time but I should not allow the oscillations to an extent that they violate the requirements of the amplitudes of the signal and hence in the process may make the system non linear or may even damage some of the components of the system. And hence I say, qualitatively at this stage, please see, I mean, you have to observe it otherwise we will be diverting. I am going to take up this in quantitative form in details. At this juncture I will like you to have a qualitative appreciation of what I say.

Qualitatively it simply means that I will have to shape this dynamic response also. How do we shape this response, how do we quantify these measures; I think wait for a while the discussion will follow.

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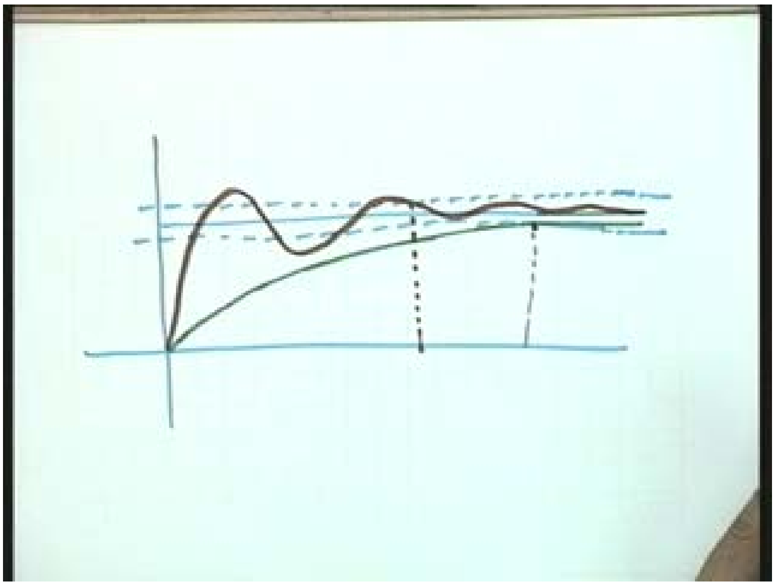

I hope the qualitative appreciation has come. And lastly after the transient response shaping let me say that it is the steady state accuracy, steady state accuracy requirement.

These are the basic requirements which are our design requirements and you will see that most of these requirements are met by feedback design but the unfortunate part is that many of these requirements are conflicted as you will see later. Again a qualitative appreciation I demand at this stage. That is why the design is a complex process otherwise you see it would have resulted into a set of equations. No trial and error was required if the conflict was not there. Unfortunately many of the requirements do conflict and therefore the design becomes a trial and error exercise.

For example, if you try to improve the steady state accuracy, as you will see shortly, the steady state accuracy improvement is a very important requirement from the point of view of user but it results into very large oscillations and even may result into instability. So it means the tendency to improve steady state accuracy, takes you towards larger amplitudes and those larger amplitudes can damage your system or can even make your system unstable. So it means there is a conflict between stability and accuracy and similarly you will find that there is a conflict between disturbance rejection and noise filtering, you will see that.

As you try to improve the filtering of the disturbances coming from the environment the effect of the noise filtering noise on the system increases. So you see that when you improve the disturbance filtering from the environment the effective the noise increases and hence a suitable compromise is required between the two situations. You cannot meet the both requirements unless you take certain precautions. So what I want to say is this that if all these requirements were nicely met without any conflict in that particular case the design was simply an exercise of solving a set of equations systematically. but this not the case, unfortunately there is a conflict and hence the design is a trial and error method, a trial and error complex method where the intuition of the designer, the experience of the designer plays a very important role. With this now I go to quantitative discussion so that all these aspects could be taken up now quantitatively as to how the feedback is going to affect the various requirements of design. The feedback verses non feedback systems will be compared now. Basically feedback verses open-loop systems I will like to compare.

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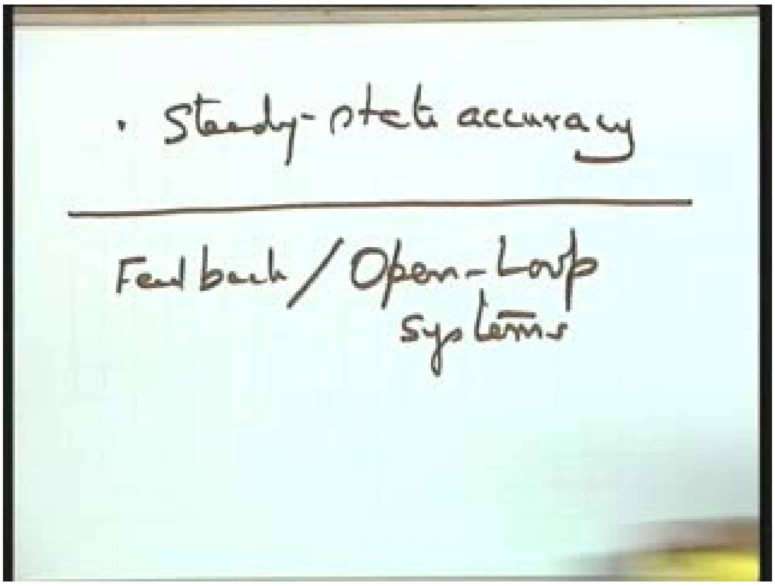

Feedback is a closed-loop system and an open-loop; these terminologies I think are quite obvious, I need not explain this again. And to explain this I first take up the sensitivity issues. You will please note I will be taking now onwards only sensitivity and not robustness in its true sense of definition. Let me say that I take certain performance measure change, let me define the way sensitivity I will be using though in some form or other the definition may be known to you.

Let us say this J is speed of response or this J is some other measure of system performance and this J is a function of some parameter theta some parameter of the plant this J is a function of that and I want to see the effect of changes of this parameter theta on J and let me assume that theta has a nominal value theta n with a deviation theta del theta. Note this point. This deviation may be coming with time, the parameter variations or this deviations we may be modeling as a result of our errors our approximations at this stage of modeling itself. So del theta is a representative of any type of error which we have cost. But you see that del theta many a times is deliberate error we have caused to simplify our design process.

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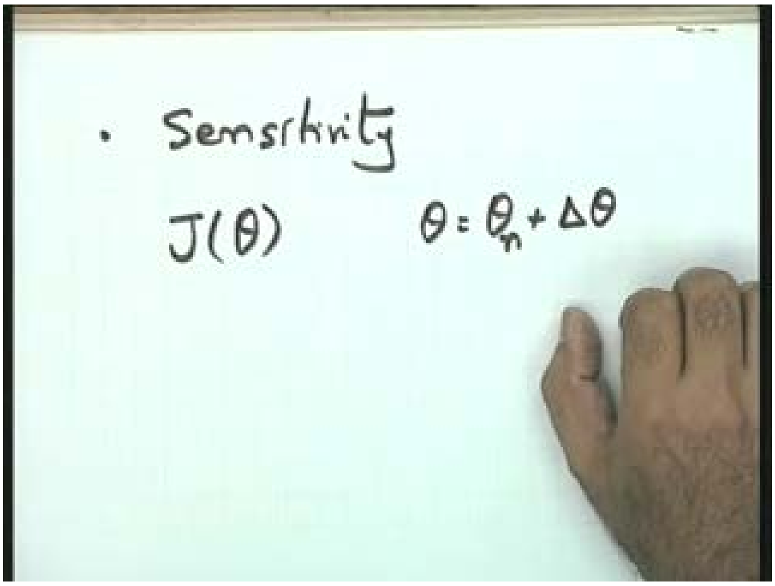

So, if this theta is given by theta n plus del theta in that particular case I say the nominal value of your performance which is the speed of response that is you may call J as a settling time, you may call J as any other performance measure; the nominal value of this is given by J as the function J n (Refer Slide Time: 38:24). Now, since sensitivity is concerned with differentially small variations I can really use Taylor series expansion for J theta around the nominal points. So what I get is the following: J (theta n plus del theta) you will help now, what is this equal to J (theta n) plus del J by del theta at theta equal to theta n into del theta plus second n higher order factors.

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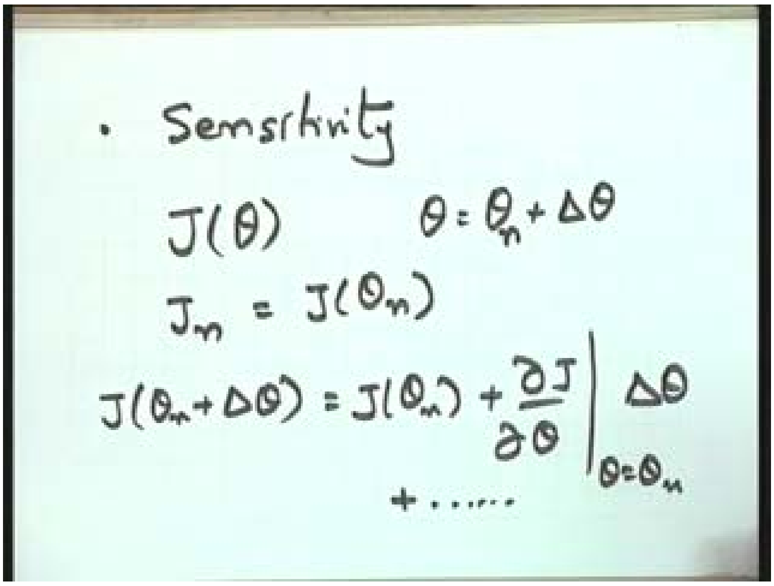

So even in this analysis I making an approximation; second n higher order factors are being neglected so I assume that only the first order variation is considered. In that particular case you find that the equation becomes: J (theta n plus del theta) equal to J (theta n) plus dJ by d theta at theta equal to theta n del theta. This becomes my equation, the second order terms have been neglected.

Now, sensitivity you see, first of all I am giving the quantitative sensitivity definition also, the nomenclature should also become very clear. I will be symbolizing it by S and this means sensitivity of J (Refer Slide Time: 39:41) with respect to variations in parameters theta this is the symbol I will be using to give you the sensitivity function. Sensitivity of J is some measure of performance and theta is the parameter whose variation is under study, this is defined as percentage change in J so it is del J by J n the nominal value divided by percentage change in the parameter which is del theta divided by the nominal value theta n.

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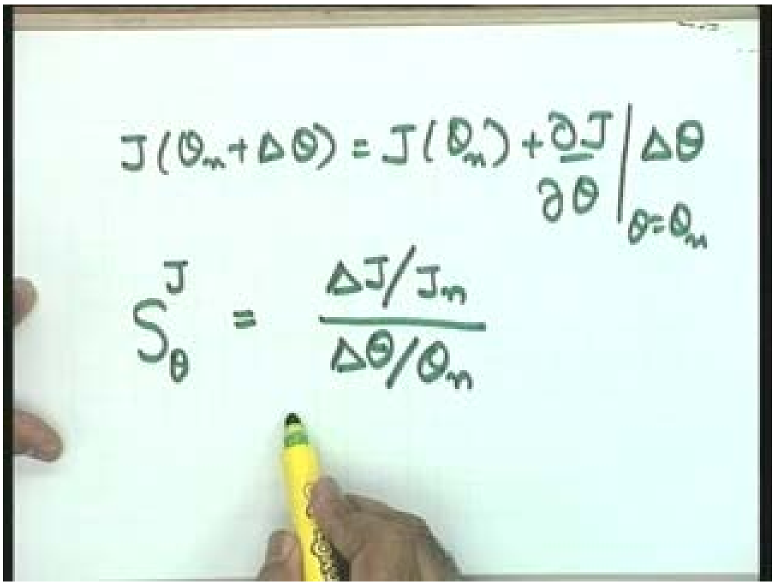

So you see that, if you look at this definition, actually I have given you an input output sensitivity model. Suppose for a particular system you have calculated this function, I mean, I am now getting this, you can use the computer software or you can use your own method of solution to get how the system responds to various variations in del theta by theta n. I can say that del theta by theta n is an input to the system because this is the parameter variation which you are going to consider, this is your model this S J theta and this model gives you the variations del J by J. So, by sensitivity analysis what do I mean, please see.

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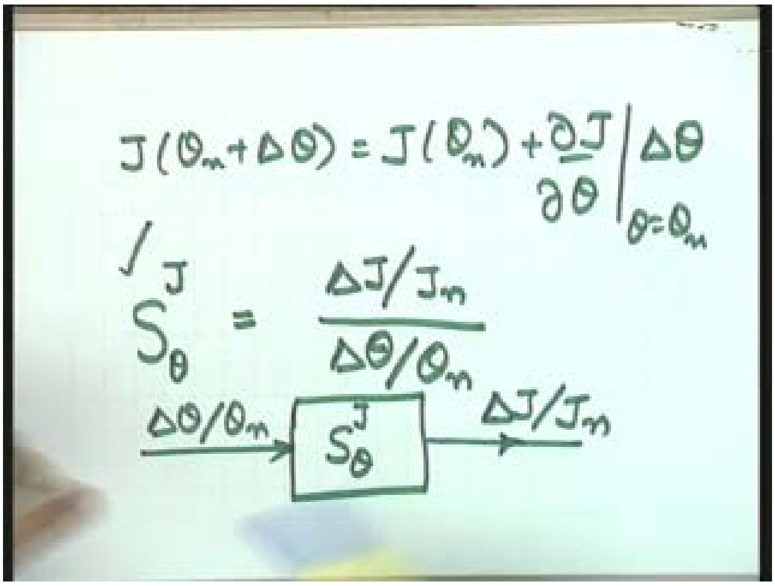

By sensitivity analysis I will require this model. Once this model is available to me then it is a simple simulation exercise as we have been doing for the plant transfer function. The simulation exercise demands that you can consider any parameter you take small variation of that parameter give that differentially small variation. Well, finite amount of variations you will have to give. Though this S J theta has been obtained under the assumption of differentially small variation but fortunately this model works even for finite small variations and therefore you will simulate and see whether this model is effective or not by taking small variations in del theta and seeing the response on del J. So this is a percentage change in the parameter and study is on percentage change in the system performance and S J theta can easily be obtained.

Give me the value please. How do you get S J theta? Let me use this expression itself. What is del J equal to? Del J equal to dJ by d theta theta equal to theta n del theta. This is my del J, just subtracting these two and therefore S J theta becomes equal to del J by del theta at theta equal to theta n multiplied by…. help me please, i hope some of you would have solved this expression, I am now asking you for the expression S J theta dJ by d theta evaluated at theta is equal to theta n into theta n by J.

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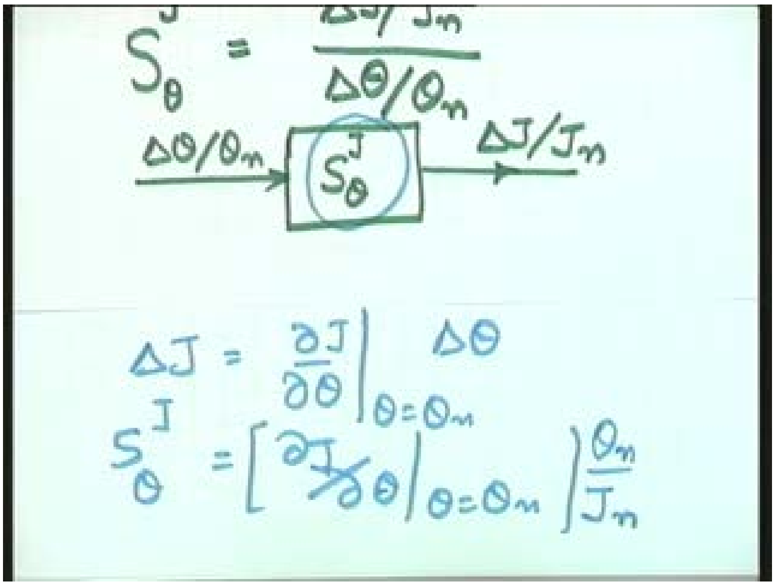

So you see that for a particular system you will have to first evaluate this value: del J by d theta study the system evaluate this sensitivity function, theta n and J n are the values at the nominal point so it means the sensitivity model is available to you. Once the sensitivity model is available in that particular case that you give del theta by theta n as the input and get the response del J by J n as the output of the system (Refer Slide Time: 43:21) this what we mean by sensitivity analysis.

Now with this definition of sensitivity I can now come to the feedback system and I want you to comment on the effect of feedback on sensitivity. I assure you with this appreciation of sensitivity function you will be able to comment on the effect of feedback concept sensitivity. and I take this as the feedback model G D is the controller plus minus R here is H. Of course all of them are functions of the Laplace variables, this is your Y. This is the basic feedback loop.

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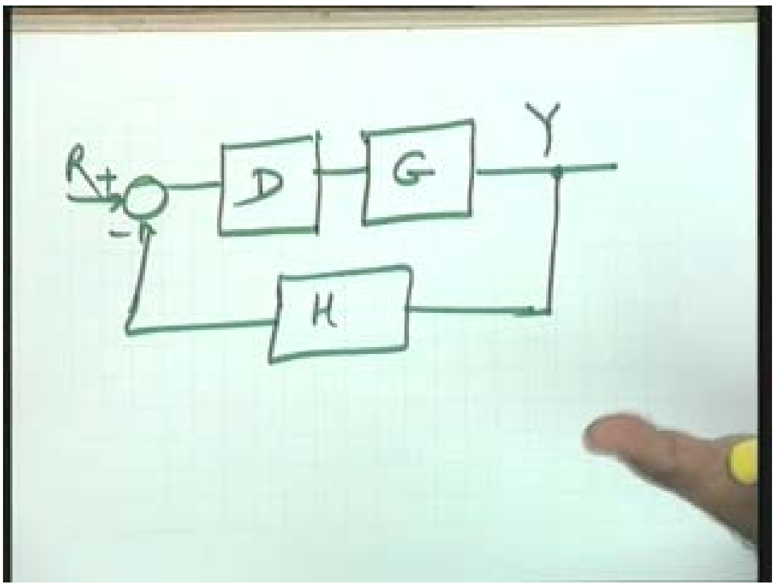

Hence, assume for the time being that there is no disturbance on the system; the basic feedback loop is given by this. So the transfer function of the system is Y by R is equal to M is equal to….. help me please it is DG over 1 plus DGH. Now you see that something which is beyond your control is G only and of course H also. The controller may be you can design properly so that the parameters do not change. These are the hardware in this system; the controller is an open-based system hoping that the controller parameters will not change you can have an appropriate design. So these are the system hardware whose parameters can change.

I really want to see the effect of the variations in the parameter on G or in H on the overall transfer function of the system because if I see the effect on the overall transfer function equivalently I have seen the effect on the system output Y. So it means you see, please note I am interested in S M G or S M H this point may please be noted, the effect on the system performance. I really want to see, let us say that G is a function of theta, H is function of alpha these are certain parameters, I want to study the effect of variations of theta on Y, the effect of variations of alpha on Y equivalently I will establish the sensitivity functions S M G and S M H to study the effect of theta on Y or alpha on Y because it is equivalent to studying the effect of variations on G, on M and the variations of H on M this may please be seen.

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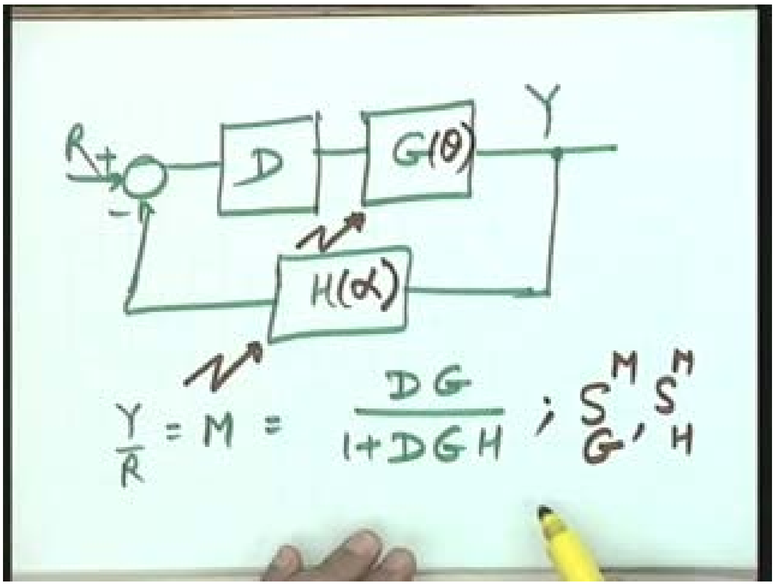

It is an equivalent because these expressions turn out to be neater; they have certain there is absolutely no problem, I may point out that you can establish equations for sensitivity of Y with respect to theta, for sensitivity of Y with respect to alpha but these equivalent expressions have something more to convey and therefore I like to concentrate on these equivalent expressions. Give me the expressions now. I have given you the basic definition of sensitivity; I have given you the type of expressions I am interested in so now I want your help please. M as a function of theta let me write down completely and as a function of s also so that at least we should be clear about what we are studying; D(s) G(theta n the nominal value, s) divided by 1 plus D(s) G (theta n, s) H(s); theta n is the parameter which is the theta n is the nominal value. So at the nominal point I have got this M.

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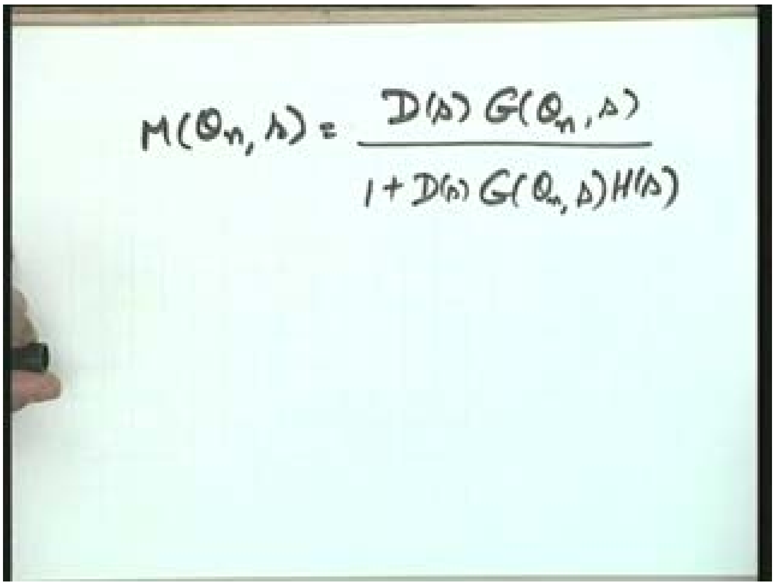

Please help me; I want the value of S M G the sensitivity of M with respect to J. First of all you give this expression dM by d theta at theta is equal to theta n I will take. You give me this expression then the other expressions will easily follow. Well, can I directly write D over 1 plus DGH squared dG by d theta at theta equal to theta n. Small exercise for you. You do it because theta appears in the numerator as well as in the denominator, please use your derivative expressions properly and rearrange, you are going to get this in the form: D over 1 plus DGH squared dg by d theta.

Anyone who is already evaluated so that we can compare if I have made an error? Yes, the expression for dm by d theta I will like to check before I base my results on this, this is okay. Please do check this. Now from this expression given the value of S M G please come on I need the result from you. So I want del M by M first; what is del M by M? Del M by M is I will divide this total expression by M (Refer Slide Time: 48:50). Total expression by M is, I will divide it by dG over 1 plus DGH so this will become D over 1 plus DGH square 1 plus DGH over DG dG by d theta theta by theta n; is it not equal to D divided by D also goes…..so 1 divided by 1 plus DGH dG by del G by G at theta equal to theta n; hopefully it is correct please check.

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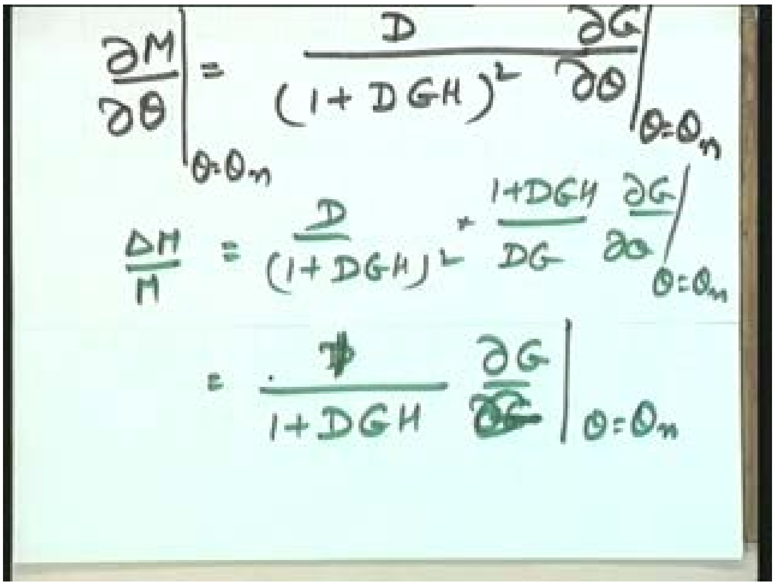

Delta theta well, you see on this side also it does not appear; as a finite value I have removed this. That is, your partial derivatives if I take del M by del theta this is del G by del theta. You see the sensitivity has been converted into finite changes. If I consider finite changes del M by del theta is equal to this expression del G by del theta then this expression results. Please see, del theta has gone from both the sides approximating it by a finite quantity. So del M by M is equal to 1 over 1 plus DGH del G by G theta is equal to theta n. this is of course to be evaluated at theta is equal to theta n.

So what is the result now?
the result of sensitivity S M G equal to…..come on please, let me say it as S a function of theta n and Laplace variable s; I need the expressions from you only now, yes please, give me the value the basic apply the basic definition. Yes, [Conversation between Student and Professor – Not audible ((00:50:51 min))] Say it loudly: 1 plus D(s) G(theta ns) H(s) please see, this is fine. You can see just I have divided the dm expression by del G by G and I get this as the sensitivity function for the system. This you can easily recheck in case there is a gap in the derivation. Please sit down calmly and recheck and you will find that this expression is right. Come on, now you give me the interpretation of this because I know only five minutes left, the interpretation of this expression should be clear.

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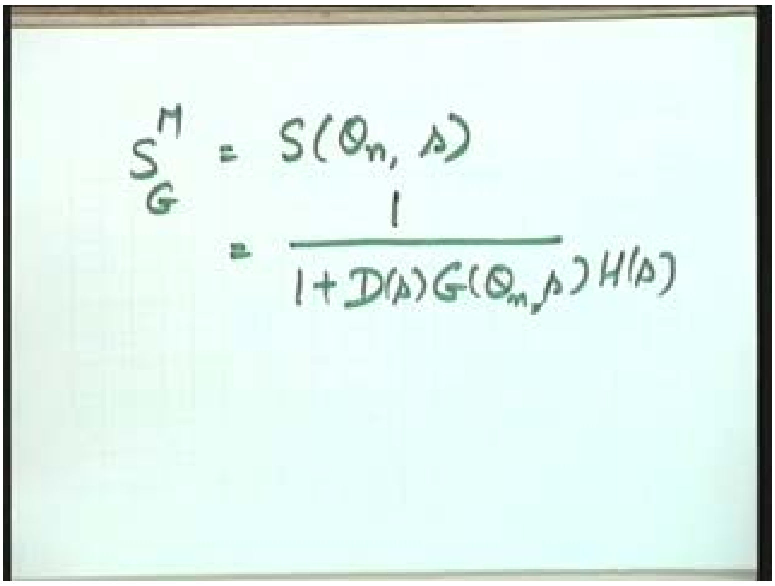

Interpretation of this is the following that this particular D (Refer Slide Time: 51:40) the controller which is in your control you can change the value of del G H which I consider as the loop gain. please note; G is not under your control, H is not under your control once the hardware has been selected; what you can control is D, if you control suitably the gain of D in that particular case this loop gain D(s) G(theta ns) H(s) can be suitably controlled so as to reduce the sensitivity to a level you want. You can see that higher the loop gain lower is the sensitivity.

Can you tell me what is the sensitivity of an open-loop system?
In the feedback diagram we have taken make H is equal to 0 and tell me the sensitivity of the closed-loop system with respect to G. Come on please, the sensitivity of the closed-loop system M with respect to G once I make H is equal to 0 I remove the feedback link is equal to 1. This you can see from this expression also set H is equal to 0 you get the sensitivity as equal to 1. Please see in this particular case: While the open-loop system is highly sensitive to the variations in G because sensitivity is equal to 1 the sensitivity of the closed-loop system can be reduced to the level you want by an appropriate design of the feedback control that is D(s) the controller is in your hand you can design this controller appropriately so that the sensitivity of the system can be appropriately controlled.

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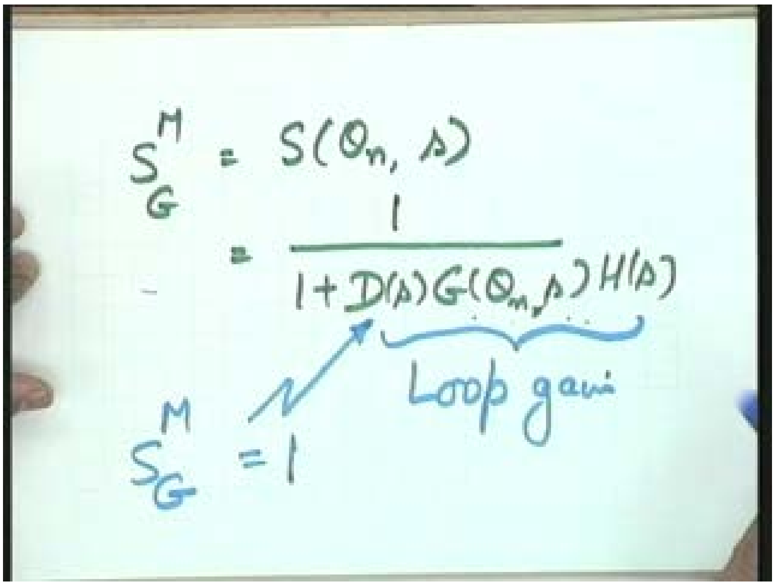

An expression I like to give and leave the derivation to you: S M H the sensitivity of M with respect to H the sensor; please see, this turns out to be minus D(s) G(s), you just on the analogous lines derive it this will be 1 plus D(s) G(s) H(alpha, s) let us say. This is the sensitivity of M with respect to H. Please now tell me, what is the effect of increasing the loop gain on the sensitivity when you consider the sensor? If you increase the loop gain you find what is the value of the sensitivity; it approaches 1, magnitude of this approaches 1. So it means the system becomes highly sensitive to the sensor when you increase the loop gain. So it means you have to pay the price for increasing the loop gain and this sensor was not there in your open-loop system so it means you have introduced an element by introducing a sensor into the system; you have introduced an element of high sensitivity and that is why I pointed out to you the sensor design has to be suitably taken care of so that at the hardware design itself you take care of the sensitivity problems.

The last point I mentioned that this H the sensor this derivation I am leaving it to you, you simply derive it on the lines of S M G and this is the value you have got dg over 1 plus DGH. Is it DGH? Let me check this again, I am giving the final expression without derivation dm by d theta it is DGH by 1 plus DGH please. This, you please derive DGH over 1 plus DGH is the expression. Now if you increase this loop gain (Refer Slide Time: 55:23) which I said in the previous case the increase in the loop gain reduces the sensitivity of the system to changes in the plant parameters.

Now increasing the loop gain means it nearly becomes equal to 1; this was an error H I had not used over here, this nearly becomes equal to 1 meaning thereby that the system is highly sensitive to sensor parameter variations. So it means, the introduction of the feedback has brought an additional problem of the variations in the sensor parameters. So I want to tell you that the sensor design has to be very appropriately taken care of so that the parameters of the sensor do not change because you see, changing the plant or designing the plant so that its parameters do not change is a very costly proposition many a times impossible proposition then changing the sensor. You may make a statement that when you are making a statement for sensor why not for the plant itself. The plant means a huge industrial environment and huge industrial plant, you cannot modify the plant but hopefully you can modify the sensor.

So it means the sensor design will have to be appropriately taken care of so that the parameters of the sensor do not change because I know that the system, by the basic nature of introduction of feedback, has become highly sensitive to sensor and a suitable filter, a suitable design of the sensor is a prerequisite otherwise you are introducing problems in to the loop through the sensor. Okay I will continue with the discussion, thank you.