Control Engineering
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Lecture - 12
Models of Industrial Control Devices and Systems (Contd….)

We can now start with the hardware of the control systems. I introduce some case studies wherein the hardware used in the industrial sector will be introduced to you. Well, naturally we cannot go to the reasonable depth of discussion in terms of the hardware details but the purpose will be their basic characteristics and how those characteristics affect a particular control system that will be the basic objective of our discussion. I take up the first control system as a speed controlled system.

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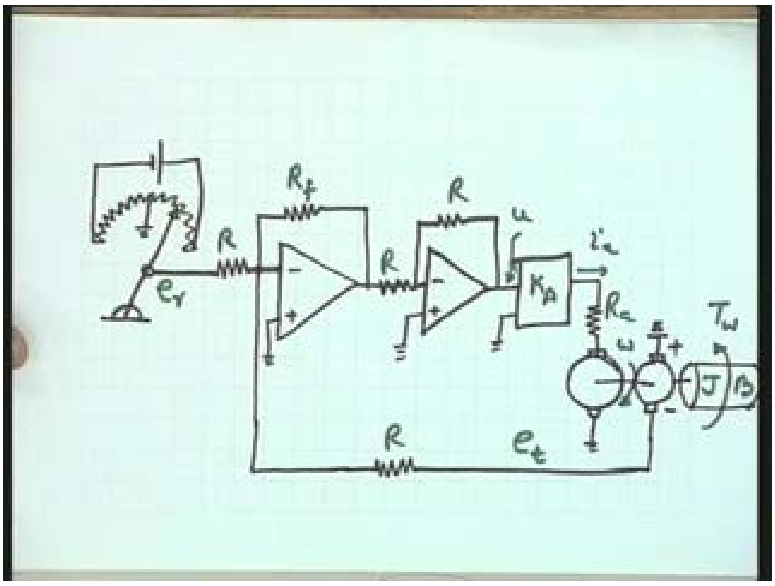

This is the speed control system and now you see how the system works and what are the basic ingredients in this particular system. In this particular case you want to control the speed of the load. so here is a load the parameters are J and B. J and B are the parameters, the movement of inertia and the viscous friction B. T w is the load torque on the system; that is it is a disturbance torque acting on this particular system. now omega being the speed so it means there is a specified omega and you want the control system to act in such a way that this particular load runs at the specified value omega in spite of disturbances T w acting on the system. This is the objective.

So in this case now, first of all, I needed an actuator, the power device in this particular case the actuator as you see is a motor with a power amplifier. So this power amplifier and the motor they constitute an actuator for this particular load because they provide the necessary torque for the load to run at the speed omega. Now this particular actuator (Refer Slide Time: 2:52) is getting the control signal u from a suitable controller, here is the controller circuit which in this particular case is a simple amplifier. At a later stage we will see that this controller could consist of complex functions like derivative of an error signal, integral of an error signal or in general any function you could take at this particular point. But here in this configuration in the basic structure I assume that the controller is a simple amplifier because the objective at this particular point is not to go to the design of the controller but the objective is to give you the basic hardware required for a control system. So you find that this amplifier now this is getting two signals: One is the signal reference signal, e r is the reference signal please see and this e r you are able to vary by varying the position of this wiper arm. So it means you will change the position of the wiper arm (Refer Slide Time: 3:56) with respect to your requirement on the commanded speed.

Omega r in this particular case, not explicitly shown. Omega r is the commanded speed and you want omega to follow the commanded speed omega r. So what I am doing is proportional to omega r I set a signal e r a larger omega r means a larger e r and I can set it accordingly by setting the wiper on the potentiometer. So this e r becomes the reference signal and where is the feedback signal; the feedback signal in this particular case is coming from a tachogenerator.

Tachogenerator as you know will generate a signal e t which is proportional to its speed the speed being omega so it means e t is proportional to omega. So it means e t has got the necessary information about the controlled variable, e r has got the necessary information about the commanded value and the difference of the two is going to give you an actuating error signal. So this actuating error signal is being amplified and this amplified signal which now becomes the control signal u is given to the actuator and the actuator is running the load. And the purpose of the controller design will be make suitable changes at this point because all others are fixed components of the system; you will not like to change the motor the tachogenerator you will rather manipulate this particular function so that omega is following the commanded value omega r. But at this point we will not go the details of the design rather I will like to express this diagram using a block diagram and from there the visualization of the feedback structure and basic elements required in a system will become very clear.

I may refer to this diagram again I need your help now to set up a block diagram. What I do is I take this as the reference signal e r this is the reference signal which is proportional to or which is a function of the commanded signal omega r. I think if you do not mind though the physical diagram does not have this component let me have this component and call this component as reference input elements. Please see, physically potentiometer is taking care of this and the reference input elements may have the signal omega r. So omega r is the commanded position and e r is the corresponding voltage.

Now let me take an error detector in the standard block diagram. This is plus and here it is minus (Refer Slide Time: 6:45). You will note, I think I can go back to the diagram, u will note that e t is a minus signal the e t plus minus over here, the e t the suitable algebra sign has been given to e t so as far as the block diagram is concerned as far as this amplifier is concerned you can see e r minus e t is being multiplied by the gain of this amplifier.

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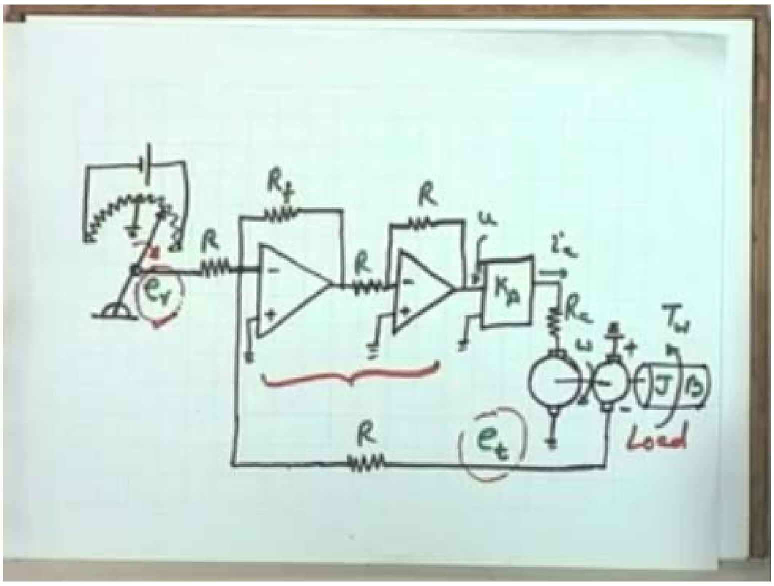

What is the gain? It is R f by R is it okay please; R f by R but with the negative sign. to take care of this negative sign I have used an inverting amplifier with a gain of unity so that the gain of this total amplification unit is R f by R with a positive sign and inverting amplifier has been used over here just to take care of the sign of the gain of the amplifier and therefore I can say that the output e this error detector output e r minus e t, so look at the unit now. What is this circle? In terms of physical system, this dotted block in terms of the physical system is your potentiometer. The circle the so-called error detector or the summing junction in terms of the physical system is an Op-Amp because the Op-Amp is getting the two signals e r and e t is generating a signal which is proportional to the difference of the two and the proportionality constant is R f by R.

So here if am putting in this particular block an amplifier gain R f by R and let me say that u is this control variable. So you will please note, the physical relevance the physical equipment also is which is being used to realize the various objectives. Now u is the control signal, this is a signal which has the knowledge about the action to be taken but it does not have the necessary power to drive the actuator. So you will like to add power to the signal keeping the basic intelligence at the same level. So what you do is you simply add here a power amplifier K A. K A is the gain of the amplifier. Now this power amplifier is able to drive the motor. So what is the transfer function of the motor I will see but at the time for the time being let me say that this block is a motor block; suitable transfer function of the motor I am going to use over here. The output here is the controlled variable omega and this omega is being sensed by the tachogenerator which is a sensor.

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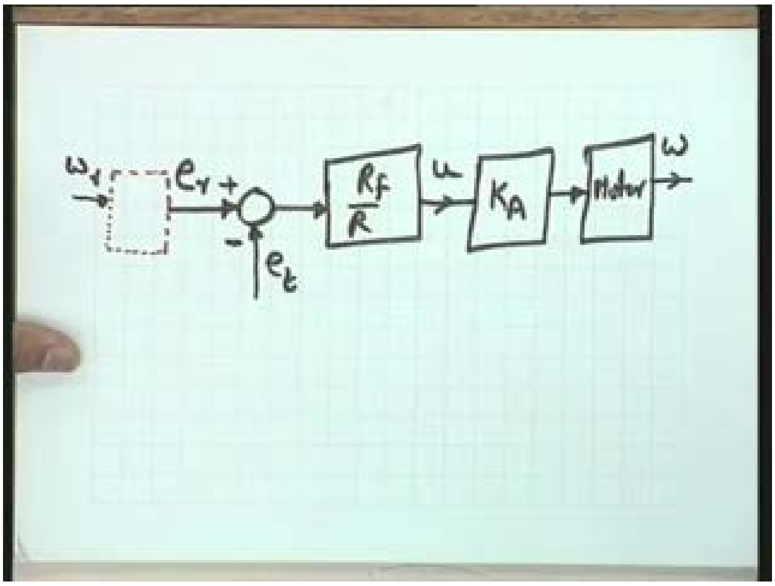

Let me say K t is the constant of the sensor so that e t is equal to K t into omega and this is being fed back here. This is the basic block diagram but for the motor block to be opened to be th detailed properly. So I am going to give you the transfer function of the motor block but before we go to the motor block let me look at this block diagram with respect to the general feedback structure which was given to you. Tell me please, this diagram this is a dotted block (Refer Slide Time: 9:59) this does not contain any physical component. If you look at the diagram from e r to omega and a feedback loop it is a non-unity feedback structure non-unity feedback structure. It is naturally in this particular case there is a sensor transfer function and this you are going to have as e cap, it is an actuating error signal and this is a non unity feedback structure.

Now suppose you see, as I told you, that for the purpose of analysis and design the unity-feedback structure may be more convenient. In that particular case I will like to transform this to a unity-feedback structure without sacrificing the information contained in it. So naturally I like that the basic equation remain the same. If the basic equations remain the same in that particular case you please note that this K t this particular block should contain K t and it has to contain K t. You please see that this K t is not a physical component there but I will set the potentiometric constant in such a way that omega r into K t becomes the desired voltage e r which you want to feed as the reference voltage.

You will see that it is obvious that the comparison of the two voltages will give you an error signal which is proportional to the errors in speed if and only if the dotted block has got K t. I hope this point is being taken very well. This e is a voltage signal it is not a speed signal the unit is not radian per second the unit is volts but this volts will be proportional to the error in speed if and only if this block has got K t and this is in your hand you see. You can set the potentiometer constant. You know that the potentiometer constant is…… what is the potentiometer constant the potentiometer? (Refer Slide Time: 11:58) this voltage equal to a constant K p into setting of the wiper arm so that is a potentiometer constant. This, you will design in such a way that the e r which you get, the K p or the potentiometer constant which you get equals K t which is the tachometric constant so that in that particular case the error in volts is directly proportional to the error in radian per second the error in speed. So, therefore the reference input elements, this is what I want to convey, may or may not be physically present in the system in the form, there is no unit like a tachogenerator. K t is the constant of the tachogenerator there is no unit like a tachogenerator on the input side but still I have a block with transfer function as K t because it makes sense, it explains the working of the system and this realization of the block is suitably done by the potentiometric unit in the system. So this becomes the reference input elements for the system. And now you see that, what I am doing is the conversion of this diagram into a unity-feedback system: omega r I take, you will now tell me whether it is okay plus minus….. I club K t here, here it is K A motor I am yet to explain omega and it goes you can just see.

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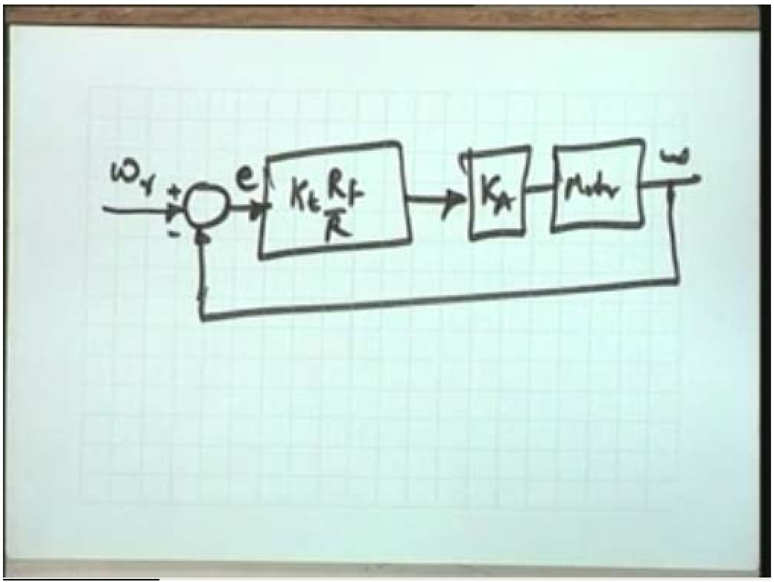

The two equations will remain the same but now it should not be confused; omega r is the reference input does not mean that physically radian per second is directly going to some unit, it does not mean that. It simply means that it is an equivalent block diagram which has got the same mathematical equation which you get from the original block diagram between omega and omega r.

Now in this particular case this is e and not e cap because e is the system error, e is the error between the commanded speed and the actual speed and the cap can now be removed and a unity-feedback system you can get e variable over here. You can see that block diagram manipulation is required to help analysis and design. I claim that this type of block diagram when we go for analysis and design is going to be more convenient. So a non-unity feedback system has been appropriately converted to a unity-feedback system for the purpose of for the usage of this particular block diagram. I hope this is okay.

Now let me come to the motor because the block diagram is not yet complete. If I come to the motor this motor used in this particular control system has got this configuration. In this case the armature voltage is e a and R a is the armature resistance, i a is the current through the armature and here is the motor shaft and I have a load here. The motor produces a torque T M and the load disturbance torque is T W, the parameters of the load are J and B and let us say the speed at which the motor runs is omega. I want you to give me the mathematical model of the system so that the motor block in the diagrams we have used could be replaced by that block diagram model or the transfer function model of the motor. These things are known to you.

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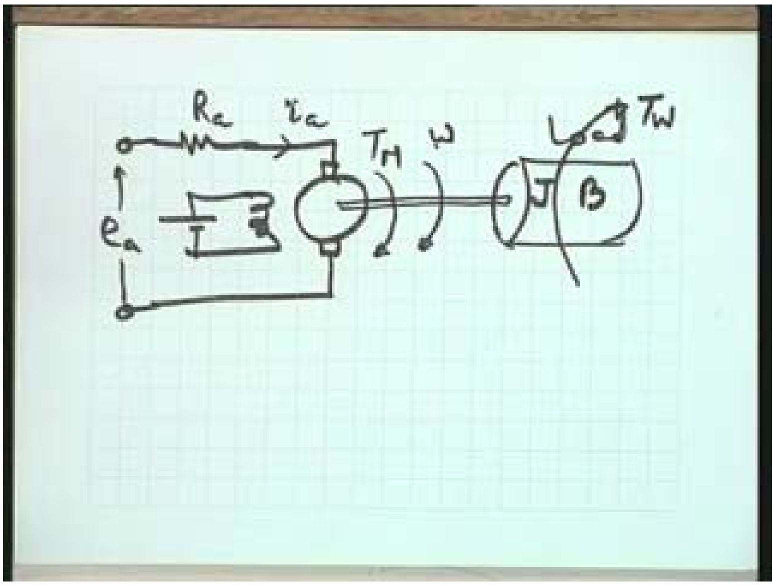

Well, I will simply revise the basic relationships for you. Let me look at the armature circuit; e a is the applied armature voltage and e b is going to be the back EMF. You will see that I have neglected the armature inductance in a simplified model. This is usually okay I mean as far as the characteristics of the motor is this armature controlled motor is concerned that the inductance can be neglected. So in that particular case the basic equation becomes i a equal to e a minus e b divided R a this is equation 1 I get for the motor model.

Now this i a you will note that if the field current is constant and i a depends upon the applied voltage. However, the torque produced by the motor as you know is proportional to the product of the two fluxes and hence the two currents. Since i f is constant I can say that the torque produced by the motor is proportional to the armature current i f being constant. So I now write T M is equal to K T into i a where K T is the torque constant of the motor, the point may be please be noted; K T is torque constant of the motor and this has come this way because i f the field current is constant and it is an armature controlled motor that is why the name is given because you are controlling the armature current through i e a and the field current is held constant. The speed of the motor is being controlled by controlling the current through the armature.

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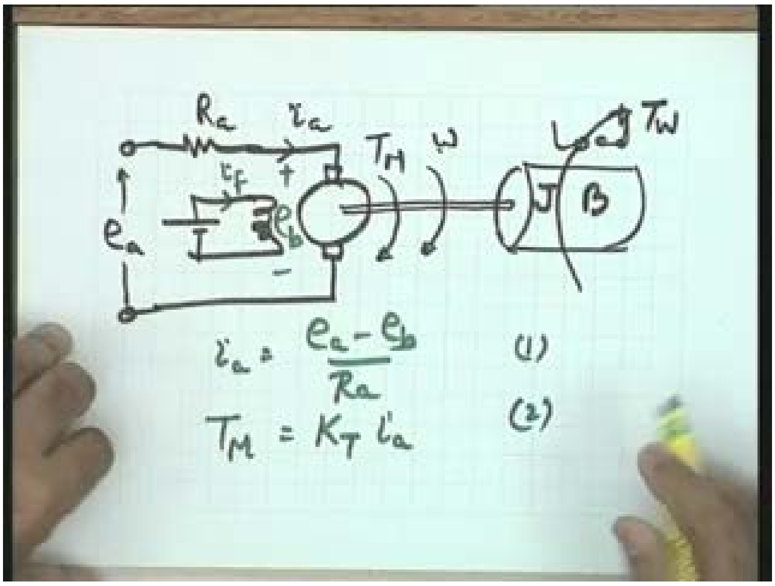

What other equations will come? What is the value of e b please? This is equation number 2. e b, I can take as proportional to speed of course and therefore I take a proportionality constant K b which is the back EMF constant proportionality constant K b into omega, omega is the speed in radians per second and this is your equation number 3. And now this torque T M what is this torque doing; this torque is driving the load. So, write an equation the force balance the torque balance equation over here, in that case I can write T M is equal to J omega dot plus B omega plus T w this your number 4. T w is the disturbance torque, J omega dot is the inertial torque opposing the motion, B omega is the frictional torque opposing motion so the torque applied by the motor is equal to all the terms opposing the motion so this becomes the basic equation.

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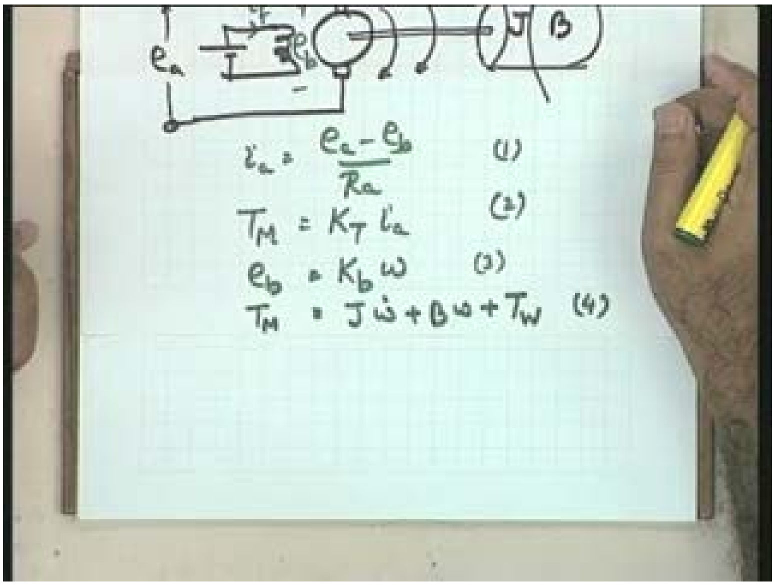

Come on please, help me now. I want you to give me these four equations in terms of a block diagram. Because as I said that manipulation in terms of block diagram will become more convenient for us in terms of the block diagram and we can avoid writing these equations in Laplace domain. You can directly give me you see. I think, now you should be equipped with the required tools that the manipulation can be done directly. So what do I do? E a is your input signal, this is the input signal so let me get started with the input signal.

This is E a; in Laplace domain I can write this as E a(s) (s) I am not writing to save space. Now this input signal you find the back EMF signal coming over here plus minus so this is E b. the difference of the two divided by 1 by R a is your armature current I a please follow up the equations. And once we have done it you see that for later applications you will be able to write the block diagram directly without explicitly writing the equations. I a you have got, multiply it by the torque constant you have got K T, can you help me here what should I do to proceed further? Now I have got the torque produced by the motor; it is I a into K T. Now what is the net torque which is available for driving the inertia, it is not T M it is T M minus T W. T W may oppose or help I do not know but you see the net torque is the algebraic sum of T M and T W.

So in that particular case I can use a summing point here. I can say that plus and here it is T W minus so the net torque available for driving the inertia is T M minus T W I think its I hope it is okay and now this torque since this is driving the inertia this particular block could be 1 over Js plus B with output as omega, you can see the basic relationships. This omega (Refer Slide Time: 21:09) now can be picked up by the sensor and here is your sensor transfer function K t and this is sorry I am sorry there is no sensor here this is the transfer function of a motor only in the open-loop mode there is no sensor as far as the motor is concerned, this is actually K b the back EMF. this feedback link please see is not being provided by any sensor it is inherent in the structure of the motor in the motor operation; K b is the back EMF so K b into omega as your back EMF and here gets the loop completed.

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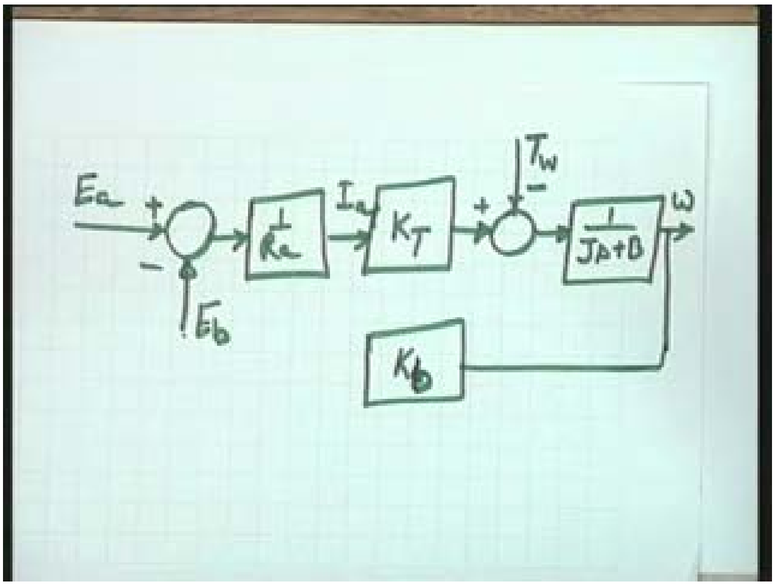

So you please note there is a feedback loop; though if you look at the motor you have not installed any sensor to give the feedback signal. This is an inherent feedback loop in the motor and this rather is an important characteristic of the armature control motor. you will see that when I take the field control mode the inherent feedback loop is not there and many a times this particular inherent characteristics of the armature control motor is a useful characteristic because it provides the required damping, this I will show later. So this becomes the block diagram structure. So let me now put it in a more convenient form which I will be using quite often. e a does not matter I can put the variables in this form as well plus minus K T by R a plus minus T w 1 over Js plus B omega K b please see that.

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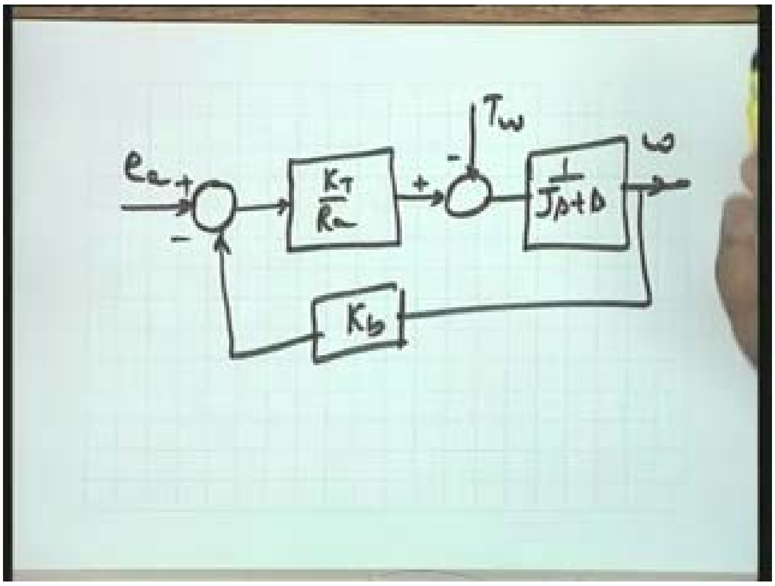

This is the block diagram model for the motor and I can place this model in the overall system model to get the block diagram model for the speed control system under consideration. Before I go to that let me study the characteristics of this motor. Please see, if I set the disturbance zero I get the transfer function between omega and e a, I need your help here; omega s over E a(s) is equal to K T by R a divided by Js plus B over 1 plus K T K b by R a divided by Js plus B. This becomes the transfer function model between omega and e a.

Please simplify this. If I simplify this model I get this as equal to K T divided by K T by R a let me take it here Js plus B plus K T K b by R a this is your transfer function model for the motor.

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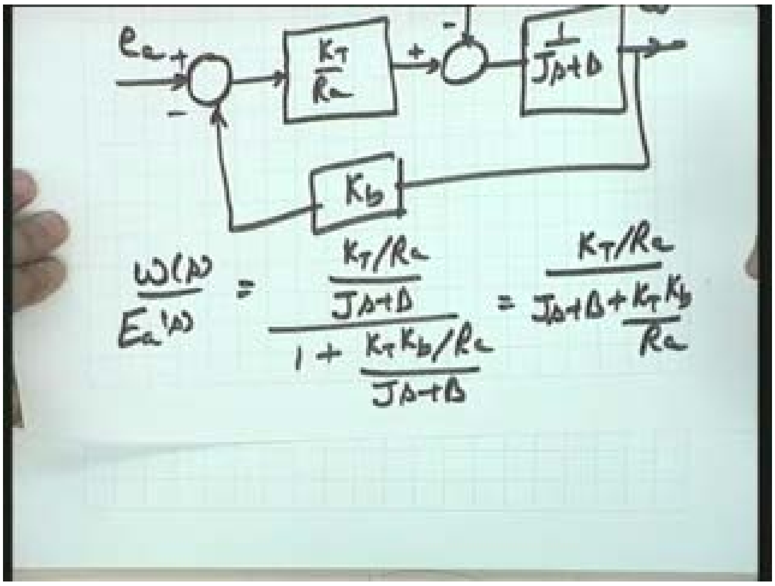

Now, in terms of the time constant could you give me the transfer function model for the motor in terms of time constant?

I will like to write this as omega s over E a(s) is equal to K m the motor gain let me call it divided by tau m s plus 1 it is a first-order transfer function model and tau m can be suitably written. please see, tau m will become J divided by this total unit (Refer Slide Time: 24:51) J divided by this total unit is your tau m and this function divided by this total unit this total term becomes your system gain or the motor gain constant K m. so the motor transfer function between the output omega and the input e a is a first-order transfer function K m over tau m s plus 1. Please store it in your memory I will be requiring this information quite often.

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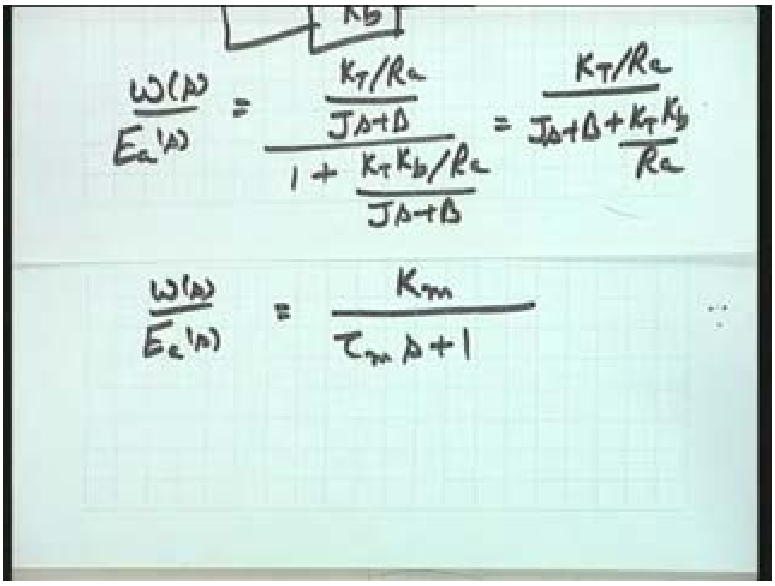

Now I want to comment on the effect of K b the inherent damping characteristics of the motor. You will please note that K b increases the value B and hence it increases the effective damping of the motor. So it means it is because of K b the oscillations of the system can be quickly damped out. The feature which will not be available as you will see when we come to the field control mode. Equivalently in the time constant form, what happens to the time constant because of K b, does it increase or decrease the time constant because of K b? It decreases and as you know smaller the time constant faster is the response. So quick damping means the oscillations are damped out quickly and the motor comes to the steady value quickly and this is the characteristics of this inherent loop which is there in an armature controlled motor. I hope this is okay.

So now let us go back to the original system wherein I have to place the motor and give you the overall transfer function of this overall block diagram of the system. I can take up the unity feedback system if you understand this very well now. Omega r is the input I am taking. I know that physically it is E r which is being given as the input but in the block diagram for the purpose of analysis and design I could take omega r as the input plus minus here K t R f by R a is the gain here K t small t the tachometric constant, this is your e a this is your e a the voltage please see, this is the voltage available from the amplifier this voltage is going to the power this is not e a this is going to power amplifier (Refer Slide Time: 27:20) this is u signal I am sorry here at this point this is u signal the power amplifier output is e a the voltage which is going to the armature controlled motor. Fine.

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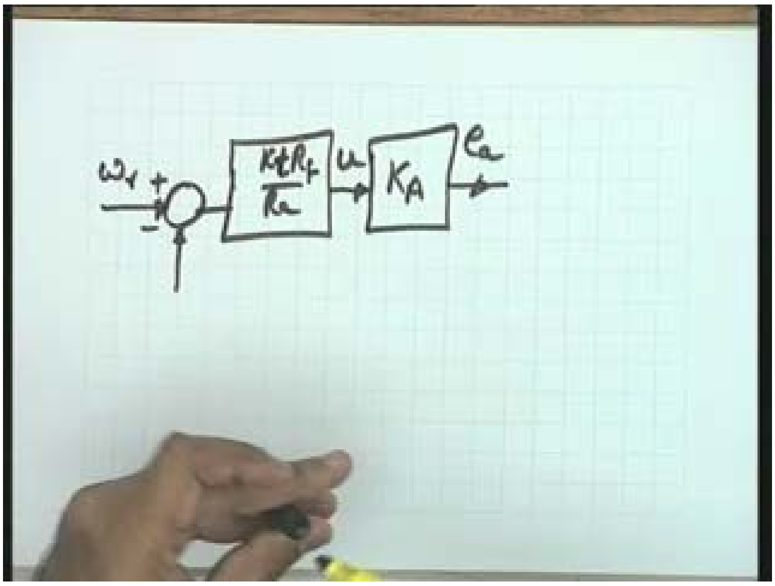

Now I will like to put the summing junction here because at this particular e a you have got e b the back EMF of the motor. Now this has a block K T over R a here so that at this particular point here we have got the torque, this is torque generated by the motor. Put another summing junction with a negative sign here this is your T w the disturbance torque on the motor. Yes…….[28:12], this torque is available for driving the inertia so this is 1 over Js plus B this is omega the speed. From omega I take a loop to generate the back EMF signal, from omega I take another loop it is a multi-loop configuration now through the sensor K t so that it is going to this particular summing junction and is giving you the primary feedback loop. I will put the diagram this way (Refer Slide Time: 28:52) K t is going to the summing junction base this summing junction and is giving you the primary feedback loop and this becomes

[Conversation between Student and Professor – Not audible ((00:28:59 min))] oh I am sorry I am sorry K t has been taken over there it is a unity feedback system you are very right. It is a unity feedback block diagram now; K t has been taken in the forward loop.

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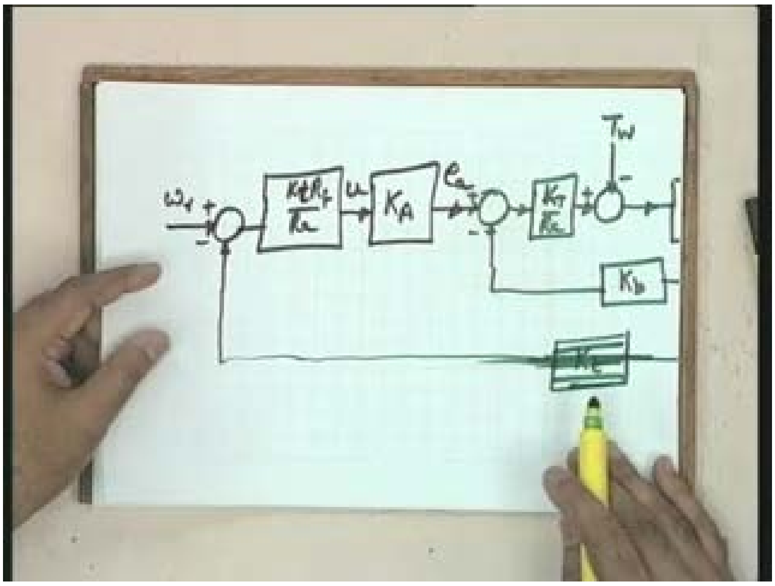

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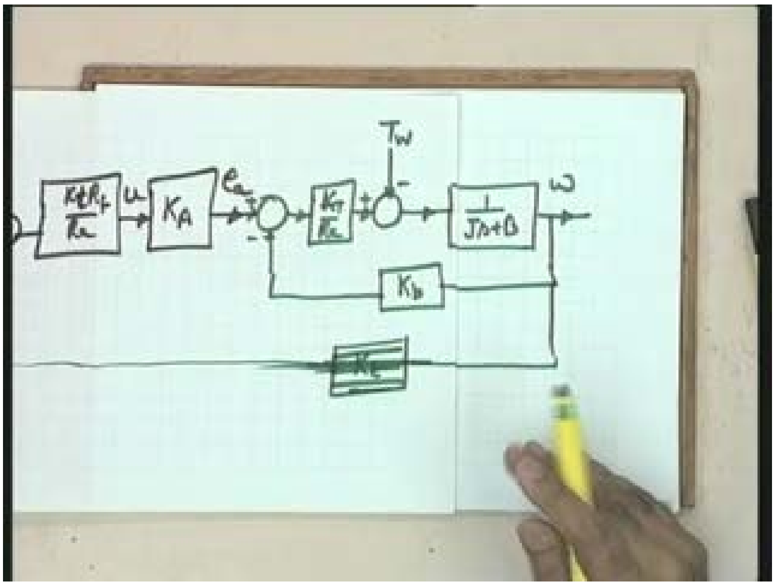

So, as far as this diagram is concerned it will go through unity so this becomes the total speed controlled system block diagram; the design and analysis aspects will come later. Now you know the block diagram reduction tools. Since the block diagram reduction tools are known to you, using those tools either through signal flow graphs or directly you can get me the transfer function between omega and omega r; you can get the transfer function between omega and T W it depends upon the interest you have. if you want to study, if you want to analyze the system with respect to the commanded position get the transfer function between or in general omega(s) is equal to M(s) omega r(s) plus M W(s) T W(s) could be obtained by block diagram reduction. That is why the reduction method Mason's Gain formula and direct methods were given to you because these will be subsequently required. And I leave this as an exercise to you, for this particular block diagram get me an equation of this form which gives you an output variable in terms of the commanded speed and the disturbance signal. It is a two input one output system.

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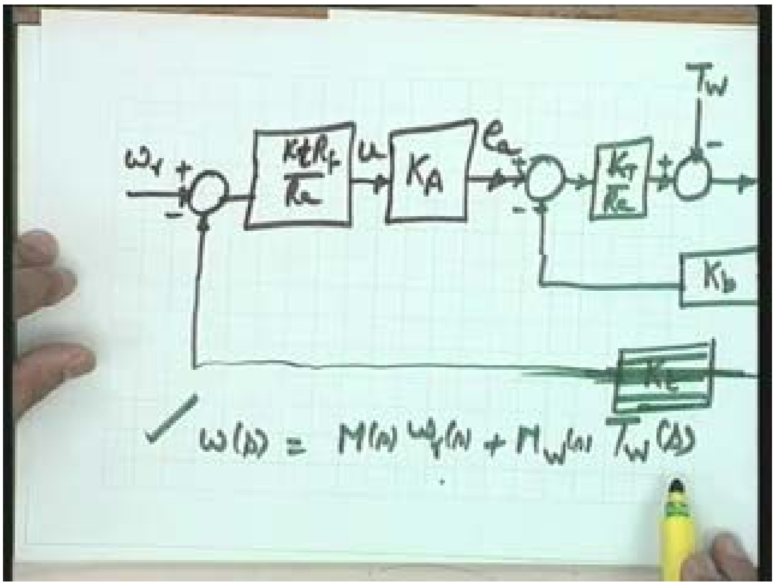

One more diagram I wanted to take and that is a position control system. Once you have got the speed control system very clearly there will not be any problem here it will follow very easily. See the objective of this function this system how does it work.

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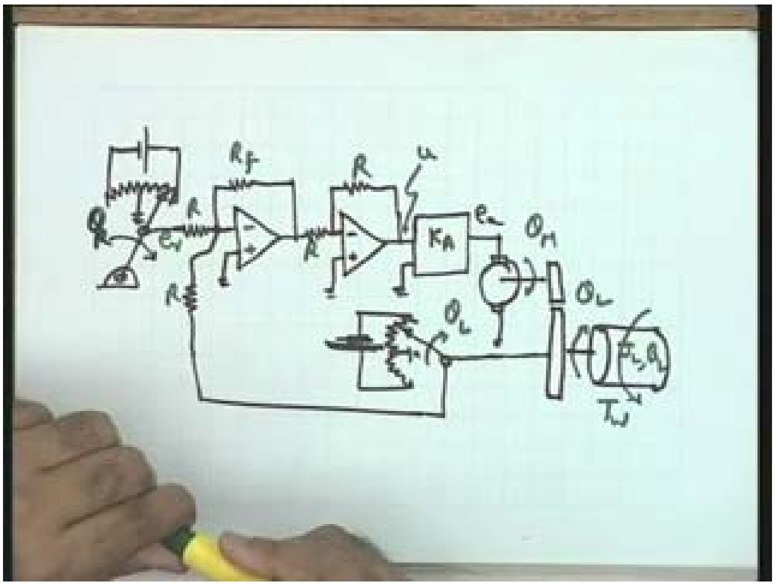

In this particular case again I have a reference input e r in terms of voltage and this reference input is the commanded position. I want to command the position of the load it is a position control system now in contrast to the earlier system which was a speed control system. So now e r…… again summing junction is the two voltages but the meaning of the two voltages has become different. This e r is a signal which represents the desired position and this desired position can be set on the potentiometer so that e r is equal to K p into theta r where K p is the potentiometric constant. I hope you get it.

Now let me see the feedback signal at this stage. Feedback signal is this particular load position theta L. this load position theta L could also be converted into a voltage signal by a potentiometer. Now I will like to take the potentiometric constant identical. Please note that this assumption will make a possible to convert the block diagram of the overall system into a unity-feedback block diagram. If that is not there you may not you will not be able to get a unity-feedback block diagram and there is no problem in taking two identical potentiometers with the same potentiometric constant.

So e r here is K p into theta r let me call this e 0 so e 0 here is K p into theta L where theta L is the load position it is the controlled position which is required to follow the commanded position theta r. So see the variables please: theta r and theta L are the two variables and you will note the sign the sign I have inverted, minus plus and minus plus taken this way you will please note the wiper on the potentiometers have been taken in this way that at this particular point the input of the Op-Amp is e r minus e t input to the Op-Amp e r minus e 0 where e 0 is the output voltage of the potentiometer this you can see with respect to this potentiometer wiper arm movement.

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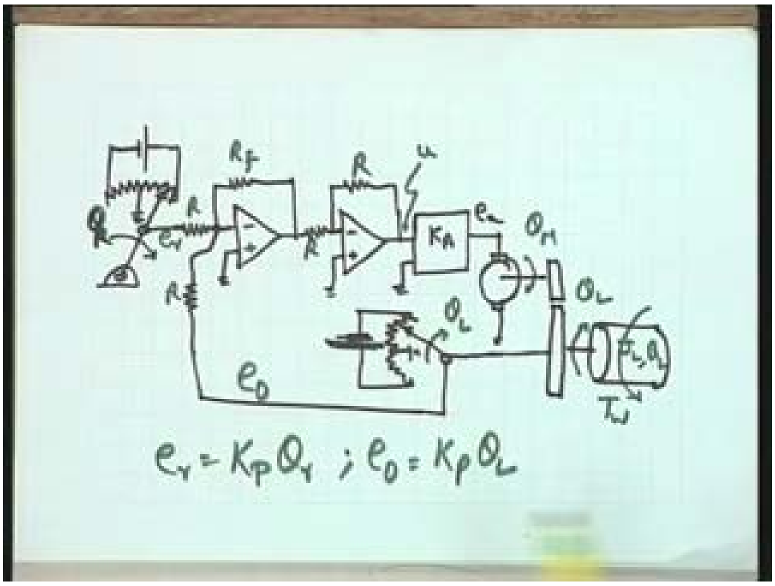

Now this voltage e r minus e 0 is being amplified. So it means the error detector, this particular amplifier is an error detector which gives you a voltage proportional to the difference of the two. Here again I have an inverting amplifier so that the minus sign is taken care of. I have a power amplifier over here so that the motor this particular signal is to drive the motor. This motor produces the torque and this particular torque is required to drive the load. now a point to be noted that I have not coupled the load directly to the motor shaft. I need your attention here. I have not coupled the load directly to the motor shaft, there is a beam that many a times for these particular for the movement of the load I need the speed reduction and the torque magnification.

You will find that this particular gear set let me call it, this is set of two gears there may be a set of more gears, so let me call this as a gear train a set of gears suitably connected suitably installed is a gear train this gear train is providing the required amount of torque magnification if the torque required at this particular the torque generated at this particular point (Refer Slide Time: 34:25) is lower than the torque required at this particular point the load is heavier in that particular case you will like to magnify the torque and this particular gear train is giving you the torque magnification. We are going to derive the model for the gear train. there from you will find that a suitable torque magnification is possible and in the process speed reduction will take place that is theta M dot and theta L dot and torque ratio will be in such a way that the gear train reduces the speed and magnifies the torque.

The objective becomes very clear; it as per the load requirements you will install a suitable gear train. So, after that, this is the load position, this is the motor position, the load position is being sensed and is being fed back. I hope the basic configuration of a position control system becomes clear. And the transfer function of each and every model will become is clear except for the gear train which I have to explain.

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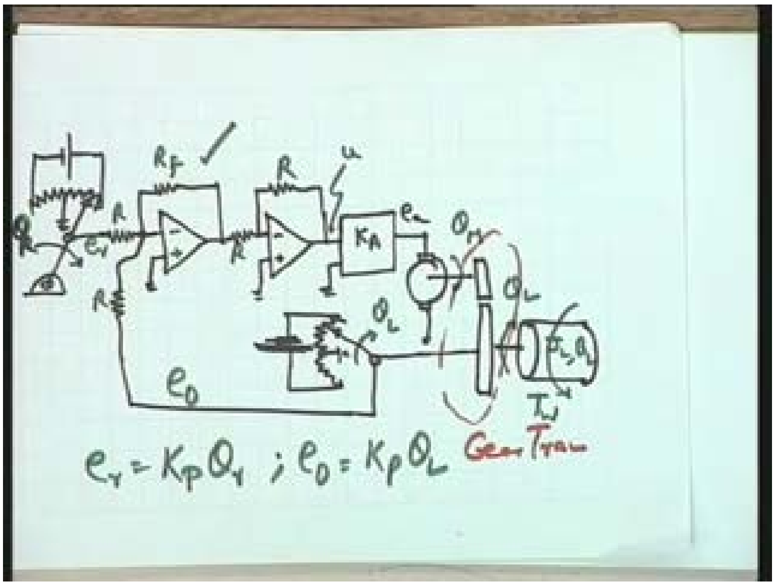

So now let me look at the gear train first so that the complete block diagram could be obtained. A set of two gears I am taking if the gears are more than two accordingly you can generate a suitable transfer function model for the gear train. Let us say that T M is the torque generated here the motor torque, theta M is the position, N 1 number of teeth on this gear, N 2 number of teeth on this gear and here is a load; naturally the reversal in direction will take place so naturally I am writing theta L this way and this drive torque is going to drive the load in this direction, so, if I want to show disturbance I must show a disturbance this way (Refer Slide Time: 36:23) because the drive torque on this particular load is coming this way so this is your disturbance T W.

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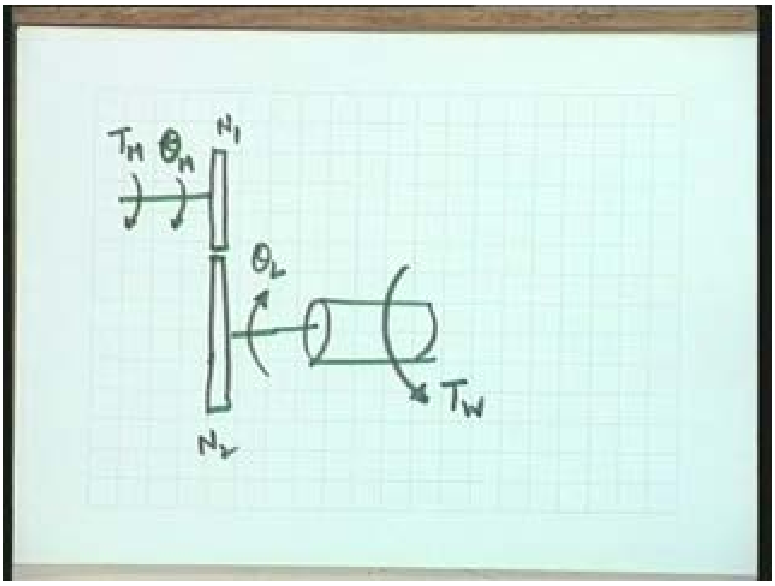

Help me to derive a suitable expression giving you the relationship between T M and the torque here and the speed here and the speed there, I want a suitable relationship for that. Well, it follows this way. I take r 1 as the radius of this gear, r 2 as the radius of this gear, you will note that the number of revolution of the two gears will be different because the linear distance traveled by the two gears has to be the same. Therefore I can write theta M r 1 is equal to theta L r 2. I hope his point is well taken. Theta M r 1 has to be equal to theta L r 2 because the linear distance traveled has to be the same. Assuming that r 1 and N 1 are proportional naturally they will be; r 2 and N 2 are proportional in that particular case I am writing N 1 by N 2 equal to theta L by theta M this I term as my first basic equation N 1 by N 2 is the teeth ratio and theta L by theta M is the ratio of the two positions; and you will not mind if I put it this way that this is equal to theta L dot by theta M dot as well that is the ratio of the two velocities because you can take the derivative of this equation and you can arrive at this particular expression if you take the derivative of this equation N 1 by N 2 is equal to theta L by theta M is equal to theta L dot by theta M dot is the expression you get which gives you the ratio of the two positions or two velocities as equal to the teeth ratio, the gear ratio that is called N 1 by N 2 let me say is the gear ratio n small n.

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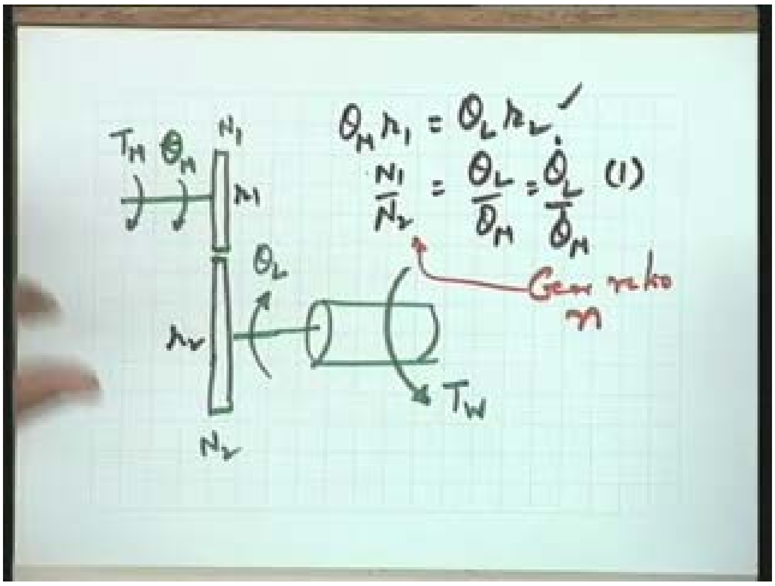

Now look at the torque expression. Now you see TM is the torque generated by the motor. Now let me assume that the torque transmitted to this particular shaft is T 21. T 21 is the torque transmitted through the gear train on to the load shaft. So T 21 is responsible for driving the inertia against the disturbance T W. You see this point: T 21 is responsible for driving the inertia against the disturbance T W. How about the gear one? Please see that if I remove gear two the total load torque on gear one will be less compared to the situation when gear two is present. It means, as far as the motor shaft is concerned the gear two and the load is loading the motor shaft.

Just look at the motor shaft only. the motor shaft is being loaded you can see the two situations; without the gear two and with the gear two the situation on the motor shaft becomes different and let me say that the total load provided on the motor shaft by the gear by this particular load and the gear is T 12. So I put it again; T 12 is the loading of the motor shaft due to the load and the gear the secondary gear or gear two and T 21 is the drive torque transmitted to the load shaft by the motor shaft. So in this case if I want a relationship between the two I know that at this point of contact the forces developed should be the same they cannot be different and therefore I write a relationship T 12 by r 1 is equal to T 21 by r 2 because the force developed at that particular point is T 12 by r 1 and the force developed if you look at T 21 it is T 21 by r 2 and this gives me the relationship T 12 is divided by T 21 is equal to N 1 by N 2. That is the ratio of the two torques is equal to the gear ratio.

This point needs your attention, please see. N 1 by N 2 is the gear ratio and this provide this T 12 by T 21 the ratio of the two is equal to the gear ratio. And now look at the point I made that the gear train magnifies the torque and it reduces the speed provided your n is less than 1 and n as for the gear train shown over here (Refer Slide Time: 41:25) will be less than 1 because N 1 is smaller than N 2. typically n could be 1 by 30 let us say, typical value you take n is equal to small n is equal to 1 by 30, if you take 1 by 30 you find theta L dot is equal to 1 by 30 into theta M dot it means the load speed has reduced by effecter equal to the gear ratio.

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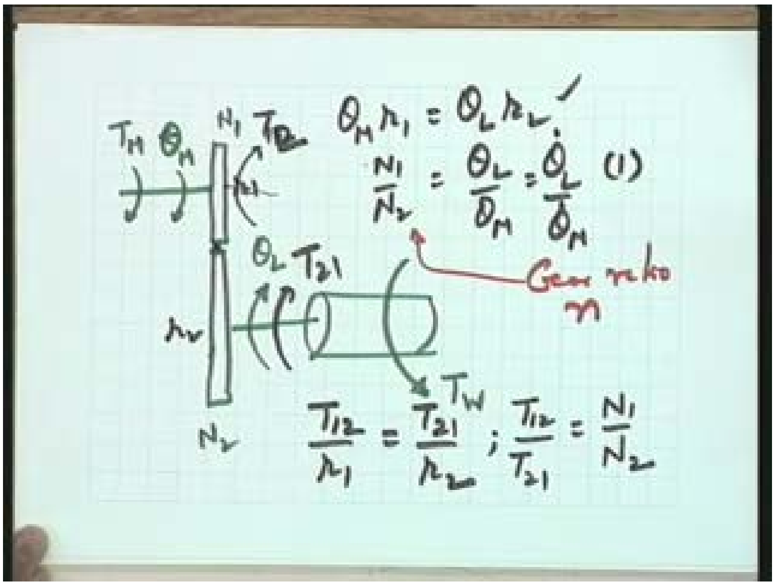

T 21 equal to…. T 21 now you take in this, T 12 is 1 by 30 so T 21 is equal to 30 into T 12 the torque available at the load shaft has been increased by effecter of 30 which is inverse of the value n the gear ratio, your result. So it means the gear train here provides the required torque magnification and corresponding speed reduction and how do you fix up the gear ratio naturally depends upon the characteristics of the motor and the load. These two units will dictate the value of the gear ratio and accordingly you will design a suitable gear ratio for the system.

Now let me come to equations. Please see here is a motor, this is the load here, come on give me the equation at the motor shaft. I assume that this particular motor itself has got certain moment of inertia J M and viscous friction B M. the motor itself has got moment of inertia J M or let us neglect it for the time being (Refer Slide Time: 43:05) TM is the torque developed by the motor I take it this way. Let us say J L is the moment inertia here and B L is the viscous friction and T W is the load torque. I need your attention here please. I am now setting up the equations. Please set up the equations for the secondary term.

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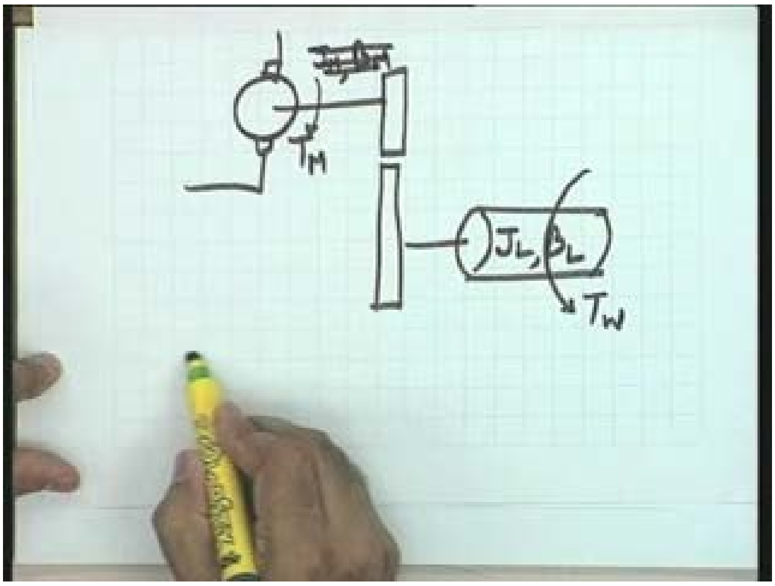

For the secondary term the equation becomes: T 12 for the secondary gear and the load, T 12 is the torque transmitted by the motor this is equal to J L theta L double dot plus B L theta dot plus T W this becomes the equation at the load shaft. Please give me the equation at the motor shaft. What is the applied torque T M equal to? Yes, we said that the position due to the secondary gear and the load on the motor shaft is represented by T 12. Let me at this stage itself make the discussion complete: J M and B M could be taken as moment of inertia and viscous friction of the motor shaft there is a friction in bearings of the motor shaft there may be friction in this part of the system which has been captured into a coefficient B M.

Help me please; give me the equation for the motor shaft T M is the developed torque, theta M is the angular displacement of the motor shaft in that particular case T M is equal to J M theta M double dot plus BM theta M dot plus T 12. Now you see that what I want to do is I want to eliminate the variables T 21 and T 12. Help me in that. If I want to do that let me retain these equations here.

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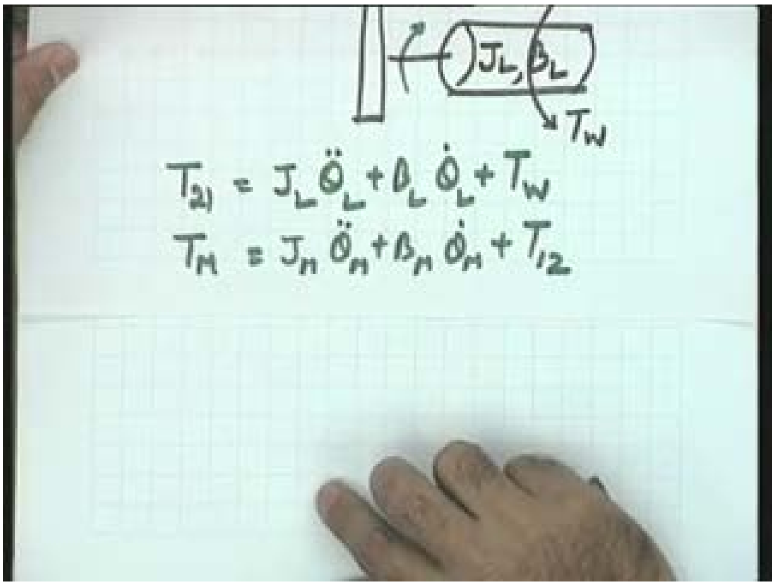

Eliminating the variables T 21 and T 12 I can write the expression as, a very important expression I am going to give you at this stage: T M is equal to J M theta M double dot plus B M theta M dot plus what is T 12 T 12 if you recall is nothing but N 1 by N 2 into T 21 (Refer Slide Time: 45:43). So, if you want to replace this expression I can write this as: N 1 by N 2 into T 21 which could be written as J L theta L double dot plus B L theta L dot plus T W. These variables T 21 and T 12 have been eliminated.

Now you see one more point that the variables theta L and theta M (Refer Slide Time: 46:17) are not independent; they are related by a constant. So your equation having both the variables theta M and theta L is not logical. Your equation should have one of these variables either theta M or theta L and mostly the basic equation is written in terms of theta M. So I want you to eliminate theta L as well. Come on please get started. You eliminate this and I also do it and then compare the result.

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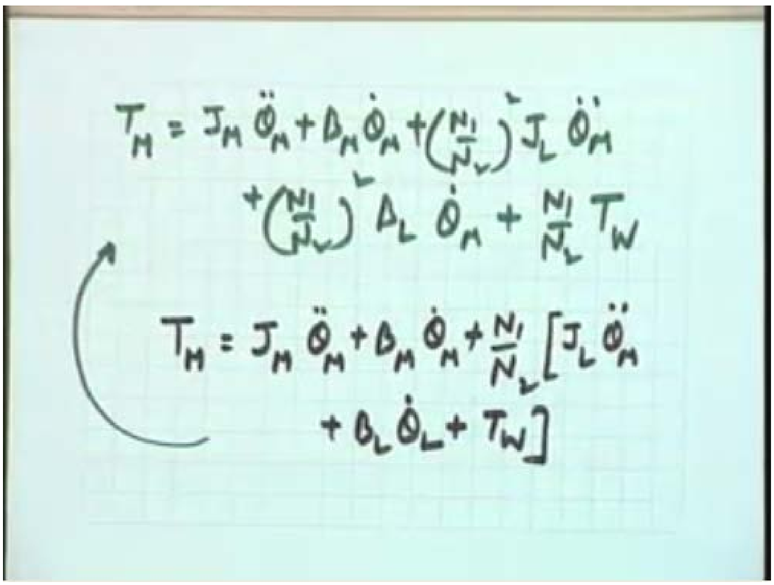

Let me write it this way. I want to eliminate theta L and write the basic equation. This becomes T M equal to J M theta M double dot plus B M theta M dot plus N 1 by N 2 squared J L theta M double dot plus N 1 by N 2 squared B L theta M dot plus N 1 by N 2 into T W. You will see this equation please: Theta L dot and theta L have been replaced as equal in terms of theta M, you know the expression: theta L by theta M is equal to theta L dot by theta M dot is equal to the gear ratio N 1 by N 2. So you see the interesting interpretation of this equation is the following: I can club these terms corresponding to theta M dot J M plus this term J M plus this term I am going to get as the total inertia term I can club these two terms B M plus this term I am going to get as the total frictional term and this is the equivalent disturbance on the load and hence the expression T M is equal to J M plus N 1 by N 2 squared J L into theta M double dot plus B M plus N 1 by N 2 whole squared B M B L sorry yes, this is okay (Refer Slide Time: 48:36) theta M dot plus N 1 by N 2 T W.

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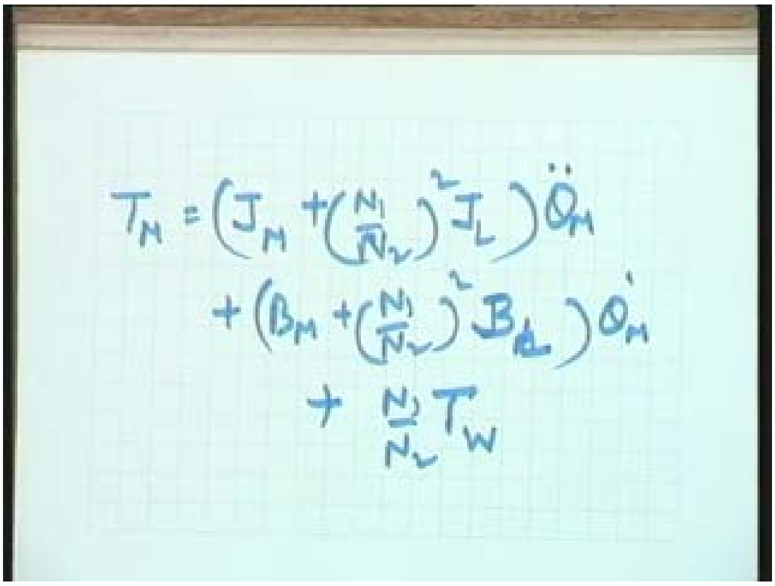

Please see what is the interpretation of this. The interpretation of this particular equation is this that, mathematically you see, you have been simplifying the block diagrams here also. Let us do some simplification. Mathematically your system which we have described as position control system is equivalent to the following situation: a motor, this is load, forget about the gear train altogether. You assume that a particular load is there directly on this particular motor.

What are the parameters of this particular load?
The parameters of this particular load are J equivalent; what is the value of J equivalent? You will find that J equivalent is given by this expression (Refer Slide Time: 49:27): J equivalent and you can take the load on this particular motor to be equal to B equivalent which is given by this particular expression and what is the disturbance directly coming on the motor shaft? Actually, physically speaking, the disturbance is coming on the load shaft but if you take the disturbance as n into T W where n is the gear ratio is equal to N 1 by N 2 these but becomes the representation; and why did we take this representation is because writing the mathematical equation for this representation becomes simple. So, with this representation for the motor shaft now the situation of the overall control system which we have taken becomes identical to the situation we have handled for speed control system and I leave this

as a simple exercise for you to give me the block diagram of the system. Or, well, I know the time limit but just let me quickly give you the block diagram and that is the end.

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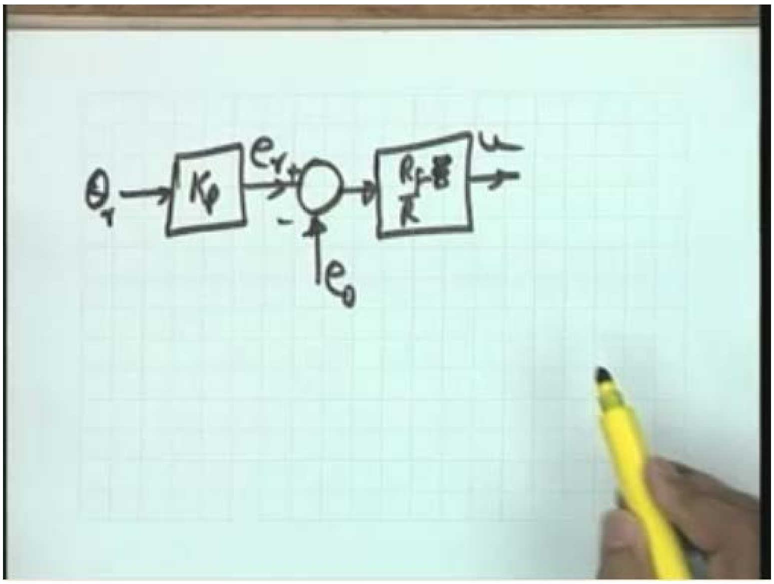

Theta r is the reference input, this is your K P which you can take inside, e r is the this voltage plus minus this is e 0 the feedback this is being compared R f by R into K R f by R gives you u here the controlled signal. What is this controlled signal doing? This controlled signal multiplied by K A gives you e a. let me save time over here and I put the motor here; the complete block diagram of the motor could be could be put over here at this particular point.

What is the output of the motor?
Output of the motor in this case is motor speed omega. Motor speed omega into 1 by s an integrator gives you position theta M. This position theta M from this particular point now I think you can connect the block diagram, you can convert it to theta L, take a suitable feedback loop through the output potentiometer and the inherent loop on the back of EMF also will come.

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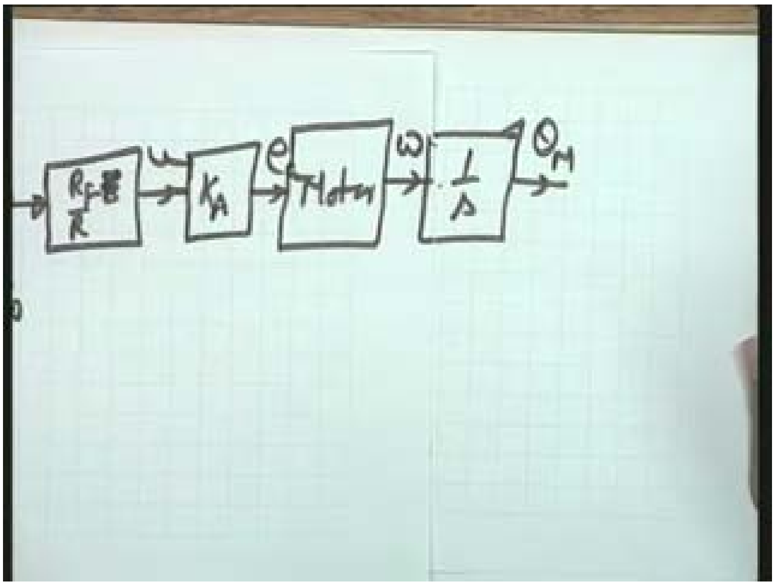

I leave this as an exercise for you to complete the block diagram for the system. There is a pressure of time, thank you very much.