

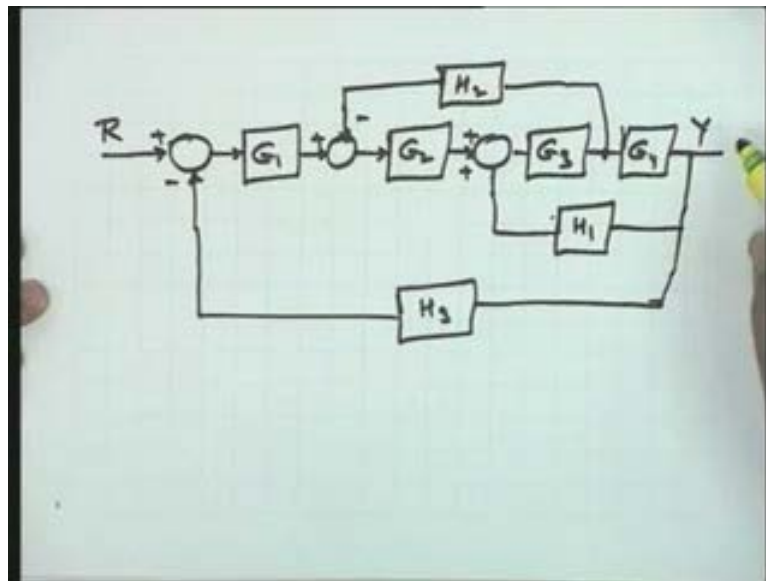
Control Engineering
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Lecture - 11

Models of Industrial Control Devices and Systems (Contd....)

Last time we were discussing about the block diagrams and the block diagram reduction procedures. Any complex situation, any practical control system for the purpose of analysis and design is normally first converted into a block diagram because visualization of the system becomes easier. As I said last time block diagram is nothing more than schematic representation of system equations. We should not view it more than that you see, with that representation helps us very much to visualize as you will see and the design and analysis tools are also developed around the block diagrams.

So the block diagram terminology the block diagram reduction procedure where I hope more or less clear last time and I will now illustrate with the help of an example. I have a simple example with me it is a multi-loop example as you see.

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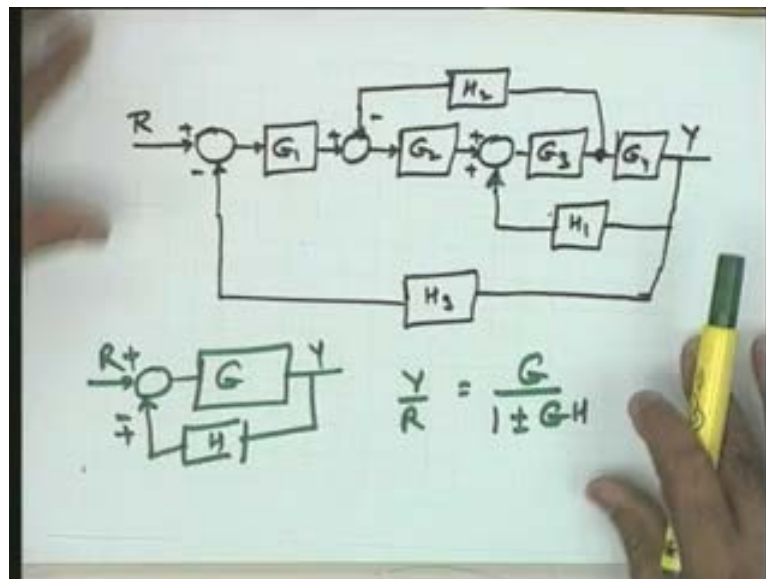


In this particular case R is the reference input, Y is the controlled output and as is normally the terminology the controlled output is being fed back through the primary loop, this loop through which the controlled variable is fed back is referred to as the primary loop. In addition to the primary loop you find that there are more loops in the system. A minor feedback loop is here, another loop is here which is also a feedback loop. So this is a multi-loop configuration. At this juncture, let me not give you the corresponding physical diagram of a control system. Let me see, whether by simple manipulation of a block diagram can we get the relationship between Y and R because for the purpose of analysis I will require a transfer function between the output and the input so that for a given input the time variation of the output could be obtained. So, for that purpose I require the relationship between Y and R.

You could very easily see that, well, if the system equations were available you could have manipulated the differential equations directly in time domain or in Laplace domain to get the relationship between Y and R. I hundred percent agree with that. The only thing is that I am giving you an alternative and hopefully you will like the alternative. that is, first writing the equations in block diagram and then **manipulating the block diagram to get the equivalent** translate to get the overall transfer function between any variable of interest to you and any specific input variable.

Now look at this diagram. I hope you have noted down the diagram by now. Now, in this particular case if I am interested in relationship between Y and R what is the primary tool available with me? Please see, the primary tool available with me is the basic feedback loop plus minus or it could be plus as well. This is the basic feedback loop. I know that Y by R equal to G over 1 plus minus GH. This is the relationship. I can resolve this basic feedback loop into a single loop and that single loop has got this transfer function. So, in this particular case unfortunately you find that there is no physical there is no basic feedback loop and hence immediately you cannot resolve this into a single block. If you look at this loop it is not a basic feedback loop because there is a take off point here. A signal is being taken off here (Refer Slide Time: 4:54) from here so similarly if I consider this loop this also is not a basic feedback loop because of a summing junction in between. And same is the story as far as the primary loop is concerned.

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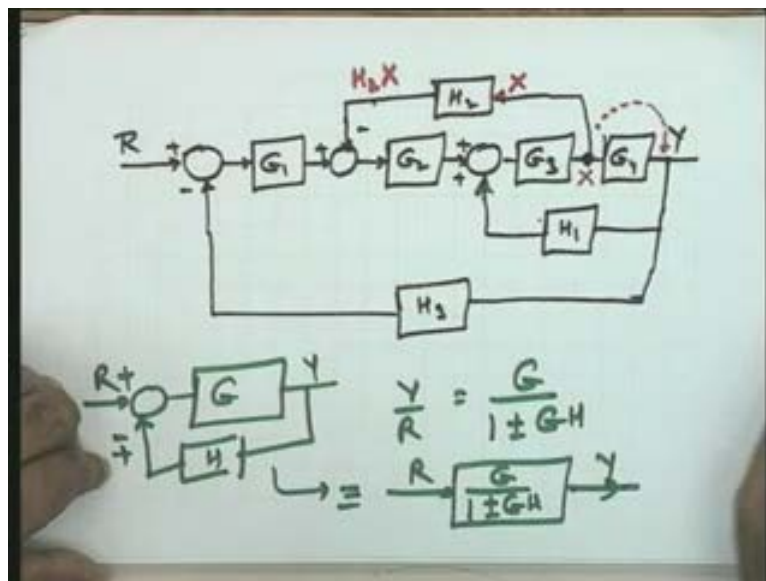


So what is normally done when you manipulate a block diagram?

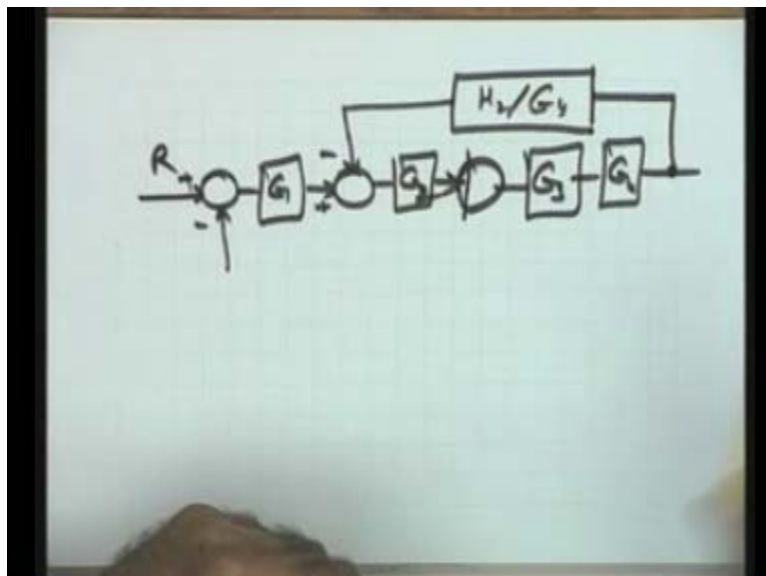
You see that you simply change the summing points or take off points appropriately so that the basic feedback loops result. Then you know that a basic feedback loop can be resolved into a loop of this nature. That is, let me draw at this diagram now, becomes equivalent to a single block with input as R, transfer function as 1 plus minus GH and the output is Y. So now this is the reduction. A basic feedback loop has been reduced to a single block. So the procedure is simple you see; it is just an intuitive feel as to what type of manipulation should be done so that a basic feedback loop results. **I am sure you can help me right away as far as this diagram is concerned.**

If I want to get this type of basic feedback loop (Refer Slide Time: 6:00) see, there may be many possibilities please see. The sequence is not unique. As you will see many alternatives in this simple question also are possible. So if I pick up from here if somehow I am able to transfer it to this point in that particular case you find that a basic feedback loop results. You see that the block diagram is a representation of system equations. If I name this variable as X so it means the input to this particular block is x . There is absolutely no problem if I shift it here provided you can assure that the input to this particular block will continue to be X ; rather I can say the output of this particular block will continue to be H_2 into X . After all, this is the variable which is being injected into the loop into the block diagram. If this variable is same, whether this comes from this point, this point or any other point it does not matter because as far as the input signal to the block diagram is concerned as far as this signal to the summing junction is concerned it continues to be $H_2 X$.

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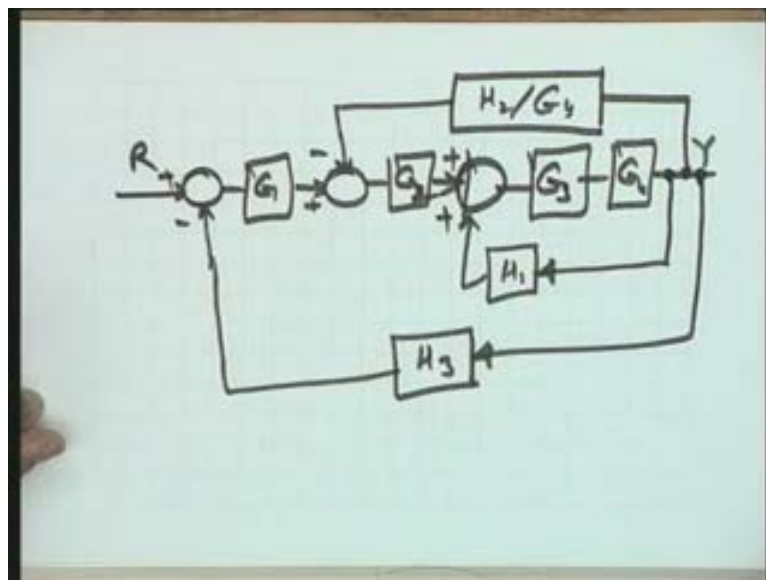


Yes please, help me, what should I do?

R plus minus G_1 another block here G_2 sorry a summing point here G_3 G_4 please see I am interested in this, now help me whether this is alright; I am putting here as H_2 by G_4 and I claim that the output signal will continue to be the same. H_2 by G just see, this particular signal is Y (Refer Slide Time: 7:58) so Y is equal to X into G_4 Y is equal to X into G_4 now to this particular block you are feeding into this block Y ; X into G_4 multiplied by H_2 by G_4 will give you same as H_2 into X . So it means, if I transform my original block diagram into this form then the answer remains the same plus plus H_3 please see. Well, the diagram has not come out neatly does not matter, the message should be clear.

One more trick I have played. One more adjustment, one more manipulation I have done. Please see, again there is no difference. Look at this particular point (Refer Slide Time: 8:51) the Y signal is being fed back through this particular loop as well as through this particular loop.

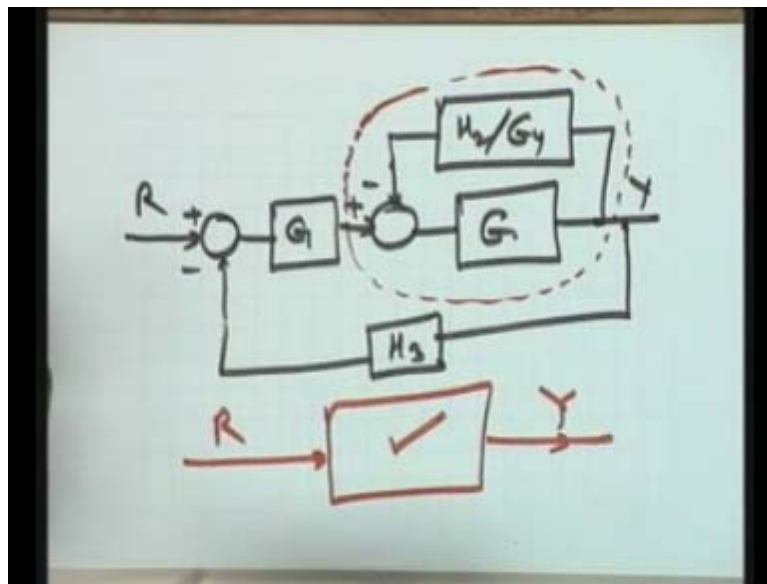
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You will not mind if I change this to this form. After all, this signal is Y this is going as an input to this particular block, this is going as input to this particular block. So it means I have simply separated out the take off point. It could be a common point but I am taking three points over here all the three points correspond to the same signal Y . the advantage will be that I will be able to identify the basic feedback loop very clearly. So you could very easily see that I have shifted this point to this point with the objective of reducing this particular loop. Now you can see that. What is the value please? $G_2 G_3 G_4$ in cascade so $G_3 G_4$ divided by $1 - G_3 G_4 H_1$ this being a positive feedback loop is the reduced block as far as this diagram is concerned.

I am putting it I am encircling it here; this entire thing now can be replaced as let us say G equal to $G_3 G_4$ divided by $1 - G_3 G_4 H_1$. The objective has been realized because my objective was to reducing it to the basic feedback loop. If this G you will recall you will keep in mind that the reduced block is G then the situation is now available to you in a simplified form.

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The new transfer function here is G_2 by G_4 and let me put a single block G here this becomes Y and this is the signal going through H_3 . I think I need not carry this problem further if you understand the situation because the situation has become too clear now. The advantage of reducing this has been that you have another feedback loop. This is the feedback loop now. You can reduce this feedback loop and let us say let me call it G dash. If I call this transfer function as G dash then G dash is in cascade with G_1 and the feedback block is G_3 you have got the major loop now; the primary loop also has come now in the basic standard feedback loop form and hence you can apply the result again and you have a relationship between Y and R ; the complete transfer function between Y and R can be obtained.

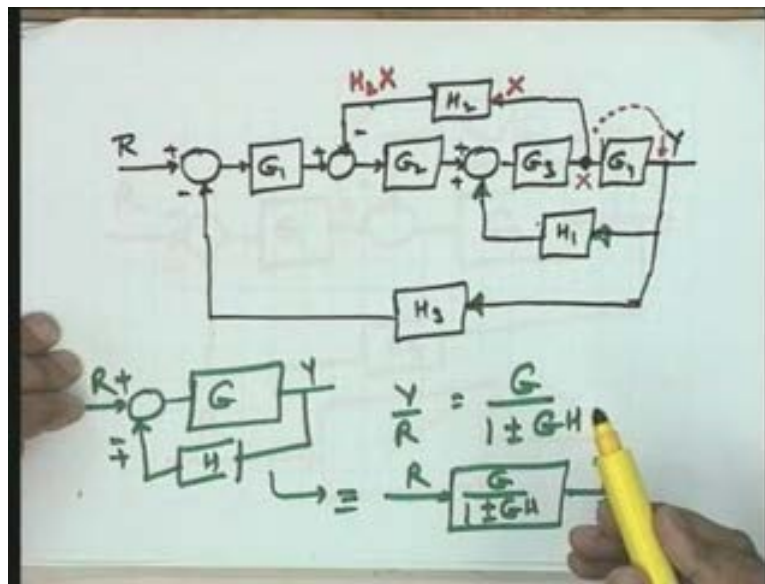
If you allow me I can leave the problem here instead of giving you the complete answer because the complete answer is now straight away available to you just by applying the basic feedback loop formula three times. One time you already applied, second time you apply it on this loop cascade it with G_1 and then apply it on G_3 you get the relationship between Y and R and hence a close loop transfer function for the system with R as the input and Y as the output is available to you. So this way any type of manipulation you see is possible.

Actually in your text book or any book you pickup on control you will find a table given in that table the basic rules of manipulation of block diagrams are available. But all those rules do come by simple intuition by simple manipulation of the basic equations there is hardly anything to explain those. If you look at those rules you will see that well those are directly applicable and you will be able to apply it on that block diagram, any block diagram you are working with.

Now, before I leave this problem, to give me confidence that you have got my point you give me some alternative to reduction, please. I have given you one method; I want you to give me an alternative method an alternative method of reduction.

[Conversation between Student and Professor – Not audible ((00:13:00 min))]

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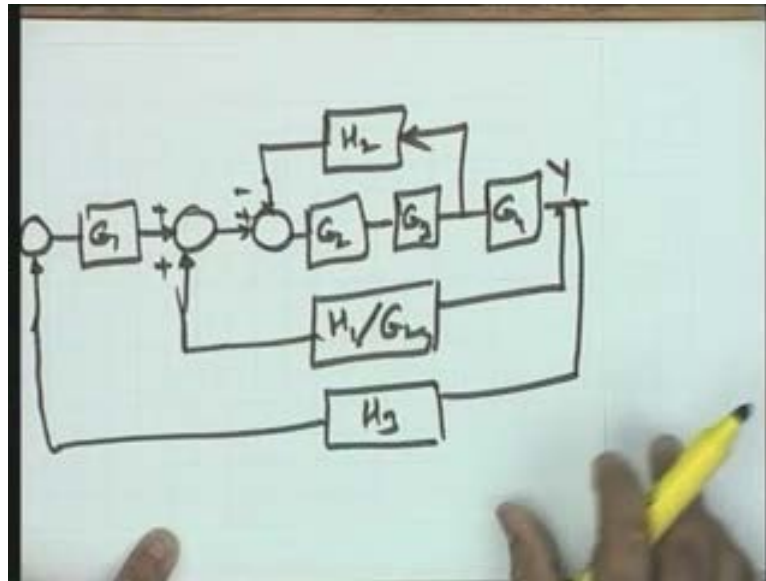
Yes, okay. Let us take this point. His suggestion, change this summing junction. Let me change this to this please. If I change this summing junction to this place what will happen, rather even let me say to this place in that particular case you will find this will be a clear loop. If I change the summing junction to this point this will be a clear loop and once I reduce it, well, the subsequent reduction will be easily as a visible. So now let us see how do I change the summing junction. Again the clue will be the same. The signal at this particular point let me say is Z . This Z signal see (Refer Slide Time: 13:47) is H_1 into Y which is going on; H_1 into Y is the signal which is being fed back and this signal is coming here. You should see that through this particular loop the signal at this point should be H_1 into Y in that case any manipulation is possible. You have to come to this point you see, you should not leave it at this point here because finally in the original block diagram this signal at this particular point is with a plus sign Z and Z is equal to H_1 into Y .

Come on, help me please. If I place this summing junction here I will make a neat diagram later. If I place a summing junction here what is happening Y into this block into G_2 is going on and reaching this point, is it alright? (Refer Slide Time: 14:39) so it means additional gain of G_2 is coming. So, if you take care of this gain so that the signal at this point is H_1 into Y well, then the manipulation is valid.

So what should I do for that?

[Conversation between Student and Professor – Not audible ((00:14:54 min))] Yes, in the feedback path let me divide it by H_2 . So just for the sake of completeness **though** since you have only answered the point and therefore you know it very well but just for the sake of completeness **I will leave** I will make the block diagram and leave it there. G_1 , I am making a block here plus a plus and another summing point here is plus with a minus. Here now it is H_2 only it is G_2 and here I am writing as G_3 I pick up a signal from here.... let me take this a little this side, this is G_4 , this is my signal Y a signal picked up from here it is H_1 by G_2 goes here. Let me use a different point here and H_3 is the primary loop.

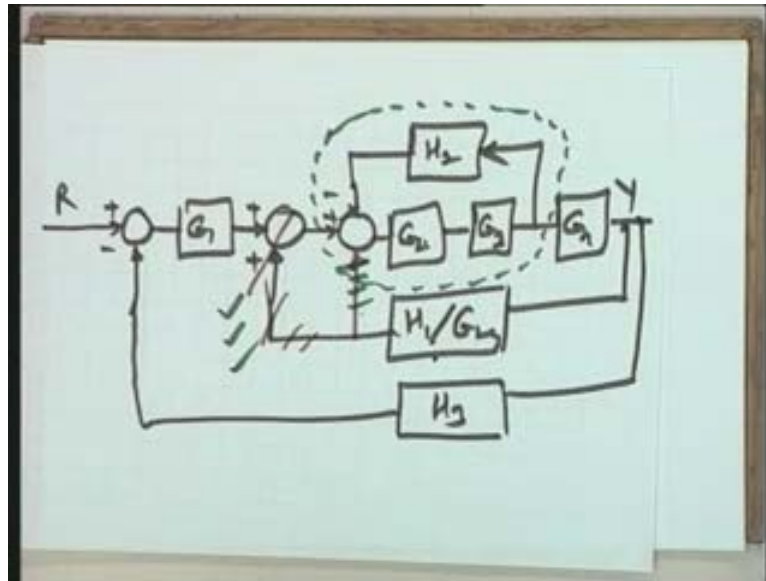
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Well, okay there is absolutely no problem. I could have fed in.... you see, his point you see it should be understood. There is absolutely no problem; as far as the block diagram is concerned it is complete if I put it here and remove it here remove this point. The block diagram is valid but the only thing I have done is I have put two summers so that your basic block diagram should be easily identified that is all. Because putting two summers or one summer does not matter because these are not the physical devices. As I told you, the block diagram should be viewed as a pictorial representation and a graphical representation of system equations. Whether you put it this way or the way I had earlier given you the basic equations remain the same and hence both the block diagrams are valid. The thing which I have done is with the only view point that your basic block should be easily identified.

So if I put it the way I did it earlier remove this particular point, make this as valid (Refer Slide Time: 16:58) and hence you find that this is your basic block. Rather many a times if a summing junction with three inputs is given you will like to split it into two summing junctions so that you can identify the basic feedback loop and you can apply the basic feedback loop formula. Now if this is okay now you find that these two are in cascade H_2 you get this block. this block and this block in cascade, this is the feedback another feedback loop then the resulting transfer function and G_1 in cascade you come to primary feedback loop and you get the answer.

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So, this way you find that..... well, some of you may come forward even for the same example.... some more different methods and may turn out to be even better methods. what I want to say is this that the block diagram manipulation is just manipulation of a set of equations so that the resulting closed-loop transfer function is available to you.

Now do not be very happy that, look, with this diagram you could give two alternatives surely if the situation is reasonably complex. The manipulation of the block diagrams may become difficult. Surely it may become difficult for a reasonably complex situation. For that I give you an alternative. I do not know whether this alternative you have studied somewhere or not, in circuit course or somewhere and that alternative is Mason's Gain formula. For that you will find that no block diagram manipulation is required.

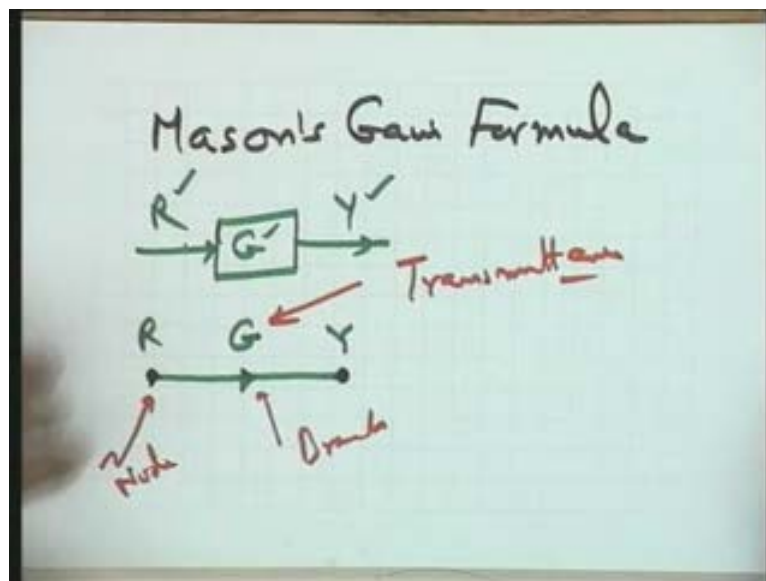
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Mason's Gain Formula

Mason's Gain formula can directly be applied on a block diagram though it is easily visualized in terms of signal flow graphs. Please see, it is really not necessary to convert a block diagram into a signal flow graph, you can apply the result directly. But since the visualization in terms of signal flow graph is more convenient I will like to first convert a block diagram into a signal flow graph and then apply a Mason's Gain formula to get the overall transfer function between the output and the input. By the way please tell me whether it has been done already in some course or not Mason's Gain formula. **No, okay in that case I will give only that much of introduction which is required for my purpose you see; I will not go to the depth okay, that point should be clear.**

I take a single block here this is input R and the output Y (Refer Slide Time: 19:26). You see, it is a variable, R is a variable, Y is a variable and G is the system gain. In the signal flow graph, well, the same equation is represented this way that is all. It helps in visualization when I come to Mason's Gain formula. The variable is represented by a node so it is an R node now. This node corresponds to Y and the system gain is now the transmittance G this is a branch. So I think the words they are quite evident also; a node, a branch and here is a transmittance of the branch or you can say even say the gain of the branch and this R is a node variable and Y is another node variable. So this is equivalent representation of this system in what is called signal flow graph.

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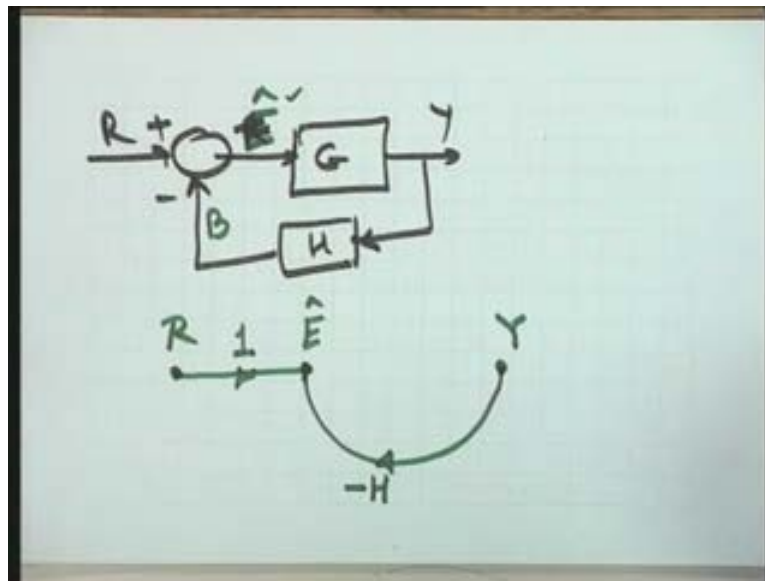
Let me take a little difficult situation a feedback loop R Y through an H I take a summing point here let me take R here please plus minus. Now you give me the corresponding signal flow graph for this. Let me say that this variable is E cap. Now, the variable should be well set; I am taking E cap because there is an H otherwise I would have taken E , the feedback variable let me take as B . so how many variables are there? R Y B and E cap. How many transmittances or how many gains are there? G and H . Now let me put it this way a node R and a node Y these are the two variables of interest to me directly.

Now this summing point this is the additional point needed otherwise the signal flow graph becomes clear. The summing point gives you a signal E cap which is the difference between R and B . You know that a signal in a signal flow graph a variable in a signal flow graph is

represented by a node. So naturally I will take a node here and call this node as an E cap node because the variable is represented by a node in a signal flow graph so it is an E cap node.

Now the basic definition: the value of the node variable is equal to the **signals coming from all the incoming** incoming signals from all other nodes. So let us see what is the signal coming over here. The signal coming from here is R so I can connect it this way through a transmittance 1 it means a signal; 1 into R is reaching the node E cap. Now I am putting a branch over here and you have to tell me what should be the transmittance at this point, should it be H minus H? This point please may please be noted, it is minus H because at this particular node the signal coming from the Y node is H into Y but there is a minus sign here **so this** since there is no provision of taking plus or minus at the node so I am clubbing that minus sign with the transmittance so the transmittance from Y to E cap becomes minus H. You could have taken a B node please there was no problem absolutely so I could have taken a B node here then between B node and Y it is H and between B node and E cap it is minus 1 but I have avoided because there is no need of an additional node; the system equations become evident with the help of this diagram. There was absolutely no problem in adding a node B between B Y you take H, between B and E cap you take minus 1 the diagram will be equivalent.

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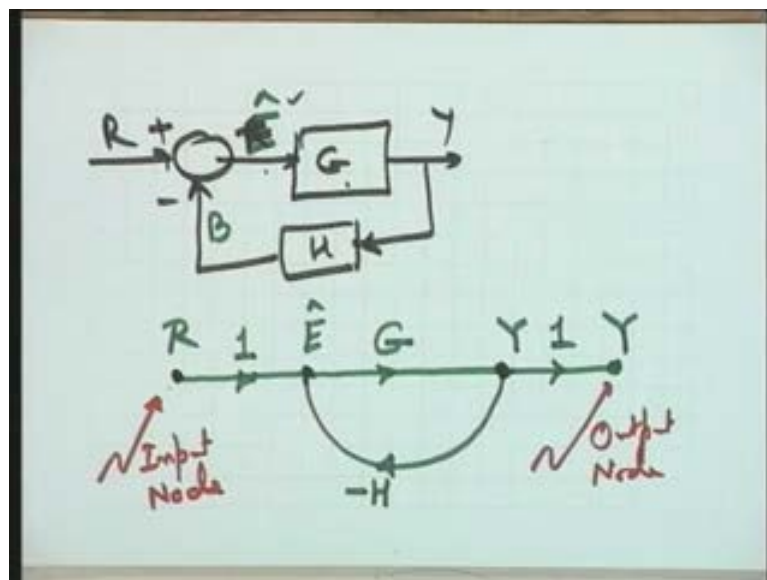
Normally I will avoid taking B node because this gives an adequate representation of the system I am interested in. now E cap is the node variable whose value is R minus H Y is the node variable whose value is R minus H Y is the node variable. Now this E cap multiplied by G gives you the value of Y. So what is this node variable? The node variable Y is equal to all the incoming signal, this point may please be noted. When you identify or when you define a node variable the value of the node variable is equal to all the algebraic sum of all the incoming signals. The node variable is not calculated in terms of the outgoing signals rather the outgoing signal is equal to the value of the node variable. So it means, on this particular branch a signal E cap is travelling in the process gets multiplied by the transmittance G and a signal E cap into G is delivered to this particular node. So I hope the terminology is now clear.

R is the node over node variable over here there no incoming, if there is no incoming signal to a node in the see in our terminology we will call that node as an input node if there is no incoming signal it is an input node, it is an external signal defined that is why the value of this node R depends upon the value you define for it. Now this particular signal is travelling on this particular branch (Refer Slide Time: 25:01) in the process gets multiplied with the transmittance 1 and is delivered to this particular node. This signal Y is travelling on this branch in the process gets multiplied with the transmittance minus H and gets delivered to this node and hence this node variable is the sum of the two.

Now the E cap signal is travelling on this particular branch in the process gets multiplied to G and is delivered to this particular node and therefore the value of the output or the node variable Y is equal to G into E cap so that way the complete system equation is represented like this.

Just for the sake of better visualization if I put one more branch which is not available here in the block diagram put Y here and call this also Y I hope it does not matter as far as system equations are concerned. So what is this equation? Y is equal to Y. there is no problem as far as system equations but it gives me one advantage. Well, I will call this as an output node. So let me say that output node is a node from where there are no outgoing branches. Input node is a node to which there are no incoming branches. So I can easily define an output node, the only thing I tell you that it is only to separate out the attribute of interest. you are interested in Y, it is clearly visible to you that look this is your output variable and I have done nothing, Y is equal to Y equation I have written for which you have no objection; the only thing is this that visualization of the diagram becomes easier and Y as an output node, R as an input node are clearly available and that is why input node output node in any diagram if the translation of your block diagram does not give you these nodes you put appropriate transmittances may be always unity transmittances and write your R and Y nodes separately.

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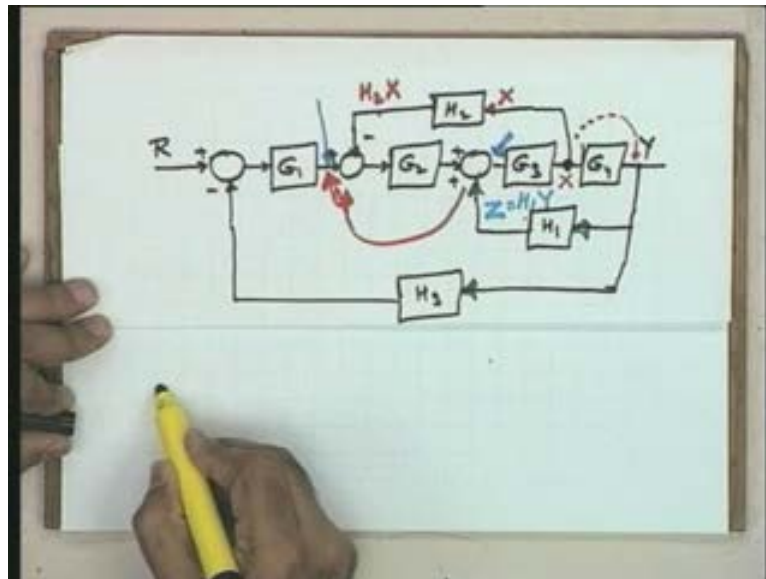


If this has been accepted, in that case I will like to take your help on this diagram and you give me the signal flow graph corresponding to this then I will give you the basic relationship

the Mason's Gain formula and we will see that how quickly we get the overall transmittance for this particular system.

Come on let us make an attempt. For this particular system I want though I told you I mean, with little experience you will apply the Mason's gain formula directly on this diagram but to get to know the gain formula very clearly this transformation to signal flow graph is helpful. [Sir put it little a lower it is not visible, is it okay now okay fine.... 27:46]

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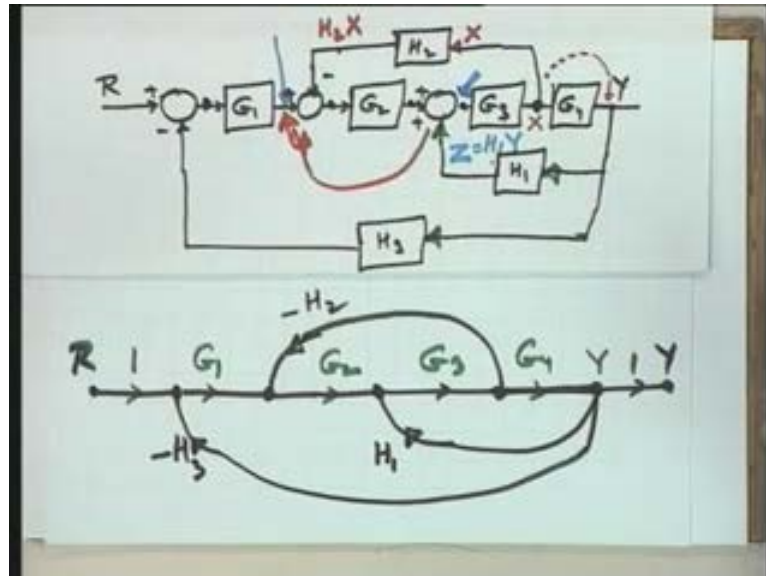


Please see that R is an input node, let me put a transmittance 1 over here, R is an input node 1 is a transmittance so I need not give all the variables now if you have followed my point. So, in this particular case now there is a signal coming from here I will put the appropriate transmittance later. I get this node variable now, this point now is available to me, here, this point (Refer Slide Time: 28:17) and hence I multiply it now by G 1. I am at this point now. I know that a signal is coming this way I will put an appropriate transmittance later, I am at this point now. This particular signal gets multiplied by G 2 and this is this variable. At this point I have a feedback signal coming and hence I reach this point and from here I have G 3 the transmittance and this is the node corresponding to x here, let me complete this path sorry I was using this color; let me complete this path and let me say that this is G 4 and this is the variable Y.

And as you have agreed to that you will not mind my putting a 1 over here so let me take another variable Y okay does not matter Y so this is the forward path. Now let me complete the links. You find that a feedback is being taken after this block that is X. What is X corresponding to? X is corresponding to this point. So let me take a feedback from here and this becomes your minus H 2. See whether you agree with this minus H 2. This is your Y output. I will not take the feedback from here because it is an output node and by definition output node has no outgoing branches so I like to take the feedback from here. so you see that I will now take and help me please what is transmittance; H 1 only because it is a positive feedback loop and lastly from the same point I take a branch here and let me call let me take this to be equal to minus H 3 this becomes your complete signal flow graph corresponding to this.

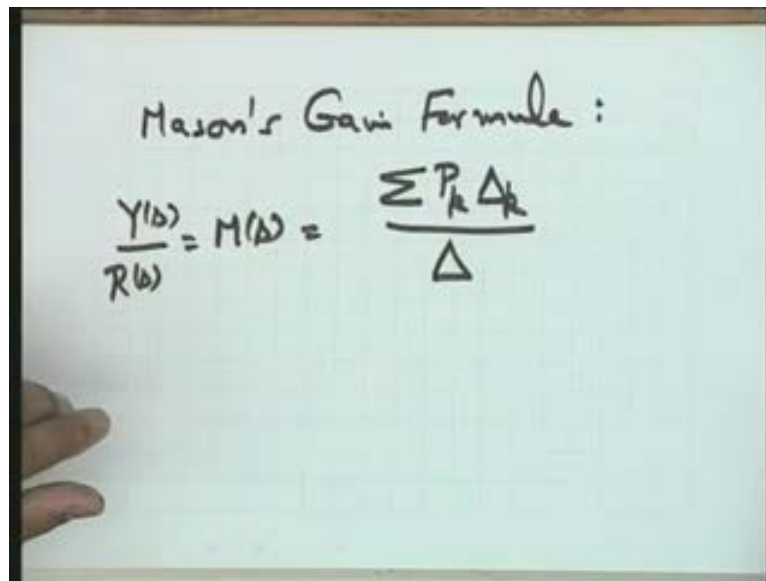
The information contained in these two diagrams is the same; the representation is different: A block diagram representation and a signal flow graph representation. This way you can convert any block diagram into an appropriate signal flow graph. I hope this is okay.

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In that case now I can give you the formula; the Mason's Gain formula as I had written. I will not give the proof here, I will give the result. It says that the overall transmittance of the signal flow graph, this is in terms of signal flow graph terminology. In control system terminology it is the closed-loop transfer function. After all overall transmittance is nothing but the output variable divided by the input variable and if all the variables are in Laplace domain the overall transmittance of the signal flow graph and the closed-loop transfer function of the system are equivalent terms. So the overall transmittance of the signal flow graph is $Y(s)$ over $R(s)$ where the $M(s)$ have been used for representing the closed-loop transfer function $M(s)$ is equal to..... you please take this point very clearly $\sigma P k \delta$ k over δ I am going to define these terms and once I define these terms I will take your help to apply this formula on the problem under consideration and you will give me the final answer I have to define these terms. To define these terms I get started with δ .

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Mason's Gain Formula :

$$\frac{Y(s)}{R(s)} = M(s) = \frac{\sum P_k \Delta_k}{\Delta}$$

Delta equal to..... please see 1 minus (sum of gains of all independent loops). See, this point sum of gains of all independent loops this is 1 minus plus (sum of gain products of pairs of non-touching loops). As I said that this will become more clear when we take up take it up to the example; (sum of gain products have pairs of non-touching loops) minus since there is no space I will speak it out minus (sum of gain products of triplets of non-touching loops) and so on. So it means what you are going to do 1 minus take individual loops take the loop gain. What is the loop gain? By definition, a loop gain will mean the multiplication of transmittances of all the branches in a loop (Refer Slide Time: 33:39).

What is loop by the way?

I think it is clear from our block diagram description that is why I have not defined it. A loop actually is a traversal of path from one point, follow the arrows and come back to the same point. So in this particular case, for example, this is a loop and what is a loop gain? G 2 into G 3 into H 2 with a negative sign is a loop gain.

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Mason's Gain Formula:

$$\frac{Y(s)}{R(s)} = M(s) = \frac{\sum P_k \Delta_k}{\Delta}$$

$\Delta = 1 - (\text{Sum of gains of all independent loops}) + (\text{Sum of gain products of pairs of non-touching loops}) - (\text{Sum of gain products of triplets of non-touching loops}) + \dots$

So in this particular case now I will say that (sum of gains of all independent loops) plus (sum of gain products of pairs of non-touching loops) minus (sum of gains gain products of triplets of non-touching loops) and so on. What is a non-touching loop? That also let me define. **a non** A pair of non-touching loop are the two loops which do not have anything in common; by anything I mean any node or any branch. Two loops are referred to as to be non-touching if they do not have anything in common. They are touching if a node or a branch is common between the two loops or triplets of loops as the case may be.

Yes please, sir what are independent loop?

Independent loops, independent loops yes, independent loops I will like to define it this way: A loop I have already defined, number of independent loops it means if I am taking X loops then all the loops should have at least something different than the other loop a node or a branch. You see, for example, you may say that this is a loop (Refer Slide Time: 35:18), some of you may take this a loop again but in this particular case you see that a loop again.... in this case **they can** there are how many independent loops; one two and three because in all these three loops there is at least a node or a branch which is different; those loops will be referred to as independent loops.

So with this now I have given you the value of delta and this I assure you will become clear through the couple of examples I am going to take. But you just store in your memory the basic definition. I am going to make it clearly. **I force it to you, you will see that this particular definition through couple of examples.** So in this particular case I hope the delta has become very clear.

Now let me go to P_k and Δ_k ; k it is with respect to k only so it going to be $P_1 \Delta_1$ plus $P_2 \Delta_2$ plus $P_3 \Delta_3$ and so on. What is P_1 ? P_1 is the gain of the first forward path. So P_k actually is the k th forward path.

What is a forward path?

Forward path (Refer Slide Time: 36:26) is the gain when you travel from the input node to the output node. So in this particular case from R to e if you travel along this line G_1 into G

2 into G 3 into G 4 is your forward path and this is the gain of the forward path. And since you cannot go from R to Y through any other root there is one and only one forward path in this particular case. So $P_k \Delta_k$ the k variable corresponds to the number of forward paths. The example before you is too simple; there is only one forward path so k is equal to 1 and P_1 is 1 into G 1 into G 2 into G 3 into G 4 that is the gain of the first forward path.

Now let me go to Δ_k that last term. Δ_k please see is the value of Δ not touching the kth forward path. It means this complete definition (Refer Slide Time: 37:29) is applicable for Δ_k . Keep in mind that all these terms not touching the kth forward path. By not touching the kth forward path I mean that they do not have anything in common in terms of branch or a node with the kth forward path that becomes the value of Δ_k .

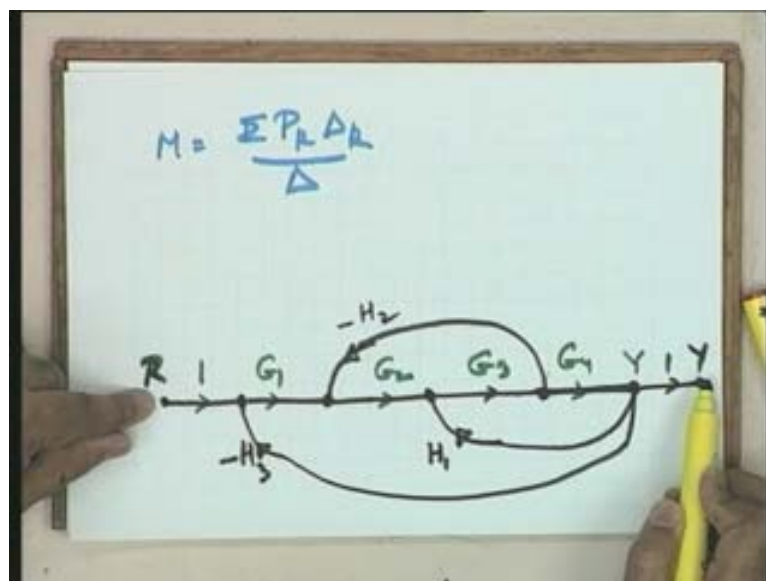
For example, what is Δ_1 equal to?

Δ_1 equal to 1 minus (sum of gains of all independent loops not touching the first forward path) plus (sum of gain products of pairs of non-touching loops not touching the first forward path) and so on. I understand there will be gaps but I will like to bridge up those gaps with the help of examples and this very simple example I first take, okay, help me please.

In this particular case I want M which is equal to $\sum P_k \Delta_k$ over Δ . I want to apply this definition on this. **Is it visible please the signal flow graph?**

$P_k \Delta_k$ upon Δ I want to use. So, first of all you must identify k. What is the value of k let us identify that first. k is one; it is you just try to traverse along the arrows from input to output. **You have not** to phase the opposite direction along the arrows you travel if you are able to reach that is your forward path and if you are able to reach one and only one way you please note..... **you have you** for example you may say that I am going this way, well, I go this way and I reach this way this is my forward path but this is not forward path. Please see that, that is why the word independent I used. In a forward path a node or a branch is traversed once and only once that point may please be noted.

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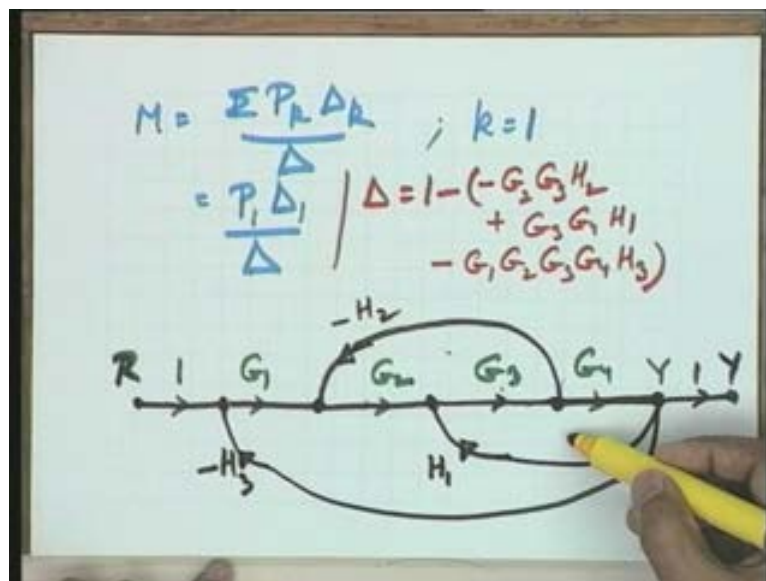
You may say that, well, you will loop around and go to the same point but I will not call this as a forward path. A forward path, by definition, is a traversal from input to output node

wherein a node or a branch is traversed once and only once, repetition is not allowed. And therefore in this particular case there is only one forward path so I identify k is equal to 1. I hope you are getting it well. Now, through the example k is equal to 1 and therefore my formula now becomes $P_1 \Delta_1$ over Δ .

Look at Δ now. Δ I now take it here itself 1 minus loop gains; loop gains, how many loops? This is one loop $G_2 G_3$ into minus H_2 so I am taking it minus $G_2 G_3 H_2$ this is one loop; another loop $G_3 G_4 H_1$ plus let me take it here $G_3 G_4 H_1$, third loop is $G_1 G_2 G_3 G_4 H_3$ with a negative sign minus $G_1 G_2 G_3 G_4 H_3$ with a negative sign there is no other loop there are three loops only, there is no other loop and therefore I close this bracket here. All the **independent loops** independent loops as I explained that every loop has got at least a node or a branch different from the other loops.

Now I need not take a plus sign here (Refer Slide Time: 41:15) because you find that there are no pairs of non-touching loops. Out of these three loops you take any pair any combination all the possibilities you try to exhaust and you will find that a node or a branch is definitely common. You cannot give me two loops where a node or a branch is not common and therefore there are no non-touching loops. **This definition needs your attention please** because in this particular case that is, if pairs are not there triplets and other higher order combinations are not possible. So if you stop at pairs you do not go further.

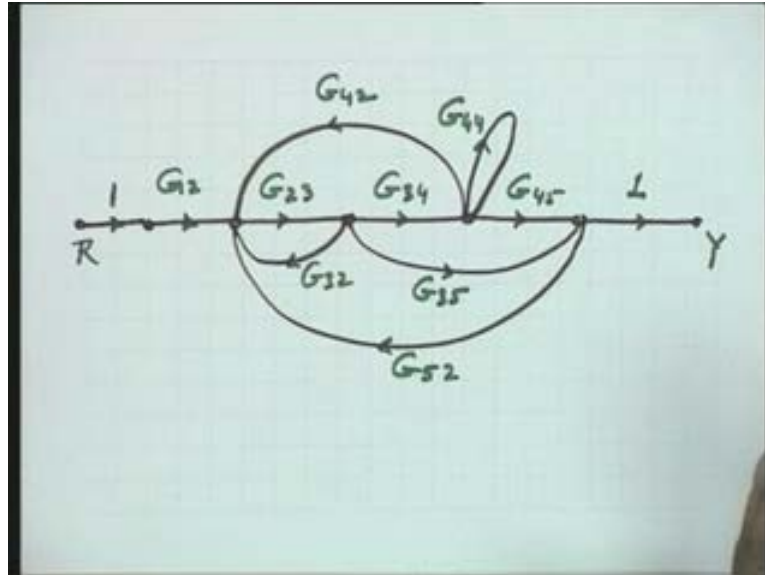
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So in this particular case, since there is no pair, this and this if you find, well, there is a branch, this and this you find well there is a branch, this and this (Refer Slide Time: 41:59) or any combination you take that there are something in common and therefore pairs are not there and this gives me the value of Δ . Now with this diagram in front of you let me look at Δ_1 because k is equal to 1. If you look at Δ_1 **you see** all the three loops are in touch with the first forward path. So what is Δ_1 equal to? 1, 1 only because even this bracket **is equal to now** is not available to you, even this bracket the first bracket itself is 0 and therefore Δ_1 is equal to one and hence I write the final answer; I do not think I need write the final answer. You write the final answer for yourself. P_1 is $G_1 G_2 G_3 G_4$ divided by 1 plus this minus this plus this if you take this minus sign inside. You get the final answer which is the

overall transmittance of the system or it is the closed-loop transfer function of the system. I hope this is okay.

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Well, it was a relatively simple situation. To give you a little complex situation I will like to take this example and you will have to give me the answer of every point regarding the signal flow graph formula. I have not given you the corresponding control system. This particular situation has been taken only to make the Mason's Gain formula very clear. Come on please. You have to help me now. First of all you can immediately take this particular diagram and then see identify the number of forward paths, the number of loops, non-touching pairs and all that. Once we have identified everything the formula you will **you will** be able to apply Y by R equal to $\frac{\sum P_k \Delta_k}{\Delta}$. Come on, get started with this problem. Give me the value of k , give me the value of k in this case please?

k equal to 2, 2 that is fine. You can just see the definition is this that you should not come back to a node or a branch; it should not be traversed more than once. You find from R to Y this forward path and from R to the second is that this forward path. these are the two forward paths so it means **i write it here itself** P_1 equal to $G_{12} G_{23} G_{34} G_{45}$ and P_2 is equal to $G_{12} G_{23}$ and G_{35} . Come on, if anyone is able to identify any other forward path we will like to examine that.

[$G_{12} G_{23} G_{44}$ and then sir then here it is running through the node 44:54]

G_{44} if you see that you are coming back to the same node. It is a self loop. G_{44} is a self-loop here. If you follow this **you are** you have reached this node. If you follow this particular point you are coming back to the same node and hence this is not allowed in the forward path. That is why this particular root will not be taken as a forward path.

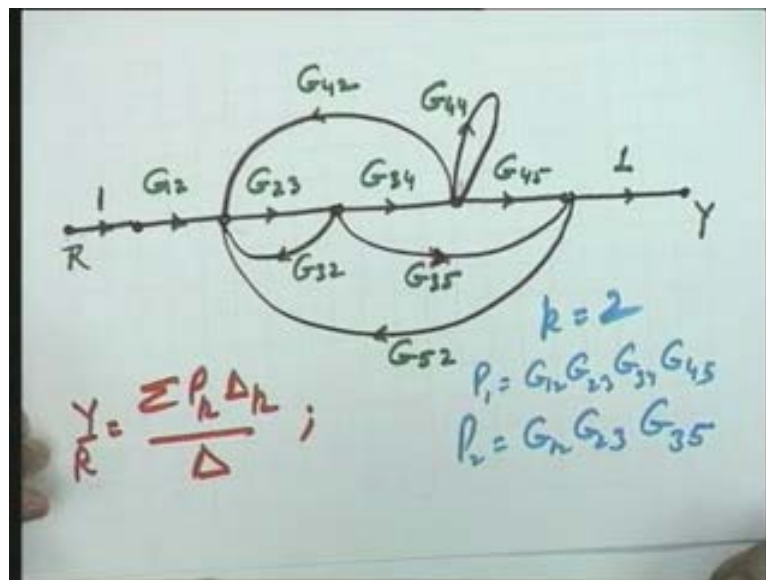
Is there a question there, any point there please?

So in this case you find that there are two forward paths only. **Come on, I hope all of you are getting it now.** Now let us see the loops, now how many loops please? Some of you might have identified by now. How many loops? The definition you know so that the loop gains will be available to us and we will then go to the delta formula. Five, five is okay. Please,

those of you who could not identify the five loops please see, let us trace those loops. One, two the self-loop, three, this one yes three, four, where is the fifth one; the fifth one is this please see, and not this (Refer Slide Time: 46:13) this is not a loop, this should be clear. Please see that this is not a loop because this is in this direction. I repeat for the benefit of those who could not identify the five. This is one, this is second, this is third the complete one, this is fourth and the fifth one is follow this path. I hope I need not write the loop gains; you can write, I can leave the problem incomplete as a home exercise for you. Just try to identify the loops and do the problem yourself. These are the five loops.

Do not make a mistake here. This is not a loop because the direction is this; it is a feed forward it is not a feedback signal this point may please be noted. Through G 35 it is a feed forward signal it is not a feedback signal that is why it is not forming a loop. So you have identified five loops so it means one minus the five loop gains will easily come there is no problem there.

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How about the non-touching loops, the pairs? Yes, how many pairs? Two one one, one pair, what is that pair?

[Conversation between Student and Professor – Not audible ((00:47:30 min))] I think I will put a different color on the pairs you are suggesting. G 44 you said okay this pair, this one and what is the other one please G 23 G23 G 32 this is very clearly a pair. There is one more, it appears. So it means the product of these two. So in the in the formula what will happen? One minus the bracket which has sum of all the loops plus start a bracket it is G 23 G 32 into G 44 plus now if you say other pair what is the other one please?

G 23 G 35, yes, that is fine. So let me put a separate blue color here. I think one is this only so rightly said please this is also a non-touching pair. There is nothing common a node or a branch. So this is a non-touching pair so there are two pairs of non-touching loops so this will give you the product of these two plus the product of these two with a plus sign outside the bracket. I am avoiding writing the complete expression.

How about triplets please? Is there a triplet possible?

No sir, it is no not possible so all other terms will disappear. Come on, now let us quickly go to delta 1 and delta 2 that is what is left otherwise the complete expression is before you.

What is delta 1 in this particular case?

You traverse this particular path and look at any loop which does not touch the first forward path.

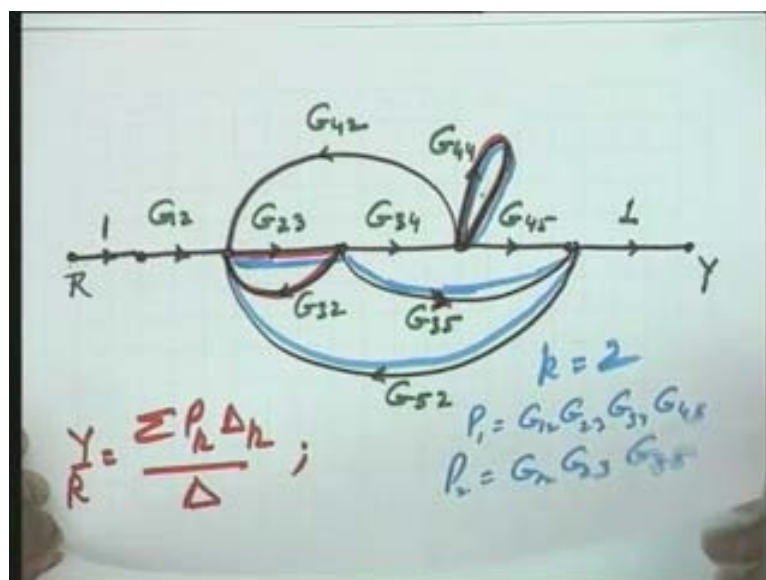
What is delta 1 equal to?

[Conversation between Student and Professor – Not audible ((00:49:14 min))] It is 1, so delta 1 is equal to 1 because all the loops are touching the first forward path. Look at delta 2 now, G 44. You see that this is the only loop so I am writing here delta 2 is G 44 and therefore numerator will become, delta 2 is not G 44 I am sorry delta 2 will become 1 minus G 44 you have to apply the basic formula. The delta formula you have to apply to get the value of delta 2 it is 1 minus G 44. I hope I can leave the problem here itself. All the terms of the basic Mason's Gain formula have been identified and therefore you can now apply this formula and get the overall transmittance of the system.

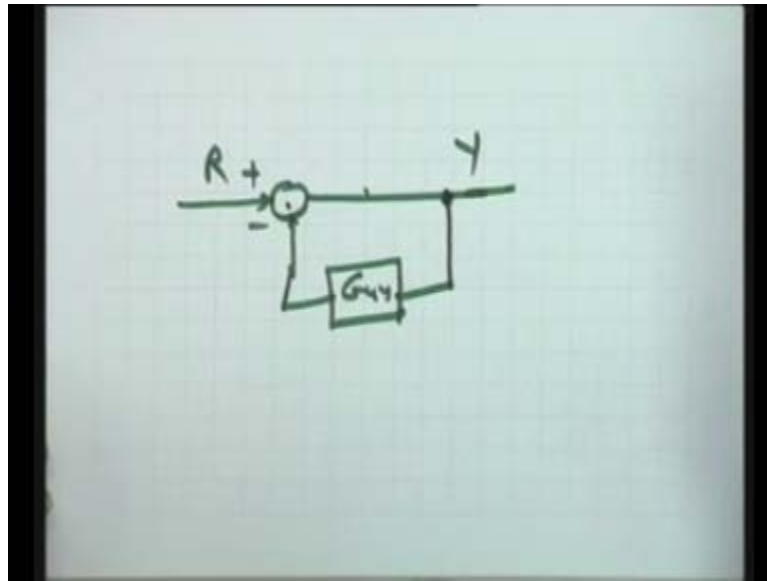
With this I can assure you that given a block diagram **you can apply** you can identify these loops directly in the block diagram, apply the Mason's Gain formula, get the results. To avoid the confusion for your benefit if you want you can transform the block diagram into a signal flow graph, identify the loops and then apply the Mason's Gain formula. Yes please?

[50:37.....sir what is G 44 doing in the actual loop diagram or in actual situation, yes for physical situation?] Okay, I will like to give the equivalent block diagram because in block diagram this loop will become very clear. But remember, I told you that this is a created situation so that the signal flow graph formula becomes more clear that is why its corresponding physical control system was not given. But yes visualization will become more convenient if I look at this particular loop in terms of its block diagram.

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So in this particular point I will say, fine, let me say this is R plus this is a feedback point and let me take a block here corresponding to G_{44} , this is let us say a signal Y . Well, I mean, in a control situation where do you require it, it is a different situation. you see that this particular signal Y if it is fed back through G_{44} you are going to get the loop you give me the equivalent node for this, you give me the equivalent load for this (Refer Slide Time: 51:27) the equivalent node for this if it is 1 because the transmittance at this point is 1 this point and this point is the same in that particular case this is realizing the self-loop that is the same. You give me the equivalent signal flow graph.

Well, whether in a system we require these things are not, whether we will require a feedback of this signal or not; this will become evident when we come to the physical controlled situations. But yes, the **signal flow graph** particular signal flow graph was given to you so that non-touching pairs are available and you are able to get the delta in a bigger way that is at least the second bracket should become applicable otherwise the **example** earlier example will terminate it at the first bracket itself that was the very specific objective. I hope this is okay. Fine then, I think next time we take up the practical examples of speed and position control systems on the coming Friday and I terminate my discussion on this, thank you.