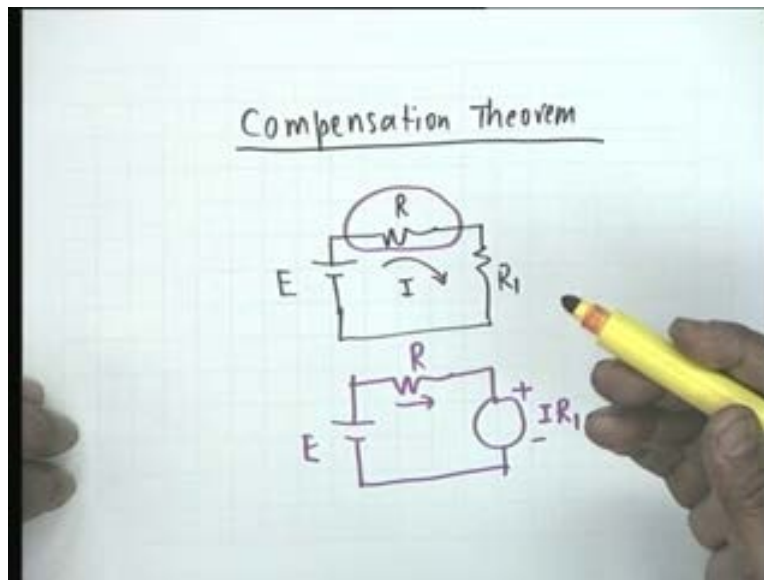


**Circuit Theory**  
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**Lecture No - 9**  
**Network Theory and Network Functions**

This is the ninth lecture. We discuss some network theorems and network functions in this lecture. One of the network theorems, that is not very familiar to many of the electrical engineers is an elementary one, which we shall use today and this is called the compensation theorem.

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Have you heard of this earlier? No? Well, it has 2 versions, compensation theorem. We require the elementary one. The elementary version of compensation theorem can be appreciated, if we take a simple circuit. We first take an example. Suppose we have a battery  $E$  a resistance, let us say,  $R$ , and the resistance  $R_1$ . The current through the circuit is capital  $I$ . Then, as far as capital  $R$  is concerned, we could replace capital  $R_1$ , that is, the rest of the circuit. You see, we concentrate our attention on this element,  $R$ . The rest

of the circuit contains a voltage source  $E$  and the resistance  $R_1$  in which the current  $I$  flows. The drop across  $R_1$  is  $I$  times  $R_1$ .

Now, if I draw a circuit like this  $R$  and a voltage source here, which is equal to  $I R_1$ , an ideal voltage source, then the rest of the circuit, that is, the element  $R$  that we have been considering, does the current change here, current or voltage drop? No, nothing happens and therefore, the compensation theorem states. This is an example of compensation theorem. It states that, any element in a linear time invariant passive reciprocal network, LLFPB network, any element can be replaced by a voltage generator whose voltage is equal to the potential drop across the element. That is as simple as that. This is the compensation theorem as far as the rest of the circuit is concerned. It does not matter.

Student: Sir, can you please repeat it?

Sir: Yes. In a LLFPB network, any element can be replaced by a voltage generator, equal to the drop across the element, we are going to use this. There is a second form of compensation theorem, which we shall illustrate, only if we need it at a later point of time and that relates to, you should know this, at least that the second form of compensation theorem relates to a change in an element.

For example, if this element  $R_1$  changes to  $R_1$  plus  $\Delta R_1$ , let me state it, at least once for all, you know this. If  $R_1$  changes to  $R_1$  plus  $\Delta R_1$ , then the effect of  $\Delta R_1$ , change can be due to temperature, can be due to humidity, can be due to ageing and all kind, 1 k resistor may become 990 ohms. So the effect of  $\Delta R_1$ , as far as the rest of the circuit is concerned can be represented by an equivalent voltage generator whose voltage is equal to the capital  $I$ , the current flowing through it, multiplied by  $\Delta R_1$ . It is a very similar form.

But, while our simple one, elementary one, concerns the total potential drop, the second form concerns only the incremental drop and that is how second form of the compensation theorem is used in sensitivity studies. That is, if an element changes by a

small amount of 10 percent, what is the effect of the rest of the circuit? The second form, we might require. If we require, then we shall go more details. But the first form simply is that, if we have an element, current flowing through it then this element, as far the rest of the circuit is concerned, can be replaced by voltage generator. An ideal voltage generator, whose voltage is equal to the drop across the voltage drop across the element.

Student: Sir what if E was changed?

We shall use this. What if E was changed, no, rest of the circuit remains the same. You see, it is a network analysis problem. In which, the effect of any element can be replaced by a voltage stabilizer. Nothing else changes it does not, if e changes, of course, the current changes.

Sir: Sir, if R1 changes, then I will also change.

Yeah, but that can be taken care of. This is why I am saying, the second form is slightly more involved. We will not enter into proof of that but the second form, I changes, but to keep I constant, an equivalent voltage source can be added,  $\Delta R_1$ , as far as the rest of the circuit is concerned. That means the current I, the new current I, shall be obtained by considering the change in  $\Delta R_1$ , as equal to voltage source whose drop is the previous current, multiply by delta.

Student: Sir, could we use a current source?

Sir: Could I use a current source? Yes.

Sir: In this case sir, increase in, change in R1 would also affect I.

That is what the question was. So what the second form of the compensation theorem states that the new current, in the rest of the circuit, can be taken care of by an equivalent voltage generator whose value is equal to  $\Delta R_1$  multiplied by the previous current.

This is how the compensation theorem becomes important. But as I said, we shall look into that in more details when it comes to the point. At the present time, we shall only use this particular form that any element can be replaced by a voltage generator equal to the drop occurs.

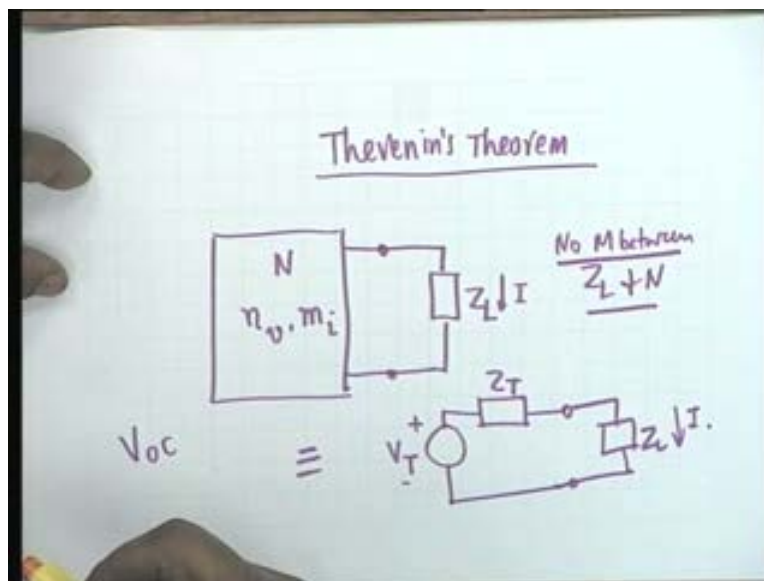
Student: Sir, will not this require that  $\Delta R_1$  tends to 0?

Sir: We are not considering that form at all. So do not consider this as result of the second form of compensation theorem. No, this is an elementary compensation theorem. Forget about  $\Delta R_1$ .  $\Delta R_1$ , I simply mentioned that you should be aware that the compensation theorem had the second form. We shall deal with that later.

Student: Sir  $L$  and  $Z$  can also be (...)

Sir: Any element. I simply say the respective resistance but any element  $Z$ , If a current  $I$  flows, then for rest of the, the rest of circuit is concerned, it can be placed by a voltage source whose voltage is equal to  $I$  times  $Z$ , with the same polarity. That is equal to picture. We shall use the compensation theorem in proving the Thevenin's theorem.

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I am sure, you are acquainted with this statement of Thevenin's theorem, but we will prove it, in this class. I do not think it was proved. It might have been verified, taking a resistive network or some other network, but proof requires the help of the compensation theorem. While, as you know, the statement of the Thevenin's theorem is that; if you have a network  $N$ , having  $n$  number of voltage sources and  $m$  number of current sources, this is how we indicate, a network  $N$ , linear passive reciprocal etcetera, not passive because it contains active sources.

So a general linear network containing  $n$  v, containing  $n$  number of voltage sources and  $m$  number of current sources, that is what indicate by  $n$  v and  $m$  i which delivers current or power to a load  $Z_L$ . Now we are being quiet general and you are working in the frequency domain, the  $S$  domain. No longer the time domain.

Capital  $N$  may contain linear elements only, linear bilateral elements. Capital  $N$  may also contain voltage sources, current sources, it may contain control sources also. That is, a source whose current or voltage is controlled by some other voltage or current, but the only thing that is barred is that the load and the network. The load is a typical element in the whole network. It is a typical element. We are interested in finding out the current that flows through the load. We are interested in finding a current through any element which we call the load.

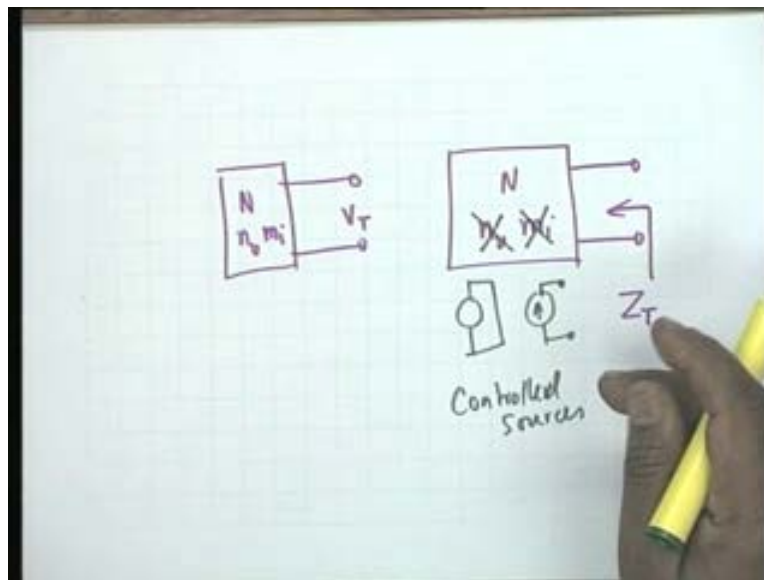
The only restriction is that  $Z_L$  and  $M$  must not have any magnetic coupling. No, magnetic coupling. No  $M$  between  $Z_L$  and  $N$ , this is the only restriction, then Thevenin's theorem states that as far as, this is very important no mutual inductance. There cannot be magnetic coupling. Then the Thevenin's theorem states that as far as the load current is concerned, capital  $I$ , the whole network can be replaced by an equivalent voltage source  $V_T$ , capital  $T$  for Thevenin's in series with an equivalent in impedance  $Z_T$ , again  $T$  for Thevenin's. It states that this is equivalent to this simple network.

The current in the circuit, the current in the load shall be the same. This is the statement of Thevenin's theorem. Where, now we shall have to specify what  $V_T$  and  $Z_T$  are.

Thevenin's theorem states that the whole network  $N$  can be replaced, in so far as the current through  $Z_L$  is concerned, the whole network can be replaced by an equivalent voltage source  $V_T$  and in series with an equivalent impedance, the Thevenin impedance  $Z_T$ .

The value of  $V_T$ , as you know, is the open circuit voltage across  $N$ . That is, if  $Z_L$  is open circuited, the current is made equal to 0, then the voltage that appears here is the  $V_T$ . So  $V_T$ , sometimes, is  $V_{oc}$ , open circuit, and  $Z_T$ , that is more important,  $Z_T$  is the impedance. Looking back into the network,

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Now let me draw this, capital  $N$ ,  $n, v, m, i$ , this is  $V_T$  open circuit. No current flows and, as far as  $Z_T$  is concerned, it is the impedance looking back into the network with the voltage sources and current sources killed or inactivated, that is, inactivation means that voltage sources, if they are ideal, that is, replaced by short circuits, ideal voltage sources are replaced by short circuits and ideal current sources are replaced by open circuits. This is what killing the sources means.

A point of caution, controlled sources cannot be touched. Controlled sources must be left as they are. Controlled sources are sources whose current or voltage is controlled by another current or voltage in this circuit.

Student: Sir, we cannot open it with a.

Sir: You neither open, nor close it. No short circuited. You just leave them untouched, controlled sources. This point is extremely important. No control sources should be touched, only independent sources. That is, voltage generator whose voltage does not depend on any other current or voltage in this circuit. Current generator, whose current is, does not depend on any other current or voltage, in this circuit, only those have to be replaced, like this. Ideal voltage sources short circuited, ideal current sources open circuited, if they are not ideal. If they are not ideal, then we replace them by their equivalent internal impedances.

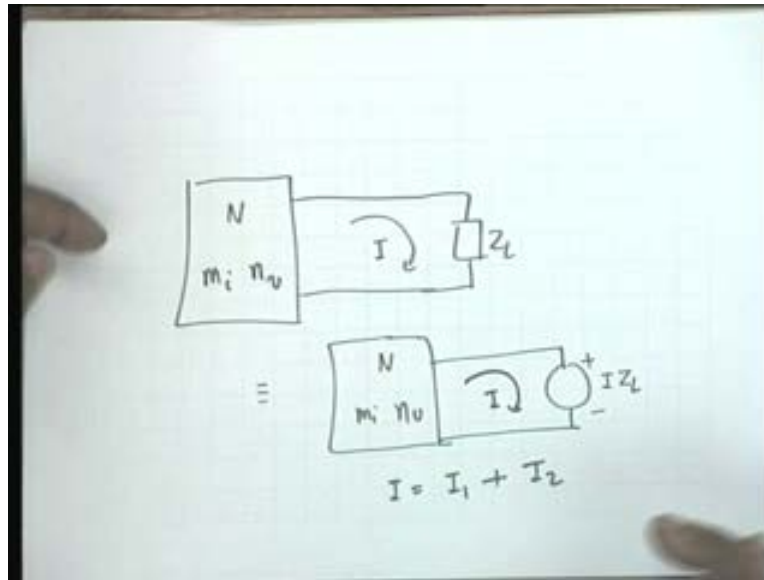
Student: Sir could you give us examples of controlled sources?

Sir: Yeah, I will. But first, let me take control sources out of the picture, first, and let me first prove Thevenin's theorem, then I will go to an example of controlled sources.

Student: Sir, in case of magnetic coupling, can we replace n (...)

Sir: Provided, it is a 3 terminal network, you cannot change the physical structure of the network. Then you are changing everything. If it is a 3 terminal network, what is saying is, if I have a load, which is magnetically coupled, can I join equivalent circuit? Yes, you can join equivalent circuit, provided, you do not destroy the architecture, architectural niceties or peculiarities of the structure. It is the 4 terminal network, you should live it as a 4 terminal network. If you can find out equivalent circuit perfectly not other works. Now the proof of Thevenin's theorem proceeds like this.

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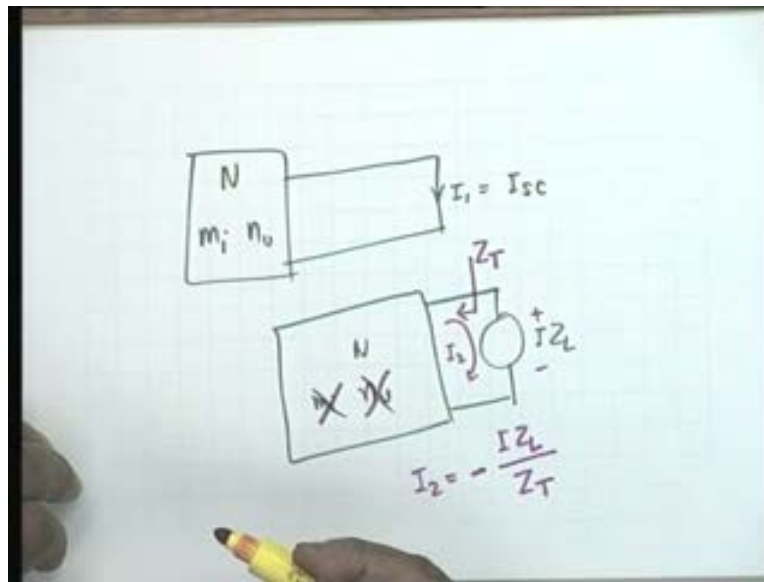


We have the network  $m$  number of current sources  $n$ , number of voltage sources and we have the load  $Z_L$ . This current is  $I$ , so what we do is, now we use compensation theorem. To find the current  $I$ , to find the current  $I$ , we place  $Z_L$  by an equivalent voltage source. In other words, this will be equivalent to capital  $N$   $m$   $i$   $n$   $v$ , then a voltage source whose value is  $I$  times  $Z_L$  and this current is  $I$ , agreed?

The current  $I$ , now invoke linearity. The current  $I$  therefore, is a result of sources internal to  $N$  and this source,  $I$  times  $Z_L$ . The current  $I$  is a result of sources internal to the network and this source and therefore,  $I$  can find them independently and apply the principle of superposition. In other words, capital  $I$  shall be equal to  $I_1$  plus  $I_2$ , where  $I_1$  is the current that flows due to  $m$  number of current sources and  $n$  number of voltage sources internal to capital  $N$  with the second, inactivated. Inactivated means, it should be replaced by a short circuit and therefore, the interpretation if  $I_1$  is that,  $I$  have  $m$   $i$  and  $n$   $v$ .



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This is  $I_1$ ,  $I_1$  is therefore, equal to  $I_{sc}$ . We call this short circuited current,  $I_{sc}$ , and the effect of  $I_2 Z_L$ , effect of  $I_2 Z_L$ , that is  $I_2$ , what we do is, we kill these sources and then, what would be, if I kill these sources, the current that flows is equal to  $I_2$ . Now  $I_2$ , obviously, would be equal to, what is this impedance? If I look into the network  $Z_T$ ,  $Z$  subscript  $T$ , this is Thevenin's equivalent impedance.

Therefore,  $I_2$  would be equal to  $I_2 Z_L$ . This voltage source divided by  $Z_T$ , but with a negative sign. That is correct, with a negative sign because  $I_2$  opposes  $I_2 Z_L$ .  $I_2 Z_L$  tends to send the current into the network, whereas  $I_2$  has been shown out of the network, coming out the network. Therefore,  $I_2$  is equal to this and therefore, my capital  $I$ , which is the addition of the 2. The current  $I$  therefore, becomes equal to  $I_1$  plus  $I_2$  which is equal to  $I_{sc}$  plus, not plus, minus,  $I_2 Z_L$  divided by  $Z_T$ .

Now this should be, I have made a mistake, it is okay? We call this now, is equal to  $V$ , some voltage  $V$ .  $I_2 Z_L$  equal to  $V$ . Now I can, this equation should be valid, I am doing something which you should understand now.

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$$\begin{aligned} I &= I_1 + I_2 = V \\ &= I_{sc} + \frac{I Z_L}{Z_T} \quad V_{oc} \neq V \\ I &= I_{sc} - \frac{V}{Z_T} \\ 0 &= I_{sc} - \frac{V_{oc}}{Z_T} \\ I_{sc} &= \frac{V_{oc}}{Z_T} \end{aligned}$$

I have decoupled now the current  $I$  from the voltage  $V$ , it was an independent voltage source  $V$ , now this equation should be valid even if I open the load. That is, if I make  $I$  equal to 0, then this equation should be valid. And if I make  $I$  equal to 0, then  $I_{sc}$  minus, the  $V$  that I shall get now is the open circuit voltage that is  $V_{oc}$  divided by  $Z_T$ . Is that clear?

Student: Sir, explain this again sir.

Sir:  $I_{sc}$  should be equal to instead of  $V$  I will write open circuit with  $V_{oc}$  by  $Z_T$ .

Student: Is not  $V$  a function of  $I$ ?

Sir: That is the question,  $V$  is a function of  $I$ , but  $V$  will exist even if  $I$  is equal to 0. And that there condition you see,  $V_{oc}$  is not, in general, equal to  $V$ . In general, no.  $V_{oc}$  is the special value. This is the  $V_2$  of the compensation theorem, whereas the rest of the circuit is concerned, you can replace it by voltage generator. Now you disconnect the component, the voltage generator remains, but it becomes open circuit voltage. That is, it draws no current from the load. This is the application of the compensation theorem and

therefore,  $I_s c$  equal to  $V_o c$  by  $Z_T$ . Now let us go back to the equation, that is, what I get is,

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$$\begin{aligned}
 I &= I_{sc} - \frac{I Z_L}{Z_T} \\
 &= \frac{V_{oc}}{Z_T} - \frac{I Z_L}{Z_T} \\
 I \left(1 + \frac{Z_L}{Z_T}\right) &= \frac{V_{oc}}{Z_T} \\
 I &= \frac{V_{oc}}{Z_L + Z_T}
 \end{aligned}$$

I equal to  $I_s c$  minus  $I Z_L$  by  $Z_T$  and  $I_s c$  is equal to  $V_o c$  divided by  $Z_T$  minus  $I Z_L$  divided by  $Z_T$  and therefore,  $I \left(1 + \frac{Z_L}{Z_T}\right)$  is equal  $V_o c$  by  $Z_T$  and therefore, I equals to  $V_o c$  divided by  $Z_L$  plus  $Z_T$  and that proves Thevenin's theorem. That is, as far as the current is concerned, current is given by a voltage source  $V_o c$ , then a series combination of  $Z_T$  and  $Z_L$ .

Student: Excuse me sir, explanation for the replacing the voltage source by an open circuit (...)

Sir: Okay, for the compensation theorem?

Student: Sir, you replaced the compensated voltage source by an open circuit.

Sir: I do not replace it by an open circuit. What I said was, this is a voltage generator capital V, where capital V exists even if I becomes equal to 0. Under that condition, we

call it  $V_{oc}$ . You see, the network contains voltage sources and current sources and therefore, if the load is infinite, even then a voltage exists and that voltage, we are calling  $V_{oc}$ . Because  $I$  is equal to 0, the voltage is not 0, the voltage  $V$  equal to  $I Z_L$  is a general condition, but when  $I$  equal to 0, the voltage  $V$  assumes a value which is equal to  $V_{oc}$ . It is not 0. Is that clear?

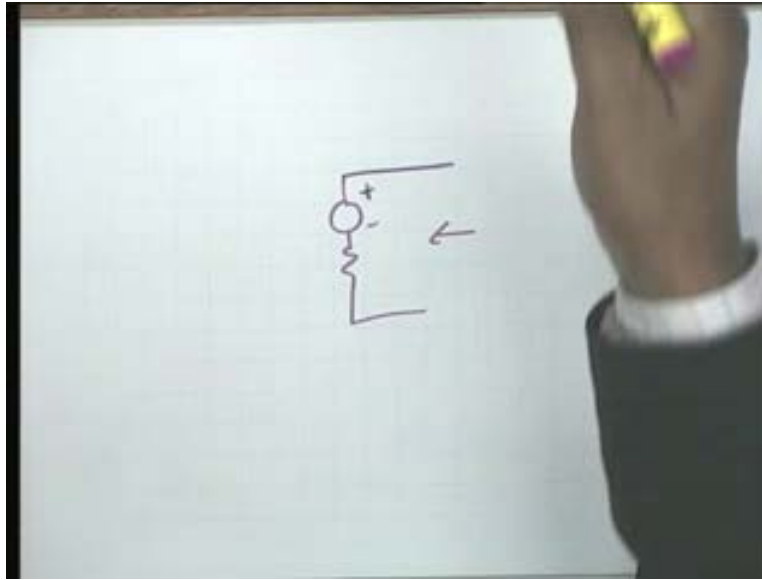
Let us take a simple example. If I have a battery, same  $R_1$ , the voltage here is  $I R_1$ . Now if I open circuit this, the voltage was  $I$  times  $R_1$ .  $I$  becomes equal to 0, but  $R_1$  becomes infinite. The product can still be finite and that product, we have called  $V_{oc}$ . Is that clear? We have not violated any condition.  $Z_L$  becomes infinity, capital  $I$  becomes 0, the product of 0 and infinity can still be a finite quantity and that is what, if we measure here, obviously, it will be equal to battery and therefore, the product still remains finite.

Now Thevenin's theorem therefore, states that as far as the current in a particular element in the network is concerned, consider that element as a load. Find out the equivalent Thevenin's source, which is equal to the open circuit voltage and the equivalent Thevenin impedance, which is the impedance looking back into the network, with independent sources replaced by their internal impedances. Independent, is an important word. Now we will illustrate with the help of 2 examples, the application of Thevenin's theorem to a situation when there are no dependent sources and to another situation where there are dependant sources. Let us look at them.

Student: Sir, if there is an independent source and there are resistors, and we are seeing into the circuit, then what will be the resistance?

It will be the resistance only if it is a voltage source, independent voltage source. What you are saying is, if I am looking at this, to calculate the Thevenin's equivalent impedance, it will be simply equal to this resistance.

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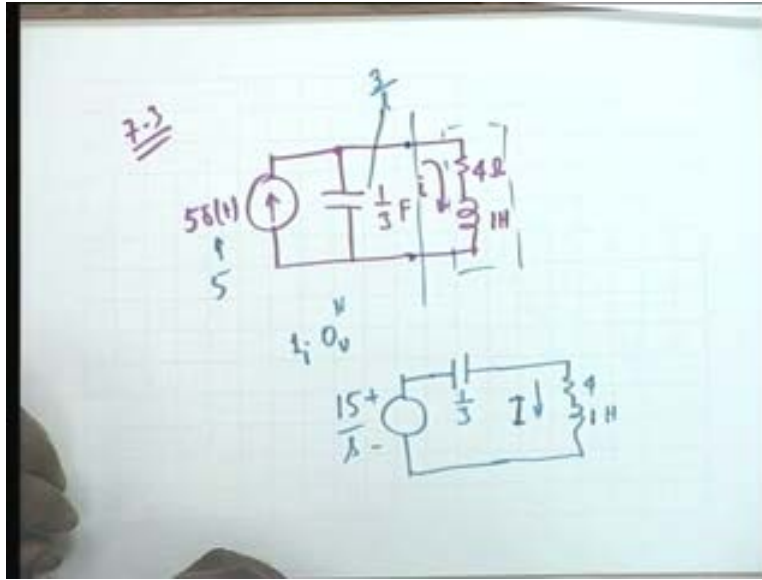


Student: No Sir, if it is controlled source?

Sir: And this, if it is control source, then obviously, this source is controlled by another voltage or current. We shall find out that voltage, that will be apparent when you do the example. Let us take 2 examples. I am proceeding very slow because Thevenin's theorem is one of the most important theorem and usually, we do not prove it in the class. You know, we simply, straightly apply and we must verify. For the first time I gave a clue, so let me proceed carefully with an example, 2 examples as I said.

The first example is example is 7 point 3 in the text book, where we have a current source  $5 \delta$  of  $t$ , a  $1/3$  farad capacitor, then we have a 4 ohm resistor and a 1 Henry inductance. It contains resistance inductance and capacitance and it is this current  $i$  of  $t$  which has to be found out by application of Thevenin's theorem. Therefore, Thevenin's theorem, as far as Thevenin's theorem is concerned, this is my  $Z_L$ , 4 ohm and 1 Henry, and this is the rest of the network  $N$  which contains 1 current source, 0 voltage sources.

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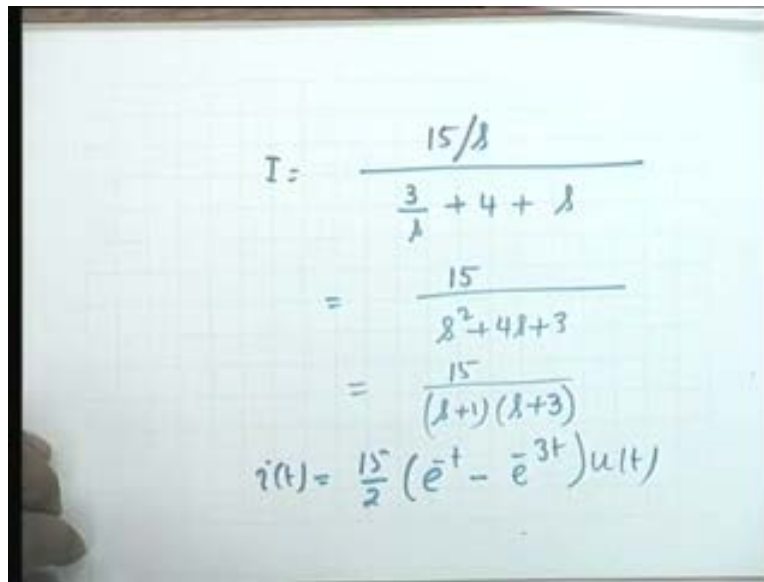


Now obviously, if I open circuit this, if I remove that, the whole current will flow through on third farad capacitor and if I go back to my transform domain, then I have, with voltage source would be 5 times 3 by S. So it is 15 by S, that would be the voltage source. The transform of this is 5 and this, the impedance of this is 3 by S and therefore, this current flows through 3 by S and therefore, the voltage source is 15 by S.

Then the impedance, looking back, obviously, current source has to be opened and therefore, the impedance is simply, 1 third farad capacitor, and then we have the 4 ohm and 1 Henry inductor and this is the current, it becomes the single loop equation. I should write capital I and so it is very easy to find out what capital I is, let us put it.

Capital I should be simply equal to 15 divided by S divided by 3 by S, from the capacitor, plus 4 plus inductance S. So that is equal to 15 divided by S squared plus 4 S plus 3, that is, 15 divided by S plus 1 S plus 3.

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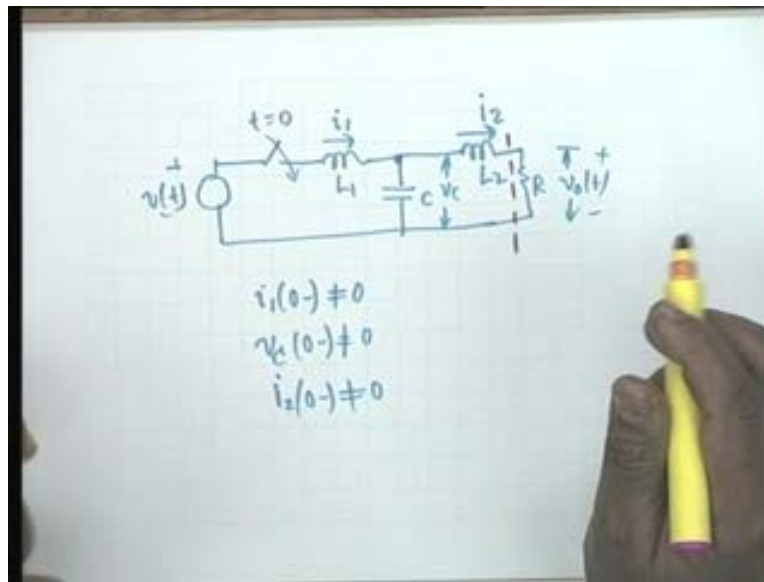

$$\begin{aligned} I &= \frac{15/s}{\frac{3}{s} + 4 + s} \\ &= \frac{15}{s^2 + 4s + 3} \\ &= \frac{15}{(s+1)(s+3)} \\ i(t) &= \frac{15}{2} (e^{-t} - e^{-3t}) u(t) \end{aligned}$$

Which I can write as, or we can break it into partial fraction and final result is 15 by 2. By solution,  $e^{-t} - e^{-3t}$  times  $u(t)$ , from this step going here is  $u(t)$ . There is nothing very brilliant about it. I have founded I of t. A question that can rise here, I have taken this example intentionally to raise this question. Let us see that if that question arisen yet in any of the minds. I intentionally took this problem, in order to get a question from the students.

I did not say anything about initial conditions in the circuit. I went straight ahead with Laplace domain. In other words, I assumed that initial conditions are 0. I did not assume a  $V_c(0^-)$ , I did not assume an  $I_L(0^-)$ . So the question that should arise at this moment, I expected, is Thevenin's theorem valid only for initially relaxed circuits? No, the answer is no. You can take care of the initially, initial conditions in exactly the same manner as voltage sources and current sources.

And independent voltage sources and current sources, I will take the second example to illustrate that particular point, a fairly involved example.

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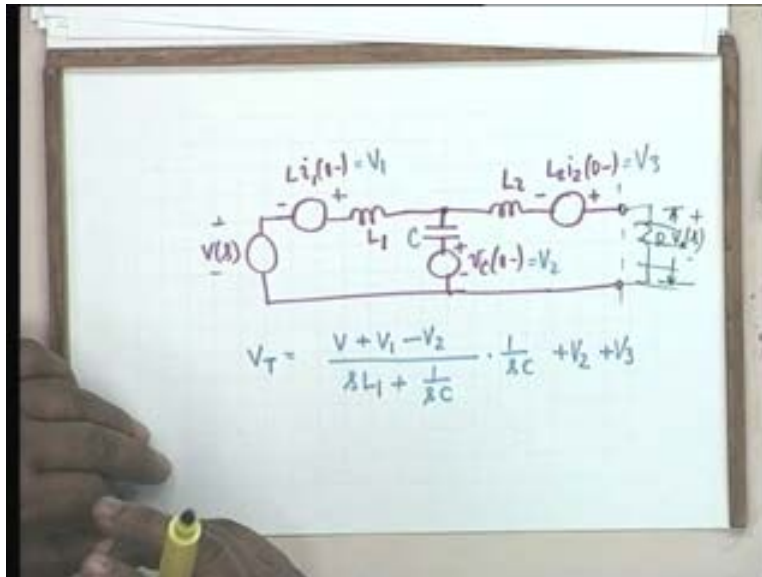
Let us say, we have a  $v$  of  $t$  and then a switch which opens a  $t$  equal to 0, which closes a  $t$  equal to 0. We have an inductor  $L_1$  the current is  $i_1$ ,  $i_1$  of 0 minus is not equal to 0. It is given,  $i_1$  of 0 minus is given, then a capacitor  $c$ , the voltage across switch is  $V_c$ .  $V_c$  of 0 minus is also given. It is not equal to 0.

Let us make it quite general, then we have another inductor  $L_2$  in which the current is  $I_2$  and  $I_2$  of 0 minus is also not equal to 0 and finally we have a resistance  $R$  whose value, the voltage across which is  $V_0 t$  and it is  $V_0 t$  which you have to find out. It is this resistance, it is this voltage that you have to find out. That is, this is the load by application of Thevenin's theorem.

Well, the first thing you do is, to draw an equivalent circuit in the frequency domain. An equivalent circuit in the frequency domain and it will look like this. If you proceed carefully, it will look this, that is, you shall have, let me draw it separately, we shall have a  $V$  of  $s$ .



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Which is the Laplace transform of  $V_T$  multiplied by  $u(t)$ , because the switch was closed, then as far as the inductor  $L_1$  is concerned, its initial conditions can be taken care by a voltage source, in opposition minus  $L_1 i_1(0^-)$ . So it would be a minus plus  $L_1 i_1(0^-)$ . Is that okay? Then an inductor  $L_2$ , whose impedance is  $sL_2$ . This takes care of the inductor  $L_2$ , the capacitor can be taken care of by a relaxed capacitor, in series with a voltage source, opposition or for, plus minus and this would be  $V_c(0^-)$ .

The third inductor can be similarly taken care of, by the inductor  $L_2$  in series with a voltage source which would be in opposition, that is, this should be  $L_2 i_2(0^-)$  and this is what the network  $n$  is. This is, to these points shall be connected, the resistance  $R$  and this is  $V_0(s)$ . What we have to find out is  $V_o(s)$  and the impedance  $Z_T$ . Now to simplify matters, let us call this voltage as  $V_1$ , let us call this voltage as  $V_2$  and this voltage as  $V_3$ .

Then is  $V_T$  obvious? I do not have to write in a loop equation or node equation. You see that, how I do by observation. You see  $V_T$ ,  $V_T$  obviously, is this voltage, this voltage, minus or plus? Plus  $V_3$ , this voltage plus  $V_3$ . And if you open circuit this, no current

flows in  $L_2$  and therefore, current only flows in this loop. If I open circuit this, that is,  $R$  is removed, you remove  $R$ , then this voltage, what will this voltage? The current in this circuit, the current in this loop shall be  $V_1 - V_2$  divided by,  $V_1 - V_2$ , this is  $C$ , divided by  $sL_1 + 1/sC$ , agreed? This will be the current.

So the drop in  $C$  shall be this multiplied by  $1/sC$  and this drop adds to  $V_2$ . This will be the voltage here, then plus  $V_3$ , this will be  $V_3$ , is that clear? This is step by step observation, but very careful observation. Not seen, there is a difference between seeing and observing. Seeing is what a news used to do and observing is what Sherlock Holmes used to do. Have you read Sherlock Holmes, Arthur Conan Doyle? Holmes always used to say what some, you do not see, you only observe here. You have to observe here and a very systematic observation. Is this clear? That  $V_T$  can be written down by inspection.

Similarly,  $Z_T$  can be also be written by inspection. For finding  $Z_T$ , you short circuit this, you short circuit this, you short circuit this, you short circuit this, because they are all voltage sources and independent voltage sources and therefore,  $Z_T$  will be simply as  $L_2 + 1/sC$ , in parallel with  $sL_1$  and once you find out  $V_T$   $Z_T$ , you can now find out the current in  $R$  or the voltage across  $R$ . The voltage across  $R$  be 0 will be simply  $V_T$  divided by  $Z_T + R$  multiplied by  $R$ . That is all. You see, a fairly complicated circuit, 2 meshed circuit solved by inspection. You do not have to write mesh equations, you do not have to write node equation.

This is the beauty of Thevenin's theorem, that if you do not want the currents in voltages, in all the branches of the network, it is only 1 branch, then you can avoid the writing those messy mesh equations or nodal equation. You can simply apply Thevenin's theorem and most of the time, the result will come out by inspection. I did not solve the equation here. I just wrote down by inspection. This is the beauty of Thevenin's theorem. So Thevenin's theorem can be applied to networks containing initial conditions. There is no problem.

Student: Sir, this is in frequency domain?

Sir: This is in the frequency domain, yes, in the frequency.

Student: It changes in the time domain?

Sir: That, in the time domain, you have to write the corresponding equations, we will have to solve the differential equations.

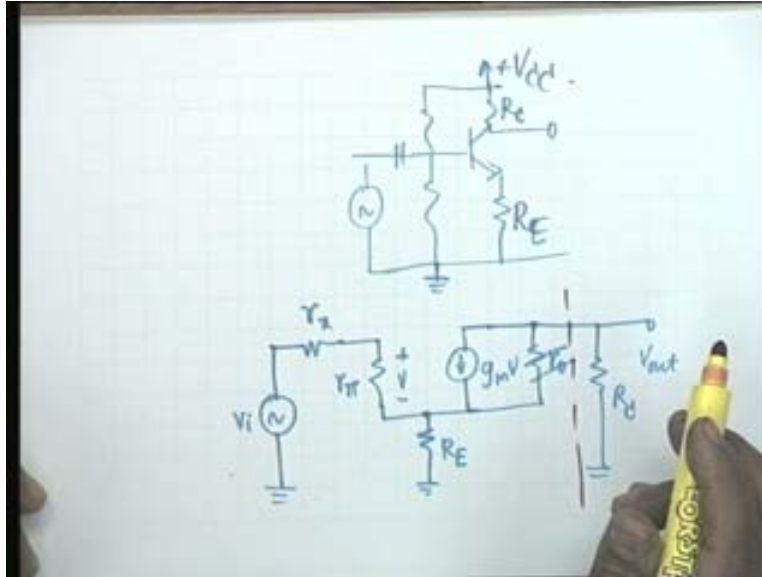
Student: Sir, is that required?

It is not required, if you are good in frequency domain.

Student: So sir, can we leave it like this, if the output is 0?

Sir: No, you have to find it, after you find out  $v_0$ . You have find out small  $v_0$  of  $t$ , yes, but you see, the  $V_T$  of the frequency domain is, you do not handle any differential equation. All equations are algebraic. Now let us take the, another example, in which, there is a control source and the most common example is that of a transistor, transistor amplifier. Let us say, with an emitter resistor, to make things complicated, which is an un bypassed.

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So the equivalent circuit, I am talking of a circuit like this. The signal source is here, of course, there are coupling capacitors and the output is, let us say, taken here. This is plus  $V_{CC}$ . Let us take a simple transistor amplifier with this unbypassed, the emitter resistor unbypassed. Then the equivalent circuit, if this is let us say  $R_C$ , the equivalent circuit is the following. You have  $r_x$ , the base spreading resistance, the input source let us say,  $V_i$ , going to ground. We call this as ground. We ignore the biasing resistors. We think that they, we consider them to be large compared to  $R_{\pi}$ .

Had you accounted with these symbols? And we ignore low frequency analysis, we ignore the internal capacitances. So let us say, this voltage is  $V$  then as you know, this is the emitter terminal from which goes a source, a resistance  $R_E$  and this current source is  $g_m v$  between the collector and the emitter and between the collector and the emitter, you also have the internal resistance of the transistor which is very large or 0 or maybe you have ignored it so far. We can ignore it, let us ignore it.

To make life simple, then you have the resistor  $R_C$  which goes to ground and this is  $V_{out}$ . Suppose, we want to solve this circuit by Thevenin's theorem, then what we have to do is, we open circuit  $R_C$ , find out  $V_{oc}$  here. In finding out  $V_{oc}$ , of course, there is no problem, finding out  $V_{oc}$ .

Student: Sir, What is this  $R_Z$  exactly?

Sir: What is  $R_0$ ?  $R_0$  is the collector to emitter resistance.

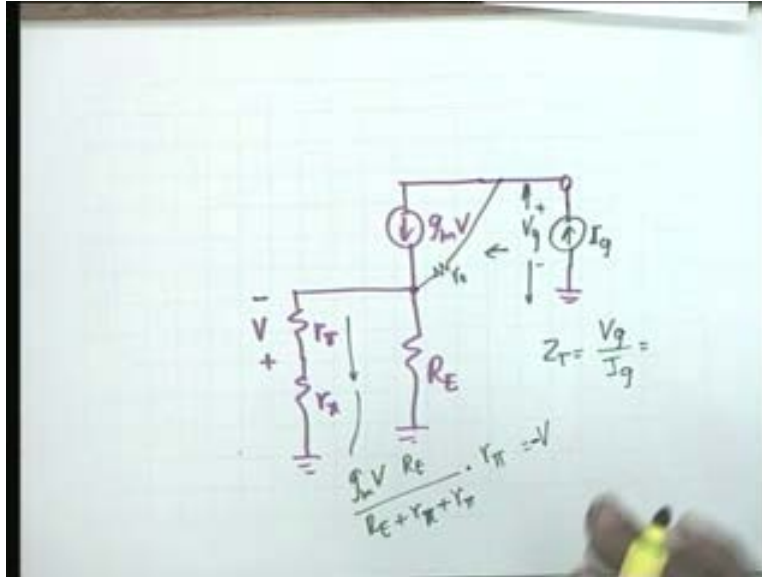
Student: Sir  $R_Z$ .

Sir: Oh  $r_x$ ,  $r_x$  is the base spreading resistor, that is between the base and the internal base  $B'$ . This is the small resistance of the order of tens of ohms, 10 to 100 ohms. That also is sometimes ignored. But what I am pointing out is, in the analysis of this circuit which you have done, by mesh analysis or node analysis, you could also do by Thevenin's. By simply open circuit it this, finding out voltage here and then to find out  $Z_T$ . To find out  $Z_T$ , you do not kill this source. What you do is, you kill this source,  $V_I$ , but not this one. This must be left intact.

To find out what this source is, you have to find  $V$ , that is the drop across  $r_{pi}$  and the simplification would be that if this source is killed then  $r_{pi}$  and  $r_x$  coming in series to the ground and therefore, what you have to do is, a current source  $g_m V$ , goes into 2 resistances. One is  $R_e$  and the other one is  $r_x$  and  $r_{pi}$ . So the drop across  $r_{pi}$ , that would be capital  $V$  and then you can find out what the impedance here is. Is that clear?

You find out  $V$ , you find out  $g$  and  $V$  then therefore, for the impedance consideration, the value of  $V$ ,  $V$  shall disappear. It will be simply an impedance. Is that okay? Or you want me to work this out? I want it to avoid that but let us do it, it does not matter.

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There is a  $g_m V$ , then an  $R_E$  and in parallel I have  $r_{\pi}$  and  $r_x$  and this voltage is  $V$  with, you must be careful about the polarity. This is minus and this is plus. I am trying to find out the impedance, looking here. Now since this current source, how do I do that? since there is a current source here, let me use a current source capital  $I_g$  and then what we have to find out is  $V_g$ . The impedance, looking back, would be  $V_g$  divided by  $I_g$ . Obviously,  $I_g$  is equal to  $g_m V$  and

Student: Sir, why do we put  $I_g$ ?

Sir: Why? You see, how do I find out the impedance looking here? I connect a source here and find out the response. I could connect a voltage source, find the current or I connect a current source, find the voltage. Now since there is a series in current source, I use a current source. That is the simplest thing to do. So my  $Z_T$  shall be equal to  $V_g$  divided by  $I_g$  and this is  $I_g$  is equal to  $g_m V$ . Now this current, obviously, shall be  $g_m V$  multiplied by  $R_E$ , divided  $R_E$ , plus  $r_x$ , plus  $r_{\pi}$  and this current multiplied by  $r_{\pi}$  is equal to  $V$ . No, it is equal to minus  $V$ .

So now what is, I know  $I_g$  is  $g_m V$  and the drop, how do I find out this drop? This drop is, that is where I needed this. This drop is 0, across the  $g_m V$ , it has become a

degenerate case now. This is why I needed that  $R_0$ . You remember  $R_0$  we had ignored? If I have  $R_0$ , then it is easy to find out this drop. But let us leave it to you. I do not want to spend time on this calculation. I set this as an assignment to you. You have done this earlier. You have done this by different method, by using the mesh analysis or node analysis, but it can be done. I have told you sufficient details about this. Even if I notice not there, even if I am not is absent, one can calculate, there is no problem.

Student: Sir will not it be easier if we, in this case, we find out the short circuit current and the find  $Z_T$  by  $V$  open circuit?

Sir: Okay, that is an easier method, yes, but you are required to calculate  $Z_T$ . You can do that  $Z_T$ , you can calculate it,  $V_{oc}$  by  $I_{sc}$  short circuit.

Student: Sir, how will we find  $V_{oc}$ ?

Sir: How will you find  $V_{oc}$ , open circuit this, a source is there, find out the output voltage.

Student: Sir, what is the voltage drop across the current source?

Sir: What is the voltage drop across current source, all that would be determined by the rest of the circuit that, see when you calculate  $V_{oc}$ , this is not absent. This is still there

Student: Sir, but if we open circuit it, we will not get any  $i_m$ , any current at that branch.

Sir: If you open circuit this, we shall not get a current here, so that is a clue.

Student: Yes sir, the voltage drop across  $R_e$  is equal to the voltage drop.

Sir: Quite so. Thus simplifications, you look at them. It is not a difficult task. The rest of the time, I have another 5 minutes, rest of the time I want to talk about the Norton's theorem.

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The image shows a handwritten derivation of the current  $I$  in a circuit. The derivation starts with the equation  $I = \frac{V_T}{Z_T + Z_L} = \frac{V_T/Z_T}{1 + \frac{Z_L}{Z_T}}$ . This is then simplified to  $I = \left(\frac{V_T}{Z_T}\right) \frac{Z_T}{Z_T + Z_L}$ . Below the equations is a circuit diagram consisting of a current source labeled  $\frac{V_T}{Z_T}$  in parallel with an impedance  $Z_T$ , and a load impedance  $Z_L$  connected in series with the  $Z_T$  branch. The current through  $Z_L$  is labeled  $I$ . A hand holding a yellow marker is visible on the right side of the whiteboard.

As you know, Norton's theorem is the dual. If I had told you once and I repeat, I will not tell you this again. Take necessary as early something to you, do work it out. Norton's theorem says the statement is, that is, exactly dual, like Thevenin's theorem. It says that as far as N with m number of current sources, n number of voltage sources, as far as the current in a load is concerned, I, the network can be replaced by an equivalent current source  $I_N$  in parallel with an impedance  $Z_N$  and then  $Z_L$ . This current is  $I$  where  $I_N$  is the short circuit current, that is,  $I_N$  is  $I_{sc}$  and  $Z_N$  is the same as  $Z_T$  and this, Norton's theorem can be proved very simply, if you have proved Thevenin's theorem. That is, we proceed indirectly from Thevenin's to Norton.

Now Thevenin's theorem says that the current  $I$  is equal to  $V_T$  divided by  $Z_T$  plus  $Z_L$ , which I can write as  $V_T$  by  $Z_T$ , look at this proof.  $1 + \frac{Z_L}{Z_T}$  and this, I can write as  $V_T$  by  $Z_T$  times  $Z_T$  divided  $Z_T + Z_L$ , which means that the current would be given by a current source  $V_T$  by  $Z_T$  divided between  $Z_T$  and  $Z_L$ . This will be the

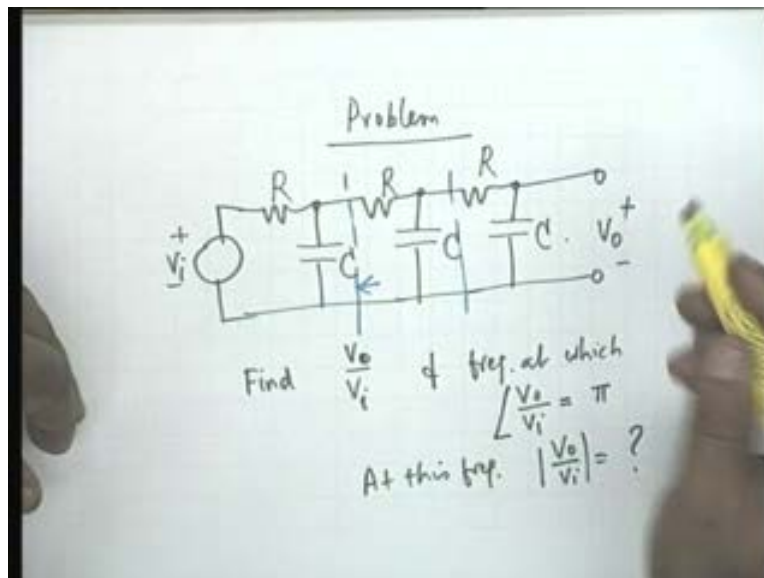


current  $I$  and agreed? This equation says that there is a current source  $V_T$  by  $Z_T$ , which splits in to 2 currents and the current to the load is  $Z_T$  divided by  $Z_T$  plus  $Z_L$ . This is the current division theorem.

Now, if you compare this with the Norton's equivalent circuit, obviously,  $I_N$  is nothing but  $V_T$  by  $Z_T$ , which is equal to  $I_{sc}$ , a short circuit current, and  $Z_N$ , the Norton equivalent impedance, usually stated in terms of admittance, but you can work in terms of impedance, there is no problem. So Norton equivalent impedance is simply equal to Thevenin's equivalent impedance and the current of the load is the same and therefore, whether it is Thevenin's or Norton, they are duals of each other, they are tools in your hand for simplification of a circuit and I repeat and this is applicable, this is great advantage when you do not require currents and voltages in all the branches of the network. You require just 1 or 2, then Thevenin's theorem or Norton's theorem comes to rescue, I shall close this class with a problem.

You have done in 110 oscillator circuits. You have phase shift oscillator? No?

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Does not matter, whether you have done or not, it does not matter. I have 3 identical sections R C, R C, R C. I have connected a voltage source,  $V_i$  here, and I am interested in finding out the voltage  $V_o$  here.  $V_o$  by  $V_i$ , this is the function, network function that I want to find out. Obviously, it will across the way all capacitors are initially relaxed and we have to find out frequency at which  $V_o$  by  $V_i$ , this angle is equal to  $\pi$ , that is, they are 180 degrees out of phase.

At this frequency, what is  $V_o$  by  $V_i$ , what is the value? And I want you to find this out. You see, if you go by ordinary, you understand the problem. If you go by ordinary analysis, you will write 3 enough equations, 3 mesh equations? And you will have to solve for 3 by 3 determinant, agreed? If you write node equations, again 1, 2, 3. 3 nodes, 3 by 3 determinants but even here, when I want to find out the network functions, the individual currents in the elements are not important. I just want to find out open circuit voltage circuit here.

Even here, Thevenin's theorem comes to great help. You see, you can apply Thevenin's theorem once here, then you get an equivalent source in series with an impedance, by inspection. No solution of differential equation or algebra equation, no, nothing, then we apply Thevenin's theorem again here, second time, then what you will have? Then you have a simple 1 mesh, 1 loop circuit and therefore,  $V_o$  can be obtained and you can do this analysis almost by inspection, careful inspection. We have to observe not seeing, agreed? I want you to solve this application of Thevenin's theorem and see you again on Tuesday.