Circuit Theory

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Lecture - 08

Problem Session 2: Step, Impulse and Complete Response

This is the eighth lecture and we are going to have our problem set 2, problem session 2 on the same topic, that is, network analysis and the time and frequency domain, basically time and we will start with problem 5.11. I will first read out the problem and then solve it.

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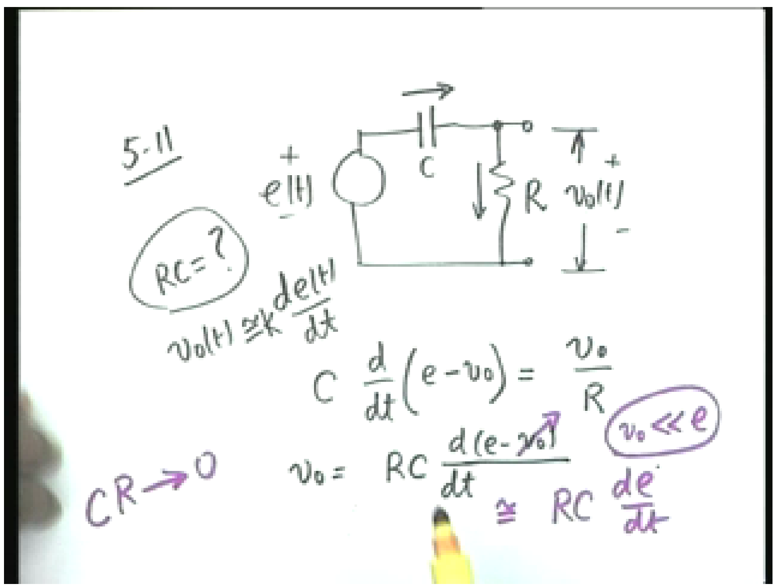

The problem 5 eleven says: An R C differentiated circuit is shown in the figure. You have an e of t, is the input, a capacitor C and a resistance R. This voltage is V 0 T. Find the requirements for the R C time constant, such that the output voltage V 0 t, what we require is, V 0 t should be approximately, the derivative of the input voltage, d e t d t. This circuit, as you know, acts as the differentiator for a certain condition on the time constant, C R. This is what you are required to find out in this problem. Now this problem can be solved in either the time domain or the frequency domain and as you will see, the frequency domain solution is more instructive, more revealing here. Let us first work it out in the time domain.

In the time domain, you see the differential equation is, that the current through the capacitor is the same as the current through the resistor and the current through the capacitor is C d d t of the voltage across this, which is e minus V 0. Therefore, e minus V 0, this is the current from the capacitor and this should be equal to the current from the resistor and therefore, this would be equal to V 0 by R and you see, you understand this, One line differential equation? So, V 0 equals to RC d d t of e minus V 0. What we want is that, it should be equal to, well, proportional to or equal to some constant K multiplied by de d t. We want the output to be equal to or proportional to the differential coefficient of the input and therefore, the only disturbing feature here is V 0.

This is V 0. If we can make V 0 much less than e, then obviously this would be approximately equal to RC de d t. The condition, therefore, is that the output voltage should be much smaller compared to the input voltage and that can be ensured by making small R, by making the resistance R as small as possible. Therefore, this approximation would be valid when R tends to 0 or the product, C R tends to 0. That is the condition. Now this is not as revealing as the frequency domain approach.

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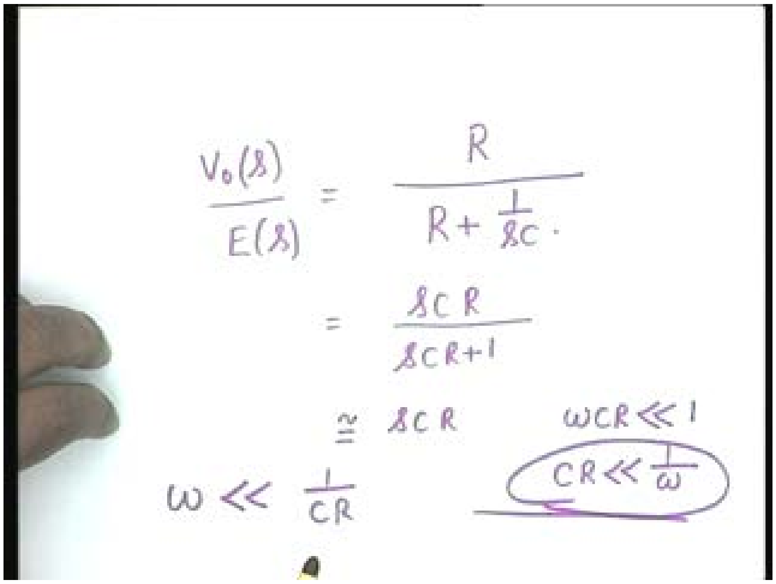

In the frequency domain approach, we simply write V 0 of s divided by E of s, is simply equal to R, it is a potential division, R divided by R plus 1 over s C and this is equal to s C R divided by s C R plus 1 and it would be, approximately, equal to s C R, a transfer function which is proportional to s corresponds to differentiation. As you know Laplace of d d t means multiplication by s and therefore, this will be approximately equal to s C R, provided you can ignore this s C R, in the denominator, as compared to 1, which means that, if you work in the frequency domain, omega C R should be much less compared to 1.

In other words, C R should be much less compared to 1 by omega. This is the condition. This condition is more revealing in the frequency domain approach, rather than the time domain approach. The time constant C R, should be much less compared to 1 by the reciprocal of twice phi times the frequency, in hertz. Another way of saying this is that, this would be a good differentiator for frequencies omega, which are much less compared to 1 by C R. These are 2 ways of utilizing, saying the same result. The product C R should be much less compared to 1 by omega. Only then, it will act as a differentiator.

Now, you can ask me, our input is not necessarily sinusoidal, right? But any wave form, as you know, can be expressed in terms of sinusoid. A summation, a combination of sinusoid either discrete ones or a continuous one, as in the case of a Fourier transform. And therefore, this condition is a much stronger condition, than the one that we derived from the time domain and in that sense time domain and frequency domain are complementary to each other. Some information is revealed quite easily in the frequency domain. Some information is revealed quite easily in the time domain and you must not lose sight of one or the other and this is why, I am totally opposed to blind working. An engineer cannot afford to work blindly or mechanically, whether it is an algorithm could be in the United States, it does not matter. You must not lose sight of what is going on in the whole game.

Student: Excuse me sir?

Sir: Yes.

Student: In this position, we do not leave the approximately the derivative of the input voltage.

Sir: That is correct.

Student: And what we have found is that V 0 t is approximately proportional to the derivative of input voltage.

Sir: Yeah, that proportionality constant, we can always adjust. Is it is the time, it is the relationship with time, that is important. Multiplication by a constant is of no concern. It will always change the multiplication.

Student: But sir, as we have derived that R C is very small and so we want is equal to R C d e by d t

Sir: So the output voltage will also be very small.

Student: Yes sir,

Sir: That is correct. It will be very small. In all probability, we shall have to add an (...) after this, to get a size of the output volt. Any other question? We skip one and we work out the next one.

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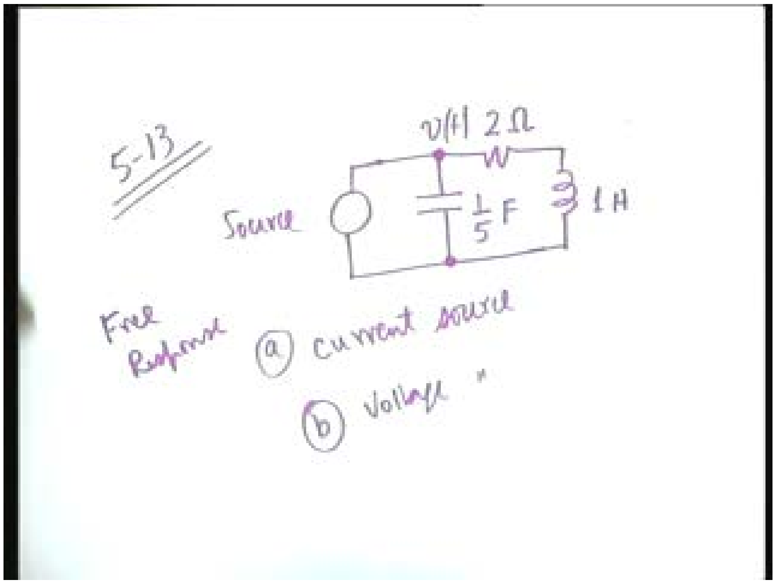

5.13 says, find the free response for i t. Find the free response for current i t, in the figure shown, and the figure is this. There is a source. This problem would be very interesting and I want you to follow this carefully. It is a demonstration of how physical insight can lead to an illuminating solution. 2 ohms and 1 Henry this is the circuit. Now the question is, you are required to find out the free response. What is a free response? That is, complementary function of the differential equations.

So you are required to find the free response, actually the form of free response. That is what is wanted. For this source under a, it is a voltage, its current source and part b, it is a voltage source. The question is, given this circuit, the source is not specified to start with the source is connected across this circuit you are required to find out the form of the free response when; this source is a current source, when the source is a voltage source. And we shall see that the 2 free responses are quite different from each other.

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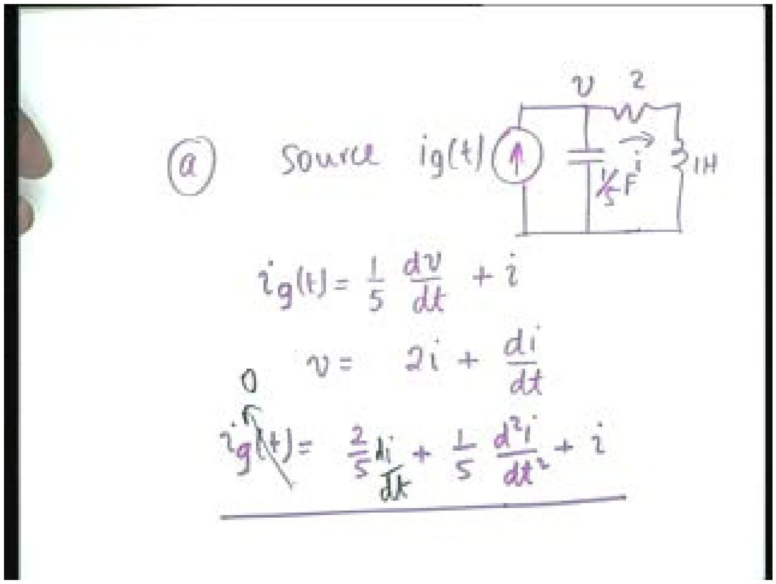

One is a second order, the other is a first order. To solve this circuit, let us work, we have to work in the time domain. We are required to find out the free response, so we must put the right hand side equal to 0 and then find out the form of the response. Now let this voltage be equal to V t and Let us first consider that the source is a current source. Source is i g t, a current source, then obviously, let us say, this is i g t. We have the capacitor, 1 fifth farad. We have the resistance 2 ohms and the inductor 1 Henry, this is v. Obviously, i g t, let us say that this current Free response for what? The current i. This is the current that we have to solve for, free response for i of t, the current.

This is the current i. Now you can, obviously, see that i g of t is the current through the capacitor, plus i. What is the current through the capacitor? C d v d t so it is 1 fifth d v d t, plus i. Agreed? Now what I want is, free response for i. So I must eliminate V. Now look at what is V. V is twice i, the drop in this, plus L d i d t so it plus d i d t, agreed? And therefore my differential equation becomes i g t equals to 1 fifth of d v d t. So 2 fifth of i, I substitute for V plus 1 fifth of d 2 i dt 2

Student: This is d i by d t.

Sir: This would be d i d t. Thank you, this would be d i d t, plus?

Student: (…)

Sir: Plus?

Student: Sir, L is 1.

Sir: Which is 1?

Student: L is 1

Sir: L is 1, yeah L is 1. That is why I did not write, I am sorry. No it is okay. You are trying to confuse me? L is 1, this equation is okay. What I have to do now, is to find the free response, which means that I put i sub g t equal to 0 and therefore,

Student: Sir, was there any constant i g minus something then we have put that (…)

Sir: That is right the driving term is equal to 0 in absence of a drive. In absence of drive means that, if the drive was a current source then it should be opened. If it is a voltage source, it should be shorted.

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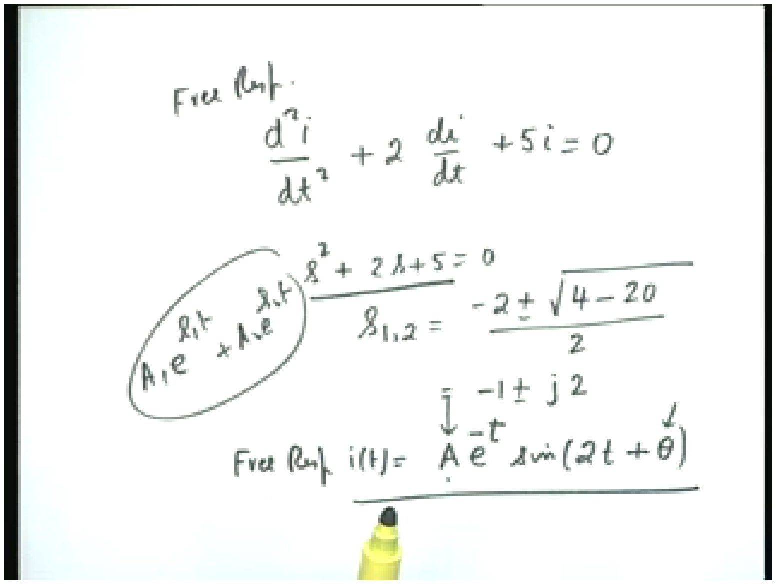

So my differential equation is, if I take care of the, if I multiply both sides by 5, I get d 2 I, let us say free response is governed by the differential equation d 2 i d t 2 plus 2 d i d t plus 5 i equal to 0. This is the this is the differential equation governing the free response, and you see the characteristic equation, put i equal to e to the s t and see that the characteristic equation is s squared plus 2 s plus 5 equal to 0. So the natural frequencies, which are the routes of the characteristic equation, they are given by, there are 2 routes; minus 2 plus minus square root of 4 minus 20 divided by 2, quadratic equation, which is equal to minus 1 plus minus j 2, is that okay?

So the free response would be of the form, a 1 e to the power this multiplied by t plus a 2 e to the power minus 1 minus j 2 multiplied by t. Which, if you do a bit of algebra, you can write free response i t would be of the form some constant A, multiplied by e to the power minus t, then sine of 2 t plus theta, some theta. So this will be the form of the free response, it would be a damped sinusoid.
I am omitting all those steps. As soon as we find out the 2 roots satisfying the 2 natural frequencies, I can write A 1 e to the power Let us say natural frequencies are s 1 t plus A 2 e to the power s 2 t, then I combine these 2 terms to put it in this form.

Student: Can we directly use this formula?

Sir: You can.

Student: Excuse me sir, if there is already some current due to some initial conditions, do we used that also, include that also in the free response or not?

Sir: If there is, yes of course. If there is some initial condition, that will not, the initial conditions will determine these constants A and theta.

Student: Then sir you just said that but suppose there is initially a current some form a current in the inductor.

Sir: Let me explain this. You see, as far as the characteristic equation is concerned, it is independent of initial conditions. The characteristic equation is independent of initial conditions. You see, what we have to do is; minus s i of 0 minus, this we have to transfer to the right hand side, and put the right hand side equal to 0. So you find the characteristic equation by de-energizing the circuit, even if it has the initial energy and find the form of the free response, A e to the minus t sin 2 t plus theta. Then if you want to solve this completely, then what we do is, you bring in the driving sources, find out the particular integral, add the particular integral to the complementary function and evaluate the constants from the initial conditions.

We have done, we have solved this kind of a circuit early. Is the point clear? What this question once was only the form of the free response. Now why does it want the form of free response for 2 sources will now be obvious when you put these as a voltage source?

Student: Sir, we do not have the initial condition here.
Sir: No, we cannot solve this we are not also given the form of the driving point function driving function so we cannot solve it completely. We just require the form.

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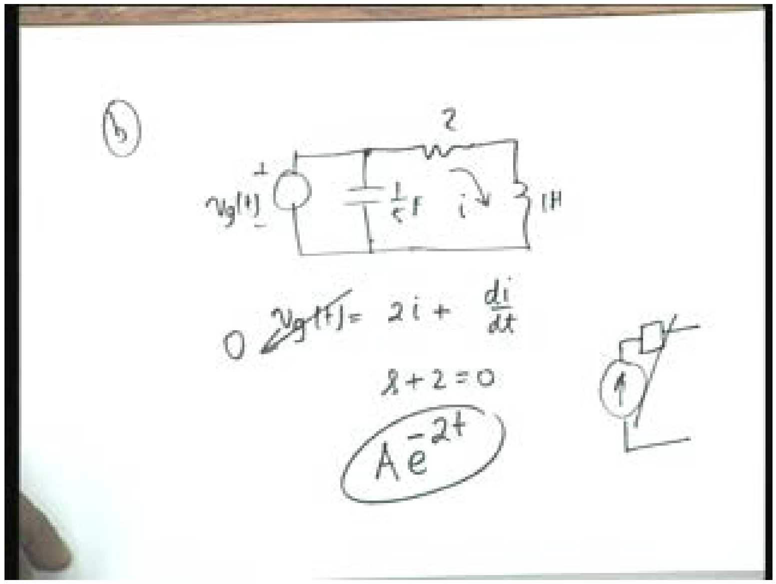

Now if it is a voltage source, suppose I have a V sub g t, then we have a capacitor of value 1 fifth farad. I am working of part b. Then we have a 2 ohms and 1 Henry and this is the current i. You see the differential equation now is V g t equals to twice i plus d i d t. This is the differential equation. The capacitor is ineffective. It does not affect the differential or the free response of the circuit. Why not?

Student: Voltage is constant.

Sir: Because this is a voltage generator. Any element, one way is to look at look at is natural response, is formed by short circuiting of voltage source which short circuits 1 fifth farad, the other way to look at it is that a any element connected across a voltage source cannot affect its voltage. By definition, a voltage source is one which maintains its terminal voltage constant irrespective of the current run by it. So irrespective of I, the voltage across 1 fifth farad remains constant and therefore, 1 fifth farad capacitor is ineffective, in determining the free response of this circuit, when the source is a voltage source.

Similarly, in the previous case, if we had an element a resistance or something in series with the current source, it would not have affected the free response, is that clear? So a free response here is obtained by putting this equal to 0 and you can easily see that the characteristic equation is s plus 2 equal to 0 and therefore, the free response would be of the form A e to the minus 2 t, that is all. You see that when it is a voltage source, the circuit is first order. When it is a current source, the circuit is second order. When it is a voltage source, it is a simple exp1ntial. When it is a current source, it is a exp1ntial multiplied by a sine and therefore, it is a decaying sinusoid.

That quite different, the free response of a circuit depends, very crucially, on the generator and this is the point that was brought out by this particular example. Any question? We skip the next and go to 5.15.

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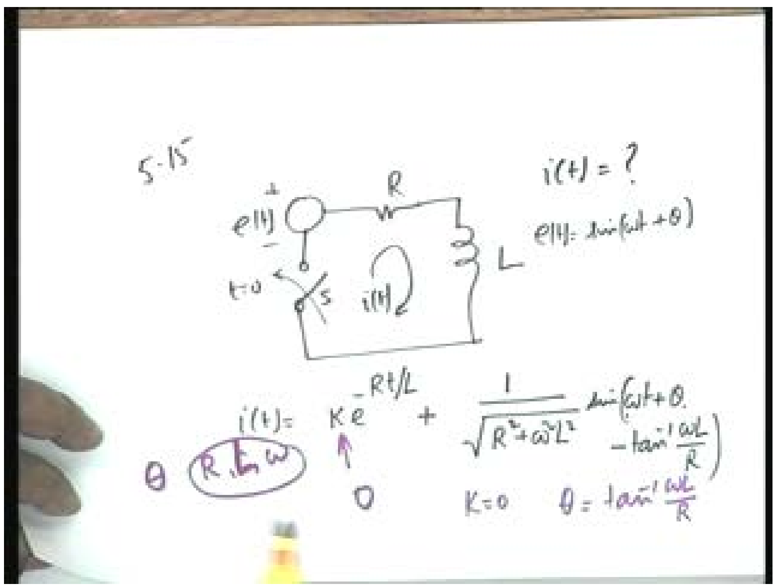

Student: Sir, 5.14, that completed response?

Student: 5.14 is a question, asking for complete response.

Student: Complete response.

Sir: Yeah, we take 5.15. 5.15 says, let me read the question. For the circuit shown, let us draw this circuit, e of t, then you have a switch as which is closed at t equal to 0 and then you have a resistance and then inductance, L. The current in this circuit is i of t. The question says the switch closes at t equal to 0. Solve for i of t, when the initial conditions are 0 and the voltage source e of t is equal to sine of omega t plus theta. Now we have worked out such examples so we know the solution. Second part, so I will not work out this part I think we can write it down.

i of t would be equal to, the free response would be an exponential e to the minus R t by L, that is ,small t divided by the time constant. So it would be of the form k e to the minus R t by L, this is the solution of the first part, plus the steady state response. Steady state response would be 1 by square root of R squared plus omega squared L squared, then sine of omega t plus theta.

Student: Minus tan

Sir: Minus tan inverse omega L divided by R, wonderful. This part you can do yourself and it should be obvious, it should be able to write this solution, immediately, by looking at the circuit.

Student: Can we write it directly?

Sir: No, it says specifically, solve for i of t when the initial conditions are 0 and the voltage source is this and therefore, you will have to find a complementary of a function in the particular integral.

Student: One way is, solving the whole response we can take that in the response will be of the form a sin omega t plus 5 and then find a and the phi, which is the same thing.

Sir: So long as you reach God, the path is not important. Your path should be logical. In reaching god, of course, many of times the path is not logical.
Student: Then the logic is only that the response of sine omega t is of the form

Sir: Would be of the form of a sine. Yeah, that is it, that is correct, provided the circuit is linear. In non linear this does not happen. Now the second part, second part says what should be theta in terms of R C and omega, so that the coefficient of the free response term, is 0? What should be theta in terms R C and omega such that, k, the coefficient of the free response term is 0?

Student: (…)

Sir: Pardon me.

Student: R C and omega, it should be L, R L.

Sir: Oh, R L, thank you. What you want is, theta in terms of this constant so that k becomes equal to 0. Now you notice that if the initial condition is 0, when the initial conditions are 0, so i of 0 is 0. That means, k would be equal to minus this quantity with t equal to 0 and is not it obvious that K would be equal to 0, if theta equals to tan inverse omega L divided by R. Is this obvious? Yes of course it is obvious. Put t equal to 0 here and put i of t equal to 0 then k equal to minus this quantity sine of, this is 0, theta minus this and it can be equal to 0 if theta is equal to tan inverse omega L by R.

However, the solution is not unique. If theta is tan inverse omega L by R plus any multiple of phi, any multiple of phi, then also its, this solution is valid. But this is the primary solution, that is, the lowest value of theta, agreed? So I need not work it out in further details. 5.15, we skip one go to 5.17.

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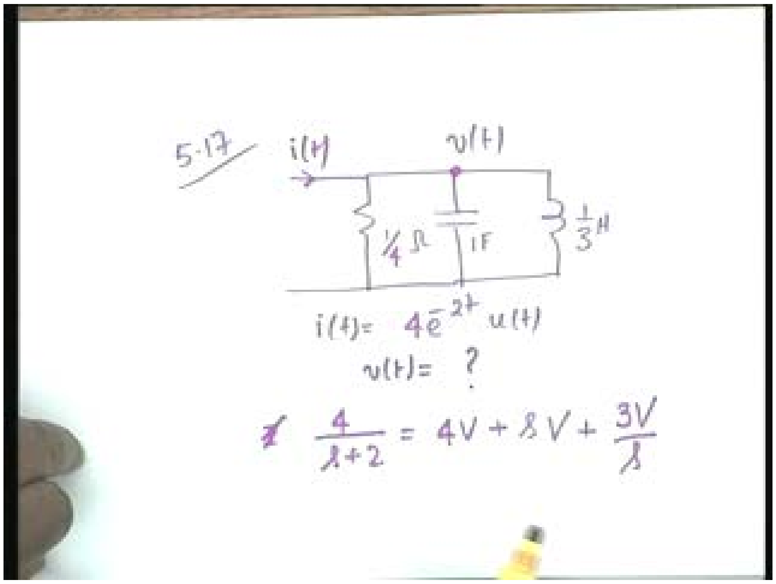

5.17, let me read the question. For the circuit shown, let us draw the circuit. It is a 1 quarter ohm resistance, then a 1 farad capacitor and then a 1 third Henry inductor and this voltage is V of t. The current is i of t and the current is given as i of t equal to 4 e to the minus 2 t u t. You are required to find out V t. Obviously, the simplest thing to do is to apply the transform technique.

In the transform domain, 1 sec, is that the question 5.17? Yeah. In the transform domain, you see, if take the, now I will skip a couple of steps. I can write the differential equation and then I can write i of t equal to small v, divided by 1 quarter plus 1 d v d t, then plus

Student: Integral of

Sir: Pardon me. Integral of 0 minus to t and so on so forth.

Student: Sir, initial conditions are 0?

Sir: It is not given.

Student: So we will assume it?

Sir: We will assume. If it is not given, whatever is convenient, we will assume. But if you do Laplace transform, then you need not worry about it. So if I, let us do Laplace transform. Obviously, i of s would be 4 divided by s plus 2. Is that okay, the transform of the input current? This current is the sum of the 3 currents and obviously, the current to the resister will be 4 times V, through the capacitor, it would be s times V, agreed? And through the inductor, it would be capital V divided by 1 third s. So 3 V by s. You have to fill in the steps, but I think these are pretty obvious. So I am skipping those.

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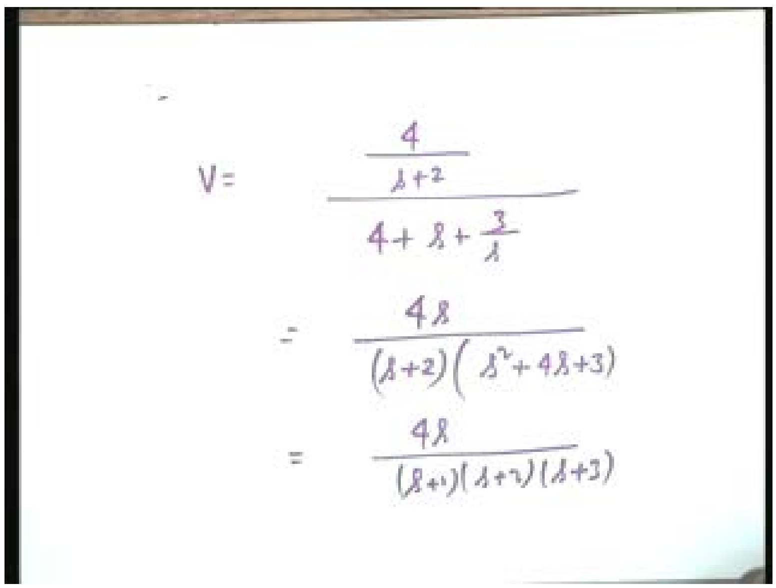

And now if I collect the terms in capital V, then I get capital V equal to 4 divided by s plus 2 divided by 4 plus s plus 3 by s, which I can write as; 4 s divided by s plus 2, then multiplied by s squared plus 4 s plus 3 and you see, fortunately, this can be, this has real roots, so s plus 1 s plus 2 s plus 3 and now you can do partial fraction and invert, term by term. Am I, if you permit me to skip those steps, the final solution is given by, my solution, I do not guarantee that my solutions are correct. You must check. This is what I got; 6 e to the minus 3 t, of course, multiplied by u t. This I got directly from the transform domain and writing the equations, directly.

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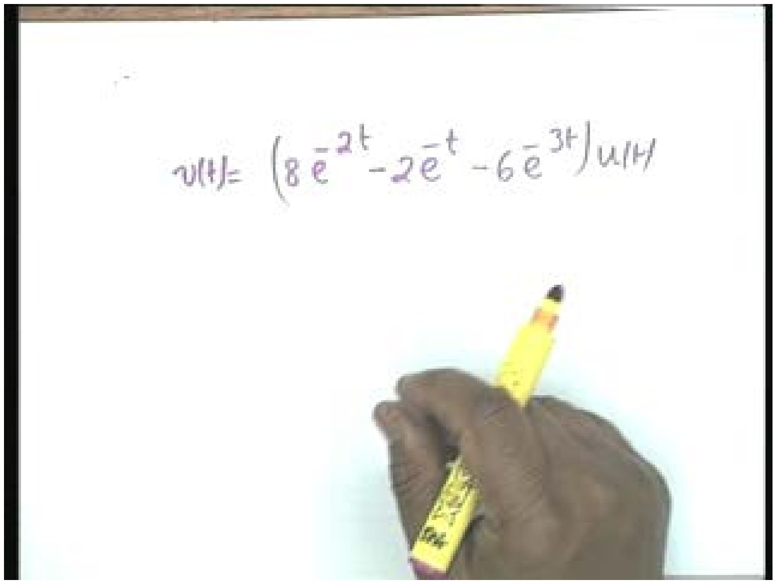

The last 2 problems, where we skip one, 5.18, we go to 5.19, the last 2 problems would prove to be a bit involved. So I will take a little more time for this. Is there any question on the previous one 5.17? We go to 5.19. 5.19 says, and let me read the, 5.19 gives a circuit. The circuit is 100 volt. The switch is thrown in, at t equal to 0 and then you have 10 ohms, 1 Henry, 10 ohms 1 Henry and 10 ohms. This is the circuit. The switch is closed at t equal to 0, find i 1 t and i 2 t for 0 minus to infinity and assume 0 initial energy.

So the usual thing that we do is, we write the loop equations, the mesh equations to be particular and I get 100 u t equal to 10 and 10, 20. 20 i 1, is that okay? No. Let me write in that form. 10 i 1 plus d i 1 d t plus 10 i 1 minus i 2, is that okay?

Student: Yes.

The second mesh. Second mesh is 0 equal to d i 2 d t plus 10 i 2 plus 10 i 2 minus i 1. Now you take the transform, Laplace transform of these 2 equations and you get 100 by s equals to 20 plus s times i 1. You know where this 20 comes from? 10 I 1 and 10 I 1 20 and s I 1 comes from d I 1 d t minus 10 I 2 and the other equation is 0 equal to minus 10 I 1 plus 20 plus s I 2.

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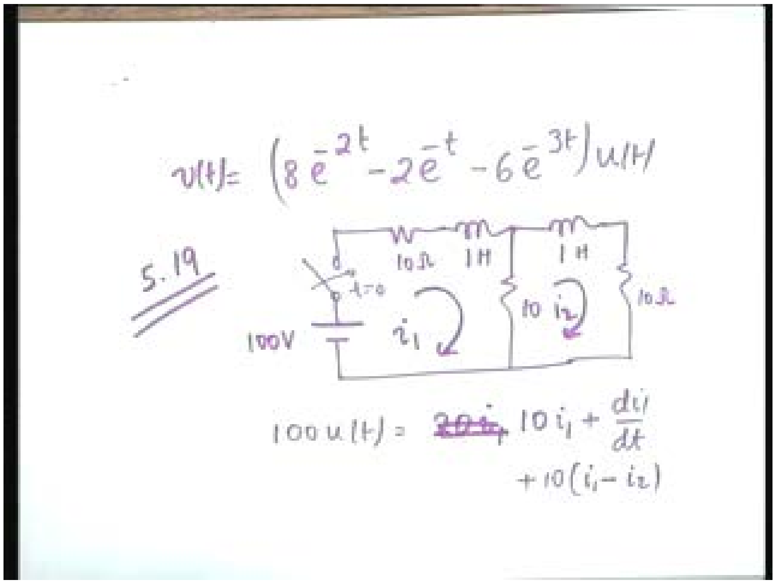

Now you have a choice. You can go to the time domain, you can find the natural frequencies, write down the corresponding equations and then make your life miserable by finding I 1 0 plus and I 2 0 plus. You have to find them out.

Student: (…)

Sir: Pardon me.

Student: Will not the characteristic equation (…)

Sir: I can write the equations.

Student: Characteristic equations at 20 plus s minus 10 and?

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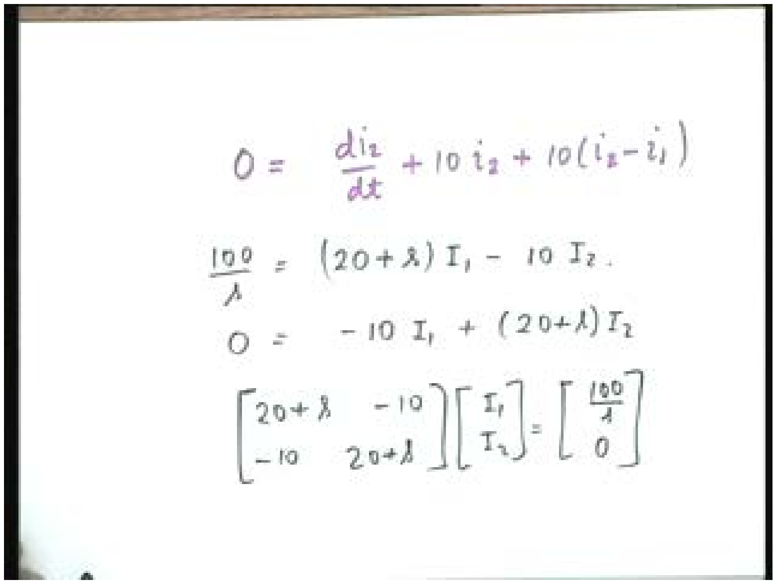

Sir: That is right. That is what I am saying. After writing the characteristic equation, you solve for the roots. Then you can write the equations directly for I 1 t and I 2 t and find out K 1, K 2, K 3, K 4, whatever the constants are. From the initial conditions, which will be I 1 0 plus and I 1 I 2 0 plus. Whereas, if you solve in the frequency domain itself, then you get simply this equation; 20 plus s minus 10 minus 10 20 plus s. This matrix multiplied by the vector I 1 I 2 is equal to the vector 100 by s 0 and you can solve for I 1 and I 2.

I did that and the solutions, the solution for I 1, is given by 100 by s 0 minus 10, 20 plus s the determinant of this divided by, the usual thing, determinant of 20 plus s minus 10 minus 10, 20 plus s. The simplification gives me, I omit a couple of steps. I have done here, but because of time, I omit this and I get 100 s plus 20 divided by s times s plus 10, s plus 30. Fortunately, the roots are real. Well, there is nothing fortunate about it. If the roots are not real, if they are complex, then you know how to write A to the minus alpha t sine of omega t plus theta. You do not have to write the form of the, A 1 e to the power something a 2, you can write the form directly.

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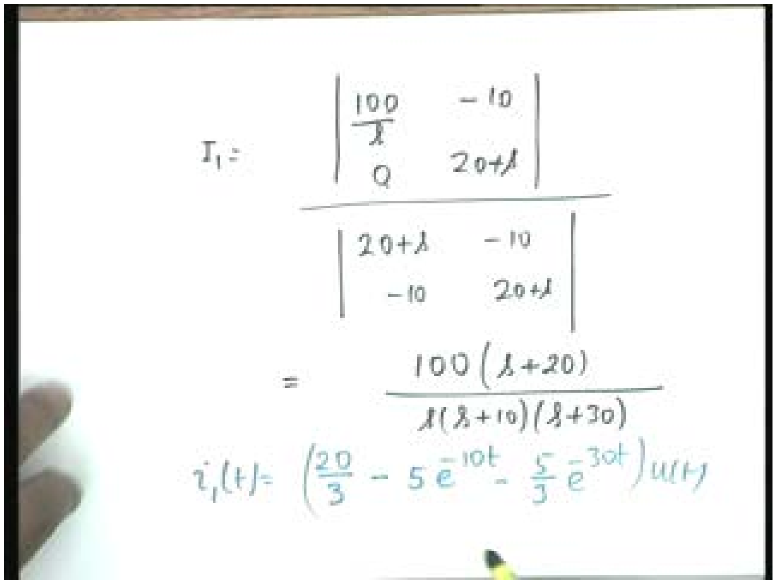

So you have to expand this impartial fraction, evaluate the constants and then find the current. If I omit those steps, if you permit me, my solution becomes 20 by 3 minus 5 e to the minus 10 t minus 5 by 3 e to the minus 30 t times u t. this is my solution

Student: how come that minus ten and 20 plus as in the determinant?

Sir: Oh, I 1 is the same matrix with the first column replaced by, you do not know this? (…) first column replaced by the right hand side, that is why 100 by s 0. If I find I 2, then this will go to the second column and the first column will remain as it is. This is the solution of simultaneous equations.

Student: Excuse me sir.

Sir: Yes.

Student: For this expression of I 1 t at t equal to 0, we are getting a constant, sir.

Sir: You are getting a 0.

Student: How come this?

Sir: 5 by 3 is 15 by, I beg your pardon, 5 is 15 by 3 and this is 5 by 3. This is one thing, very interesting point very raised him. You see, i of 0, i 1 of 0 is 0 inductor current, initially de energized, and therefore, at t equal to 0, this must be equal to 0, obviously 0. 20 by 3, this is 5 by 3 and this 15 by 3. 15 plus 5 is 20 and 20 minus 20 is exactly 0. It has to be. This is a check. What about the infinity? Infinite time values?

Student: (…)

Sir: 20, does it check? 100 by, how much was it?

Student: (…)

Sir: 100 by 5 is not 20 by 3

Student: 100 by 15.

Sir: 100 by fifteen is 20 by 3, exactly. So these are the checks. Now the last problem is 5. 21. 5.21 Let us see where I have it, yeah.
5. 21 says that there is a current source, I will simply indicate how to solve the problem and then leave the rest to you. I will also give the final, my solution. Just to convince you that I did spend time on solving the there is a switch and this switch is opened at t equal to 0 that means, at t equal to 0 minus, the source was short circuiting. Then you have problem and getting the answer. There is a 2 ampere current source, across which, there is a source, a 3 ohm and a 1 Henry. 1 Henry with a dot here and a 6 ohm and this voltage is V 2.

For this circuit, for the transformer circuit shown, the switch as opens at t equal to 0. Find the voltage V 2 t for t between 0 and infinity. Assume this is important. Assume 0 initial energy. Which means, 0 initial energy means that this current i 1, i 1 of 0 minus is equal to 0 i 2 of 0 minus is equal to 0. If this condition were not given, this is what I want to discuss, if this is condition was not given, suppose this sentence was not there, then what would have been i 1 of 0 minus?

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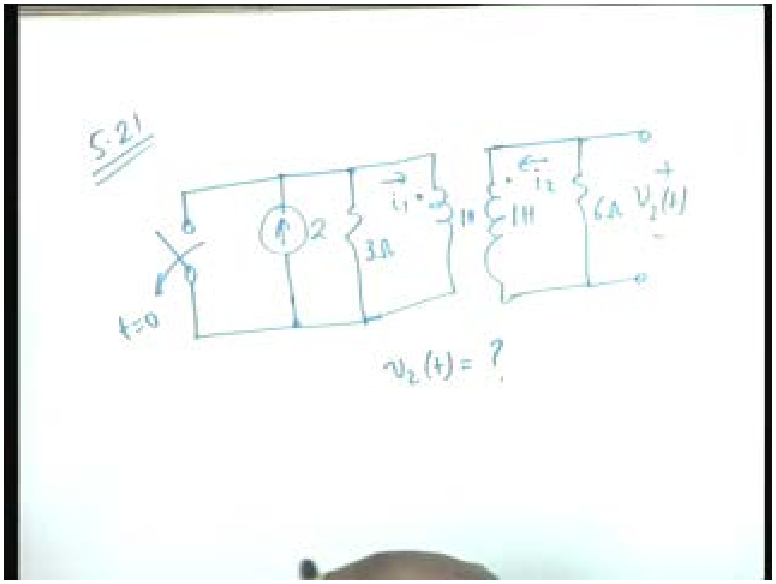

1 ampere. Is not that right? Because at 0 minus, everything is steady; this is a short circuit, this is also a short circuit. No current will flow through 3 ohms and half of this must flow here, half of this must flow here. But it says, that the inductor has been so constrained that its initial energy is 0. So what it means is that, there must be the switch here which was opened at t equal to 0 minus. At t equal to 0, it is closed. Is that clear? It makes a hell of a lot of difference, whether that sentence is there or not, agreed?

Now your requirement is to find i 2. The mutual inductance is given as 1 Henry. What does this mean? It means M squared is equal to L 1 L 2, that it is a perfect transformer and if it is a perfect transformer, then you know you will have to, if you work in the time domain, you will have to find out i 1 0 plus and i 2 0 plus. On the other hand, if you work in the frequency domain, you do not bother about this. But what do you think will be the order of the circuit? Would it be, what order do you expect?

I think we have done this, that is, if L 1 L 2 is equal to M squared, then it becomes a first order circuit. So if one asks you without calculating anything, can you write the form of the, i 1 t and i 2 t? Obviously, it will be of the form K 1 plus K 2 e to the power minus some constant, let us say, alpha t, agreed?

Student: Yes sir.

Sir: Because at infinity, there would be a current, infinity, the current would be, what will be the current? Through 1 Henry.

Student: 2 amperes.

Sir: 2 amperes, right. Therefore, K 1 is 2 K 2 you have to find out from i 1 of 0 plus, 0 plus. 0 minus is 0 and i 2 t, obviously, at infinity when the fluxes stabilizes no change of flux, the current i 2 shall be equal to 0 and therefore, it would be simply of the form K 3 e to the minus, now what will be the time constant here? Would it be the same as alpha or different? It is a first order circuit. It has only one natural frequency and therefore, it has to be alpha. No question.

Student: (…)

Sir: It is a first order circuit. The equation, if you find the characteristic equation, it is a first order equation. So whether it is this current or this current or this voltage or this voltage, all of them will have the same time constants.

Student: Transients should die at the same time?

Sir: Transients will die at the same, same rate. Now the other point about this circuit is, that this a current generator and we have always written for a transformer, a loop equation. But that is not a problem because this can be converted into its (…) equivalent, that is, 2 u t, in series we take 3 ohm and that is what I prefer. Instead of writing the current equation, I write a loop equation.

There is nothing sacred about loop equation or node equation. You can write either. They are equally convenient, well, I did it by making into an (…) equivalent. Is the point clear?
Now the total procedure, if you go through the final solution, I think I leave the rest of the working to you. The final solution is

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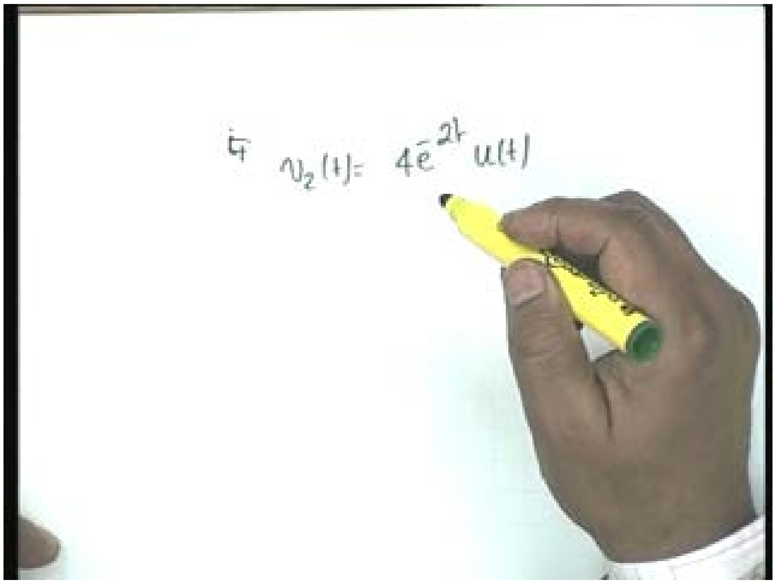

i 1 t, now what did you want to find out? V 2. V 2 t is equal to 4 e to the minus 2 t u t. This is the solution. You see, the time constant, obviously, here is half and it is not obvious from the circuit. Is not that clear?

It is not obvious, which resistance and which inductance? There are 3 inductances; 1 Henry, 1 Henry, 1 Henry which are equivalent to 2, because the coefficient of coupling is 1 and these 2 inductances and 6 ohms and 3 ohms combine in such a manner, there is no easy way of observing and finding the time constant.

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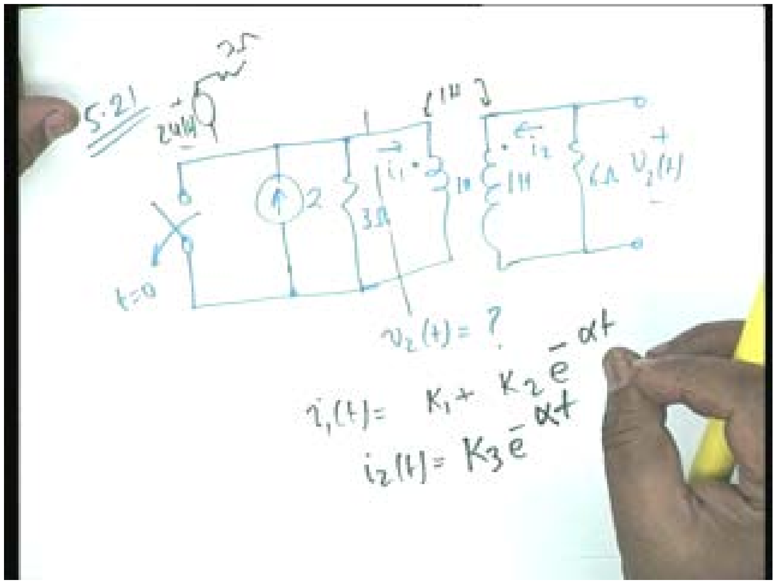

You have to go through the way. You have to write the characteristic equations and then solve for the natural frequency. That will give you this coefficient, this coefficient, alpha. Alpha happens to be 2 here, but do not trust me completely. Sometimes, I make mistakes intentionally to be pointed out later. If you cannot point out, I will have the last laugh. More tomorrow.