Circuit Theory
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Lecture No. 07
Transformer: Transform Domain Analysis

Seventh lecture and we are going to complete the discussion on transformer that we had started last time and also enter into a discussion on the transform domain circuit analysis. That is, circuit analysis in the S domain by taking Laplace transform.

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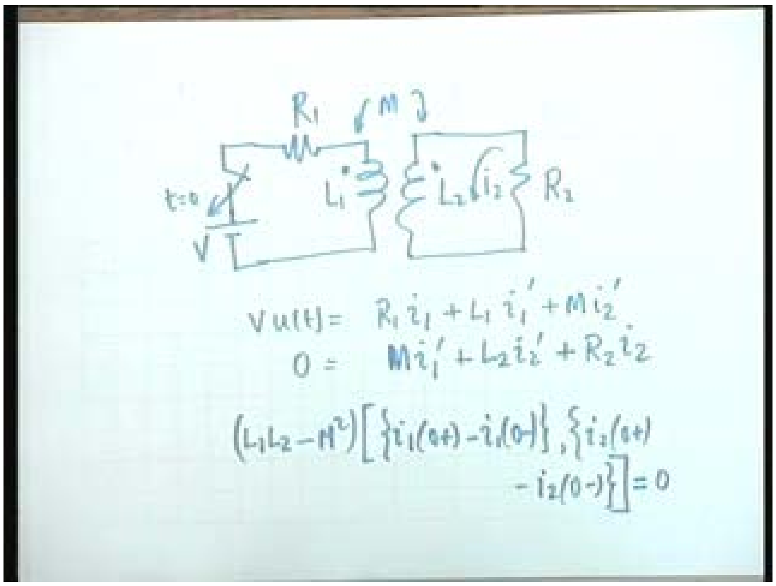

To continue the discussion on transformer, we were considering a circuit of 2 magnetically coupled coils, where the inductances are L 1, L 2 resistances R 1, R 2 and we had considered a switch closing at t equal to 0 in series with a battery capital V and the dots were like this, the mutual inductance M is this and our equations were V u t equal to R 1 i 1 plus L 1 i 1 prime plus M i 2 is in this direction. That is, i 2 enters the dot. That is the convention M i 2 prime and 0 equals to L, I beg your pardon, M i 1 prime plus L 2 i 2 prime plus R 2 i 2, these are equations and by integrating the equations from 0 minus to 0 plus, we had shown that the condition, that is to be satisfied is L 1 L 2 minus M squared multiplied by i 1 0 plus, minus i 1 0 minus, as well as, i 2 0 plus minus i 2 0 minus. Both of these quantities have to be equal to 0.

The two equations written in the form of one. This comma indicates that it is either this or that and we argued that if L 1 L 2 is greater than M squared, that is, if the coefficient of coupling is less than 1, then the currents in the 2 coils have to be continuous as t equal to 0. That is, i 1 0 plus must be equal to i 1 0 minus and i 2 0 plus would be equal to i 2 0 minus. This case, we have considered in details.

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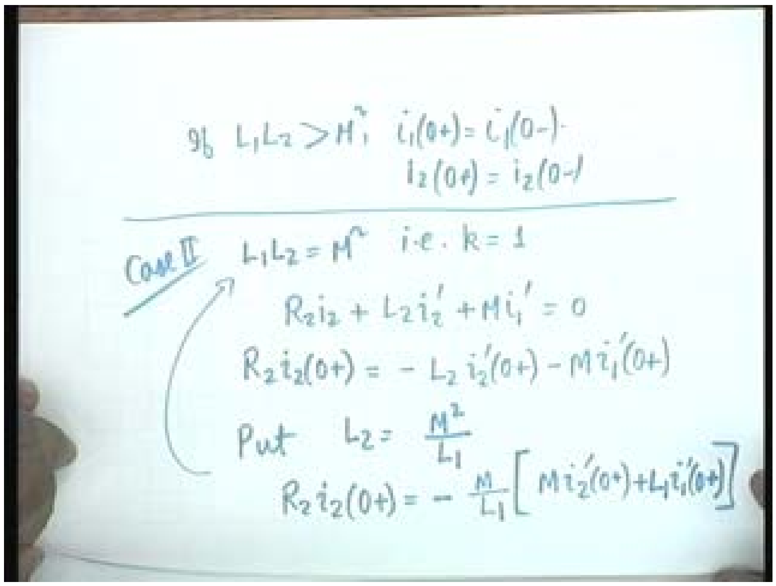

To summarize, if L 1 L 2 is greater than M squared, then the currents would be continuous. i 1 0 plus would be i 1 0 minus and i 2 0 plus would be equal to i 2 0 minus. However, this is not the case if L 1 L 2 is equal to M squared. That is, if the coils, if the transformer is a perfect transformer, that is, the coefficient of coupling is equal to 1, then we shall show that these 2 shall not be valid. To this end, we had taken the values at 0 plus and substitute in the two equations. We had taken the secondary equation first, that is, R 2 i 2. This is case two.

We take the equation of secondary, that is R 2 i 2 plus L 2 i 2 prime plus M i 1 prime equals to 0 and if you put t equal to 0 plus here, then I get, R 2 i 2 0 plus would be equal to minus L 2 i 2 prime 0 plus minus M i 1 prime 0 plus. In this we put L 2 equal to M squared divided by L 1. Put

L 2 equal to m squared divided by L 1. This is the given condition and if I do that, then I can write this equation as, R 2 i 2 0 plus equal to minus.

Let us take M by L 1 as common factor. Then, inside the bracket I shall have, M i 2 0 i 2 prime 0 plus. This is from the first quantity. L 2 is M squared by L 1 so i have taken M by L 1 out M remains M times i 2 prime 0 plus because minus sign has been taken out plus instead of M i had taken M by L 1, so I must substitute L 1 i 1 prime 0 plus. Let me repeat this equation in the next sheet, and let me change the color of the pen.

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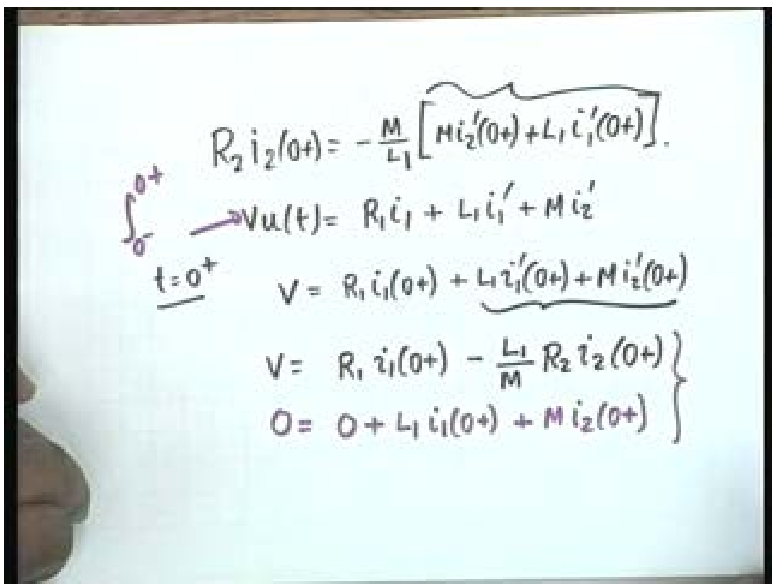

I have R 2 i 2 0 plus equals to minus M by L 1 M i 2 prime 0 plus, plus L 1 i 1 prime 0 plus. Let us take the first equation, that is, the equation to the primary that is V ut is equal to R 1 i 1 plus L 1 i 1 prime plus M. What was it? i 2 prime? And in this equation, I put t equal to 0 plus. Then I get on left hand side capital V. If I had taken 0 minus, left hand side would have been 0, it because t equal to 0 plus it is V. Then I get R 1 i 1 0 plus plus L 1 i 1 prime 0 plus, plus M i 2 prime 0 plus and you notice that, this two are the same as these two, right? This is the same as this and therefore, I can write V equal to R 1 i 1 0 plus, minus L by M R 2 i 2 0 plus. Agreed?

Student: the L 1.

Sir: Pardon me.

Student: L 1 by M.

Sir: L 1 by M. Thank you. I missed that one. This is one of the equations containing i 1 0 plus and i 2 0 plus. We wanted to find out the initial condition, that is, i 1 0 plus and i 2 0 plus and we want to check whether the current is continuous or not. The currents at 0 minus were equal to 0. The other equation, naturally we require 2 equations of to solve for 2 unknowns. The other equation is obtained by integrating the first equation, that is, this equation, integrating this equation, integrate this from 0 minus to 0 plus.

If we do that, the left hand side shall be 0, left hand side shall be 0 equal to R 1 i 1 integrated from 0 minus to 0 plus will again be 0, plus L 1, I shall have i 1 0 minus i 1 0 minus, but i 1 0 minus is 0. Therefore, I shall get i 1 0 plus M, in a similar manner. This integral of this integral of this will be i 2 0 plus minus i 1 0 minus and therefore, it would be simply i 2 0 plus. These are the two equations. Do you understand how I derived this pink equation?

Now if I solve these 2 equations, say simultaneous equations, I can now find out i 1 0 plus and i 2 0 plus. Obviously, neither of them is 0. Is not it right? Obviously, neither of them is 0 and solution of this is elementary. I can simply write down the final results. What I get by solving this? We said 2 by 2 equation, 2 by 2 matrix. So that is not a difficult problem. I get i 1 0 plus equal to V L 2 divided by R 1 L 2 plus R 2 L 1 and i 2 0 plus is equal to minus V M divided by R 1 L 2 plus R 2 L 1 and you see, these are not equal to i 1 0 minus, not equal to i 2 0 minus. Now this is very good as far as mathematics is concerned. We have shown mathematically that the currents in the two coils are not continuous at t equal to 0, if the set of coils constitutes a perfect transform. That is, if the coefficient of coupling k is equal to 1, the currents are discontinuous.

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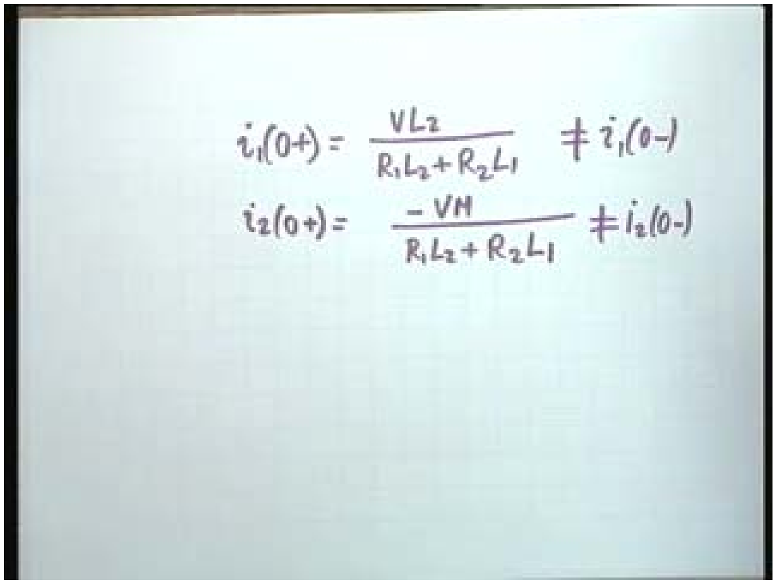

Now, physically, can we explain this negative sign? Can we explain this negative sign? Let us go back to the circuit. Let us go back to the circuit. t equal to 0 plus is an instant immediately after this switch is put on. When the switch is put on, this voltage suddenly rises and you know that the dot means that this voltage, the potential of this point also rises, simultaneously, and if this rises, then obviously the current through the resistance would be in this direction.

Whereas our convention was that i 2 flows into the dot and that is where i 2 has come negative, is that okay? Whereas here, current is forced through L 1 and therefore, the direction of i 1 is the same as that of i 1 0 plus. But i 2 0 plus is opposite to i 2 0 minus and that is why, the negative sign comes is that. You must lose sight of the physical picture and that is the strongest equipment

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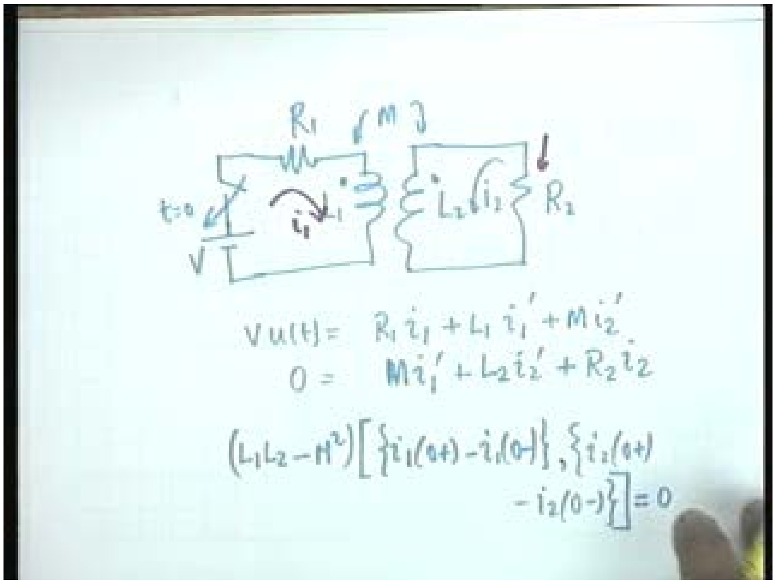

Student: Excuse me sir?

Sir: Yes

Student: Sir, from i 1 0 plus should we not have it 0 at immediately after putting on the switch because on (...) is an open circuit.

Sir: No, but this is a degenerate case, this a degenerate case, in which the coefficient of coupling is exactly equal to unity. Current is forced; it cannot remain 0. It is forced because current flux has to be established. There is no other way. This is a degenerate case.

Student: You stated that in an inductor (…) changes continuously but here it is changing.

Sir: That is correct, if the circuit is not a degenerate circuit. For example, when a capacitor is charged, a capacitor is charged and then connected across and connect uncharged capacitor. The uncharged capacitor, immediately becomes charged. There is a delta function, there is a delta impulsive current that flows. Here also, the same phenomenon occurs. There is no other way. A current, I think I have d1 this earlier in an example.

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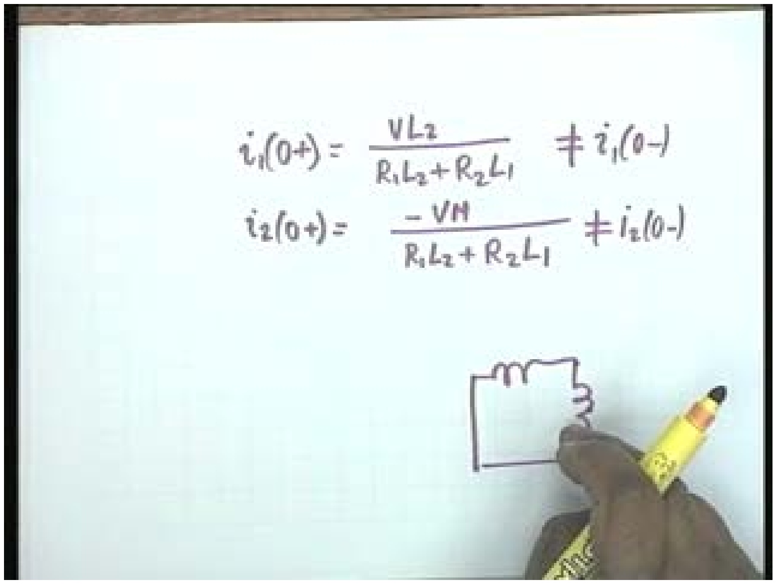

If I have two series inductors, this carries an initial current and suddenly, this did not carry. Suddenly I close this. The current has to flow, the flux has to and therefore, an impulsive voltage appears across the inductor, to force a current through L. These are degenerate cases and therefore, they cannot, they do not obey the continuity of current or continuity of charge in a capacitor and perfect transformer is also a degenerate case. You see why it is degenerate? There many ways that you can look into it.

Ordinarily, a transformer requires three parameters - L 1, L 2 and M or L 1, L 2 and the coefficient of coupling. Here, in a perfect transformer you just require two parameters. Is not that right? k equal to 1. if you L 1 L 2. Then, you know M if you L 1 and M and then, you know L 2 and therefore, this is indeed, a degenerate case. In fact, if you write the equation, this is what I am going to do, if you write the characteristic equation normally because there are 2 inductors and magnetic coupling between them it should be a second order differential equation. As you will see, this will become a first order. It is a degenerate case. Let us look at that. Let us look at the characteristic equation.

Student: Sir, how long does this current i 2 flow in the reverse direction towards the taken into the, Sir: That, you have to solve for the.

Student: (..)

Sir: To solve for, you see, ultimately current i 2 shall become equal to 0 and it can be shown that i 2 continues to flow in the negative direction.

Student: Then, why do we say degenerate case?

Sir: Degenerate means, it cannot be treated. That is, the redundancies. If there are three inductors here L 1, L 2, M but one is redundant, because it can be specified in terms of the other two. Similarly, if there are two capacitors in parallel, for example, you can combine them into a single capacitor. So it can be another resistor in the circuit. The differential equation of the circuit will not be a second order one, it will be a first order one. That is, why you call this a degenerate case. That is, what meets the i, is not the truth. Is that clear? The truth is that two capacitors can be combined here also. Three inductors combined into two, just two and it appears it terms out that the characteristic equation of the circuit is a first order equation. Let us look at that.

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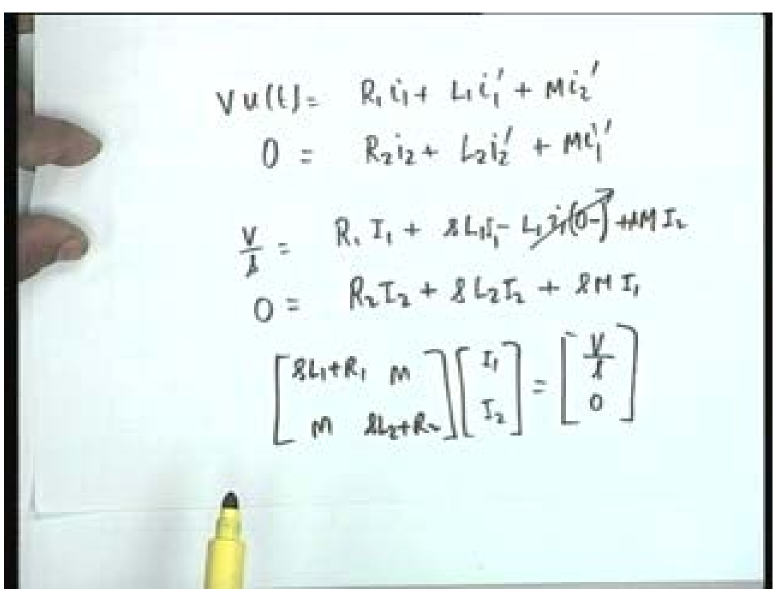

You see, you take this equations V u t equal to R 1 i 1 plus L 1 i 1 prime plus M i 2 prime and the other equation was, 0 equal to R 2 i 2 plus L 2 i 2 prime plus M i 1 prime. These are the two equations. Now if I take the Laplace transform, in transform domain, the question of continuity need not be explored. You see, if you are in the time domain, then the differential equations have to be solved and differential equations require initial conditions, the initial conditions are not 0 minus. The initial conditions are 0 plus and therefore, you had to find out i 1 0 plus and i 2 0 plus.

If you work in the transform domain, no such complication arises and this is the advantage of transform domain that you can go straight from the equations to the transform domain without , whether the current was continuous or discontinuous. As we shall see here, if we take the Laplace of both sides, both the equations, then I get R 1 capital I 1 plus s L 1 minus L 1 i 1 of 0 minus, because Laplace is 0 minus to t and this is 0 and therefore, you need not bother about it.
Plus, similarly s M i 2 and then you have 0 equal to R 2 i 2 plus.

Student: (…)

Sir: s L 1 i 1 ye plus s L 2 i 2 plus s M i 1 and the matrix could be s L 1 plus R 1, s L 2 plus R 2 M and M multiplied by, notice this carefully, I am not playing any tricks, straight forward calculation. You know that i 1 and i 2 shall have, in the denominator, the determinant of this matrix. The determinant of this matrix equated to 0 gives you the characteristic equation.

Student: Is this a, some sort of hexa matrix that the determinant is equated to 0 (…)

Sir: Well, this is one of ways to find the characteristic equation. The characteristic equation, you have to go back to the differential equation, try a solution of the form e to the s T.

Student: (..)

Sir: Then, the algebraic equation that you get is the characteristic equation. This is a simpler way of making the characteristic equation.

Student: Take a determinant is equal to 0

Sir: That is, it will give you the same thing if you had treated the original equations. Put e to the s T i 1 equal to K 1 e to the s T i 2 equal to K 2 e to the s T. Then you will get the same thing.

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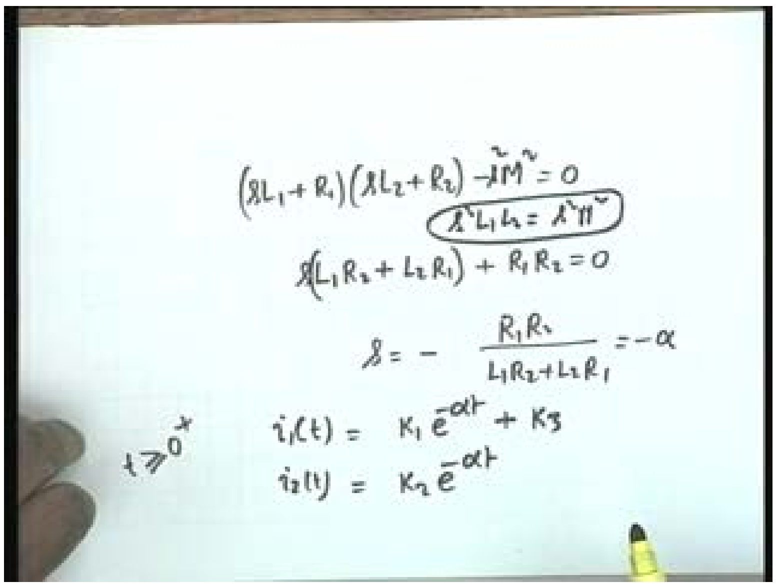

So, the characteristic equation now becomes s L 1 plus R 1 multiplied by s L 2 plus R 2 minus M squared equal to 0 and you see that, this, the second power term in s, it is s squared L 1 L 2. I have made a mistake. This is, in the matrix itself, s times M and s times M. Could you correct this s times M? So you get s squared M squared and since M squared equal to L 1 L 2, you get a first order characteristic equation. That is, u s L 1 R 2 plus L 2 R 1 plus R 1 R 2 equal to 0. This is the equation that you get. Why? Because s squared L 1 L 2 is equal to s squared M squared because of perfect coupling and therefore, the square power terms cancel each other.

This is how the degeneracy shows into the picture. It is a first order equation and therefore, the natural frequency, that is, the natural frequency would be just 1 and this is given by minus R 1 R

2 divided by L 1 R 2 plus L 2 R 1. call this equal to minus alpha and therefore if you know the natural frequencies, you can write down the equations for the currents immediately. You can write i 1 t and i 2 t i 1 t would be some k 1 e to the minus alpha t. Is that okay? And i 2 would be some K 2 e to the minus alpha t. No, this is not

Student: (…)

Sir: We are always considering t greater than equal to 0 plus, but in i 1 it, no it is not simply K 1 e to the minus alpha t, plus some constant k 3. Why, because at infinity the current is not 0. This comes from physical considerations. You must not lose sight of that. In i 2, as t tends to infinity, obviously, the current would be 0 because at t equal to infinity the current establishes into a steady stabilized state in the primary and unless the current changes, there cannot be any current in the secondary, this is why i 2 solution is correct to find K 1, K 2 and K 3. To find K 3 1 simply refers to the physical circuit. Did you have a question?

Student: Sir, if we consider an equation, we also have a 4 (…) s is equal to 0 so are we ultimately get a K 3 equation?

Sir: That is correct. What he points out is, had we not bothered about time domain, if we had solved in the frequency domain only, then i 1 is equal to the determinant of V by s 0 s M s L 2 plus R 2 divided by the determinant of this. If you proceed blindly, you shall get a term like this. On the other hand, if you want to work in the time domain, then K 3 comes because of the physical consideration that at t equal to infinity the current is not equal to 0. I must tell you that you cannot divorce time domain from frequency domain and the third component here is the physical considerations.

You must keep all the three in your mind and all the three considerations must lead to the same result. For example, t equal to infinity, the physical circuit shows this. If you solve from Laplace transform, it will also show the same thing. If you solve from the differential equation, it will give the same thing and these three considerations must check with each other. In solving a certain problem, these are the checks and balances, whether we made a mistake or not. Is this point clear, that, if you had proceeded blindly in the Laplace, this is the advantage of Laplace domain that you do not have to consider continuity of flux, continuity of current, whatever it is, you just write the equation, transform it and be done with it.

Transform it, solve for the transformed variable, then invert in Laplace transform technique. Inverse Laplace transform, the current domain, the time domain solution shall come out automatically without any caution except algebraic caution. You must not make a mistake. But in the time domain solution, you have to take account of the fact, whether it is perfectly coupled or not perfectly coupled, as in this case. K 3 is equal to V by R 1 and to find out K 1 and K 2 now, we take care of i 1 0 plus, we have found out. i 1 0 plus would be K 1 plus K 3 and i 2 0 plus would be equal to K 2. We know i 1 0 plus, we know i 2 0 plus and therefore, find out K 1 and K 2 and substitute in the equation.

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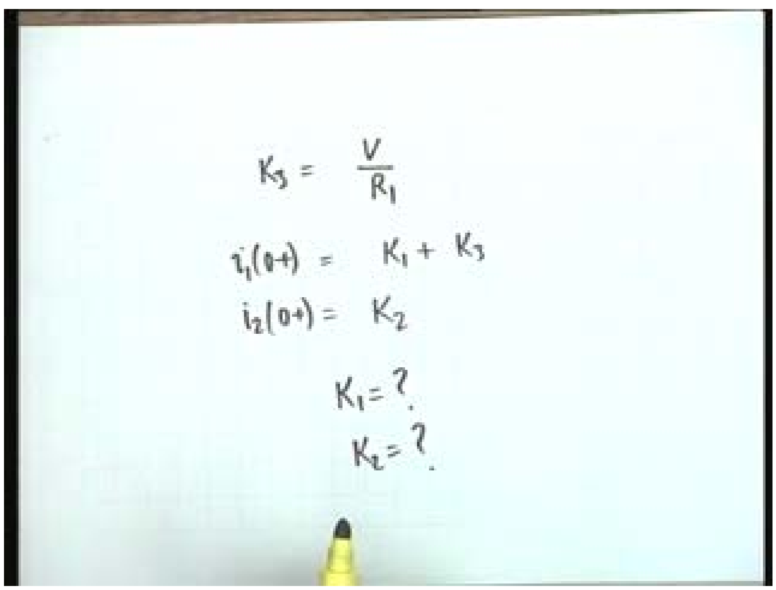

That gives you the final solution to the transformer with perfect coupling. Now, so far we were going back and forth between time domain and frequency domain. Wherever convenient, we brought the time domain into the picture. Wherever convenient, we brought the frequency domain into the picture. Now let us see, if we can remain completely in time domain, in the frequency domain with initial conditions. As I said in the frequency domain, the Laplace domain, initial conditions take care of themselves. Let us see how we can proceed, purely in the frequency domain without bothering about the continuity or otherwise, of fluxes and charges. And to this end, we review the equations to the elements. That is, if you have a resistance then the frequency domain equation is V equal to R I, a frequency domain equation.

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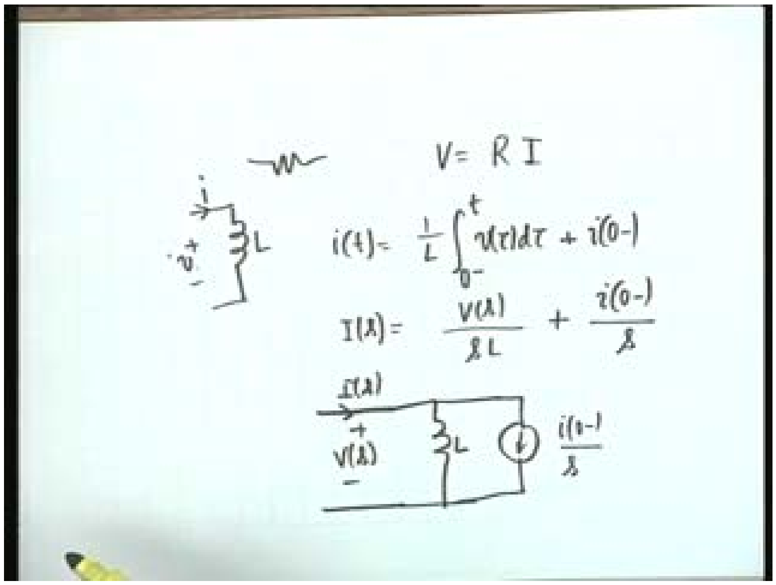

If you have an inductance now, you shall have to be careful. If it is an inductance V and i are the currents, then you know, you can write it in two forms. One is, the current can be written as 1 by L 0 minus to t V tau d tau, then plus i of 0 minus. This is the time domain equation and its frequency domain counterpart is I of s is equal to V of s divided by s L plus. This quantity gives you, i of 0 minus divided by s and it clearly shows that this equation can be represented by an equivalent circuit like this, equivalent circuit in the frequency domain.

Now, equivalent circuit is, you have an inductor without any initial conditions. This will take current V of s by a cell and this can be taken care of by a current generator, whose value is i of 0 minus divided by s. This is the frequency domain equivalent circuit of an inductor. It is in the form of a current generator in parallel with an inductor. This was the same type of equivalent circuit in the time domain. The exception is that the current generator now, is divided by s. In previous case, it was not, it was simply i of 0 minus. It is important to distinguish between the time domain and frequency domain equations.

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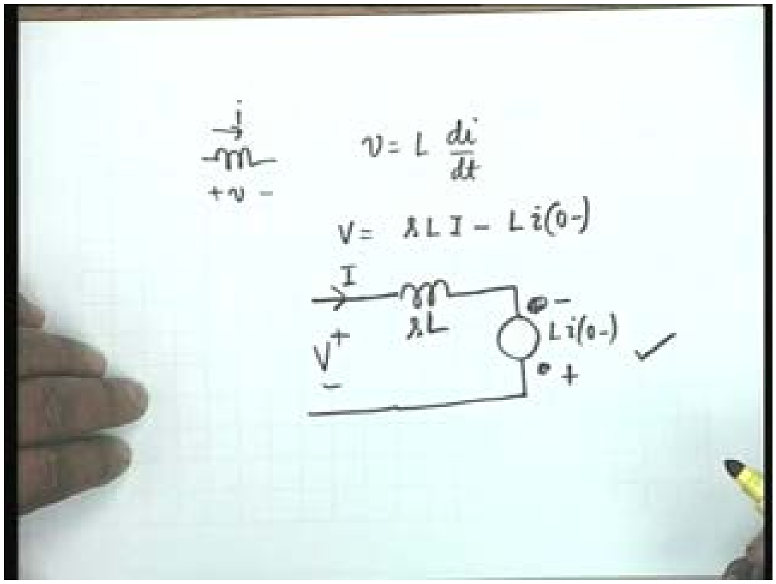

The other form of equivalent circuit that we can write for an inductor is by taking the current voltage equation in this form V equal to L d i d t, which means that capital V equal to s L I minus L i 0 minus. If I take the Laplace transform of both sides, then minus L, L is a common factor, s I minus i 0 minus and this can be written, in terms of a series equation like this, that you have a V I you have drop in L, s L I, L is initially relaxed. In addition, you have a voltage source which is in the opposite direction, minus Li 0 minus. So, it would be plus here and minus here.

Student: That would be S L minus.

Sir: Yes, the impedance is SL. It should be other way down?

Student: Yes sir.

 Sir: Okay. Is that clear? V equal to I times s L minus L i 0 minus. We shall utilize both these forms in solving problems. Wherever convenient, we shall use this circuit. Wherever convenient, we shall use the parallel circuit. One is, the series circuit is in terms of a voltage generator. The parallel circuit is in terms of a current generator and you know, series and parallel circuits are duals of each other and that is why, a voltage generator in the series circuit becomes a current generator in dual.

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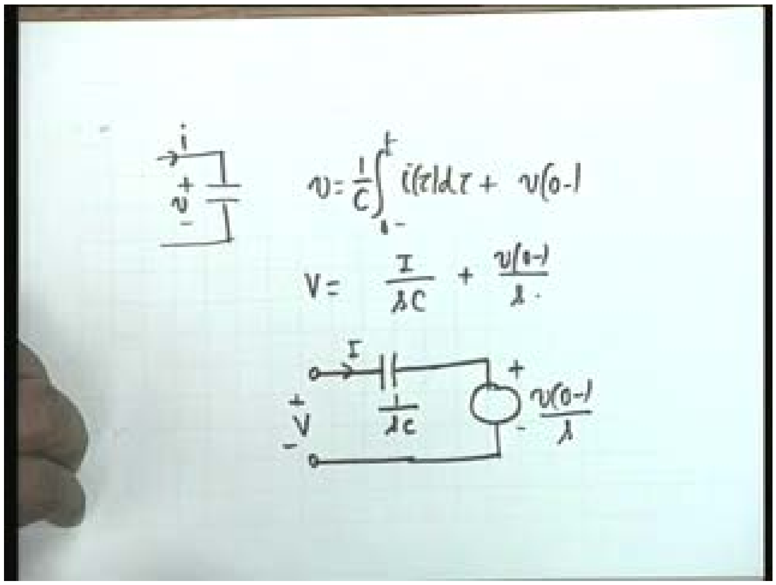

In a similar manner, we can treat a capacitor. In a capacitor V and I, the equation is V equal to integral 0 minus to t i tau d tau plus V of 0 minus and a Laplace transform. I made a mistake.

Student: C times.

Sir: C times or 1 by C times?

Student: 1 by C

Sir: 1 by C and therefore, it would be, I divided by s C plus V 0 minus divided by s. That is, this gives rise to a series equation, in which we have a V and then a capacitor, whose impedance is 1. Over s C, this is the current I. In addition, we have a voltage generator with this polarity; V 0 minus divided by s, is that okay? This is the equivalent circuit in the transform domain with initial conditions. Now, the initial conditions are at 0 minus, if you work in the transform domain.

We can also write a dual circuit of this, dual equivalent circuit of this, by taking the other equation for a capacitor. That is, i equal to C d v d t. Then, I have capital I equal to C s times V minus V of 0 minus, which means that, I equal to s C multiplied by V minus C V 0 minus which means that I have, the current i contains 2 components. One is for the, through the capacitor, initially relaxed, which is equal to s C times V, I would follow this convention that I would not write impedance or admittance. I will simply write C, the value of the element. I will follow this convention throughout.

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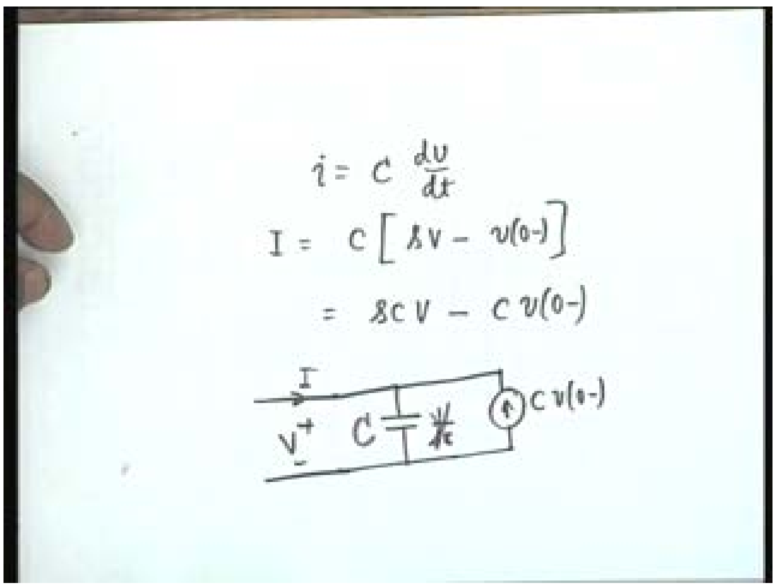

Then, there is a current generator which negates. In other words, the current generator will now come up.

Student: Opposite.

Sir: Up. This would be C v 0 minus. These are two terminal elements the three, two terminal elements. We have also talked about a four terminal element, namely the transformer. Let us see what happens in a transformer. We have L 1, L 2, the mutual inductance M. Let us say, these are the dots this is V 1, this i 1 V 2 i 2. Then my equations are V 1 equal to L 1 d i 1 d t plus M d i 2 d t and the corresponding equation is V 2 equal to M d i 1 d t plus L 2 d i 2 d t.

Now, if I transform the two equations, the first one gives you capital V 1 equal to L 1 s I 1 minus L 1 i 1 of 0 minus. This is the L 1 d i 1 d t and the second equation gives s M I 2 minus M i 2 0 minus, term by term. For the second equation, I get V 2 equals to s M I 1 minus M i 1 0 minus plus s L 2 I 2 minus L 2 i 2 0 minus and these equations written in pink color, can now be represented by an equivalent circuit, an equivalent circuit like this. I will reproduce these equations in the next slide.

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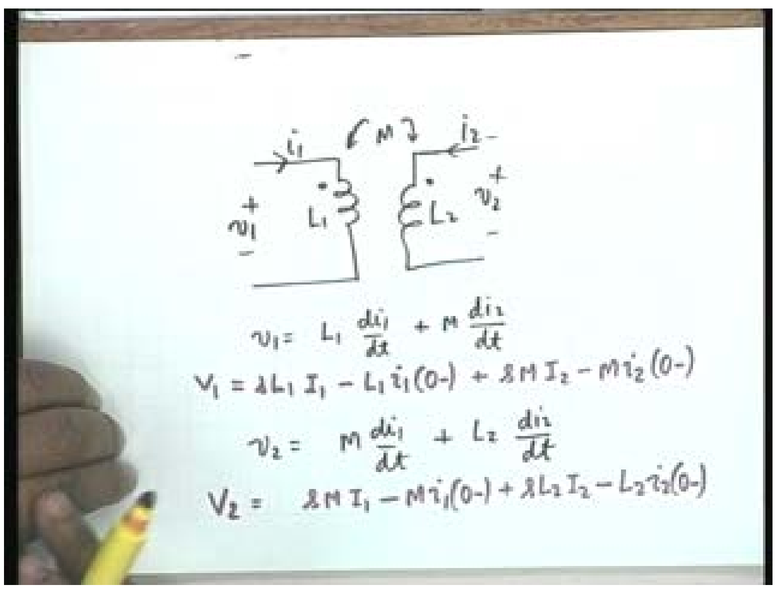

V 1 equal to s L 1 I 1 minus L 1 i 1 0 minus plus s M I 2 minus M i 2 0 minus and V 2 equal to s M I 1 minus M i 1 0 minus, plus s L 2 I 2 minus L 2 i 2 0 minus. You see, when I write the equivalent circuit, the magnetic coupling has been taken care of. No longer do we require a coupled circuit. Well, it is a coupled circuit but no, not linked by magnetic flux, as you will see. We have V 1 and I 1 and on the other side, we have V 2 and I 2. Now let us construct V 1. V 1, first is, drop in drop in L 1. So we have an inductance L 1. Then, a voltage generator which opposes the flow of current, namely this would be negative and this would be positive and the value of the voltage generator be L 1 I 1 0 minus, agreed? This is first equation. Then, we have another inductor but the current is not I 1.

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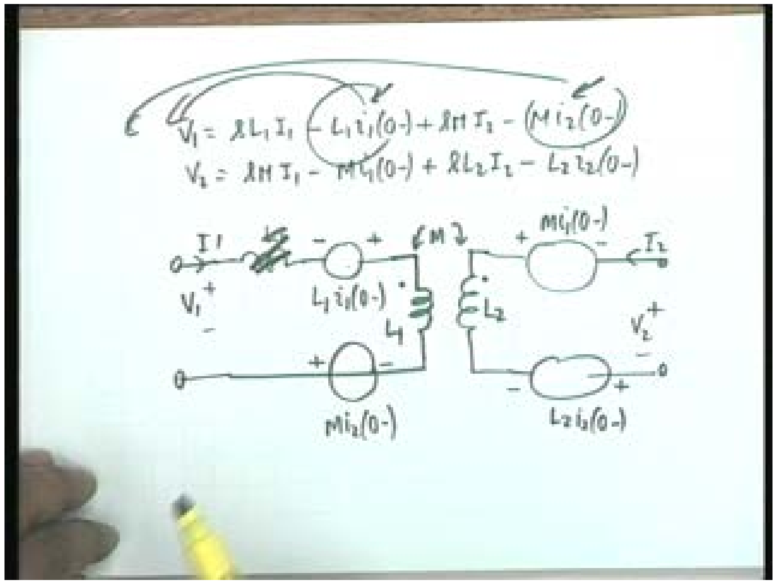

Student: (..)

Sir: So, we cannot do it this way. What we do is, we take an L 1 here. We do not remove the coupling. The coupling still remains. I will show you how. We take an L 1 here and in addition, we take a voltage generator of opposite polarity, namely minus plus, so, minus plus. This is M i 2 0 minus and the voltage here would be L 1 s L 1 I 1 plus s M I 2. Now how do we take care of this? We take the second inductor, put the dots here, and a mutual inductor. So, you do not remove magnetic coupling. Magnetic coupling is still there.

Student: Excuse me, sir. We can take L 1 minus M in that circuit and M in between common for both with and remove the.

Sir: I shall come to that. I shall come to that in a minute, the equivalent circuit that is suggesting. I will come to that in a minute. Now, so, that will take care the current I 2 flows here. So the voltage here would be L 1 s L 1 I 1 plus s M I 2. In a similar manner in the secondary, we shall have two generators. One would be minus plus. The value would be M i 1 0 minus and we shall have a voltage generator here. With what polarity, current flows like this? So, it will be minus plus and this would be L 2 i 2 0 minus. Far from being a nice equivalent circuit, but we cannot help it. Is that clear?

What I have done is, I have taken the initial conditions away from the coupled coil. This coupled coil has no initial condition. That is, initially it is de energized. The 2 voltage generators that arise L 1 I 1 0 and M i 2 0, I have separated them out. Another way of looking at it, the variety is the sense of circuit theory. There are many ways that you can look at a particular thing. You can combine these two generators along with the external generator and say that, in a coupled circuit, the external voltage sources are complemented, I am sorry, are supplemented by initial conditions, which in terms of equation, means that this is turn taken to the left hand side. This is taken to the left hand side and therefore you have instead of V 1, you have V 1 prime which is equal to V 1 plus L 1 I 1 0 minus plus M i 2 0 minus. Now, the question that one of students asked is a very interesting question. We will solve the circuit. We will take an example but let me answer that question first.

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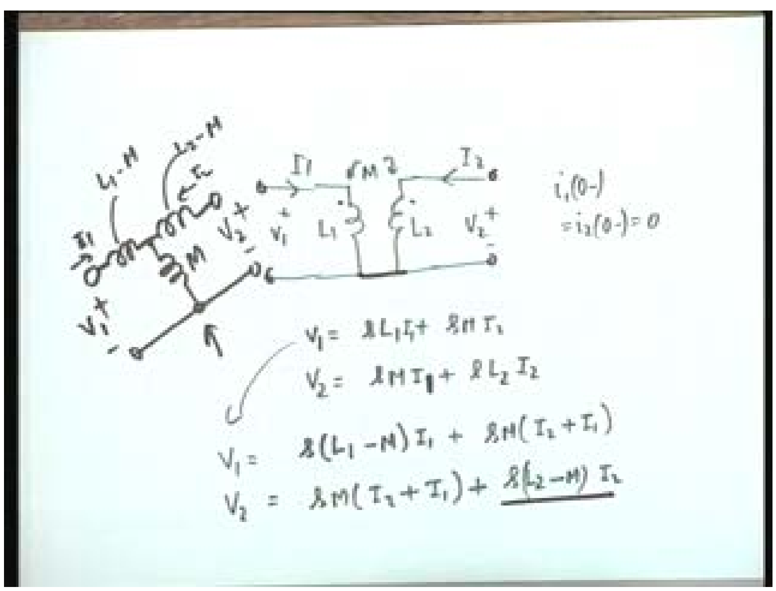

Suppose we have L 1, L 2 and M. These are, let me write in terms of transform domain quantities, V 1, V 2, I 2 and I 1 and suppose, for simplicity, suppose i 1 0 minus equal to i 2 0 minus equal to 0. Then, my equations are V 1 equal to s L 1 plus I 1 plus s M I 2 and V 2 equal to s M I 2 plus s I 1. That is right. Thank you, s L 2 I 2. Now, be with me. I am trying I am going to do some manipulations. I can write the first equation in a slightly different manner like this.
s L 1 minus M I 1. I am introducing term, minus M I 1. Then, I should write s M I 2 plus I 1, is that okay? I am subtracting a term here, then adding that term. I have a purpose, as you will see in a moment. Then, similarly the second equation, I can write as s L 2 minus M I 2 and this would be sM I 2 plus I 1. Now, look at this equivalent circuit. Look at this circuit.

If I have a circuit like this, V 1, V 2, I 1, I 2 and these inductors are L 1 minus M, this inductor is L 2 minus M and this is M and you write the equations, for V 1 and V 2. You see V 1 would be as I 1 L 1 minus M the first term here plus the drop in M, which is the current in M is I 1 plus I 2. So, s M i 1 plus i 2. Agreed? In a similar manner, V 2 is equal to I 2 times s L 2 minus M, which takes care of the second term here and the drop in M, once again, is s M I 2 plus I 1. This is an equivalent circuit, in which the coupling has been removed, in which the coupling has been removed. Is that clear? However, there is an objection. Are these 2 circuits completely equivalent? The answer is no and this answer comes from physical considerations. How many terminals does this circuit have?

Student: 4

Sir: and how many terminals does this circuit have?

Student: 3

Sir: 3. So, a 4 terminal network cannot be physically equivalent to a 3 terminal network, unless this terminal is connected to this. If that was so, if they had a common terminal, then they would have been perfectly equivalent to each other. However, this equivalence is mathematical equivalence and this is where mathematics can be dangerous to a circuit theory. For example, if these two potentials in an actual circuit differ by, let us say, 1 kilo volt and you are designing a circuit on this basis, then short circuit in this might lead to a very large spark and a very dangerous situation. One must not lose sight of that. Yes, what this gentleman said? These are equivalent, but they are only mathematically equivalent. On the other hand, if these two are connected together, then there they would have been exactly equivalent to each other. We shall use this equivalent circuit later, in some other context. We shall use this equivalent circuit. Now, let us work out an example.

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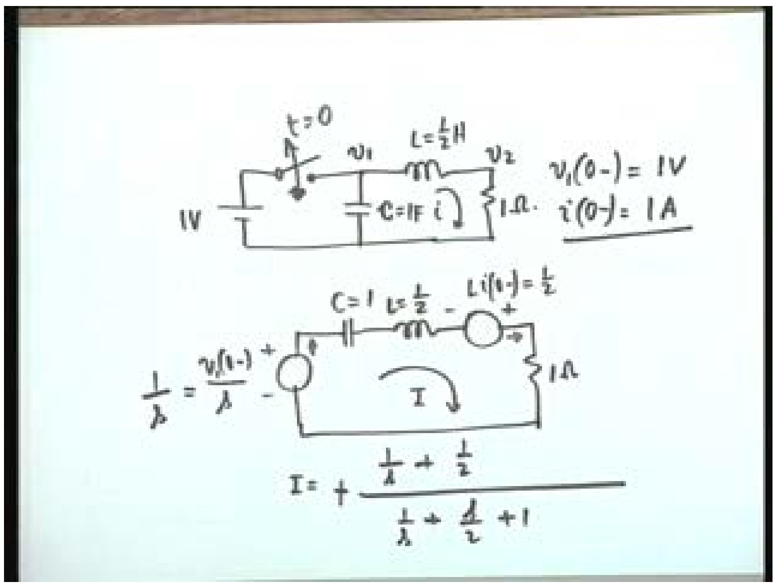

And this example is a battery of 1 volt switched on to a circuit, I beg your pardon, it is the other way down, switched off from a circuit, in which there is a capacitance of 1 farad and there is an inductor of half Henry and a resistance of 1 ohm. The problem is to find V 1 and V 2 and here, we shall work, the problem is to find V 1 and V 2 in the time domain, but we will prefer to work in the frequency domain. The only thing that we need is, we want to we have to find out if this current is i, we have to find out V 1 0 minus. You see, before t equal to 0, this switch was in the on position and therefore V 1 0 minus would have been?

Student: 1 volt

Sir: 1 volt and in the on position, when the current is stabilized in the circuit, then obviously i of 0 minus would be equal to 1 volt divided by 1 ohm, so, 1 ampere. These are the initial conditions. That is all that we need to know. Now, we draw the transform domain equivalent circuit. What kind of circuit should we prefer, in this particular case? I can draw an equivalent of capacitor in terms of a parallel circuit. That is, a current generator and a capacitor, or a series circuit in terms of voltage generator and a capacitor. Similarly,

Student: series

Sir: I will prefer a series, why, because, I have to solve only 1 loop. I want to find out the current. I will have to solve only 1 loop. If I know the current, I know V 1, I know V 2. So I will use this series circuit, not the parallel. It is not that, if you do not, if you use the parallel circuit, it will not work. Yes, it will work but you will have to do a little more algebra.

Student: So, that means we can do that. We will write the differential equation for the circuit and take the.

Sir: No, we do not write differential equation. We want be in the time frequency domain only.

Student: Then, we will take the Laplace transform and

Sir: We do not do that. Either we go by replacing each element by its equivalent circuit. That is, the simplest thing to do.

Student: Will not it be same?

Sir: It would be the same it is like God. There are many ways but you reach the same point. It is also like infinity. I will tell you about infinity some other day. Today, let us draw the equivalent circuit.

Student: sir, we could as well replace L by parallel (…)

Sir: They would give exactly the identical result, if you replace L by a parallel of L and a current generator, similarly, C by a parallel of C and a current generator. You have to solve the circuit, but the solution will contain a little more effort than if I take series circuits for both and this is what I did, I took a equivalent circuit V 0 minus divided by s, V 1 0 minus divided by s, then the capacitor C, which is equal to 1. For the inductor also, I draw the series equivalent circuit. It is half; inductor is half and then I have the voltage generator plus minus L i 0 minus.

This is equal to simply half, because i 0 minus is 1 and L is half Henry and what is this equivalent to? This is simply 1 by s and finally, the current the resistance 1 ohm and this is the current capital I. Just one stroke, I replace each element by its Laplace domain equivalent circuit and the current, it is obvious that the current would be one by s plus half, this generator plus this generator. They help each other, do they? Or have I made a mistake plus minus, plus minus.

Student: 1 by s, C is equal to 1 by s.

Sir: No, I will simply write the elements, instead of writing impedance or admittance. No, what I am doing is, I am trying, I am finding the volt, equivalent voltage in this loop. This is half plus 1 by s, but they drive a current in the other direction. If this is my direction of current, obviously, I should put a negative sign divided by the total impedance of the circuit, which would be 1 by s for the capacitor, then s by 2 for the inductor plus 1. Agreed?

Student: (…)

Sir: Oh, there should not be the negative sign. That is right. This current generator tends to send a current in this, the voltage generator. This tends to send in this direction, which agrees with I, done. And if you now simplify this, you get i equal to s plus 2, I leave the algebra to you, divided by s plus 1 whole square plus 1. I have written this in this form with a particular motive. The motive is that, I will simply find out the inverse transform by inspection. I will write this as s plus 1 plus 1 divided by s plus 1 whole squared plus 1 and therefore, small i t would be equal to the inverse of this is cosine t.

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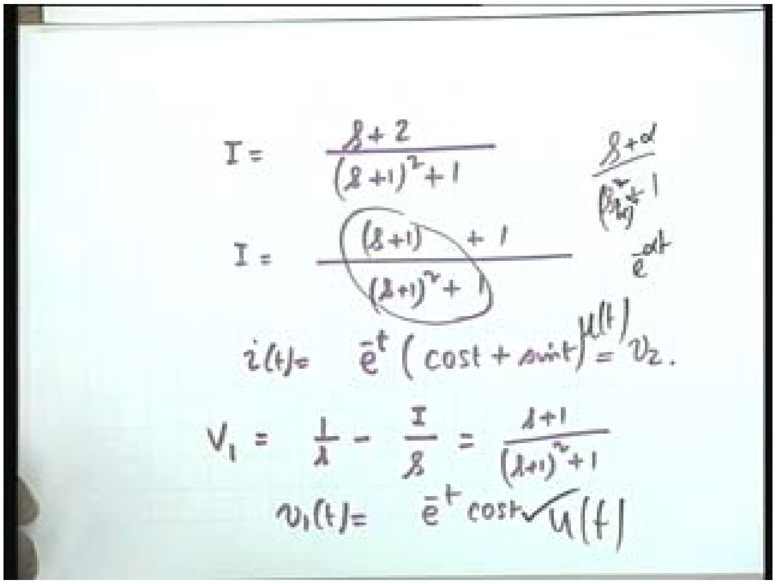

Student: e minus t.

Sir: e to the minus t, cosine t inverse of this and the inverse of 1 by this is a sine t. So, e to the minus t cosine t plus sine t and since V 2 is the drop across 1 ohm, this would also be equal to V 2. How do you find V 1? How do you find V 1, capital V 1? Let us look at the circuit. This is V 1. No, I do not integrate, I do not do anything. This is V 1 and obviously, V 1 is equal to 1 by s, this voltage generator minus the drop in the capacitor. So V 1 is equal to 1 by s minus I divided by s and I already know I, substitute this, and simplify. What you get is, the final result is, s plus 1 divided by s plus 1 whole squared plus 1 and therefore, V 1 t is simply equal to e to the minus t. Yes, cosine or sine?

Student: cosine.

Sir: cosine t. You should remember this, these elementary transforms cosine t and sin t. Actually the transform of cosine t is s divided by s square plus 1, and if you shift s by the amount alpha, then this would be s plus alpha squared. Then the transform is e to the minus alpha t times cosine

t. So, this is as simple as that. The advantage of Laplace domain, as I have already mentioned is that you need not take care, you need not calculate V 1 or I 1 0 plus. The quantity is, 0 plus need not to be calculated and therefore, the continuity of flux, continuity of charge, all those physical considerations need not be brought in. This is the advantage, you can work blindly, but I strongly advocate against any blind approach. If you lose sight of the physical condition, it is very easy to make mistake.

Student: Sir, wont there be u t?

Sir: Of course, there will be u t.

Sir: u t shall multiply. ut is ub q t s it is omnipresent because we cannot work from minus infinity to plus infinity or t, because this an infinite interval of time. Our life time is finite and so is the duration of current or voltage in this particular equation. We will meet at, 1 hour later.