Circuit Theory

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Lecture 51

Network Transmission Criteria

this is the fifty-first lecture, and our topic of today would be network transmission criteria

before we go into this a few words about the bridged T network the constant resistance

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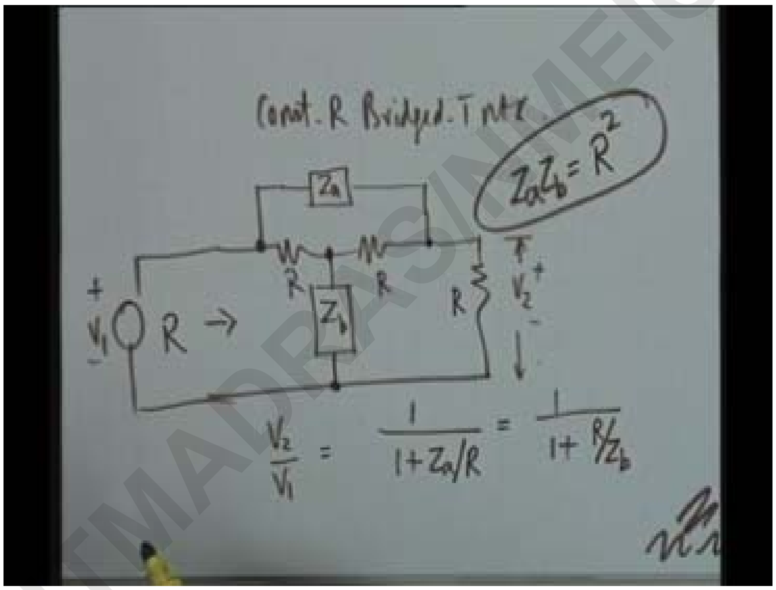

that we discussed as one of the examples

while we recall that the bridged T network is like this you have two resistances and an impedance here zb and an impedance here za with resistance R and this is a constant resistance bridged T network which means that if either of the ports is terminated in a resistance R then the input impedance looking at the other port is R

the network is symmetrical and therefore either port terminated in R shall give you an input resistance of R and if this voltage is V two and this is connected to a voltage generator V one then V two by V one is equal to one by one plus za divided by R which is the same as one plus R divided by zb because the the condition for constant resistance network is that za zb should be equal to R square this is the main result

[Conversation between Student and Professor – Not audible ((01:58:00 min))]

Student: sir this resistance R which is inside the network correct is that equal to the terminating (( )) ((00:03:03 min))

correct correct they have to be identical yes they have to be identical okay

this is the main result that you can prove this i think part of it we proved what you want to do is to illustrate this with an example [Vocalized – Noise]

and as i said constant resistance networks are most useful in cascade synthesis that is break up the given transfer function into component ones simpler ones and realize each simpler transfer function by a constant resistance network and then cascade network that is what you do is

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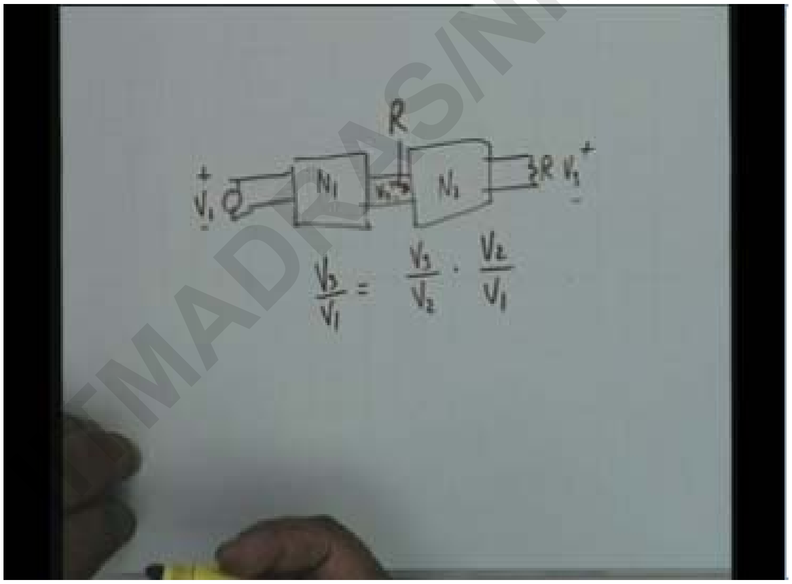

you have a network N one you have a network N two both of which have constant resistance and then terminate by R okay

drive it by voltage source V one if this is V two and if this is V three then V three by V one is equal to V three by V two that is the transfer function of this multiplied by V two by V one and because the input impedance here is equal to R the transfer function of N one is not altered is not changed when N two is cascaded to it this is the beauty of constant resistance network and they are the answers for the lack of buffered devices like op-amp in passive circuits

the cost that you pay you have to pay price for every advantage that you accrue the cost that you pay is the increased number of elements

it is expansive constant resistance networks are expansive in the number of elements

the power losses

the power losses of course power loss possibly is not of much concern not of much concern particularly in filtering the applications okay but simply in filtering filtering the applications the main concern is frequency filtering it is not power but if power can be preserved it is a good point it is a plus point all right

let's take an example

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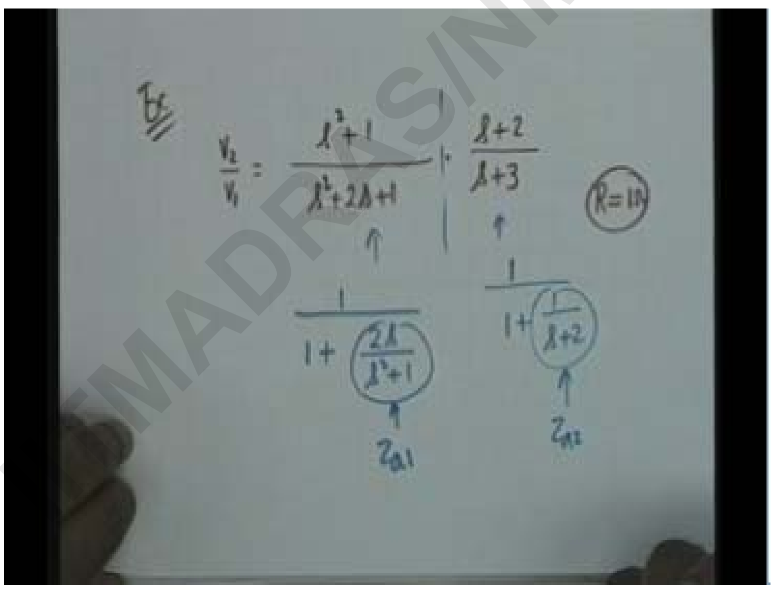

suppose i have to synthesize a transfer function V two by V one equal to s squared plus one divided by s squared plus two s plus one times s plus two divided by s plus three let's say okay

now [Vocalized-Noise] by a constant resistance network and a constant the terminating network the terminating resistance given is R equal to one ohm all right R equal to one ohm

then what i do is i look up on this as the product of two transfer functions one is this and the other is this and this obviously can be written as one plus two s divided by s square plus one which means that za for this network this is one plus za by R so za is this za of the first network where as this can be written as one over one plus one by s plus two

so za two za two for the second network it is this and one can now draw the two networks immediately if you know za then you know zb also zb shall be simply the reciprocal of za

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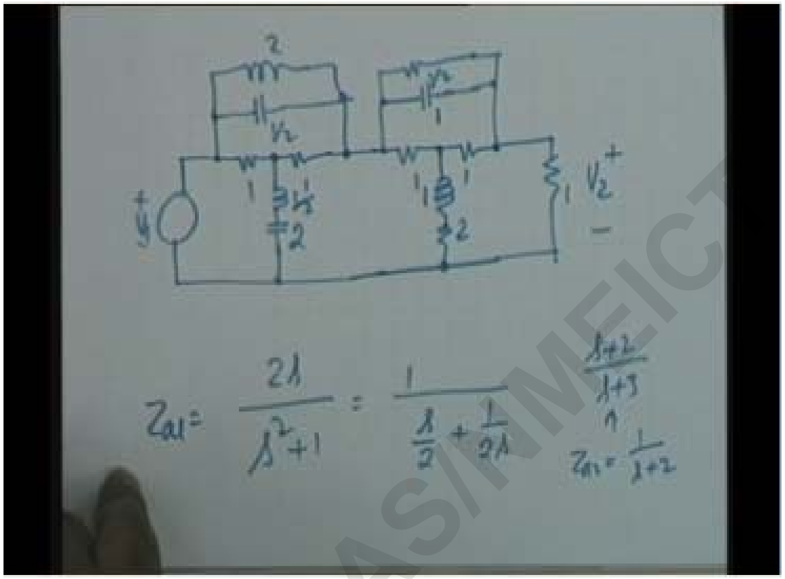

and therefore our network will now be one ohm one ohm the input is the voltage source V one then [Noise] then za za is two s divided by s squared plus one which i can write as s by two plus one over two s

this is an impedance this is an impedance of one by this must be an admittance in other words we have a capacitor of value half in parallel with inductor of value two two Henry and that's it

then what we have here is the reciprocal of za zb is r squared by za r is one therefore is the reciprocal in other words treat this is an impedance that will give you zb which means that we have a half Henry inductor in series with a capacitor of value two Farads okay this realizes the first transfer function

on the on the condition that the termination is one ohm right the condition is that the termination should be one ohm instead of one ohm we cascade a second network okay

and the second network realizes the transfer function s plus two divided by s plus three which gives you za two is equal to one over s plus two and therefore i have the second network one and one and these two must be there then in parallel i have one over s plus two which means an admittance of value s plus two that means a capacitor of value one and a resistor of value half okay

so that's it for the second network and as far as this this ((arm)) ((00:07:52 min)) is concerned zb should be simply equal to S plus two which means an inductor of value one and a resistance of value two

the final network is now to be terminated in one ohm and this should be V two this is the complete synthesis

all right there are difficulties difficulties for example if the transfer function was uh let's go back to this

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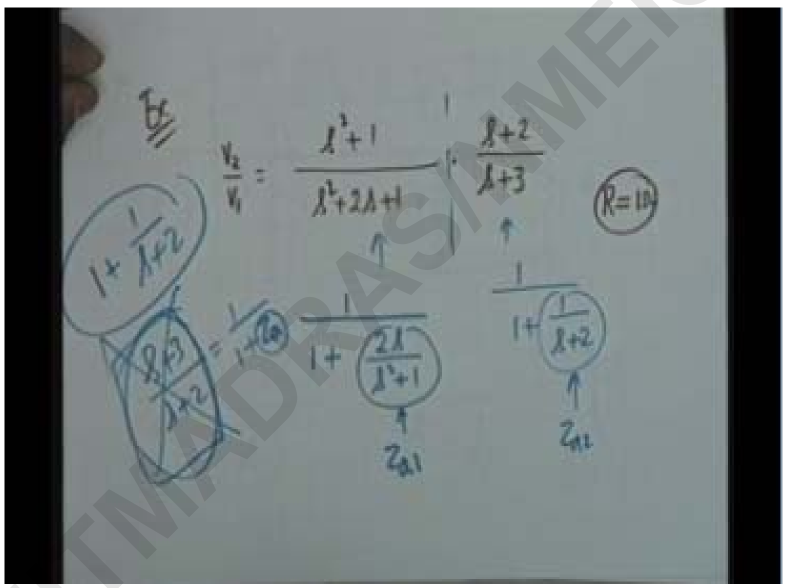

suppose the transfer function was s plus three over s plus two all right s plus three over s plus two could you have realized this could you have realized the transfer function [Noise]

if it was s plus three by s plus two can you express this as one by one plus za

[Conversation between Student and Professor – Not audible ((08:55:06 min))]

Student: but we can do one thing

what can you do just tell me just tell me can you express this as an one by one plus za by za where za is a peer function no

[Conversation between Student and Professor – Not audible ((09:11:00 min))]

Student: sir instead of taking V two by V one we can take V one by V two and for s plus two by s plus three

you can take V one by V two that is not what is wanted

we cannot change the specs

[Conversation between Student and Professor – Not audible ((08:55:06 min))]

Student: sir but for the second function for the second network

correct

Student: we can

[Conversation between Student and Professor – Not audible ((09:21:06 min))]

it is symmetrical fine

[Conversation between Student and Professor – Not audible ((09:30:06 min))]

Student: we can invert that

no no no if you invert that the transfer function is changed

there is a fundamental reason why you cannot realize this can you tell me what the [Noise] fundamental reason is

there is a fundamental reason for which you cannot realize this whatever network you whatever network you attempt no it is not a question

[Conversation between Student and Professor – Not audible ((09:56:06 min))]

no there is no instability s equal to minus two

[Conversation between Student and Professor – Not audible ((10:03:06 min))]

Student: one plus one by s plus two

one plus one by s plus two okay which means that the DC response is greater than one

can you think of a network which can multiply DC unless it is an amplifier

so activity is a part of it this transfer function cannot be realized

so have a close look at the transfer function before you attempt any of the realizations it is just a caution

now the main topic for today [Noise] is network transmission criteria

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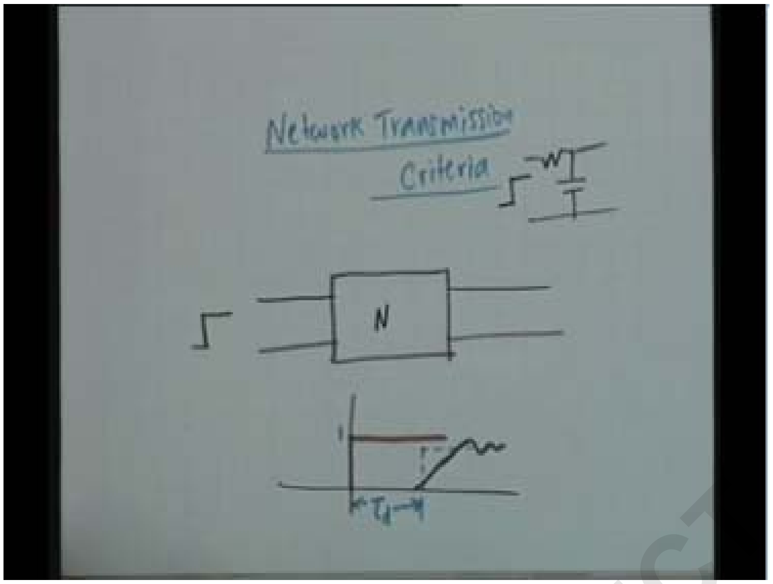

most of the networks most of the networks in communication channels that you see in practice are low pass structures

that is their high frequency response is limited okay

so our discussion shall be concerned with low pass structures only but [Noise] with a little bit of modification they can be adapted to other structures also is the point clear

an amplifier an audio amplifier that you make hardly ever even a high quality stereo amplifier hardly ever goes to a three degree point of twenty kilohertz for example

sixteen hertz to sixteen kilohertz is okay there are people who can listen to more than sixteen kilohertz and to them the audio quality is important not the watts not the loud ((press)) ((00:11:44 min)) or the volume okay if the quality is important well uh the even the high quality amplifier cannot go up to twenty kilohertz it stops somewhere around fourteen point something okay

a communication channel a telephone channel for example

a telephone channel is a low pass channel that is very high frequencies cannot be transmitted

a telephone transmission line cuts off and so what happens is if you have all these can be treated as a network

a telephone line for example is a distributed type network instead of a lumped network it is a distributed network

now such networks therefore most of the networks that you encounter in practice are low pass so if you feed this with a unit step you don't get a unit step here what you get although this is what you want for faithful transmission for faithful transmission is what you want is the following

that if this is the input then this is the input then the output [Noise] that you want is something like this

it may not go to one it may not go to one may be the final value is slightly less than one because of attenuation in the channel but what you want is that the the shape of the input should be preserved it should it would occur after a certain delay

delay delay is a fact of life you have to put up so this much is the delay let's call this **{**thou d} ((13:21:01: min)) it is an ideal transmission channel

where i can tolerate a delay that is perfectly all right because uh signals have to travel from one place to another be it a lumped network or a distributed network it does require a certain amount of time

what we cannot tolerate is the distortion in the waveforms what you will see in practice will not be like this what you will see is something like this

may be of an overshoot and undershoot an oscillation what you see in practice is something like this okay therefore if the if the waveforms were not distorted if there was no signal distortion you would have said the delay introduced by the channel is tau t

now how do you define delay in such a case how do you define delay in such a case that is that is a problem

now you know that in the laboratory you define delay as the time for the waveform to rise to fifty percent of its final value this is the rule of thumb and there is nothing sacred about it it could also be seventy percent for example all right

have you ever measured delay time in the laboratory uh you must have in the digital electronics laboratory

okay so what if you say is uh for example if i take a simple RC network and i uh i feed a unit step here okay then what i get here is actually you don't we don't feed a unit step you feed a pulse okay and then look at how the pulse rises

the pulse will rise like this so if this is the final value then you go to fifty percent and measure with the help of a CRO what is the time in that is what you call as i said there is nothing sacred about fifty percent it could be seventy percent for example if let's say uh the input voltage is one volt that is one volt step and point seven volt triggers a certain a uh logics circuit then what you require is seventy percent delay okay seventy percent response and you will call that as a delay so there is nothing sacred about it

number two it is okay in the laboratory to see on the oscilloscope and measure but suppose you want to design a circuit you are an electrical engineer and specifications have been given you want to design a circuit

so you would like to know the effect of the elements of the circuit on the delay right

in other words is what you want is an analytical definition not an experimental definition like you go to the lab and measure the CRO that you means you have already made the system

it is only under that condition that you can measure all right or maybe you simulate this on a computer a digital computer find its response measure the response and if it is not satisfactory go back and change some of the design this is an iterative procedure okay right it may convert or it may not convert

and a circuit may contain a a number of elements let's say twenty elements which one would you change would you change one at a time or two at a time three this it's a messy effort

so what is required is an analytical definition if i can have an analytical definition then i should be able to judge how good a particular circuit is or how do i change this circuit so as to satisfy a certain prescribed requirement all right

and this analytical definition was given by a gentleman by the name Elmore W. Elmore in nineteen forty-eight and this definition has become a standard definition

the definition was used very sparingly till uh about end of seventies very sparingly wherever you require to design a circuit for a certain prescribed delay and as i am going to introduce the rise time and rise time well uh Elmore definition used to be used but uh [Noise] the advent of VLSI design and a design of integrated circuits that requires that this interconnections small interconnections that you make in the integrated circuit they act like transmission lines at very high frequency they act like transmission they cause a delay and therefore and it is a mess VLSI and a typical VLSI may contain several tens of thousands of transistors and tens of thousands of interconnection how to get what the delay would be introduced and due to the delay what is the distortion all right

so Elmore's definitions are the the uh ((oft)) ((00:18:13 min)) coated words from any VLSI design and if you look at any VLSI design you will see there is a definition of delay there is an estimate a statistical estimate of what the delay could be and how to change the delay and so and so

the name Elmore has become very famous now and it is important that you understand what Elmore's definitions are

now before we uh take these definitions [Noise] uh i have to tell you about two delays did i tell you earlier about phase delay and group delay did i no i didn't i thought i did but anyway a few minutes a few minutes

suppose you have an ideal transmission channel an ideal transmission channel then what would be its transfer function

there is there is certain amount of delay and there may be a certain amount of attenuation

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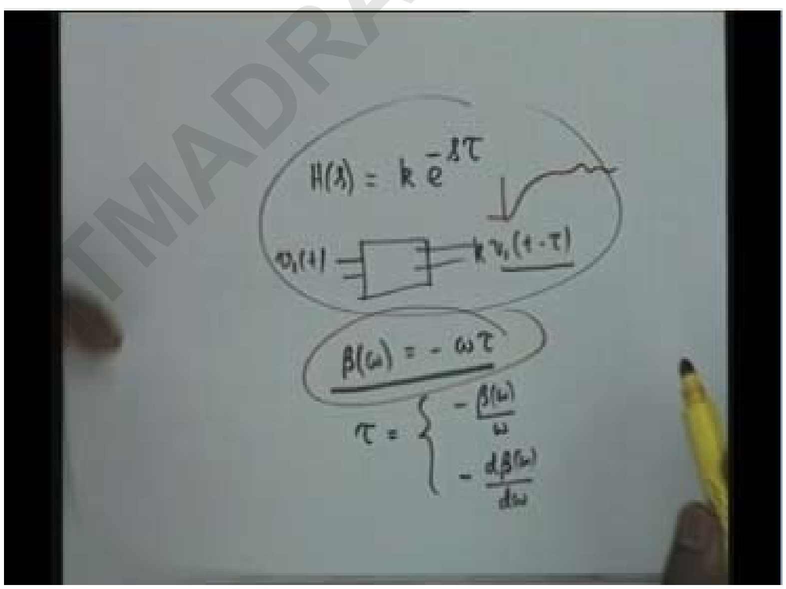

let's say h of s is equal to k e the minus s tau this is the transfer function you understand why this is the transfer function

if V one t is the input then the output output should be V one sum k times V one t minus tau all

all that you require is the waveform should be preserved the shape of the waveform it may be reduced in magnitude or increased if it is an amplifier there is an amplifier or a repeater in the channel it may be increased but what we want is that the shape of the waveform should be the same that is V one t minus t is simply the delayed version of V one T okay

and you can see that the transfer function of this ((Laplace of this divided by Laplace)) ((00:19:53 min)) of this simply equal to the k e to the minus s tau okay

so uh the phase shaped beta omega ah phased shaped of this is simply equal to minus omega tau and you see that tau has two significances here tau can be either looked upon as minus beta omega by omega or minus d beta omega d omega

that is you can either look up on this as minus beta omega by omega or the differential coefficient with a negative sign differential coefficient in a phase with respect omega

beta omega is the phase shift of this transfer function obviously the magnitude is k and the phase shift is minus omega tau we put s equal to j omega so j omega tau okay

now in this ideal channel ideal channel minus beta omega by omega and d beta omega by d omega they are the same and it is very easy to define the delay delay is tau it is very easy to measure the delay and uh

but this is not in generally true because all channels in practice distort the signal

that is the this is not k v one t minus tau it is something different okay it is not a step anymore if this is a step it come like this and perhaps has oscillation also

all right so these are in general different they are the same if the phase varies linearly with frequency so it’s a linear phase system it was in that context that i had introduced this to ((tau)) ((00:21:39 min))

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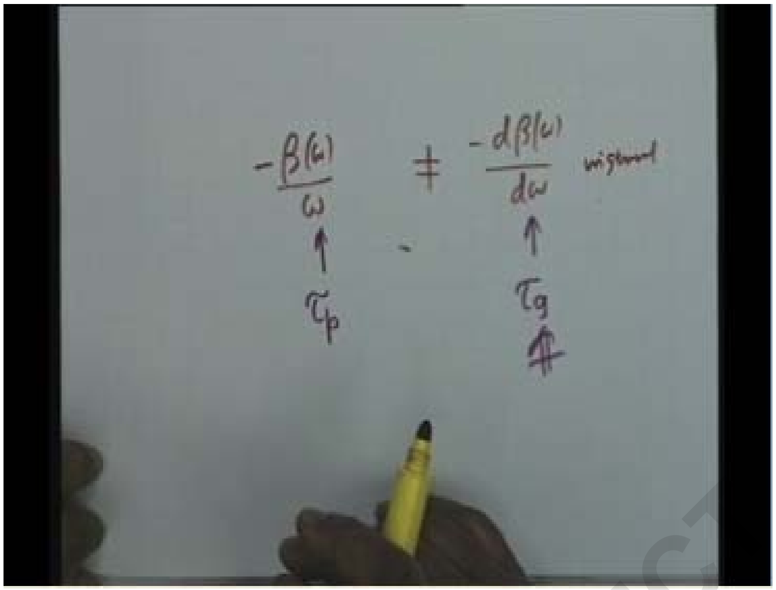

now if it is not linear phase then minus beta omega by omega is not the same as minus d beta omega d omega in general

and this quantity minus beta omega by omega is called the phase delay tau p and it relates to a single frequency a single frequency omega

on the other hand the negative gradient of phase negative gradient of phase is called the group delay tau g and it relates to a small band of frequencies around omega

to calculate d beta d omega we require a small band of frequencies and therefore no information can be transmitted without a band of frequencies

a single frequency is absolutely worthless as far as information transmission is concerned that means you cannot transmit information with zero bandwidth

if it is a single frequency zero bandwidth you require a non zero bandwidth for transmission of information this is why the bandwidth is so costly and there are CCITT regulations and all kinds of regulations on the bandwidth that can be used for television channel

if a new television channel is to be introduced in Delhi it cannot use any bandwidth that it likes it has to be regulated there is a regulatory mechanism all right that is a different study

so we have two definitions of delay one is the phase delay and one is the group delay and it is this delay that is of most practical importance

the group delay you shall also encounter perhaps you have already encountered in electro what is it electromagnetic theory or n theory course okay all right

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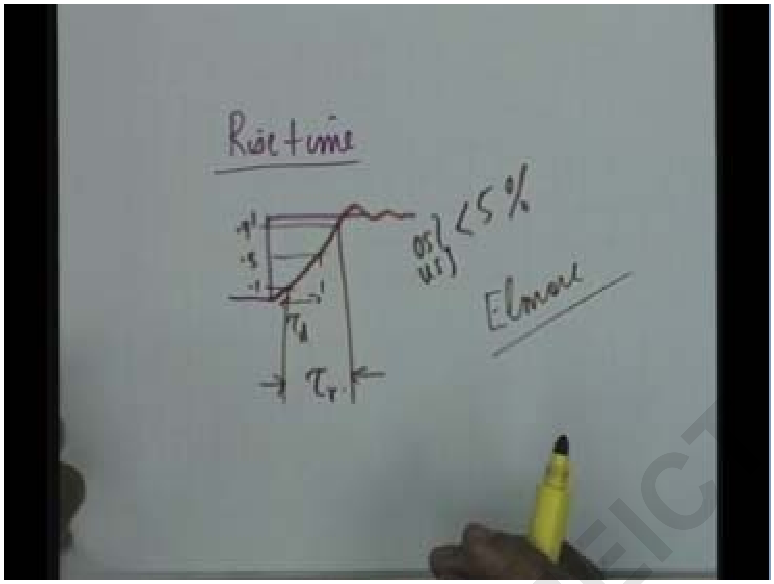

then uh the next point uh the next quantity that is of interest is the rise time okay the rise time

i know that if i apply a unit step well the output output shall not be like this output will be like this let's say let's say it is like this okay and i said that in the laboratory you define fifty percent of rise but it may go to unity and it may not go to unity

let's normalize the output response such that the final value is unity

say you find out fifty percent and you call this as tau d a delay time and in the laboratory you measured the rise time that is fifty percent delay time only measures when the waveform goes to the fifty percent of the final value but if you wanted an integrated view of the waveform how fast does it rise

well what you do is you go to point one and point nine and measure the difference that is ten percent to ninety percent

this is called a rise time tau R okay

it is meaningful it is meaningful only if there are ((nonviolent)) ((00:25:08 min)) oscillations is that right it is meaningful strictly it is meaningful for monotonic step response monotonic that is if there are no oscillations but a small amount of overshoot and a small amount of undershoot a practical figure is five percent

if the overshoot or undershoot is less than five percent well the rise time is taken to be a meaningful point we have again the same problem with regard to the rise time if we can simply measure it by a definition and arbitrarily adapted definition

one can ask why not nine to ninety-nine or one to ninety-nine well perfectly all right you can adopt any definition this is what uh people usually use in industry ten percent to ninety percent

but as i said if you are a circuit designer then this definition is useless because you don't know you don't know how to design the circuit such that a particular ten percent to ninety percent rise time is met

so you want an analytical definition and once again it was Elmore in the same paper Journal of Applied Physics in nineteen forty-eight he gave a definition of rise time which has now become a standard an analytical definition

[Conversation between Student and Professor – Not audible ((26:16:07 min))]

Student: sir that ten percent to ninety percent it is of the input maximum or the output maximum

output maximum that's what i said that output may not rise to one it may rise to point eight then you normalize this point eight two one okay that means a ninety percent of point eight point seven two or whatever it is okay

so we do require a definition of rise time and analytical definition and this was given by Elmore

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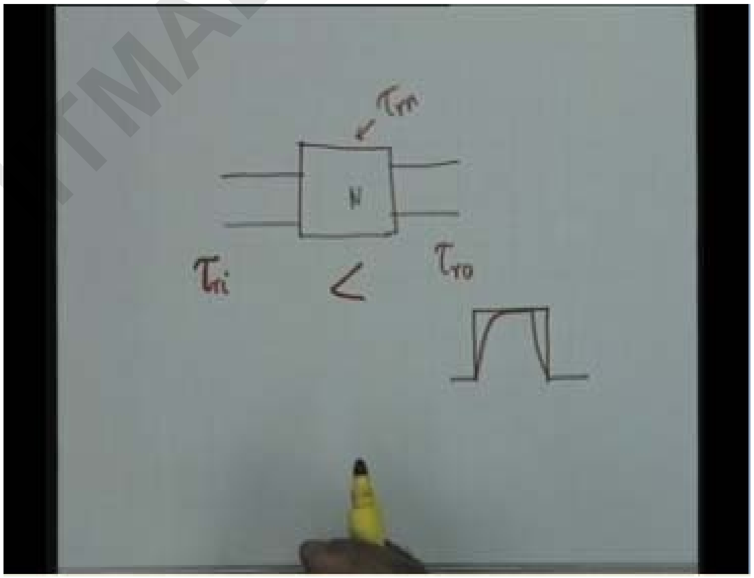

now hardly ever in practice you can generate a unit step what you generate in the laboratory is a pulse which it looks like this on the oscilloscope but if you zoom that is if you expand the time base you will see that what you see as a square wave or a pulse is really something like this something like this if you expand on the oscilloscope if you zoom all right therefore the input pulse also has a rise time

the input that you apply here let's say let's say the rise time is t r i the input rise time the output rise time is tau r zero now what can you say about the relative values of these two

which one is bigger

[Conversation between Student and Professor – Not audible ((27:44:07 min))]

obviously because the network causes further distortion of the signal

now if tau r zero and tau r i if these two quantities are known we will show later that the network rise time tau r n can be determined by a very simple formula which we shall find later

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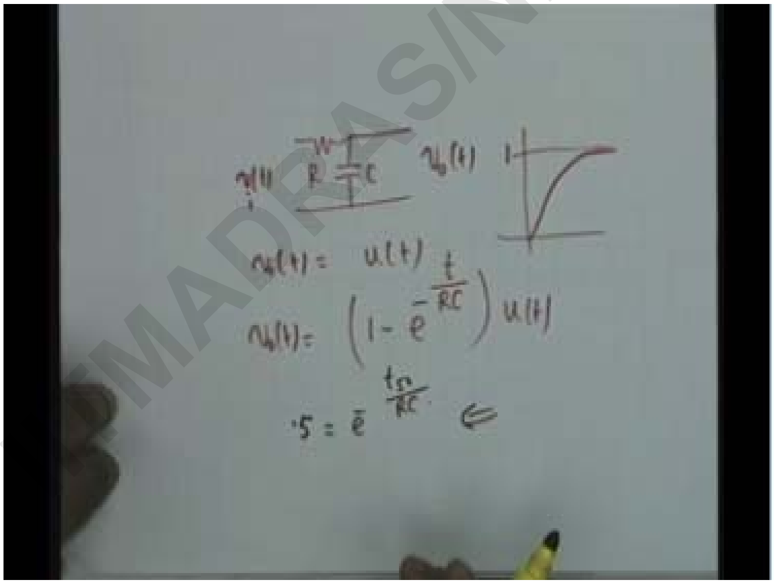

uh before we go further let's take a simple example of the uh RC network RC network simple low pass RC network in most of the channels most of the communication channels in practice can be modeled by uh a simple RC network

now you know that the input is a unit step V i t if the input is the unit step V i t is equal to u t then you know that v zero t for this network is simply one minus e to the minus t by RC multiplied by u t this you of course know all right it rises monotonically like this

and it does indeed go to the value one when t goes to the infinity all right

now obviously the fifty percent delay time is obtained by making point five equal to e to the minus t fifty divided by RC okay you can find a t fifty from here

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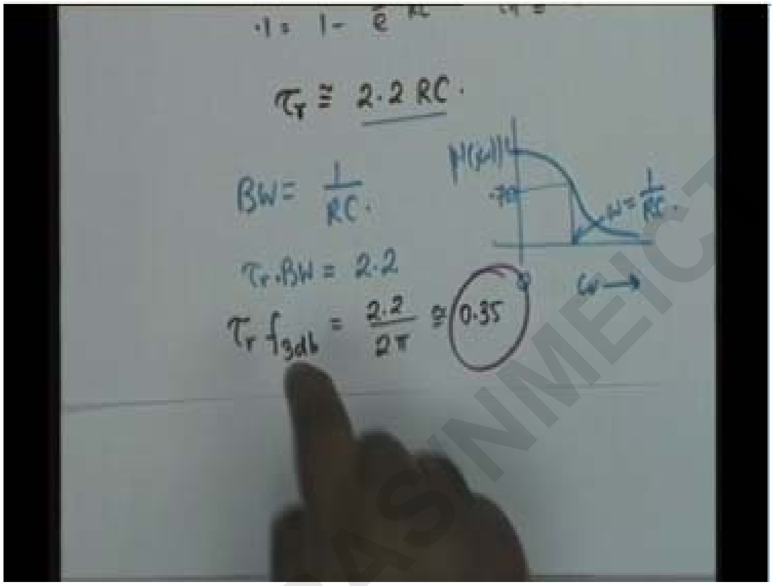

all right if you want to find the rise time by using the usual definition then what you do is you write point nine equal to one minus e to the minus t point nine by RC t point nine is a a uh constant and the other one is point one equal to one minus e to the minus t point one by RC

and if you solve this then t point nine is approximately equal to two point three approximately and t point one is approximately equal to point one

so that the rise time tau r for the simple RC network is equal to two point two i have made a mistake here RC okay i must multiply by RC so it is two point two RC the rise time is approximately two point two RC okay

most of the communication channels can be modeled roughly by a simple RC network like this so this figure is also a very sacred figure two point two RC

you also know that in the the frequency response h of g omega magnitude it starts from one and then goes to zero as omega goes to infinity for this simple low pass filter and you know that if this is one the three d view point of point seven o seven is reached when omega equal to one over RC and this we take as a the bandwidth of the RC network right bandwidth is equal to one over RC and you notice that tau r multiplied by bandwidth this bandwidth is in radians per second this is in radians omega you see that this is equal to two point two a constant all right for the RC network it change the values of r and c doesn't matter the product of tau r and bandwidth is a constant

in communication system practice one uses the hertz frequency in bandwidth that is one says the bandwidth in hertz instead of radians radians

so this would be two point two divided by two pi and this is approximately equal to point three five and this figure should be remembered at all costs

you see uh the rise time and three degree bandwidth in hertz the product is a constant point three five and in most of the low pass situation whether it is a telephone channel or any other kind of channel uh uh this figure point three five is the a very sacred figure

it it can deviate point three four nine or point three five two but it is around point three five and the simple RC network therefore models and this uh this shows this shows a certain kind of uncertainty principle

this is a reflection of the well known uncertainty principle it says that you can't have the what is it called cake and eat it too it is that kind of it okay that's what it says

it says that you cannot improve things in the time domain as well as in the frequency domain that is if you increase the bandwidth then your rise time shall be small

if you decrease the bandwidth the rise time would be higher that is wherever the bandwidth that you use okay for better resolution the higher will be the rise time all right you cannot improve things in a time domain as well as the frequent there are many uh other political analogies of this but i am not going to this

[Conversation between Student and Professor – Not audible ((33:24:07 min))]

rise time is going to be short

[Conversation between Student and Professor – Not audible ((33:27:04 min))]

isn't that good the cost one of would not like to use a large bandwidth because it is very costly one who would like to use as narrow a bandwidth as possible okay and therefore the rise time suffers

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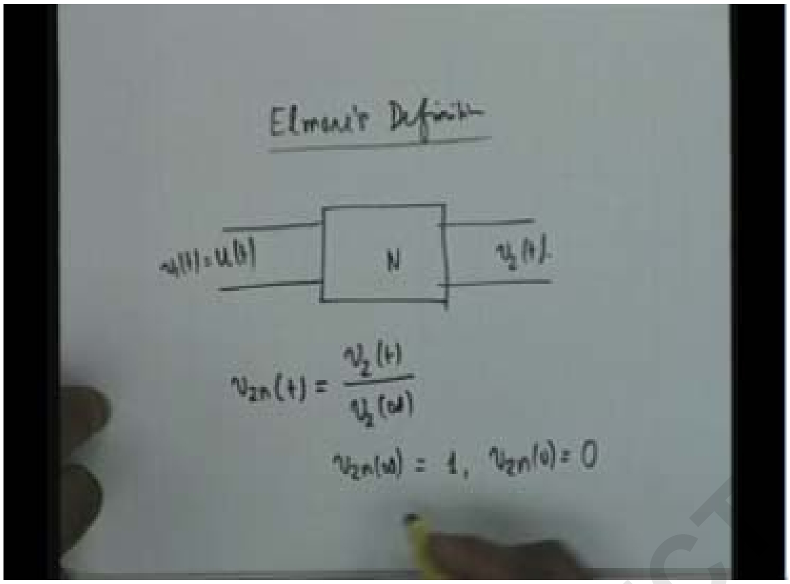

all right now we come to Elmore’s definition [Noise]

suppose you have a network n whose input is v one t which is a u t and suppose the output is v two t okay and as i said the network could be such that v two t does not rise to the value one even if the input is u of t

so we talk in terms of not v two t but v two t is normalized with respect to v two of infinity all right

this we call as v two n t v two n t normalized output response

obviously obviously v two n infinity is equal to pardon me one and v two n zero is zero okay

do you understand this

we simply normalize this whatever v two infinity is we will call this uh we will normalize this with respect to v infinity so that v two n the normalized value goes to one

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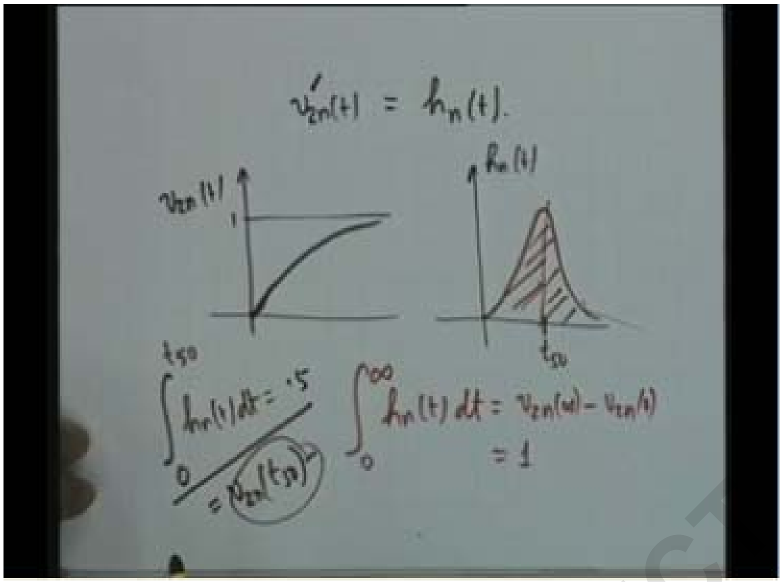

now if v two n t is the unit step response then obviously if i differentiate this with respect to t this would be the unit impulse response that is h n of t normalized unit impulse response okay is that clear

the differential coefficient of the unit step in signals and system because the inputs are uh related by differentiation output should also be related by differentiation we are talking of a linear system

if the input is differentiated

Student: sir here which one is the output

v two n is the output normalized output and its differential coefficient shall be the unit impulse response again normalized normalized okay

now therefore if v two n t is of this shape let's say it is of this shape then h n of t the unit impulse response we are talking of all normalized values this should obviously be a bell shaped curve isn't it maximum slope would be reached somewhere here then the slope goes to zero here monotonically the slope is zero here also

so i i have not shown it correctly it is like this it starts in the zero slope so you get an impulse response like this agreed

what is the area under the curve integral zero to infinity h n t dt one why because this is v two n infinity minus v two n zero which is equal to one is that clear

so the area under the curve is one all right now how do you find the fifty percent rise time

obviously fifty percent rise time would be zero to t fifty let’s say h n t dt should be equal to how much

come on this is not a difficult question half that’s right it is equal to point five

so what is t fifty then

t fifty is obviously a value of time such that the area on the left hand side and the area on the right hand side is equal isn’t that right

the fifty percent delay time therefore divides the area under the normalized unit impulse response into two equal halves and this was the point at which Elmore said oh i can give now a very beautiful analytical definition

[Conversation between Student and Professor – Not audible ((38:11:07 min))]

Student: sir this need not be at the peak

no it need not be at a peak because this goes up to infinity it may be slightly further away but it it gives a clue to Elmore’s definition

this is this is what inspired Elmore to define uh [Noise] delay time instead of fifty percent delay time as the centroid of this curve

[Conversation between Student and Professor – Not audible ((38:32:04 min))]

oh because this is equal to v two n t fifty minus v two and zero and this is equal to point five

so the area under the curve is half all right

so Elmore said instead of taking the fifty percent which is a uh rule of thumb why don’t we take the centroid of this bell shaped curve in other words he defines tau d as equal to integral zero to infinity how do you define the centroid

the first moment that is t h n t dt divided by normalized this is respect to h n t dt this is the definition of Elmore

do you understand the logic of the motivation

logic is t fifty divides into two equal halves instead of doing that we take the centroid

Student: sir zero to infinity

pardon me zero to infinity we take the centroid of the curve as the time delay okay but you know that this is equal to one and therefore tau d is simply equal to zero to infinity t h n t dt agreed this is the definition of Elmore and then Elmore found once he obtained this he found that tau d is very simply related to the frequency domain this is a time domain definition you have to find out the normalized unit impulse response multiplied by t take the area under the curve okay from zero to infinity

now he showed that this simply related to the transfer function okay

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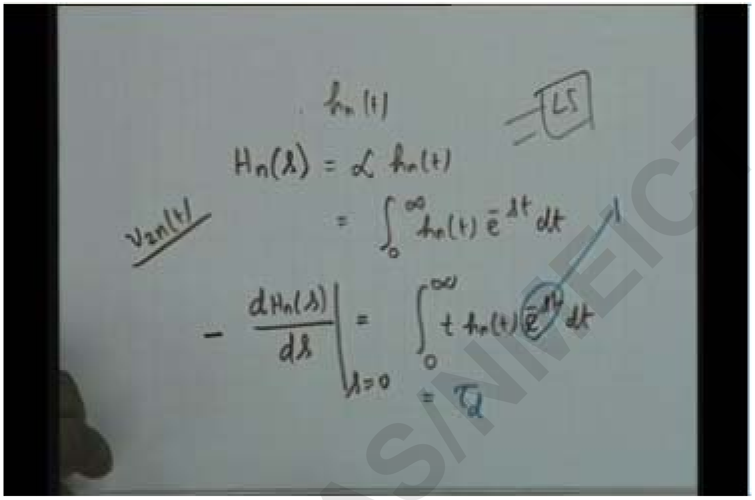

now you know that h n t if h n t is the normalized unit impulse response then the normalized transfer function would be simply Laplace transform of h n of t is that okay Laplace transform h n of t it is equal to zero to infinity h n of t e to the minus st dt agree

[Conversation between Student and Professor – Not audible ((40:54:00 min))]

Student: v to n and impulse response

oh v two n t is the normalized unit step response and therefore its differential coefficient unit will be normalized due to impulse response because ((phase)) ((00:41:13 min)) and impulse are related by one is the differentiation of the other and since this is the linear system if the input is differentiated the output will also be differentiated as simple as that

all right now this is a million dollar equation because if you differentiate h n s with respect to s not t this integral is with respect to t so you can differentiate inside the integral because s as far as the integration is concerned is a constant but you can differentiate and you can see that if you take a negative sign here then it is simply zero to infinity t h n t e to the minus st dt okay and then you put s is equal to zero you put s equal to zero so this term into the minus st this goes to one and this becomes exactly equal to tau d

isn’t that right so given the transfer function you don’t have to go back to the time domain to find tau d you can find it in the frequency domain itself all that you do is uh differentiate the transfer function with respect to s and put s equal to zero

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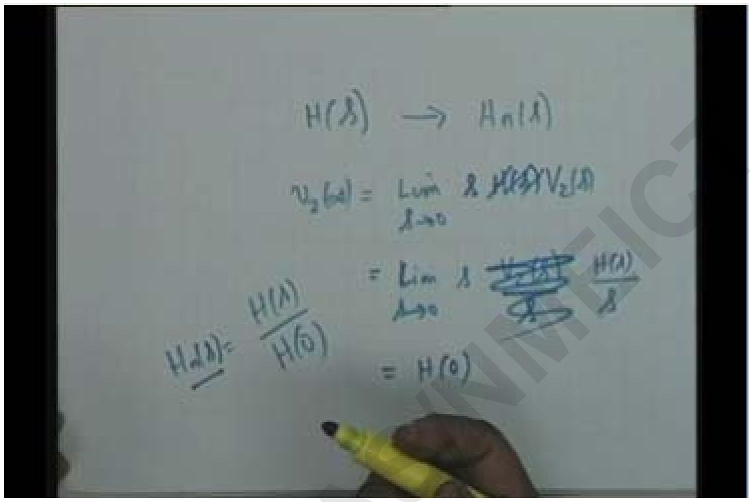

now a question arises if the transfer function h of s is given actual transfer function how do you find the h n of s how do you find the normalized transfer function

[Conversation between Student and Professor – Not audible ((42:50:00 min))]

maximum value no how do you know the maximum value

final value theorem

what is the final value theorem

v two infinity is equal to limit s tend to zero s h of s correct

now what is h of s what is h of s

you remember what is was v two of s it is unit step respond so if we divide by s that would be h of s okay

so if you multiply this you get v two infinity that is the final value theorem and don’t you see this is simply equal to [Noise]

pardon me

no i have made a mistake here limit s times v two s i made a mistake here and this is limit s tend to zero

what is v two s

v two s is obviously h of s by s and therefore it is equal to h of zero all right

all that we have done so far is to normalize with respect to v two infinity v two infinity is a constant and this is exactly equal to h of zero and therefore our h n of s would be equal to h of s divided by h of zero that’s it extremely simple you don’t have to do the Laplace inversion you don’t have to go to the time domain given the transfer function you divide by its DC value that is the normalized transfer function and then you differentiate this with respect to s and put s equal to zero

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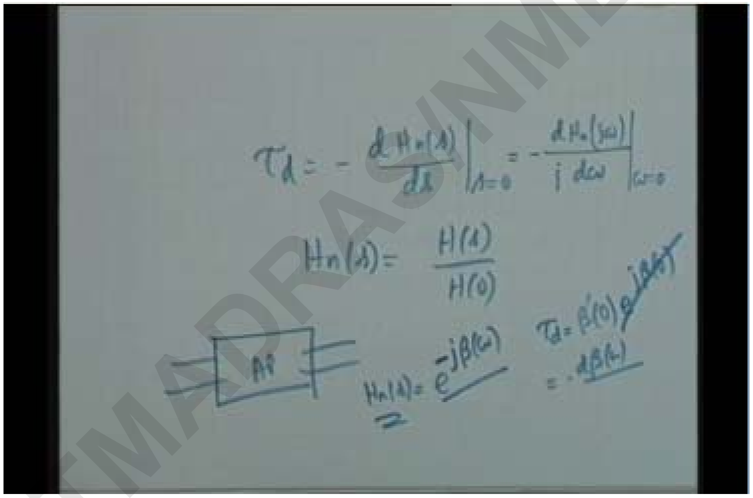

let me write it again tau d is equal to minus d h n s divided by DS s equal to zero h n s is equal to h of s divided by h of zero all right

now isn’t it clear that you can also write this as minus d h n instead of s if you put j omega then d g omega

so i can write this as j d omega j is a constant at omega equal to zero in some cases this is more appropriate to use than this function all right

for example if you have let’s say an all pass network you know in an all pass network the transfer function magnitude is a constant and the phase varies with frequency okay if it is all pass

then what would be the normalized transfer function h n of s what would be the form of this yeah

[Conversation between Student and Professor – Not audible ((45:59:04 min))]

e to the power j beta omega the magnitude would be one because we have normalized

so this would be the form and you can show by applying this definition that tau d can be written as [Noise] this algebra i leave out tau d can be written as beta prime zero e to the power j beta zero have patience for one more minute then a very interesting result comes out

what is beta zero what is the DC value of beta pardon me zero why

your answer is correct but why because beta omega phase is an odd function and therefore for omega it must be zero therefore this is one which means that tau d is d beta omega

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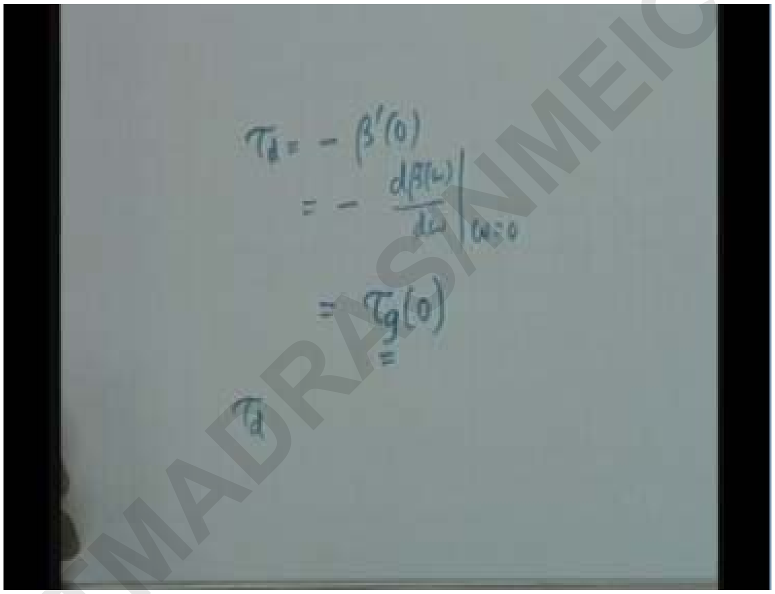

i made a mistake can you tell me what the mistake is i should would have taken a negative sign here yes or no e to the power j beta no no no hm wait a second perfectly all right

there is a minus sign here because of this minus sign okay because of this minus sign there is a minus sign here

so this is minus d beta omega d omega at omega equal to zero all right let me write it again

tau z d comes with minus beta prime zero which means minus d beta omega d omega at omega equal to zero what is this quantity

[Conversation between Student and Professor – Not audible ((47:59:09 min))]

no j will cancel

this is the group delay and therefore this is tau g zero all right and this is another strong motivation for Elmore using the Elmore's definition

the Elmore's definition has a physical significance this is the group delay at DC

it is the group delay at DC

so tau d has several interpretations it it minus d h n s ds at s equal to zero tau d is the centroid of the unit impulse response curve tau d is also equal to the group delay at DC and this is where we shall start from in the last lecture