Circuit Theory
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Lecture - 5
Step, Impulse and Complete Responses

We discuss today: step response, impulse response and complete response. You have been acquainted with these terms in the signals and systems. In the context of circuit theory, we shall illustrate this. Since you know the general principles, we shall illustrate this with the help of a few examples, a series of examples.

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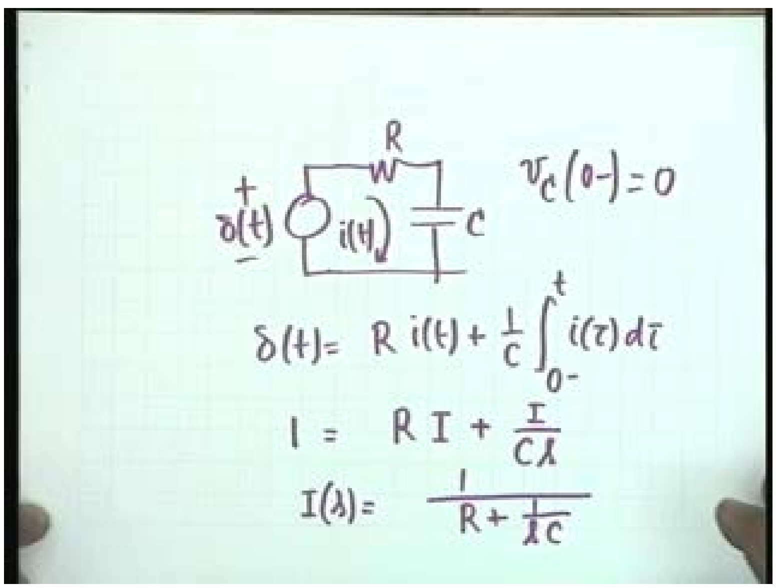

First we take the well known R C network and we assume that V sub C 0 minus is equal to 0, that is, the capacitor is initially relaxed and we connect an impulse voltage source delta t here and we wish to find out the current i of t. As you know, the only 1 mesh equation delta t would be equal to R i of t plus 1 by C integral 0 minus to t i tau d tau and the simplest way of doing this is to take the Laplace transform of both sides and you get 1 equal to R capital I, capital I is the Laplace of small i, plus I by C s and therefore, capital I of s is equal to 1 divided by R plus 1 over s C.

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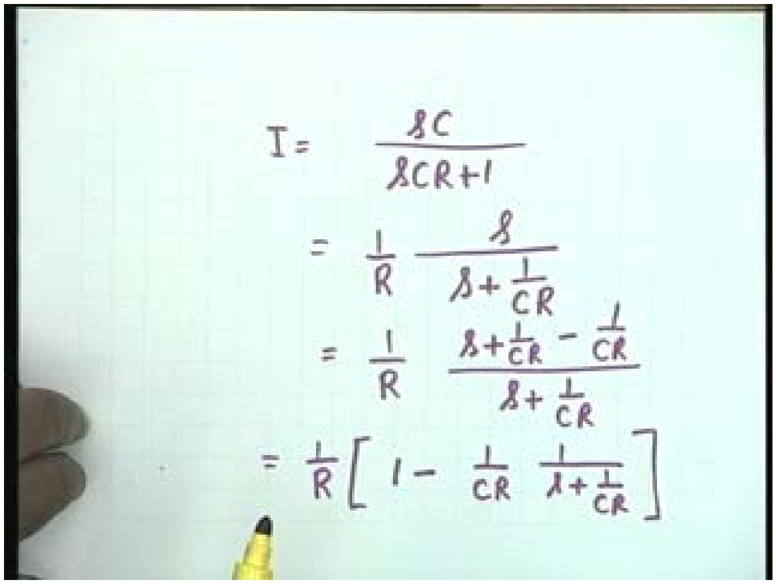

Or capital I is equal to s C divided by s C R plus 1, which I can write as, I always write the denominator as s plus something. Coefficient of s is made unity, so we write s plus 1 over C R and then we shall have s and here we shall have 1 by R. Have I made any mistake? C and C cancels, and this I can write as 1 over R s plus 1 over C R minus 1 over C R divided by s plus 1 over C R.

I am just making some simple algebraic manipulation, so that I can write this as 1 over R 1 minus 1 over C R, 1 over s plus 1 by C R, agreed. Now the inversion of this is extremely simple. The inverse of 1 is delta t and the inverse of 1 by s plus alpha is e to the minus alpha t times u of t u of t, of course, has to be there and therefore my i of t becomes equals to 1 by R delta t minus 1 over C R e to the minus t divided by C R. The whole thing has to be multiplied by u of t.

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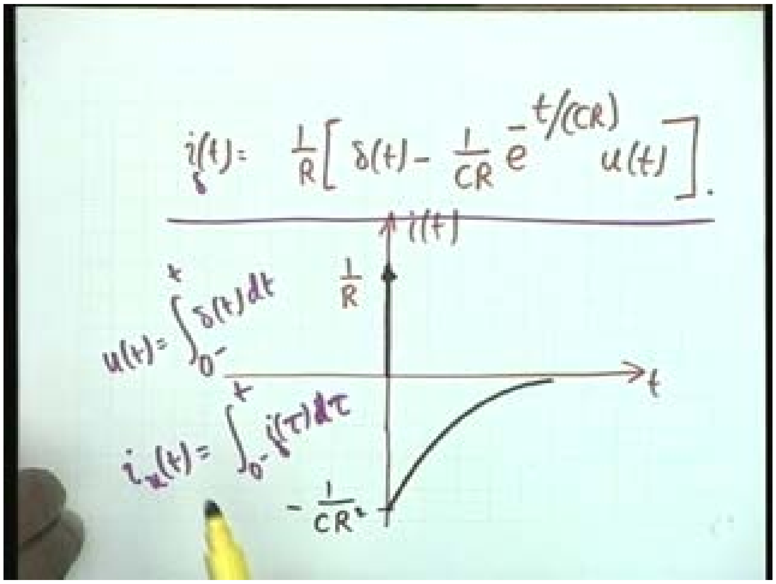

This is what the current response is and you notice that it consists of 2 terms. There is an impulse function, there is an impulse function in the term, if I plot this, then I shall have an impulse here and an impulse is represented by an arrow with the value of the, with the strength of the impulse written by its side. The strength of the impulse is 1 by R and the second term, the second term, obviously, is negative to start with. t equal to 0, it is 1 by C R squared this R also comes into picture C R square and then it goes down exponentially. So this level is minus 1 by C R square. This is the plot of i of t verses t.

The question now is, the impulse is effective only at t equal to 0 because it exists at t equal to 0 only t equal to 0 plus it is 0 t equal to 0 minus it is 0, it is exactly at t equal to 0. Now the question, the next question that arises is, suppose, we wanted to find out this unit step response. Then do we have to proceed (…) or does this have a, does this give a clue to the solution? You see, unit step u of t by definition is 0 minus to t delta t d t. Unit step is the integral of the unit impulse and therefore, because it is a linear system, the unit step response, that is, i u t, if I call this as i delta t, that is, the current response due to a delta function excitation, then the unit step response excitation would be simply integral 0 minus to t i delta tau d tau.

If the excitation is integrated, the response should also be integrated. This is the basic principle of a, say, linear system, and therefore, what I can do is, I can integrate this and get the unit step response.

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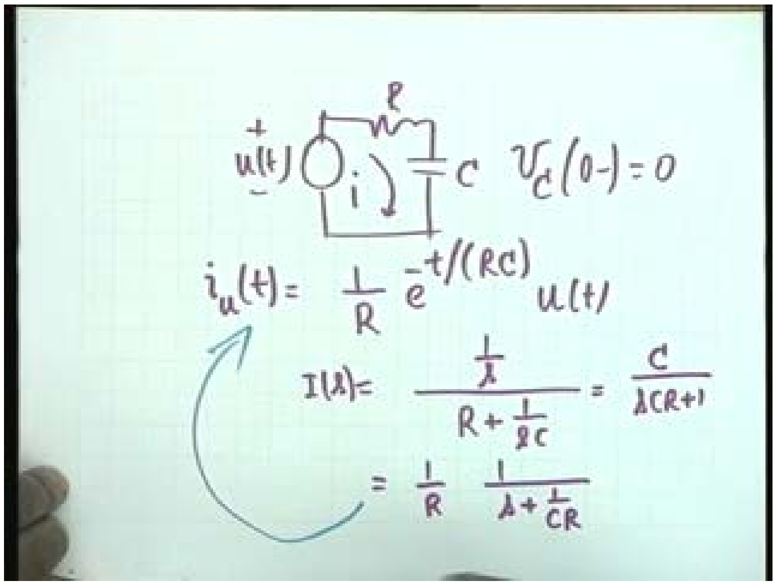

That is, if the excitation was u of t, R and C with V sub C 0 minus equal to 0, then the response, current response i u of t shall be equal to the integral of that and I have carried out the integration. It turns out to be a very simple thing. 1 by R e to the minus t by RC multiplied by u of t, this is what it turns out to or else, if you do not like to integrate, well, this integration involves cancellation of terms and so on. You might like to do it directly and directly means that capital I of s now shall be 1 by s, that is, the Laplace of u of t and divided by the impedance R plus 1 over s C and therefore, this simply becomes C divided by s C R plus 1, which is equal to 1 over R, 1 by s plus 1 over C R.

So this is no more complicated than finding the response to an impulse function. Then all that you have to do now is to invert this and, obviously, the inverse would be the same as this 1 by R then 1 by s plus C 1 by C R is e to the minus t by R C times u of t. So either way, whatever is convenient for you, it is either, you find the impulse response, then you integrate it to get the step response or you proceed directly in the frequency domain, the Laplace transform domain

Student: (…)

Sir: 0 minus to t, that is right. On the other hand, if you know the step response, then you can find the impulse response by simply differentiating. If you know the step response, then you can find the ramp response. A ramp is linearly raising function with an angle of 45 degree which is exactly the integral of u of t and therefore, if you know the step response, if you know i u of t, i r of t, that is, the response to unit ramp would be simply integral of i u of tau d tau. These are the, 0 minus to t. These are the simplifying relationship that holds only for linear systems, if the system is non linear such are done away with. By definition, non linear is also a time consuming affair. Solving any non linear network is a time consuming affair.

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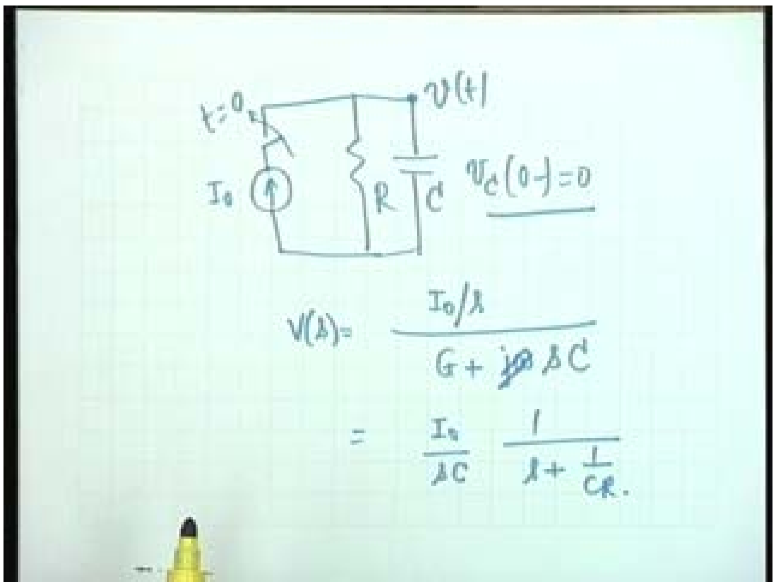

Our second example is to make a, to bring variety into life. We now consider a step response excitation, that is, this switch is closed at t equal to 0. A current generated and instead of a single circuit, we take a parallel circuit. The same R and C elements, same R and C elements are, they are V sub C 0 minus equal to 0 and this voltage we treat as V. It is our intention to find V of t, that is the second type of response and you see, I can either write the differential equation, solve it, or I can go directly into Laplace domain because the initial condition is 0. I do not have to write the differential equation. But mind you, if the initial condition is not equal to 0, then you must write the differential equation and take Laplace transform turn by turn, because initial condition turn, imposes a driving function, equivalent of a driving function.

So here, to bring variety, once again I can write V of s as equal to the Laplace of the current which is i 0 divided by s, because it is a unit step, not unit step, a step of strength i 0 divided by multiplied by the impedance, which is equal to G divided by the admittance G plus j omega, no s C, Laplace variable, not the sinusoidal, not a phaser, g plus s C and this I can write as i 0 divided by s and I can take C common. Then I get s plus G by C which I can write as 1 over C R. Have I done this correctly?

Students: Yes sir.

Sir: Okay.

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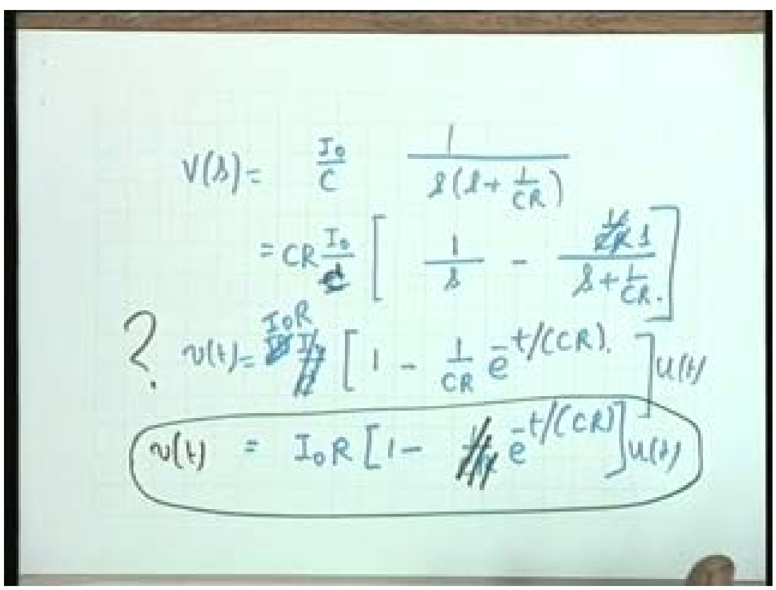

So in order to invert it, I write it like this, V of s is equal to i 0 by C, then 1 over s times s plus 1 over C R and the partial fraction expansion is extremely simple. I write simply as I 0 by C. I can write this as s minus S plus 1 over C R, then I can have 1 and 1 by C R. Such things will come from experience. I do not have to do the formal partial fraction. Is that okay? s plus 1 by C R and

C R cancels. So, and therefore, V of t would be equal to i 0 by C, 1 by s is u of t. u of t, I will take out minus 1 over C R e to the minus t divided by C R.

Student: Sir, partial fraction is not correct.

Sir: partial fraction is not correct?

Student: (…)

Sir: This is also 1.

Student: (…)

Sir: here? How does this? Wait a second, s plus 1, I think this is correct. This is correct. Do correct me if I make a mistake. So this will be i 0 times R.

Student: Sir C is also there, I naught plus C is there.

Sir: No, oh yes I 0 by C is there and therefore, C and C cancels. I get simply i 0 R. Let me write this correctly, i 0 R. Now tell me if you have an objection. Is that okay? I hope so. Now you see that it contains 2 terms 1 is a constant and the other is, other is an exponential term and you can notice, you can see that this, the t equal to 0 the value is 0. Well is it after you get a solution to a network by whatever means is possible. As I said, there is no replacement of common sense and <a_side> (( )) (14:12) <a_side> 1 by C R is not there, anything else?

Student: Second step sir, it should be 1 by C R i 0 by C R

Sir: i 0 by C R, no, i 0 by C i 0 by C and then C R, I have taken out and therefore, C and C <a_side> (( )) (14:38) <a_side> okay you do correct, that is, this result.

Students: Yes

Sir: Is this result okay? This result is okay. Which stage are you objecting to this one?

Student: Third one.

Sir: Third one, okay, you correct it. Let me leave something for you too. Now you notice that t equal to 0, now I want to bring physical concept. As I said, there is no replacement for physical concepts. No mathematics, no amount of brain washing can replace physical concept.
Physical concept is that V t is 0 at t equal to 0. Now, is this, does this stand to logic?

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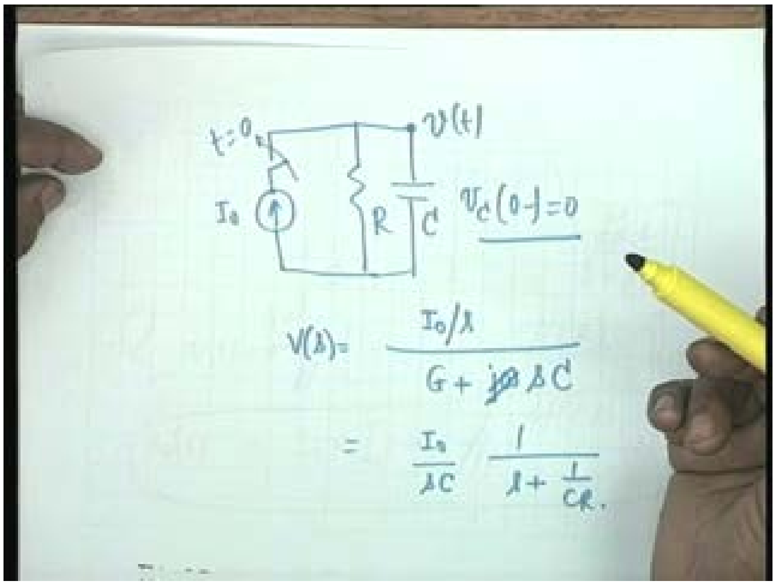

Look at the circuit, V C 0 minus is 0 and V C 0 minus is V of 0 minus. The capacitor cannot change its voltage instantaneously and therefore, V of 0 plus is also equal to 0. This stands to result. Now if this current stands for a long time, then the capacitor will be fully charged and will not allow a current to pass through it. So the total current i 0, shall pass through R and therefore, at infinity, V of infinity should be simply equal to i 0 multiplied by R. Now is this borne out by mathematics, yes, this terms goes to 0 as t tends to infinity and therefore a t equal to infinity V of infinity would be simply equal to i 0 by R. So the result is correct.

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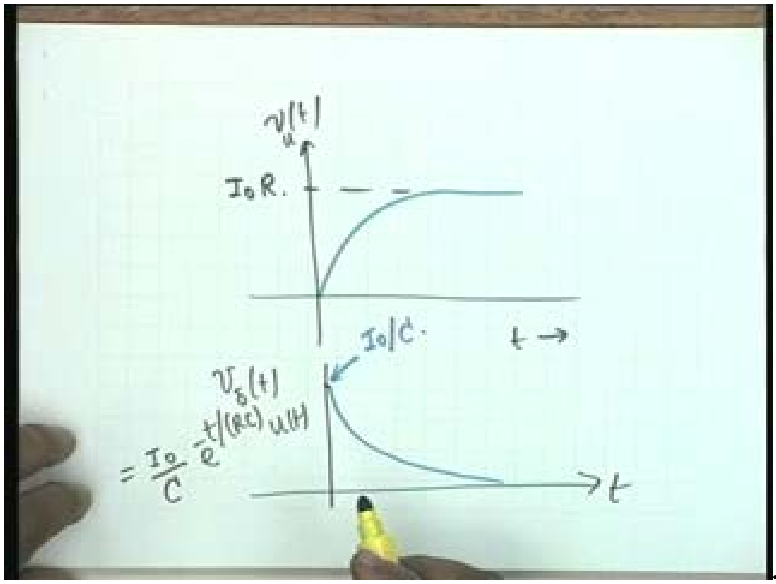

If I make a plot of V of t verses t, then the plot would be like this, it starts from 0 and goes to i 0 R at t tends to infinity. This is i 0 R. Now this is my view of t in response to a unit step current. Suppose I want V delta t, then I can differentiate this. I can differentiate V u t and find V delta t and you will notice that if I do this, well, I can write the expression V delta t, if you carry out this differentiation.

As I said, you can either differentiate or go back to the Laplace domain, if you so desire. If you do that, then it is simply V delta t is i 0 by C, multiplied by e to the minus t divided by R C times u of t, that is, it starts from i 0 by C and falls exponentially, like this. This is the unit impulse response. You can do this by differentiation or you can go back to the Laplace domain and carry out this step.

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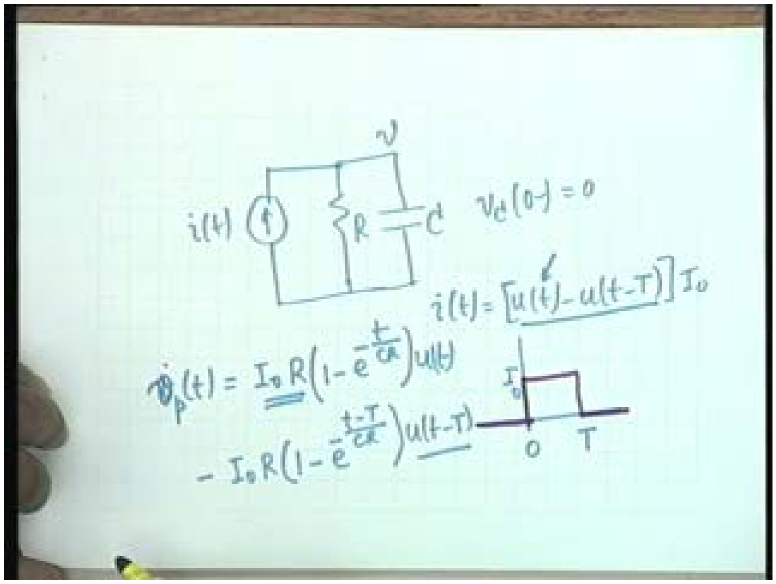

The third example that we take; we will illustrate the principle of super position, that is, I consider the same current generator. Let us say, i of t same current generator and the same resistance and capacitance, same resistance and capacitance with V sub C 0 minus equal to 0. But I now consider i of t as a pulse, rather than an impulse or unit step. It is u of t minus u of t minus capital T and as you know, the plot of this is like this; it is a pulse, it is a rectangular pulse of unit height and duration 0 to capital T.

u of t is a unit step from which is subtracted another unit step at t equal to small t equal to cap T and therefore, this is the pulse. We want to find out the response to this. Now, obviously, because the excitation is a superposition of 2 unit steps, the response, I will call this, i p of t the response to a pulse. The response of the super position of 2 responses, that is, we go back to the unit step response of this, which was equal to i 0 R 1 minus e to the minus t by C R multiplied by u t. This is the response to u of t.

Student: V of t, sir.

Sir: I am sorry. This is V p. This is the voltage response V, V p of t, this is the response to the first term, that is, i u of t. This is multiplied by i 0. The current is, a pulse, a unit pulse, multiplied by i 0. Yeah, amplitude would be i 0. Just to be general, it could be unity also, it does not matter. i 0 R multiplied by 1 minus e to the minus t by C R u t and then the response to the second term. Well, would be minus i 0 R 1 minus e to the minus, instead of small t, this is what time invariance brings in, that if you delay the input, the output is simply delayed by the same amount, nothing else changes. So minus t minus cap T divided by C R multiplied by, instead of u t, it would be u of t minus capital T now.

Why is this? How come you multiply this by u of t minus capital T? That is because of causality, that is, the system is non-anticipatory. It cannot respond to u of t minus T, before u of t minus T occurs in this scene. The previous response is due to u of t, so linearity, super position, time invariance and causality, all of them come into the picture and this is why this example is a very interesting example. You cannot combine these 2 terms for small t, between 0 and capital T. Only the first term dominates. It is only at capital T that the second term shows its step.

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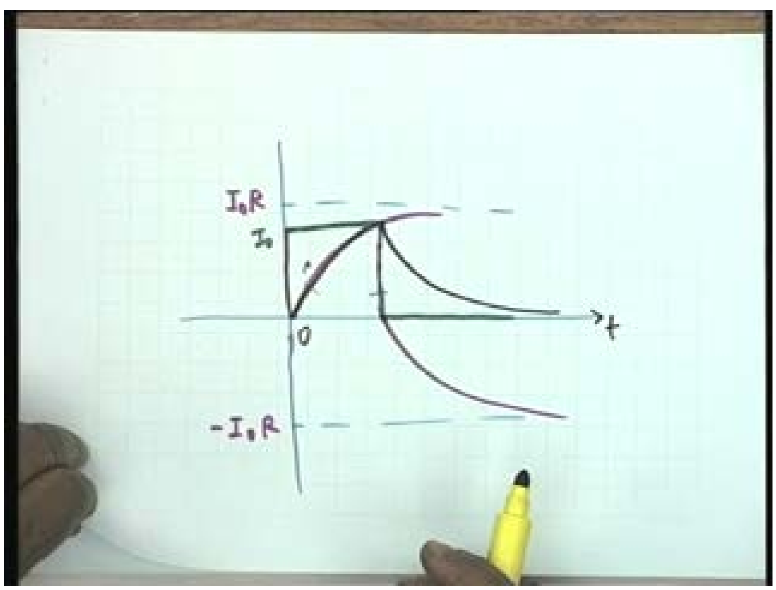

The response can be the total response, can be drawn like this, as far as the first term is concerned, that is, the response to u of t while it goes like this, the second term starts at t equal to small t equal to capital T and then it is negative and therefore, it goes like this and if it allowed to go to infinity, then it would have reached, what level? This would have been minus i 0 R and this is plus i 0 R.

So it is the combination of these 2. It is the combination of this, plus this. Between 0 and between 0 and capital T, it is only this term that dominates. And beyond this, the negative the cancellation starts its step and therefore, as t goes beyond capital T, it must come down like this. Ultimately, at small t equal to infinity, the value would be 0, because this will be reaching i 0 R, this will be reaching minus i 0 R. So this is the total response to a pulse. The excitation was like this, excitation was i 0, this was the excitation and to the excitation as long as the excitation is there, you notice this that the voltage is rising, the voltage rises exponentially.

As soon as the excitation is taken off, now comes to physical concept. What happens? Excitation is taken off. Now comes the physical concept. The capacitor has charged to i 0 R, not i 0 R, less than this value and then it discharges. The excitation is drawn away, so the capacitor charge gradually discharges and this is why, this exponential decay. Ultimately, when we allow sufficient time, the capacitor discharges completely and the voltage across it becomes 0. You must not divorce physical concept, because circuit theory is a mathematical subject. In fact, in no mathematical subject related to engineering, you should lose sight of what is happening, physically. That is your best weapon. The best weapon of an engineer is physical insight.

Student: (…)

Sir: Open, open circuit, because that is where the switch and a current generator removed, means it is replaced by the infinite. A voltage generator removed means, it is removed by short circuit. Now we have talked about step response, impulse response, we have also talked about sinusoidal response. What about complete response?

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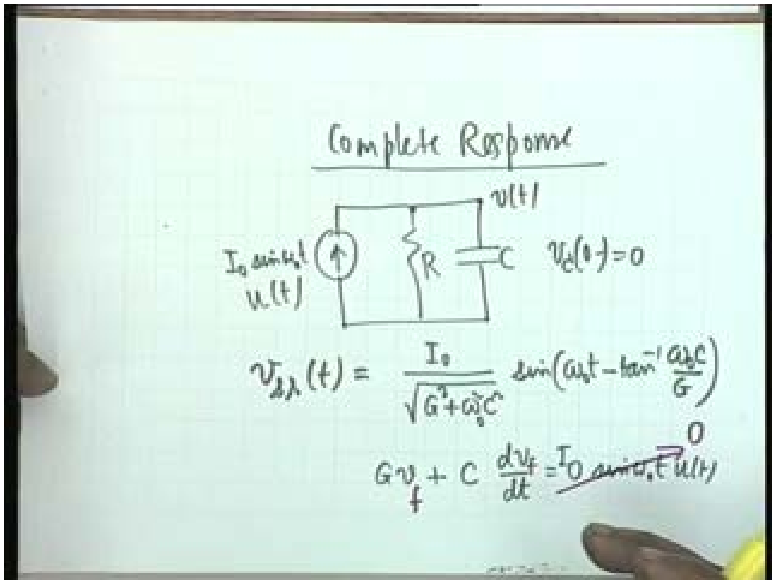

Now, complete response, as you know, a complete response of a network consists of 2 parts: the free response and the forced response or what we call in terms of differential equation terminology, the complementary function and what is the other part?

Student: Particular integral.

Sir: Particular integral. So we will take 2 examples to illustrate how we derive complete response and both of them would be interesting. Suppose I have an i 0 sine of omega 0 t u t. Now we are not finding the steady state response. We had found earlier, we want to find out the complete response now and this is applied across a combination of resistance and the capacitance with V sub C 0 minus equal to 0. We have already found out the steady state response for v of t. We have already found that out, the steady state response V s s steady state of t, if you remember, it is i 0 divided by square root of G squared plus omega square C square. This is done in the last class, in the third lecture, multiplied by sine of omega 0 t minus tan inverse s omega 0 C divided by G. This is the steady state response or the forced response.

To find out the complementary function, you have to write the differential equation and put the right hand side equal to 0. Obviously, the differential equation is G V plus C d v d t equal to 0, equal to the differential equation is i 0 sine of omega 0 t u t and to find out the complementary function or the free response, we have to put, let me use a colour. If you want to find out the free response, let us call this as v f. Then, the right hand side is to be put equal to 0. The free response means, the complementary function and you know the solution to such an equation, that is, G V plus C d v f d t.

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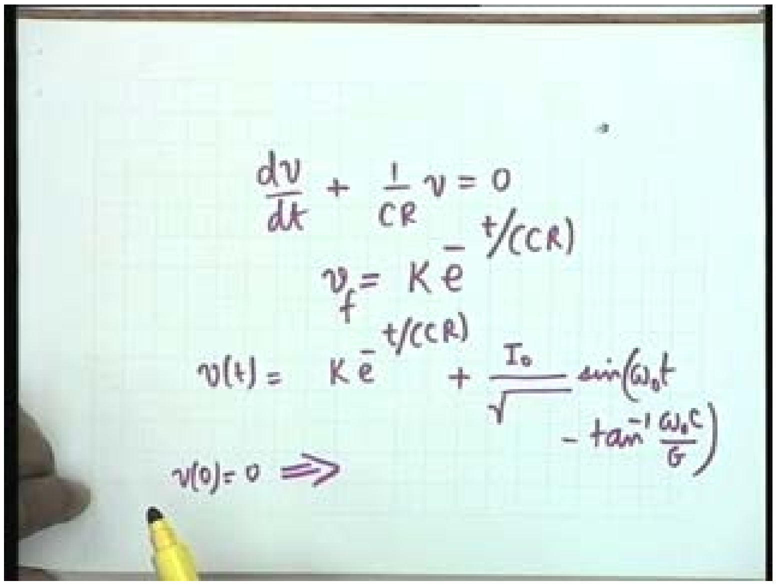

You can write this as d v d t. Let us ignore the subscript f for a moment, d v d t plus G by C, which is, 1 by C R V equal to 0. The solution to this equation is of the form v equal to K e to the minus t divided by C R. The solution to this equation is of this form. Now the question is how to find out K? Let us put f again a free response. How to find out k? You cannot find k by taking, this is a very important point, by taking this equation by itself and putting the initial condition. No, initial condition has to be put on the complete response.

You see, if you put initial condition equal to 0, obviously, k would be equal to 0, is not that right, whereas you know that, when a transient, when a sinusoidal voltage source is applied across an R C circuit, there shall be a transient. So what you have to do is, to find the complete solution. First, the complete solution, that is, the complete solution would be the sum of the 2. So it would be K e to the minus t by R C plus i 0 divided by square root of, I will not write the complete thing, sine of omega 0 t minus tan inverse of omega 0 C divided by G.

You must remember this, to put initial conditions, to find the complete response, initial conditions have to be found on the total solution, not part of the solution. This is where you have to put the initial condition that is, you put V 0 equal to 0, then find K

Student: Sir, V f here is the free response?

Sir: It is the free response, yes. If there was no excitation, if i 0, if there are no current source and the capacitor was charged at 0 minus, then this would have been the total solution. But the total solution here is that there is an excitation and therefore, you have to write the complete solution, then find the initial condition. If i do that v 0 equal to 0 then you get the following equation.

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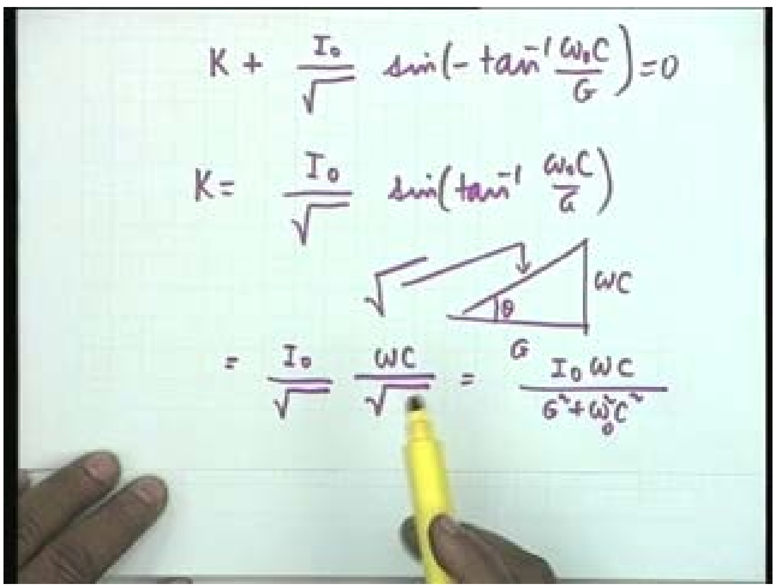

K plus i 0 divided by square root of G square plus omega square C square, then sine of minus tan inverse omega 0 C divided by G. Now, if you want to find that, this would be equal to 0 therefore, k would be equal to i 0 divided by square root of this sine r of tan inverse omega 0 C divided by G and finding this is not a problem. This is omega C and this is G, if theta is the angle, then the perpendicular must be omega C and the base must be G and therefore, the hypotenuse is square root of G square plus omega 0 square C square, agreed, and therefore, sine is perpendicular divided by hypotenuse and therefore, K would be equal to i 0 divided by square root of this, then omega C divided by square root of this.

Which means that it would be equal to i 0 omega C divided by G squared plus omega 0 squared C squared. Am I going too fast? A bit? Okay, tell me if there is a problem here. Is this okay? Finding the tangent, the tangent is given, you have to find the sine. Now I combine, now I substitute back this in the complete solution.

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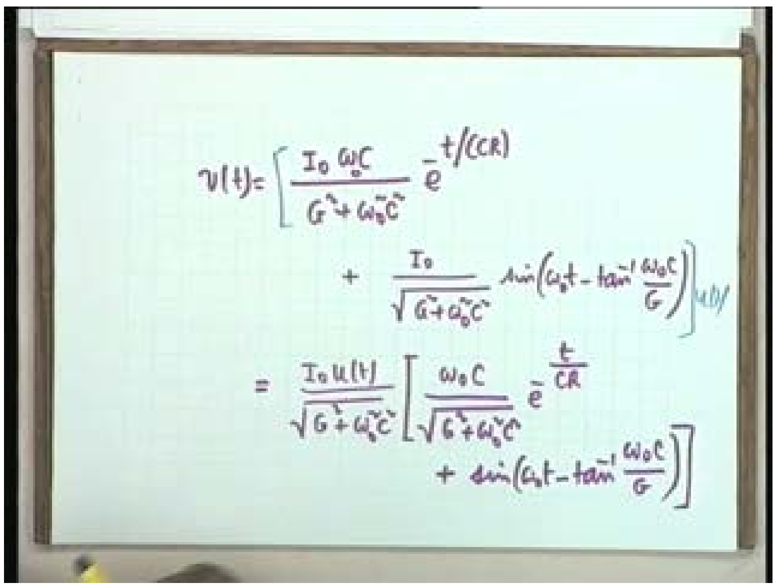

The result becomes V of t is equal to i 0 omega C divided by G squared plus omega 0 squared C squared e to the minus t by C R plus i 0 divided by square root of G squared plus omega 0 squared C squared sine of omega 0 t minus tan inverse omega 0 C divided by G. This is the total solution. It can be written in a slightly nicer form, but before that, do you agree that this is the correct solution?

Student: Yes sir.

Sir: We have made a mistake. Do not trust me always, I sometimes, intensely make mistakes

Student: Sir u t.

Sir: That must be there, the whole thing is to be multiplied by u t. Now, it can be written in a slightly nicer form like this; i 0 u of t divided by square root of G squared plus omega 0 squared C squared. If I take this common, then I get omega 0 C divided by square root of G squared plus omega 0 squared C squared e to the minus t divided by C R plus simply sine of omega 0 t minus tan inverse omega 0 C divided by G, so just a different form to combine the terms together. Then we take next example, so far we have dealt with only first order network, that is, 1 capacitance, 1 resistance or in one of the lectures, we took 1 inductance and 1 resistance.

Let us see what happens when both the energy storage elements are taken into consideration, that is, we shall get then, a second order equation. A second order equation is one in which the highest order of differential coefficient is 2. This happens when an inductor and capacitor both are present in a circuit. Inductor, the voltage is d i d t, whereas, a capacitor voltage is integral I t d t and therefore, if I differentiate once more, I shall get second differential coefficient of i t and the example that we shall take is one which you shall come across again and again.

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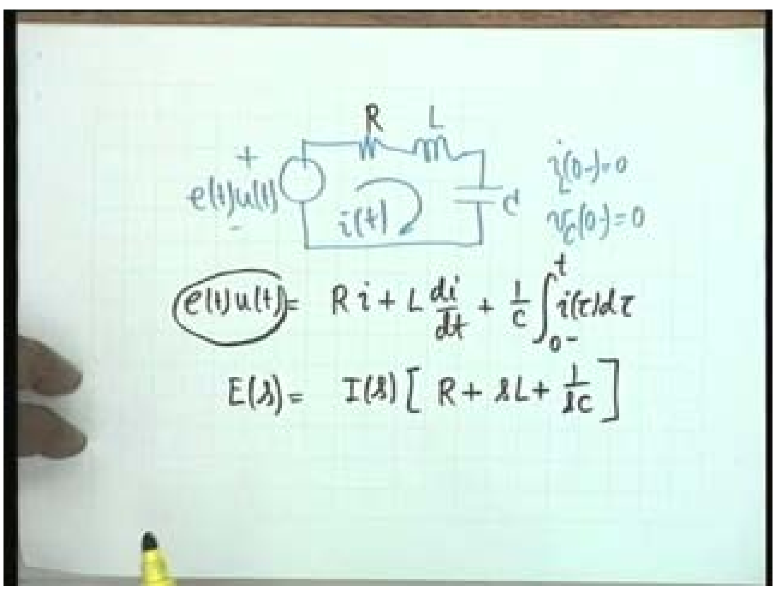

That is, we consider a voltage source e of t u of t connected to a series combination of resistance inductance and capacitance and the problem is to find out the current i of t. We have, you have done this exercise in w 1 1 0. We will do it slightly differently. We will take Laplace transform and then try to solve it. In the process, we will bring certain terms into focus. We assume that i L of 0 minus equal to 0 and V sub C of 0 minus also equal to 0. Then our differential equation becomes e t u t, differential equation becomes e t u t R i plus L d i d t. You see, i L of 0 minus is of no concern, but 1 over C integral i tau d tau, here if VC 0 minus was not equal to 0, we should have written it. We would have to write it specifically, 0 minus to t and the simplest thing that one can do is to take the Laplace transform, that is, you get E of s equal to I of s multiplied by R plus s L plus 1 over s C.

Student: (…)

Sir: No, we are making E of s represent, we do not know what is e of t. There is, I think this requires a bit of consideration.

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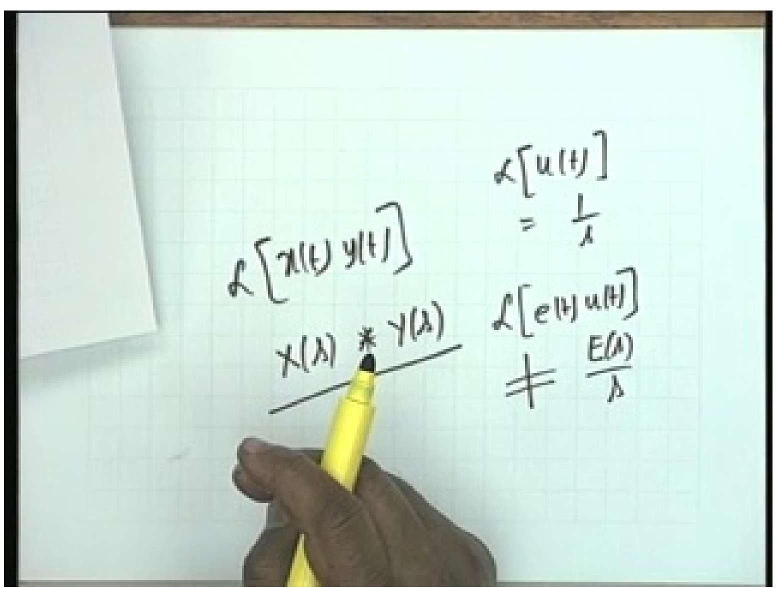

You see, Laplace of u of t is equal to 1 over s Laplace of a general function. e of t u of t is not equal to e of s by s, no, it is not. There is a problem if e of t is a constant S capital E by s but not otherwise. How do you find out the transform of this? Suppose you have x of t multiplied by u of t, you want to find the Laplace of this,

Student: Complex convolution.

Sir: Complex convolution, that is, you do x of s star capital Y of s, not the ordinary convolution and there is factor of 1 by 2 pi.

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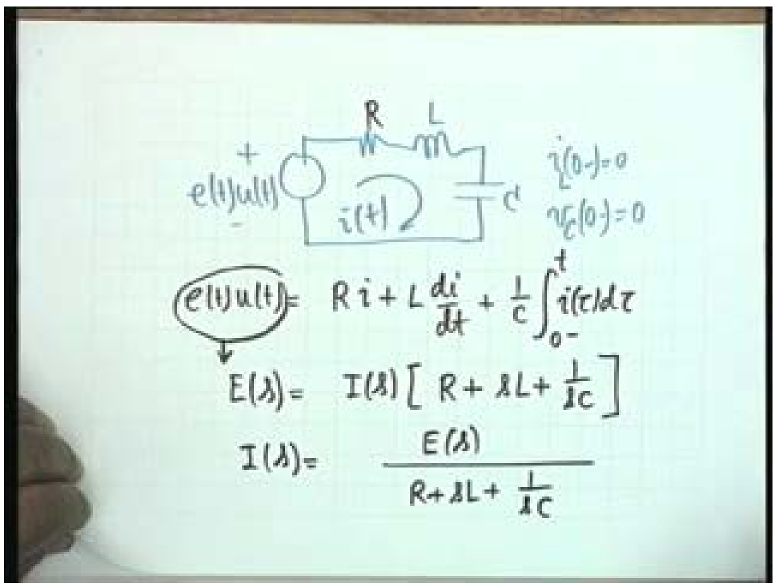

So please do not make this mistake. I have said e of s is the Laplace transform of e of t u t, capital E of s. Now therefore, capital I of s equals to E of s divided by R plus s L plus 1 over s C. I can write this, let me write it again. I of s equal to, I can simplify this as, s C E of s divided by, the denominator I want to write in the form of a polynomial, divided by s square L C plus s C R plus 1. This, I can write as s E of s. Now I want to write the denominator with the highest power term coefficient equal to unity.

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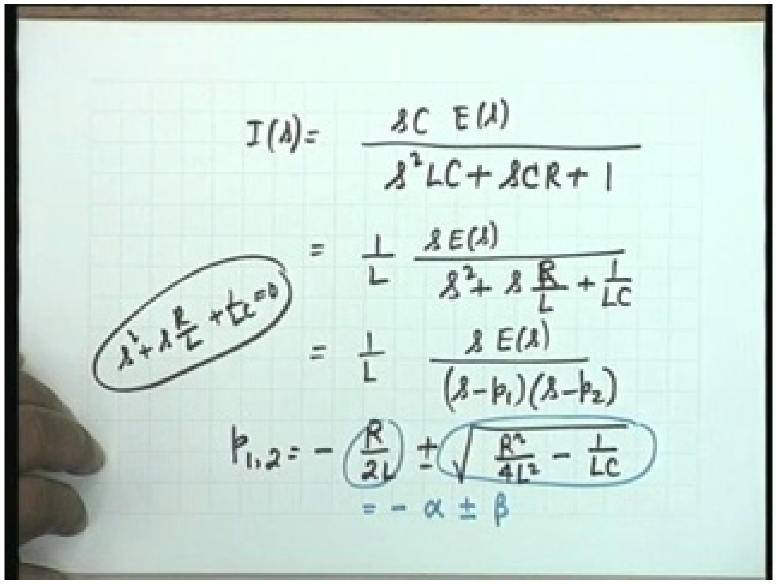

So I take L C out, then I get 1 by L, s E of s, s squared plus s, R by L plus 1 over L C. That, I can write as 1 over L s E of s divided by s minus p 1 multiplied by s minus p 2. It has 2 routes, this quadratic can be factored, will have 2 factors and the factors are s minus p 1, s minus p 2 where p 1 and p 2 satisfy the quadratic equation s square plus s R L R by L plus 1 over L C equal to 0 p 1 and p 2, are the routes of this equation, and as you know p 1 and p 2 are called the poles of the function, poles.

The values of s at which the function blows out, it becomes infinite, are called the poles. The values of s at which the function vanishes are called the zeros. How many zeros does this function have?

Student: 1? More than 1?

Sir: Depending on E of s. Suppose E of s is a constant, then how many zeroes?

Students:1.

Sir: No.

Student: if E of s is constant?

Sir: If E of s is constant, then how many zeros does this have?

Student: Infinite zeros.

Sir: That is correct. Infinity is another point at which the function vanishes. So in generally, you must remember that all functions, no matter how it looks like, have the same number of poles and zeros; the number of poles is equal to number of zeros. Here, the poles are finite at p 1 and p 2. The zeros are, one of them is finite s equal to 0 the other is it infinity. The number of poles must be equal to number of zeros. Now the values of p 1 and p 2 p 1 2 would be equal to minus R by 2 L plus minus square root of R squared by four L squared minus 1 over L C. That is very simple, if the solution is to be quadratic equation, these are the routes and we shall denote this by minus alpha plus minus beta, that is, this quantity we call beta and R by 2 L, we call alpha.

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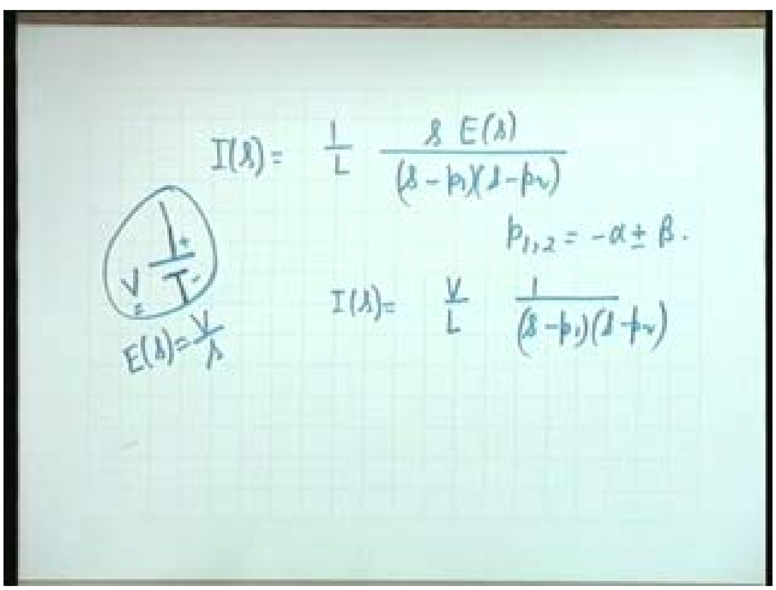

Then my solution for current is 1 by L s E of s divided by s minus p 1 s minus p 2. I have written this again, where p 1 2, we have written minus alpha plus minus beta alpha and beta have been properly defined. Alpha is R by 2 L and beta is square root of R square by four L square minus 1 by L C. Now, to be specific, let us now specialize the excitation. Let us say, the excitation E of s simply arises due to a battery V, capital V. Then what is E of s, V by s is not it? E of t u t would be capital V multiplied by u t and therefore, E of s is equal to V by s.

Student: Sir, what is this, V by t?

Sir: No, this is a battery, plus minus. This is a battery. If E of t u t is this, then E of s is this and therefore, i of s. We are now considering a special case, i of s would be equal to V by L, s will cancel and therefore, we shall have simply s minus p 1 s minus p 2. We want to find the current but before we find the current, once again, we pause to look at the physical circuit.

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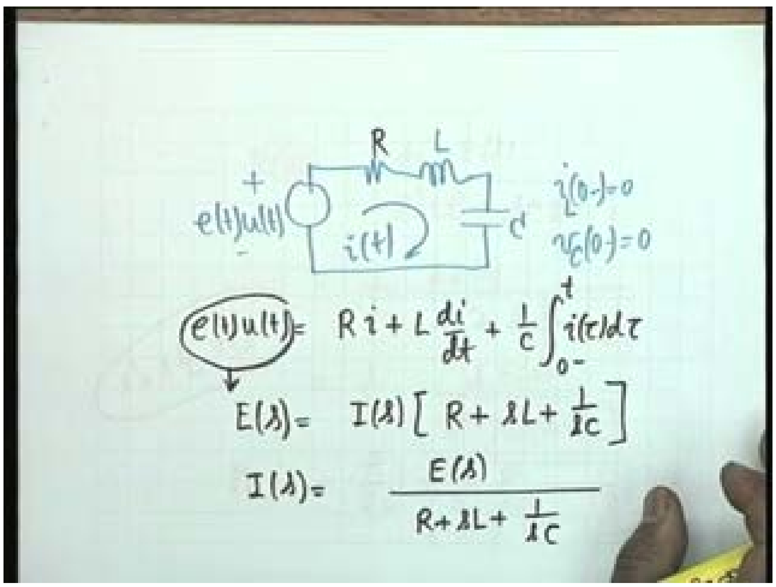

The circuit is this and this is a battery switched on at t equal to 0, the initial conditions are the initial current in the inductor is 0, initial voltage across the capacitor is 0. So what would be i of 0 plus? It must be 0. What would be i of infinity? If you look at the physical situation?

Student: 0.

Sir: 0, so it is only between the 0 and infinity, the current is allowed to play whichever way it wants to go. It can go positive, it can go negative and so on, but 0 and infinity the limits are constrained to be 0. The current must vanish.

It is like a child which is born and the old man it becomes an old man and then dies between birth and death. These 2 are constraints, every individual must be born, must be die. Here also, the current is born at t equal to 0 and then it must die at t equal to infinity. Let us see what happens in between and this in between story depends on the value of beta. You recall beta is equal to square root of R square by 4 L square 2 minus 1 by L C. Therefore, depending on the relative values of R L and C, beta can be real, can be equal to 0 or can be purely imaginary. We shall consider all the 3 cases, separately.

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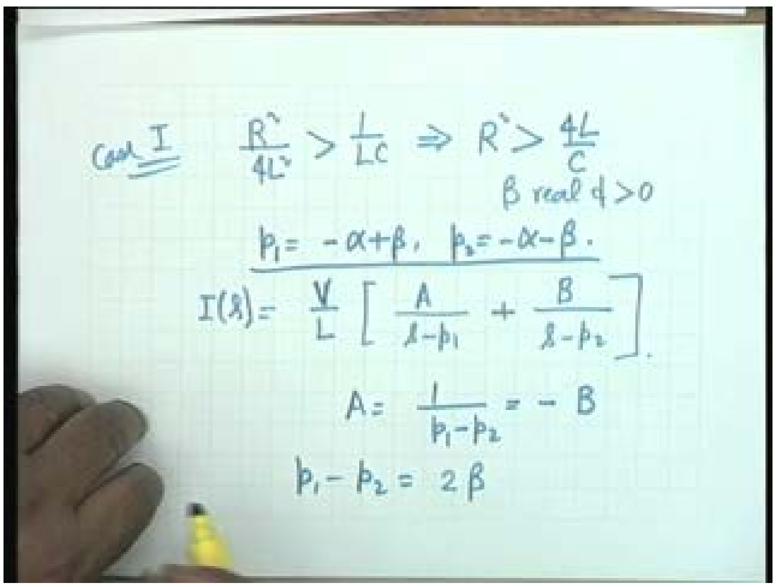

First let us say that R square case 1 R squared by 4 L square is greater than 1 by L C, which means that R square is greater than 4 L by C. That is, beta is real and greater than 0. Of course, we have taken plus minus beta. Is not it? So p 1, therefore, is equal to minus alpha plus beta and p 2 is equal to minus alpha minus beta and our equation i of s is given by 1 over L. Now it would be V by L. Then A, by now you make partial fraction expansion, s minus p 1 plus B by s minus p 2 and you can easily see that capital A would be p 1 minus p 2.

Student: Sir capital A can be p1 and B can be minus p2

Sir: No, what is capital A, capital A is 1 over p 1 minus p 2 and this would be equal to minus B, agreed. That is very simple to see. How do you find capital A? You multiply this by s minus p 1 and put s equal to p 1. That is how you get 1 by p 1 minus p 2 and B would be exactly the negative of that, but what is p 1 minus p 2? If you look at the values of p 1 and p 2, it is simply equal to twice beta, agreed? p 1 minus p 2 is twice beta and therefore, if I omit a couple of steps, I can write i of s.

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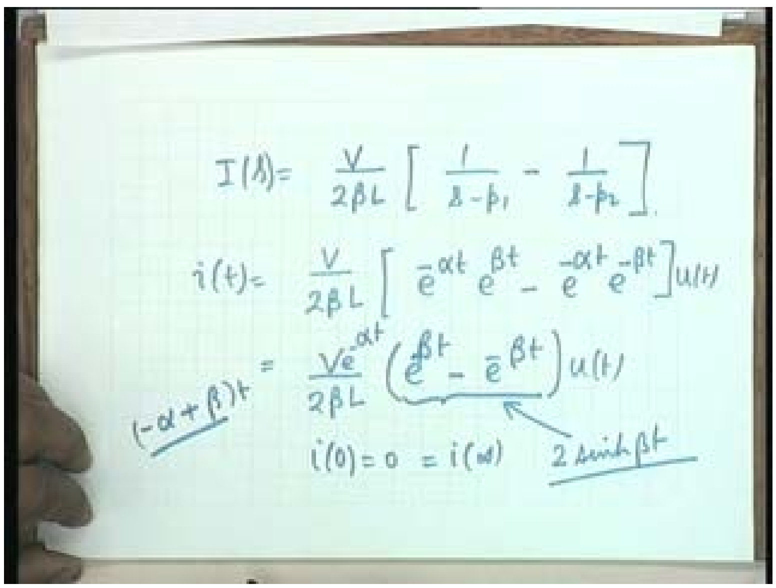

No, just 1 step I omit V by twice beta L. I take a and b out, multiplied by 1 by s minus p 1 minus 1 over s minus p 2 and therefore, i of t. If I take the Laplace inverse, it would be V by twice beta L e to the minus p 1 t, e to the plus p 1 t, yes, which would be e to the minus alpha t e to the beta t minus, agreed. This is minus alpha plus beta minus e to the again, minus alpha t but e to the minus beta t, multiplied by u t, which I can write as V by 2 beta L, e to the minus, e to the beta t minus e to the minus beta t. No, I have missed a term, u of t, I have missed the term, e to the minus alpha t. Now this, obviously, physical concepts physical considerations let us check against this mathematics at t equal to 0 this is indeed 0 at t equal to infinity this is 0. Now this term e to the minus alpha plus beta times t depends whether beta is greater than alpha or alpha is greater than beta. Here, beta is greater than or less than alpha.

Student: Sir, greater than.

Sir: Less than alpha because alpha is R by 2 L and beta is R squared by 4 L square minus something, square root of this. So this also will go to 0 and therefore, i of 0 is equal to 0 i of infinity is also equal to 0, which is correct and in between, can it go in between, can it go negative? Well, if you recall the definition of hyperbolic functions, this is simply twice sinh of beta t and sine can never be negative and therefore, this whole quantity is positive.

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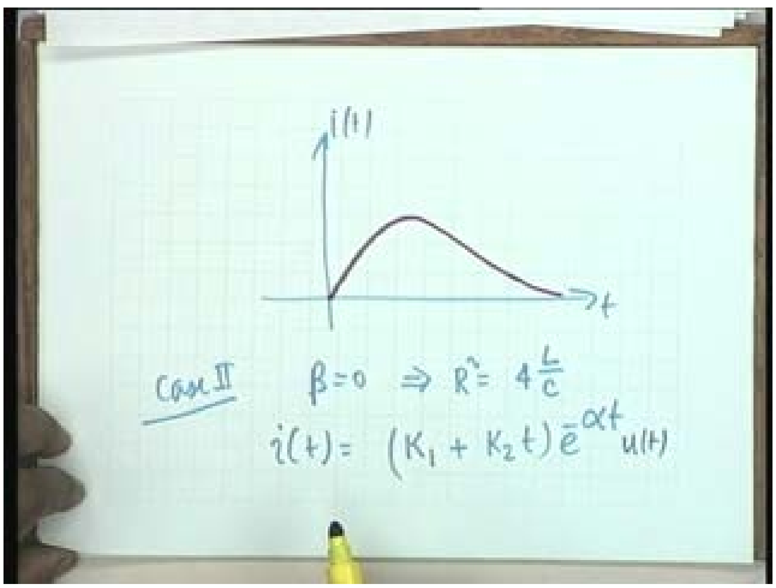

In other words**,** the variation of i of t, verses t, shall be of this form. It will rise from 0, it will get a maximum and then fall to z. Now we consider the next situation, that is, case 2. Case 2, in which beta is equal to 0, which means that R square equal to 4 L by C. If beta equal to 0, then you sees p 1 and p 2 are both equal to minus alpha and therefore, the solution is a case of repeated routes. The solution for current i of t will now be of the form, K 1 plus K 2 t multiplied by e to the minus alpha t. Is it agreed? u t okay, u t. Now the problem is to find K 1 and K 2. Where are the initial conditions? How did you get this thing?

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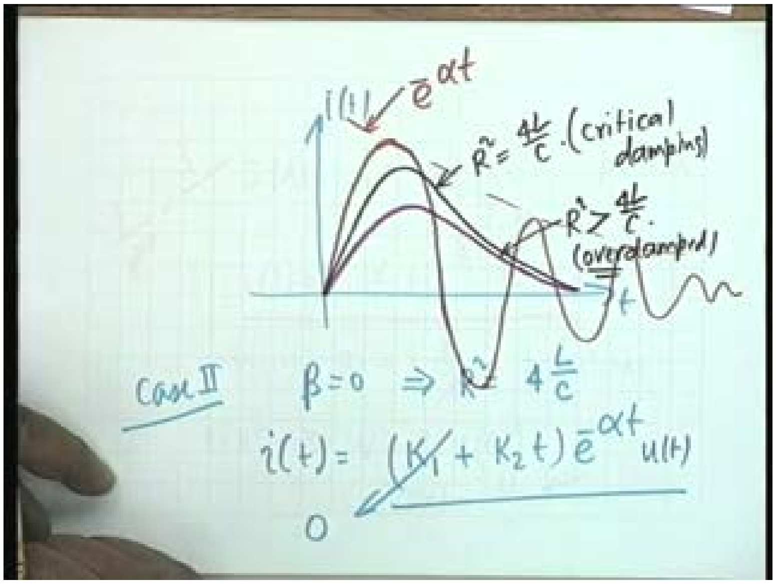

Well, it is a differential equation, or you take the, you take the original i of s. Just a minute, i of s, let me check, the expression is equal to V by L 1 by s minus p whole squared, where p equal to minus alpha. I take the inverse of this. Well I can get K 1 and K 2 directly from here right? Can you tell me what the inverse of this is? V by L e to the power pt. what s squared? t, agreed?

Now I wanted to go from the differential equation. Suppose, I do go from, I do not take the inverse of this, I go from a differential equation or from previous knowledge. Since routes are repeated, my solution would be of this form. Now, how do we find the initial conditions here? One initial condition we know, i of 0, that is, 0 and therefore, K 1 should be equal to 0. There is no other way. How do you find K 2? t equal to infinity? It is again 0 but that does not give K 2. Derivatives, you must be aware of this. The short cut is, you take a Laplace transform table and invert it.

Suppose, if the table is not available and you have forgotten what the inversion is, when you go back to the rules, how do you find out K 2? Well, if you take, if you write the equation, if you write the equation V u t, the original differential equation. What is it R i plus L d i d t plus 1 over

C integral 0 minus to t i tau d tau, and put here t equal to 0 plus, then the left hand side becomes V equals to i 0 plus is equal to 0 therefore, L i prime 0 plus and what is this quantity then? 0 and therefore, i prime 0 plus is equal to V by L. Is the point clear?

To find K 2, we have to take the first differential coefficient value at t equal to 0 plus, and this is how we find i prime 0 plus and if you substitute this in this, take i prime, take the differential coefficient put the equal to 0 and put that value equal to V by L. You shall get exactly this solution. So these are the different ways of arriving at the same solution. Nothing succeeds like success, the final result must be correct. If you have made a mistake in between, fortunately, for engineering, as I said, physical concepts, there is no substitute. For engineering, here are at every step you can check. Whether it is it stands to reason from physical consideration? The result that you have got, does it stand to reason from physical consideration?

Now we go back to the plot, you see, my solution is that, where is the solution? This is the solution, now obviously, at t equal to 0, well what is p? p is minus alpha? At t equal to 0, this is 0, at t equal to infinity, does it become 0? t times e to the minus alpha t. The exponential decay is faster than linear rise, you know that? If you do not, you can expand t by e to the alpha t, expand this, divide by t, then you will see that it becomes, 1 over sums of powers of t at t equal to infinity, obviously, that goes to 0.

So our physical conditions are satisfied, that it starts from 0, it dies at infinity. In between, can it be negative? No, it has to be positive. Now therefore, the curve, the plot shall be similar to this. It shall be quite similar to this; would it be above or below this? That is the maximum. You see, this case is for R squared greater than 4 L by C. Would the maximum value be greater? Physical concept. You can verify later, you can find out what this maximum is, what the other maximum is by differentiating, putting it, 0 and so on. But physical, it should be greater.

Ultimately, of course, it dies down to 0. So this is the case for R square equal to 4 L by C and this case is called the case of critical damping. This is the critical value of resistance, below which, the response would be damped. This case is called over damped response that is over the objective over stands for more than critical, that is, the resistance in the circuit is more than the critical value and as we shall see in the next lecture, if it so happens that R square is less than 4 L by C, well, the response is still of general shape. But it will rise, it will rise further but it crosses 0 earlier than infinity can become negative, and then goes on oscillating in a damped manner. The decay of the peaks is exponential and the rule is e to the minus alpha t.

Student: Sir, do all the peaks occur at the same point?

Sir: Do all the peaks appear? That, I leave it to you to find out. I might ask you this question in one of the minor. Things I leave to you, remain in my mind while setting exam papers and therefore, take them seriously.

Student: Sir, what was the question?

Sir: The question was, do all the peaks occur at the same point? Well, all the peaks, its magnitude is different, that is obvious. Do they occur at the same point? I will not answer. I leave the answer to you, you find out. But interesting point is that the value of R, the value of R causes the circuit response, changes the circuit response tremendously.

You see, as long as capital R satisfies this relation, R square equal to or greater than this, the current can never go negative. It shows the maximum, then comes down. But when as soon as the R satisfies, the R goes below this value, that is, R square is less than 4 L by C. The current behaves like a delinquent child. It goes on oscillating between positive and negative till it is forced by this damping factor e to the minus alpha t to ultimately come to 0, at t equal to infinity. That must be satisfied. Birth and death, these are constraints. In between, it can either remain positive or it can oscillate. The third case is the most interesting case, and we shall consider this in the next lecture. Thank you.