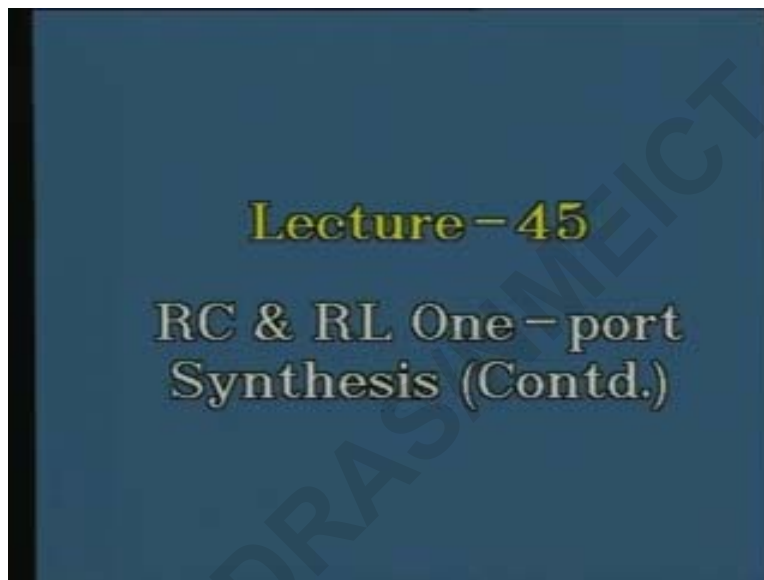


Circuit TheoryProf. S.C. Dutta RoyDepartment of Electrical EngineeringIIT DelhiLecture 45

RC & RL One-port Synthesis (Contd.)

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this is the forty-fifth lecture and we are to going to discuss the rest of RC synthesis and introduce also RL one port synthesis two element kind

let's recall the properties of RC impedance and admittance because they are not similar they are not identical

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Z_{RC}
 $\frac{K_0}{s} + K_\infty + \sum \frac{K_i}{s + \sigma_i}$
 $K_0, K_\infty, K_i, \sigma_i$
 real +ve.

Y_{RC}
 $K_0 + K_\infty s + \sum \frac{K_i s}{s + \sigma_i}$

$s Z_{RC} \stackrel{f}{=} Y_{RC} : \frac{Y_{RC}}{s} \stackrel{f}{=} Z_{RC}$

the properties are different

let's make a make a short review of the properties of RC impedances and admittances [Noise]

you recall that starting from the most general faster one form of RC impedance that is impedances of this type and the most general form for RC admittance which is R in parallel with C and then a number of RC in parallel

starting from this we derived the general forms for RC impedance and RC admittance

the general form was k zero by s due to this capacitor plus k infinity due to this resistor plus summation of terms like k_i divided by s plus σ_i where k_i 's and σ_i 's are expressible in terms of the individual resistors and capacitors and therefore all these coefficients k infinity k_i and σ_i they are real end positive

on the other when you took an RC admittance we saw that it is of the form k zero because of this resistance plus k infinity s because of this capacitor plus summation of terms like k_i divided by s plus σ_i

this is the difference there occurs an s here because R and C in series if you take the admittance where admittance is of this form

in other words these k_i 's are not the residues of Y_{RC} and you also notice that Z_{RC} and Y_{RC} only differ by a multiplication by s

that is if you multiply ah Z_{RC} by s the form is the same as that of Y_{RC}

another way of expressing this is that YRC by s is of the same form same form a ZRC all right and from these two expressions from these two expressions and ah this similarity we derived several properties of RC impedances and RC admittances and these are as follows [Noise]

i keep the expression here

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Z_{RC}	Y_{RC}
$\frac{K_0}{s} + K_{\infty} + \sum \frac{K_i}{s + \sigma_i}$	$K_0 + K_{\infty}s + \sum \frac{K_{id}}{s + \sigma_i}$
poles on -ve real axis	poles on -ve real axis
No pole at ∞	
K_{∞} or 0 } at ∞	pole or const } at ∞
pole or const } at $s=0$	const or zero } at $s=0$

so is to facilitate appreciate appreciation of the properties that i am going to mention k zero by s plus k infinity plus summation ki by s plus sigma i

then k zero plus k infinity as just multiply this by s and you get the general form for YRC it is kis divided by s plus sigma i and [Noise]

the first thing you notice is that poles of ZRC that is this sigma i since they are real in positive and the other pole is possible at s equal to zero

the origin so poles must be on negative real axis and the same applies here poles on negative real axis and what are poles of YRC at the zero's of ZRC therefore i can say poles and zero's of RC admittances must lie on the negative real axis all right

then we look at the we look at the situation at infinity [Noise] there is no way that ZRC can have a pole at infinity there is no way

<a_side> (()) (00:04:47) <a_side>

yes

<a_side> (()) (00:04:48) <a_side>

fine that's also negative real axis

infinity is a single point whether you go from this side or that or this side or that side

infinity is a single point and when convenient we consider it on the negative real axis

when convenient in the Lc case for example we consider it at at plus j infinity okay

since we don't know it we can {pu} (00:05:15) put it anywhere it's exactly like a concept of God or Heaven okay

infinity is a very similar to that concept in various religions and so on

ah [Noise] what you said was if what are poles of YRC at the zero's of ZRC

so we we conclude that poles and zero's of RC admittances all lie on the negative real axis

then the situation at infinity there is no way that ZRC can have a pole at infinity no pole at infinity pole at infinity is not permitted all right

at infinity what can happens is that be value can be ah

<a_side> (()) (00:05:53) <a_side>

either k infinity either a constant or zero at infinity

on the other hand YRC can have a pole at infinity pole or if it as no poles then it is a constant that infinity okay and similarly [Noise] similarly at the origin it can ZRC can have a pole because of this term or a constant at origin at s equal to zero

on the other hand for YRC at s equal to zero it is either a constant constant which is equal to k zero or zero at s equal to zero

while if you look at ah look at the properties of ZRC this property at YRC is obvious okay if ZRC has a pole at infinity at s equal to zero obviously YRC has to have a zero right

so it is the reciprocal property but never the less we list them out separately

<a_side> (()) (00:07:12) <a_side>

yes

<a_side> (()) (00:07:15) <a_side>

okay this point we did mention earlier

k zero can is not a residue because no pole there there cannot be a pole at the origin pole at the origin is not permitted k infinity is a residue

this k infinity is the residue of the pole at infinity

however these k_i 's are not residue's they are residue's of YRC by s not of YRC all right okay

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$Z_{RC} = \frac{K_0}{s} + K_{\infty} + \sum \frac{K_i}{s + \sigma_i}$
 Residues are real & +ve
 $\frac{dZ_{RC}(s)}{ds} < 0$
 Poles & zeros alternate

$Y_{RC} = K_0 + K_{\infty}s + \sum \frac{K_i s}{s + \sigma_i}$
 Residues of Y_{RC} real, -ve
 $\frac{dY_{RC}(s)}{ds} > 0$

we continue this discussion ZRC YRC k zero by s plus k infinity plus summation k_i

this is the ah kind of a Bible for uh RC impedance or admittance synthesis if you remember this everything else follows that's why i am repeating this in every slide k zero plus k infinity s plus k_i divided by s plus sigma i

now ah since the question of residue came residue's here k zero k infinity k they are all residue's residue's are real

<a_side> ((k is not a residue)) (00:08:24) <a_side>

is not a residue okay

so k zero and k_i 's they are all real and positive

whereas if you find the residues of YRC not YRC by s you can show that they are real but negative

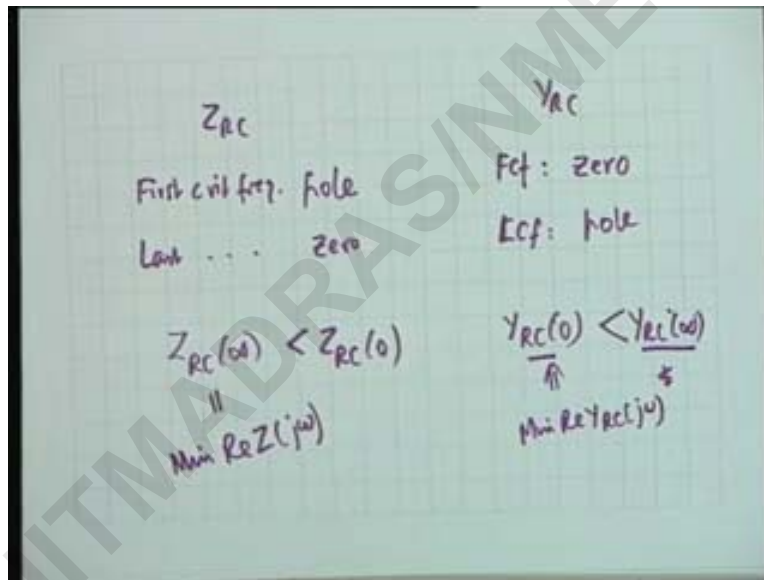
we shall demonstrate this with the help of examples but we will not prove it it will come out to be negative there is no no other way [Noise]

we also showed by taking the value ZRC on the real axis that is sigma and by differentiating this with respect to sigma we showed that this has to be strictly negative if you do the same thing with respect to this that is you put s equal to sigma and differentiate you can show that YRC of sigma with respect to sigma would be strictly positive

it would be strictly positive and as a corollary to this property that is the slope is strictly monotonic monotonic it is either strictly negative or strictly positive it follows that poles and zero's in either case have to alternate there is no other way

poles and zero's alternate this is a consequence of the this slope property to have to alternate in the addition [Noise] in addition ah

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for ZRC the first critical frequency first critical frequency has to be for ZRC has to be <a_side> ((a pole)) (00:10:08) <a_side>

a pole first critical frequency has to be a pole and it follows that for YRC the first critical frequency must be a zero

last critical frequency must be a zero and last critical frequency here last critical frequency Lcf this must be a pole all right

if you plot the sketches of YRC verses sigma it is very easy to show that the first critical frequency must be a zero for YRC and it must be a pole for YRC and it also follows that

the infinite frequency value of ZRC ZRC infinity is less than ZRC zero and because admittance is reciprocal YRC of zero must be less than YRC of infinity

if the two values are constants okay if ZRC at ZRC has a pole at the origin obviously it is infinity the value is infinity

if YRC has a pole at infinity obviously it is infinitely large now not only that

<a_side> (() (00:11:28) <a_side>

yes so the value is infinity which you don't know what the value is

nevertheless never the less (00:11:39) if it is a constant then this constant much bigger than this constant that is what it means if it is a pole then we don't bother okay

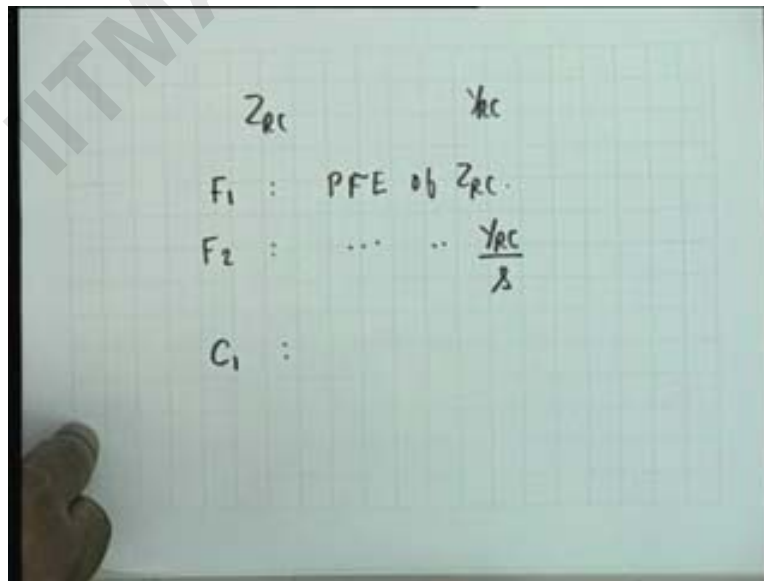
now not only that for ZRC we showed that ZRC of infinity is the minimum value of the real part of Z of j omega

we showed this and in an exactly similar manner we can show that YRC of zero is the minimum value of the real part of YRC of j omega

which means that in the synthesis problem you can always remove ZRC infinity without converting the rest of the function or the remainder function non positive real

similarly here from YRC you can always remove the dc value the rest of the remainder function shall still remain positive real okay and then [Noise] as for a synthesis concern

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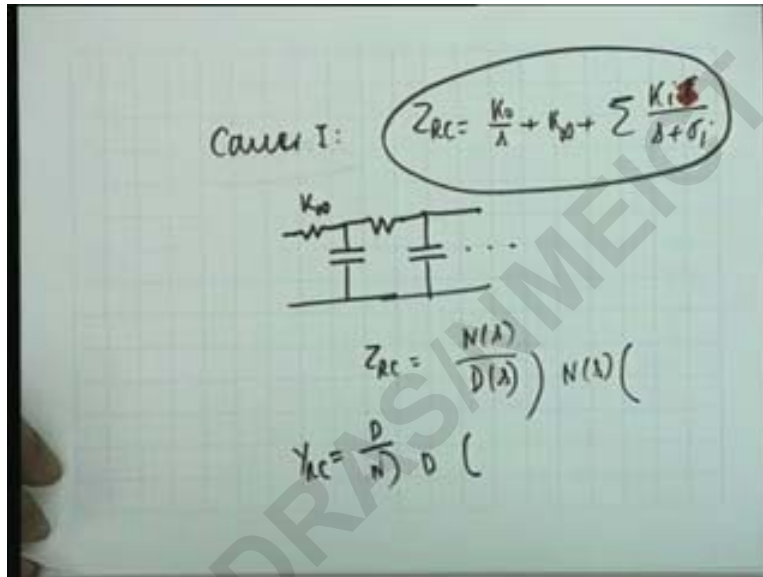
ZRC YRC the faster one form can be synthesized from the partial fraction expansion of ZRC where is faster two requires the partial fraction expansion of YRC by s this has to be remembered

cover one did we discuss cover one in the last class

we concentrate our attention on the pole at infinity or the point at infinity okay

let's look at cover one and cover two then we shall illustrate by means of an example

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cover one synthesis (()) (00:13:23) i am in repetition even if i even if i did this okay

ZRC ZRC cannot have a pole at infinity right RC impedance cannot have a pole of infinity

recall the form ZRC is of the form k zero by s plus k infinity plus summation k_i divided by s plus σ_i

this is the form if you remember this all properties all synthesis are should be obvious

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oh how can i make that mistake no s [Laughter] all right

now cover one concentrates on the point at infinity is at infinity what ZRC can have is either a constant or a pole

now this constant can be removed without affecting the rest of the function

so if I remove k at infinity if I remove this then what will be the behavior of ZRC at infinity it will be a zero and therefore YRC shall have a pole at infinity which I can remove as a capacitance all right

if I remove that as a capacitance then I take the reciprocal of that function

that will again be a constant and which I can remove and I can {repre} (00:14:46) proceed this way till the function is exhausted that is simple cover one

cover one simply focuses on the point at infinity

now the starting point is important do you start from RC impedance or RC admittance

you see if you remove the infinite frequency value of RC admittance then you have made a great mistake because you cannot remove them from RC admittance what you can remove is YRC of zero not infinite frequency value

so one must remember that if you want to develop in cover one you can remove two things are permitted either the constant value from ZRC the the infinite frequency constant value from ZRC or a pole from YRC

these are the two things that have to be done alternately

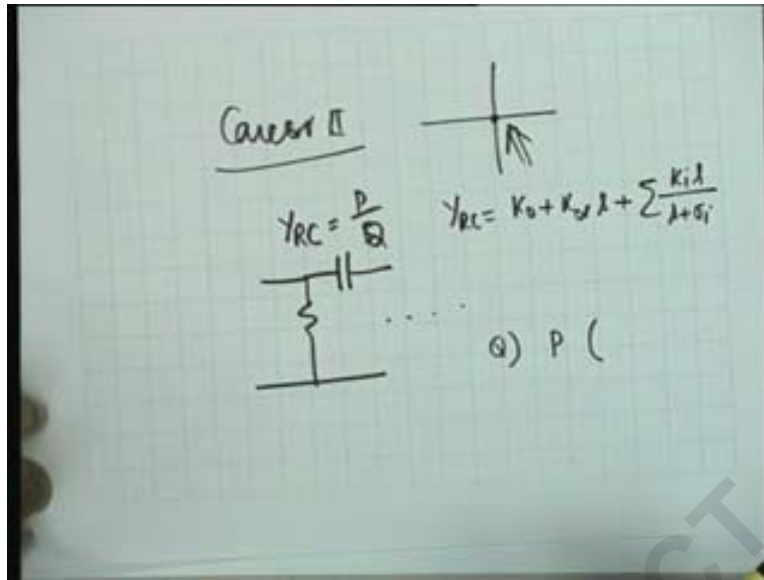
which means that what you actually do is to expand ZRC in continued fraction starting with the highest powers all right starting with the highest powers

the caution is that you you can do this for Lc functions either in impedance or admittance it doesn't matter but in the case of RC you must start from the impedance function all right

this is cover one it would be more obvious when we take an example

as for as yes

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<a_side> (() (00:16:19) <a_side>

oh the last part

question is [Noise] suppose we had a ah suppose you invert this YRC okay

YRC is D by N can i do a continued fraction expansion starting with the highest powers with YRC no

what would that mean if i do this what would that mean it would mean removing the constant value at infinity from YRC

which you cannot do right what we can remove from YRC is it Dc value not D infinite frequency okay

cover two focuses the attention on the point at the origin s equal to zero okay this is the point which you look at

now at s equal to zero what is it that we can remove from an RC remittance function obviously it is constant from ZRC or YRC

<a_side> YRC <a_side>

YRC therefore cover two must start from YRC

there is no other way what you do is you look at YRC which is k zero plus k infinity s plus summation kis divided by s plus sigma i

you can remove from YRC the constant value k zero and then if you remove this if you remove k zero then YRC has a zero at the origin

so ZRC shall have a pole at the origin which you can remove as a series capacitor and you proceed like this

so what you get is shunt resistors and series capacitor okay

this is cover two

once again cover one has to be started with impedance cover two with admittance and this can be formalized by making a continued fraction expansion starting with the lowest powers okay starting with the lowest power

that you divide P on to Q but write them in reverse order

that is the constant term comes first here also the constant terms comes first

what is the logic logic is that we are removing the Dc value

that is the constant divided by constant value

lets take an example to illustrate this and this example we shall continue ah throughout today okay our uh [Noise]

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$$Y_{RC} = \frac{(s+1)(s+3)}{(s+2)(s+6)}$$

$$K_0 + K_d + \sum \frac{K_i}{s+P_i}$$

$$Z(s) = \frac{(s+2)(s+6)}{(s+1)(s+3)}$$

$$= 1 + \frac{K_1}{s+1} + \frac{K_2}{s+3}$$

$$K_1 = \left. \frac{(s+2)(s+6)}{s+3} \right|_{s=-1} = \frac{(1)(5)}{2} = \frac{5}{2}$$

function is s plus one s plus three we carry this out all through s plus two s plus six

the question is ah realize this realize this function as a driving point driving point function it doesn't see impedance or admittance all right

the question is realizing this function as a driving point function of a two element kind network obviously since the first critical {fre} (00:19:25) first is realizeability is it realizable by an RC network okay

first critical frequency is is zero one two actually minus one minus two minus three minus six so they do alternate

the first critical frequency is zero last is a pole and therefore if at all realize it should be an YRC not ZRC because for ZRC the first critical frequency must be a pole last critical frequency must be a zero and therefore for faster one you have to take the impedance function Z of s which is equal to $s^2 + 2s + 6s + 1s + 3$ and expand in partial fraction

one has to be careful [Noise]

does the function have a pole at the origin no it doesn't have a pole at the origin

so we we have to take the infinite frequency value k infinities

so general form has to be k_0 by s if it is impedance plus k_∞ plus summation k_i by $s + \sigma_i$

what is k_∞ no i its its one okay

s^2 and s^2 that is one plus plus terms of this form we have two of them $s + 1$ $s + 3$ this is k_1 and k_2 and i told you the trick to find k_1 is multiply this by $s + 1$ and put s equal to minus one

so we get $s^2 + 2s + 6$ divided by $s + 3$ under the condition of s equal to minus one and you can do this very quickly one five two

therefore five by two two point five

we will not use the decimal fractions till we can okay because decimal might require a truncation which you don't want to do

we will continue fractions rational numbers as long as possible if it becomes very large then you go ahead

did they uh what is the danger suppose you truncate were what is the harm the harm is that in continued fraction things will not cancel

if you truncate somewhere in continued fraction things will not cancel okay

[Noise] because in faster one is an end by itself you don't have to do anything further but in cover one once you cover one or cover two once you do truncate your interest so do not truncate

similarly you can find

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$$K_2 = \frac{3}{2}$$

$$Z(s) = 1 + \frac{5/2}{s+1} + \frac{3/2}{s+3}$$

$$= 1 + \frac{1}{\frac{2s}{5} + \frac{2}{5}} + \frac{1}{\frac{2s}{3} + 2}$$

K_2 is equal to if I am not mistaken three by two okay

therefore my Z of s is an one plus five by two divided by s plus one plus three by two divide by s plus three

now do not try to place mart here because five by two may become two by five in the element value and things like that so have the patience to write this as one over two s by five plus two by five then element value shall be obvious

similarly here you write two s by three plus s here two okay then you draw the network one ohm and then you have s a parallel RC another parallel RC corresponding to this term and that goes back this is my faster one

the capacitor here would be two by five since this is impedance the reciprocal would be an admittance and the resistor here would be five by two not two by five

this is cs plus gs sc plus g similarly this will be two third and this would be

<a_side> half <a_side>

half that is the correct value all right this is faster one

let's look at faster two [Noise]

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$$\begin{aligned}
 &= \frac{1}{4s} + \frac{M_1}{s+2} + \frac{M_2}{s+6} \\
 M_1 &= \frac{1}{8}, \quad M_2 = \frac{5}{8} \\
 Y_{RC} &= \frac{1}{4} + \frac{\left(\frac{1}{8}\right)s}{s+2} + \frac{\left(\frac{5}{8}\right)s}{s+6} \\
 &= \frac{1}{4} + \frac{1}{8 + \frac{16}{s}} + \frac{5}{8} + \frac{48}{5s}
 \end{aligned}$$

the function is YRC is equal to s plus one s plus three s plus two s plus six and in faster two we have to divide by s

so we divide by s and expand this into partial fraction

the first quantity would be k zero by s k zero here is three divided by twelve so one quarter s one quarter divided by s which is one by four s is that okay

k zero should be obvious put s equal to zero except for this term then one into three three two into six twelve three by twelve in one quarter so i have written one by four s

i could have written one quarter divided by s this is k zero but i have saved one step

plus you shall have some ah let's say [Noise] M one i used k earlier bur it doesn't matter M one by s plus two plus M two by s plus six and you can find out ah i will skip this steps M one and i my case it came to one eighth and M two was five eighth

so i can write down YRC is equal to one quarter plus okay now i multiplied by s one by eight s divided by s plus two plus five by eight divided by s plus six and as i said do not try to play smart write these write this in terms of realizable quantity that is here it would be eight agree

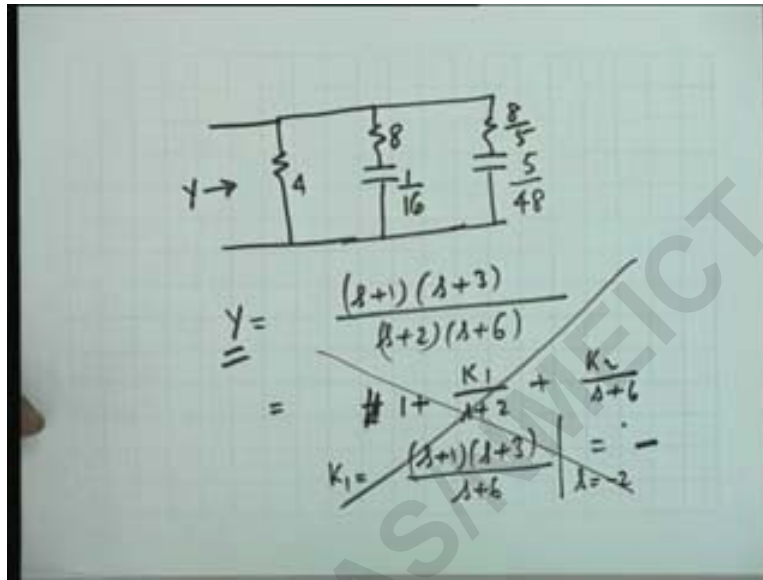
<a_side> ((five by eight)) (00:25:35) <a_side>

yes five by eight s

multiplies by s okay [Noise] thank you

so i get eight as plus sixteen divided so eight plus sixteen divided by s plus what do i get here eight by five plus forty-eight divided by five s agreed and accordingly now i can draw the faster two network as

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we always indicate resistance values

so this resistance should be four ohms and then we shall have two RC series networks in parallel

the resistance value would be eight agreed reciprocal of admittance is impedance here also the resistance value would be eight by five and then R plus one over sc so this capacitance will be one by sixteen and this would be five by forty-eight and your faster two is complete faster two

now suppose suppose we forgot that we have to divide by s let us see what happens it is worth doing at least once s plus one s plus three divided by s plus two s plus six

suppose i expand this in partial fraction then [Noise] you see if i write this as what are the things that i should write

i should write the infinite frequency value okay that means three divided by twelve which will be one i am sorry this is one plus some k one by s plus two plus k two by s plus six

let us see what k_1 comes out to be it would be $s + 1$ divided by $s + 3$ under the condition $s = -2$ which becomes negative

similarly you can show that k_2 is also negative it can be strictly through we don't need to expansion of Y

<a_side> ((sir what about k_0 zero)) (00:27:52) <a_side>

what about k_0 k_0 is one by four but shall we take k_0 out yes we can you should not necessarily [Laughter] actually this expansion should not be done so lets not discuss it further okay

is like learning spelling one doesn't learn spelling by ah giving you a wrong spelling because that gets in printed on your mind so let's not consider this further

you must remember that YRC as sacred time sacred not is to be tide around by division by s

otherwise the expansion is not valid

lets look at cover one now

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$$Z(s) = \frac{s^2 + 4s + 3}{s^2 + 8s + 12}$$

$$= \frac{1}{s+3} + \frac{\frac{s}{4}}{s+3} + \frac{\frac{16}{7}}{s+4}$$

for cover one we have to take the impedance function because it is RC we have to take the impedance function and write it as a rational function not in terms of the factor factors but in terms of a polynomial and i get $s^2 + 8s + 12$ (00:28:55) plus twelve $s^2 + 4s + 3$ is it okay

all right

now i start continued fraction expansion $s^2 + 8s + 12$

the logic is that we are removing the infinite frequency constant value from zero phases

so i get one $s^2 + 4s + 3$ let me bring the ah remainder here $4s + 9$ that divides $s^2 + 4s + 3$ do not forget to indicate the dimensions

this is a Z

then i get s by $4s^2 + 9$ by $4s$ and this an y

so i bring the remainder here sixteen minus nine is seven by four as $4s + 9$ divides $4s + 9$ [Noise] the number start getting ah

<a_side> ((sixteen)) (00:30:02) <a_side>

sixteen by

<a_side> ((seven)) (00:30:05) <a_side>

seven oh i might have made a mistake

this is an impedance so $4s + 48$ by 7 48 by 7

so $63 - 15$

<a_side> (()) (00:30:23) <a_side>

fifteen by seven i did make a mistake

fifteen by seven [Laughter] that divides uh

<a_side> (()) (00:30:30) <a_side>

seven by $4s + 3$

so i get uh {how mu} (00:30:36)

<a_side> ((forty-nine by sixty)) (00:30:37) <a_side>

forty-nine by sixty correct

yes isn't that needed s is needed

this is a y so i get seven by four as then remainder is

<a_side> ((remainder is three)) (00:30:52) <a_side>

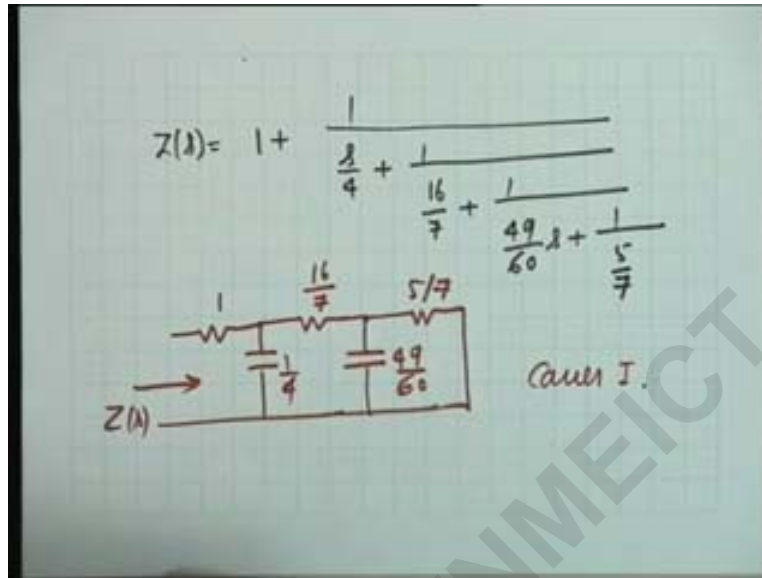
three divides fifteen by seven

so five by seven

this is a Z impedance okay

in other words [Noise] my network shall be

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while let me write this continued fraction this is one plus one over s by four plus one over sixteen by seven plus one over forty-nine by sixty s plus one over five by seven

is it correct and the network should now be obvious my network would be one then a capacitors of value one quarter then a resistance of value sixteen by seven then a capacitor of value forty-nine by sixty

then a capacitor a resistor of value five by seven and that's it

this is Z of s and this is my cover one

have you started this with y of s you would get negative values and that would show that you are you have to go back but i hope you would not make a mistake

if it is a ZRC you start continued fraction would be highest covers all right

if it is to be cover [Laughter] one no it's other way down

if it is to be cover one then you starts continue fraction of ZRC starting with highest power

let's look at cover two

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Case II

$$Y(s) = \frac{3 + 4s + s^2}{12 + 8s + s^2} = \frac{1}{4} + \frac{3 + 2s + \frac{s^2}{4}}{12 + 8s + s^2} \leftarrow y$$

$$\frac{2s + \frac{3}{4}s^2}{12 + 8s + s^2} = \frac{6}{s} + \frac{12 + \frac{3}{2}s}{12 + 8s + s^2} \leftarrow p$$

$$\frac{\frac{3}{2}s + s^2}{12 + 8s + s^2} = \frac{4}{7} + \frac{2s + \frac{3}{4}s^2}{12 + 8s + s^2} \leftarrow y$$

cover two you have to take Y of s it is RC you have to take Y of s and you write the rational function in the reverse order that is you write three plus four s plus s squared divided by twelve plus eight s plus s squared and then you start continue fraction because you can remove Y of zero YRC of zero

three plus four s plus s squared if you make a mistake if you try the other way round you will immediately be correct it but you lose time in the process okay

ah [Noise] the first quotient would be one quarter is it correct three plus two s this should be two s plus s squared twelve plus eight s plus s squared

so you get six divided by s this is y this is Z it has to be because this will be s series capacitor six by

<a_side> (()) (00:33:36) <a_side>

what is this

<a_side> (()) (00:33:43) <a_side>

oh how can we ignore that s square by four

so there would be another term here no the coefficient of s squared would be

<a_side> ((three by four s square)) (00:33:56) <a_side>

three by four s square okay

so i {multi} (00:34:03) is this okay six by s where would be twelve plus ah how much would be this eighteen by four which is nine by nine by two s

no s square so i get sixteen minus so seven by two s plus s squared divides two s plus three quarter s squared

what we are getting here four by seven and this is admittance

so i get two s plus four by seven s squared the numbers are not too bad the remainder would be

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$$\begin{array}{r} \frac{5}{28}s^2 \overline{) \frac{7}{2}s + s^2} \left(\frac{98}{5}s \leftarrow ? \right. \\ \underline{\frac{7}{2}s} \\ \frac{5}{28}s^2 \end{array}$$

$$\begin{array}{r} \frac{5}{28}s^2 \overline{) \frac{5}{28}s^2} \left(\frac{5}{28} \leftarrow y. \right. \\ \underline{\frac{5}{28}s^2} \\ 0 \end{array}$$

how much five by twenty-eight s square this divides seven by two s plus s square and the remainder is the quotient is ninety

<a_side> (()) (00:35:01) <a_side>

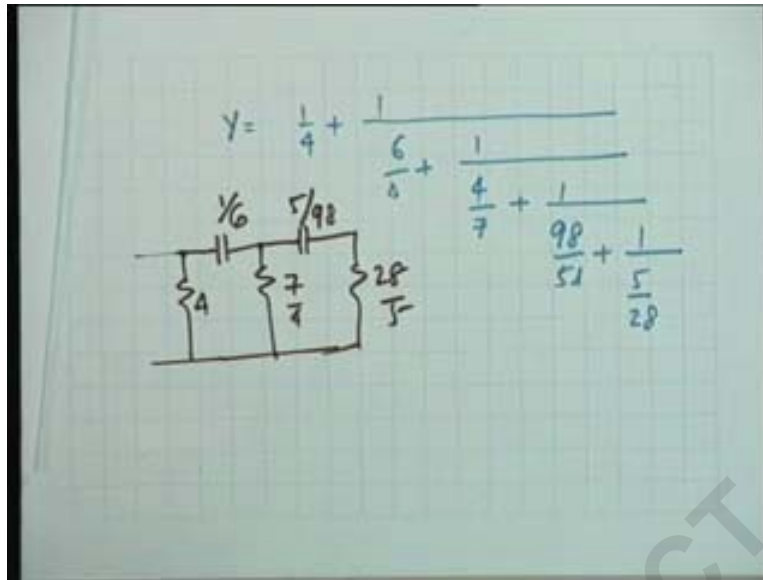
oh that's good ninety-eight by five s

yes okay

seven by two s and this is an impedance remainder s squared the last step is very nice five by twenty-eight and this is an admittance five by twenty-eight s square the remainder is zero

the network can then be immediately drawn if i write the continued fraction expansion like this

(Refer Slide Time: 00:35:30 min)



capital Y equal to one quarter plus one over six by s plus one over four by seven plus one over ninety-eight by five s plus one over five by twenty-eight

at least in the first few problems that you work out you should write this this is instructive

so that as its full prove after a little bit of experience you may not write the continued factor you may write the element values write from the continued fraction expansion

so what we get is our network becomes our network becomes a resistance of value four then a series capacitor of value one six right

<a_side> ((yes sir)) (00:36:20) <a_side>

then a resistance of value seven by four then a capacitor of value

<a_side> ((five by ninety-eight)) (00:36:27) <a_side>

five by ninety-eight and finally a resistance

<a_side> twenty-eight <a_side>

twenty-eight by it's done

we have done cover one cover two all the four canonic forms have now been done

now ah [Noise] once we have uh we know how to how to design and synthesize an RC network it turns out that its RL networks are very simple to synthesize

once we know how to do it for RC

for RL network lets look at ah the ah lets go back

(Refer Slide Time: 00:37:05 min)

$$\begin{aligned}
 & R_0 + sL_\infty + \sum \frac{s R_i L_i}{sL_i + R_i} \\
 & = R_0 + sL_\infty + \sum \frac{s R_i}{s + \frac{R_i}{L_i}} \\
 Z_{RL} & = K_0 + K_\infty s + \sum \frac{K_i s}{s + \sigma_i} \stackrel{f}{=} Y_{RC} \stackrel{f}{=} Z_{RC}
 \end{aligned}$$

(()) (00:37:06) the most general form the most general form would be this a number of R's RL networks in parallel

this should be the most general form for faster one configuration and you see that if you call this R uh zero if you call this L infinity okay and you call this L_i and R_i then the expression would be expression for impedance would be R zero plus s times L infinity plus summation the the equivalent of this is R_i sR_i L_i divided by sL_i plus R_i

which i can write is R zero plus sL infinity plus summation if i take L_i common from here then i get sR_i divided by s plus R_i divided by L_i okay

so far so good ah the general form therefore obviously is k zero

you understand why we write this is R zero at DC at DC all the inductors are short for this is the value and DC and we wrote L infinity because at infinity it is this inductance which dominates this is an open but it is shunted by R_i

so at infinity it would be sL infinity plus a resistance which is finite and therefore the value would be infinity okay

so the the form is k zero plus k infinity s plus summation k_i s divided s plus sigma_i

where all k 's are real and positive and so are σ 's and if you pause for a minute you notice this is exactly the same form as YRC and therefore whatever we have said about YRC is applicable to a ZRL impedance

for example if you want to expand an RL impedance in faster one you have to first divide by s all right

all that we have said about YRC now applies to ZRL and by the same token YRL therefore shall be of the same form as ZRC

just { recip } (00:39:57) take the reciprocal of both sides all right

therefore whatever we have said about ZRC that's apply to YRL also

for example the example that we took [Noise]

<a_side> (()) (00:40:13) <a_side>

pardon me

<a_side> (()) (00:40:17) <a_side>

faster two no for faster one we have to divide there because faster one is an expansion of impedance and since the impedance is of the form of YRC that is k_i they are not be residue

you have to first divide by s then to it okay

if a faster one is a series connection faster two is a parallel connection that is the difference between them

cover one is the point at infinity cover two is the point at the origin okay

so for example if you want to expand an RL impedance into force into cover one

then how would you start the continued fraction next one

if you want to expand a ZRL

<a_side> (()) (00:41:16) <a_side>

pardon me can you can you start with ZRL

<a_side> (()) (00:41:23) <a_side>

no no no no cover one i am talking

<a_side> (()) (00:41:29) <a_side>

you have to start from YRL

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$$Z_{RL} = \frac{(s+1)(s+3)}{(s+2)(s+6)}$$

$$= \frac{s^2 + 4s + 3}{s^2 + 8s + 12}$$

why

<a_side> ((RL has pole at infinity)) (00:41:36) <a_side>

why RL has pole at infinity or ZRL had the pole at infinity

<a_side> (()) (00:41:42) <a_side>

so can we start from with the starting with highest powers

<a_side> (()) (00:41:52) <a_side>

why lets look at this let's look at the same example may be things would be clear

our example was s okay

first thus this qualify to be a ZRL can it be can can this be a ZRL

<a_side> (()) (00:42:11) <a_side>

yes because a first critical frequency is a zero all right so okay now i want to {expa}

(00:42:17) i want to get a cover one for this

can i start expansion with the highest power that is the question i am asking s square plus

four s plus three s squared plus six no eight s plus twelve

can i go ahead and start continue fraction expansion

<a_side> no sir <a_side>

no because if i do that i shall be removing the infinite frequency value of ZRL which is not permitted

therefore i must take the reciprocal and then start continue fraction this is the point that if you do it blindly you will come back let say don't do it again [Laughter]

<a_side> (()) (00:43:01) <a_side>

because you will get negative from yes you will get eight s here so minus four s and so on then you know here made an mistake

you have to come back is say is the point clear similarly if you want to cover two cover two can you start it ZRL yes you can

<a_side> (()) (00:43:18) <a_side>

that's you can start it ZRL

so in cover one and cover two you have to be very careful

similarly faster one and faster two you will also have to be careful which one is to be divided by s

<a_side> (()) (00:43:32) <a_side>

Z no

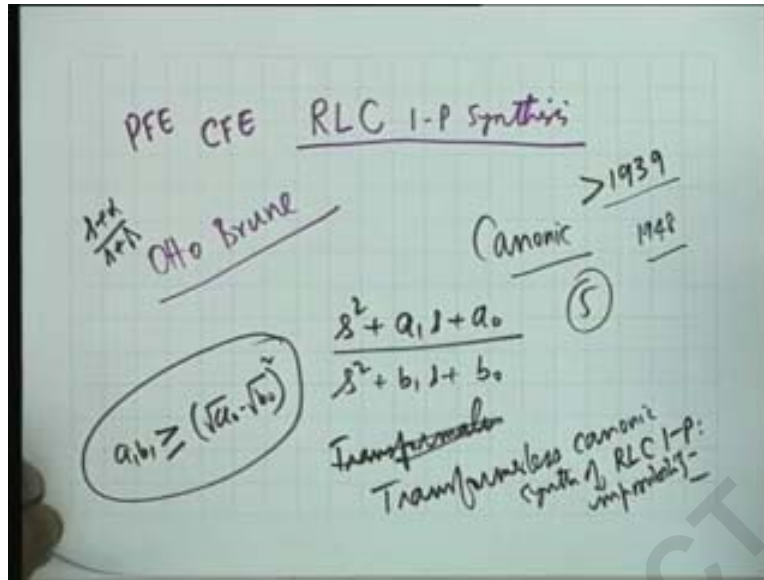
<a_side> ((ZRL)) (00:43:33) <a_side>

ZRL or Ys okay

ah without ah any further ah time spending any further time on RC and RL and we take further problems in the in the problem session

can we go to

(Refer Slide Time: 00:43:50 min)



an introduction to RLC that is if all the three elements are there then what you do in general RLC one pole synthesis

in general it has been found very unfortunately that the partial fraction expansion and continued fraction expansion which works so beautifully for RC RL and LC network was doesn't in general work for RLC and [Noise] and ah if you are lucky it may be a combination of partially fractions and continued fractions (()) (00:44:37)

or may be continued fraction ah starting with the higher power then switching over to lowest powers then switching over to highest powers again but it requires a lot of engine with you be you cannot say right by looking at the function that it will work in RLC one pole synthesis in general partial fractions and continued fractions or any combination of them may or may not work and therefore this remained a problem this remained a problem for every long time till in nineteen thirty-nine a gentleman

i think i have mentioned his name Otto Brune a gentleman settled in the United States

Otto Brune in his PhD thesis at MIT Massachusetts Institute of Technology ah proved that the most general synthesis of RLC one port most general and canonic synthesis

you see all the synthesis that you have been talking about so far two element kinds is canonic the number of elements is exactly equal to the number of specs

nothing more nothing less

you cannot do it less you can do it more of course okay

Otto Brune proved that given a let's say a bi quadratic let's say a bi quadratic like this $s^2 + a_1s + a_0$ (00:46:08) $s^2 + b_1s + b_0$ where a_0, a_1, b_0, b_1 are constants restricted only by the positive real condition that is they are all positive and real and they satisfy the real part condition

that a_1, b_1 should

$\geq \sqrt{a_0 - b_0}$ (00:46:26)

greater than or equal to square root $a_0 - b_0$ whole square

this is all that it is satisfy nothing else otherwise is completely general

Brune prove that is you require five element Brune proved that if you require a canonic synthesis you cannot do it without a transformer

you cannot do it without a transformer that is a transformerless canonic synthesis is not possible okay

let me state this this is beside this is beyond the scope of this course but uh i think you should know this you should know the beautiful elegant history that is behind one pole synthesis

transformerless canonic synthesis of of ah RLC one port is an impossibility

now ah in between ah the in between the ah advant of telephone where filters were required end nineteen thirty-nine there have been lot of attempts at doing this and there have been a PhD thesis written showing in possibility of synthesis at all okay

that ah such things under possible you cannot synthesis in RLC one pole unless you are lucky unless a very special case occur

therefore since nineteen thirty-nine since nineteen thirty-nine lot of attention gone to prove to prove or disprove Brune a more to disprove Brune but none succeeded and therefore the next step for to us [Noise] when don't insist on canonic it requires five may be we are we are willing to give see a transformer is not a very favored element

a transformer has lot of problem shielding coat it becomes heavy it becomes bulky it picks ups things ah magnetic induction and thinks likes an transformer where you cannot do it out a transformer you have power lines for example [Noise]

transmission distribution then in matching for uh audio frequencies and so but in general if you can avoid a transformer nothing like it and this is why in uh power amplifiers is complimentary transistor that was invented and it was a boom because you could produce a a power amplifier without using any transformer

and it is very small size and without any of though shielding and other problems

so people pardon me

<a_side> (()) (00:49:15) <a_side>

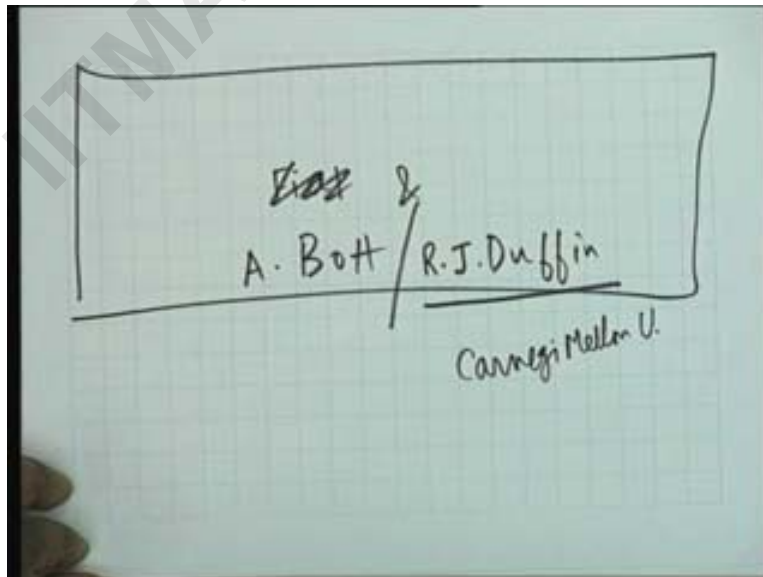
bi quadratic is the simplest form you see bilinear we know [Noise] bilinear s plus alpha by s plus beta it would be either in RC or in RL okay

so bi quadratic is the first ah complication that arises and people ah [Noise] concentrated on bi quadratic because if you know how to design a bi quadratic you can possibly find out how to design others

so since nineteen thirty-nine people had been saying all right lets uh let's leave of this first or insistence on canonicity

i am willing to you six can you give a transformer like synthesis while unfortunately this question remain i am answered till nineteen forty-eight when a two mathematicians [Noise] the names are

(Refer Slide Time: 00:50:11 min)



Fialco no i beg your pardon [Noise] Bott and Duffin

Duffins [Noise] excuse me A Bott and R J Duffin two mathematicians [Noise]

i don't remember where they are working but Duffin is a more than hundred is still alive and uh he has a special office at the University ah no at uh Carnegie Mellon at Pittsburgh yes Carnegie Mellon University he has an office and uh [Noise] Professor for life there till he lives

the beauty of this uh ah they were both of them are mathematicians they were told by some of the their electrical engineering friends that such a problems exists and they looked at it and in nineteen forty-eight they published a half a column paper half a column means if you take a quarter size paper ah a well of half of this of this size that much print with one figure in it

they prove that ah if you keep the canonicity then transformerless synthesis is possible unfortunately the number of elements that have to be used was seven not six seven and till to date there hasn't been any improvement on this

so transformerless canonic synthesis still remains a mysterious problem whereas practical networks are possible you can built network you can analyze you can built network without transform but given s is given a a function its very difficult to do it and the Bott and Duffin procedure then after nineteen forty-eight once again was subject on intensive investigation because ah did the demand for uh for filters started going up with with at expansion of communication the demand for filters grow up and uh there are many attempts and uh [Noise] under very special circumstance one could reduce the number to six but not in general it still remains a mysterious problem

we will continue this discussion in the next class at twelve [Noise]