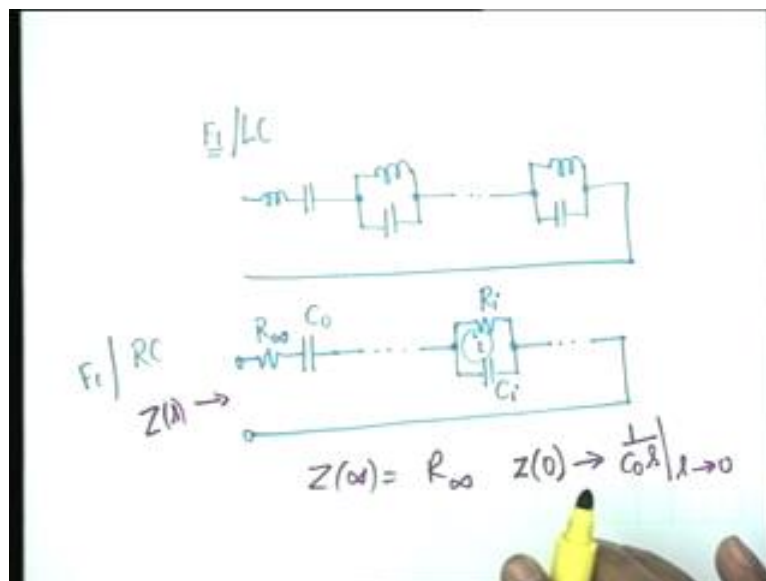


Circuit Theory
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Lecture - 43
RC and RL Driving Point Synthesis

People will also have to be told I will tell, this is the forty-third lecture and we are going to discuss today, the other 2 kinds of networks consisting of 2 kinds of elements only. So, far we discussed only LC inductance and capacitance and the motivation was that most of the practical networks which are to be used in communication should preferably be lossless or LC. However, they do arise occasions on which you require to synthesize resistance capacitance and resistance inductance driving point functions and this is what we shall discuss today.

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You recall the faster 1 for LC faster 1form for LC network is, you can have an inductance a capacitance and then a number inductors and capacitors in parallel. This is the most general form for LC or F2, F2 is also the most general form for LC. Now, we can take anyone of them, we can take anyone of this general forms. And by virtue of the knowledge that we have hopefully gained to the discussion of LC networks. We can derive the properties of RC and RL networks without going into much of derivation. All the facts that we have gathered for LC can now be transferred to RC in a very simple manner. For example, if we take an RC networks this is for LC if we take an RC

network; obviously, the most general form for RC network shall be obtained can be obtained, from this simplify removing all inductances and replacing them by resistances.

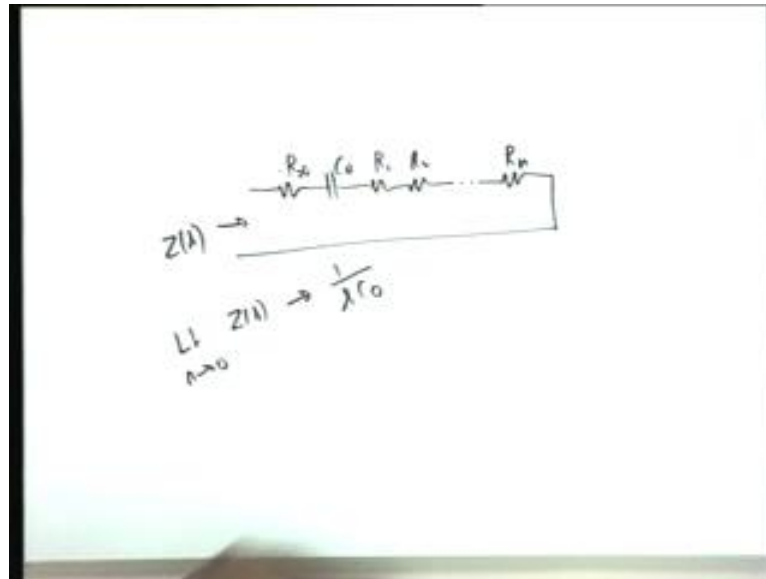
Therefore, my the most general form for RC network shall be of this form. We shall have a and resistance and a capacitance in general, then a number of resistor capacitor networks in parallel. This comes let us say, this is the i^{th} 1, this is i^{th} RC network in i^{th} RC parallel RC network in series. Let say, this is R_{∞} ; why I use the symbol infinity shall be clear in the minute. Let us say, this is C_0 and let us this is R_i and this is C_i this is the most general form for an RC driving point function. It's not that other forms are not possible, other forms are possible and this is 1 of the forms and as I said we can start with any of the forms and go over any derived the properties. Now, to look at these properties, first let me explain why I use this subscript infinity.

You notice that at s equal to infinity all capacitors will be short. And therefore, this will be short, all resistors will be shorted by this capacitors and therefore; obviously, if this is Z of s Z of infinity is equal to R_{∞} isn't that right. And this is why the symbol the subscript R_{∞} . Why I use this C_0 is also obvious, that at s equal to 0 at dc what we have is R_{∞} . This becomes infinity, C_0 becomes the impedance becomes infinity and all other parallel branches simply reduce to resistances. So, what we have is a series connection of R_{∞} and summation R_i in series with a capacitor of value C_0 .

Therefore, Z of 0 tends to only by C_0 s , s tends to at origin, it is this capacitor which take a over. And obviously, Z of s shall have a pole at the origin. And this is why the subscript C_0 here, but we shall derive this in more details. I just wanted to make an observation because it's obvious here. At a DC all we have the equivalent impedance is that of a capacitor of value C_0 . At infinite frequency the equivalent impedance is that of a resistance of value R_{∞} . Now, if I write an expression.

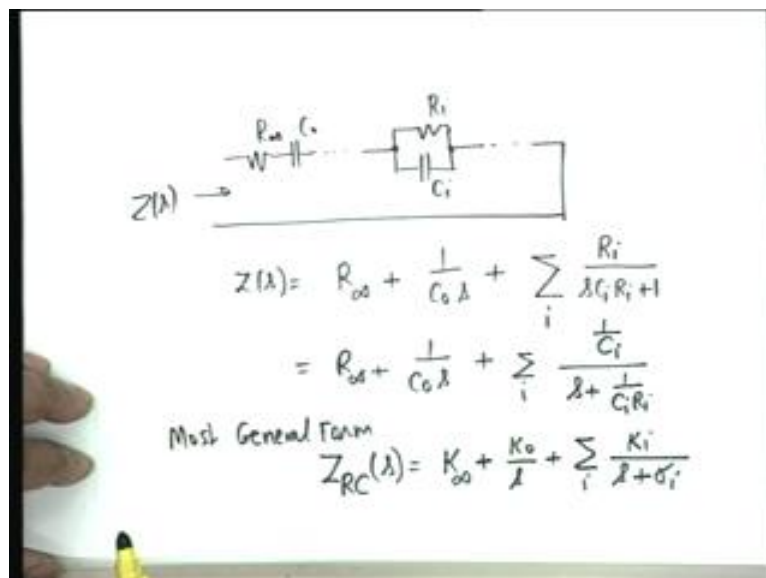
At s equal to 0 what we have is you see each of this, parallel branches this is open and therefore, takes over.

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So, at s equal to 0 the equivalent circuit would be R infinity C 0, and then this is what it is, R_1 , R_2 and let say R_n . So, at dc this is a series RC combination, when compare to the compare to infinity there resistances can be ignored. So, the function Z of s blows up at s equal to 0 and Z of s limits s tend to 0 tends to 1 by $s C_0$. In other words, it has a pole at the origin; of residue 1 by C_0 is that correct? Residue is one by C_0 not C_0 , but if you look at this, this network once again.

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The most general RC network R infinity C_0 and then you have a number of R_i 's and C_i 's in parallel. If we write an expression and analytical expression for this you see that Z of s is equal to R infinity plus 1 by $C_0 s$ plus summation over i . What is the impedance of this? If this is $C_i R_i$ divided by $s C_i R_i$ plus 1 as simple as that, R_i parallel 1 by $s C_i$ is this; which i can write as R infinity plus 1 by $C_0 s$ plus summation i . I want to write this in the form of an a polynomial, in which the leading coefficient is 1 . That is, I want to make the pole obvious.

So, I want to write this as s plus 1 by $C_i R_i$ then the numerator; obviously, shall be 1 by C_i is that. So, the most general form from this observation, we say the most general form of an RC impedance Z subscript RC of s would be K infinity. Now let's, do away with the elements and write in the general form of residues and poles. I shall have K infinity plus K_0 by s plus summation over i K_i divided s plus σ_i . Where σ_i is a real positive quantity and is equal to 1 by $C_i R_i$. One of the property is of an RC impedance is obvious now, that its poles must be on the negative real axis there can be a pole at the origin and there can be poles at s equal to minus σ_i where σ_i has to be positive. And therefore, 1 of the property is that is obvious, from this form.

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Handwritten notes on a whiteboard showing the general form of an RC impedance and its properties:

$$Z(s) = K_{\infty} + \frac{K_0}{s} + \sum_i \frac{K_i}{s + \sigma_i}$$

① Poles on the -ve real axis only
 K_{∞} not a residue
 K_0 is a residue
 K_i

② Residues must be real & +ve

K infinity I am talking of RC only K infinity plus K_0 by s plus summation K_i divided by s plus σ_i over i . 1 of the properties is that poles on the negative real axis, is this point understood. Only if poles cannot be anywhere else. So, this is the most general form the

poles are obvious. What are the frequencies at which the function blows up; obviously, 1 it can blow up at s equal to 0? Now, origin is also a part of the real axis it is as good a part of the imaginary axis as of the real axis. So, poles on the negative real axis only this negative real axis now includes s equal to 0. And what can you say about K_∞ K_0 and K_i they are truly the residues. Unlike LC functions in which you wrote $K_i s$ divided by $s^2 + \omega_i^2$ those $K_i s$ were twice the residue here these are truly residues. K_∞ , is K_∞ a residue?

No there is no pole at infinity, there cannot be a pole at infinity do not you see that at infinity the value is simply K_∞ . At infinity this becomes 0 all of them become 0. It does not have a pole at infinity. So, K_∞ is not a residue, you see, the difference between LC and RC networks. LC networks must have either a pole or 0 at infinity. Here, this is not the case at infinity the RC impedance has a constant. Now, it is a matter that this constant can be 0 K_∞ can be 0. If it is 0, then the function has a 0 at infinity, but under no circumstances can be ZRC of s have a pole at infinity.

If this is the general expression it simply cannot blow up at infinity. And therefore, ZRC for ZRC pole at infinity is barred. It cannot have a pole at infinity. K_∞ is not a residue is K_0 a residue, yes it is K_0 is a residue it is a residue of the pole at the origin, if this pole is there if it is not there, then at the origin what side the situation. If there is no pole at the origin if K_∞ equal to 0, Then what is the situation at the origin, it is a pole is it a 0.

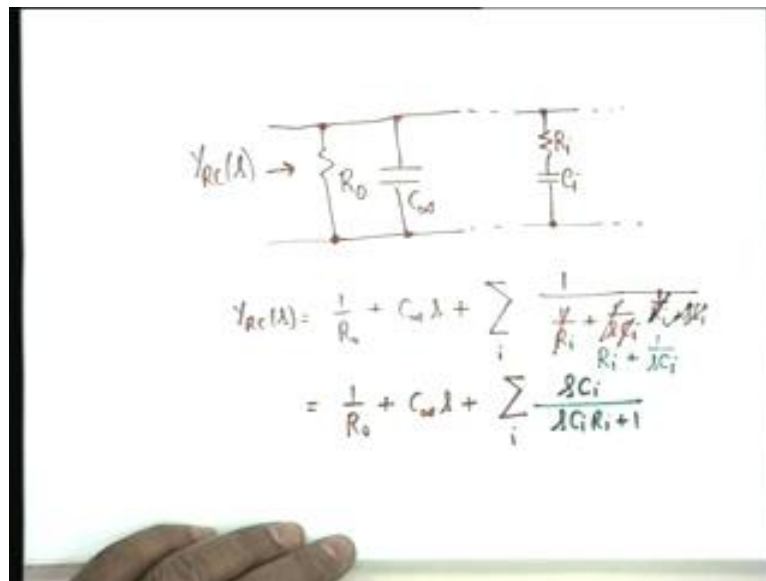
No it is a constant, do not you see? If this term is not there and you put s equal to 0. Then you simply get $K_\infty + \sum K_i$. So, at the origin RC impedance can have either a pole or a constant value. At infinity an RC impedance let me repeat, at origin an RC impedance can have either a pole or either a pole if K_0 is not equal to 0 or a constant value. What is the constant value $K_\infty + \sum K_i$. At infinity this ZRC of s can have either a constant value which is equal to K_∞ or a 0 if K_∞ is equal to 0. And you see, the difference between LC and RC. K_0 is a residue K_i is also residue.

So, what can we say about the residues, about the nature of the residues. How did we identify, what is K_0 , K_0 was $1/C_0$ and therefore, K_0 must be real and positive.

Similarly, K_i was $1/C_i$ and therefore, that must also be real and positive. So, all residues must be real and positive. It cannot be any other way any questions so far,

No, I could do this for an RC admittance also. If I take I am just taking slide d2 to bring another point that is what I am trying to do is. I have concluded that poles are on the negative real axis what about the 0, can be 0 be anywhere else to show to come to a conclusion about this question about the 0 of ZRC. We look at the poles of YRC. Now, what would be the most general form for an YRC

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Obviously, we take the faster 2 form. Faster 2 form of LC and replace each L by a resistance. So, what we get is this sum R and a reason to be made clear we call this R_0 . Then we have a capacitance we call this is C_∞ and then we shall have a parallel connection of several branches, each consisting of a series connection of a resistance and a capacitance C_i . This is the most general form for an YRC. Now, therefore, I can write YRC of s as equal to one over R_0 plus $C_\infty s$ plus summation admittance of this series combination. What is the admittance? $1 / (1/R_i + 1/s C_i)$, which I can write as $1 / R_0 + C_\infty s$ plus

How can I make, so many mistakes at 1 go am I right now, thank god.

So, I can write this as $s C_i$ divided by $s C_i R_i + 1$ agreed. Let us do this let us do a further bit of simplification.

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$$Y_{RC}(s) = \frac{1}{R_0} + C s + \sum_i \frac{\frac{1}{R_i} s}{s + \frac{1}{C R_i}}$$

$$Y_{RC}(s) = K'_0 + K'_\infty s + \sum_i \frac{K'_i s}{s + \sigma_i}$$

$$Z_{RC}(s) = \frac{K_0}{s} + K_\infty + \sum_i \frac{K_i}{s + \sigma_i}$$

K'_i are not residues of $Y_{RC}(s)$ but they are residues of $\frac{Y_{RC}(s)}{s}$

I must have your undivided attention, total attention. Because, things can become confusing. RC impedances do not have the same properties of RC admittances. Unlike LC this is ZLC and YLC they have the same partial fraction expansion. It is not the case here, you see the difference. I can write this as s plus 1 over $C_i R_i$. And I can write this as 1 over R_i multiplied by s . I have written the denominator in terms of a polynomial the leading power of which is unity. And you notice, that the most general form for an RC admittance would be a , by the way do you understand why I have used the symbol R_0 . You see at dc everything is open, capacitance open, these are open and therefore, it is only this resistance R_0 .

So, R_0 , 1 by R_0 is the value of Y_{RC} at s equal to 0 . And you can also appreciate why have you C infinity. Because at infinite frequency; this is a short impedance short. So, the admittance blows up, admittance is a pole at infinity with a residue of C infinity. All others, these resistances all other resistances they are large compared to a short circuit and therefore, at infinity Y_{RC} of infinity is dominated by C infinity. And this why, this subscript C infinity. This is just an observation on the way side observation. Now, the most general form therefore, shall be of the form K_0 plus K infinity s . If you want we can use a different symbol because we used K for the residues, but I think it should be clear in the context. Plus summation i $K_i s$ divided by s plus σ_i , this is the situation if you let us use a prime.

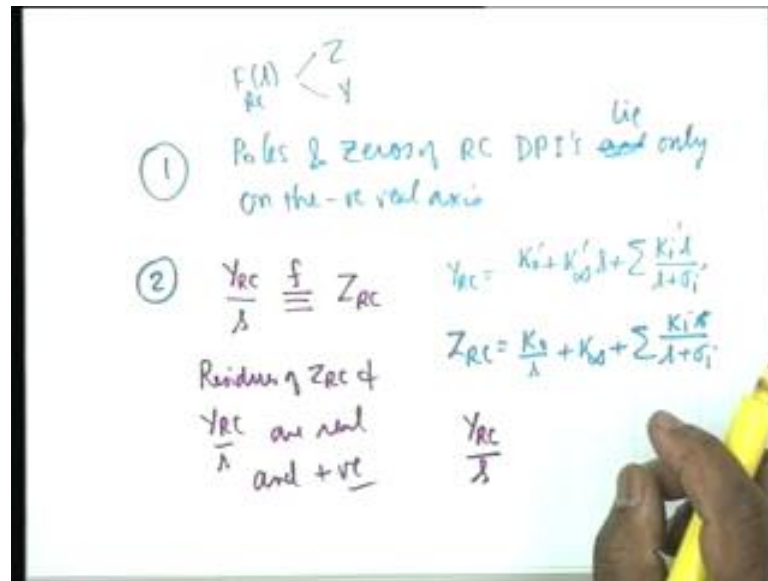
If you compare this with ZRC of s is of the form $K_0 + \sum_{i=1}^{\infty} \frac{K_i}{s + \sigma_i}$. If you compare these 2 forms; obviously, you see that these are not the same forms. These are not the same forms not only that, these K_i prime. K_{∞} prime is this a residue yes it is, is K_0 prime a residue. No because at dc, you look at YRC as it cannot have a pole at the origin. At this dc s equal to 0, you put this group blow this vanishes and you are simply left with K_0 prime. So, at dc YRC can only have a constant value or if this constant value is 0. That is; if the initial R naught is not there. Then YRC can have a 0 at the origin.

So, the behavior at the origin for YRC and ZRC are different, ZRC can have a pole YRC cannot have a pole. On the other hand, at infinity is ZRC cannot have a pole. Impedance cannot have a pole at infinity no way, but YRC can have a pole, because of K_{∞} prime s . K_{∞} prime is truly the residue of YRC at a s equal to infinity. If it does not have a pole, if K_{∞} prime is 0, then what is the value at infinity constant? It is K_0 prime plus $\sum_{i=1}^{\infty} K_i$ prime, no σ_i because at high at s equal to infinity it is the highest power term K_i prime s and s .

So, at infinity the value would be simply YRC infinity shall be K_0 prime plus K summation K_i prime, but the most important deviation is that this K_i prime is not a residue. Do you understand this? It is not a residue. If you write a partial fraction expansion; obviously, partial fraction expansion should have been of this form that is a constant divided by the pole factor. It is not in that form it is multiplied by s , but can you identify K_i prime as residues of some quantity. Obviously, if you divide YRC by s then K_i prime would be residues. So, you make the observation here, K_i prime are not residues of YRC of s , but they are residues of YRC of s divided by s if you divide by s .

The major conclusion that we wish to derive from this expression is about, the poles of YRC. Where are they located; obviously, on the negative real axis, could there be 1 at the origin. No. There cannot be a pole at the origin of YRC, but its 0 are still on the negative real axis. The pole of YRC at the 0 of ZRC isn't that right. Because Z and Y are reciprocals of each other, poles of YRC must be the 0 of ZRC and the conclusion therefore, is that poles and 0 of RC driving point functions either impedance or admittance it does not matter must lie on the negative real axis. So, let us make this statement poles.

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Okay.

So, there are specific locations; on the negative real axis which are bared for poles. There are specific locations which are bared for 0, but no pole or 0 can be outside the negative real axis they cannot on imaginary axis they cannot be complex. So, the statement is poles and 0 of RC DPI's are on the negative real axis. This is 1 of the properties of RC DPI F of s FRC s which is either Z or Y poles are 0 of RC driving point driving point admittances are on. Now, this is not a strong statement, let us make it stronger only or only on the lie not R lie only on the negative real axis nowhere else. Let us make it strong statement.

Poles and 0 of RC driving point admittances lie only on the negative real axis. Second: I get by looking at the expressions for YRC which is K_0 prime plus K_∞ prime s plus summation K_i prime s divided by s plus sigma i prime. And at ZRC that is K_0 by s plus K_∞ plus summation K_i divided by s plus sigma i no s. We have written this again and again. Now, if i look at this you notice that if I divide YRC by s. What are the terms; that I get. I get K_0 prime by s plus K_∞ prime plus summation K_i infinity prime by s plus sigma i prime. Which is exactly the same form a ZRC, is it not that right? Therefore, I can make this statement, that YRC by s and ZRC are formed equivalent. They are exactly of the same form. And this I write as triple equality sign, triple line,

triple horizontal line with an f, f stands for form. YRC by s and ZRC are exactly equivalent in form

This is a statement, which has million others inside it. What it means is; that the residues this is equivalent to stating that residues of ZRC and YRC by s are real and positive because they are form equivalent. And we also showed independently that case of i primes, K infinity prime, K0 prime they should also they should all be real and positive because they relate to a component value. Now, Ki primes are residues of YRC by s question suppose I find the residues of YRC not YRC by s. Suppose i find the residues of YRC would they be a real, would they be positive. It turns out that they are real, but strictly negative. This we shall show by demonstration now we will not prove it.

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Handwritten notes on a whiteboard:

$$Z_{RC}(s) = jX(\omega)$$

③ Residues of YRC are real and negative

$$Z_{RC}(s) = \frac{K_0}{s} + K_\infty + \sum \frac{K_i}{s + \sigma_i}$$

$$\frac{dZ_{RC}}{ds} = -\frac{K_0}{s^2} - \sum \frac{K_i}{(s + \sigma_i)^2} < 0$$

$$\frac{dY_{RC}}{ds} > 0$$

That is residues of YRC, simply YRC if you do not divide by s are real and negative. Now, because they are negative they are useless for us we cannot utilize that. Because they are negative we cannot use that partial fraction expansion. So, it also follows that if you wish, to synthesize a given YRC in faster 1 form. Then you have to first divide by s and then expand in partial fraction. You understand what i mean? If you have a function if you have an a realizable a positive real RC admittance function and you want to realize in faster to form. Then you must first divide by s carry out the continued partial expansion. You divide it by s carry out, partial fraction expansion to find out. K0 prime,

K infinity, prime K_i prime then you multiply by s . This is faster 1 faster 2 synthesis of an RC DPI.

Faster 1 is simply expand ZRC faster two is expand YRC by s . You see the difference between the 2 cases. The next property that we derived which is also useful, remember that in the case of LC the poles and 0 alternate. And the first critical frequency, is the origin, origin is either a pole or a 0. The last critical frequency is infinity which is either a pole or a 0. You will see that, both of them differ in the case of RC to a derive that let us look at ZRC of sigma. Since, poles and 0 of RC impedances as well as admittances lie only on the negative real axis, real axis. It makes sense to talk of RC impedance or admittance on the negative real axis.

You see in the previous case, in the LC case or poles 0, where in the Z omega axis. So, what we did was we took a ZLC of Z omega and put that is that equal to JX of omega X we called. What reactant, not susceptance reactance and similarly, YLC we took we put s equal to j omega. Why s equal to j omega, because the critical frequencies are on the j omega axis. And then you put that equal to j time's b omega, where b is susceptance. Now, in this case naturally it make sense to talk of the impedance functions or admittance function on the real axis. So, let us take ZRC sigma, that is K_0 prime plus, K_0 by sigma plus K infinity plus summation K_i divided by sigma plus sigma i .

Now, if you differentiate this.

Yes, I am taking an impedance function. If you differentiate this with respect to sigma, In the previous case in the LC case, we showed that $dx/d\omega$ or $db/d\omega$ are strictly positive.

Yeah, that is what I wrote.

Impedance functions it can have a pole at the origin, the value at infinity is infinity k infinity. And then K_i divided by sigma plus sigma i and what is the objection about this? I have simply put s equal to sigma nothing else. Now, if i differentiate notice, that it will be minus K_0 by sigma squared. K infinity differentiated would give 0 and what about this term, it would be minus summation K_i divided by sigma plus sigma i whole square. And you see that, each of these quantities are positive and therefore, this slope of ZRC on the real axis would be strictly negative strictly negative. It was strictly positive in the

earlier case and if you take YRC put s equal to σ and take the differential coefficient with respect to σ you can similarly show, what is your guess?

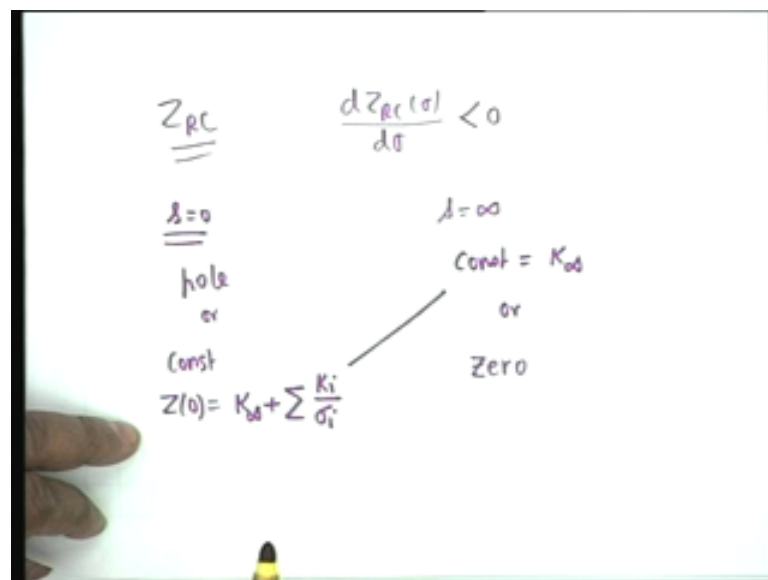
This would be strictly positive, see impedances and admittances differ the behavior differently.

Why can it not be 0 good question.

If it has to be 0 since, this cannot be 0, this cannot be infinity that K_0 and K_i s all of them have to be 0.

All of them have to be 0, then we are talking of a pure resistance. Pure resistance is a degenerate case of an RC impedance that is not something we are worried about. We are worried about, an RC when the capacitance comes in that is why, we said this is strictly. What is said is right; we are actually we should be less than equal to 0, but equal to 0 is a trivial case and therefore, we take that output of consideration. Similarly, if you differentiate YRC it is very easy to show that it is strictly positive.

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Now, if that is so let us take ZRC to be specific $dZRC/d\sigma$ is strictly negative. And the behavior at s equal to 0 is either a pole or a constant pole or a constant. What is the constant value? It is K_{∞} plus summation K_i divided by σ_i this is the constant value. At s equal to infinity it is either a constant equal to K_{∞} or what is

the other possibility 0 occurs, if K infinity is equal to 0 or therefore, there are 4 possible combinations for an RC impedance.

You see, there are 2 possibilities here, if you make combination I can have, constant at the origin, constant at infinity constant at origin 0 at infinity pole at the origin constant at infinity. There are 4 such cases of these 1 case is interesting that is constant at origin and constant at infinity.

Which constant is greater?

No, if it is constant then; obviously, you see this is, the dc value is greater than the value at infinity. Not only that, if you go back to the expression for ZRC.

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The image shows a handwritten derivation on a whiteboard. It starts with the expression for the impedance $Z_{RC}(s) = \frac{K_0}{s} + K_\infty + \sum \frac{K_i}{s + \sigma_i}$. Below this, it shows the real part of the impedance on the imaginary axis, $\text{Re } Z_{RC}(j\omega) = K_\infty + \sum \frac{K_i \sigma_i}{\sigma_i^2 + \omega^2}$. A horizontal line is drawn under this expression. Below the line, it states $\text{Min Re } Z(j\omega) = K_\infty$, also underlined. To the right of the text, there is a small hand-drawn graph showing a curve that starts at a high value and decreases towards a horizontal asymptote, representing the real part of the impedance as frequency increases.

It is K_0 by s plus K_∞ plus summation K_i divided by s plus σ_i . This was the expression, if I take ZRC of $j\omega$ and the real part of that. Can you tell me what this is, this is K_∞ , this will be purely imaginary. This K_∞ plus summation what should I get here K_i .

σ_i divided by σ_i^2 plus ω^2 . What is the minimum value of this? What is the minimum value of this expression?

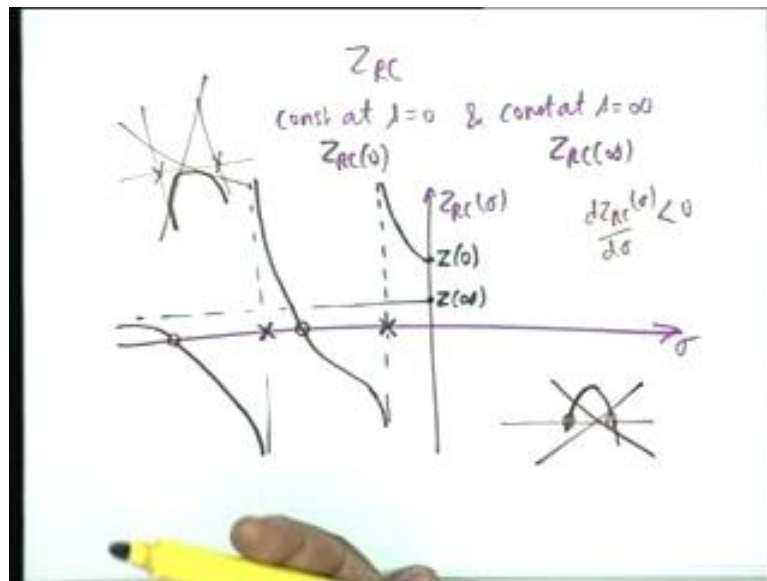
Obviously K_∞ , which occurs at ω equal to infinity isn't that right. As ω increases this term continues to decrease monotonically to 0 at infinity. And therefore, the

minimum value of the real part of Z of $j\omega$ is equal to K infinity. K infinity therefore, has a very special significance not only it is the value at infinity, It is the minimum value of the real part.

S equal to σ , that was the to find this slope. If you want to find the real part on the $j\omega$ axis, this is what you have to do. And we showed in positive real function testing that this is an important quantity. This has to be non-negative and you also showed, that from any positive real function, if the real part on the $j\omega$ axis varies like this.

Then global minimum value we can remove without the function become a non-positive real. I brought in this only to tell you that, this constant infinity can be removed easily, can be removed without destroying the positive real character of the function, but you cannot remove $Z(0)$. You cannot remove this because if you do that, then you are making then at some frequencies the real would be negative. So, let us look at this, let's take a specific case, let us say the function is constant.

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At s equal to 0 , take Z_{RC} which is constant at s equal to 0 . And constant at s equal to infinity. This value is Z_{RC} of 0 and this value is Z_{RC} of infinity, then let us plot Z_{RC} of σ versus σ . We have taken this case, constant at s equal to 0 and constant at s equal to infinity. If the constant at s equal to 0 is this value let us say Z of 0 . Then the constant at s equal to infinity must be lower than this, must be lower than this suppose

this is the value. Suppose this is Z infinity both of them must be real and positive must be real and positive because Z of s is a positive real function.

Now, the slope of ZRC of σ is strictly negative. So, starting from here as we go towards the negative real axis how can it do, can it do down? No, it has to go up there is no choice for it, it has to go up. If it goes up, where should it go? It has to go up monotonically; How far, can it go it can go up to infinity. Which means that, the first critical frequency must be a pole, is that clear? No. You see, the function I am plotting ZRC of σ on the negative real axis. The function has to go up, because the slope is negative it has to go up. It cannot come down, if it could come down, then we could have the possibility of a 0.

The function has to be strictly slope has to be strictly positive, strictly negative. What strictly negative means, that it cannot have a maximum or minimum. And therefore, it has to go monotonically. How far can it go? It can go to infinity. Let it go to infinity, if it goes to infinity; obviously, there must be a pole there. You know that, at a pole the function changes sign and therefore, after the pole it must rise from here. It must rise it cannot fall from minus infinity it must rise. So, the next critical frequency must be a 0. And then it should be continue to rise.

So, it rises which means that the next critical frequency must be a pole.

Yes, it can take negative value. If σ is negative; obviously, it can take negative value, you see positive real function simply say, if s is real Z of s is real it does not play. What can happen next? Next it can repeat, it will rise from here then it can go through a 0. It can go to another pole, but suppose that is the end. Suppose, if there is no further poles then; obviously, there will be 0 here and the function will asymptotically go towards Z of infinity. The figure says, that we get from here is, that first is poles and 0 must alternate.

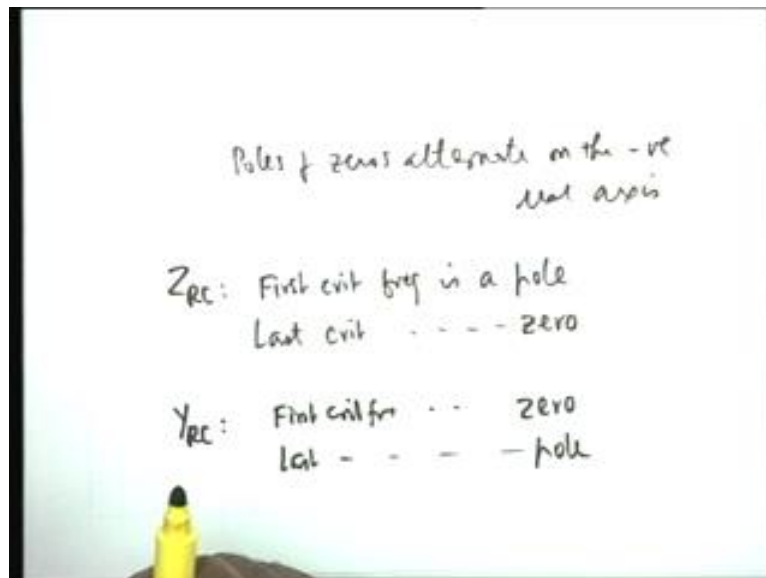
We cannot have 2 adjacent poles or 2 adjacent 0, why not because then the slope will not be strictly negative. If there are 2 adjacent 0 for example, then it must go up and then it must come down. So, the slope has to be go through 0 and then positive which is not permitted. Similarly, we cannot have 2 adjacent poles. Why not, because then what is happening?

Slope has to be positive.

So, it must come down and then it must go like this; it must come up and then go without crossing a 0. So, this is not permitted either. Number 1 poles and 0 must alternate. Number 2 observation; the first critical frequency has to be a pole 1 might argue that, this is not the only situation the function could have a pole at the origin. The behavior at origin is only, 1 of 2 possibilities 1 of the possibility is that it might have a pole at the origin or it is a constant at the origin. If it is constant at the origin, then we see that first critical frequency is a pole. If it is a pole at the origin; obviously, the first critical frequency is a pole. So, in either case it is a pole.

The last critical frequency is a 0 irrespective of whether the function attains constant value at infinity or what is the other possibility; 0. If it attains a 0 values at infinity; obviously, the last critical frequency is 0. If it attains at constant value then also the last critical frequency must be a 0.

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So, our conclusion from this diagram is first poles and 0 alternate. Alternate poles and 0 are only on the negative real axis. So, you can add this on the negative real axis. And number 2, for ZRC first critical frequency is a pole, last critical frequency is a 0. Can we without further derivation can we say what will be the first critical frequency of an YRC.

First critical frequency with YRC is a 0 and last critical frequency YRC is a pole. These are the differences and I should not get lost in these differences; you must after a while you will remember. Now, if you remember the first critical frequency of ZRC is a pole, then you can remember everything else. If the first is pole, last has to be a 0. And for YRC it is exactly the reverse. So, if you remember just one of the statements and I find it very convenient to remember this; that the first critical frequency pole. Now, these peculiarities of description of ZRC and YRC that puts a lot of restrictions on the synthesis. Let us take some examples;

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$$\frac{(s+1)(s+4)(s+8)}{s(s+2)(s+6)} \quad Z_{RC}$$

$$\frac{(s+1)(s+8)}{(s+2)(s+4)} \quad Y_{RC}$$

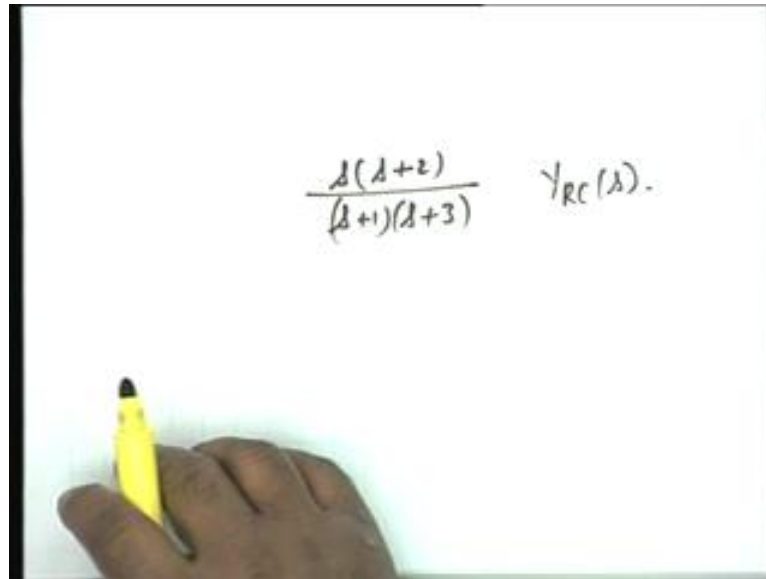
$$\frac{(s+2)(s+4)}{(s+5)(s+8)}$$

Let us say $s + 1$, $s + 4$, $s + 8$ divided by s times $s + 2$ times $s + 6$. The function is given, it is not said whether it is an impedance or an admittance. It is not said whether it is a RC or not, first thing is you find out whether it is RC. If it is RC poles and 0 must be on the negative real axis poles and 0 at 0 minus 2 minus 6 minus 1 minus 4 minus 8. So, the poles and 0 satisfy that combination do they interrelate 0 minus 1 minus 2 minus 4 minus 6 minus 8 yes they do interrelate, then is it an impedance.

It has to be an impedance, because the first critical frequency is a pole. If the first is a pole last must be a 0, last is minus 8 it is a 0, so this is indeed a ZRC. Let us take, the next $s + 1$, $s + 8$, $s + 2$, $s + 4$, can it be an RC immittance function.

Poles and 0 do not alternate and therefore, this is not an FRC. The next $s + 2$, $s + 4$, $s + 5$, $s + 8$ yes. No. 2 minus 2 minus 4 minus 5 minus 8. It cannot be.

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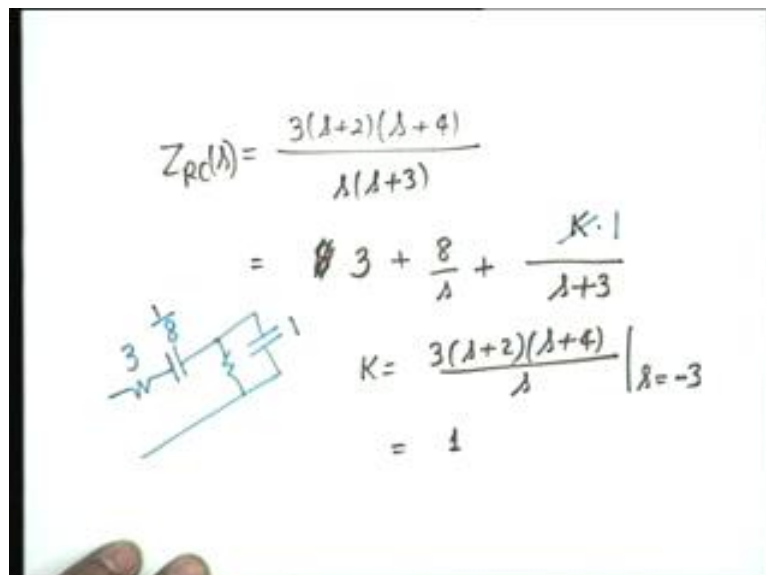
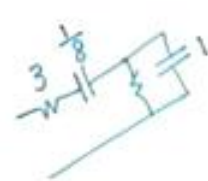

$$\frac{s(s+2)}{(s+1)(s+3)} Y_{RC}(s).$$

Next s times, s plus 2 s plus 1 s plus 3. What about this?

This has to be.

YRC s.

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$$Z_{RC}(s) = \frac{3(s+2)(s+4)}{s(s+3)}$$
$$= 3 + \frac{8}{s} + \frac{K}{s+3}$$

$$K = \frac{3(s+2)(s+4)}{s} \Big|_{s=-3} = 1$$

3 times s plus 2 s plus 4 s time plus 3.

Yes ZRC.

Okay

Let us look at this is a ZRC, let's look at a synthesis of this. If I want to synthesize this then I write this in terms of its poles; does it have a pole at infinity? It cannot have a pole at infinity if it is ZRC. So, at infinity with a little practice you will be able to write down the value directly.

What is the value at infinity?

No, $3 \times 2 \times 4$ divided by 3×8 .

Okay

It is $3s^2$ divided by s^2 , Plus what is the value at 0? What is the residue at 0 origin is a pole.

8.

So, it is $8/s + 1$ pole, only 1 internal pole on the negative real axis $s + 3$ and the numerator shall be some constant K. What is K? K you find like this multiply this by, $s + 3$ and put s equal to minus 3.

It is 1.

Therefore, the circuit is simply 3 ohm, resistance a capacitor of value one-eighth. Then a parallel combination of a resistance and a capacitance, the capacitance is 1 and the resistance is 3 or 1 by 3? Come on this is an impedance a 1 by it must be an admittance. How much?

1 by 3 and the synthesis is complete. We could do this by faster 2 also.

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$$\begin{aligned} Y_{RC} &= \frac{s(s+3)}{3(s+2)(s+4)} \\ \frac{Y_{RC}}{s} &= \frac{s+3}{3(s+2)(s+4)} \\ &= \frac{K_1}{s+2} + \frac{K_2}{s+4}. \end{aligned}$$

If we do it by faster 2, then we take YRC. YRC is s into s plus 3 s plus 2 s plus 4 and there comes a problem. We expand YRC.

We expand YRC by s therefore, this is s plus 3 divided by 3 s plus 2 s plus 4 and i expand this K_1 divided by s plus 2 plus K_2 divided by s plus 4. I would argue that you try expanding this in partial fraction. You will see that, you get negative residues; it is only this way that you can do and finally, you find out YRC and synthesize this. We will continue this in the next lecture.

Thank you.