Circuit Theory

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Lecture 41

LC Driving Point Functions

our topic today would be properties of LC driving point functions that is networks containing only inductors and capacitors and their synthesis that is given an LC driving point functions how to determine the network

we had started this discussion on the in the fortieth lecture or the thirty-ninth i think [Vocalised Noise] and we are good that if a network contains only LC elements

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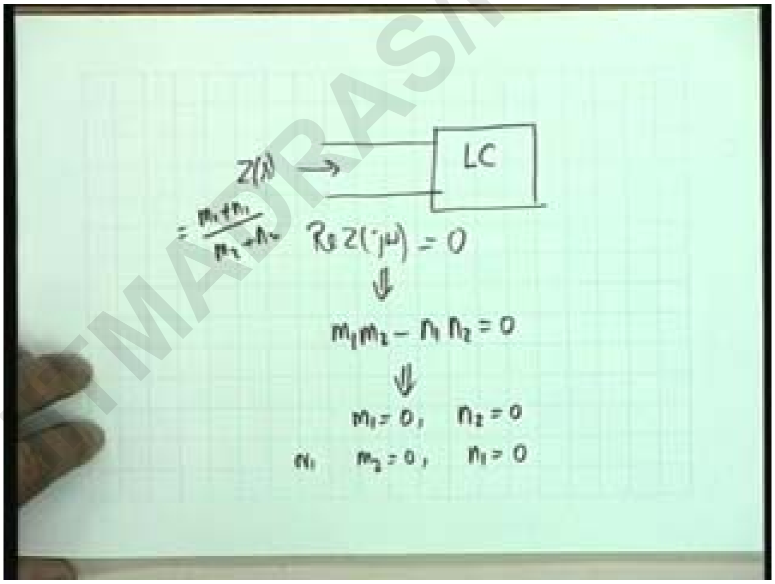

then the input impedance the real part if this is Z of s the real part of Z of j omega should be equal to zero because there is no power that is dissipated in the network and from this we concluded that if Z of s is written as m one plus n one divided by m two plus n two then m one m two minus n one n two has to be equal to zero and the only way only two alternative ways that can be that can be satisfied is either m one equal to zero and n two equal to zero both terms should be individually zero or m two or m two equal to zero and n one equal to zero which means that the input impedance that is Z of s shall be a ratio of even to odd polynomial or odd to even polynomial

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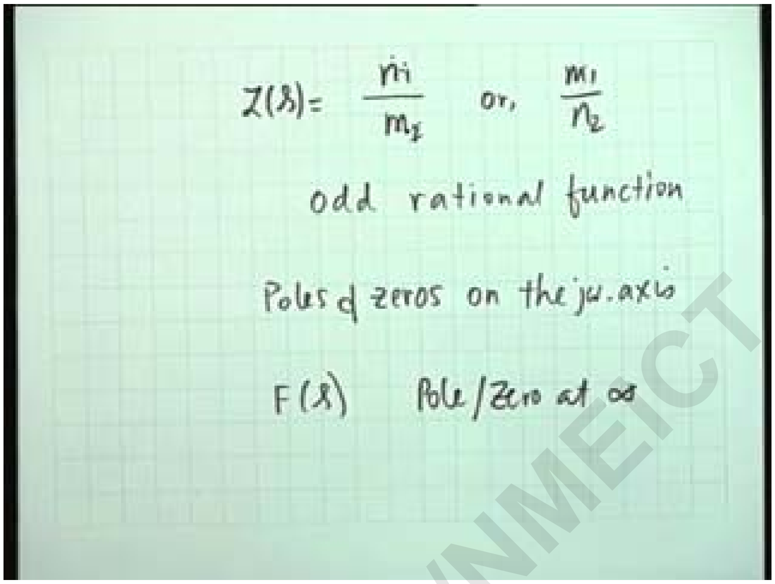

that is the only possibilities are that Z of s would be equal to n one by m two or m one by n two it should be a ratio of even to odd or {even to} (00:02:03) or odd to even polynomial and such a rational function is called an odd rational function odd rational function

a ratio of even polynomial to odd polynomial or odd polynomial to even polynomial is called an odd rational function and if the ah function is odd then naturally n ah since Z of s is positive real it has to be positive real otherwise we we cannot realize this

if it is to be positive real then obviously numerator as well as the denominator must be Hurwitz that means they cannot have roots in the right of plain

similarly here also the numerator and the denominator both have to be Hurwitz and if they are Hurwitz Hurwitz and even polynomial or Hurwitz and odd polynomial [Vocalised Noise]

naturally the roots must occur on the j omega axis that means we conclude the poles and zeros of LC Driving Point Functions shall all be on the j omega axis

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and if there are poles on the j omega axis or zeros on the j omega axis you know that for the function to be positive real these poles and zeros must be simple and considering the poles the residues at this poles shall be real and positive okay [Vocalised Noise]

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no no we are not saying that we are saying that both [Vocalised Noise] n one and m one both n one and m two have to be Hurwitz because Z of s is positive real it cannot have poles or zeros in the right of plain okay and since this is purely odd or purely even it must have zeros on the j omega axis

it cannot be anywhere else

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[Laughter] well it must have zero Z poles okay a polynomial of degree two must have two roots okay so it must have zero it must have poles and this poles and zeros must be on the j omega axis alright

now one of the consequences is of this ah derivation is that the degrees cannot be equal by definition the degree of an odd polynomial cannot be equal to the degree of an even polynomial alright even and odd they are of different characters they cannot be equal the degrees cannot be equal the degrees cannot be equal

if the degrees cannot be equal they can differ only by one they cannot differ by zero because the degrees cannot be equal and therefore the numerator and denominator of an LC driving point function what we are speaking about ah impedance is also applicable to admittance because reciprocal of an impedance [Vocalised Noise] reciprocal when f p r function is p r alright

so ah if for a driving point function of an LC network the degrees must differ by one the degrees of numerator and denominator must differ by one and if the degrees differ by one it follows that the function must have either a pole or a zero at infinity pole or zero at infinity this is a must okay because the degrees differ by one

if the denominator degree is one greater than the numerator then it is a zero at infinity if it is the other way around then it is a pole at infinity it is also follows it also follows that

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because it is odd to even or even to odd it also follows that s equal to zero belongs to this same category that means at s equal to zero the function must have either a zero or a pole this one for example shall have a zero at the origin because the numerator is odd

an any odd polynomial in s times in an even polynomial therefore it must be zero at s equal to zero where if it is this form then it must to be a pole therefore there must be a pole or zero at s equal to zero also alright that means the pole

i am sorry the the point at s equal to zero origin and the point of infinity both of them are critical frequencies critical because the function must either vanish there or it must be blow up either the function vanishes or it's reciprocal vanishes

so they are two critical frequencies in general all poles and zeros are also referred to by the {ss) (00:06:58) same phrase then be critical frequency

a critical frequency is the frequency at which the either the function blows up or it's reciprocal blows up alright

so ah both the points at the origin and infinity are critical frequencies for an LC function [Vocalised Noise] let's take a specific case

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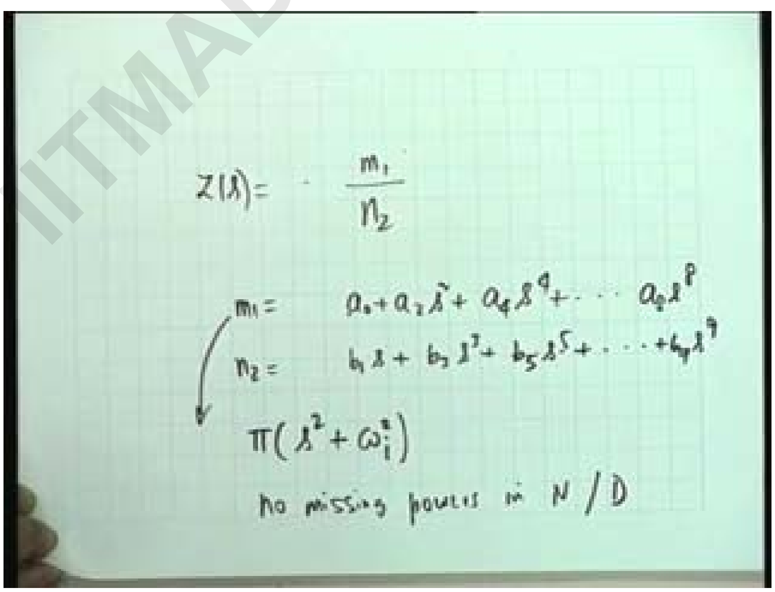

let's say Z of s equal to ah to the specific let us say it is ah m one by n two in other words it's a ratio of an even polynomial to an odd polynomial wherein next question that one asked is um i know that m one has to be of the form a zero plus a two s square plus a four s four and so on m one has to be of this form and n two [Vocalised Noise] has to be of the form b one s it cannot have a constant term b one s plus b three s cubed and so on let's write another b five s five and so on

the highest power here if it is eight then the highest power here shall be either

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seven or nine okay agreed

the question that one ask is between a zero and let's say ah a eight let's say it is eight and let's say this is b nine s to the nine between a zero a eight there can there be a missing power

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no what obviously all odd parts are missing but in an even polynomial in an even polynomial it is on the even powers of s that should be present can there be a coefficient which is missing

no because an even polynomial even Hurwitz polynomial must have all it's roots on the j omega axis that means it must be product of terms like {thit} (00:08:52) this s squared plus omega i squared and if you multiply terms like this all even parts have to be present there cannot be a missing term missing term means there is cancellation and therefore there must be negative signs in the factors alright

if you don't allow that obviously no missing powers in m one by in by a similar total [Vocalised Noise] no missing powers in n two okay so no missing powers in numerator or a denominator in numerator or denominator okay no missing powers [Vocalised Noise]

on the basis of this discussion we have not ah we have not exhausted the property we have discussed some properties but on the basis of this discussion it follows

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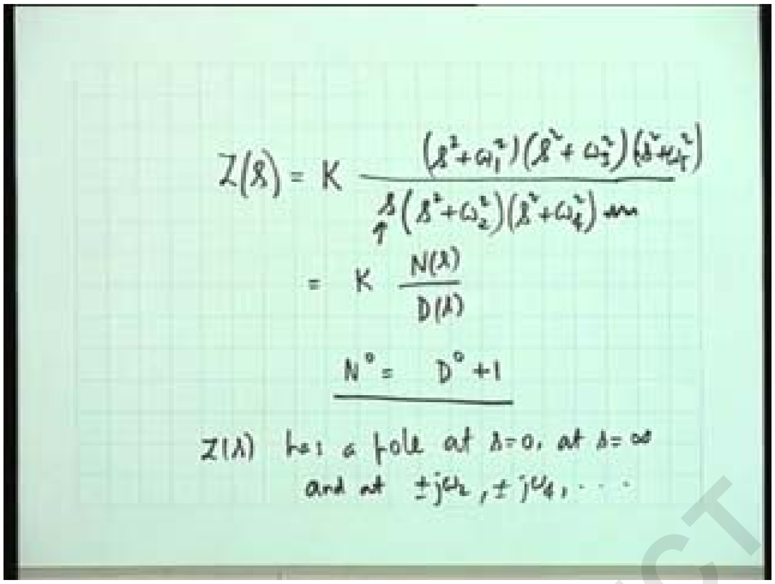

that ah an LC impedance function must be of this particular form that is it can have a multiplying constant k then it can have let’s say a {zero at the}(00:09:56) a pole at the origin it can have it may not have okay

it can have {a ze} (00:10:03) pole at the origin then the next {crtica} (00:10:09) oh that we have not discussed so far but let’s say the numerator the denominator must be of this form s square plus omega two square s square plus omega four square and so on alright

the the numerator [Vocalised Noise] the numerator then must be of the form s square plus omega one square s square plus omega three square and so on now there is a specific way i have written this all the denominator ah poles i have written with even subscripts all the numerator zeros i have written with odd subscripts

there is a significance which we’ll come to a little later but you you um shall agree with me that Z of s must be of this form that is some k multiplied by let's say N of s by D of s where both N and D are written with living coefficient equal to unit alright

it also follows that the degree of N must be either one greater than degree of D or one less then degree of D to be specific let us say that the degree of N is equal to degree of D plus one then this function has a pole at infinity is it okay

if the degree of N is one greater than the degree of D then the function has a pole at infinity alright

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so the Z of s that we have taken has a pole has a pole may have taken only an example has a pole at s equal to zero alright what are the question is any question at s equal to zero

then one at s equal to infinity and at points on the j omega axis [Vocalised Noise] in between that is between s equal to zero and s equal to infinity there are other frequencies at which is vanishes that is at plus minus j omega two plus minus j omega four and so on

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there is a pole s {equals}(00:12:26) s occurs in the denominator so at s equal to zero the function blows up it becomes infinity

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why [Laughter] i can have another term here and i can terminate this here can i now i can have s square plus omega five squared here and this end set omega four

let the degree of the denominator would have been five [Vocalised Noise] and the degree of the numerator would have been six that is a dot dot means [Vocalised Noise] it’s continued we are not taking a specific case but here we have said if the degree of the numerator is one greater than the degree of the denominator then the function is a pole at infinity

what we are going to say now is yeah

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no

put s [Vocalised Noise] equal to zero here you see this is omega one square this is omega three square omega two square omega four square so these were all be constants it is this s which will cause the function

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no not necessary because s could occur in the numerator

if it occurs in the numerator then the function has a zero at the origin

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no not necessary you can have a zero at the origin and a pole at infinity

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in this case i have not specified all i have specified is that there is an s in the denominator and that the numerator degree is one greater let's be specific let's say s square plus omega five square this is creating confusion this is a perfectly valid LC function right this is purely even this is purely odd

the numerator degree differs from the denominator degree by one numerator degree is one greater and therefore there is a pole at infinity put s equal to zero here the value is infinity and therefore it has a pole at the origin

it has a pole at infinity and it has two poles at plus minus two pairs of poles plus minus j omega two and plus minus omega two agreed let's consider this ah in a little more detail

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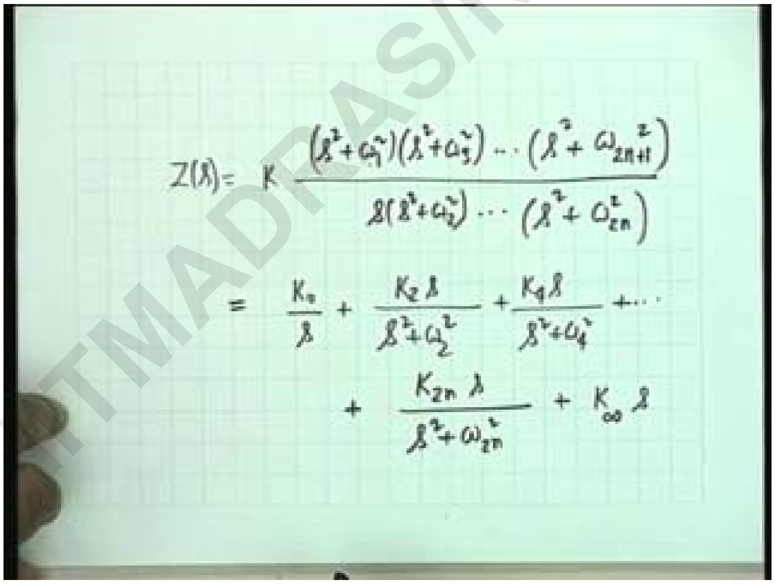

no let's ah [Vocalised Noise] let's generalize this let's say we have s squared plus omega one squared s squared plus omega three squared etcetera up to let's say s squared plus omega um

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two n minus [Laughter] one square ah no two n plus one square let’s say and in the denominator we have s times s squared plus omega two squared and so on up to s squared plus omega two m squared alright

let's generalize this then {we are} (00:15:30) we have made sure that the that the numerator degree is one greater than the denominator degree and because the denominator is odd it has a pole at infinity and it has pairs of poles at plus minus j omega two plus minus j omega four plus minus j omega two n and so on okay

now we also know that Z of s is here and therefore if it is positive real it’s poles on the j omega axis must be simple which we have taken care off none of them are multiple and the residues there must be real and positive

in other words if i make a partial fraction [Vocalised Noise] expansion of this then i should get terms like [Vocalised Noise] K zero by s plus plus K two s divided by s squared plus omega two squared alright

actually i should have written twice K two if i write in this form then the residue is [Vocalised Noise] K two by two do not forget this okay instead of writing two K do i prefer to write K two okay plus K four s divided by s squared plus omega four squared and so on so on plus K two n s divided by s square do not forget this s

if this is s is not there the function will be non pair okay okay so s squared plus omega two n squared then finally i must have i must take care of the pole at infinity that would be let's say let the residue there be K infinity then K infinity s agreed

the pole {tha}(00:17:19) that we assume that the function is a pole at infinity [Vocalised Noise] alright therefore K infinity is must come

there is a difference between the characters of K zero K infinity K two K four K two n while K zero and K infinity are truly residue K two K four K two n [Vocalised Noise] are not residues there twice the residue okay one must remember this

how do you find out K zero K two etcetera K zero is obvious K zero is you simply put s equal to zero then K times omega one squared omega three squared omega two n plus one whole squared divided by omega two squared etcetera up to omega two n square

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so K zero is obvious K infinity is also obvious what is K infinity in this form

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simply equal to K isn't am right because at infinity the living power here living coefficient is one so this tends to s to the power n this takes to s to the power n plus one [Vocalised Noise] and therefore K infinity is simply to K to find out the other residues [Vocalised Noise]

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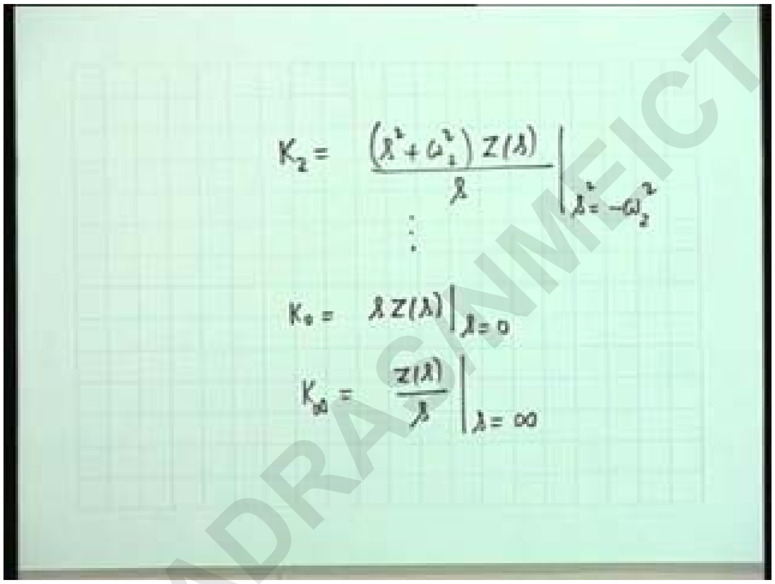

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for example K two what you do is multiple the function by s squared plus omega two squared multiply the function by this and divide by s alright and put s squared equal to minus omega two squared and so on for all other residues

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this is no this is twice

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twice the residue okay twice the residue alright

i can write the other terms also like this K zero for example is s times Z of s with s equal to zero this is truly a residue K infinity is Z of s divided by s with s equal to infinity

[Vocalised Noise] these are general formulas but K zero and K infinity should be obvious by inspection

whereas K two K four etcetera may require some amount of calculation but is it this calculation is not difficult because we are only handling real numbers we are not putting s equal to plus j omega or minus ((j omega)) (00:19:39) we are putting s squared equal to minus omega two squared and what [Vocalised Noise]

why are {ss} (00:19:44) why is one so sure that one we handle only a real numbers because we are dividing by s therefore we will get an even polynomial divided by an even polynomial and put s squared equal to minus omega squared alright okay by ah the hypothesis the Z of s is positive real

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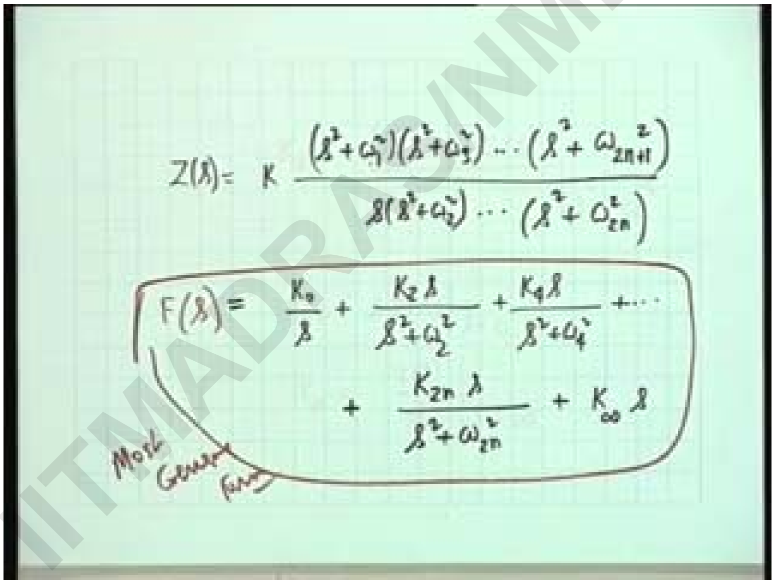

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K zero K two K four K two n K infinity they must all be real and positive alright

now i also claim ah (( )) (00:20:17) i also claim that this is the most general form of an LC driving point impedance this is the most general form

well how how can you differ how can another LC driving point impedance differ from this in character the only difference will be that instead of a pole at infinity it has a <a_side> ((00:20:41)) <a_side> zero well if the function if there is a function [Vocalised Noise] which has a zero at infinity all that it will happen is K infinity shall be equal to zero alright so{the}(00:20:51) in this general form if you put K infinity equal to zero you get a zero at infinity isn’t that right

if K infinity is zero and you put s equal to infinity here the {whoe} (00:21:01) the rest of the function is zero agreed

in a similar manner in a similar manner if the function has a zero at the origin all that we’ll happen is K zero shall be equal to zero now put this term away and look at the rest of it at s equal to zero this is residue and therefore i claim that this is the most general form of partial fraction expansion of an LC impedance and what i have said about impedance is also true about admittance and therefore in general LCDPI one must be able to express it in this form this is the most partial fraction expansion of an LCDPI driving point {imitance} (00:21:49)

it could be impedance or it could be admittance is it point clear the most general form [Vocalised Noise] what was the logic [Vocalised Noise] only deviations that can occur at that instead of a pole at infinity there can be zero at infinity instead of a pole at the origin there can be a zero at the origin or a combination of them how many combinations are possible

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four because for each of them there are two possibilities

so zero at infinity is zero at the origin zero at infinity pole at the origin pole at infinity zero at the origin pole at infinity pole at the origin {there is} (00:22:27) only there are four cases and all cases are covered by this partial fraction expansion alright okay

if this point is clear then {we be} (00:22:37) let’s be a little more specific and let's see we are considering an impedance

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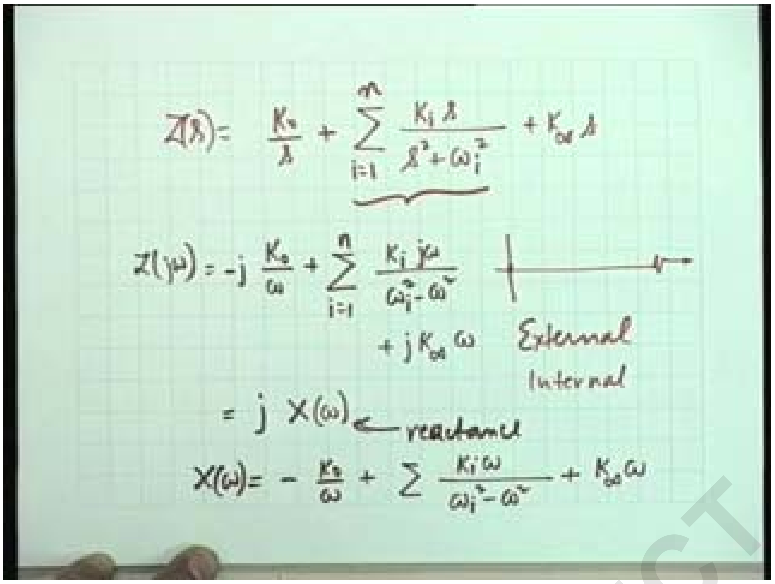

there is no loss of generality because we shall show what happens if we consider an admittance [Vocalised Noise] for an LC impedance the most general form is K zero by s plus summation [Vocalised Noise] Ki s divided by s squared plus omega i squared let's say okay

forget about those even subscripts and odd subscripts these are the poles let's say if there are n number of poles i equal to one to n plus K infinity s [Vocalised Noise] the commonsense question [Vocalised Noise] and i have said n number of poles is that correct

it has this function has n plus two poles because there is one at the origin one at the infinity now these poles the poles at origin and infinity poles or zeros the two points origin and infinity they are considered as external points external this is just a nomenclature

they are considered an external points in other words this function has two external poles okay or external {eve eve} (00:24:04) every LC function for every LC function the origin and infinity they must be critical frequencies both of them can be poles both of them can be zeros one can be zero one can be a pole

there are four such possibilities but we say nomenclature just a nomenclature just a language that both origin and infinity are [Vocalised Noise] critical frequencies and they are external critical frequencies

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if these two poles are external poles obviously these you should call internal poles so there are n number of internal poles and two external poles okay this is just a business of nomenclature the text book use it so i am also obliged to use it there is no other significance to this uh terminology

now if we consider the value of this function as i said this is the most general form of the LC impedance function if i consider the value on the j omega axis Z of j omega now since it is an LC driving point function it's real part should be equal to zero or which is indeed brought out you see that Z of j omega ah is equal to K zero by j omega which is minus j K zero by omega alright [Vocalised Noise] plus summation [Vocalised Noise] i equal to one to n [Vocalised Noise]

we shall have Ki j omega divided by omega i squared minus omega squared plus j K infinity omega okay as you see each time is imaginary each time is imaginary as it should be there is no other way because it's a LC there is no real part resistive part is zero it absorbs no power and therefore it must be purely imaginary

the {imagin}(00:26:09) the real part of Z of j omega is zero the imaginary part as you know is called the reactance of this impedance so if i put this as j times X of omega then capital X is the reactance the reactance of the LC network reactance the imaginary part of the impedance function on the j omega axis is the reactance and you see that capital X of omega is simply minus K zero by omega plus summation Ki omega divided by omega i squared minus omega squared plus K infinity omega alright okay is that okay alright now let's look at this a little more closely

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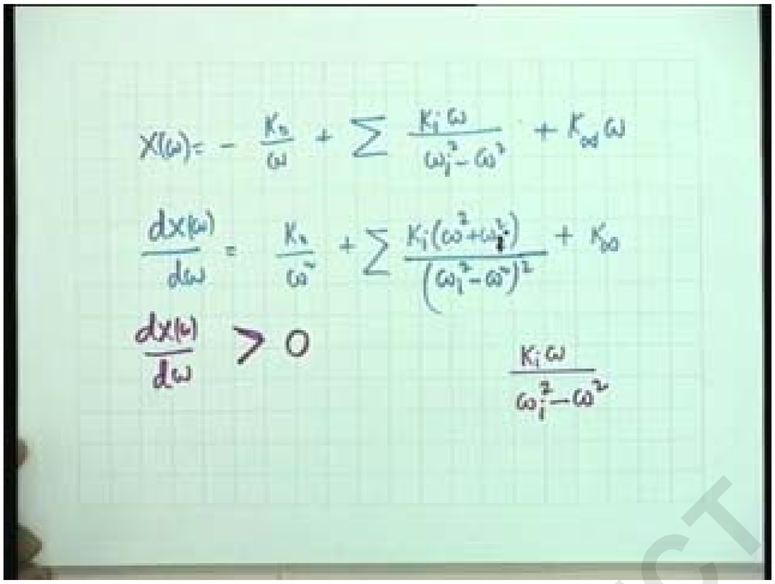

i repeat this here X omega is minus K zero by omega plus summation Ki omega divided by i omega squared minus omega squared plus K infinity omega

suppose i differentiate this with respect to omega [Vocalised Noise] i take the differential coefficient now you must understand one thing that X of omega as far as the points of blowing up is concerned we can’t say we can’t say X of omega has poles okay

Z of s had poles X of omega blows up at the same points as Z of s that is X omega blows up at omega equal to zero at omega equal to infinity and omega equal to plus minus [Vocalised Noise] omega okay and loosely we call them poles of X of omega

so we say X of omega has poles that omega equal to zero omega equal to infinity and omega equal to plus minus omega i alright [Vocalised Noise]

just a matter of terminology [Vocalised Noise] but as far as this differential coefficient is concerned [Vocalised Noise] you notice that the first term shall give you K zero by omega square the last term shall give you simple K infinity and each term in this if you carry out the differentiation [Vocalised Noise] it should be [Vocalised Noise] Ki omega squared plus omega two squared divided by omega i squared minus omega squared whole square

i leave the algebra to you you can show very easily that the

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ohh i am sorry it should be i [Vocalised Noise] let me correct it omega square plus omega i squared divided by omega i squared minus omega squared at the whole square

if you look at this if you look at the individual terms you see that each of them you see K zero Ki K infinity they are real and positive omega squared can never be negative omega squared plus omega i squared is also positive

this whole squared omega i squared minus omega squared the whole squared is also positive K infinity is a positive quantity and therefore this must be strictly positive this is a the most important property of LC driving point impedance that the reactance function slope of the reactance functions is always positive can it be equal to zero <a_side> ((00:29:56)) <a_side> no

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[Laughter] why not <a_side> ((00:30:01)) <a_side>

if all the residues at zero if all residues are zero the function is identically equal to <a_side> ((00:30:07)) <a_side>

zero [Vocalised Noise] therefore [Vocalised Noise] therefore it’s a trivial case and you need not consider this

therefore we conclude that dx d omega that is the slope of the reactance function must be strictly positive now this creates problems and it also offers summations the problem is the following

let's take this function a typical function let’s say Ki omega divided by omega i squared minus omega squared let's take a typical turn inside the summation [Vocalised Noise] i repeat this Ki omega divided by omega i squared minus omega squared and let us say we are interested in plotting capital X of omega versus omega

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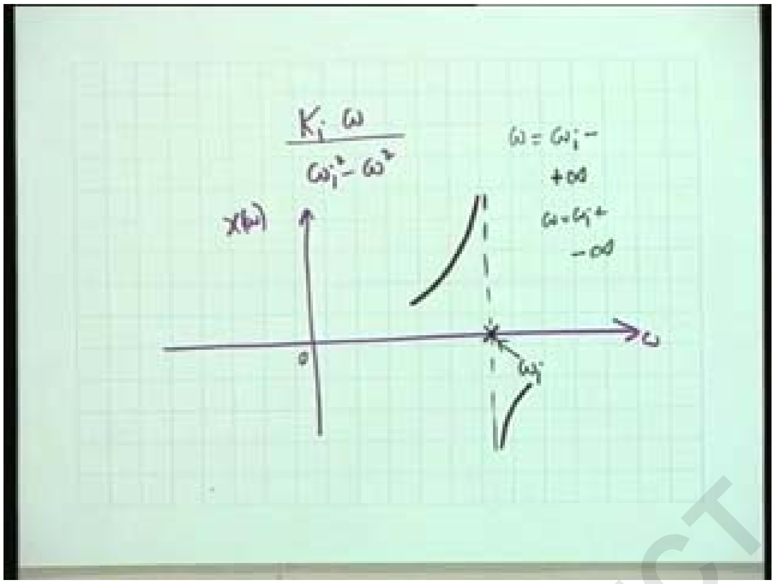

and omega i is somewhere here [Vocalised Noise] omega i is somewhere here and you see at omega equal to omega i the function blows up and therefore it is a pole and you know poles are denoted by crosses

so let's say this is omega i on the negative side off course there is a minus omega at which also and it’s ah it blows up but we’ll show only the positive side from zero to infinity now if omega is less than omega i that is on this side if omega is less than omega i obviously the function is positive okay and omega greater than omega i the function is negative alright

so um but at omega equal to omega i it blows up and therefore at omega equal to omega i there must be a discontinuity a discontinuity you see at omega equal to omega i minus the function is plus infinity [Vocalised Noise] right at omega equal to omega i plus the function is minus infinity and therefore if we sketch this function it must go to infinity like this with a positive slope

the slope has always to be positive and it must come up like this at omega i omega i plus is this point clear that at omega equal to omega i the the function has a discontinuity it goes from plus infinity to minus infinity [Vocalised Noise] can it be other way can it come from minus infinity can it be minus infinity here at omega i no

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[Vocalised Noise] alright therefore suppose we have a function

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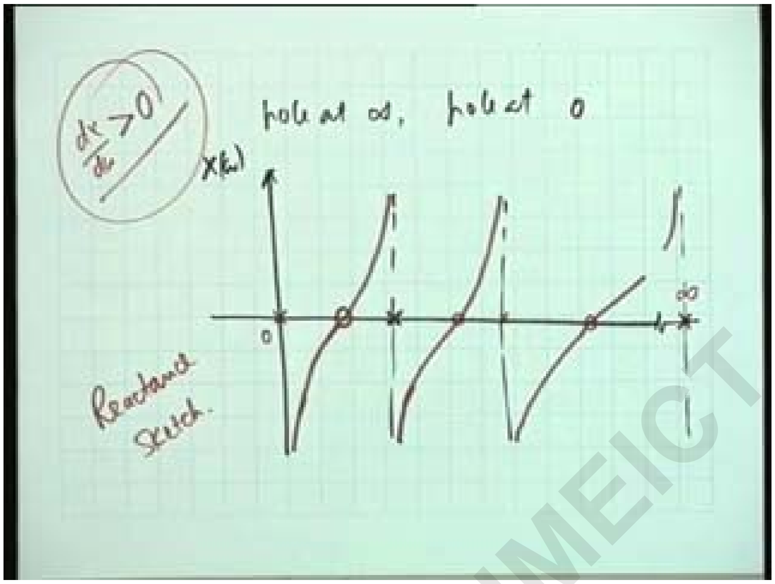

which has a pole at infinity pole at origin pole at origin alright then and we want to plot x of omega the slope is strictly positive there is a pole at the origin there is a pole at infinity which we show by mention of break and let's say point infinity here this is origin this is infinity okay now [Vocalised Noise] how should the function how should the function come from zero

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from minus infinity why

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okay can can it go like this from minus infinity it must rise but then what should it <a_side> ((00:33:50)) <a_side>

can it can it go to another infinity

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before passing through a zero

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no

therefore the next critical frequencies starting from the origin must be a zero

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okay must be a zero it must cross zero and then it must go to infinity at the next pole and the next pole it must go to infinity like this which means [Vocalised Noise] then this will be a point [Vocalised Noise] this will be a pole this will be a pole and this would be a zero agreed next [Vocalised Noise]

suppose next it must rise from minus infinity

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yes

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oh because the slopes dx d omega has always to be strictly positive

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continues this <a_side> ((00:34:48)) <a_side>

oh like this

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oh that's what i have done

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[Vocalised Noise] oh the curvature like this

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yes ah in theory yes but ah the curves [Vocalised Noise] if you plot it the curves are [Vocalised Noise] like the tangent curve like the curves of tangent theta versus theta okay

the curvature is like this it it just happens it could be other way round for another type of network alright i don't we don't know

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now similarly the curve now must rise from minus infinity with a break here and the next critical frequency must be a zero okay must be a zero and it goes to the next critical frequency so this is a zero and this is a pole the next one is a pole

the the [Vocalised Noise] infinity point is also a pole [Vocalised Noise] okay now suppose what can there be in between between this pole and this pole

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there must be a zero at least one zero okay let's this zero be here then obviously the curve goes like this and then there is a break and it goes like this [Vocalised Noise]

this is a typical plot of the reactance function that is capital X versus omega and this is called a reactance sketch a reactance sketch there are three other possible cases three other possible cases depending on whether the origin is a is a pole or a zero and the infinity is a pole or a zero and i need the rest of the sketching to you it would be a fun

for example if the origin is zero the next critical frequency must be a

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must be a pole and it raises like this if the infinity point was a zero instead of a pole then it must come like this and go to zero at infinity okay [Noise]

so the three other possible sketches i leave it to you and in each case you can verify that poles and zeros must alternate must alternate this is a reflection of the positiveness of the slope property therefore for an LC driving point impedance function

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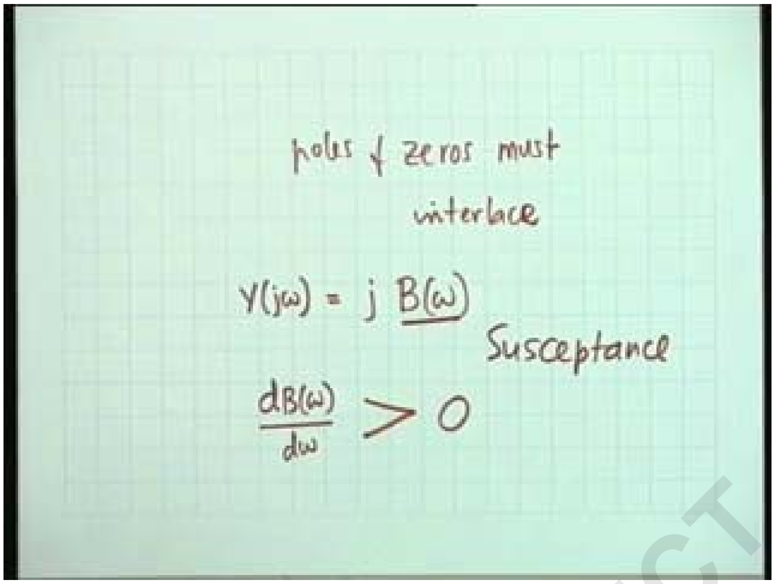

we conclude that poles and zeros must there is a there is a there is a poetic word for it instead of alternate must interlace [Vocalised Noise] is slightly better use the English poet's uh quite a bit okay

poles and zeros of an LC driving point function must interlace and what we had said so for about impedance function should also be true for admittance function that is [Vocalised Noise] if i write an Y of s and then an Y of j omega Y of j omega [Vocalised Noise] should also be purely imaginary

so we write this as j times B of omega where B what is the name for B there is a name for B susceptance why you don't know about this susceptance Y of j omega is the real part is capital G which is conductance and imaginary part is the capital G happens to be zero here because it's an LC network

so on the j omega axis the admittance is purely imaginary [Vocalised Noise] and the imaginary part is known as susceptance and by the same argument we can show that dB omega d omega also has to be strictly greater than zero strictly positive both slopes are strictly positive [Vocalised Noise]

let's state

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yes

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same analysis there is no difference same analysis let's take a few examples suppose we have a function

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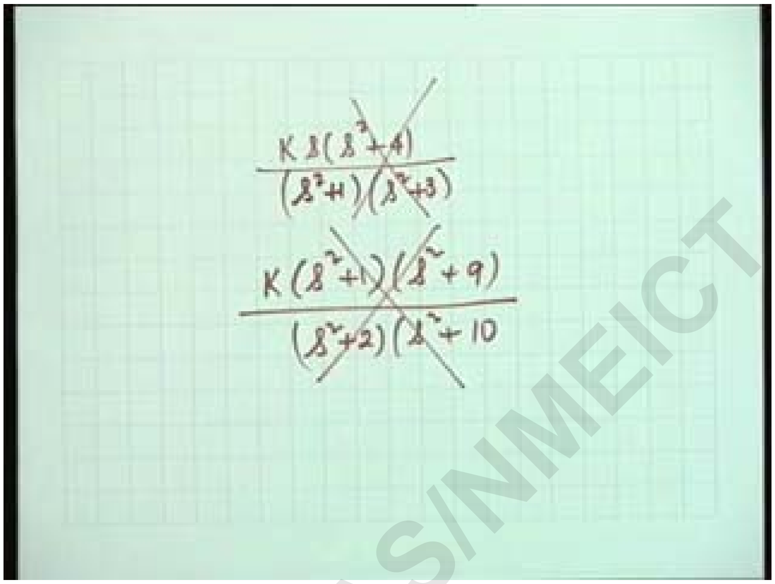

K s squared plus four divided by s squared plus one s squared plus three is it an LC [Vocalised Noise] can it be an L C

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the answer is no [Laughter] this is zero at the origin pole at square root of one then there is a pole at square root of three

so two poles come adjacent to each other so this is not LC okay poles in zeros must interlace there is a zero at the origin s equal to zero the next is a pole at one next is also a pole at root three root three is one point seven three two which is less than root four and therefore there are two poles adjacent to each other so this cannot qualify as an LC function

let's take another K times let’s say s square plus one [Vocalised Noise] s square plus nine divide by s square plus two s square plus ten

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both the even it is not an odd function and therefore this cannot be an LC can you bring an s somewhere so that it becomes LC where should s be

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denominator alright next question this is not (( )) (00:40:44) i have i don't show the poles

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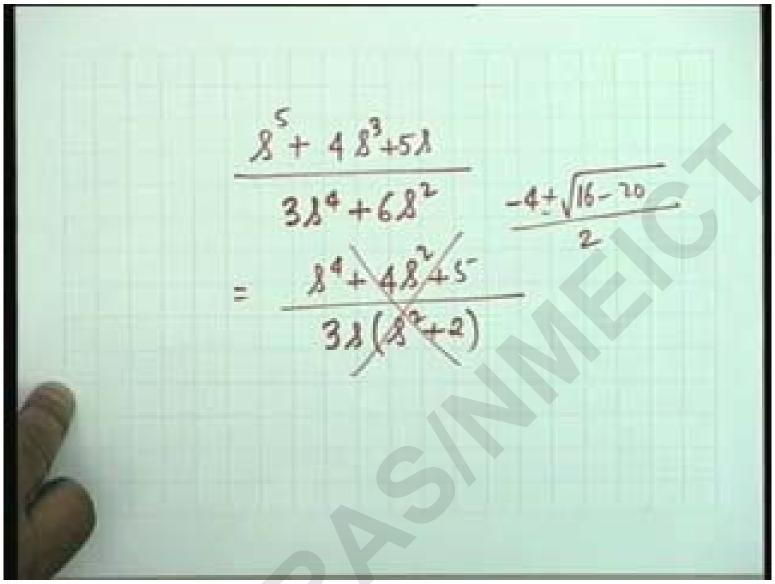

i have s five

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pardon me

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that's right s has to be in the denominator other they would not interlace

now i don't show the poles directly [Vocalised Noise] i s five plus four s cubed plus five s [Vocalised Noise] divided by three s to the four plus six s squared [Vocalised Noise] how do i test whether this is LC or not [Vocalised Noise] well you see the the it is not the function can you cannot make a judgment by looking at the function because it is not in this in the lowest order

you see there is an s there is a factor of s cancelling from the numerator and the denominator so before making any conclusion don't make a conclusion that it is a double

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order pole at the origin obviously if you look at this function it has a double order because of s square but there is an s in the numerator so first you write it in this form in the lowest form that in the form in which the numerator and denominator what is the mathematical term numerator polynomial and denominator polynomial have no common factors

what are such polynomials called

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no

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another guess

not proper

nothing is improper [Laughter]

they are primes with respect to each other alright three two numbers let's say three and twenty-one they are not primes with respect to each other but three and seven are primes with respect to each other three and eight are prime similarly with respect to a polynomial [Noise]

if two polynomials have no common factors they are said to be primes with respect to each other so i reduce this to primes four s squared plus five divided by three s cubed plus six s i can write this as three s times s squared plus two okay

now the question does it qualify as an LC you see the numerator is an even polynomial the denominator is purely odd the poles are on the j omega axis one at the origin one at plus minus j root two one pair and there is a pole at infinity

so almost everything is satisfied

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pole at infinity (( )) (00:43:17) four and s cube okay but [Vocalised Noise] the problem comes here and you see this is obviously this is obviously Hurwitz there is no missing term but even then alas the zeros are not in the j omega axis

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you see the zeros are at minus four plus minus sixteen minus twenty divided by two <a_side> ((00:43:39)) <a_side>

so these are not on the j omega axis the zeros are complex at complex value of s square therefore these doesn't qualify as an LC function [Vocalised Noise] alright

so one must be careful this this said all the ingredients of qualifying for an LC functions but ah unfortunately it is no now let's go back to a most general partial fraction expansion of an LC function

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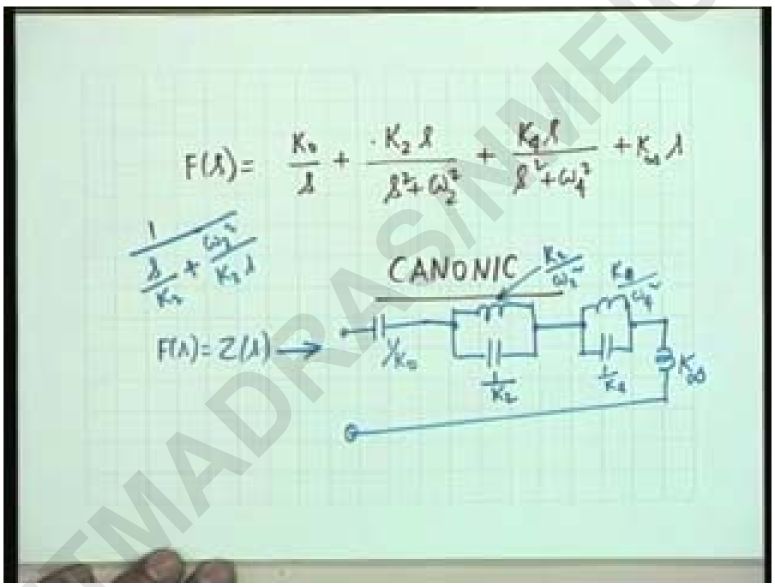

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what we had was K zero by s plus let's take a specific case let's say K two s divided by s squared plus omega two squared plus let's say K four s divided by s squared plus omega four squared and let's say we have K infinity s a specific case okay

now [Vocalised Noise] how many specs are there in this function in this expression how many specifications are there what is the total number of ah data or information that is to be supplied to specify this function obviously

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six what are these six K zero K two K four K infinity omega two and omega four and if we can synthesis this we have six elements and no more that would be the best

you cannot hope to synthesis with less than six because there are six pieces of information you required six independent elements alright you might be able to do it in more than six more than six is no problem

you have more money you put ah instead of a one in the inductor you put point five and point five in series

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oh there are six constants K zero K two K four K infinity and you change any of them obviously the network shall differ

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for satisfying six specs we required at least six elements at least six different values are needed and such a synthesis is given the name of CANONIC CANONIC synthesis

CANONIC is given the network function the number of pieces of information required to specify it if the number of elements is exactly equal to that then it is called CANONIC and the synthesis should be obvious

suppose F of s is an impedance [Vocalised Noise] if F of s is an impedance then obviously it has been put the partial fraction expansion puts the impedance function as the sum of one two three four times and each time each time the synthesis is obvious

the first one for example is

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a capacitor one K by naught that is correct the next one obviously is a parallel connection of an inductor and a capacitor similarly the next one is also an inductor and capacitor and finally we have an inductor K infinity let's see the element values

what is this elements values

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K two by

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omega two square and this one

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one by K two that is correct do you see how we did this no while we wrote K two s by s [Vocalised Noise] we wrote this as one by s by K two plus omega two squared by K two s and if this is an impedance obviously the denominator is an admittance so it is sc plus one over s l

s c c must be one by K two and l must be K two by omega two squared is that clear similarly this should be one by K four and this must be K four by omega four square and we'll see that we require exactly six elements alright if F of s was an admittance

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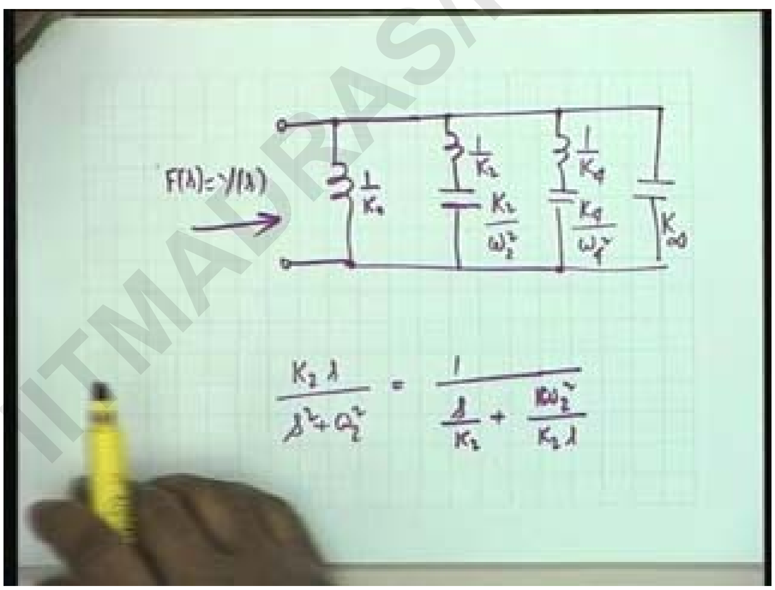

if F of s was an admittance F of s is Y of s then obviously all these elements should come in parallel and K zero by s will now represent

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an inductor of value

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one over K zero and each partial fraction term that is K two s by s square plus omega two squared if this is to be an admittance then obviously it is a series connection of l and c

so it is K two omega two square by K two s and therefore what we have is an l and a c in the series and [Vocalised Noise] the value of the inductor will no be

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one by K two and this would be K two by omega two squared

the next one would be similarly one by K four and K four by omega four square finally we shall have

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a capacitor of value K infinity and if you compare these two networks if you compare is the whole thing on the screen

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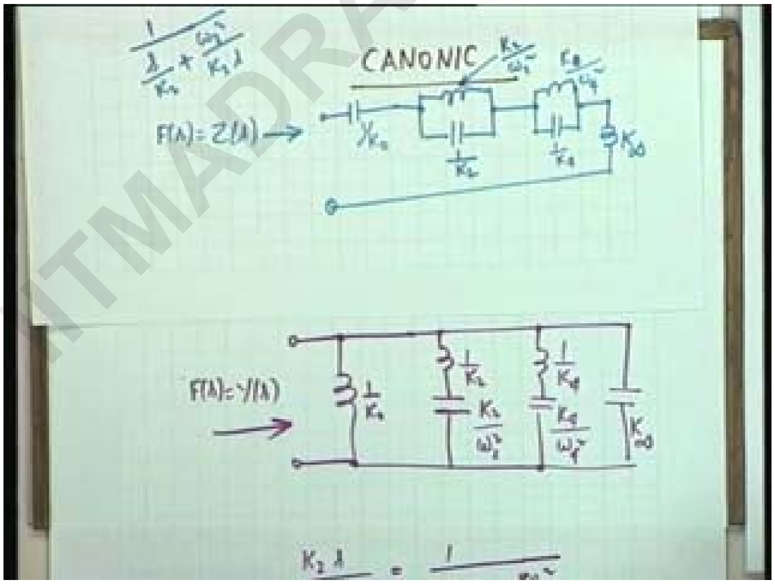

if you compare these two networks (( )) (00:49:45) exact duals of each other is a series network is a parallel wherever there is a parallel connection of the series connection this is the impedance this is the admittance and the element value [Vocalised Noise] are also duals ofof each other

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one by K zero is the value of a capacitor here it is a value of a inductor okay K infinity is an inductor K infinity is capacitor similarly this inductors and capacitors also interchange so that exact duals of each other and this synthesis of an LC impedance was first given by a gentleman by the name R M Foster Ronald M foster

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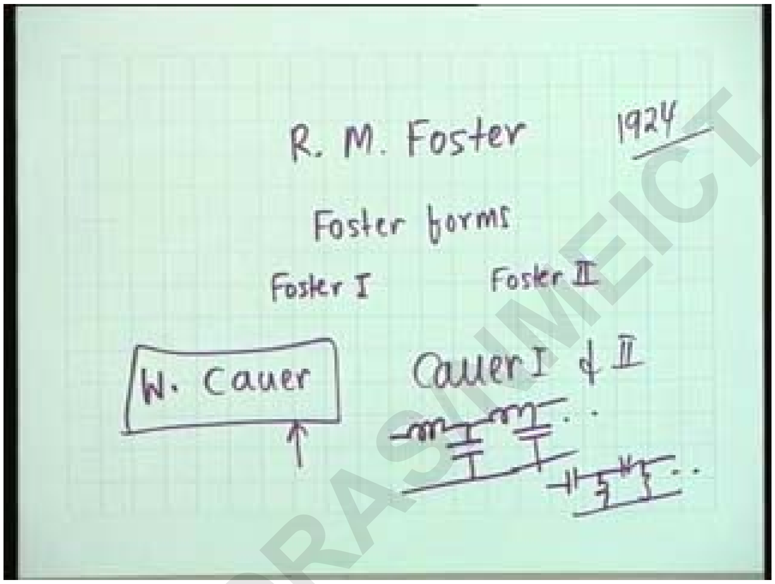

i haven't heard about his death and i believe he is still alive this was given in nineteen twenty-four more than seventy years ago the man must now be more than hundred say must be he is alive he was to work with {in the} (00:50:50) MIT Massachusetts Institute of Technology and this was part of his PhD thesis and this synthesis this synthesis which gives the synthesis in terms of a series connection or a parallel connection

they are known as Foster forms Foster forms and this series connection sometimes is called Foster I and the second the parallel connection is called Foster II Foster I and Foster II it also incidentally [Vocalised Noise] incidentally makes a limited verification of the statement that you made at the beginning that if a synthesis problem has one solution it has indefinite number at least we have got two

the other [Vocalised Noise] form and you see that in the in the second form also Foster II is CANONIC so these are two CANONIC forms of synthesis due to Foster there there exist another two basic CANONIC forms which are due to a German by the name Wilhelm Cauer unlike Foster Cauer is dead long ago Cauer died during the second world war by he died at the hands of Nazis he must have been achieved i am not sure don't take my words for it but during the second world war the only thing that is known is that after the second world war was over

you see during the second world war there was intense activity in electrical engineering particularly circuits circuit theory intense activity because telephone communication was on the anvil and people needed filters people needed filters to separate speech from noise and things like that intense activity throughout the West

where UK United States Germany and the whole of US and this gentleman working in Germany he derived independently of anybody else independent of Foster he didn't know the existence of Foster similarly Foster didn't know the existence of Cauer he wrote a a book those days it used to be called a ((tritice)) (00:53:14) or something a thick book called synthesis of linear communication networks

it was written in German and the manuscript was discovered only after the world war was over and they found that there is a beautiful synthesis of many kinds of networks including the simplest the LC network

LC network synthesis in fact it is one of the simplest synthesis problems okay and those two forms are known as Cauer forms very simple minded logic but they are ah a testimony to the brilliant mind that Wilhelm Cauer had and the Western world ah it required quite a bit of time for the English speaking word to discover the work of Cauer as late as ah as the end of ah beginning of fifty or may be end of forties that the English speaking word discovered this and then they translated this the book is available it's a thick book one copy is available in the library and these are called Cauer also gave two forms [Vocalised Noise] Cauer one and Cauer two he didn't call them Cauer one and Cauer two he simply said it i can synthesis in this form unfortunately he didn't discovered the Foster form

he discovered Cauer I and Cauer II and both of these are in the form of a ladder is a beautiful synthesis Cauer I for example is series inductance and shunt capacitance this is not a pure series connection or pure parallel connection and Cauer II is series capacitor and shunt inductor it is of this form and these are the two forms that we shall discus tomorrow at five pm here

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