Circuit Theory

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Lecture - 39
Problem Session 9: Realizability, Hurwitz Polynomials and PRF's

Thirty-ninth lecture problem session 9. Our problems today will be concerned with realizability theory Hurwitz polynomials and positive real functions, the first problem 10 1.

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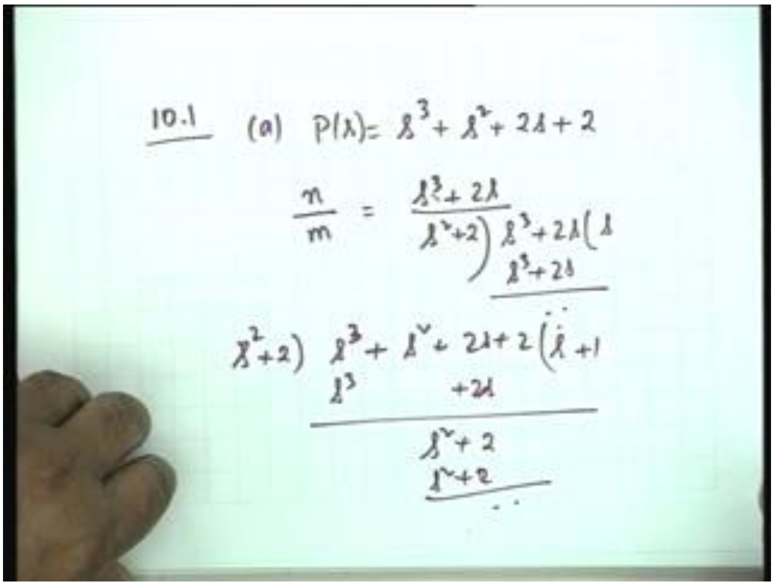

We will do some of them 10 1a let us say where P of s is equal to s cubed plus s squared plus twice s plus 2. If I do this blindly we have to form the ratio of n by m because, the odd power what we have to do is to test if test the polynomial for Hurwitz property. Odd it's the degree is odd. So, you take the odd part to the even part which is s cubed plus twice s divided by s squared plus 2. So, s cube plus twice s s s cube plus twice s.

So, this is premature termination and the last device Premature. Because, we expected 3 quotients we have got only 1 and the last quotient therefore, must be a factor of both the even part and the odd part. In other words, s square plus 2 must be a factor of P of s. So, we divide although it is obvious we carry out the procedure s s cubed plus 2s and s square plus 2. So, plus 1 s square plus 2. This is not a continued fraction expansion, this is a simple division, and therefore our P of s.

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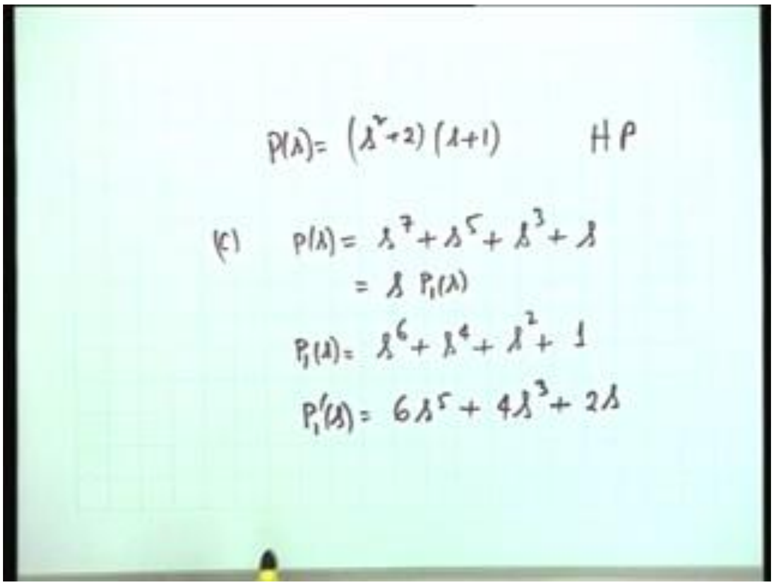

P of s is therefore, equal to s square plus 2 multiplied by s plus 1 and obviously, this is the Hurwitz polynomial. Its roots are obvious 1 is at minus 1 and the and there are 2 1 the j omega axis plus minus j root 2. We skip b then look at c Can anyone tell me by looking at b whether, it's Hurwitz or not?

Because there is a missing term. C is a purely odd polynomial and therefore, we have to use P prime s. P of s is s to the 7 plus s to the 5 plus s cubed plus s this is a purely odd polynomial. There is a slight simplification that is possible. We have to test this for Hurwitz. So, why do not we test? Why do not you see? That s is a factor which is permitted in the Hurwitz Polynomial. So, call this s times P1 of s where P1 obviously, will be purely even. P1 would be s to the 6 plus s2 the 4 plus s squared plus 1. Is that clear?

I have discovered that there is a 0 at the origin since, it is purely odd there has to be a root at s equal to 0 which is perfectly all right for Hurwitz polynomial. So, we will have to test only the rest of the polynomial P1. And since, this is purely even we take its first derivative P1 prime s is become 6s to the 5 plus 4s cubed plus twice s all right and we have to know carry out, the continued fraction expansion.

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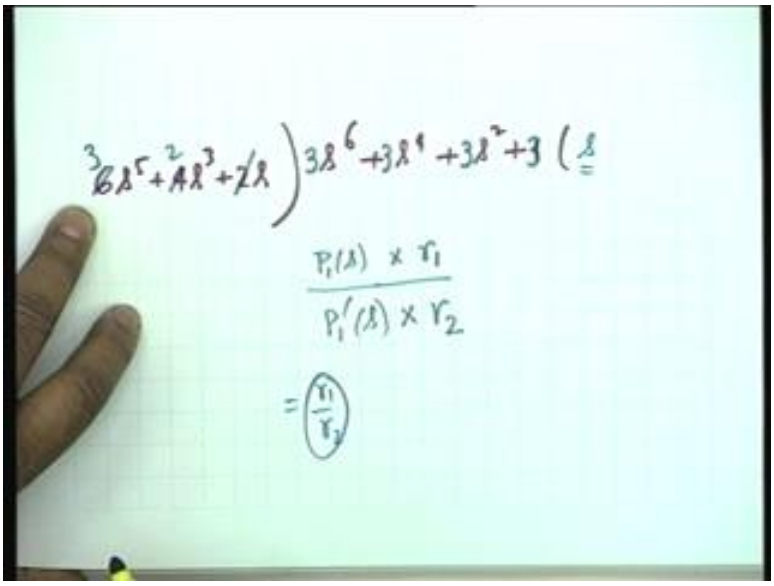

So, we do that you see what we have to do is s to the 6 plus s to the 4 plus s square plus 1. Let's, do some more tricks of detrain what we have to divide is by 6s 5 plus 4s cube plus 2s. The Hurwitz test is not disturbed if either the divider, divisor or the dividend are multiplied by a positive constant. So, we can change this to keep number small we can divide by 2 let say, this is 2 and this is 3.

Then, again to keep number small and manageable we could divide this whole thing by 3, we could multiply this whole thing by 3. Is that clear? Why we can do that? Because, we want to keep our numbers, nice numbers. All that we are concerned about is the sign of the coefficient of the quotient. So, it does not change if I multiply or divide by a positive integer. We will simplify that also, if it does effect further division. We will simplify that also we will multiply or divide by a positive constant.

No, you see just a minute what we are doing is we are making a continued fraction expansion of P1s by P1 prime s. What I am saying is I could divide this by an arbitrary positive constant R1. I could multiply this by another arbitrary positive constant R2 without effecting the signs of the quotients. Is that clear? So, it will not affect, it will effect. This signs of the coefficients of the quotients all that it effects is the the quotient the value of the quotient.

No. Why should it? Because, it will depend on my value of the quotient. It will not effect. You see this is simply R1 by R2 multiplied by whatever, the CF is, but R1 by R2 is a positive number. So, it will not effect. This we do to keep numbers small if you are fussy you go ahead with the original 1, I have no objection end up in one-sixth 1 thirty-fifth and so on and so all right .

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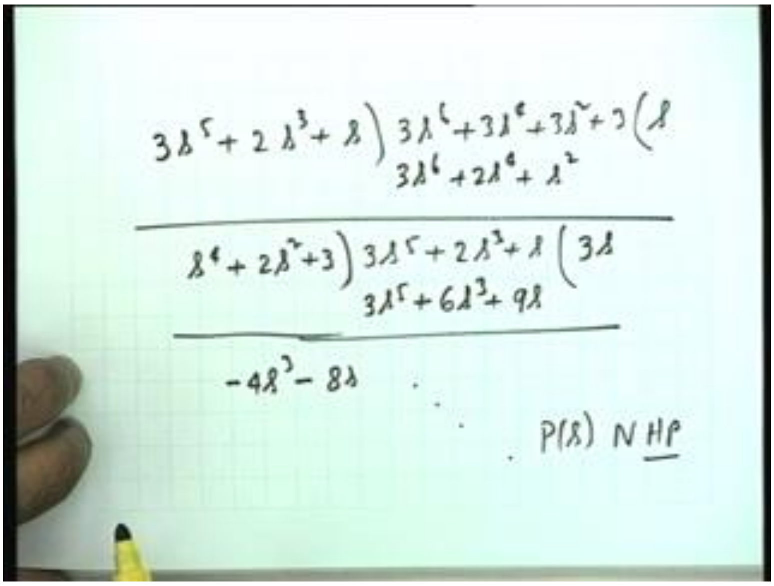

So, what we are doing is 3s to the 5 plus 2s cubes plus s that divides 3s to the 6 plus 3s to the 4 plus 3 squared plus 3. So, s is the quotient I get 3s to the 6 plus 2s to the fourth plus s squared. And. So, the remainder is for reasons that I have told you simple reasons I bring the remainder here. So, I get s to the 4 plus 2s squared plus 3 the numbers are not too bad. So, 3s to the 5 plus 2s cubed plus s my quotient is 3s.

So, 3s to the 5 plus 6s cubed plus 9s. And the remainder is minus 4s cubed minus 8s and I do not have to proceed further. As soon as I get a negative coefficient here I know that the next quotient shall have to be, shall have to have negative coefficient. And therefore, I conclude P of s is not Hurwitz polynomial all right. We do not have to carry out, we do not need to complete it as soon as, we get a negative coefficient you know that the job is done.

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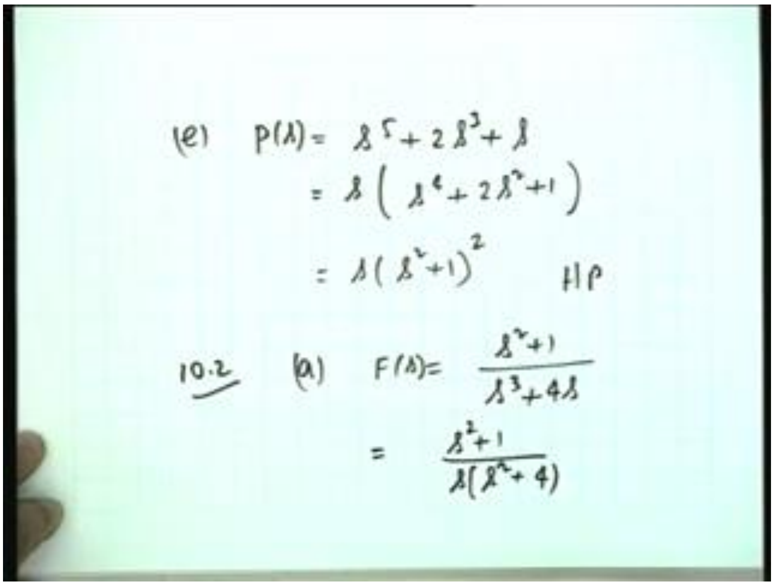

We skip d we look at e. Our e is also purely odd e is s to the 5 plus 2s cubed plus s. We do not, we do not differentiate this first what we do is we simplify this by taking a s common and I get s4 plus 2s squared plus 1. Do I have to carry out continued fraction? And since, it obvious that this is s times s squared plus 1 whole squared. Had I done the continued fraction expansion what you expect of this that would have been?

Premature termination either got 1 quotient. Anyway, you can do this therefore, this is Hurwitz. This is a Hurwitz polynomial F I have already done in the class. So, we skip that. 10 2, 10 2 says determine whether the following functions are PR. For the functions with the denominator all ready factored perform a partial fraction expansion first.

For example: a function is F of s equal to s squared plus 1 divided by s cubed plus 4s it is says determine whether this function is PR or not. For these functions denominator already factor well this is also approximately factored. Because, you know s is a factor then s square plus 4. So, make a partial fraction expansion s square plus 1s into s square plus 4.

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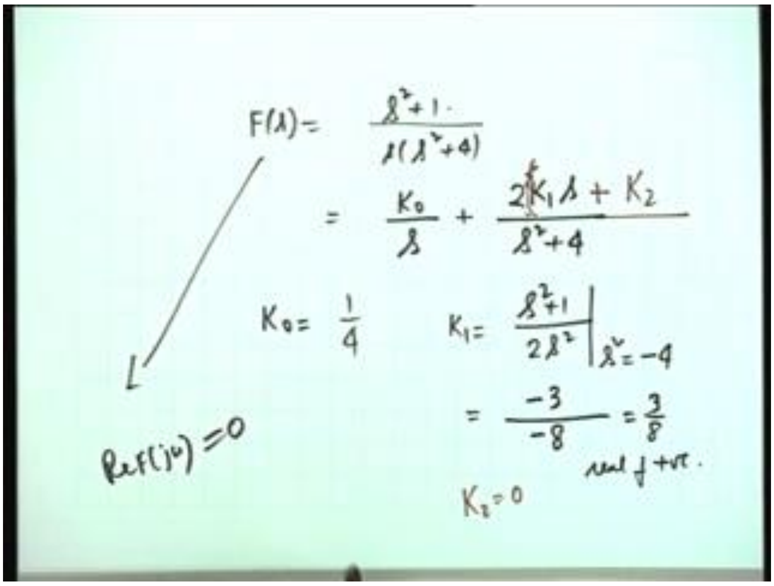

Now, in this partial fraction expansion F of s equal to s square plus 1 divided by s into s square plus 4. The partial fraction expansion do not bring in the complex poles. The imaginary poles you make a partial fraction expansion in terms of s square plus 4 in the denominator. That is you write there is a pole at the origin, corresponding to this you write K0 by s.

And corresponding to the next term if the function is to be positive real then the residue must be positive must be real and positive. But we do not know what it is. So, we assume that the residue is let us, say K1 twice K1s divided by s square plus 4 some residue. And you notice that K0. Can you see what K0 is? 1 by 4 because, at s tends to 0 it is 1 by 4s.

And what is K1? K1 is s square plus 1 multiply by this and divided by twice s. So, 2s squared 2s squared equal to minus 4 that is K1. So, this is minus 4 plus 1 that is minus 3 and minus 8 which is equal to 3 8 real and positive.

Do I have to do anything else? Do I have to check the real part on the j omega axis? What we have done is discovered that there are poles on the j omega axis we have verified that none of the poles are multiple they are simple poles. And we have verified that the residues at the poles K0 and K1 they are real and positive. The next part should be, the real part of F of j omega. Now, do we have to check that? Do not you see that the real part of F of j omega what is the value?

Identically equal to 0. Because, this is a odd rational function, this is even divided by odd. So, it is an odd rational function. At s equal to j omega it is purely imaginary. So, the real part is 0. In other words, the conclusion is that the function is positive real. Can we that is a good question. Why do we write 2K 1s? Because, yes. No, but there was a constant term. You recall, that this was K1s minus K2 times omega 0 that is 2K2.

So, in all in all Faraday's you should have written this as 2K1s plus K2 and discover that K2 equal to 0. It is a very good point it should be done, because you do not know whether the function is PR or not. You have to find out K1 and K2, You can easily see that K2 is equal to 0 here. How do you see? There is no term of coefficient s. Does that? Does that satisfy everybody?

Do the way you like, but what I say it is a purely odd rational function numerator is purely even. Now, by combining this 2 if K2 is nonzero we shall not get any even polynomial in the numerator. A very simple common sense are to be used all right therefore, K2 must be equal to 0. The next, the next term that you take is we will skip b and c we will look at d. We will look at 10 2d.

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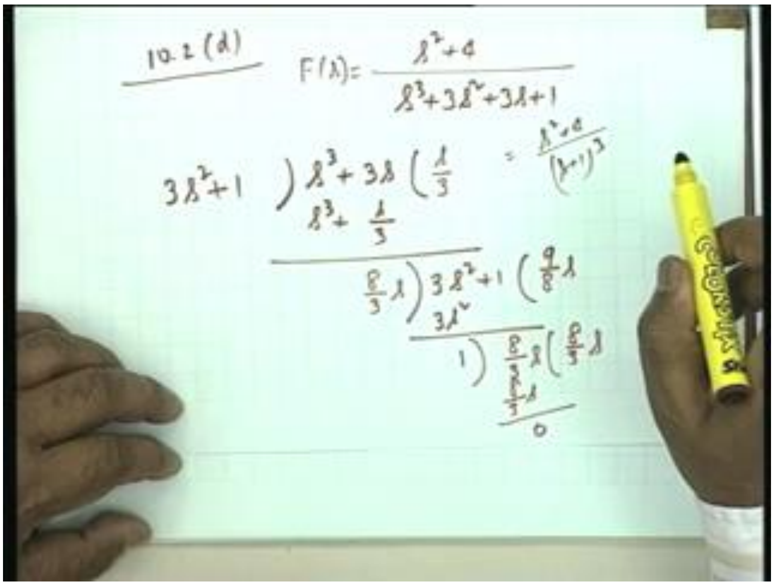

The function is F of s equal to s square plus 4 divided by s cubed plus 3s square plus 3s plus 1. The denominator here is not factored. So, we will have to go we will have to go step by step. First it is obvious that F of s is real when, s is real first part is. Now, to see whether it has right half plane poles are not we must make a continued fraction expansion of s cubed plus 3s divided by 3s squared plus 1 right.

We must check d of s the denominator polynomial whether it is Hurwitz or not. And for that we take the higher power term, that is the odd part divide by the even part and find the continued fraction expansion. I have s by 3 I could have multiplied this by 3 I could have done that. Let see s by 3. So, we get s cubed plus s by 3 this is 8 by 3s is that clear 3 times 3 9 minus 1. 8 by 3s, 3s square plus 1. So, my quotient becomes 9 by 8s 3s squared 1 8 by 3s 8 by 3s 8 by 3s 0.

The degree is 3 and I have got exactly 3 quotients no premature termination therefore, no j omega axis poles. And the denominator is indeed a Hurwitz polynomial therefore, therefore, it has no poles the function has no poles in the right half plane that part is. We do not have to check any residues. I could have done that also if I have discovered it earlier I would have done. If it is s plus 1 whole cube then there are multiple poles at s equal to minus 1. Does that violate the positive realness? No, not necessarily.

You must check the real part. You see what Mayank is saying why could not someone else saying, why could not you see that this is s square plus 4 divided by s plus 1 whole cubed indeed it is, which means that, there is a pole at minus 1 of multiplicity 3. Now, does this violate positive realness? No, not necessarily multiple poles on the j omega axis are not allowed.

Now, because there are no multiple poles on the j omega axis does not guarantee that the function is pure we still have to check. Let us, check. We check the real part now.

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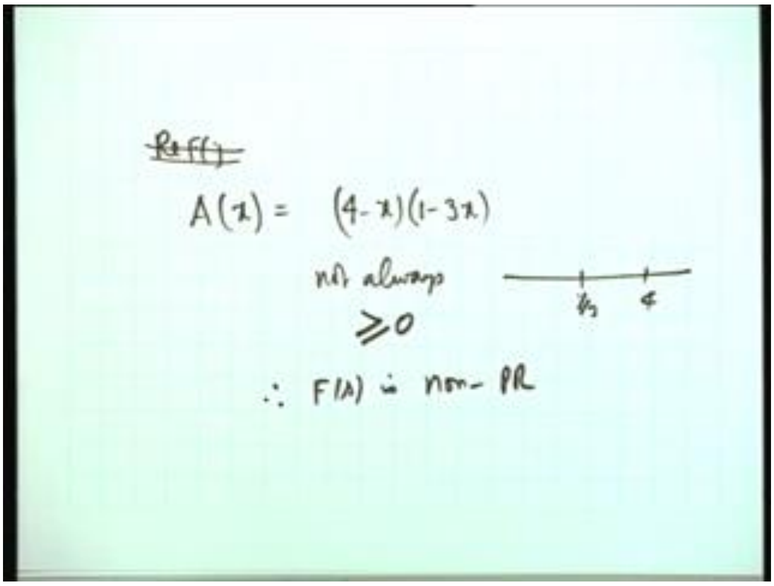

That is real part of F of j omega no we do not write real part F of j omega. We write the polynomial A of x directly. A of x becomes 4 minus x from the numerator then 1 minus 3x then that’s all. Because n1 is equal to 0 the numerator odd part is equal to 0. And obviously, this will this will not always be positive. Obviously, isn’t that right? It is positive only when, both terms are positive or both terms are negative that cannot be guaranteed.

So, not always. We do not have to check which range, you know which range it is positive, which range it is negative we do not have to find that out. As soon as we see, there is root on the positive real axis x equal to 4 there is root at on the positive real axis at x equal to one-third. So, at one-third and four at both places the function changes sign and therefore, not always greater than equal to 0. Therefore, F of s is non-PR is it now all right .We shall skip e also and go to 10.3 is simple we have all ready discussed in the class 10.4.

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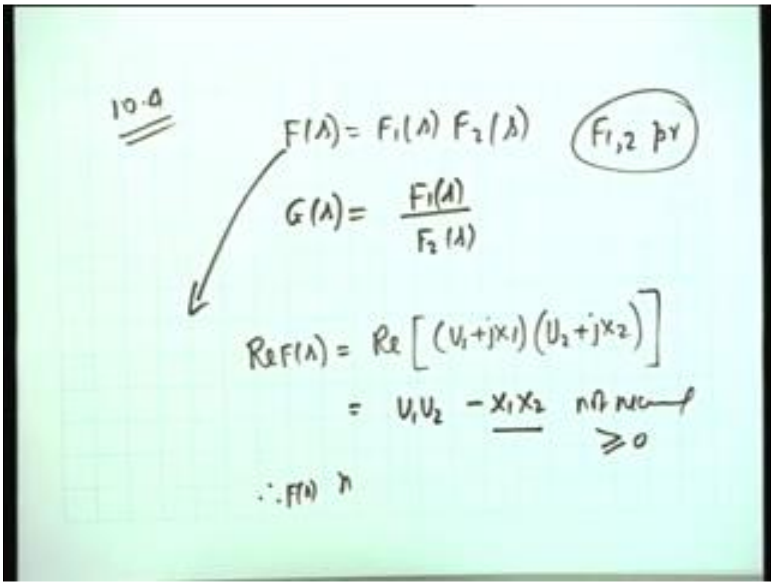

So, that the product of 2Pr functions need not be Pr. So, that the product of 2PR functions need not be Pr F1, 2 are PR you have to show that this is not necessarily PR. The first part also show, that ratio of 1Pr function to another Pr function. That is let us, say another function which is F1s divided by F2s this is not necessarily Pr you have to show. And then give 1 example of each, give 1 example of each.

That is give an example in which the product is not Pr although individually, they are PR the ratio is not PR although, individually they are Pr. We will prove, we will show for this for the other 1 it should be absolutely clear. You see what we have to do F1 and F2 are Pr that you have to show that the product is not necessarily Pr. Now, as far as the first condition is concerned in the definition that is s real F of s real that is satisfied. So, it is the second condition.

So, here you do not have to go to the j omega axis because, nothing is known about F1 and F2. You do not know whether they have poles in the j omega axis you do not know whether, well we know that they are if they have poles in the j omega axis. They are simple, but these 2 poles maybe coincident all right F1s may have a j omega axis pole at the same point as F2s then they will have a multiple pole at s equal to j omega. So, this is 1 reason.

But even if they do not, even if the product does not have a multiple pole there is this real part consideration. That is suppose, real part of F of s I am taking the complete s plane now. Complete right half s plane I do not have to go to the special case. It can be proved in a more general terms. Real part of F of s is the real part of the products of the 2. So, let us say 1 is u1 plus jx1 and the other is u2 plus jx2 all right.

All that you know is u1 is greater than equal to 0 and u2 is greater than equal to 0. Why? Because, F1 and F2 are PR all right. So, this is equal to u1 u2 minus x1 x2 which is not necessarily, not necessarily greater than equal to 0. It will depend on the value of x1 x2 as compare to u1 and u2. All that we know is u1 u2 is positive, but x1 x2 maybe positive and greater than u1 u2. So, it is not necessarily therefore, F of s is not necessarily Pr is the proof ok? Not necessarily, it could be it could be Pr also, but it is not necessarily Pr. Because, x1 x2 may be greater than u1 u2. Similarly, you can prove for G of s. But now, an example.

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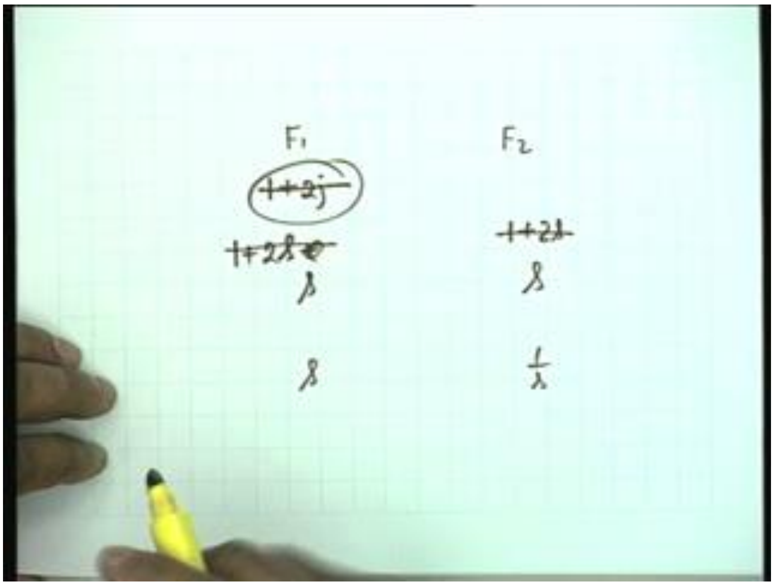

Give a simple example in which the product of 2Pr’s is not Pr give an example. 1 plus 2s and F2 same thing. How do you know this is? How do you know this is not Pr?

Multiple pole at…

Which location on the j omega axis, j omega axis is the ocean.

At infinity. But why did you have to go to 1 plus 2s. Why not simply s? Simply s and s. Now, give an example of F1 and F2 which are individually Pr, but there ratio is not Pr. Ratio F1 by F2 simplest example that is right. The ratio is s squared which is double poles at infinity. An engineer always goes by that kind of intrusion, but when, someone says prove 10 thousand examples are not enough to prove, but 1 counter example is enough to disprove. So, if it is proved, A counter example is enough here?

Because, it is not necessarily Pr.

Wonderful. Because, it is necessarily Pr. Because, proof is a qualified proof. So, a counter example shall do here all right. With the next that we work out is we skip 10 5 then 10 6 both of them is very interesting. We go to 10 7.

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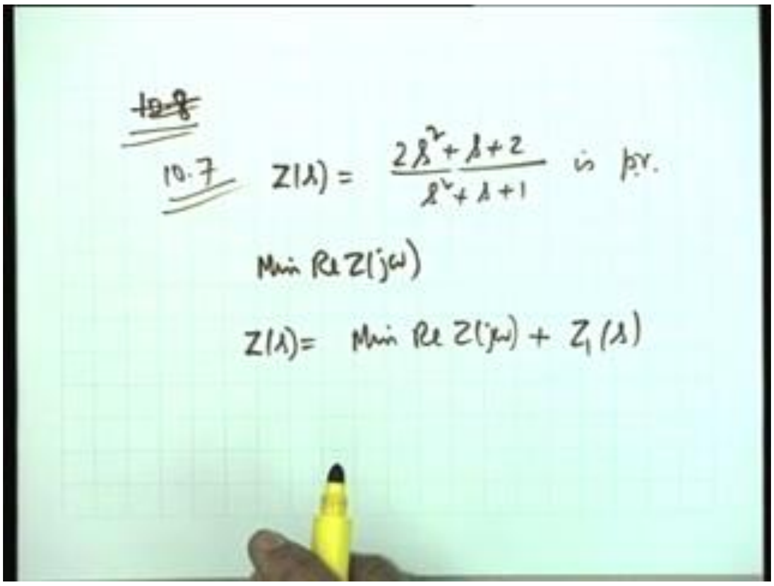

No we skip 10 7 also let us, keep it for you or maybe there is a point in 10 7 which I want to illustrate. 10 7 says given Z of s is equal the 2s square plus s plus 2 divided by s square plus s1. It is given that this is Pr, it is given that this is Pr. Determine the minimum part of real part of Z of j omega and synthesize Z of s by first removing this minimum part. That is synthesize Z of s as minimum real Z j omega plus Z1 of s let us say.

Let me, tell you what this problem involves. It is given that this is Pr. So, you need not test whether, and it is Pr or not. What it says is remove the minimum part of the real part of Z of j omega. Now, what would this be in terms of an element, network element? Resistance. Because, it is a real part it is a real quantity remove the minimum value. Can you remove the minimum value? You can, without destroying the positive real character of the function. For example.

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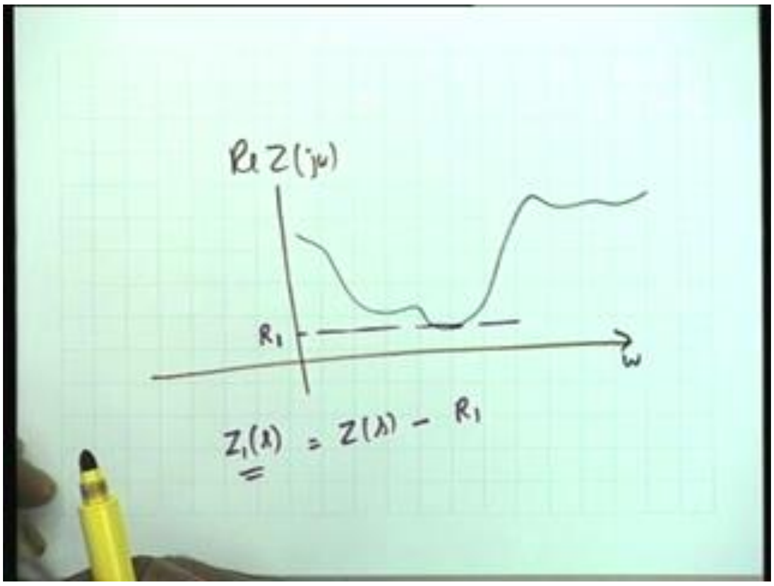

If the real part of Z of j omega let us say, it varies like this, it varies like this. Minima real part of Z of j omega has always to be positive, always to be non-negative; not positive, non-negative. Now, the minimum value is somewhere here, minimum is somewhere here. Let us, say this minimum value is R1. If I remove if I form a function Z of s minus R1. Will this still be positive real? Yes, it is shall still be remains remain positive real because, the minimum value of this shall now be 0.

So, therefore, what this problem says is 1 of the ways of simplifying a function is to take out a constant from it. If you take at you know, what value of constant. That constant says take the minimum value of the real part of Z of j omega. If you do that perhaps the function would be simpler. Let us, see if we can do that.

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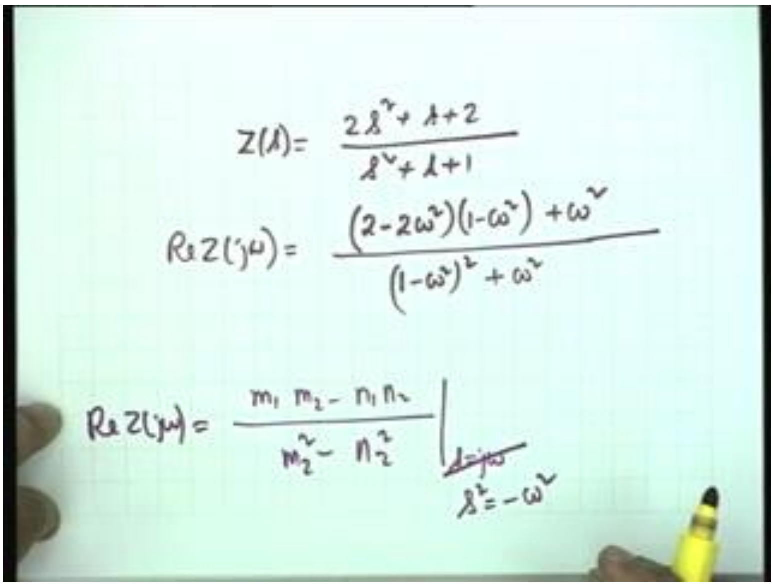

Our function is Z of s equal to 2s squared plus s plus 2 divided by s squared plus s plus 1. So, real part of Z of j omega. I am not taking A of x now why not? Because, we are not testing, we not testing it for positive realness. I have to find out the minimum value of the real part of Z of j omega and therefore, I have to find the total function. The total function would be m1 m2 m 2 minus 2 omega squared.

Do you see that i am putting s squared equal to minus omega squared immediately? So, this is m1 and m2 is 1 minus omega squared then minus n1 n2 that is minus s squared with s squared equal to minus omega squared. So, this is plus omega squared.

Is that ok? Is this what I have written here?

The numerator should have been m1 m2 minus n1 n2 and the denominator is m2 squared minus n2 squared with s equal to j omega all right. This is the real part of Z of j omega. Now, m1 is 2 plus 2s squared and s squared equal to no I do not want s equal to j omega, I want s squared equal to minus omega squared. Therefore, 2 minus 2 omega square m2 is 1 plus s square and s squared equal minus omega square. So, 1 minus omega squared. Then, minus n1 n2, so minus s multiplied by s. So, minus squared which is equal to plus omega squared. And in the denominator I shall have 1 minus omega square whole squared.

Plus omega squared. So, let see what this function is then you have to minimize this.

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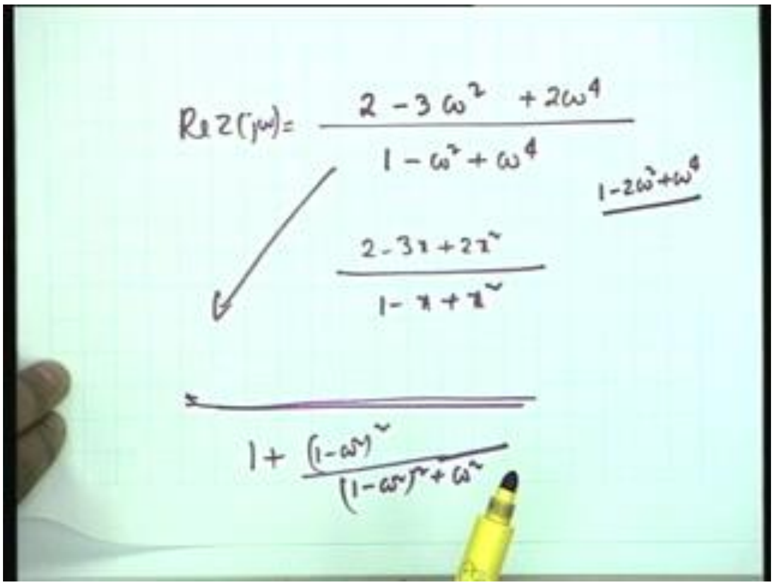

So, I get real part of Z of j omega. In the denominator I get 1 minus omega squared plus omega to the 4. Is that ok? 1 minus omega square the whole square. So, 2 omega square plus omega square. And in the numerator I get the constant will be 2 the fourth power will be twice omega to the fourth. Then, here it will be minus 2 minus 2 this is actually minus 3 omega square. Not minus 2 it will be minus 3. Because, it that all right? So, how do I find the minimum value of this is it obvious, where the minimum is omega square equal to 1? How do you know? How do I find the minimum value?

Let us, put omega squared equal to x squared then 2 minus 3x plus 2x square divided by 1 minus x plus x square. Quit. So, is that is that clear? No, it is not very he is very fast, very quick to see. I did not see that. So, fast I think he is right. You see, what he does is he takes 1 minus omega squared plus omega 4 out from the numerator. So, what you are left with is 1 minus 2 omega square plus omega to the fourth. Is that ok? Is that ok?

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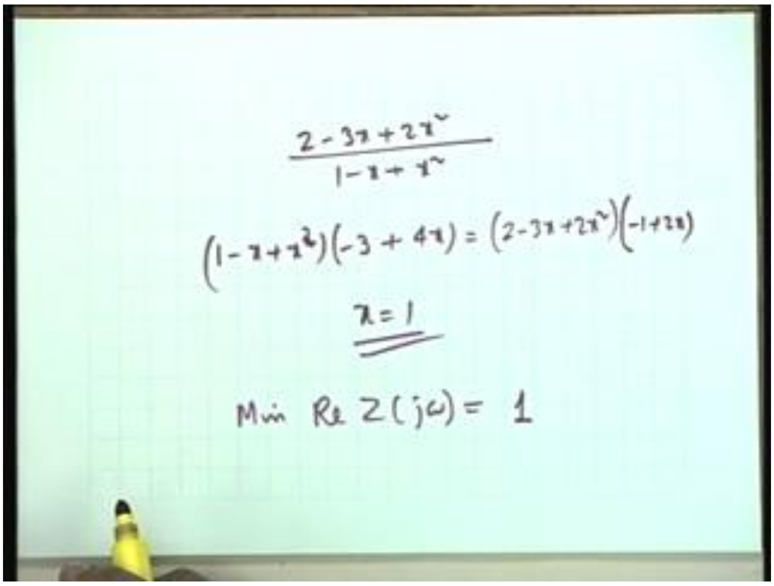

The numerator is denominator plus this and therefore, it is 1 plus you see the numerator 1 minus omega square whole square. Here there is not trick it is a very simple observation, but if you do, if you do not see it then you do the usual thing that is 2 minus 3x plus 2x square divided by 1 minus x plus x square. The minimum occurs when, 1 minus x plus x square denominator multiplied by the differential coefficient to numerator.

So, which is minus three plus 4x is equal to the denominator 2 minus 3x plus 2x squared multiplied by the differential coefficient of the denominator which is minus 1 plus 2x. And make your life miserable by simplifying this and finding that indeed x equal to 1 is the answer. You have to simplify this.

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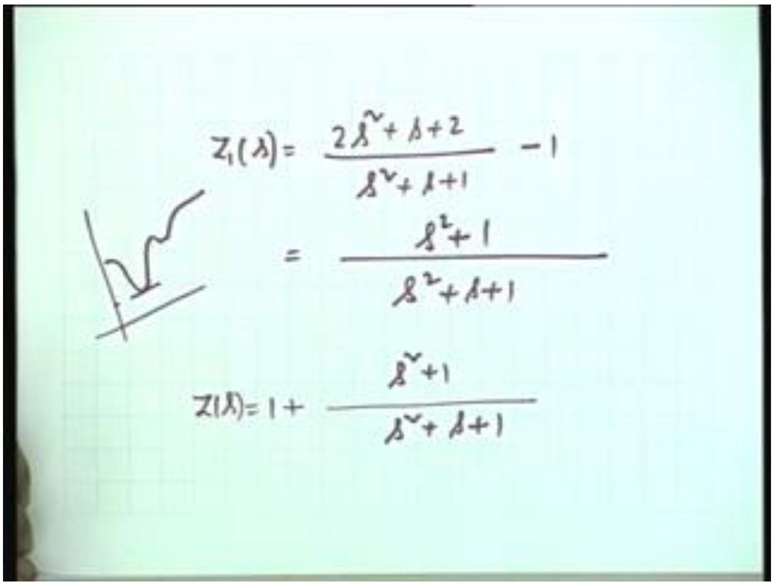

Now therefore, our sum in substances that the minimum of real part of Z of j omega is equal to 1 and I remove this, which means that my Z s would be equal to Z of s 2s square plus s plus 2 divided by s square plus s plus 1 minus 1 that is equal to s square plus s plus 1 what am I left with s square plus 1. If there are 2 minimals then obviously, you have to take the global minimum.

Absolute minimum. You see you can subtract this no you cannot. Because, then the remainder function will be non-Pr. Is not it? So, you can subtract only this much you have to take the global minimum. Yes it is a very correct question there maybe 2 minimal. Then, you have to check what is the value at these minimal and you have to take the lowest of them you have to take the lowest of them Very correct question.

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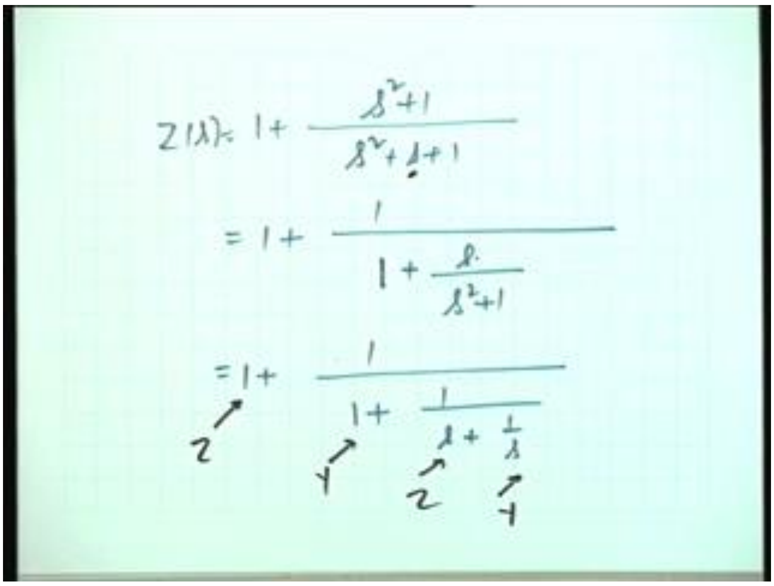

Now, in this particular problem therefore, I have Z of s equal to 1 plus s square plus 1 divided by s square plus s plus 1. And the problem is that I have to synthesize this. I have to synthesize this. Now, now some tricks of the trade. What i have is Z of s equal to 1 plus s square plus 1 divided by s square plus s pus 1. And 1 should be able to see that i can write this as. 1 no I do not divide by s, I divide by s square plus 1.

So, I get 1 plus s divided by s square plus 1 all right. Which I can write as 1 plus 1 by 1 plus 1 over s plus 1 over s. Do not you see that I have expanded into a continued fraction? And I have done this by inspection. Now, since this is an impedance function: this is impedance, this is admittance, this is impedance, and this is admittance. So, what is the network then am I going too fast.

This function I see that this term is there in the denominator also s square plus 1. So, i divide. There is no guarantee that this procedure is going to give you correct results in the general case, but here it is extremely simple. What I find is that 1 plus s remains and divided by s square plus 1. So, s by s square plus 1. Then, I treat this 1 I divided by s. So, I get 1 here 1s and 1 by s, which is exactly in the form of a continued fraction.

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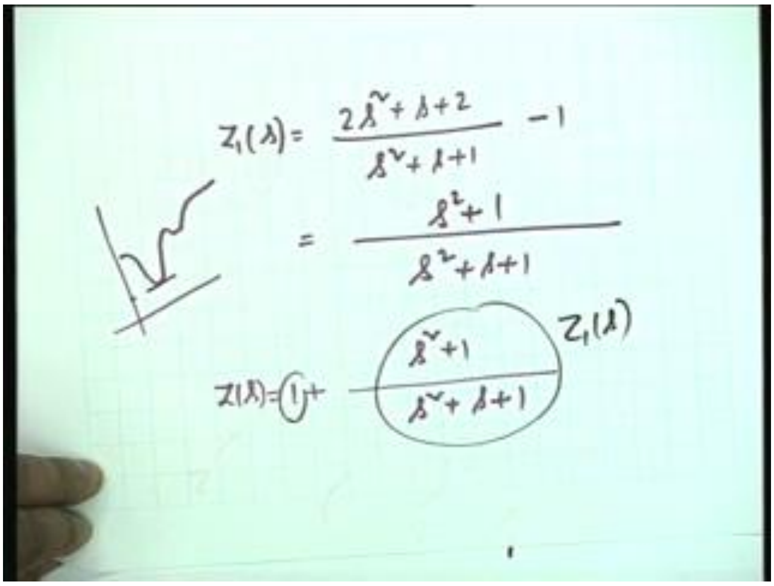

And the network now shall be a Ohm resistor and impedance. Then, an admittance of ohm which means another 1Ohm resistor. Then, in parallel we have an impedance of 1Henry and and a capacitance of 1Farad. So, this is my Z of s and the synthesis is complete we have actually found a network. Because, this function was simple that is it. No which is not correct. Because, we had here a 1Ohm and a 1Ohm. We do not have a pure single tuned circuit. It is not that we have a series LCR.

No, the Second part is a parallel tuned circuit. Isn't it? There is a resistance then in parallel way it is not 1 of those circuits that you have dealt with earlier. No it is not. You see what we did was we had a series tuned circuit in which R came in series with L with a parallel tuned circuit in which a capacitor came in parallel with a resistance and a inductor it is none of them you have to be careful.

The rest and then added 1. This 1 was the 1 that was removed. What we did was we wrote Z of s is 1 plus something. So, this is my Z1. You have to bring it back you removed only to facilitate handling the rest of it. Why did you remove? Because, a function the remainder function becomes simpler than the original function. And that is the strategy given a complicated problem you convert it into simple problem.

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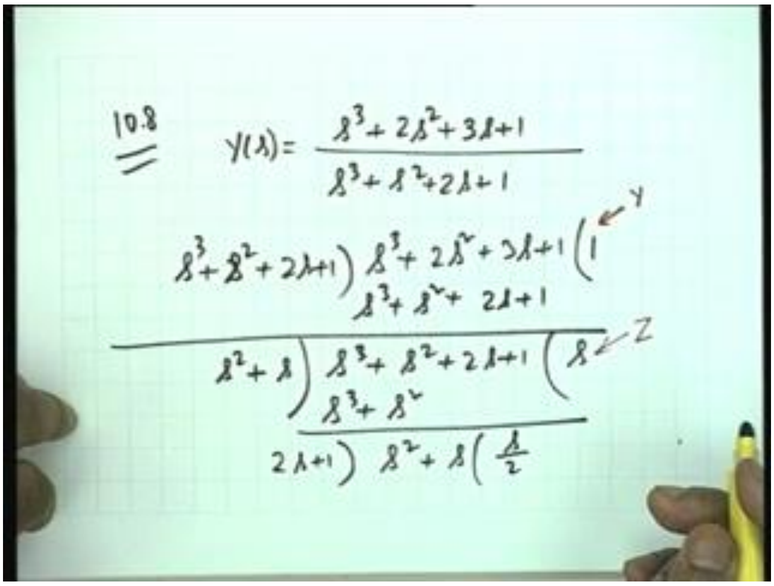

My next problem is 10 8; 10 8 perform a continued fraction expansion of the given ratio Y of s equal to s cube plus 2s squared plus 2s plus 1 divided by s cubed plus s squared plus 2s plus 1. It says perform a continued fraction expansion on the ratio what does the continued expansion imply if Y of s is the driving point admittance of a passive network. What does it imply?

Draw the network from the continued fraction. So, let us do the continued fraction then see what the implications are. The continued fraction is s cubed plus s squared plus twice s plus 1s cube plus 2s squared plus 3s plus 1, so you take 1. What is the dimension of this? This quotient?

Admittance say it is Y you must keep a track. I get s cubed plus s squared plus 2s plus 1. Now, in continued fraction, in continued fraction expansion here because, we are going to realize a network we are not permitted those liberties. Which liberty? Multiplying by positive constant. No, because that will change the network elements we are not allowed that liberty. If it was simply to find out whether it is Hurwitz or not. Then, of course you do whatever you like. So, my remainder is s square plus s s square plus s that is all. Then, I get s cube plus s square plus 2s plus 1. So, s is the quotient s cube plus s squared and this s is now an impedance.

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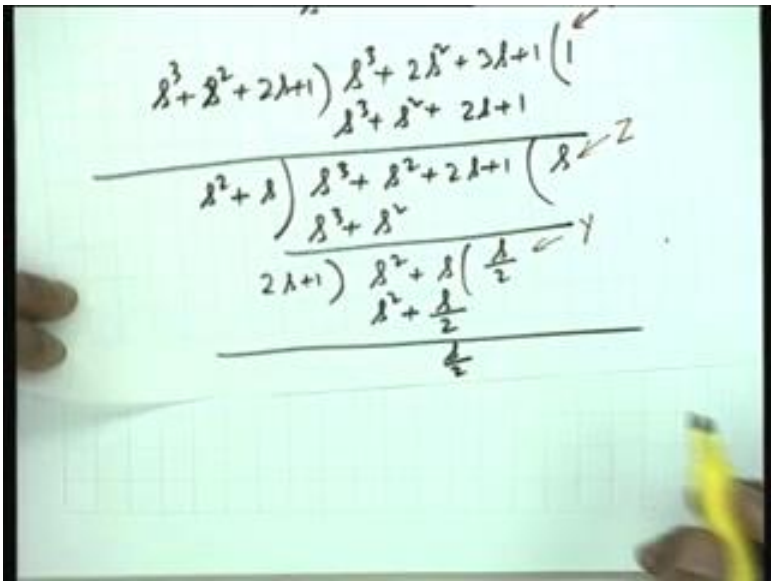

I am left with 2s plus 1s square plus s s by 2 s squared plus s by 2 this is an admittance Y.

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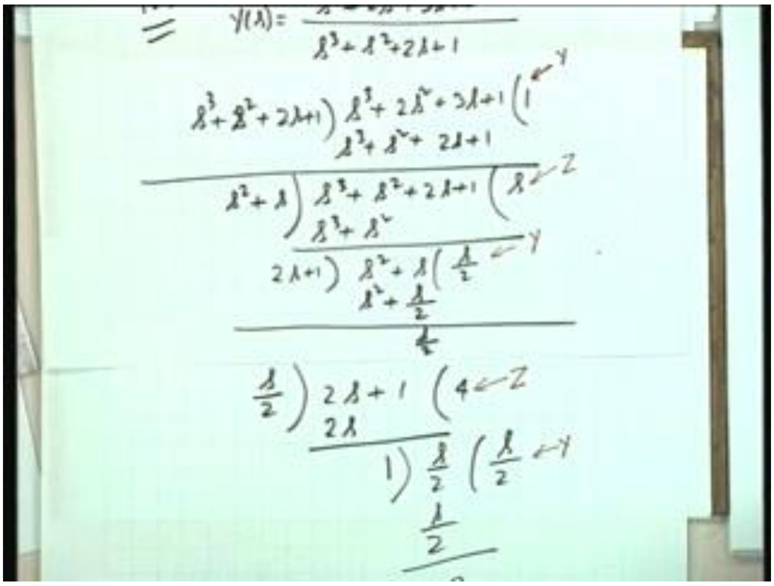

Then I am left with s by 2, I am left with s by 2 and s by 2 divides 2s plus 1 4s. 4s or 4? 4 simply 4.2 s1 and this Z impedance 1 divides s by 2s by 2s by 2 0 and this is an admittance. Now, if you look at the total continued fraction I do not know if you can see it. Total continued fraction is that no let me, do it this way. Can you see the whole thing on the screen? Yes. The next question is we have done the continued fraction expansion. What does it imply? What does it imply? What is the implication? If there are more than 1 implication tell me. So yes, 1 of the implications is since all this quotients are have positive coefficients.

Obviously, it is realizable therefore, Y of s is Pr. Agreed? Any other conclusion that you can draw from the continued fraction expansion, any other conclusion? Yes. About the nature of the network. How can we say that Y of s is Pr? Because, the continued fraction expansion yields terms likes one s s by 2 4s by 2 which are resistors, inductors or capacitors right.

Therefore it is Pr, therefore it is realizable. In fact the network is obvious I will draw the network in a moment network is obvious, but I want you to make another conclusion about the, about the result that you have got. That it is not it is a RLC network. As simple as that it contains resistors, inductors as well as capacitors. Let us draw the network.

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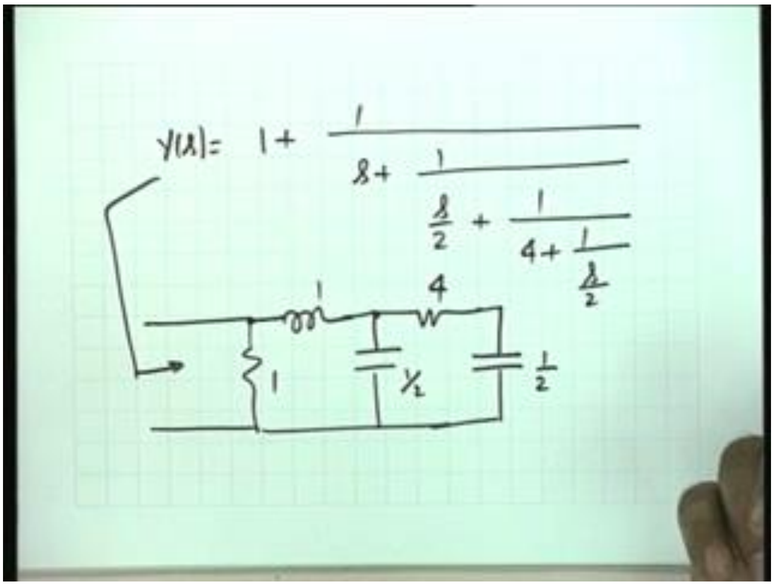

What we see is that Y of s is equal to 1 plus 1 over s plus 1 over s by 2 plus 1 over 4 plus 1 over s by 2. Which means, that the network would be 1Ohm Registor, the next 1 would be 1Henry Inductor , the next 1 would be a shunt element half Farad Capacitor, next 1 would be a 4Ohm registor and finally, half Farad capacitor and this is Y of s.

It is an RLC network that is all that we. Now, you can say there are few other things that it is an equal capacitor network 2 2 capacitors are equal resistors are not equal. But let me, question you let me question you this is nowhere simplified example is this cannot be used in in general. In general, continued fraction. No, this is an admittance. This is a admittance; this is impedance, this is admittance; this is impedance, this is admittance. So, it is a half Farad capacitor.

What I am going to, what I wanted to tell you is that, this is not in general applicable that this continued fraction does not necessarily give you a realizable network or a realization. For example: none of you questioned me as to why I carry out a continued fraction expansion only starting with the highest powers? Why not lowest powers? What is wrong? If you do that here you will see, that after a few states you get negative. That is also not general there are situations where continued fractions with highest powers will give positive quotients. Continued fraction with lowest powers gives negative. There are situations where continued fraction with lowest powers works that with the highest powers does not work.

How do you do? You write it in the reverse fashion. That is start with the lowest power you write this instead of s cube plus s square plus 2s plus 1 you write 1 plus 2s plus s square plus s square. Similarly, here also 1 plus 3s plus 2s square plus s cube. That’s right. And 1 upon s you know, if it is an impedance then you know it is an inductor. If it is a capacitor you know, it is a capacitor.

So, coefficient continued fraction expanfsion with lowest powers does not work in this case. There is no general rule as to which 1 will work whether, both will work or not whether 1 will work or not there is no general rule. This is a simple example that is why it happened to be so.

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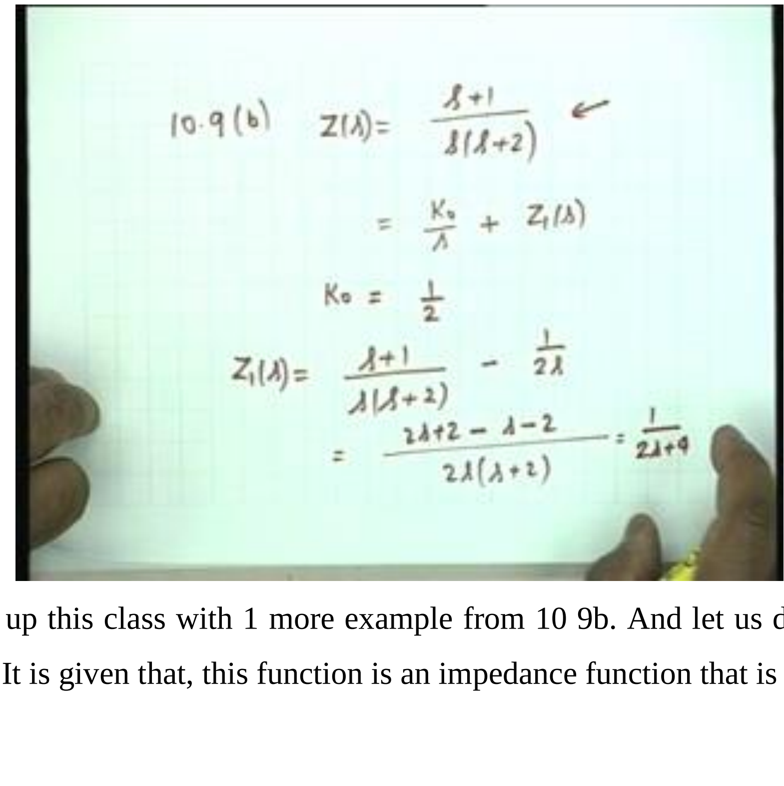

We shall end up this class with 1 more example from 10 9b. And let us do b 10 9b just one problem. It is given that, this function is an impedance function that is Z of s equal to s plus 1 divided by s into s plus 2. Synthesize the impedances by successive removals of j omega axis poles or by removing minimum real part on the j omega axis. We have to synthesize this by successive removal of poles on the j omega axis well as far as or by taking the minimum value of the real part.

That’s why if this is concerned. Does it have a pole on the j omega axis? Yes or no. At the origin. So, you can write this as K0 by s plus some remainder impedance Z1s.What is K0? 1 by 2. Therefore, Z1 is equal to s plus 1s into s plus 2 minus 1 by 2s. Let us, see what this is? 2s s plus 2 2s plus 2 minus s minus 2. So, s and s cancel, this also cancels. What we get? 1 by 2s plus 4 is it ok? Have I made a mistake? Yes I have.

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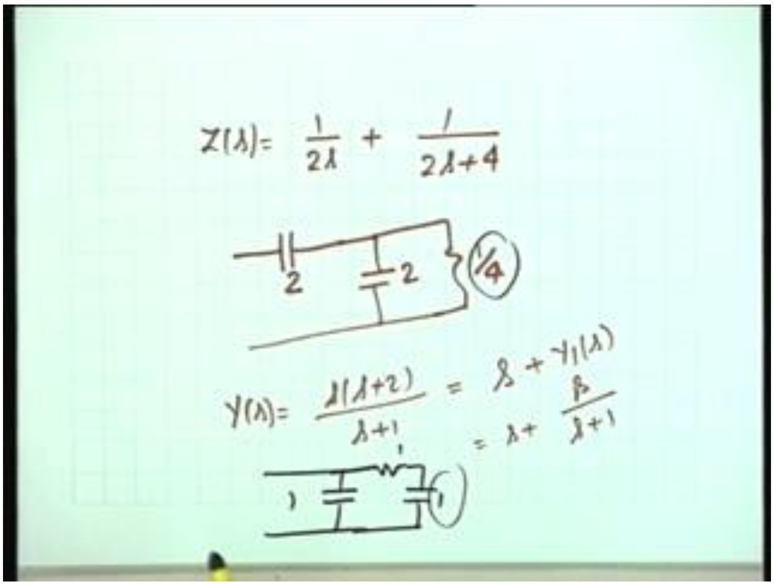

Therefore, I get Z of s as equal to 1 by 2s plus 1 over 2s plus 4. Do I have to do anything else? No. So, my Z of s becomes first a 2Farad capacitor all right and then a parallel combination of.2Farad capacitor no Henry. This is impedance. So, the denominator is an admittance and 1 by 4. Question is there any other way you could do it? Just by looking at poles at on the j omega axis. Is there any other way?

Let us, look at the function again can we could we do it in some other way, some other obvious way. Instead of Z suppose, we took the Y reciprocal of this. Then, what would we have got. Y of s would be equal to s into s plus 2 divided by s plus 1 obvious this has a pole where. On the j omega axis. You see you have to remove poles on the j omega axis only no real axis. Now, it has a pole at infinity and what is the residue at infinity?

So, it is simply s plus Y1 of s. Now, if you remove s from here what is Y1? So, it is s plus 1 by s plus 1. Is that clear? S square plus s, no that does not sound well. S by s plus 1. Is that correct? S square plus s. Yes, that is correct. And now, the network is also obvious, what we have is Y of s equal to s. So, I have a 1Farad capacitor then what? 1Ohm and 1Farad says right.

So, at least there are 2 networks that we have found out immediately. Which 1 last question. If these are at your disposal you know, that the same function is realized by this as well as this these are both RC network. Now, which 1 if any if any would you prefer. Smaller capacitance, larger the capacitance the price goes up this purely economic consider nothing else. 1 quarter Ohm register with 1Ohm resistance does not matter price will be now I think it is about 50 paisa or 35 paisa.

If we want a quarter Watt resistance, Losses in this network would be more. Why? But the total effect is the same. No he has, he has something, he has brought out a point what you said is not correct losses in this would be less, not less losses in the total shall be the same, but there is 1 more consideration besides cost, 1 more consideration for using lower valued capacitors.

Can anyone tell me what this is? Size yes. Size of course, is important. As the size increases, the cost also increases. But there is 1 more consideration very obvious and very simple consideration larger the value of the capacitor, larger is the leakage or the impurity. Therefore, 1 shies away from large capacitors. Whether, it is a power system laboratory or electronics laboratory. You should choose the lower value of capacitor. Lower the capacitance lower are the losses that is all for today.
Thank you.