Circuit Theory
Prof. S.C. Dutta Roy
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture - 38
Testing of Positive Real Functions

Thirty eight lecture and we are going to talk about testing of positive real functions. As I said there were some mathematical concepts which I have which you find. It appeared to me that you found if it difficult. So, we will go over it again. As radical our definition of positive real function oh no this is not the, our definition of positive real function.

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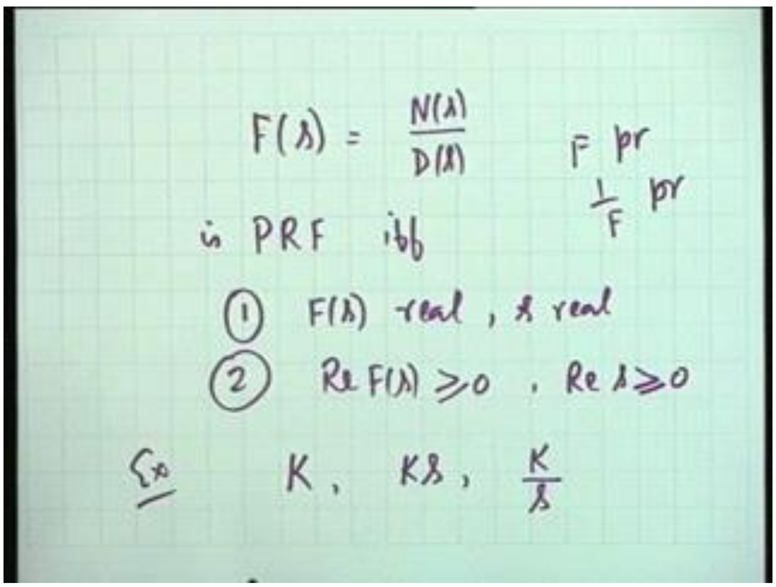

Is well we also assumed that this is rational. So, it is N of s by D of s and is PRF if and only if 2 conditions are met. That is F of s is real for s real and the second condition is real part of F of s nonnegative for real part of s nonnegative this is the definition. From the definition itself we showed simple examples of positive real functions. For example examples are K which is real and positive real and positive K.

This is a positive real function ks is positive real function K by s is a positive real function. And you can see that these 3 elements correspond to resistor inductor and capacitor impedances of resistors capacitors and inductors or admittances. So, any combination of them we also showed that sum of 2 PR functions is PR.

So, any combination of them either in series or in parallel or parallel series would be PR. And for that we showed that if F of s PR F is PR then, so is 1 over F. And therefore, parallel combination which consist of sum of 2 admittances is also PR. And then we shown then we try to motivate why we should study PR functions.

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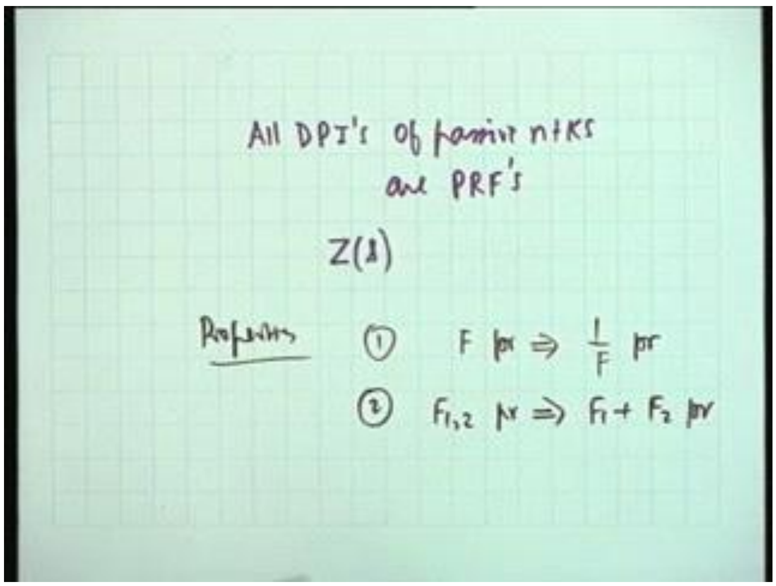

We said that all the DPI’s all driving point (( )) of passive networks of passive networks are PRF’s which we proved we stated this is a theorem. And then we proved that all Z of s driving point impendence driving point impendence satisfy the property that Z of s real for s real. And real part of Z of s is nonnegative for real part of s nonnegative we proved that. And we also proved that if F is PR then 1 by F is PR therefore, Z of s is PR. 1 by Z of s which is admittance should also be PR.

So, that was the second part of the study and then we considered some properties. And this is where much of the difficulty arose the properties of PR. First we have already shown that if F is PR then 1 by F PR F PR means that 1 by F is PR. And then if F 1 2 are PR then. So, is F 1 plus F 2, but not F 1 minus F 2 necessarily. F 1 minus F 2 will depend on the relative values of real part of F 1 and real part of F 2. For the right half plane, but F 1 plus F 2 is definitely PR. In terms of a circuit it means series combination or parallel combination of 2 elements or 2 positive real functions.

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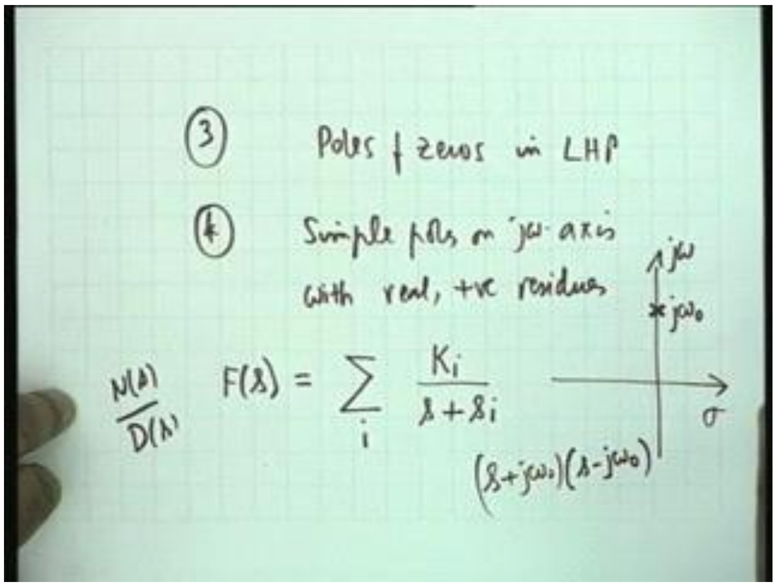

The third property we stated that poles and zeros now zeros are included poles and zeros of PR functions must be in left half plane, because right half plane will lead to instability. And therefore, a violation of the real part condition poles and zeros should be in left half plane. And then we allowed simple poles simple poles on j omega axis. If there are poles on j omega axis they must be simple with real positive residues. And there were considerable discussion on this business of residues.

First what is a residue any any rational function when expanded in partial fraction would be of the form. Let’s say Ki divided by s plus si what do I mean by poles on j omega axis. That is in the s plane there exists a point 1 or more points at which the function blows up that is a function becomes infinity this is the pole j omega 0.

Which means that if I write F of s as N of s by D of s then D of s has a factor s plus j omega naught. And necessarily s minus j omega naught, and therefore at s equal to j omega naught or minus j omega naught. The function blows up and that is a pole did your question have any other implication. That’s right. If s plus j omega naught is a factor s minus j omega naught it should also be a factor, because the function has to real for s real. And. So, this is one of the poles the pole must be here.

Now, if there exists a pole on the j omega axis then this pole must be simple and the residue there must be real and positive. What is a residue someone ask yesterday now I suppose that should have been done in signal and systems but.

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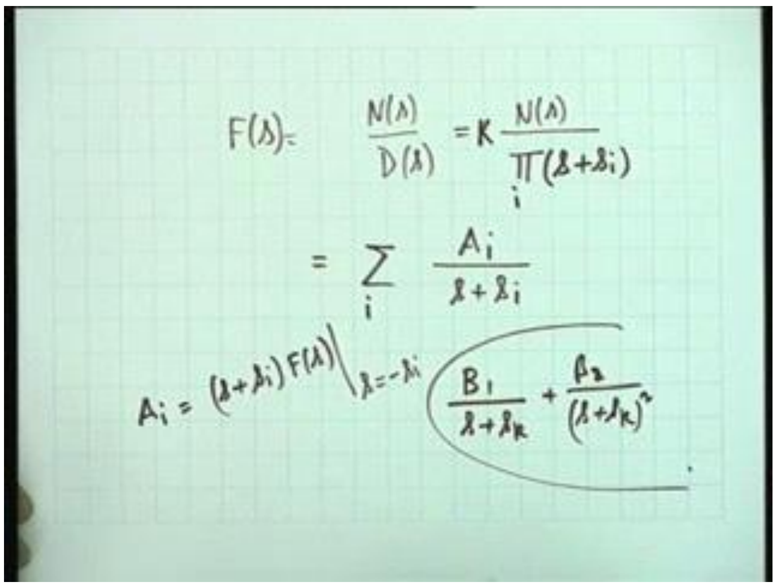

We will consider this we have an F of s which is equal to N of s by D of s. A rational function and you know that any polynomial any polynomial of degree N shall have n number of roots. And therefore, I can write this as s plus si continued product over i all right. There may be a some constant let us say some constant K the denominator can be factored into n number of roots. If the degree of D of s is small n it can be factored into so many roots.

This then can be expanded in partial fraction that is I can write this as sum Ai divided by s plus si summed up over i. this is the form that I have written here shows simple poles, but there may be multiple poles also. In other words s plus si could be raise to the to the factor to the power Pi let us say. There can be multiple poles, but it suffices to to as far as the definitions are concerned it suffices to consider simple poles. If there are multiple poles let us say at sk then I shall have factors of the form. Let us say B 1 divided by s plus sk plus B 2 divided by s plus sk whole square.

If there is a double pole then i shall have 2 terms like this. If there is triple pole then there will be 3 terms like this and so on and so forth. Now, these constants Ai which occur in the numerator are called the residues. And the reason is the following why they are called residues is that if you want to find out this constant Ai. What do is you multiply both sides by s plus si multiply F of s and put s equal to minus si. What remains on the right hand side just Ai isn't that right. All other factors multiply by s plus si put s equal to minus s si values and this is 1 physical meaning of residues that is what is left.

Now, residue in the complex variable has a slightly different intonation. Which we do not wish to enter into for our purpose its if I as to say that these are the constants which occur in the numerators of the partial fraction expansion. One can ask at this stage what would you do if you have multiple pole. Let's say sk is a double pole there is a double pole at sk. Then there are 2 residues B 1 and B 2 B 1 is called first order residue. And B 2 is called the second order residue; obviously, B 2 can be found out by multiplying by s plus sk whole square both sides and putting s equal to minus sk. And B 1 to find out B 1 you will have to differentiate the function 1.

That must have been done while doing Laplace transform right. This constant or these constants are called residues all right. Now, a in our case we said if there is a pole on the j omega axis.

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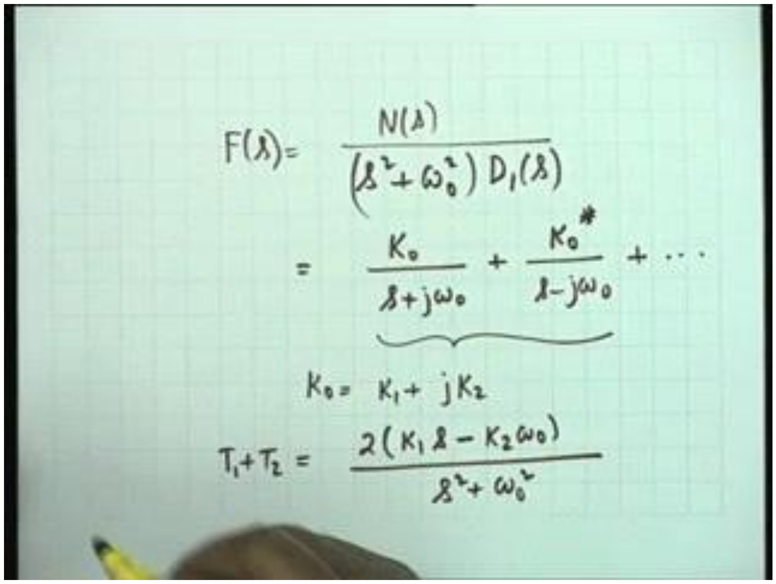

Suppose to be specific let us say that our F of s is N of s divided by let us say let us consider a pair of poles on the j omega axis at plus minus j omega naught. That is we have a factor s square plus omega naught square multiplied by D 1 of s all right. Is the point clear we have assumed the existence of pair of poles at plus minus j omega.

Therefore I can expand this in this form K 0 divided by s plus j omega naught. One of the poles plus s minus j omega naught and from the discussion that, I held yesterday. The residue here the constant here must be complex conjugate of this constant why because the function is a real function. So, it would be K 0 star plus other terms plus other the terms due to other poles. Now, if you combine these 2 you assume that K 0 is equal to K 1 plus j K 2. Where, K 1 and K 2 are real quantities if K 0 can be complex.

You see our our hypothesis our statement was that if there is a simple pole on the j omega axis. The residue must be real and positive let us prove whether it should be real or not. Let us prove that it is real by contradiction that is if possible let the residue be complex. Let the residue be complex then the sum of these 2 terms. That is the T 1 plus T 2 the sum of these 2 terms if you if you carry out the calculation. It simply twice K 1 s i think someone mentioned it yesterday I hope he is right. Twice K 1 s minus K 2 omega 0 divided by s square plus omega naught square agreed.

It is sum of these 2 terms now if F of s is to be positive real. Then its real part real part in the closed right half plane must be nonnegative that is correct. Its real part must be nonnegative if real part of s is nonnegative. Now, we do not have to test this over the complete right half plane. Let us test it on 1 particular axis that is j omega axis. Let us see what the value is on the j omega axis let us call this as T. Then you notice that T. No there is a negative sign there is a negative sign. It does not matter even if it is positive sign then it is happens to be negative. Even if it is positive it does not matter i, let let us say let us see.

So, what he said was let us test at a particular on a particular axis j omega axis for which sigma is equal to 0.

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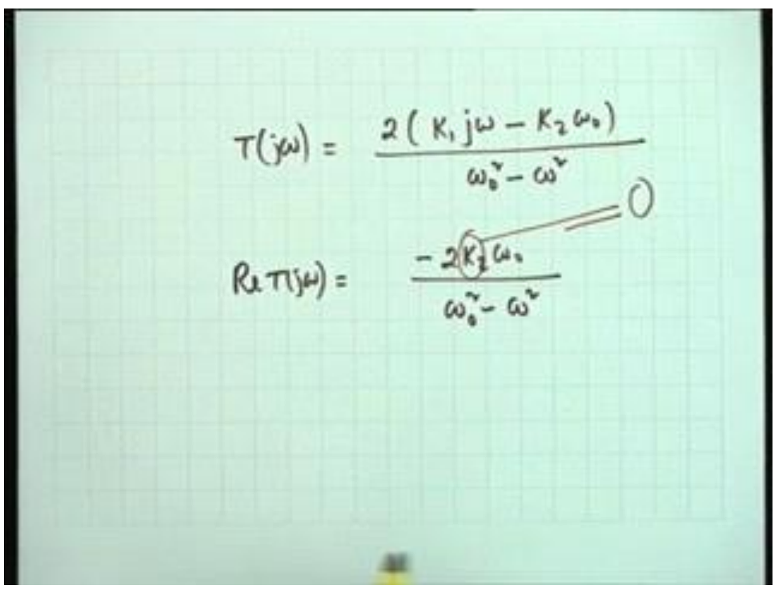

The value of T j omega is twice K 1 j omega minus K 2 omega 0 divided by omega naught square minus omega squared is that right. We have put s equal to j omega and therefore, the real part of T j omega is equal to minus 2 K 2 omega 0 divided by omega naught square minus omega square agreed because this is imaginary this term K 1 j omega is purely imaginary therefore, we ignored it. Now, you notice that when omega if K 2 is positive if K 2 is positive. When omega is less than omega 0 the value is negative the real part is negative all right.

If K 2 is negative then the denominator is negative for omega greater than omega zero. So, there exist values on the j omega axis at which the real part can be negative. And the only way that you can avoid it real part being negative is to make K 2 equal to 0 is it argument clear. That is the argument that i gave yesterday, but i did not carry out this calculation. Therefore we prove that K 0 has no other alternative, but to become real all right.

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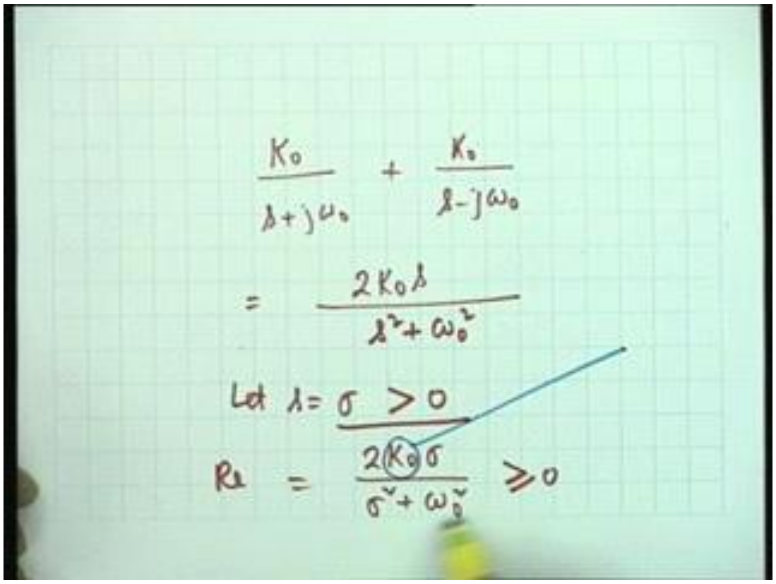

Now, if it is real if it is real then the sum of these 2 terms that is K 0 divided by s plus j omega naught. Plus K 0 divided by s minus j omega naught or next point of study is whether it is positive whether it can be it can be negative because our contention was that the residue must be real and positive. Let us if it is negative what harm what is the harm again we will prove by contradiction. If you if you if you combine these 2 terms it simply becomes 2 K 0 s divided by s square plus omega naught square all right.

Now, suppose the real part of this should be nonnegative for real part of s nonnegative. Let us test now not on the j omega axis, but on the real axis that is let us assume. Let s be equal to sigma which is greater than 0. Then the real part of this; obviously, the whole thing is real then isn't that right. The function itself should be real that is that is the part of the definition of positive real function. And you see that the real part of this is equal to 2 K 0 sigma divided by sigma squared plus omega naught square.

Now, this is require to be nonnegative, because sigma has been assumed to be greater than zero. And what is the condition under which it can be nonnegative the only condition is that K 2 0 because the denominator is positive. Sigma square plus omega naught square. So, K 0 must be greater than 0. Can it be equal to 0? It can be if K 0 is 0 is there is a pole at plus minus j omega naught. If K 0 is 0 then there is no pole are plus minus j omega naught all right.

Therefore it need not bother us there are infinite number of such poles in any given function for which the residue is 0 right. So, our K 0 must be strictly positive. So, we prove we prove therefore, that a positive real function. This is the point about which i made a fun there are infinite number of such poles in any given function. For where the residue is 0 if the residue is 0 it does not show at all. We are not concerned about it. For example Dinesh Pratap Singh is absent here. He is not present we need not ask him a question because we know we know he is not there. So, K 0 cannot be 0 if it is 0 we do not bother all right.

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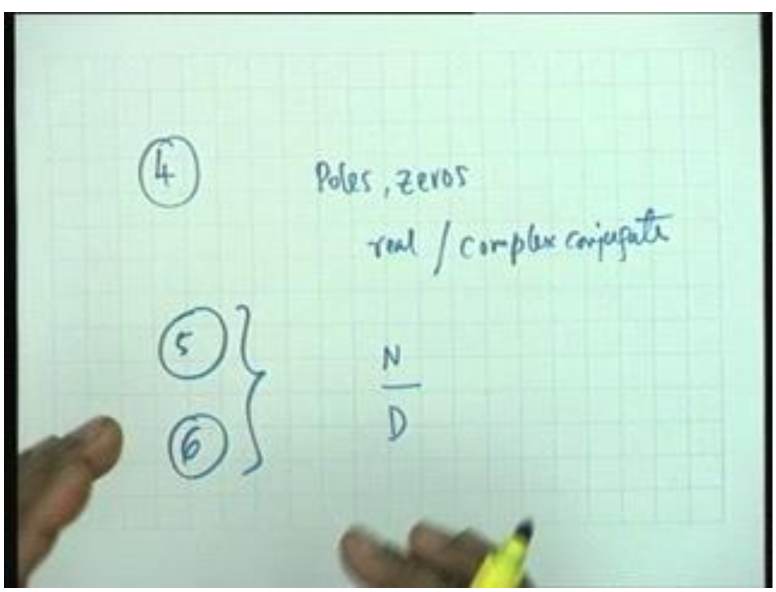

Next 1 we said property number 4. That poles and zeros are either real or occur in complex conjugate pairs. Well poles and zeros are either real or complex conjugate. And this is a property this is a consequence of the fact that the function itself should be real for s real. This we know and then we said well then we talked about 2 properties. Which we combine into 1 that is the highest powers as well as the lowest powers in N and D cannot differ by more than unity more than unity that is the degrees of N and D can be equal.

At the most they can differ by 1 and the reason is that if we differ by more than one. We shall have multiple poles either at the origin or at infinity which is not permitted. And therefore, this property should be true that the degrees of N and D degrees of N and D. And also the lowest parts in N and D cannot differ by more than one. These are all properties and I said they are all necessary properties, but there are sufficient. For example for example, if you simply see that the degrees do not differ by more than 1. It does not guarantee that the function is pure.

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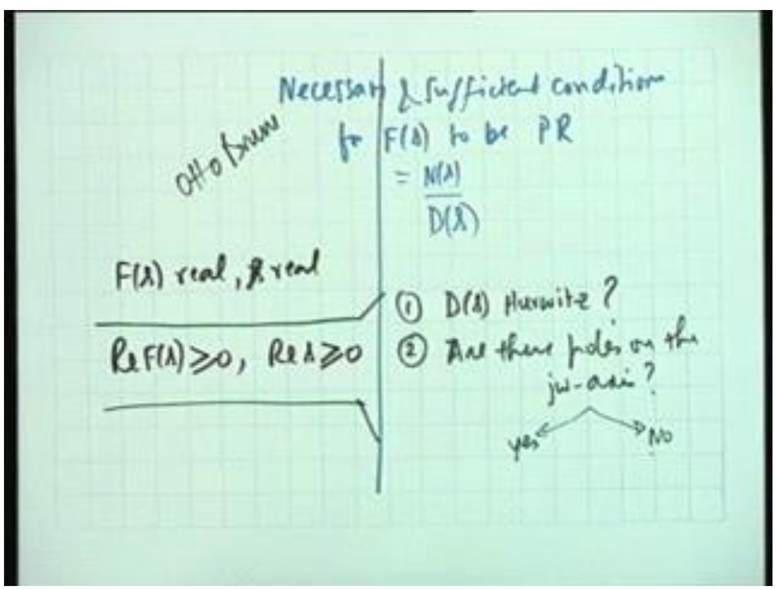

So, the question that arises is what are the necessary and sufficient condition. And; obviously, you know one of the answers. What are the necessary and sufficient condition while if the function satisfies the definition. These are necessary and sufficient all right and therefore, necessary and sufficient conditions necessary and sufficient condition for a function F of s to be PR. And we assume that F of s is N of s by D of s that is it is rational function and 1 of the 1 of the conditions is that the definition F of real. For s real and real part F is of s nonnegative for real part of s nonnegative.

Now, if we if we want to test the positive testing even function. To see whether it is positive real or not the definition is difficult to apply because this takes that you shall have to you shall have to explore all points in the right half plane. All point all s in the right half plane which is an extremely difficult task that is you will have to put s equal to sigma plus j omega consider all possible values of sigma. And all possible values of omega in very simple cases it may be possible, but not otherwise.

So, what has been found out is people have worked on it throughout many years. And it was a nineteen thirty nine a gentleman of Germany origin by the Otto Brune he wrote PhD thesis at MIT Massachusetts Institute of Technology which for the first time gave the testing procedure. For positive real functions a simpler 1 then exploring then investigation the total s plane.

Yes you have a question not. When you point your pencil towards me its an indication that you are stuck all right.

So, Brune said it can be done in a simpler fashion this condition this is the difficult condition because this is almost obvious. As soon as look at the function if there is a complex coefficient you throw it out. Now, as far as this is concerned he said do 3 things. Number 1 first find out if all poles are the left half plane that is how do you find this. Not zeros he says take the function and test whether all poles are on the in the left half plane or not close left half plane. How do you test that?

You perform Hurwitz test on the D of s on the denominator poles are the zeros of D of s. So, first question is you you ask is is D of s Hurwitz is D of s Hurwitz Now, if it is Hurwitz if it is Hurwitz that is you you divide the even part by the odd part perform continue perform continued fraction expansion. Look at the questions are they all positive if they are all positive then it is having. If it passes this test then you ask the question does the function have poles on the j omega axis. As I said earlier, we will be discovered; if the function is poles on the j omega axis.

It will be discovered during the Hurwitz test that is it will end prematurely and last devisor shall have these factors. So, the next questions you ask is are there poles on the j omega axis. If the answer is yes then you will have to test whether these are simply or multiple. If the answer is yes, if the answer is no. Then you go to the next step if the answer is yes then you say then you ask poles simple or multiple. If they are multiple then your testing stops there is not it then the function cannot be fail, but if the poles are simple.

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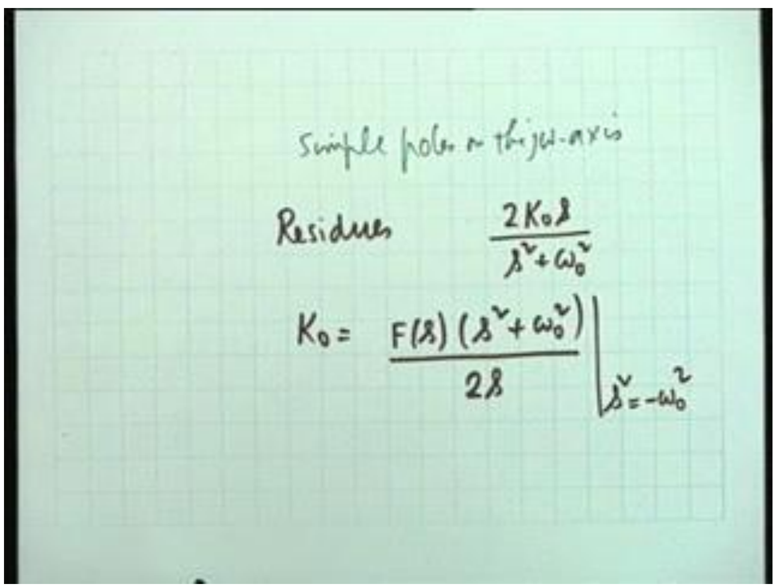

If the poles are simple poles on the j omega axis, then you test the residue then you find out the residues then you find out the residues. How do you find the residues? You know corresponding to a pole a pair of poles at plus minus j omega naught. The term shall be 2 K 0 s divided by s square plus omega naught square right. So, how do find K 0 K 0 is equal to we multiply the given function by s squared plus omega naught squared. We do not want to handle complex numbers we do not want to find out K 0 by multiplying by s plus j omega naught.

Then putting s equal to minus j omega naught. Why because handling of complex numbers is more difficult. Then handling of real numbers we can do by handling real numbers only because i multiply by s square omega naught square. And then divide by 2 s all right and then put s square equal to. Minus omega naught square that is correct. So, that is how i can find out the residues. If there are simple poles on the j omega axis then determine the residue and check whether the residue is real and nonnegative.

If it happens to be complex then you stop testing if it happens to be complex. Or if it happens to be negative then also you stop testing, but if it is positive and real at all the poles from the j omega axis.

Not 1 all of them then you have go to the next step.

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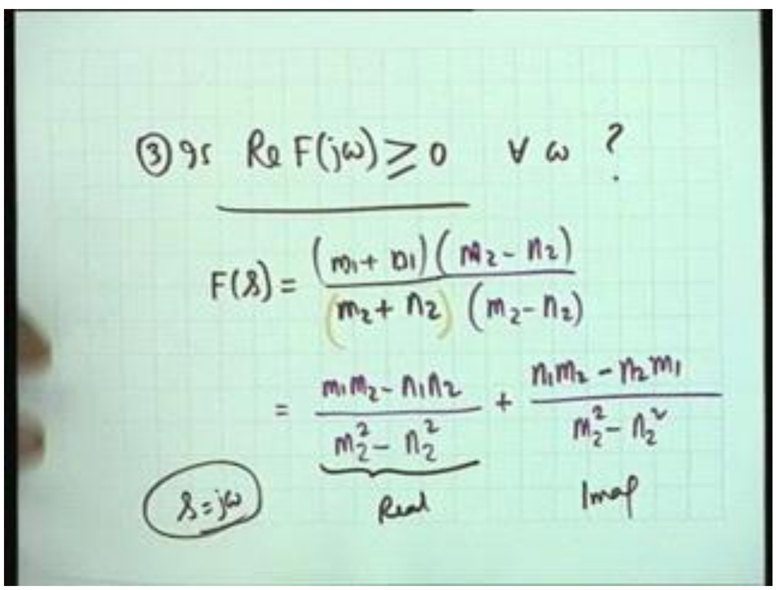

And the next step the third step. Is to test the real part all this does not observe you from the responsibility of testing the real part. The real part has to be tested, but fortunately Brune shown that you do not have to test the real part on in the complete right half plane. You just have to on the test the j omega axis that is the next part would be real part of F of j omega only on the j omega axis. Is it greater than 0 for all omega if it is. So, if it is. So, then the function is positive real.

Now, 1 might because, of the number of words that i use to 1 might think that this is most complicated. Then testing real part of F of s over the complete right half plane that is not. So, because these are mechanical and 1 can even program it on the computer a give the function. And the computer says whether its positive real or not 1 can write algorithm. There are programs available, is couple of points on this testing this testing. Although, it contains an infinite number of points from minus infinity to plus infinity on the j omega axis the testing is in fact, very simple because if you recall we had done this.

That if F of s i want you to recall that F of s is written as m 1 plus n 1 divided by m 2 plus n two. Where small m stands for the even part and n stands for the odd part all right. Then you recall that by multiplying the numerator by the denominator and numerator this is too light. If I multiply this by m 2 minus n 2 and this also by m 2 minus n two. Then I can break this up into its even part and odd part and even part is m 2 square minus n 2 square.

Do you see that this is even and in the numerator I shall have m 1 m 2 minus n 1 n 2 this will be the even part. M 1 m 2 and the product of 2 odd parts n 1 n 2 plus the odd part which will be m 2 squared minus n 2 squared. And the numerator terms will be formed by. N 1 m 2 n 1 m 2 minus n 2 m 1. This is same thing I have said it in an order. So, this is the even part and this is the odd part why is it odd, because the numerator is odd the denominator is even. If both numerator and denominator are odd then the function would have been even.

Now, our purpose is to find the real part of F of j omega. If I put s equal to j omega; if I put s equal to j omega in this then you see that this would be purely real. Is it not that right? It is an even polynomial divided by even polynomial. So, the powers of s are s squared s to the fourth s to the six and so on. So, this would be this would be purely real and this would be purely imaginary.

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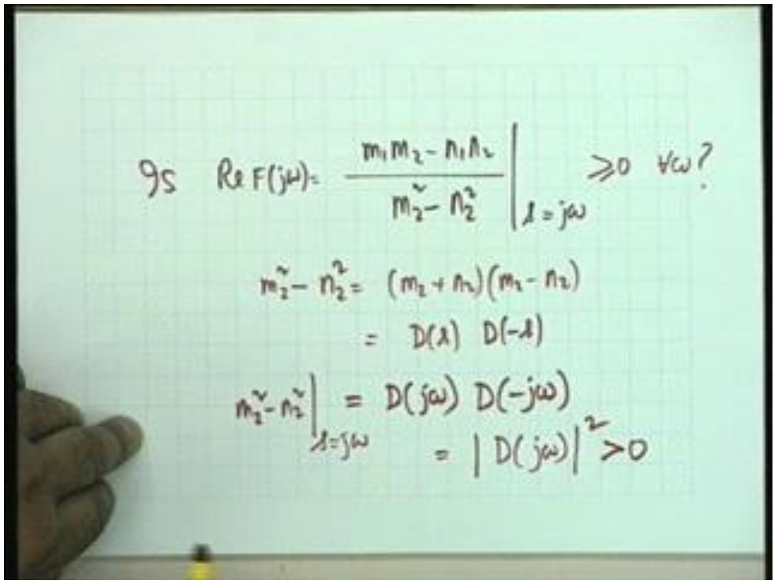

Therefore, if follows that the real part of F of j omega is equal to m 1 m 2 minus n 1 n 2 divided by m 2 squared minus n 2 squared with s equal to j omega. And it is this function that we have to test we have to see whether the question that we ask is is this greater than equal to 0 for all omega. That's why it will be asking right given the function i can identify m 1 m 2 n 1 n 2. I can form this and then test this, but this testing is further simplify by the fact that the denominator of this. That is m 2 squared minus n 2 squared is m 2 plus n 2 times m 2 minus n 2.

Which is equal to the denominator of the given function D of s multiplied by D of what is this? In terms of D.

D of minus s not conjugate no not yet this is D of s then this is D of minus s because if we change s 2 minus s the only thing that changes is m 2. The odd part changes it sign agreed. Now, this point is not agreed. Therefore m 2 squared minus n 2 squared under the condition s equal to j omega is simply equal to D of j omega multiplied by D of minus j omega. Now, these 2 terms are complex conjugates of each other agreed.

Therefore this is equal to magnitude j omega whole square. 1 is the complex conjugate of the other. So, the product of them is magnitude square. And a magnitude squared is always greater than 0.

Can it be equal to zero?

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Can we can we permit magnitude D omega square equal to zero.

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Then the real part is infinity. So, the function does not exist all right. So, it is strictly greater than 0 if it is greater than zero. Then; obviously, we do not have to test the denominator. We only have to test the numerator agreed is this point clear. This is a rational function real part F of j omega. There is a numerator which is in even polynomial in omega there is a denominator which is an even polynomial in omega. And we proved that the denominated polynomial is always positive.

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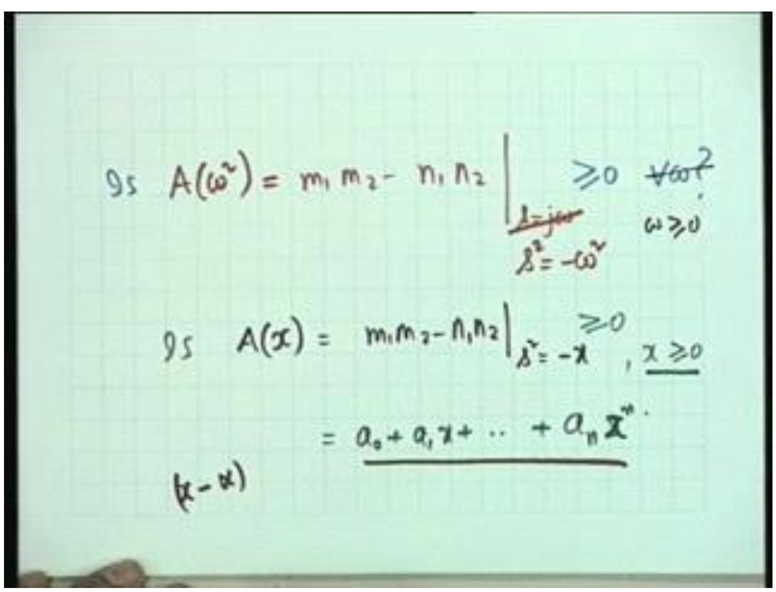

If that is so all that we have to test is only this part that is we have to test m 1 m 2 minus n 1 n 2 under the condition s equal to j omega. Now, would you like to put s equal to j omega in this to find out what this is or would you like to evaluate it in some other fashion. I have already given you the clue we do not like handling complex numbers right because they create complexity. By definition complex numbers have to be complex isn't it there is no other way they cannot be simple.

Then handling of complex number is going to be complex, but you notice that m 1 m 2 minus n 1 n 2 does not contain any odd power of s they all contain even power. So, we could as well instead of this we could as well put s squared equal to minus omega square. And therefore, this polynomial this will now be a polynomial it would be a polynomial in omega squared A of omega squared all right is that point clear. And the question that we have to ask now is, is this greater than equal to 0 for all omega?

Now, further simplification you see we do not have to test for all omega because only positive values or only negative values. So, we say no we do we wont test for all omega we will only test for omega greater than equal to 0. But let us avoid that question also because it is a function of omega squared because it is an even polynomial why do not you simplify by saying no. We would not test A of omega square we will test we put omega squared equal to some other variable let us say x all right.

And test how do I find A of x? I take m 1 m 2 minus n 1 n 2 and put s squared equal to minus x.

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And then test what do I test is this greater than equal to 0 for x greater than equal to done. This is the lowest form we cannot simplify it further. A of x will now be a polynomial in x. In x it is neither purely even nor purely odd, but we have to test only for x greater than equal to 0. That does the job all right. So, it would be of the form a 0 plus a 1 x plus etcetera plus let us say a n s to the n. x to the n x to the n.

So, this is the polynomial that you will have to test. You see from the real part Z of s nonnegative for all values of s in the right half plane. We have come a long way we have come a long way. Only thing we have to check is whether the denominator polynomial was Hurwitz or not. And then we have to test if the Hurwitz does it have poles from the j omega axis. If so, are these poles simple if these are simple poles are the residues real and positive? If all the questions are answered in the affirmative.

Then we come directly to this polynomial capital A of x the long way. Before we subject this to further test let me ask a question. Suppose this in this polynomial suppose we discover. That, there is a factor like x minus alpha suppose I discover that there is a factor x minus alpha; where alpha is positive what conclusion can you make.

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The function original function was not clear because if x the original function was not clear. This is jumping many barrack heights because if there is a factor x minus alpha. Where alpha is greater than 0 then for some values of x this will be negative isn't that right. X equal to 0 for example, it with be negative is the point clear.

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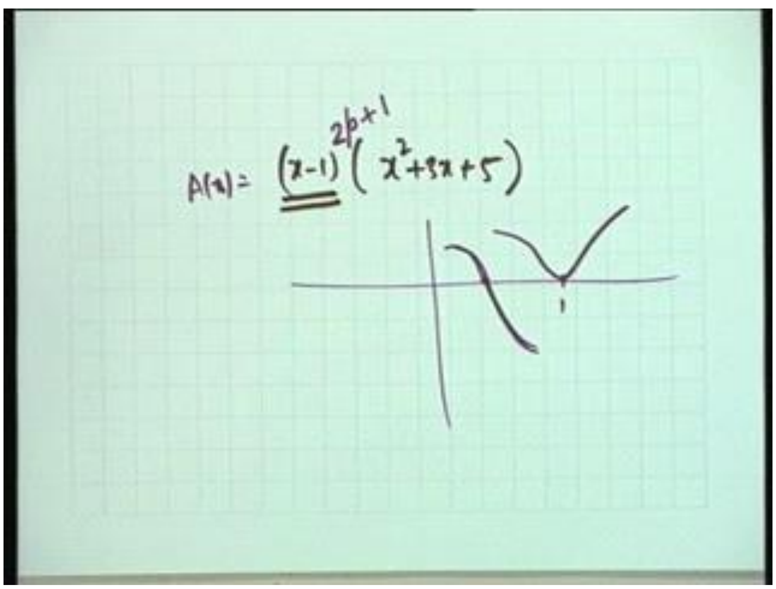

X minus 1 multiplied by a polynomial which is no negative covers. Obviously for x less than 1 the function would be negative isn't that right. For example let the other power x square plus 3 x plus five let us say. Obviously if we have a factor like this then we do not have to proceed further. If we can discover that there is a factor like this. Then we do not need to further, but suppose it is A x plus 1.

Then we need not worry agreed then we will have to check the rest of the polynomial , but there is an exception. Suppose the power suppose x minus 1 is not just 1 root, but there is a double root. Then we need not worry because squared cannot be negative. Similarly, it is not just 2 it could be twice p where is p any integer agreed. in other words if we discover that there are multiple there are even number of roots on the real axis. X equal to 1 is a point here if there are multiple if there are even roots here.

Then what will happen is a of x will go like this it will go through 0 and then go like this, but if it is odd if the power x minus 1 is odd that is 1 or 3 or five or seven. Then what it means is that if this is point the the polynomial goes through a change of sign is that clear the point clear. There is 1 of the observations that we made sometimes it is possible to discover whether, just by observation. It is possible to discover whether a of x nonnegative or not, but enough of theory.

Let us work out some simple problems.

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This point if we discover that A of x has a factor x minus 1 with a power of 2 p. Then; obviously, this factor will not give rise to negative value because x minus one. Even if it is negative raise to an even power is always positive. On the other hand if it is 2 p plus one. Then; obviously, at x equal to 1 there is a change of sign it remains then becomes negative. Sometimes it is useful, but let us look at some graded problems some problems of graded difficulties.

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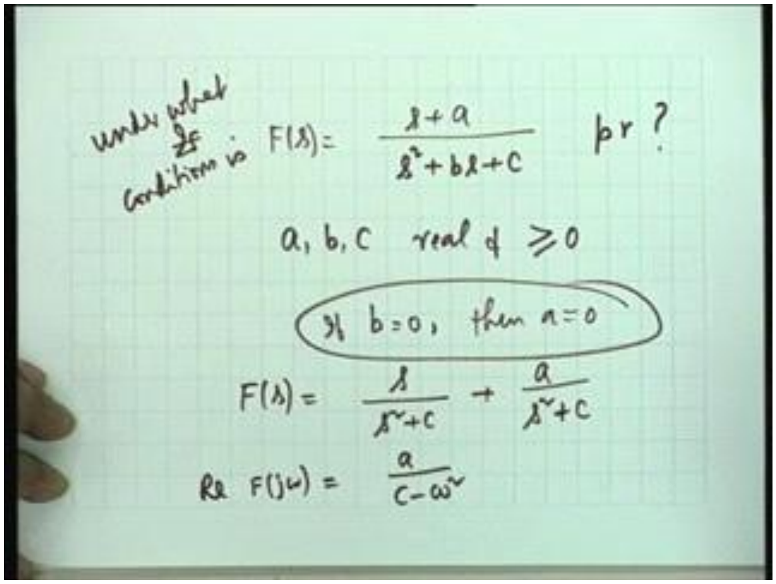

Let us say that we have a function F of s equal s plus a divided by s squared plus bs plus c; obviously, the first thing that you look for is we have to test this. That is we ask the question is the PR or let us say what are the conditions? I think that is a better way. Under what conditions is this function F of s PR under what conditions. The first thing we say is that a b and c should be real and real and.

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Not necessarily nonnegative isn't that right some of them 1 of them can be 0 why not. Nonnegative for example, if a is a can be 0 here. Even then the function could be positive real. So, you must allow this not all powers have to present the numerator could be simply you see all that we need is that the numerator and denominator degree should be differ by more than one. And therefore, a could be equal to 0.

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If b and c are 0.

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Then they are multiple poles at...

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At the origin and therefore, the function would be. So, that is one of the restrictions that b and c cannot be simultaneously 0. We will come to all this we will come to all this. Yes, can b be 0 a and c are not 0 b can be 0 b if b equal to 0. Then are there any restrictions on a and c.

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Can we have a b and c as negative?

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What can cancel?

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No please reflect this can be b negative if b is negative; obviously, this polynomial we will not be having. If c is negative; obviously, this polynomial we will not be having. If b and c are both negative then also its not having right. So, there is no other alternative we are being biased towards negative quantities. We have being negative towards negative quantities, but there is not alternative because we are talking of positive real function. Definitely positive numbers are all bias.

If b is 0 does it put a constrained on other constants? You see let us look at this that is what said some fun. If b is 0 then F of s can be written as s by s square plus c plus a by s square plus c.

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It turns out yes a is positive real no your statement turns out to be wrong. Let us see let us see suppose we want to test on the j omega axis F of j omega. Then you notice that this quantity the first term shall be purely imaginary. And therefore, the real part of F of j omega is a divided by c minus omega square, which can be negative?

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Therefore the only condition that the condition that has to be satisfied is that a should be equal to 0.

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So, if b equal to 0 then a must be equal to 0. Suppose b is not 0 is this point clear.

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If b is 0 then a must be equal to 0 that is the factor would be or the function would be of the form s by s square plus c yes that is positive here. Now, let us see g in general if b not equal to 0 if b not equal to 0. Do you have to test the multiplicity of poles on j omega axis no there can be only 1 pair. If b 0 then s square plus c there is a simple poles. And what is the residue then?

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No.

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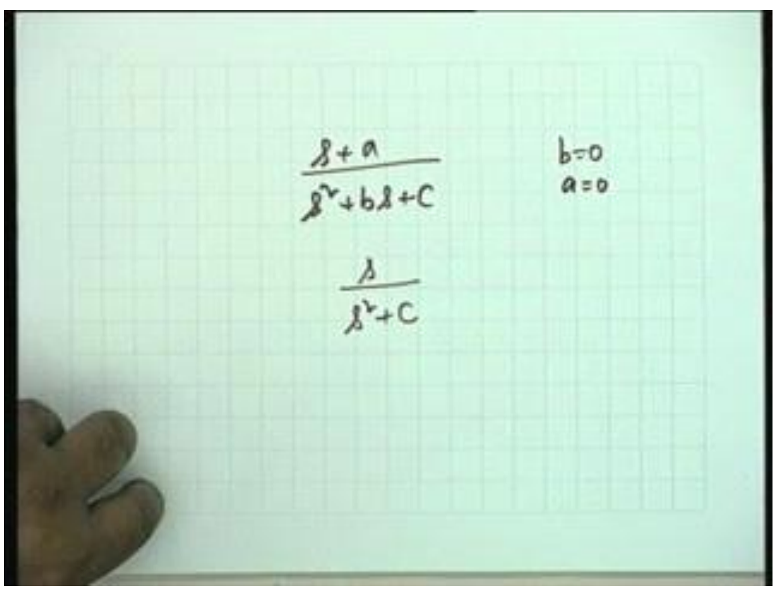

Our function is let us me ask the question again. Be careful about what i am asking b is 0 and we have shown that if b is 0 then is also 0. So, what is the function then s by s square plus c it has pair of poles on the j omega axis these are simple poles what is the residue.

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That is the correct answer you see because it should be of the form twice K 0 s divided by s squared plus omega naught square. Obviously, the root the poles are at j square root c and K 0; obviously, is equal to half. That is the correct statement all right. But suppose b is not equal to 0.

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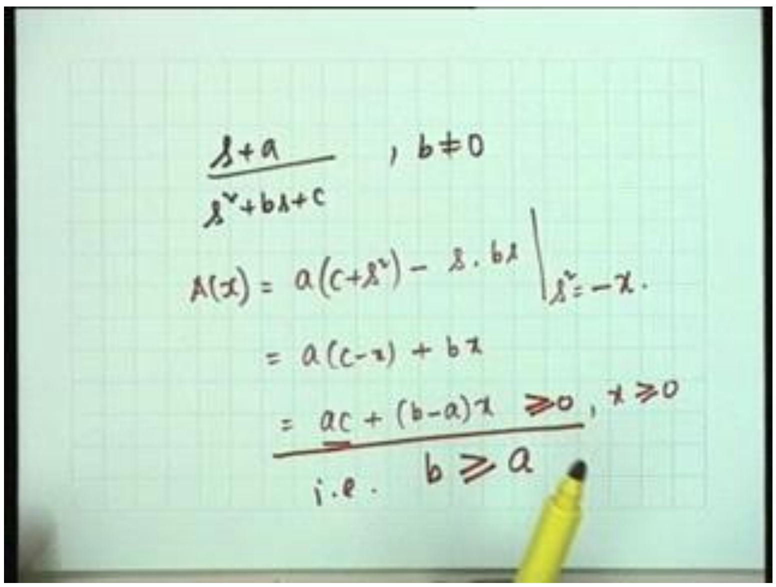

Our function is s by a s square plus b s plus c b is not equal to zero. If that is. So, then we we go right away to the real part testing that is we test A of x which is with some experience you be able to do it instantly. Which is m 1 what is m 1 a m 2 c plus s squared m 1 m 2 minus n 1 n 2 that is s times bs. Under the condition s square equal to.

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Minus x do not forget this. So, this is a c minus x plus bx. That is equal to ac plus b minus a times x and if this has to be greater than equal to 0 for x greater than equal to 0.

Obviously the condition is that b must be greater than or equal to a right. You see ac is a positive quantity. If b, this is the only quantity which can lead to negative signs and negative signs can occur only b is less than a.

Therefore, in order that this remains nonnegative we must have b greater than equal to a This also shows that if b is 0 then a has no other alternative, but to comply with b It has to follow b it has to be also equal to 0 all right.

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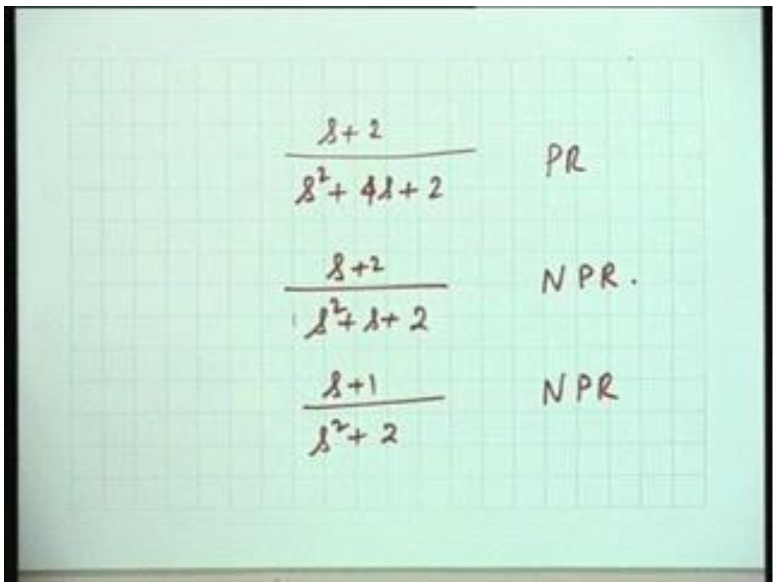

Let's take another function. Now, conflict examples s square plus four s plus 2 is this PR Just by looking at it.

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It is PR.

Suppose I have s plus 2 divided by s squared plus s plus 2 yes.

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No non PR.

I have s plus 1 divided by s square plus 0.

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No because b is 0 and a is nonzero.

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Now, we in the in the remaining time let us complicate the function.

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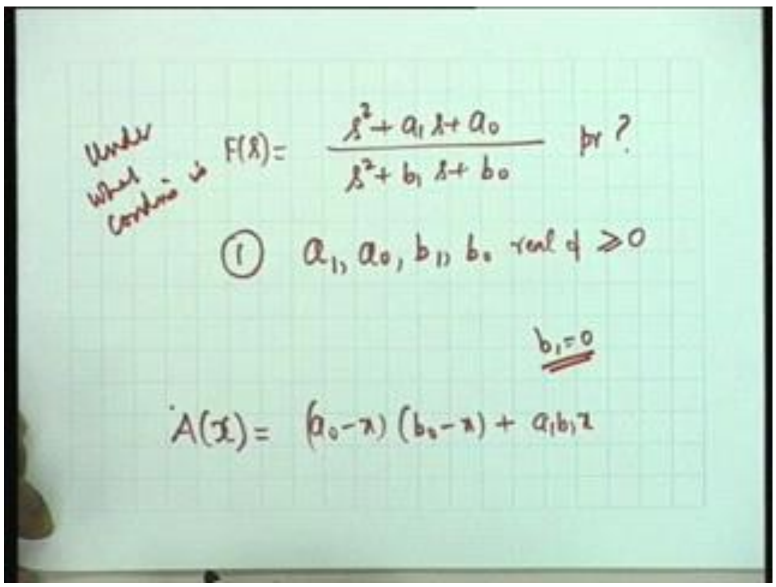

That is we have F of s equal to a general biquadratic s squared plus a 1 s plus a 0 divided by s square plus b 1 s plus b 0. Now, the numerator has been made into a quadratic in the previous case. If in the previous case if t numerator was linear and the denominator was a quadratic. We ask the question what are the conditions under what conditions is this PR. You see the first thing we say is that a 0 a 1 b 1 b 0.

They must be real and nonnegative not strictly positive some of them can be 0, but you have to be careful. Which 1 can be 0 which 1 cannot be suppose b 1 is 0. Under this condition what are the constraints on a 1 a 0 and b zero. Well if b 1 is 0; obviously, we shall have poles on the j omega axis. And these poles are simple right either a plus j square root b naught or minus j square root b naught. And the residue is there how do you find the residues.

If b 1 is 0 what are the residues what is the residue at the complex pole at the imaginary axis pole. What is the residue a thing is obvious when you seeing at why do not see it. What is the question if b is 1 0 we have pair of poles on the j omega axis. What is the residue there that is the question as simple as that all right? I leave this question.

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Is it a 1 by 2 by any chance you can show that it is a 1 by 2.

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Yes right therefore, K naught is a 1 by 2. It should be obvious, but there is a condition its not that obvious. You can be for finding out that condition we will have to find we will have to test A of x. Well what A of x can i write without going through s square can I write this as a 0 minus x multiplied by b 0 minus x plus a 1 b 1 x. Can I write like this directly with a little experience you will be able to do that? And what we want to do is we asked the question is this greater than equal to 0 for x greater than equal to 0.

We shall look at this carefully next time. The text book makes a mess of this testy; I will give you a very simplified procedure next time.