Circuit Theory

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Lecture 36

Elements of Realizability Theory (Contd.)

[Noise] this is on elements of realizability theory is being continued

last uh lecture we ended up in a bit of discussion about realizability theory

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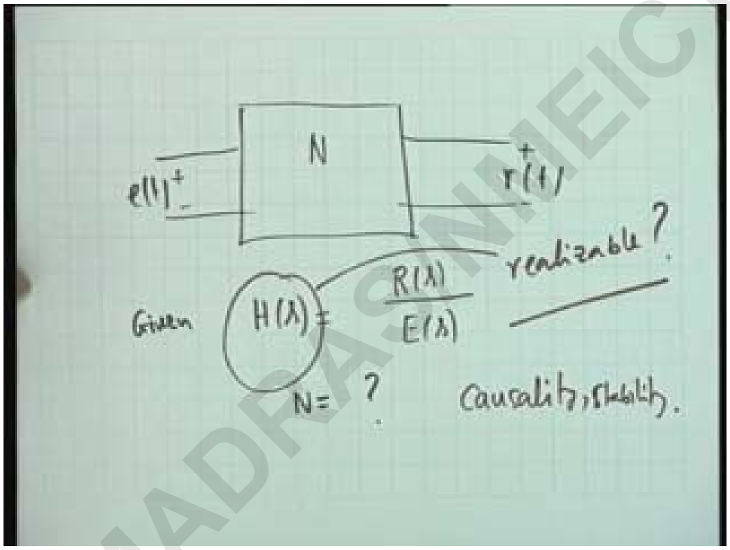

and to recall what we did was that the network synthesis problem is that the excitation and the response they are given excitation and the response r some relationship between them

and the usual relationship that is given is the transfer function or the Laplace transform of the output divided by the Laplace of the input under the conditions of zero initial conditions if this is given the problem is to find N that is the network synthesis problem

and as i said the first thing that we ask is is this is a solvable problem

in other words is the given H of s realizable this is the first question that we ask

and there are various realizability constraints for linear time invariant passive networks the constraints are that they should be causal and also stable [Noise] these are the basic two constraints that we will make as far as this course is concerned

and H of s realizable causality and stability

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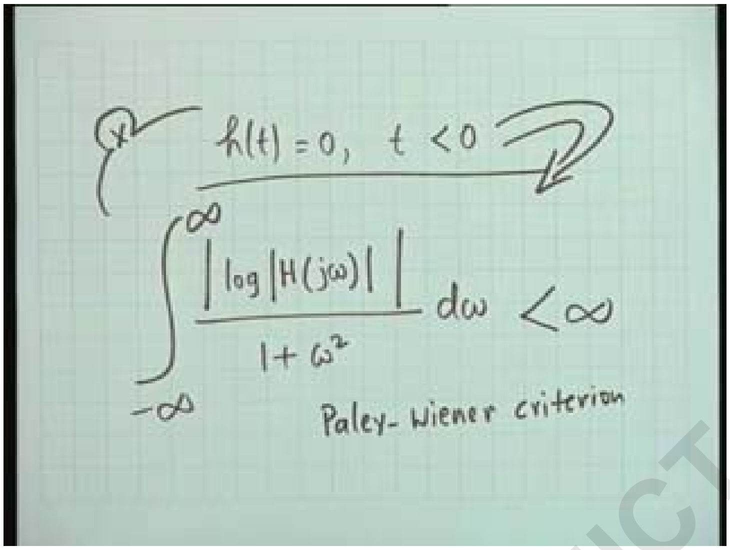

as you know causality implies that the output cannot precede the input that is the and it is stated in terms of unit in pulses

((once)) (00:01:55) h of t should be equal to zero t less than zero because delta t is applied at t equal to zero this is the causality constraint

and [Noise] and we also mentioned last time that in terms of the frequency domain the causality constraint can be transformed into

H of j omega where H of j omega is the Fourier transform of h of t

and the constraint is that if we take log of H of j omega magnitude and then take the magnitude because log could be positive or negative

divide by one plus omega squared and integrate this from minus infinity to plus infinity [Noise] d omega then this should be finite

this is the this is the equivalent of this simple relationship h t equal to zero t less than zero in the frequency domain that is

<a_side> (( )) (00:02:57) <a_side>

no no it’s only log of magnitude H

well if we take the magnitude squared it is only multiplied by two and therefore if this quantity is less than infinity ah um divided by two also makes it less than infinity okay so it clearly doesn't matter whether you take squared or not but [Noise] we are not deriving this [Noise] we are simply saying that this is the so called Paley Wiener criterion

but we can have a deeper look at what it implies

for example if there is a frequency at which the magnitude of H of j omega is zero

as an example you take the parallel t network parallel t network or the bridged t network which has a zero which has a transmission equal to zero at some frequency that is a frequency of zero transmission

now if magnitude H of omega equal to zero what is log

log is minus infinity all right

now [Noise] the value of the integrant equal to minus infinity is tolerable at a particular value of omega all right which simply means that at that frequency this quantity shall have a pole agreed

all network functions and analytic functions they are differentiable everywhere except at a finite number of singularities all right

so singularity doesn't matter [Noise] if there is singularity and we want to make a contour integration then what we actually do is we make a (( )) (00:04:29) like this you know this contour integration business

but suppose the magnitude is zero over a band of frequencies over a band of frequencies

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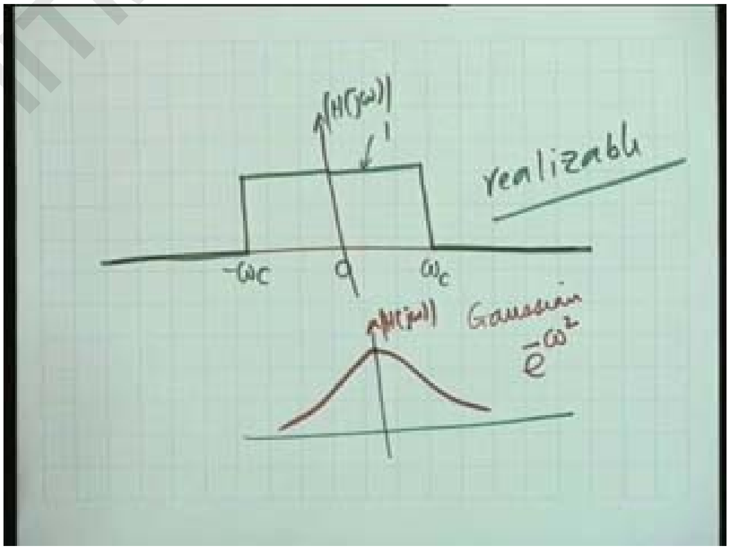

for example [Noise] you have the ideal low pass filter [Noise] let's look at this

ideal low pass filter is it is unity let's say between some frequency omega c and zero minus omega c and it is zero elsewhere this is the ideal low pass filter

now in the ideal low pass filter you see the magnitude of the transmission this is magnitude H of j omega this is zero over a band of frequencies now if it is over a band of frequencies then obviously infinity [Noise] multiplied by any finite non zero quantity will be infinity

and therefore this Paley Wiener criterion constraints that magnitude H of j omega cannot be zero over a non zero band of frequency all right

in other words if it restricts that this is not a realizable realizable magnitude function

an ideal low pass filter any ideal filter is not in fact realizable because [Noise] it is required to be zero over a non zero band of frequency okay this is one one of the restrictions that this Paley Wiener criterion implies

and the other restriction is that if magnitude H of j omega falls with frequency as a low pass filter should let's say it falls like this on the other side also it falls like this then this fall should not be mark my [Noise] words more rapid than an exponential fall it cannot decay more rapidly than how an exponential decays okay

now to to make this a little more specific let's take the Gaussian curve the bell shaped curve which can be represented by e to the minus omega squared okay

suppose my H of j omega is equal to e to the minus omega squared then the fall with increase of frequency is more than exponential right [Noise]

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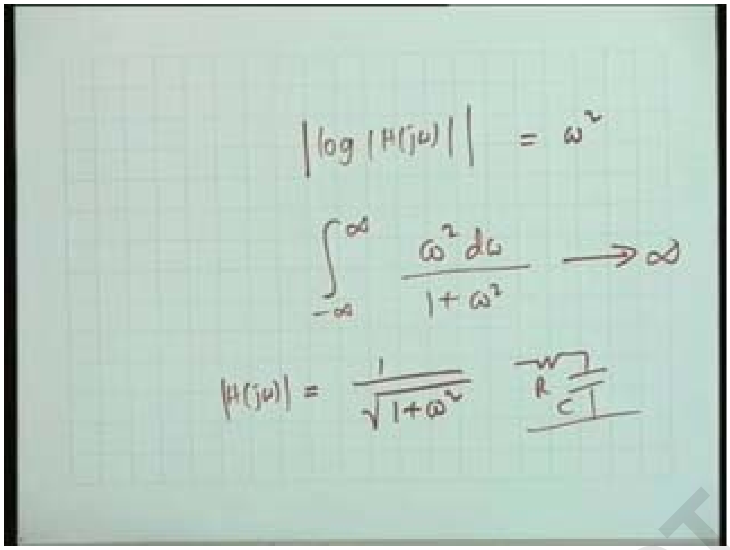

now [Noise] if you take the ah if you take the ah log magnitude H of j omega and magnitude of {thus} (00:07:23) this this is simply equal to omega squared

an integral minus infinity to infinity omega squared d omega divided by one plus omega squared goes to infinity it is not a finite quantity it is one example

but one of the interpretation one of the ah consequences of the Paley Wiener criterion is that the magnitude squared function or the magnitude function [Noise] cannot decay more rapidly than an exponential

similarly it cannot rise more rapidly for high pass filter it cannot rise more {ex} (00:07:57) more rapidly than [Noise] an exponential okay

so there are two ah consequences of the Paley Wiener criterion which we should be aware of there are many others and you have to ah actually test by Paley Wiener criterion

but the easier way is the h of t and for us for circuit theory purposes h of t would be good enough

on the other hand suppose magnitude [Noise] H of j omega

let me ask you a question suppose magnitude H of j omega is square root of one plus omega squared well is it realizable this is a trivial question

it is realizable why because we know the network i think it's obvious when we see it we know the network what network does realize this a simple RC where R and C are both equal to unity or the product is unity that’s good enough sc one by sc r plus one and if the product is unity then the magnitude squared function is this so you know

and if you if you substitute this in the Paley Wiener criterion you will see that the integral is a finite quantity okay

so ah um [Noise] in summary we have not proved anything we have only stated the Paley Wiener criterion and we have seen two consequences of the Paley Wiener criterion and these are good enough for an electrical engineer [Noise]

that is

<a_side> suppose (( )) (00:09:25) why we can't take ah ah [Noise] (( )) (00:09:30) zero or not um (( )) (00:09:32) <a_side>

okay the question is ah why can H of j omega why is it constrained to have ah not why it is constrained not to have a band of frequencies for which the value is zero

you see if the value is zero then log is infinity log is minus infinity and minus infinity magnitude is infinity so of necessity this integral is going to be infinite because the integrant itself ((blows up)) (00:10:00) all right

that's why however if it is only at a particular frequency then infinity multiplied by zero because the band width is zero integral means ah an area the area will be equal to zero okay

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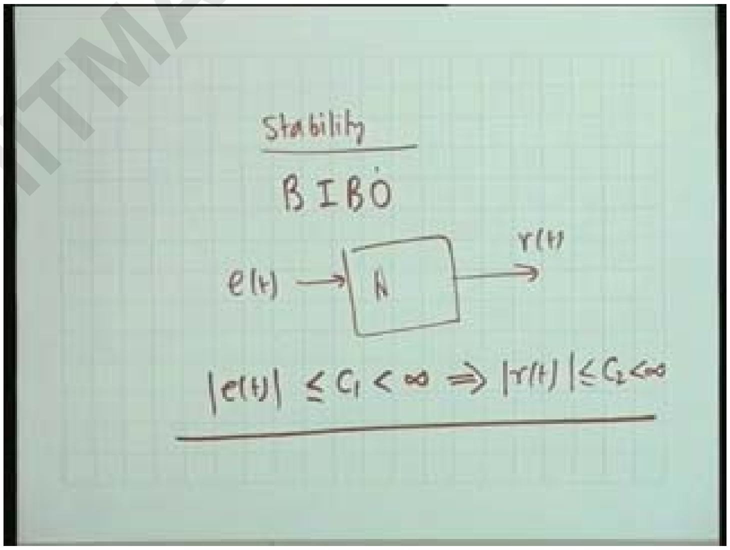

now the second question for realizablity then that is that is more important in the context stability and the kind of stability that we bother about in electrical engineering is the B I B O stability

that is if the input [Noise] e of t is bounded

if e of t is bounded less than equal to C one which is less than infinity some C one some constant C one

then B I B O stability says that r of t magnitude should be less than equal to some other [Noise] quantity C two which is finite this is the criterion for stability

and you know that for a linear time invariant system causal system now r of t and e of t they are related by the convolution integral okay

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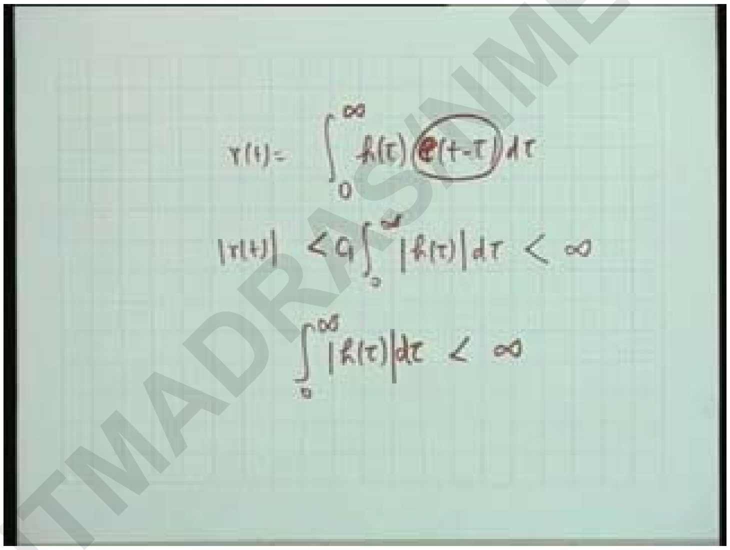

so [Noise] the convolution integral is that r of t is equal to integral zero to infinity [Noise] why zero because we [Noise] assume [Noise] causal okay

h tau r of i am sorry e of t minus tau d tau this is the convolution integral

and the mod r of t shall be less than integral zero to infinity mod h of tau d tau

<a_side> (( )) (00:11:48) <a_side>

in general zero to infinity because we have not assumed e of t as causal signal in general zero to infinity but you if this is also causal that is e of t doesn’t exist for t less than zero then the upper limit shall be equal to t this is more general okay now if e of t minus tau is bounded by C one then it is less than C one multiplied by this and this is required to be less than infinity therefore this is a (( )) (00:12:25) a revision of what you did in the signals and systems course

that is stability requires that h of tau d tau integral should be less than infinity this ((is this)) (00:12:37) you must have done in signals and systems that is the impulse response should be absolutely integrable the absolute value [Noise] of this integrated from zero to infinity should be finite

now this is in the time domain

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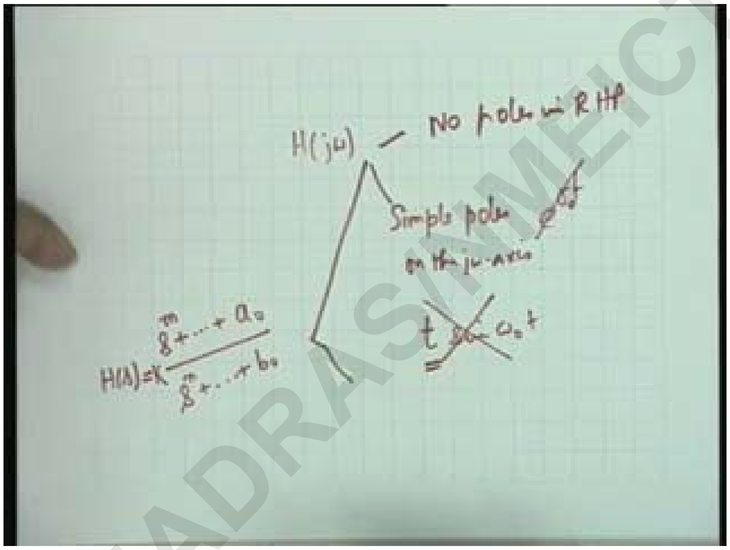

the frequency domain criterion is that the corresponding transfer function

that it is H of j omega must have poles or the other way around no poles in right of plane what will happen if we have poles in the right of planes

then the time domain response would have would increase indefinitely with time okay

the time domain response would have factors of the formation [Noise] of sigma t sigma zero t where sigma zero is positive and therefore it would grow indefinitely

no poles in righter plane

if there are poles on the j omega axis if there are poles on the j omega axis they must be simple (( )) (00:13:41) why what is the restriction why is this restriction simple poles on the j omega axis why is it so

<a_side> (( )) (00:13:49) <a_side>

okay

if there are multiple poles [Noise] then we shall have the time domain [Noise] response would be would contain product of t and sin or cosine of omega zero t okay

if there are simple poles of the j omega axis the corresponding impulse response shall be either sin or cosine okay

it can be explained

if there are multiple poles let's say second order pole that is pole of multiplicity two then we shall have a term of the of this form t multiplied by this and this grows indefinitely as t increases

so we we can have only simple poles on the j omega axis

this also implies if there are simple poles on the j omega axis

and we assume that H of s is a polynomial in s divided by another polynomial [Noise] that is something of the form K let's say s to the power m plus etcetera plus a zero

and if the denominator is s to the n plus etcetera plus b zero let me write it again [Noise]

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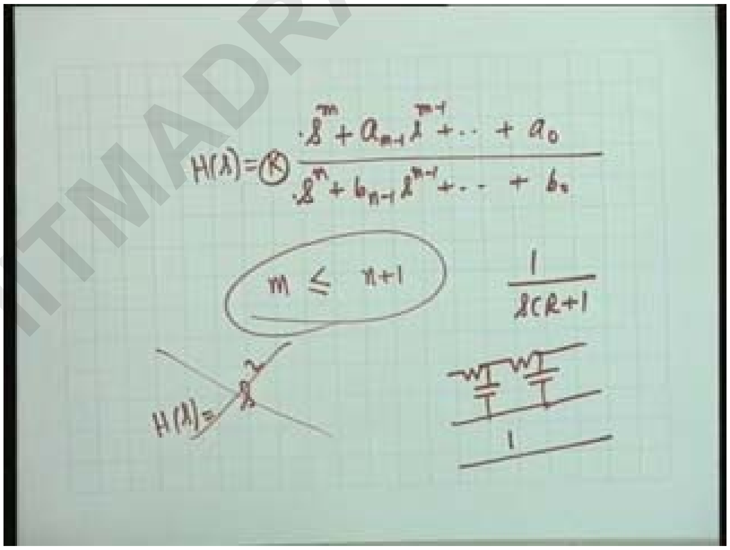

if our H of s is of this form K multiplied by s to the m plus let's say am minus one s to the m minus one plus etcetera plus a zero

divided by s to the n plus bn minus one s to the n minus one plus etcetera plus b zero okay then there is a restriction on the degrees of m degrees of the numerator and denominator [Noise] you understand how how we wrote this transfer function

this is in general a ratio of polynomials n of s divided by d of s

and what we have done is the living terms here and here we have made into unity the constants they have observed in K all right this will be a standard form for writing the transfer function

now [Noise] if that is so then what what is the restriction between m and n

which we will say that if there are poles on the j omega axis [Noise] they must be simple

that means how can there be pole on the at infinity infinity after all is a point on the j omega axis

<a_side> and can we (( )) (00:16:08) <a_side>

m should be greater than n but can m be greater than n by more than one that is the point

no

therefore m should be less than or equal to n plus one this is the restriction

then the numerator degree can be the less than the denominator degree it can be it can be constant for example the numerator can be a constant

for example in one by SCR plus one [Noise] well the numerator is not a polynomial it's a constant so it's a zeroth degree polynomial

now [Noise] if you take a second order network for example two RC in (( )) (00:16:53) then numerator can still be a constant unity the denominator will be a quantity okay

so the numerator can the degree of the numerator can be less than that of the denominator but if it is greater then it cannot be greater by more than one that is the restriction

this is why if you take an H of s is equal to s squared this is not realizable okay because the degree the the because there are two poles at infinity you can only allow a simple pole

[Noise]

now to this ah [Noise] realizability criterions

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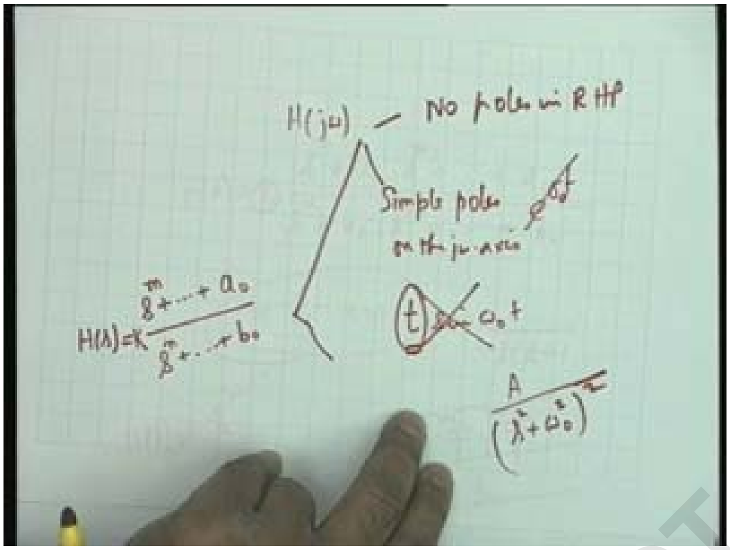

<a_side> excuse me sir <a_side>

yes

<a_side> in this how did we prove that (( )) (00:17:40)) <a_side>

how did we

<a_side> yes sir <a_side>

okay

if there are multiple poles on the j omega axis then the time domain response shall contain terms of the form t times [Noise] sin or cosine

you see if we have A factor of the form s plus omega s squared plus omega not squared let's say whole squared okay then we shall have terms of the form t times sin omega zero or cosine omega zero

and as t tends to infinity this term (( )) (00:18:21) it grows indefinitely so the system becomes unstable

any other question

<a_side> excuse me sir <a_side>

yes

<a_side> let me have ah m greater than n by one degree <a_side>

okay

<a_side> then we will have a uh power of s ah so we will have a term at quotient in s <a_side>

correct

<a_side> sir if we can work it to uh the time domain <a_side>

right

<a_side> we get a delta dashed e <a_side>

okay

<a_side> sir uh but delta (( )) (00:18:49) <a_side>

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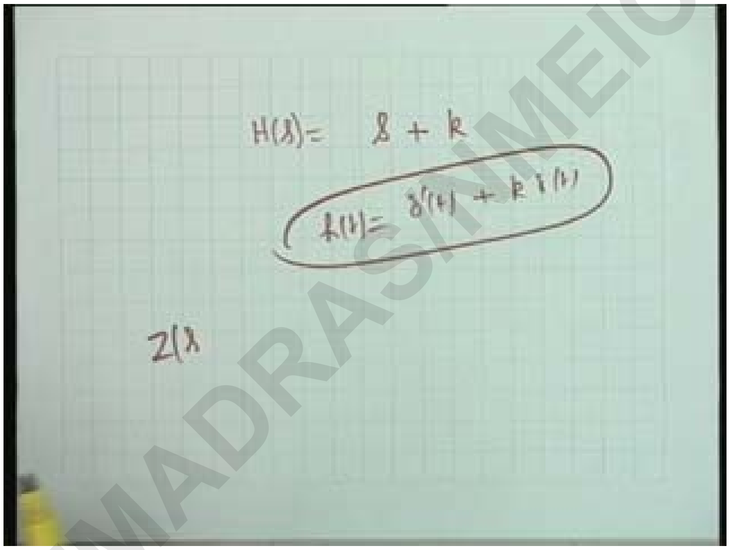

H of s equal to s

<a_side> yes sir s plus something else <a_side>

all right

<a_side> and sir if we convert this uh we get one term as <a_side>

delta prime t

<a_side> yes sir <a_side>

and the other becomes K delta t

<a_side> moreover the delta prime t is not uh realizable sir (( )) (00:19:02) so it doesn't mean anything <a_side>

all right it doesn’t mean anything

well we know that if we have a if we have this is an impedance function

<a_side> yes <a_side>

s plus k [Noise] i know that an inductance realizes s isn’t that right an inductance the impedance is s times L

now if you if this is a transfer function that is a voltage uh ratio then we shall have problems we will come to that

but obviously a network function H of s which is an impendence function can be can be of this form s plus k okay

and our function H of s that we wrote [Noise] was of a general form it's not a it's not a true transfer function it is a network function all right

any other question

[Noise]

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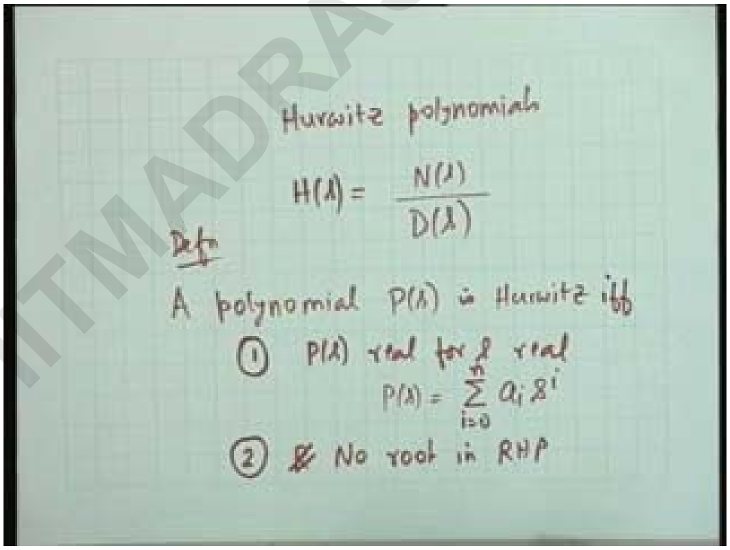

intimately related to this ah question of realizability ah an intimately related to testing for realizabilty is a kind of polynomial which go by the name of a gentleman by the name Hurwitz Hurwitz polynomials [Noise]

and ah [Noise] and lets study what they are because they are very intimately related to the testing for realizability suppose we have a a network function it is N of s by D of s okay

then [Noise] realizability demands that H of s poles must be in the left of plane okay open left of plane that is they can be on the j omega axis also but no none on the in the right of plane okay

D of s therefore and [Noise] the poles of H of s at the zeros of D of s okay

ah a a polynomial which closely obeys this condition is the Hurwitz polynomials okay

so one says that a the D of s the denominator polynomial must be a Hurwitz polynomial

now let's see the Hurwitz polynomials are

<a_side> (( )) (00:21:10) <a_side>

okay

conditions at for for realizabiltity of H of s we want it's poles to be in the left of plane including the j omega axis okay that is closed left of plane

such polynomials polynomials whose [Noise] roots are in the closed left of plains are called Hurwitz polynomials

and therefore the realizability constraint can be stated in terms of Hurwitz polynomials rather than saying where it's poles should be if poles occur on the j omega axis they should be simple and so on and so forth

<a_side> (( )) (00:21:47) <a_side>

poles of H of s are the zeros of D of s

<a_side> (( )) (00:21:53) <a_side>

pardon me

<a_side> sir poles of numerator (( )) (00:21:59) or <a_side>

okay

the numerator as you will see in the case of an impedance function the numerator also has to be Hurwitz's because you know [Noise] {bar a} (00:22:07) for a driving point function poles as well as zero should be in the left of plane [Noise] so we will apply that to N of s also okay when it comes to

but let me define first what a Hurwitz polynomial is a polynomial this is the definition a polynomial P of s in general is defined to be Hurwitz if and only if two conditions are satisfied [Noise]

that is it should be a real polynomial that is P of s should be real for s real what does that mean that means that since it is a polynomial it should be of the form ai s to the i let's say i is equal to zero to n

then it P of s real for s real means that the coefficients ai must all be

<a_side> real <a_side>

real that's all as simple as that okay

and the second condition is that roots of P of s should be

or put it this way no root in RHP P of s cannot have any roots in the righter plane [Noise]

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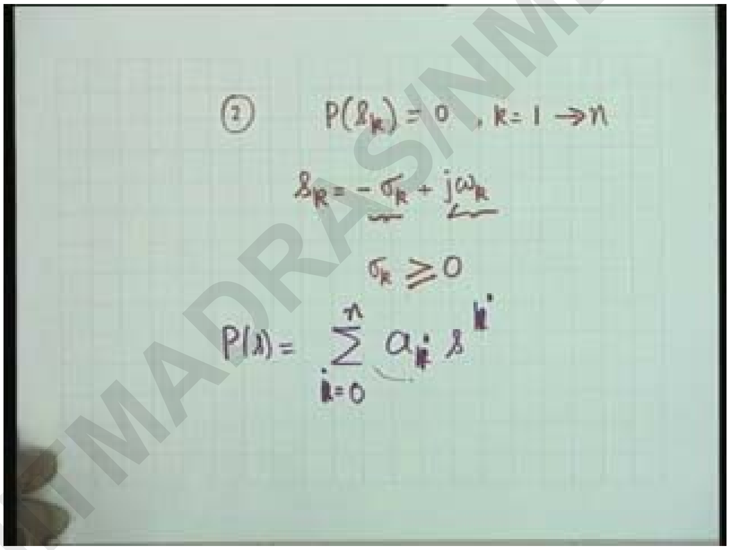

this statement that no root of RHP can be ah {amp} (00:23:37) amplified like this that is suppose P of let's say sk is equal to zero [Noise] okay

P of sk is equal to zero k equals to let's say one to if n is the degree of the polynomial then the fundamental theorem of algebra states that every polynomial of nth degree will have

<a_side> (( )) (00:24:00) <a_side>

n roots therefore k will go from one to n

so the second requirement that no root in right of plane means that

if P of sk equal to zero that is sk is one of the roots where the subscript k goes from one to n and sk equal to let‘s say minus sigma k plus j omega k where this is the real part and this is the imaginary part okay then what does it mean

it means that sigma k must be

<a_side> greater than <a_side>

greater than or equal to zero because it is the close left of plane

the real part of the roots must be must be non negative

one of the ways of expressing this greater than equal to zero is that the real part should be non negative all right [Noise]

any question

our polynomial we took as summation ak [Noise] s to the k

k equals oh no i took i isn’t it

ai s to the i where i is equal to zero to n all right

[Noise]

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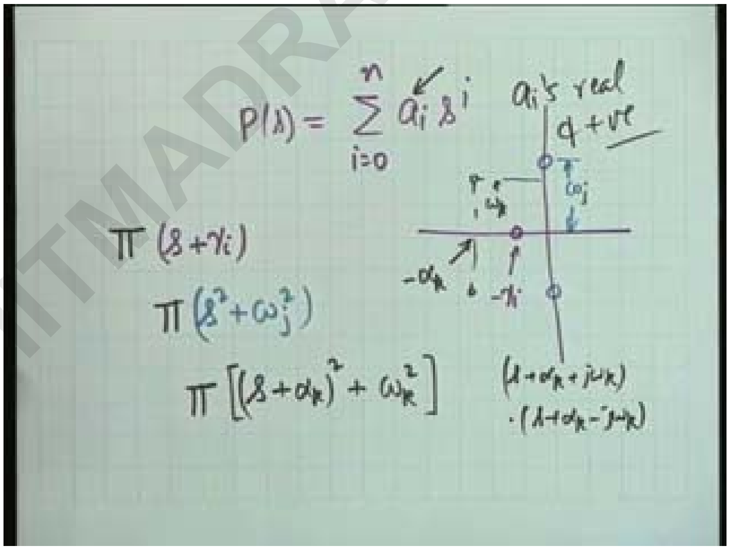

let’s look at this a little more closely

P of s is equal to ai s to the i i is equal zero to n [Noise]

now i have already said that this ai these coefficients must be real because Hurwitz polynomial demands that the polynomial be real [Noise] for s real [Noise] if s is real [Vocalized-Noise] (( )) (00:25:34) obviously have to be real okay

what about sin {cand} (00:25:39) can some of the coefficients be negative

the answer transfer to be no

we cannot have a negative (( )) (00:25:47) let's see why let's see why [Noise]

the roots that are permitted the roots that are permitted for P of s are belong to these locations there can be a root on the negative (( )) (00:26:01) axis okay

let's say this point is minus gamma i okay

so this root will contribute a factor of s plus

<a_side> gamma <a_side>

gamma i isn't it right this root will contribute a factor of s plus gamma i

this is one possible root [Noise] another possible root could be a pair on the j omega axis let's say this is omega j

why two

<a_side> (( )) (00: 26:35) <a_side>

there must be ((complexed)) (00:26:36) on the way because

<a_side> (( )) (00:26:39) <a_side>

because the coefficients are real questions correct

so what do this factors what do this two roots contribute what factor obviously s square plus omega j square all right these two roots contribute to a factor like this

and the only other possibility only other possibility is that we have roots like this complex conjugate all right

let's say this is this distance is omega k [Noise] and this distance is let's say minus alpha k all right [Noise]

these two roots contribute to a factor of s plus alpha k whole square plus omega k square is this clear these two roots [Noise] contribute to a factor of this form because what you have is

s plus alpha k plus j omega k multiplied by s plus alpha k minus j omega k and if you multiple this out this is precisely what you will get all right okay so [Noise] in general these are the only three possible forms of way you can’t think of any other

and the polynomial must be a product of factors like this that is you can have continued product of factors like this continued product factors like this and continued product factors like this

in theses component polynomials in this component polynomials all coefficients are positive and by multiplying a number of component polynomials in which the coefficients are positive you can get a you can never get a negative term isn’t that correct

this is right or wrong

which means which means that the coefficients ai’s must be positive ai’s are real and positive agreed

if you see a [Noise] a network function ((in it)) (00:28:55) a denominator contains a complex coefficient immediately you can throw it out not realizable if you see ah all coefficients are real but one of them is negative immediately you can throw it out okay

now next question

<a_side> we are talking of denominator <a_side>

denominator yeah in general for impedance functions the numerator has also to be tested

<a_side> sir excuse me <a_side>

yes

<a_side> (( )) (00:29:20) non negative <a_side>

{sh} (00:29:23) that is the question i am going to ask you now okay whether its {sh sh} (00:29:28) whether we should call it as positive or non negative

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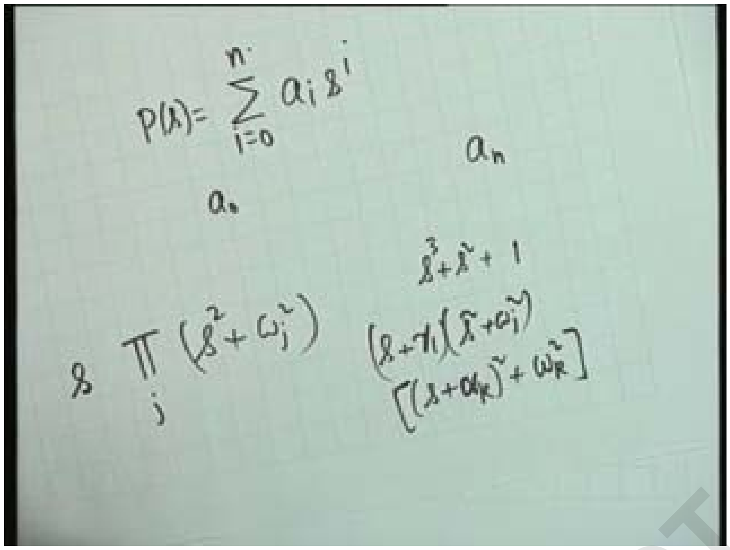

let’s say P of s [Noise] once again ai s to the i i is equal to zero to n

a question that i am going to ask you is [Noise] between a zero and an between a zero and an can there [Noise] be a missing term

<a_side> yes sir <a_side>

it can be

now how can a missing term be created from product of factors like this how can you think of a missing term

there cannot be the answer is there cannot be unless you have a term like s minus gamma i when because missing terms requires cancellation and cancellation can occur only if in uh in one of the components there is a negative term

obviously there cannot be a negative term here unless this is minus alpha k

there cannot be a negative term here at all these are both positive

<a_side> (( )) (00:30:35) <a_side>

okay okay that’s a good question [Noise]

<a_side> (( )) (00:30:44) <a_side>

good question what he says is [Noise] if i have terms only of this form continued product over j yes there are missing terms possible but it’s a very special case what is the special case

that it is a purely even polynomial it's a purely even polynomial and it is Hurwitz because it’s roots are not in none of the roots are in the righter plane okay so you must qualify [Noise] that in general there cannot be a missing term between a zero at an unless [Noise] unless all odd terms are missing that is unless the polynomial is purely even [Noise]

and we are being a bit ((bias)) (00:31:32) when we are saying purely even because such a polynomial if multiplied by s also shares the same property if you multiply by s then the polynomial [Noise] becomes purely

<a_side> odd<a_side>

odd and all even parts are missing

therefore you you you should say that in general no term between a zero and an should be missing unless all odd terms are missing or all even terms are missing all right that is unless i unless the polynomial is purely even or purely odd

now i uh [Noise] look at the question i ask you a question [Noise] unless you have a question

<a_side> sir how it makes difference that some other (( )) (00:32:18) <a_side>

how it makes a difference [Laughter] okay [Noise]

if if let's say [Noise] you have a term s cube plus s square plus one

then the coefficient of s is missing how can that coefficient be missing unless there are [Noise] cancellations unless there are cancellation

you see our only term are s plus gamma i s square plus omega j squared [Noise]

and s plus omega k squared no alpha k squared plus omega k squared by multiplying such terms you can never get a missing term unless unless either this gamma i is negative or alpha k is negative that means roots in the right of plane [Noise]

any other question

now let me ask my question [Noise]

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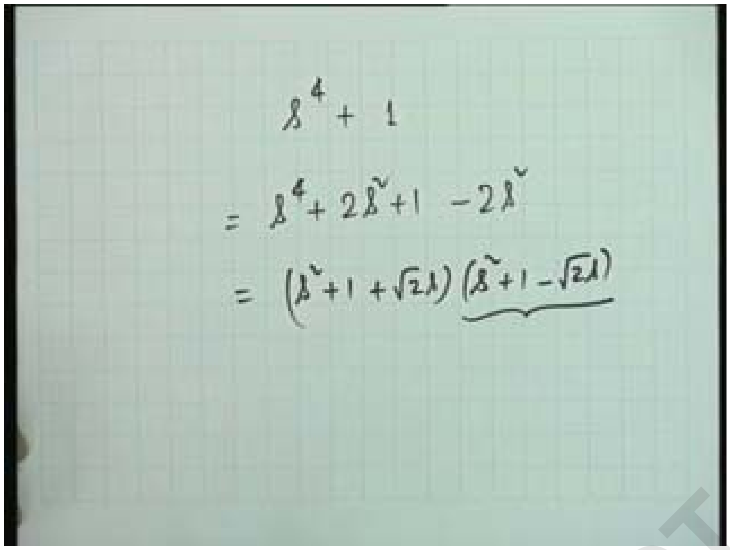


suppose you have a polynomial s to the four plus one [Noise] it's purely even it is purely even

<a_side> (( )) (00:33:28) <a_side>

is it Hurwitz

<a_side> no<a_side>

no because

<a_side> (( )) (00:33:32) <a_side>

one of the event terms is absent

there can you see you can allow the polynomial to be purely even or purely odd provided there are no missing terms again you can allow all odd terms to be missing but no even terms should be missing if it is even polynomial all even terms should be there

missing means that there are [Noise] there are roots in the righter plane can you show this here it's very easy

you can write this as twice s squared plus one minus twice s squared

so this is s square plus one plus root two s s square plus one minus root two s and you can see that this is a [Noise] non Hurwitz polynomial because there is a

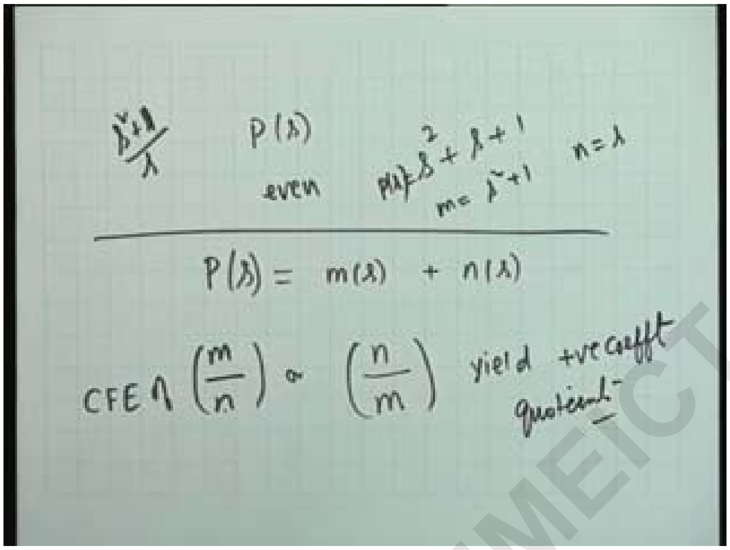
<a_side> negative coefficient <a_side>

negative coefficient

is that clear

we shall have lot of fun with Hurwitz polynomials [Noise] and Hurwitz polynomials pay ah play a very important role [Noise] in the uh circuit theory in network synthesis

(Refer Slide Time: 00:34:39 min)



now suppose i have a P of s which is purely even no missing terms where do you think it’s roots shall be

<a_side> it's purely even <a_side>

purely even with no missing they must be (( )) (00:34:53)

similarly if it is purely odd then also its roots must be on the j omega axis only with

<a_side> and zero <a_side>

yeah with one root at the origin

can there can be two roots at the origin

<a_side> no <a_side>

no if it is odd then there cannot be

<a_side> one uh <a_side>

can you have three roots on the origin

<a_side> yes <a_side>

yes you can but if there are three roots at the origin then the then the function is useless for network synthesis why

<a_side> multiple <a_side>

multiple poles (( )) (00:35:24) absolutely wonderful that means Hurwitz polynomial has (( )) (00:35:27) okay

suppose P of s [Noise] in general it's a Hurwitz polynomial with its even part denoted by small m and it's odd part divided denoted by n of s small n of s

then one of the properties and this is the test for a Hurwitz polynomial the test for a Hurwitz polynomial is that

the ratio m to n the ratio of the even part to the odd part or the odd part to the even part depending on which one has a higher degree

suppose the even {pa} (00:36:11) suppose P of s the highest degree highest power is even then you should take the ratio m by n

<a_side> (( )) (00:36:18) <a_side>

[Noise]

pardon me

<a_side> (( )) (00:36:21) <a_side>

[Noise] [Noise]

oh m is the even part i think i {sh} (00:36:25) okay

small m is the even part of P of s and small n is the odd part of the P of s

for example if i have plus s square plus s plus one P of s

then m is equal to s square plus one and n is equal to s even part and odd part all right

and [Noise] the ratio that you should take is now is s square plus one divided by s that is you should take the higher degree polynomial in the numerator and the lower degree one in the denominator

what can be the degree difference between these two m and n

<a_side> (( )) (00:37:02) <a_side>

can it be two why not

<a_side> can be three <a_side>

why not two

<a_side> (( )) (00:37:08) <a_side>

because one is even and the other is the odd so the {diff} (00:37:13) difference [Laughter] must be difference must be odd okay

<a_side> sir it cannot be three <a_side>

it cannot be three it it it can be three but then we shall not be interested okay all right

now one of the properties of a Hurwitz polynomial is and this property leads to the testing of Hurwitz polynomials is that this ratio m by n or n by m putting this higher degree polynomial in the numerator

if you make a continued fraction expansion of this or this depending on which ever is applicable the continued fraction expansion of this shall yield yield positive quotients it can be rigorously proved but we are not interested in the proof [Noise]

the testing of a Hurwitz polynomial says that the continued fraction expansion of m by n or n by m will yield positive quotients okay positive quotients

now quotients obviously {posit} (00:38:24) and i am i am not this is not quite correct what will be the form of quotients

suppose suppose i divide s square plus one by s if i expand this into continued fraction the first quotient will be

<a_side> s <a_side>

s

now what do you mean by positive s [Noise] no positive

<a_side> positive {co co} (00:38:430) coefficient <a_side>

coefficient quotient all right [Noise]

the continued fraction expansion should yield positive coefficient quotient

before i take an example [Noise] ah tell me will this continued fraction expansion will it always be finite or is there a possibility that it can go on and on like a recurring decimal continued fraction expansion will it be finite or shall it continue indefinite

<a_side> sir if it is realizable (( )) (00:39:16) <a_side>

it has to be finite how many quotients do you expect

<a_side> one <a_side>

how many quotients continued fraction expansion well all you have forgotten what is continued fraction expansion it would be

<a_side> (( )) (00:39:31) <a_side>

difference

<a_side> no sir degree of difference (( )) (00:39:36) <a_side>

depending on the degree of the numerator which is also the degree of the polynomial [Noise] isn't that right

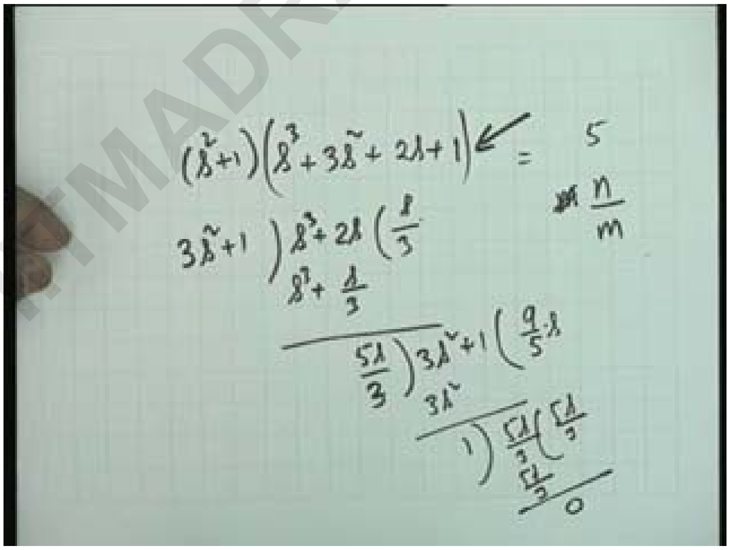
therefore continued fraction expansion one of the properties is not a Hurwitz polynomial any continued fraction expansion shall be finite and the number of quotients that we expect is equal to the degree of the polynomial

for example if you have a cubic polynomial like this

<a_side> suppose you give only finite only the uh it is a realizable (( )) (00:40:09) <a_side>

no

(Refer Slide Time: 00:40:07 min)



any it is realizable or unrealizable it doesn't matter Hurwitz or not it doesn't matter

if you have a cubic polynomial for example the continued fraction expansion would be of this form

s cube plus two s you shall divide by three s square plus one all right s by three s cube plus s by three what am i left with

six

<a_side> (( )) ((00:40:39)) <a_side>

five s by two three s square plus one

<a_side> s by three <a_side>

five s by three okay

so this would be nine by five s

so three s square

one five s by three five s by three and that's it continued fraction expansion ends

i have exactly three quotients the coefficients of which are positive and therefore i conclude that this must be a Hurwitz polynomial

the this is did only to illustrate the number of quotients is exactly equal to degree of the polynomial

now it may so happen in Hurwitz testing it may so happen that you don't get this required number of quotients in other words the continued fraction expansion may end prematurely is that possible

<a_side> (( )) (00:41:37) <a_side>

it is possible only if the even part and the odd part have a common factor

for example if this polynomial is multiplied by s square plus one all right then what is the degree of this polynomial

degree would be five but you shall get exactly three [Noise] quotients because in forming what shall we form here n by m in forming n by m n has a factor s square plus one m has a factor s square plus one and therefore and therefore the number of quotients shall be exactly equal to three not five

in other words there is a possibility there is a possibility that the continued fraction expansion will end prematurely and if it ends prematurely that shows that even part and the odd part have a common factor

and this common factor if an even polynomial let me go slow if an even polynomial and an odd polynomial [Noise] has a common factor what can be the form of this common factor you understand what i mean [Noise]

<a_side> s plus alpha <a_side>

it cannot be s plus alpha

<a_side> it has to be a s squared or a (( )) (00:43:02) <a_side>

it has to be an even polynomial and this is why you said s square plus one okay and where how do you detect it

let's before taking an example

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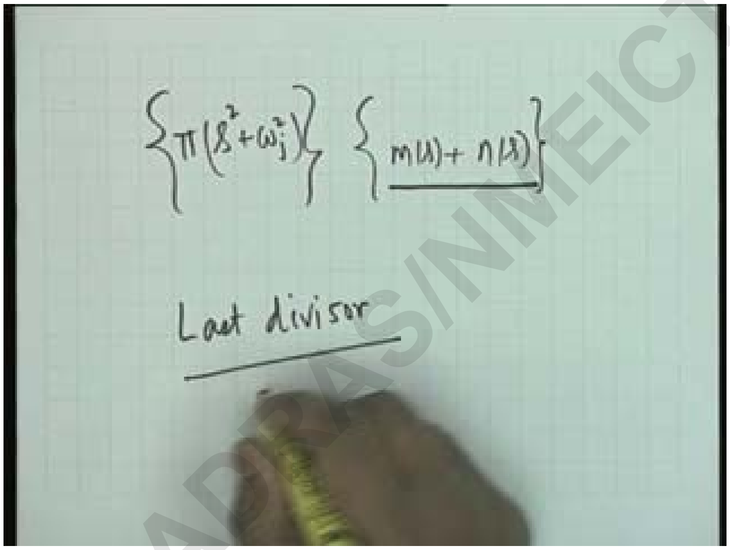

suppose i have let's say s square plus omega j squared a number of them

suppose this multiply in m of s plus n of s

suppose i have a number of factors

how do i detect these factors because in the continued fraction expansion of the even part divided by odd part or the reciprocal of it i shall get a number of quotients exactly equal to the degree of m plus n these factors how do i (( )) (00:43:49)

it it is it requires a bit of thinking to convince oneself that these factors will make their appearance during the continued fraction expansion in fact the last divisor the last divisor shall be proportional to these cancelling factor the last divisor after that there is no other divisor because the ah remainder is zero

<a_side> (( )) (00:44:21) <a_side>

okay

<a_side> could you please explain me the last (( )) (00:44:24) <a_side>

ah [Noise] may be i take an example then i will come to the theory

let me take an example [Noise]

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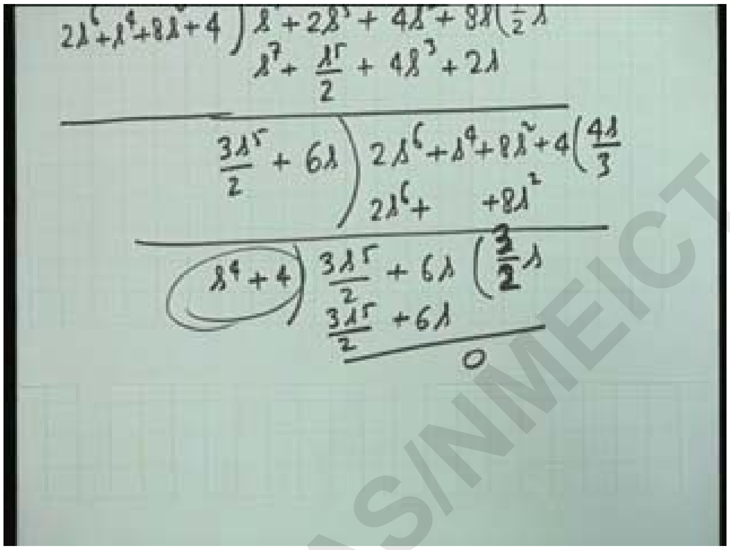

suppose i have s to the seven plus two s to the six plus two s to the five plus s four clearly non trivial example four s cube plus eight s squared plus eight s plus four this is my P of s

and the question being asked is question being asked is is this Hurwitz or not

so what we do is we make a contribute fraction expansion like this

two s to the five plus four s to the three plus eight s

i divide by by two s to the six plus s to the four plus eight s squared plus four okay

so i get half s as the first portion i get s to the seven plus ah s to the five by two then plus four s cubed four s cubed plus two s agreed

please don’t allow me to make a mistake is that okay

so i get two {ma} (00:45:49) ah this would be three s to the five by two all right

these two cancel plus six s is that right [Noise] all right

so i divide two s to the six plus s four plus eight s squared plus four

now what do i get

four s divided by three this is the quotient [Noise]

don’t allow me to make a mistake

so it's two to two s to the six plus ah eight s square eight s square

so i am left with s to the four plus four all right

so let's bring three s to the five by two plus six s all right then what do i get

two by three s

<a_side> (( )) (00:46:45) <a_side>

three by two s can i slow down [Laughter] okay of course i can

{whe} (00:46:56) where where did where did did you loose your self

<a_side> (( )) (00:46:58) <a_side>

or you might [Laughter] all right

prevention is better than cure

we shall do that three by two s

so i get three s to the five by two plus

<a_side> six s <a_side>

six s

and you see that i expected seven quotients and end up with three

<a_side> (( )) (0047:21) <a_side>

the last divisor must be of degree four i have got three i have missed four of them the last divisor must be of degree four and the last divisor is the common factor not the common factor there could have been a constant

for example two s four plus eight the last divisor is a constant multiplied the constant here happens to be one the last divisor is the common factor

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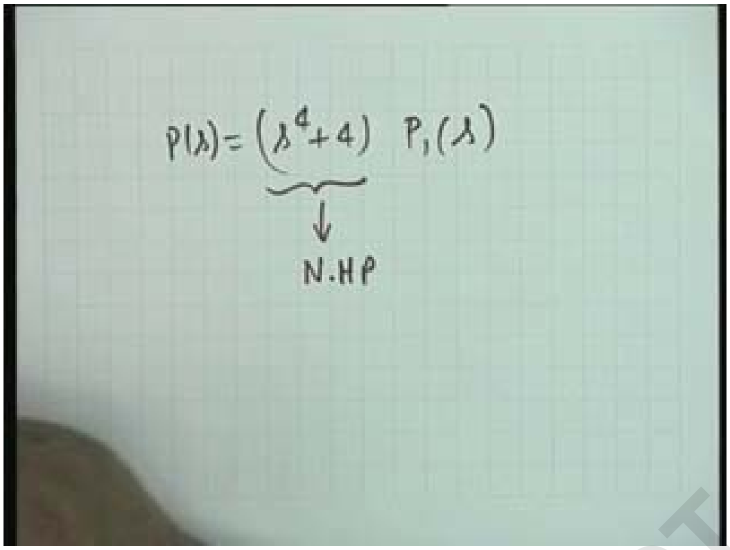

and therefore P of s shall have a factor of s to the four plus four multiplied by p one of s some p one of s okay

do we have to go further to to test whether P of s was Hurwitz or not do we have to go further

no because this polynomial is non Hurwitz

<a_side> yes <a_side>

isn't that right because there is a missing term here s square term is missing

but suppose there were no missing term

suppose this was Hurwitz then you had to test P one s for Hurwitz character is the point clear

this is only one step if it ends prematurely you discover the common factor you take the common factor call the rest of the polynomial as P one if the common factor is itself Hurwitz

<a_side> sir that means you have to (( )) (00:48:55) divide s four plus four (( )) (00:48:58) of P of s and find the <a_side>

that's right you have to divide this given polynomial P of s by s four {pul} (00:49:05) plus four find out what is P one of s then subject P one of s to Hurwitz character Hurwitz ((state)) (00:49:15) is that clear

and then only at the end of this exercise we will be able to say whether it is Hurwitz or non Hurwitz

now this is a purely even polynomial but its roots are not all on the j omega axis okay if they are all common roots common roots shall be discovered by Hurwitz testing all right and can this polynomial for example be odd can the last divisor be odd

<a_side> no <a_side>

no it shall be (( )) (00:49:51) with caution i make this statement with caution ah [Noise] you might have to disappoint yourself later

suppose now another tricky question

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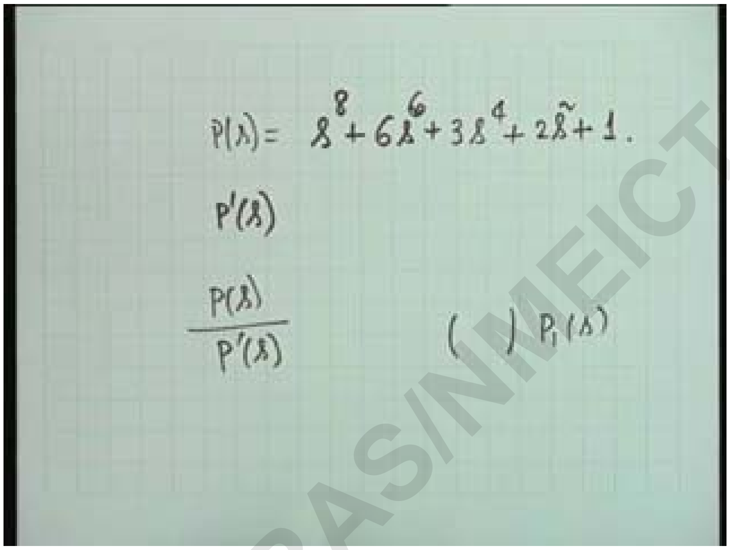

suppose P of s is given as

let's say s to the eight plus six s squared [Noise] no six s to the six plus three s to the four plus two s squared plus one

therefore this is the polynomial that is given that is it's a purely even polynomial [Noise] how do you test this because its odd part is zero

on the other hand this could be a this could be a Hurwitz polynomial if all it’s roots are in the j omega axis this could be a Hurwitz polynomial all right

then the method of testing is again without proof

if you are given a purely even or purely odd {polyn} (00:50:50) it applies to odd also what you do is

take the first differential coefficient

p prime s obviously p prime s shall be purely

<a_side> odd <a_side>

off

and then you make a contribute fraction expansion of P of s divided by P prime of s first differential coefficient

and if this satisfies the criterion that is it does not end prematurely number one number two it’s all its quotients are positive then P of s is Hurwitz

<a_side> sir if it ends prematurely it does imply that it's non Hurwitz <a_side>

non Hurwitz correct if it ends prematurely then you shall have to subject the remainder polynomial

<a_side> (( )) (00:51:34) <a_side>

correct

<a_side> sir it doesn’t take (( )) (00:51:38) <a_side>

correct correct that’s what i am saying

if ends prematurely then you have to discover the common factors you have to discover P one of s and then subject both of them to Hurwitz test

<a_side> if they come out to be ah (00:51:54) <a_side>

in this particular case in this particular case this was by inspection you could see that this is non Hurwitz so you could discover it

but suppose you cannot do that by inspection [Noise] then you have to subject both them to Hurwitz test all right

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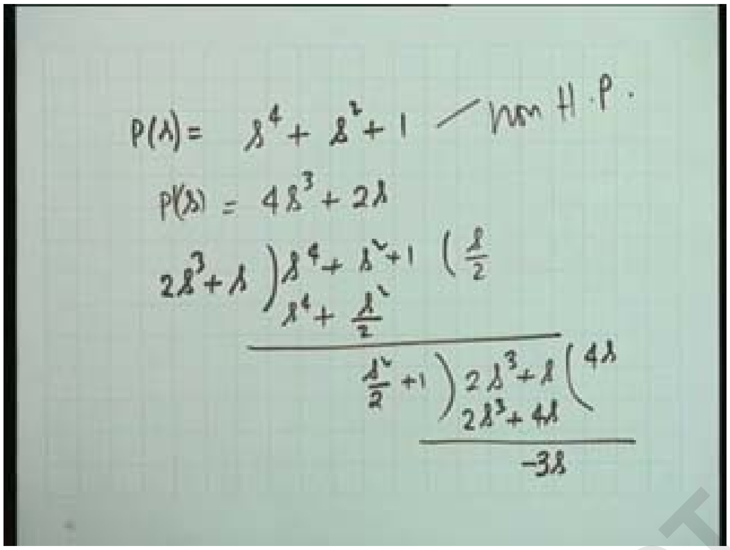

for example [Noise] if i take this polynomial s to the four [Noise] plus s squared plus one

suppose this is my P of s then P prime of s is equal to four [Noise] s cube plus twice s okay

now [Noise] shall i divide that's another another question that i want to ask shall i divide by four s cube plus two s can i divide by s ah two s cube plus s

<a_side> (( )) (00:52:46) <a_side>

two s cube plus s can i do that

<a_side> yes sir <a_side>

why can i do that

<a_side> (( )) (00:52:55) <a_side>

all i want is {whe} (00:52:59) whether the quotients are positive or not and dividing by dividing any part by a positive constant

<a_side>constant <a_side>

doesn't matter

this is the simplifying feature otherwise we will have to handle large numbers all right

so i can do that i get s by two s to the four plus s squared by two

what i am left is s squared by two plus one then i divide two s cube plus s this is how continued fraction is made

i take this as the as the ah what is it called dividend dividend so i get how much

four s four s two s cubed plus four s

<a_side> minus three s <a_side>

minus three s do i have to further

<a_side> no sir <a_side>

no because the next quotient will be negative therefore this is non HP non Hurwitz polynomial [Noise] okay [Noise]

now ah in tomorrow's class (( )) (00:54:02) [Laughter] in tomorrow's class we shall continue the discussion of Hurwitz polynomials and and their importance and their importance in testing network functions