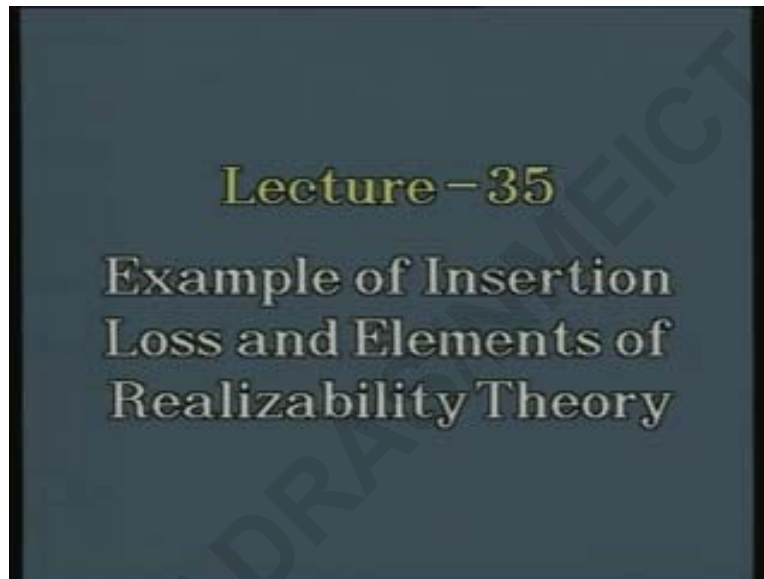


**Circuit Theory**Prof. S.C. Dutta RoyDepartment of Electrical EngineeringIIT DelhiLecture 35

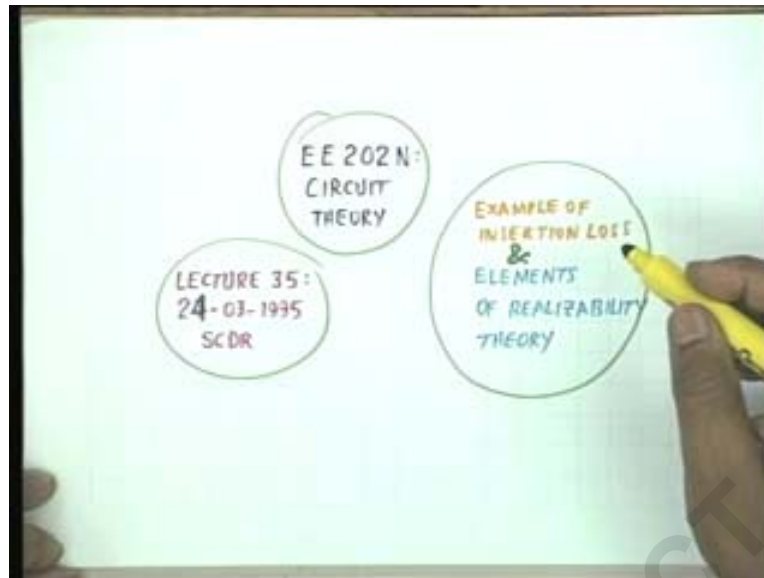
Example of Insertion Loss and Elements of Reliability Theory

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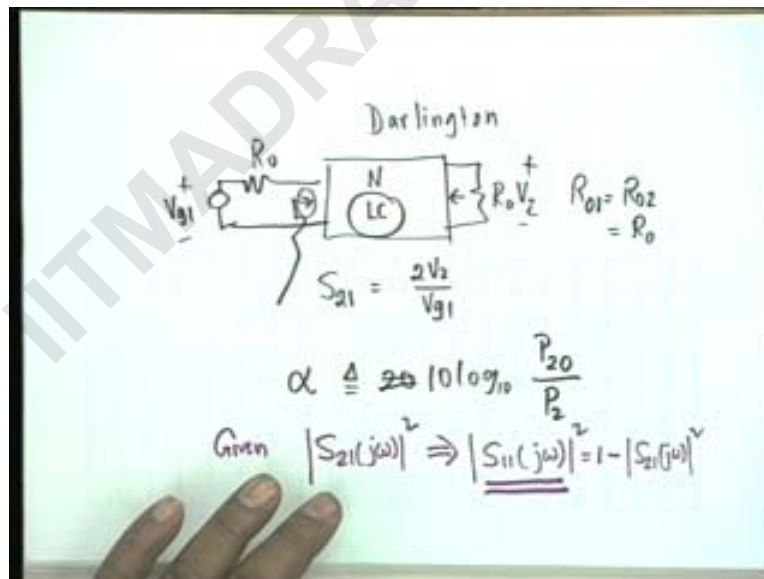
this is the thirty-fifth lecture [Noise] was suppose to be held yesterday but because of power failure this is why this correction in the date and what we are going to do

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is to complete that example of insertion loss synthesis and then take and then make a discussion of elements of realizability theory as i said this example was the first one in the series of discussions that we are going to have on network synthesis and the argument we took a very specific example of synthesis that is

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two synthesize an LC two port which is terminated in equal resistances let's say  $R_0$  and  $R_0$  and we took  $R_0$   $R_0$  can be normalized to one ((ohm)) (00:01:12) if you so desire both terminations are equal

then you have a voltage source  $V_g$  one [Noise] the voltage across this is  $V_2$  then you know that  $S_{21}$  one parameter that is the forward [Noise] transmissions catering parameter  $S_{21}$  one is equal to twice  $V_2$  by  $V_g$  one times square root of  $R_1$  by  $R_2$  by both are equal and therefore [Noise] therefore that ratio is one our reference resistances as usual  $R_1$  equal to the terminating resistances okay and the synthesis problem is this we have to synthesize an LC network why LC because we want to transfer as much power as possible to the load and observe nothing in the network

why do we insert the network to ((given)) (00:02:07) to transfer maximum possible power that means it is a matching network all right that is one objective the other objective is and this is why we have to synthesize

the other objective is we want to transfer power to the load in a frequency sensitive manner for example if there is a low frequency signal and we have a high frequency interfering noise with it for example somebody is speaking and there is shrieking sound in the adjacent room because of a machine running which is at a higher frequency than that of a speech all right we want to get rid of this so we want to transfer power from the source this could be a micro phone assembly and this could be an amplifier the amplifier is designed in such a manner

so that it rejects that high frequency noise and only transfers power at the low frequency and therefore there are two objective one is [Noise] that is [Noise] if it is a single frequency application then all we want is that the network the insertion network the network that is inserted between the sources and the load should be such that maximum possible power is transferred at port one which means that our aim is to make an such that the input impedance is approximately equal to  $R_{in}$  agree

under that condition the source gives the maximum available power [Noise] and then this network is to be (( )) (00:03:39) such that the power none none of this power is observed by  $N$  but is transferred to  $R_{in}$

in the process if there is a mismatch if for example [Noise] excuse me if this impedance is not  $R_{in}$  naught then the reflection coefficient will not be zero okay

what we want is that the input impedance here should be  $R_{in}$  so that no power is reflected back but if it is not  $R_{in}$  a little power will be reflected back

similarly if the impedance looking back into the network is not  $R_{naught}$  then there will be a reflection coefficient here okay

so the total not not all the power shall be observed here a part of it shall go back all right

so our aim in putting a network in inserting a network between a source and the load are twofold one is that maximum possible power should be transferred to the load and second is this transfer should occur in the desired frequency sensitive manner

it could be a low pass it could be high pass it could be band pass it could be band reject okay

this network LC could be any of these types

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right that it will not be frequency sensitive okay

so in this particular situation it is a frequency sensitive i was waiting for this ah statement you see if both the terminations are under then we don't and our aim is simply to transfer maximum power then we don't have to put any network all right but this is a specific case as i said and it is a special case of the procedure of Darlington of the problem the Darlington solve Darlington and therefore this problem is known as a Darlington's insertion loss problem Darlington's insertion loss i have also told you what is insertion loss insertion loss  $\alpha$  is defined as twenty not twenty ten  $\log_{10} \frac{P_{20}}{P_2}$  or the other way round this way okay [Noise]

now this is therefore the problem so what is given is what is given is either this voltage ratio  $\frac{V_2}{V_{g1}}$  or  $S_{21}^2$  or alternatively the  $\frac{I_2 P}{R}$  the insertion power ratio if any of this three is given we can find out  $S_{21}^2$

if this is given then from this we can find out because  $N$  is (( )) (00:06:37) we can find out  $S_{11}^2$  as equal to one minus  $S_{21}^2$  okay

if you subtract this for one then you will get  $S_{11}^2$  one you see this step this particular step is a very very elegant very beautiful step what it does is [Noise] the transfer function synthesis problem you see given  $S_{21}^2$  is a  $S_{21}$  after all is a transfer function right

it it relates the [Noise] the voltage at one port to the voltage at some other ports so the transfer function synthesis problem which concerns at least two ports is converted into a driving point

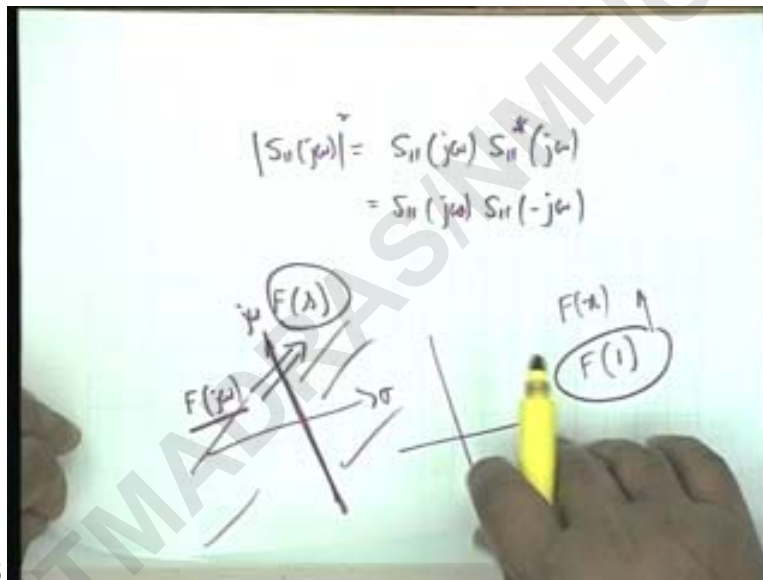
function synthesis problem isn't right  $S_{11}$  is the reflection coefficient at port number one it relates to the incident and reflected waves at port number one all right

so the transfer function synthesis problem is converted to a driving point function synthesis problem and this is the general philosophy that you will see in all synthesis problems that we want to convert a complex problem into a simple one a driving point synthesis problem is much simpler to solve than a transfer function

so ah in all synthesis situations our aim our primary aim is to convert this problem in to simpler one that a driving point function and then solve it okay

the next thing that we did was

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after you find out  $S_{11}(j\omega)$  magnitude square we write this as  $S_{11}(j\omega)$  multiplied by  $S_{11}^*(j\omega)$  and  $S_{11}(j\omega)$  you know if this is a real function then its complex conjugate is simply by making minus  $j\omega$  all right

and then we we said we use the word a big leap forward okay suppose a function  $F$  of  $S$  is given if a function  $F$  of  $S$  is given how do i find out the corresponding function on the  $j\omega$  axis that is for  $(\lambda)$  (00:08:53) all i do is i put  $S$  equal to  $j\omega$

that is i have a general field i have a general field and the function is defined at all points in the field i want to find out the function at a specific location i simply put  $S$  equal to the general variable equal to that specific location

for example if  $F$  of  $X$  is given i want to find out  $F$  of one i simply put  $x$  is equal to one got it but suppose  $F$  of one is given that is at a specific location and then you have to find out  $F$  of  $X$  in general this is an impossible task isn't it all unless you know more about the functional property of  $F$  of  $X$

even if the function is given on a particular line let's say  $X$  equal to two [Noise] or ah if it is a function of two variables if it is a function of two variables it's given on let's say  $Y$  equal to zero or  $X$  equal to zero to go to the general variable  $F$   $XY$  from a specific location is an impossible task

usually it is an impossible task that is what you are asking is to construct the whole from a part of it if i give you a piece of brick a broken piece of brick and i ask you to construct what it was originally you can only guess that perhaps it was a ah what is it called cuboid perhaps but there is no guarantee i might have on the cuboid a hemisphere okay

so one usually cannot break but in the case of a complex variable where the function is analytic and the analytic function remain the function is differentiable everywhere except at a finite number of singularities if that is so it turns out that if the function is given on the  $j$   $\omega$  axis you see after all capital  $F$  of  $S$  is a function of two variables one is  $\sigma$  the real part and other is  $\omega$  is imaginary you say function of two variables

what we are given is the value of the function at  $\sigma$  equal to zero  $\sigma$  equal to zero defines the  $j$   $\omega$  axis

now the problem now is to go from  $F$  of  $j$   $\omega$  to  $F$  of  $S$  as i said in general this is impossible task but fortunately the networks that we deal ((we din't)) (00:11:18) practice or maybe we have chosen only those networks there are networks which do not behave so nicely we are not we are not concerned with them we don't use them because we don't know how to use them here fortunately what happens is that we can go from the  $j$   $\omega$  axis to the complete ((as plane)) (00:11:35) wherever you want by simply substituting  $S$  for  $j$   $\omega$

so that is what we do capital  $S$  one one of  $j$   $\omega$  is an analytic function  $S$  one one of  $S$  and

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$$S_{11}(j\omega)S_{11}(-j\omega) = S_{11}(s)S_{11}(-s) = \frac{(s-2)(-s-2)}{(s+1)(-s+1)}$$

$$S_{11}(s) = \frac{Z_1 - R_0}{Z_1 + R_0}$$

therefore from  $S_{11}(j\omega)S_{11}(-j\omega)$  we go to substitute  $j\omega$  is equal to  $s$  and we get  $S_{11}(s)S_{11}(-s)$  okay

and the next task then is to choose an  $S_{11}(s)$  is given as a product suppose  $S_{11}(s)$  has a factor  $s+1$  in the denominator then it must also have a factor you see it's a product of  $S_{11}(s)S_{11}(-s)$

so if this quantity has a factor like this then  $-s+1$  should also be a factor

similarly in the numerator suppose you have a factor of  $s-2$  then you must have a factor of  $-s-2$  also

so if  $S_{11}(s)$  and  $S_{11}(-s)$  can be factorised into its roots of the numerator and roots of the denominator okay

as for as the denominator is concerned the choices has to be unique why because we go back to the definition of the reflection coefficient it is input impedance minus  $R_0$  divided by input impedance plus  $R_0$  okay

and the poles of  $S_{11}(s)$  which are the zeroes of  $Z_1 + R_0$  is an impedance function an impedance function cannot have zeroes in the ((right half plane)) (00:13:27) and therefore all zeroes of  $Z_1 + R_0$  shall be in the left half plane which means that in  $S_{11}(s)S_{11}(-s)$  in the denominator after you factorise out you have to choose only those factors which contribute to zeroes in the left half plane the zeroes in the right half plane shall go to  $S_{11}(-s)$  here for example  $S_{11}(s)$  is

absolutely clear okay here for example  $S_{11}$  you will have to choose only this factor in the denominator  $S$  plus one as first the numerator is concerned you have a choice numerator is difference between two impedances and different between two impedances can have poles anywhere can have zeros anywhere in the  $S$  plane it's not restricted okay

$Z_1$  is equal to  $R_0$  this defines the zeros and so you have a choice you could either choose  $S$  minus two or you could choose minus  $S$  minus two all right

let's say  $S$  minus two then this is your reflection coefficient  $S_{11}$  and from the reflection coefficient then you can find out  $Z_1$  and  $Z_1$  [Noise]

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$$S_{11}(s)S_{11}(-s) \Rightarrow S_{11}(s)$$

$$Z_1 = R_0 \frac{1 + S_{11}(s)}{1 - S_{11}(s)}$$

↓  
Synthesize.

the input impedance from  $S_{11}$  you choose  $S$  minus  $S$  then you find out  $Z_1$  is equal to  $R_0$  by  $(1 + S_{11}(s)) / (1 - S_{11}(s))$  all right and finally finally you synthesis  $Z_1$  okay finally you synthesis  $Z_1$

we have not yet dealt with the problem of synthesis of a  $Z_1$  of a drive in point impedance but in the Darlington problem in very simple cases not very complicated in very simple cases it turns out that the synthesis problem is also very simple and this is what we used to illustrate with the help of an example [Noise] yes

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correct



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there will be two solutions yes i am glad you asked that question to a synthesis problem if there exists one solution there exists an indefinite number of solutions and this is a demonstration of that that there are atleast two solutions that you can easily find out okay the problem the example that we were discussing is

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$$|S_{11}(j\omega)|^2 = \frac{\omega^6}{1 + \omega^6}$$

$$S_{11}(s)S_{11}(-s) = \frac{-s^6}{1 - s^6 \pm s^3}$$

$$S_{11}(s) = \frac{s^3}{(s^3 + 2s^2 + 2s + 1)}$$

S. Butterworth

s two one omega magnitude square is equal to one by one plus omega to the six and i told you this is a low pass filter it's obvious that the transmission is unity at one at omega it will be zero and it falls off at high frequencies it's a low pass filter it's a third order filter because it is magnitude square and therefore the magnitude would be of order three third order and this specific case is known by the name of gentleman an English gentleman by the name S. Butterworth so its called a Butterworth third order low pass filter these are names you may forget them you may remember them

for the present it is not important but i just i just thought i would mention the name [Noise] in uh with the view regards to Butterworth okay

the next ah thing that we do is we find out S one one j omega magnitude square by by subtracting this from one and therefore we get omega to the six divided by one plus omega to the six and then i find out S one one S S one one minus S by substituting omega equal to S by j and that gave us minus S to the six divided by one minus s to the six then we factorized the denominator

as per the numerator is concerned its either plus s cube or minus s cube okay accordingly we shall have two solutions we will look at that

the denominator as you saw was one plus s multiplied by one plus s plus s square which gives you s cube plus twice s square plus twice s plus one i hope i am right and minus s cube plus two s square minus two s plus one let's find out only S one one S we take this as the denominator in the numerator now we have a choice it is either plus s cube or minus s cube we shall take both of the them and see what the network is

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what should i do

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no we cannot because one is S one one S the other is S one one minus S

if you take plus S cube here you have to be fair to be fair to S one one minus S you must give that minus S cube all right you cannot take any power of S all right so let's take S one of one S

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$$R_0 = 1\Omega \quad S_{11}(\lambda) = \frac{\lambda^3}{\lambda^3 + 2\lambda^2 + 2\lambda + 1}$$

$$Z_1 = \frac{1 + S_{11}}{1 - S_{11}}$$

$$= \frac{2\lambda^3 + 2\lambda^2 + 2\lambda + 1}{2\lambda^2}$$

is equal to plus s cube divided by s cube divided by two s square plus two s plus one [Noise] now we have to find out the impedance Z one Z one for this we have to fix now R zero the terminating resistances

let us fix  $R$  zero is equal to one ohm then  $Z_1$  is simply one plus  $S$  one one divided one minus  $S$  one one and you can easily see that this would be two  $s$  cube plus two  $s$  square plus two  $s$  plus one and the denominator it would be two  $s$  square plus two  $s$  plus one agree okay

now the problem is now to synthesize  $Z_1$  in a constrained manner what is the constrained that the ((last)) (00:19:49) resistance must come out as one one and this is the beauty of Darlington synthesis that he saw into it

he saw into the depths of this problem and ensured that if you go this way the terminating resistance is bound to come as you will see its no magic we shall we shall be find the logic at a later date but it does come let us see how to synthesize

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$$Z_1 = \frac{2s^3 + 2s^2 + 2s + 1}{2s^2 + 2s + 1} \xrightarrow{s} \begin{matrix} \rightarrow s \\ \text{as } s \rightarrow \infty \end{matrix}$$

$$= s + \frac{s+1}{2s^2 + 2s + 1}$$

$$= s + \frac{1}{\frac{2s^2 + 2s + 1}{s+1}}$$

two  $s$  cube plus two  $s$  squared plus two  $s$  plus one divided by two  $s$  squared plus two  $s$  plus one you notice that this impedance has a pole at  $S$  equal to infinity isn't that right

as  $s$  goes to infinity the highest power term two  $s$  cube the highest power term is two  $s$  squared

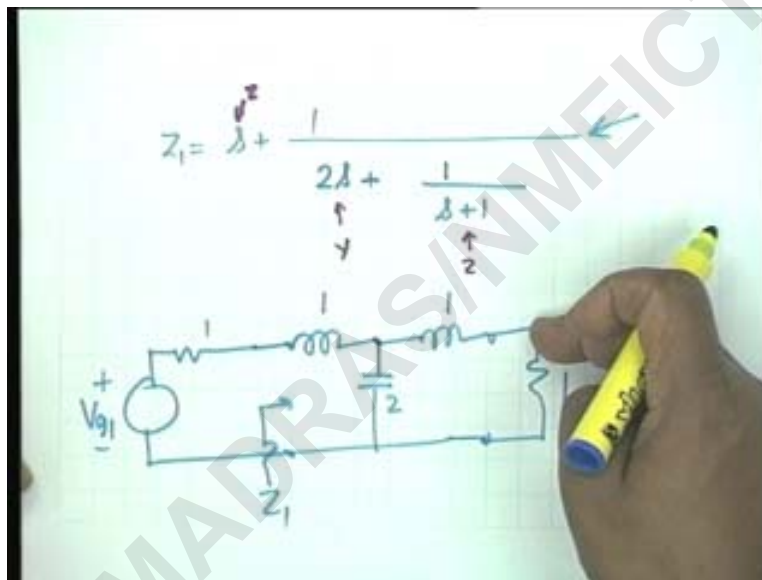
so it behaves like  $S$   $Z_1$  tends to  $s$  as  $s$  tends to infinity which means that  $Z_1$  has a pole at infinity and what we do is we remove this pole what does that mean it means that we divide the denominator divide the numerator by the denominator and find out the the remainder that means you shall have  $s$  the quotient will be  $s$  and what will the remainder  $s$  plus one all right so the next thing that we do is please follow this carefully because this is going to be a very important tool in your hand as for as the synthesis is concerned

now you see that the remainder function after we remove  $s$  the pole at infinity the remainder function cannot have a pole at infinity it has a zero at infinity isn't that right as  $s$  tends to infinity it is a zero which means that the admittance shall have a pole at infinity is that clear

if the impedance this is impedance this uh impedance equal to impedance plus impedance the impedance has a zero at infinity and therefore the admittance shall have a pole at infinity

so write this is an admittance  $2s + 1$  divided by  $s + 1$  is that okay i write this as reciprocal of impedance is reciprocal of an admittance and the admittance now has a pole at infinity which we now remove again by division

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so i write  $Z$  one equal to  $s$  plus one over what would be the quotient twice  $s$  plus one by  $s$  plus one okay now you recall that this is an impedance what is this admittance then what is this this is an impedance right admittance plus admittance one by one by admittance is an impedance and the network the synthesis is now complete

you see you are synthesizing  $Z$  one that is the input impedance you had a one ohm you had a  $v_g$  one and then  $Z$  one is this impedance  $Z$  one starts with an  $s$  which is the impedance of an inductor of value

<a\_side> one Henry <a\_side>

one Henry then plus one by admittance and therefore you shall have a shunt element a capacitor of value two farads two farads

this admittance in parallel with another admittance whose impedance is  $s$  plus one that means we have another inductor a value one and the last element is the terminating element of one ohm and the synthesis is complete okay

the mechanization of this  $s$  is obvious from this expression is to expand the given function in continued fraction this is a continued fraction

a plus one by  $b$  plus one by  $c$  plus and so on and so for its a continued fraction so the mechanization is that of continued fraction and we shall do this again with continued fraction rather than arguing from removal of poles removal of poles was done to explain to you the actual mechanization of network synthesis

we were removing poles one by one poles at infinity okay the mechanization is this we have

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$$Z_1 = \frac{2s^3 + 2s^2 + 2s + 1}{2s^2 + 2s + 1}$$

$$= s + \frac{s+1}{2s^2 + 2s + 1}$$

$$= s + \frac{1}{\frac{2s^2 + 2s + 1}{s+1}}$$

$$= s + \frac{1}{2s + 1}$$

$$= s + \frac{1}{2 + \frac{1}{s}}$$

$$= s + \frac{1}{2 + \frac{1}{s}}$$

two  $s$  cube plus two  $s$  squared plus two  $s$  plus one divided by two  $s$  squared plus two  $s$  plus one the mechanization of this synthesis is development of a continued fraction continued fraction that is you write this here and then you see the quotient is  $s$

so two  $s$  cube plus two  $s$  square plus  $s$  the remainder i don't want to write here because then i will go like this so i will keep i will bring it here remainder is  $s$  plus one and in the process we must not forget that this quotient is an impedance all right

then this divides two s square plus two s plus one you understand the mechanization of continued fraction development okay this you bring here then the quotient is two s and immediately identify that this is an admittance because we have taken the reciprocal of an impedance okay so you get two s square plus two s and you are left it one then you bring this divider here and and your your quotient is s s remainder is one and immediately identify this as an impedance then you divide one by one one one zero and the continued fraction expansion is over the last element is what

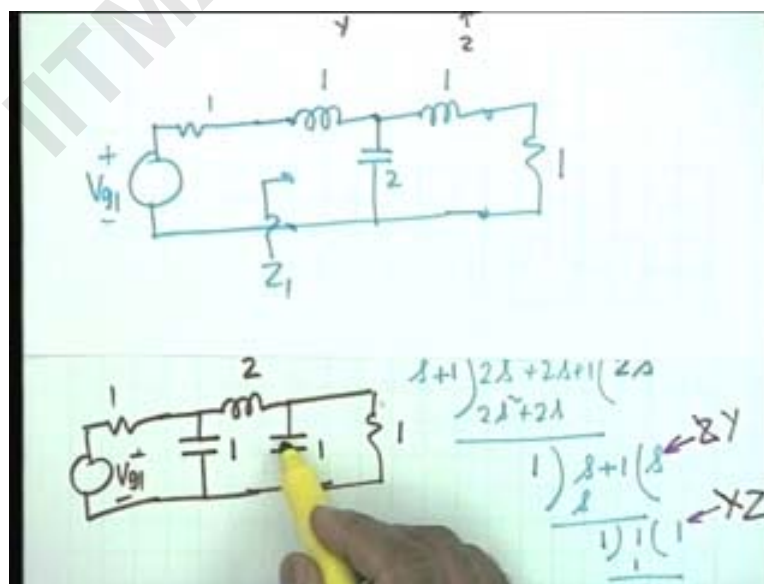
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no admittance [Noise] this is an admittance because you have taken the reciprocal it just happens that one by one is one and one ohm admittance will be same as one ohm resistance it may not be so if the terminating resistance was two ohms then the result here would have been half not two okay

so you must keep track of the dimensions and the dimensions that you see the alternate starting with the impedance why is to be starting with impedance because the expression is that of the impedance starting with impedance impedance admittance impedance admittance all right and the synthesis is complete

now let's see [Noise] any question at this point let's see what happens if we take instead of the plus sign the minus sign in the numerator

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if we take  $s = 1$  as equal to  $\frac{-s^3}{s^3 + 2s^2 + 2s + 1}$  plus one then your impedance  $Z$  shall be equal to  $1 + \frac{1}{1 - s}$

so it would be equal to  $\frac{s^2 + 2s + 1}{2s^3 + 2s^2 + 2s + 1}$  minus minus this so it would be  $\frac{2s^3 + 2s^2 + 2s + 1}{2s^3 + 2s^2 + 2s + 1}$

don't you see that this function is exactly the reciprocal of the previous function isn't that right its exactly the reciprocal so if you want to argue in terms of poles first you take the admittance that is  $\frac{1}{1 - s}$  plus one divided by  $2s^2 + 2s + 1$  plus two  $s$  plus one

and continued fraction expansion that you had done earlier shall be valid now also provided provided you consider this as an admittance agreed let's go back to that mechanization let me use a different colour [Noise]

if you consider this an admittance then obviously the first element would be an admittance [Noise] second would be an impedance third would be an [Noise] admittance and the fourth would be impedance is it okay

is exactly the same continued fraction expansion except that the dimensions change which means that the networks shall now be instead of starting with the series inductor  $i$  shall have to start with the capacitor of value one farad because the admittance is  $s$  then we have an inductor of value two Henry twice  $s$  then again a capacitor of value one farad and finally an impedance of one ohm so this is a perfectly valid alternative network to the first choice that we had made okay wherever we have found out at least two networks to synthesize the given  $s^2$  [Noise] one geo mega magnitude square

i must tell you you may not be that lucky you may not be that lucky in the general case but how to get a (( )) (00:30:11) we shall discuss later

i want you to notice two important facts ah can you see both of them can you see on the monitor both of this okay

you see both of this networks share something in common what is it [Noise] that the inductors are series elements and capacitors are shunt elements isn't that right and i said the given function was a low pass low pass function it is indeed low pass you see at  $c$  this is shut this is open this is shut so whatever is fed here goes to the load

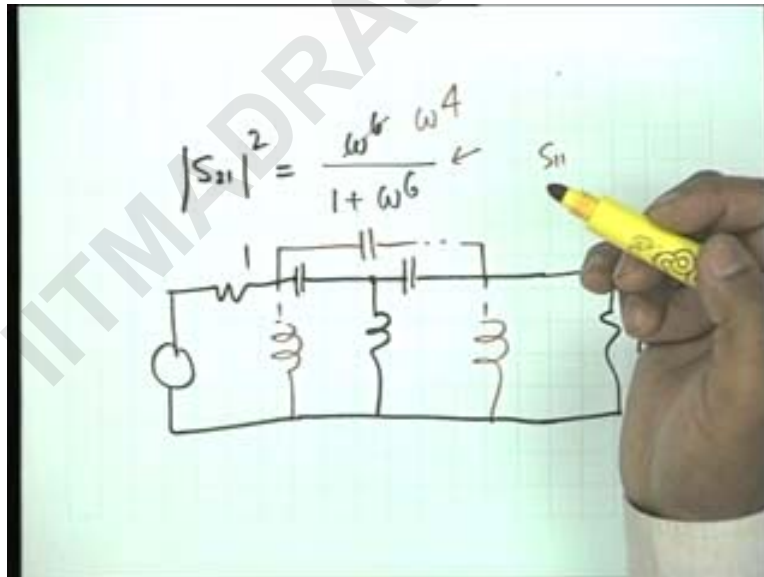
similarly here okay at this c this is shut [Noise] these two are open so they go and as frequency increases the inductor impedance increases the capacitor impedance decreases and therefore the the transfer function or the transmission goes down as frequency increases and such ah [Noise] such architectures should be obvious from the given specification such architecture should be obvious from the given specification okay

here for example it is a third order network third order function third order function to synthesize a third order function we require three energy storage element each of there was three now three energy storage elements now i am appealing to your common sense they cannot be of the same kind they cannot be all inductors okay if there are all inductors then you are say trivial networks and it cannot act as low pass therefore you must have at least one capacitors at least one inductor you see one capacitor case is this and one inductor case is this

now before i leave this problem i want to [Noise] i want to extend this common sense a little bit further

suppose to start with your function was let's say

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omega to the six divided by one plus omega to the six

suppose s two one ((j)) (00:32:30) mega squared was equal to this without any calculation can you tell me what the architecture should be okay between what the architecture of the insertions network should be



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capacitors in series and the inductor in shunt at least three energy storage elements so two capacitors one inductors or two inductors one capacitance and obviously this is one of the architectures

the other architecture would be other one would be yes

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other one would be an inductor here an inductor here and a capacitor here okay these are the two possible architectures

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just an inspection because this is high pass it is a high pass network and high pass network means [Noise] that capacitors should come in series and inductors should come in parallel why because as as as increases the inductive impedance increases so more and more high frequencies ((favoured)) (00:33:47) to be passed from the source to the low

again suppose

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you have to find out all that you have to to be able to find out the values you have to find those all we have to go to s one one find out Z one and synthesis okay what are you saying is the architecture should be obvious from the given specification let me ask you a trivial question now

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oh yes that we can be sure because there are only three poles because the [Noise] order the function is six it's a magnitude square the magnitude of the only of order three and therefore the function would be of the order three okay

now a trivial question could we have instead of omega to the six could we have a omega to the three here why not (( )) (00:34:52)

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say it again

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one and half order no no order is not determined by the numerator order is determined by the number of poles the denominator could we have here instead of six could be have three

<a\_side> ( ( ) (00:35:10) <a\_side>

tell me why [Laughter] no i know

<a\_side> ( ( ) (00:035:13) <a\_side>

((oh no)) (00:35:16) i told you this a trivial question the answer should be obvious and the thing is obvious when you see it look at the left hand side a magnitude squared how can it be odd [Noise] [Laughter] okay

so omega to the three is not permitted is omega to the four permitted yes

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yes okay now what is this function

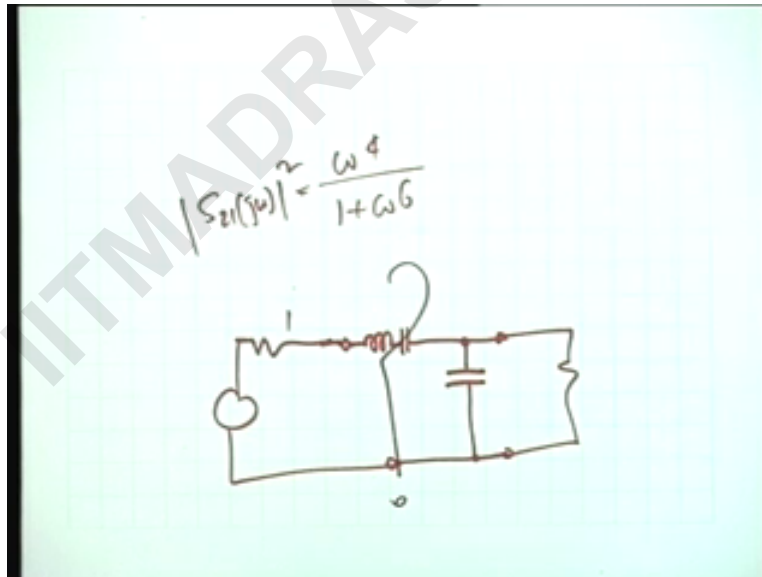
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is it low pass at omega equal to zero the transmission is zero

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it is a band pass function what does that mean can you tell me the architecture now

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if it is omega to the four divided by one plus omega to the six

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it would be an LC ((series)) (00:36:05) okay in series or in shunt

<a\_side> ( ( ) (00:36:13) <a\_side>

[Laughter] okay tell me one possible either i shall have this [Noise] there must be three elements so i must have what shall i have here a capacitor

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or an inductor okay i leave this with a question mark for you

<a\_side> why should be three elements <a\_side>

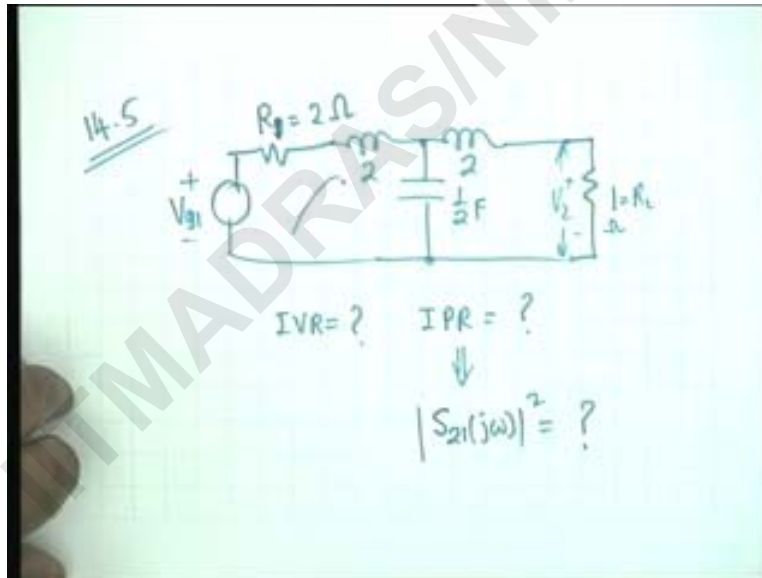
why should be three elements because they are three ports the function is of third order

so there must be three value but how are they connected that is the question i leave it for you to decide please do [Noise]

we ah conclude our discussion on scattering parameters by taking another [Noise] example another example from the book

this is an example fourteen point five

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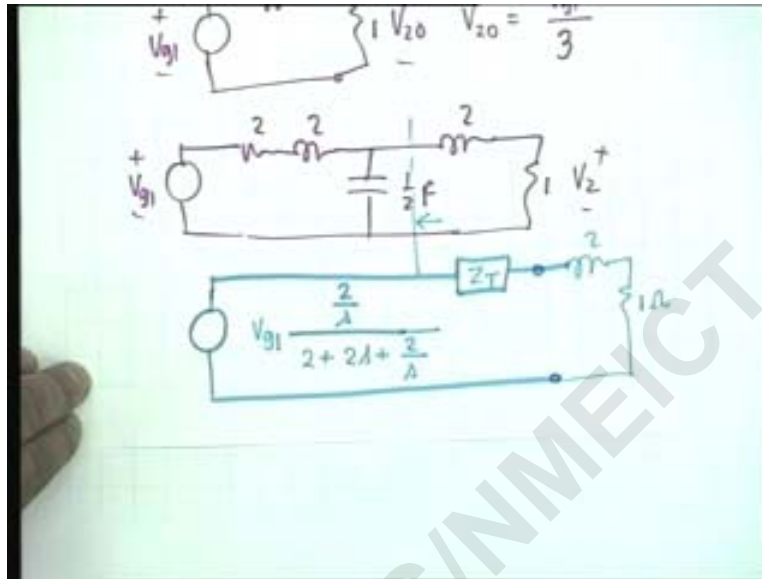
and the problems is [Noise] now we we make the thing a bit more complicated

we say it's not equal termination let's say  $R_g$  equal to two ohms and  $R_L$  is equal to one ohm they are not equal termination so you have to take here of different terminations this is  $R_1$  and this is  $R_2$   $R_2$  equal to one ohm in between you have this is  $V_{g1}$  in between you have a low pass network like this and the values are two Henry two Henry and half Farad

a problem is to calculate IVR [Noise] this voltages  $V_2$  the problem is calculated IVR and IPR and from IPR from IPR find out  $S_{21}(j\omega)$  magnitudes square okay this is the question

first to find out the instruction voltage ratio second instruction power ratio and after a founded instruction power ratio find out  $S$  two one  $j$  omega square you know that the very intimately related the product is a function of  $R$  one and  $R$  two okay

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to solve this network as for as IVR is concerned IVR is simply found out  $V_g$  one two ohms to find out  $V$  two zero you simply connect the load here one this voltage will be  $V$  two zero and obviously  $V$  two zero is equal to  $V_g$  one divided by three agree

then you have to find out  $V$  two okay for finding  $V$  two you go back to the network  $V_g$  one two ohms two Henrys half farad two Henry one ohm and this is  $V$  two you have to make an analysis of this network to find out  $V$  two and the way i do i find it convenient (( just apply thing)) (00:39:43) that is two the left of this lines then you see (( )) (00:39:50) equivalent generator would be  $V_g$  one  $V_g$  one multiplied by this impedance that is two by  $s$  all right

impedance of half farad is one by half  $s$  which is two by  $s$  divided by this the sum of these three impedances so two [Noise] plus two  $s$  plus two by  $s$  then ((Thevenin)) (00:40:19) generator and then you have a ((Thevenin)) (00:40:24) impedance Thevenin impedance would be the parallel combination of this and this okay will calculate this in a minute let's call it  $Z_T$  then you have at two ohm two {far} (00:40:37) ah two Henry and one ohm okay this is the ((Thevenin)) (00:40:42) equivalent

now [Noise] it is a matter of matter of calculation now

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$$V_{g1} = \frac{A}{2+2s+\frac{s}{A}} = \frac{sA}{s^2+2s+1}$$

$$Z_T = \frac{\frac{s}{A}(2+2s)}{2+2s+\frac{s}{A}} = \frac{2(1+s)}{s^2+2s+1}$$

$$V_2 = \frac{V_{g1}}{s^2+2s+1} \cdot 1 = \frac{\frac{2(1+s)}{s^2+2s+1}}{s^2+2s+1}$$

that the source is  $V_g$  one two by  $s$  divided by two plus two  $s$  plus two by  $s$  from which this factor two cancels out and we are left with then you multiply by  $s$  so you are left with one by  $s$  square  $s$  plus  $s$  plus one agree as simple as that and the Thivenin impedance  $Z_T$  is half farad that is two by  $s$  in parallel with two plus two  $s$

so two plus two  $s$  divided by two plus two  $s$  plus two by  $s$  and once again you can forget about this two

so you will get two one plus  $s$  divided by  $s$  squared plus  $s$   $s$  plus one all right so our equivalent ah [Noise] circuit {equi} (00:41:54) yes

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where

<a\_side> ((is in the colour combination)) (00:41:59) <a\_side>

half farad in  $i$  have two plus two yes

<a\_side> (( )) (00:42:07) <a\_side>

oh

<a\_side> (( )) (00:42:10) <a\_side>

$i$  have done there in the denominator

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(( )) (00:42:21) this is a Thevenin equivalent okay therefore my  $V_2$  shall be equal to  $V_{g1}$  i thought you were saying this there should be  $V_{g1}$  here  $V_{g1}$  divided by  $s^2 + s + 1$  divided by no this multiplied by one ohm the voltage  $v_2$  is across one ohm divided by the current in the circuit that is  $2s + 1$  plus  $s$  divided by  $s^2 + s + 1$  plus plus  $2s + 1$  plus one so i know  $V_2$  i know  $V_2$  zero and i simply divide one by the other

i should leave that algebra to you and my IVR my calculation shows that it is  $2s^3 + 3s^2 + 5s + 3$  divided by three

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$$IVR = \frac{2s^3 + 3s^2 + 5s + 3}{3}$$

$$IPR = \left(\frac{R_2}{R_1 + R_2}\right)^2 \left|\frac{V_{g1}}{V_2}\right|^2$$

$$= \left(\frac{1}{3}\right)^2 \left|(2s^3 + 3s^2 + 5s + 3)\right|_{s=j\omega}^2$$

$$\underline{|S_{21}(j\omega)|^2} IPR = \frac{4R_1R_2}{(R_1 + R_2)^2} = \frac{8}{9}$$

this insertion voltages ratio and now the next question is how to find out IPR

IPR as you know is  $R_2$  divided by  $R_1 + R_2$  whole squared this you have derived yesterday multiplied by  $V_{g1}$  by  $V_2$  magnitude squared

now  $V_2$  by  $V_{g1}$  we have already found out and there and i know  $R_2$  i know  $R_1$  and therefore this is simply  $R_2$  is one by three whole squared multiplied by magnitude of  $2s^3 + 3s^2 + 5s + 3$  under the condition  $s$  equal to  $j\omega$  magnitude square  $V_{g1}$  by  $V_2$  magnitude squared okay is that all right

we leave this simplification the  $s^2 + s + 1$   $j\omega$  squared how do i find that you know that this multiplied by IPR we have derived this as  $4R_1R_2$  divided by  $(R_1 + R_2)^2$  whole square and this in this case should be eight by nine is that right eight by nine

so you can find out  $s^2 + s + 1$   $j\omega$  square all right

a few minutes that are left is there any question

we are not going to come back to scattering parameters again

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what ( ( ) ) (00:45:17) three because it is  $V_g$  one by  $V$  two this is not an insertion voltage ratio this is simply the reciprocal of  $V$  two by  $V_g$  one it is nothing do in insertion voltage ratio

IPR is not in terms of IVR okay that is the reason [Noise] any other question okay as an introduction to synthesis this was the glimpse into synthesis of the most usual situations most usual situation is i have the source and i have the load for a given source is a microphone the load is a loudspeaker i have to design an insertion network i cannot connect the microphone to the loudspeaker why not why not

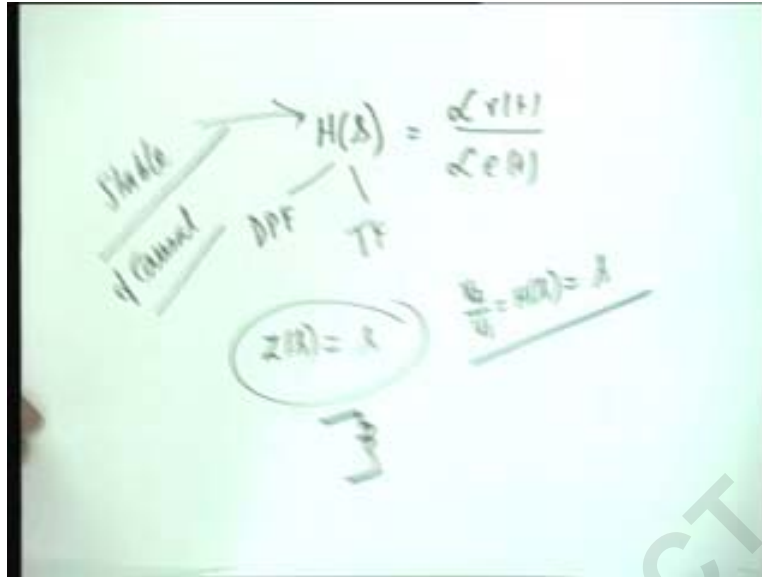
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impedance matching nothing will come in the loudspeaker all right because in microphone is a high impedance source and the loudspeaker in a low impedance slope typically four ohms or eight ohms speaker all right

so i have to design an insertion network such that maximum power from the source is transferred to the load all right in between of course i have an amplifier in the usual situation i have amplifier it push up the power but in in a passive case for example in a passive case i can design an insertion network such that maximum power is transferred to the load

this was as i said usual situation now we shall try to approach the problem of synthesis in a systematic manner and one of the problems is given a transfer functions

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or a driving point function given a function in  $s$  domain which can be either DP or transfer either DPF driving point function or transfer function the task is to find a network where  $H$  of  $s$  obviously the ((Laplace)) (00:47:23) transform will be response to Laplace transform of the (( )) (00:47:28) with initial condition equal to zero this is the definition of a transfer function or a function characterizing the network or any system for that mater

now [Noise] the problem is to find a network at least one network and you know if one network exists there exists an infinite number of network to find at least one network which realizes this the first question that one ask is is this realizable is this a realized function for example suppose i am i am ask to design a network which realizes a simplest synthesis problems which realizes  $Z$  of  $s$  is equal to  $s$  obviously it is realizable why why

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no question on ports  $s$  is the impedance of one Henry inductor i don't care about theory its realizable okay if it is one by  $s$  even then its realizable all right

one by  $s$  is a farad capacity but suppose it is a transfer functions suppose its is  $V$  two by  $V$  one equal to  $H$  of  $s$  equal to  $s$  is this realizable why no

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it is an unstable function okay



so one of the criterion for realizability is that the given function must be stable it is a pole at infinity pole at infinity of an impedance function or admittance function is perfectly all right because you know how to realize

but pole at infinity of a transfer function makes the transfer function unstable

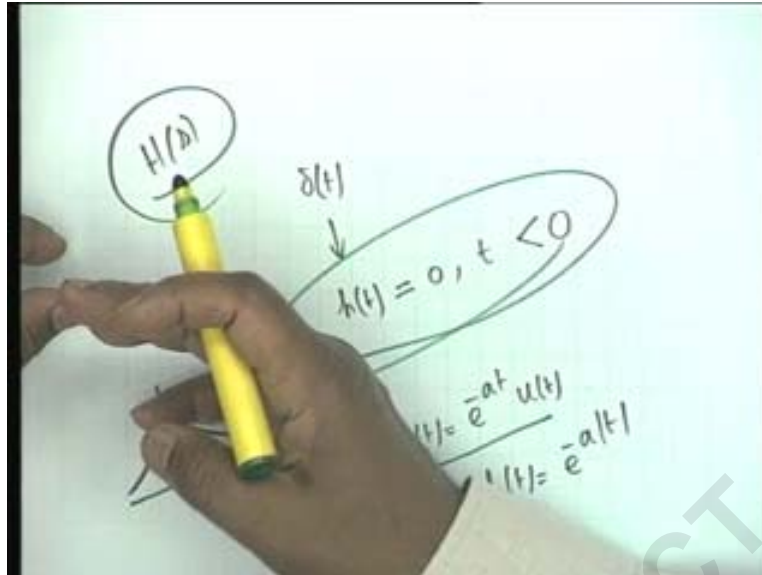
that means as frequency increases the output increases in definitely all right

so the output is not bounded even if the input is bounded which means that the function is unstable so one of the one of the criterion for a transfer function synthesis is is that is must be stable and the other criterion is there are two criterion other criterion is that if you want a real time realization that is if you want to ah find out a network in the (( )) (00:49:50) to a given excitation gives you given response obviously the network must be causal it should not be anticipatory now what is the stable and causal what is the ah criterion for causality how do you define causality or how do you how do you characterize a network to find out whether its causal and not causal

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we excide to it the unit impulse function that is we excide i am sure you notice this was done in signal in systems you see the point is okay forget forget what was done in signals system the point is the output should not proceed the input output should not ((anticipate)) (00:50:42) in input okay and is easiest way to do this is to apply unit impulse the output should appear only after the impulse is apply

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that means what is the response  $Ht$  unit impulse response it should be equal to zero for  $t$  less than zero this is the criterion for causality

for a linear time invariance system the causality is guaranteed if  $h(t)$  equal to zero for  $t$  less than zero all right [Noise] for example [Noise] if  $h(t) = e^{-at} u(t)$  if the unit impulses causal this it is to causal function is that clear

on the other in if  $h(t)$  is equal to into the minus a let's say mark  $t$  then it is not causal because  $h(t)$  exists for  $t$  less than zero before unit impulse is applied there is an output therefore it is not [Noise] it is not stable

now this is it is not causal this is in the time domain what about the frequency domain how do you find out given in  $H$  of  $s$  how do you find out whether it is causal or non causal that criterion is somewhat difficult and its outside the scope of this class but and perhaps outside the scope of the total B.tech curriculum over electrical engineer perhaps it will never be done but i will mention a criterion which you must remember throughout your life

and its its by a

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Paley

Paley-Wiener criterion:

$$\int_{-\infty}^{\infty} \frac{\log |H(j\omega)|}{1+\omega^2} < \infty$$

the gentle man by the name Paley pal no i ((beg your pardon)) (00:52:30) Paley Paley wiener criteria have you heard the name where which course

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oh Fourier transform i am glad was it proved

<a\_side> (( )) (00:52:44) <a\_side>

the result is Paley-wiener criterion says that the integral if H of s is given in the frequency domain integral from minus infinity to plus infinity log of magnitude of H of j omega divided by one plus omega square okay (( )) (00:53:12) to see why it is so it it can be proved but we shall not prove it the proof is rather involved the Paley-wiener criterion says if the network in causal then this must be finite this must be less than infinity okay we will go into ah more details of this [Noise] next time next time would be Tuesday