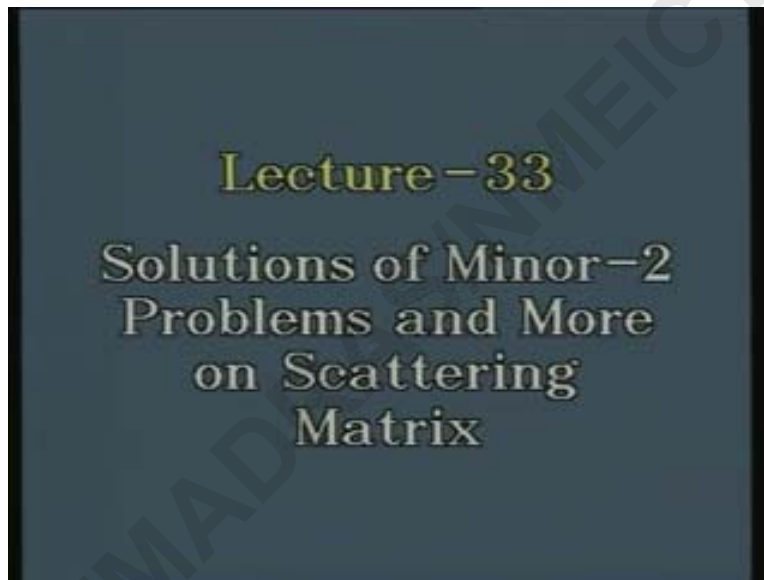


Circuit TheoryProf. S.C. Dutta RoyDepartment of Electrical EngineeringIIT DelhiLecture 33

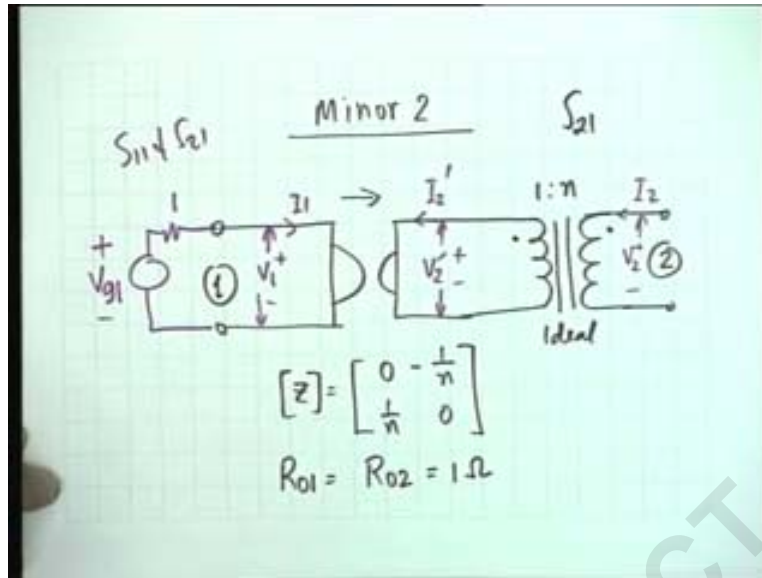
Solutions of Minor-2

Problems and More on Scattering Matrix

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thirty third lecture three three and [Vocalised Noise] what we are going to do is to solve the minor two examination problems point out the difficulties that you experienced and the common mistakes that you that you made and if time permits as i hope it will we shall do little bit more on the scattering matrix of a two port

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minor two the first problem [Vocalised Noise] the first problem was with regard to a gyrator [Vocalised Noise] this is the forward direction [Vocalised Noise] and the gyration constant is not given what is given is that the Z matrix of this is given as zero one by n minus one by n one by n zero this is the Z matrix that is given and then it is cascaded to a transformer ideal transformer one is to n this is ideal

this is port two and this is port one and what is needed is [Vocalised Noise] scattering parameters that is you have to find out S one one S one two S two one and S two two with the reference registers at both ports equal to one ohm this was the question alright now some of you [Vocalised Noise] some of you tried to ah [Vocalised Noise] solve it by finding out the abcd parameters abcd parameters of the two networks multiplying them out and then from the abcd parameters go back to the voltage current equation and then find out S one one S two two and so on and so forth

in the process one of the mistakes that you did was that you assumed $ab - bc$ for this is equal to one which is not correct i used formulas which are true for reciprocal networks only in fact [Vocalised Noise] some of you calculated S two one and said because this is reciprocal this must be equal to S one two equal to one which is not correct the non reciprocity is obvious here the gyrator is a nonreciprocal network and therefore you cannot do that

one of the other mistakes that happened was with regard to the transformer equation you see the side that has n the turns ratio it is from this side that the impedance is n squared times the impedance here not from this side and the voltage current equation many of you could not write correctly that is a pity is a open book open notes examination and you must be able to write the voltage current equation

now the simplest way that i find of doing this is to assume the voltages and current do not make any conversion just find out just write down the voltages and currents at various points and you will see how easy it becomes this is V_1 [Vocalised Noise] this is I_1 i suggest that you take this down this is let's call this V_2' I_2' these are all conventions then this is V_2 and this current is I_2 alright that's all that we need

now if you look at the gyrator if you look at the gyrator well what you have to do is to calculate S_{11} and S_{22} you have to terminate the port one into a one ohm resistor in series with a V_g one a voltage a voltage source V_g one alright

[Vocalised Noise] and

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$$V_1 = -\frac{I_2'}{n}$$

$$V_2' = \frac{I_1}{n}$$

$$Z_1 = \frac{V_1}{I_1} = -\frac{I_2'}{n} \frac{1}{nV_2'} = \frac{1}{n^2} \frac{1}{-\frac{I_2'}{V_2'}}$$

$$= \frac{1}{n^2} \frac{1}{\frac{1}{n}} = 1.$$

$$\therefore S_{11} = 0 \quad ||| \quad S_{22} = 0$$

ah if i look at the gyrator then obviously V_1 V_2' you see the the equation [Vocalised Noise] is V_1 V_2' the Z matrix is given zero minus one by n one by n zero times I_1 I_2' this is the this is the Z matrix equation V_1 V_2' these are the two voltages and these are the two currents

therefore V_1 as you can see is equal to $-I_2' / n$ and then V_2' is equal to V_2' is equal to I_1 / n that's all

so let's find out the input impedance of at port one that is Z_1 which is equal to V_1 / I_1 V_1 / I_1 now let's substitute for V_1 $-I_2' / n$ multiplied by I_1 is n times V_2' so n times V_2' that is equal to $1 / n^2$ and i can write this as $V_2' / -I_2'$ alright now $V_2' / -I_2'$ $V_2' / -I_2'$ i missed the prime

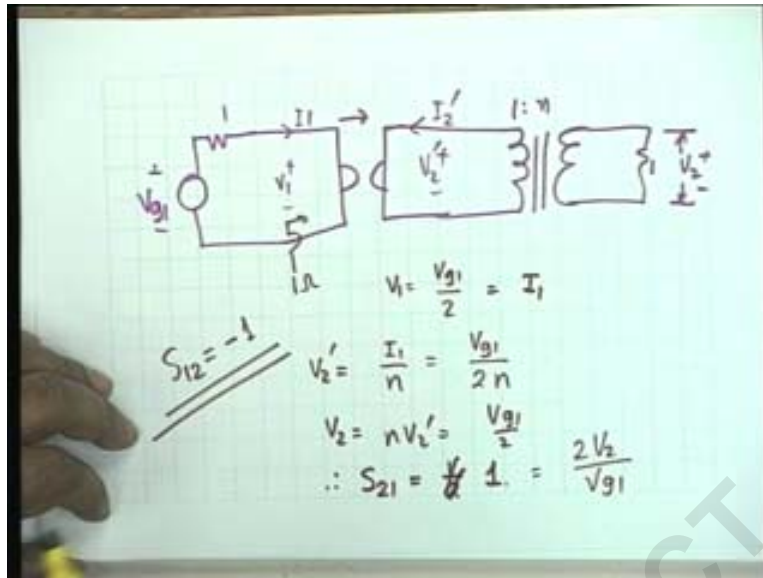
now if you look at the network $V_2' / -I_2'$ is the impedance looking here and since you have to terminate this into a one ohm resistor [Vocalised Noise] one ohm resistor the impedance looking here would be

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$1 / n^2$ not n [Vocalised Noise] squared $1 / n^2$ alright and therefore i can write this as $1 / n^2$ multiplied by 1 squared which is simply equal to 1 the input impedance is one [Vocalised Noise] and therefore S_{11} is equal to $Z_1 - R_0 / Z_1 + R_0$ that is equal to zero okay and in an exactly similar manner [Vocalised Noise]

if you go back that is terminate this in one ohm [Vocalised Noise] and find out the impedance here ((if)) (00:06:41) that will also be exactly one ohm and [Vocalised Noise] therefore S_{22} S_{22} is also equal to zero the procedure is exactly similarly let's calculate S_{21} and S_{12} for S_{21}

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let's go back one ohm V_g one then you have the gyrator and the ideal transformer one is to n this is one ohm this is what you have to calculate V_2 okay [Vocalised Noise] and to do that this is V_1 I_1 again V_2' I_2' prime

now since this impedance looking here is one ohm obviously these are simplifying things things are it should be V_1 should be equal to V_g one by two and V_1 is the voltage dropped across a one ohm resistor by the flow of a current I_1 I_1 flows I_1 sees a resistance of one ohm and therefore this must be equal to I_1 is that clear and once you pardon me is that okay

((you see)) (00:08:05) the impedance here is one ohm so V_1 is the drop in a one ohm resistor due to a current of I_1 and therefore V_1 is equal to I_1 multiplied by one and once you recognized this it will become very simple V_2' which is equal to I_1 by n from the Z matrix equation V_2' is I_1 by n therefore this is equal to V_g one by two n because I_1 is V_g one by two and V_2 is equal to V_2' this is the voltage that you have to find out that that should be equal to n times V_2' n times V_2'

so this is V_g one by two and therefore S_{21} is equal to twice V_2 by V_g one square root R_1 by R_2 R_1 and R_2 are the same therefore this is simply equal to half

<a_side> (()) (00:09:06) <a_side>

i beg you a pardon this will be equal to one because S_{21} is twice V_2 by V_1 okay

[Vocalised Noise] now [Vocalised Noise] the the total device is non reciprocal and therefore if you found out many of you did if you found out by mistake due to change of sign of a particular current that S_{12} is equal to one you should have suspected your calculation right then and there will be good

S_{12} cannot be equal to one because the gyrator is nonreciprocal and nonreciprocal device cascaded with a reciprocal device cannot make it [Vocalised Noise] {nonrecipro}(00:09:52) cannot make it reciprocal it is possible the other way round you may have combinations of nonreciprocal devices which make the total network [Vocalised Noise] reciprocal but not the other way round and therefore if you proceed in an exactly similar manner and take account of the directions of current correctly and the transformer equation correctly you should be able to obtain S_{12} equal to minus one that is the correct result that is question number one [Noise]

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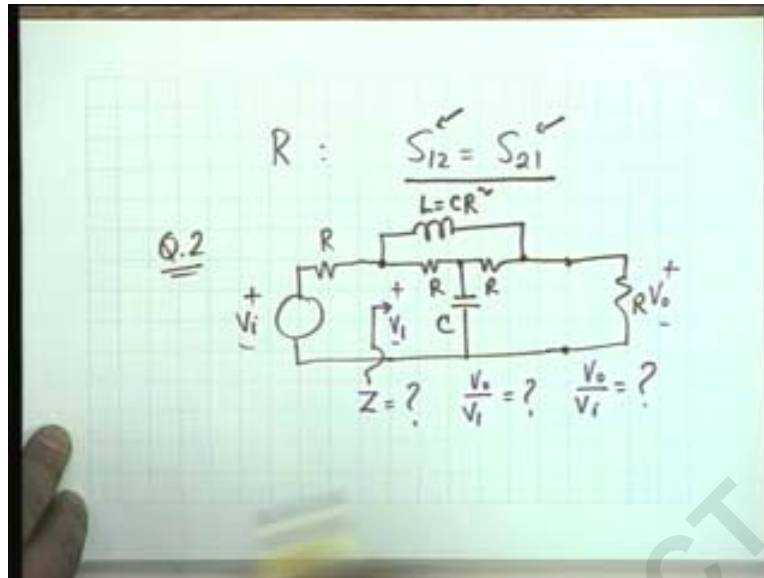
excuse me sir

yes

<a_side> ((for a reciprocal network is it necessary that S_{21} should be equal to minus S_{12})) (00:10:32) <a_side>

no for a reciprocal network

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reciprocity demands that S_{12} is equal to S_{21} not a negative no for reciprocity S_{12} should be equal to S_{21} for the definition of reciprocity because what you are doing is interchange the source and the load interchange the cause and excitation

reciprocity demands that this cross scattering parameters or the transmission parameters transmission scattering parameters let's say this is the forward transmission coefficient and this is the reverse transmission coefficient the two should be equal [Noise]

this device is nonreciprocal question number two is a network which is a parallel combination of two networks one is R and C is terminated in the resistance of capital R and then there is an inductance here of L equal to CR^2 do not tell me i didn't do such a problem in the class [Noise]

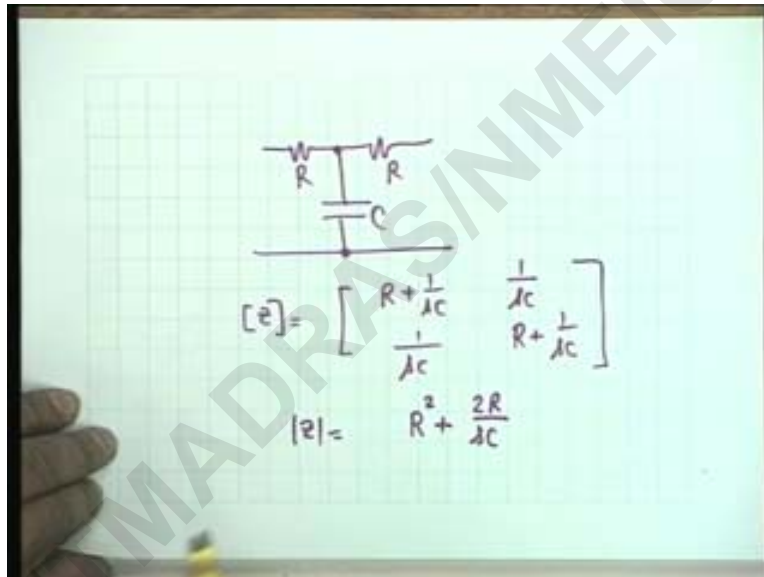
in fact in fact what i did was a bridge to T the exactly the same except that there is a small resistance here this is a simpler one not only simpler from that consideration but it is also simpler because L is related to C and R^2 and even though [Vocalised Noise] there are two energy storing elements and inductance and the capacitance it behaves ((as a)) (00:12:16) first order network there are cancellations

now the [Vocalised Noise] question says question says find out this impedance Z then if this voltage is V_1 and this voltage is V_2 you have to find out Z you have to find out V_2 by V_1 and you have to find V_2 by V_i these are the three things to be found out let me go systematically [Vocalised Noise]

to solve the network well the the constraint was you cannot use mesh and node analysis some of you played a little smart i didn't penalize it to the extent that i should have done but some of you what they did is [Laughter] assume a current here take this to be I one take that to be I two alright then again another distribution here don't you see that this is in disguise a mesh analysis a combination of mesh and node analysis okay

you were asked not to resort to that alright [Vocalised Noise] those who have done so one or two one or two i didn't tell penalize if this is a general phenomenon then i will penalize penalize heavily but since there are only one or two and i was in a good mood that day [Laughter] i didn't penalize greatly okay

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so the first thing we do is we find out the y parameters of this okay the Z matrix can be written down by inspection R plus one over sc i did this in the class one over sc one over sc R plus one over sc and the determinant of Z is equal to [Vocalised Noise] R squared plus twice R divided by sc which i can well let me leave it like this unnecessary simplifications i will not do till the last moment okay and [Noise]so the y parameters

(Refer Slide Time: 00:14:25 min)

$$[y]_T = \begin{bmatrix} \frac{R + \frac{1}{sC}}{R^2 + \frac{2R}{sC}} & -\frac{1}{sC} \\ -\frac{1}{sC} & \frac{R + \frac{2R}{sC}}{R^2 + \frac{2R}{sC}} \end{bmatrix}$$

$L = CR^2$

$$[y]_L = \begin{bmatrix} \frac{1}{sCR^2} & -\frac{1}{sCR^2} \\ -\frac{1}{sCR^2} & \frac{1}{sCR^2} \end{bmatrix}$$

of the T network y parameters of the T network therefore is equal to R plus one over sc divided by R squared plus twice R by sc Z this is what Z one one or two two

<a_side> (()) (00:14:45) <a_side>

two two

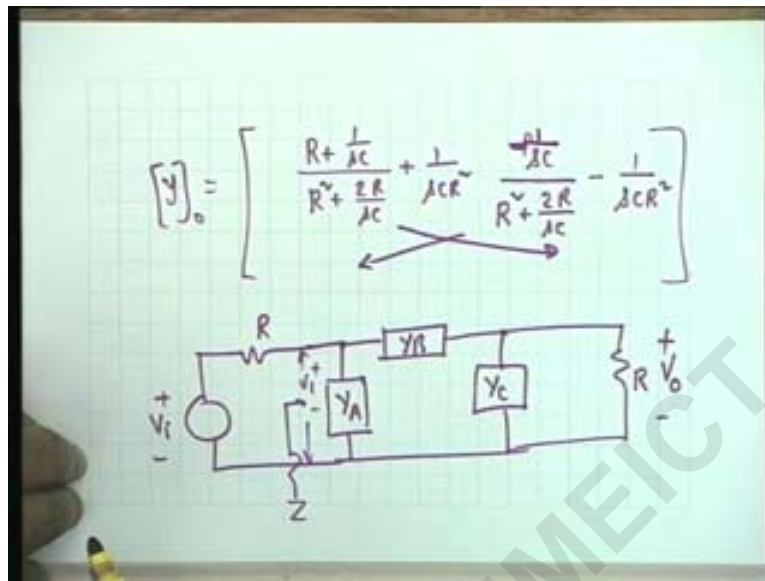
now two two and one one are the same here because of symmetry and therefore it doesn't make a difference the two two y two two parameter is the same as this but this Z two one and Z one two parameter would be R squared plus twice R by sc it would be a negative sign and one should not loose sight of that [Vocalised Noise] that would be equal to y one two also okay y one two and y two one are equal and as far as the inductance is concerned L equal to CR squared

now here at these right at this point of time because of the special relationship it should have occurred to you and there must be purpose in choosing the inductance to be equal to CR squared so there is no point in proceeding with the general terminology L we'll put CR squared for L and the [Vocalised Noise] y parameters of the L network is obviously one over sc R squared then plus or minus

<a_side> ((plus)) (00:15:54) <a_side>

no [Laughter] minus one over sc R squared y one two or y two one will be minus all over ((sc)) (00:16:03) R squared minus one over sc R squared plus one over sc R squared okay

(Refer Slide Time: 00:16:15 min)



[Vocalised Noise] and therefore the total y parameters y overall would be equal to let me write down Z y one one and y two one or y one two that should be enough it would be R plus one over sc divided by R squared plus twice R by sc plus one over sc R squared and one two parameter would be

<a_side> (()) (00:16:42) <a_side>

minus one over sc [Vocalised Noise] okay minus one over sc divided by R squared plus twice R by sc minus one over sc R squared and this and this are equal okay [Noise]

now to be able to calculate the impedance [Vocalised Noise] the voltage transfer functions etcetera we require the equivalent ((phi)) (00:17:10) networks so what we do is we find [Vocalised Noise] out the network then boils down to this we have $V_i R$ what we require is since we have found out the y parameters it is not difficult [Vocalised Noise] to find out this impedances Y_A Y_B and Y_C

then this is terminated in R ohms and this is V zero and this is what we want Z and we also want this voltage V one okay [Vocalised Noise]

now obviously because of symmetry Y_A and Y_C are equal it suffices to find out just one of them and Y_B it should be obvious Y_B is simply minus y_{12} or minus y_{21} therefore [Noise]

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The image shows a whiteboard with handwritten mathematical expressions for admittance. The first equation is $Y_A = \frac{R}{R^2 + \frac{2R}{sC}} = Y_C$. The second equation is $Y_B = \frac{\frac{1}{sC}}{R^2 + \frac{2R}{sC}} + \frac{1}{sC R^2}$. Below these equations, there is a small circle labeled v_i . A large watermark 'KINMEICT' is visible across the whiteboard.

therefore Y_A is equal to you see the simplification it is simply R divided by R squared plus twice R by sC okay because the other things cancel minus one by minus sC and minus one by sC R squared they cancel and this is equal to $Y_{sub C}$ and $Y_{sub B}$ is simply the negative of minus Y_{21} [Vocalised Noise] which is one over sC divided by R squared plus twice R by sC plus one over sC R squared do you understand this

<a_side> ((yes sir)) (00:18:45) <a_side>

(()) (00:18:45) the shunt the the bridging elements should be minus y_{12} okay or minus y_{21} and once you have done that once you have done that the [Vocalised Noise] let me go back to the network once you have done that the impedance would be this impedance would be now at this stage at this stage i would suggest that you make a simplification of this make a simplification and things shall be obvious

what you have to do is find the admittance of this combination that would be Y_C plus one over R then take the impedance of that let me write it down

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$$Z = \frac{1}{Y_A + \frac{1}{\frac{1}{Y_B} + \frac{1}{Y_C + \frac{1}{R}}}}$$

$$= R!$$

Z would be equal to one over okay let me look at this and write it down one over Y_A this admittance comes in parallel with an impedance okay so one over

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no i have done a mistake

<a_side> ((it should be one over by zero plus)) (00:19:56) <a_side>

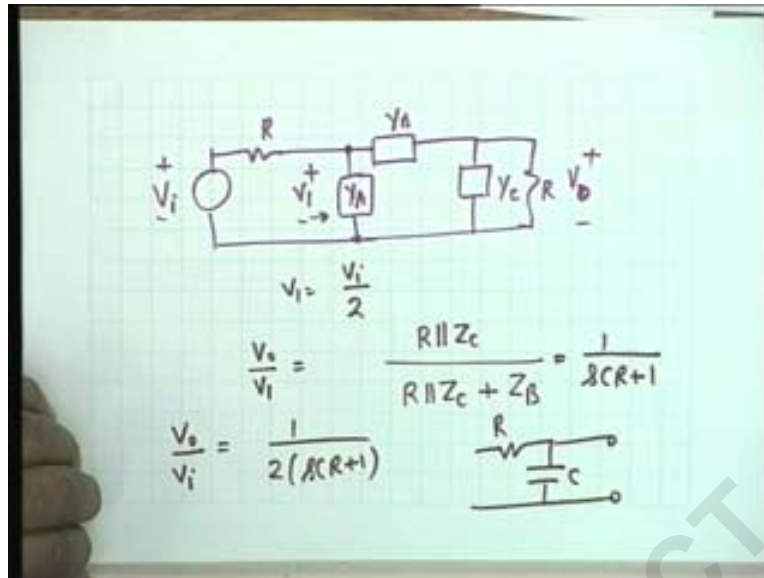
okay one by one over by Y_B plus

<a_side> (()) (00:20:03) <a_side>

plus one over Y_C plus one over R

now [Laughter] it's looks complicated but if you go systematically you will see [Vocalised Noise] that this comes simply as capital R the input {resis} (00:20:18) input impedance is simply equal to capital R and if it that is [Vocalised Noise] so if that is so let's go back to the network

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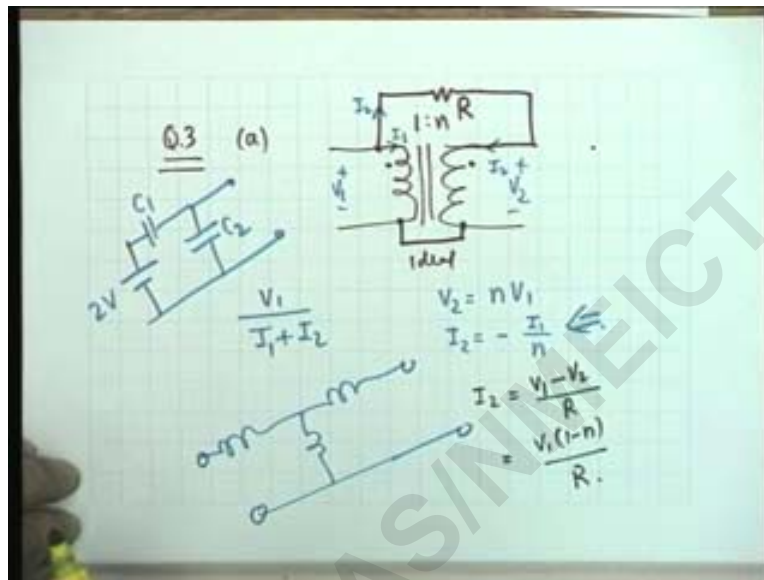
V_i R [Vocalised Noise] Y_B Y_C R [Vocalised Noise] V_0 and there is a Y_A here this is V_1 if this impedance is R then obviously V_1 is equal to V_i divided by two is that correct if this impedance is R then V_i divides into two equal resistors and therefore V_1 is V_i by two and V_0 by V_1 V_0 by V_1 does is not effected by Y_A and therefore it is simply equal to R parallel Z_C R parallel Z_c which you have already found out divided by R parallel Z_C plus Z_B and [Vocalised Noise] if you substitute the values and do a bit of simplification simplification should be obvious it simply becomes one by scR plus one

it is as if the total network you see that it is a first order transfer function the total network behaves as a first order although there are two energies storage elements okay and it is as if what you are doing is you are taking an R isn't it the same transfer function one by sc divided by one sc plus R okay

so the total network behaves like this and obviously V_0 by V_i shall be equal to half of this so one by two scR plus one that is the correct answer the mistakes [Vocalised Noise] or ah other mistakes that you many of you committed in this is ah not [Vocalised Noise] huge the mistakes that many of you have committed are silly ones they are in simplification okay you made a mess of a very long expression and you continued with that you didn't simplify and ultimately there were problems there were also problems

regarding the conversion of T into phi some of you try to try to put the formula and in a ZA ZB ZB ZB and ZC and so on so forth not from first principles i have always told you that in such simple cases do it from first principles okay and in this simplification somewhere along the line there is occurred a mistake

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question three was to calculate two input impedances and there also what i wanted the first problem was to judge whether you have understood what is an ideal transformer that's about all okay

it was an ideal transformer like this and you will see how simple the question is one is to n ideal and these two are connected these two are connected these two are also connected but through a resistance R

now [Vocalised Noise] the input impedances is to be calculated so we say this is V one now the mistake that you many of you did was to assume this current to be I one and it is the same as the current flowing through the primary which is not correct because there is a another path okay and [Vocalised Noise] some of you draw a line here and put a current I two here it was as if I two as dividing into two parts no i have given that this is that this has been left open and if this current is I two obviously this current is also I two and the input impedance now is V one divided by I one plus I two this is the input impedance [Vocalised Noise]

that's all that one has to calculate and ah well you formulate a voltage here and since it is one is to n you get V_2 equal to nV_1 and I_2 equals to minus

<a_side> ((minus)) (00:25:02) <a_side>

I_1 by n this is the equation that many of you faltered and this is why some of you got incorrect answers

you see what you should remember among [Vocalised Noise] this is that this is an ideal transfer so it's it should be perfectly lossless $V_2 I_2$ plus [Vocalised Noise] $V_1 I_1$ must be equal to zero this is the easiest way of remembering okay $V_2 I_2$ must be equal to minus $V_1 I_1$ yes what was your question

<a_side> (()) (00:25:30) <a_side>

you cannot you cannot because i was going to say this [Vocalised Noise] because L_1 and m both are infinite so $L_1 - m$ you don't know whether it is infinity or zero it is a difference between two infinities and we don't know what it is an ideal transformer has primary as well as secondary inductance equal to infinity but their ratio is finite

<a_side> ((in that case you can assume I_1 to be negligible)) (00:25:59) <a_side>

in that case you can assume I_1 to be negligible no there does flow a current there does a flow a current it's a magnetizing current what is your logic for assuming this to be zero

<a_side> ((sir we are saying n one is infinity)) (00:26:19) <a_side>

or so

<a_side> (()) (00:26:22) <a_side>

okay

<a_side> (()) (00:26:26) <a_side>

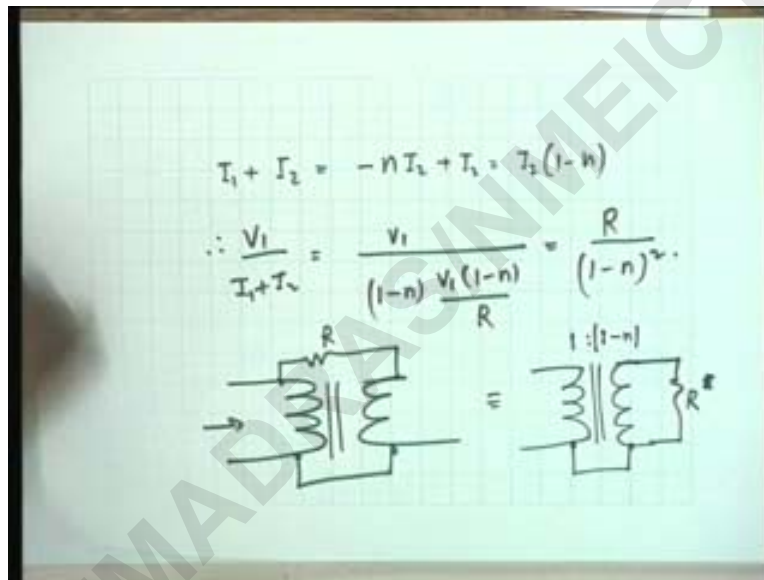
you see this is the equivalent circuit now this may be infinity this is also infinity and this is also infinity okay and therefore there occurs division between infinite inductances it's exactly like let me answer this question

if you have a battery and two capacitors let's say equal and this is a two volt battery the impedance here is infinity no current flows that doesn't mean that the capacitance do not

get charged they get charged instantaneously and the voltage here shall be one volt there occurs a division between two equal infinite impedances and therefore a voltage does appear

similarly here a current does flow okay now to to complete the question this if you can write this two then obviously there is no other problem you also noticed that I_2 this current I_2 is equal to V_1 minus V_2 divided by R which means that this is equal to V_2 is n times V_1 therefore V_1 one minus n divided R okay and all that you required to do now is to find out V_1 divided by I_1 plus I_2

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now I_1 plus I_2 [Vocalised Noise] is equal to [Vocalised Noise] minus $n I_2$ I_1 is minus $n I_2$ plus I_2 therefore this is I_2 one minus n and therefore V_1 divided by I_1 plus I_2 is equal to V_1 divided by one minus n then substitute for I_2 which is V_1 one minus n divided by R so this equal to R divided by one minus n whole squared that's it

one ah conclusion which i attempted to make is the following that if you have a circuit like this [Vocalised Noise] this two are connected and this is R as far as the input impedance is concerned isn't this equivalent to [Vocalised Noise] some resistance R' prime here

no some resist the same resistance R but it turns ratio [Vocalised Noise] instead of one is to n it is one is to

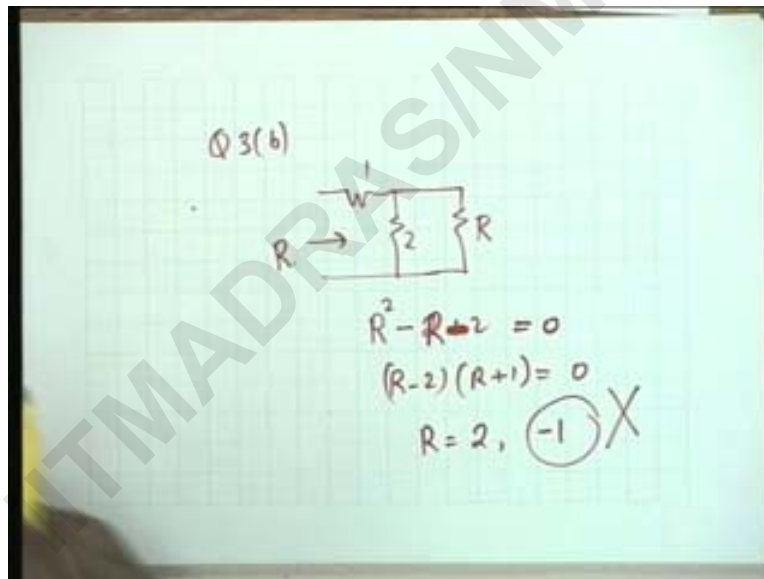
<a_side> ((n minus one)) (00:29:13)

why not one minus n

<a_side> (()) (00:29:15) <a_side>

turns ratio cannot be negative okay therefore it is one minus n or n minus one depending on whether n is [Laughter] greater than one or less than one one cannot say definitely so you see one minus n mod because when you square it when you square it it doesn't matter alright this is one equivalent which is sometimes find useful in network synthesis problem

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the last question question three B was simply if i am not mistaken i think i did this in the class

<a_side> (()) (00:29:51) <a_side>

no for an infinite ladder no but many of you most of you got it correctly

<a_side> (()) (00:29:59) <a_side>

[Laughter]

not all not all because i tell you [Laughter] as far as the phrase silly mistakes are concerned this is not a monopoly [Laughter] of any particular individual there are many individuals who do commit silly mistakes and all that you have to do here is to recognize that one two and then what you see to the right is the same as the network and therefore all that you have to do is this and the input resistance then satisfies the equation $R^2 - R + 2 = 0$ what was it

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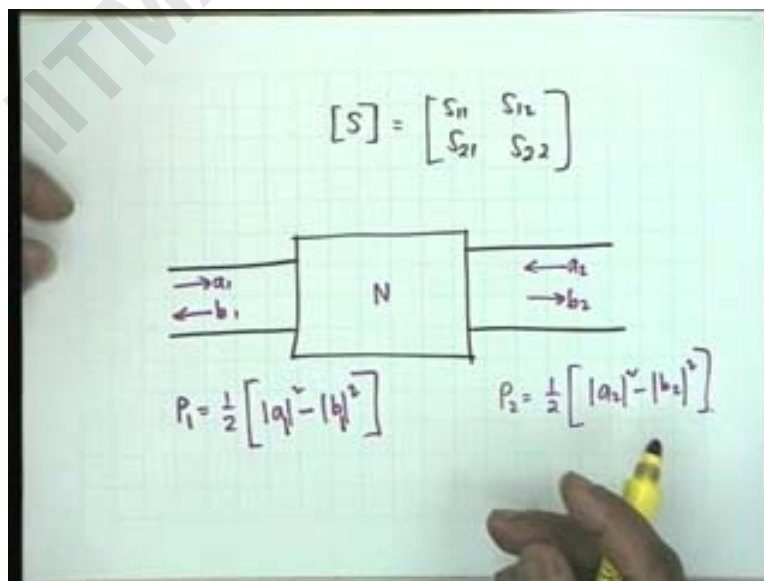
$R^2 - R + 2 = 0$

<a_side> (() (00:30:42) <a_side>

no that is not correct let me see $R^2 - R - 2 = 0$ and it was at this stage that some of you [Laughter] made a silly mistake this is $R - 2 = 0$ or $R + 2 = 0$ and [Vocalised Noise] it is two or minus one minus one is not possible because it's all positive resistor and therefore R must be equal to two but some of you made a mistake here in solving this simple quadratic equation alright in the is there any question no

[Vocalised Noise] in the remaining time that is left twenty minutes we would like to look at [Vocalised Noise]

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some properties of the scattering matrix if you recall the scattering matrix for a two port is S_{11} S_{12} S_{21} S_{22} [Vocalised Noise] and if we considered a two port a two port in which the incident and reflected parameters are a_1 and b_1 a_2 and b_2

then the power that is absorbed by the network n would be the sum of the powers fed from the two ports from the two ports the power fed to the network n from this port is given by half half magnitude S squared minus magnitude b squared if you recall is it okay

<a_side> (()) (00:32:36) <a_side>

a_1 squared minus magnitude b_1 squared alright [Vocalised Noise] why this factor half comes because we have taken a_1 and b_1 to be peak values and similarly P_2 the power that is fed at port number two is half a_2 squared minus b_2 squared therefore the total power that is observed by the network P

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$$P = \frac{1}{2} [|a_1|^2 + |a_2|^2 - |b_1|^2 - |b_2|^2]$$

$$= \frac{1}{2} [a_1 a_1^* + a_2 a_2^* - (b_1 b_1^* + b_2 b_2^*)]$$

$$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\underline{a}^{*T} \underline{a} = [a_1^* \quad a_2^*] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

shall be given by half a_1 squared plus a_2 squared minus b_1 squared minus b_2 squared this is the total power that is fed to the network now i can write this as the magnitude i can write as $a_1 a_1^*$ plus $a_2 a_2^*$ minus similarly $b_1 b_1^*$ plus $b_2 b_2^*$ where star denotes complex conjugate plus $b_2 b_2^*$ alright

now [Vocalised Noise] let us define the vector a let us define the vector a as the column vector a_1 a_2 and the vector b to denote that it is a vector or a matrix we put a $\underline{\quad}$ we put an underline is equal to b_1 b_2

then you see that $a_1^* a_1 + a_2^* a_2$ can be written as $a^* a$ is that okay this quantity can be [Vocalised Noise] written as $a^* a$ if you are not convinced let's do this what is $a^* a$ okay [Vocalised Noise] this multiplies a_1 a_2

so you get $a_1^* a_1$ multiplied by $a_2^* a_2$ is that okay simple matrix manipulation similarly this quantity $b_1^* b_1 + b_2^* b_2$ can be written as $b^* b$ alright

(Refer Slide Time: 00:35:16 min)

$$P = \frac{1}{2} (\underline{a}^* \underline{a} - \underline{b}^* \underline{b})$$

if N is passive

$$P \geq 0$$

$$\underline{a}^* \underline{a} - \underline{b}^* \underline{b} \geq 0$$

$$\underline{b} = \underline{S} \underline{a}$$

$$\underline{b}^T = \underline{a}^T \underline{S}^T$$

so our power absorbed by the network capital P is equal to half $a^* a$ do not forget the vector signs minus $b^* b$ [Vocalised Noise] and if the network is passive if N is passive then you know that this power this power has to be greater than or equal to zero P must be greater than or equal to zero

which means that $a^* a - b^* b$ should be greater than or equal to zero okay but you also know that [Vocalised Noise] the b parameters are expressed in terms of a parameters by the matrix S b is equal to [Vocalised Noise] $S a$ that

is how the parameters are defined S_{11} S_{12} S_{21} S_{22} and you also know that if a matrix is transposed that is b^T shall be equal to do you know this

<a_side> ((a transpose)) (00:36:43) <a_side>

a transpose then [Vocalised Noise] multiplied by S^T they get interchanged their positions get interchanged okay and therefore if i substitute this in this expression

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$$\underline{a}^{*T} \underline{a} - \underline{a}^{*T} \underline{S}^T \underline{S} \underline{a} \geq 0$$

$b^{*T} \quad b$

$$\underline{a}^{*T} [\underline{I} - \underline{S}^T \underline{S}] \underline{a} \geq 0$$

$1 \times 2 \quad 2 \times 2 \quad 2 \times 1$

Quadratic form $\underline{a}^{*T} \underline{A} \underline{a}$

← positive definite.
semi

then i get [Vocalised Noise] $a^*T a - b^*T S^T S a$ this will be a star transpose or transpose star is the same thing a star transpose then $S^T S a$ okay this is b^*T this is b^*T and this is b this must be greater than or equal to zero

which i can write as follows you see a star transpose pre multiplies both the quantities so i can take i have made a mistake here i didn't use the underlining to denote matrix if there is a question interrupt me if you do not understand interrupt me okay

i am doing matrix manipulations a star transpose pre multiplies both the terms and a post multiplies both the terms okay so let's take a star transpose and a out this from the beginning and this from end under bracket then what shall i be left with from this term i shall be left with

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okay i should be left with the identity matrix of dimension two by two that is one zero zero one minus i shall have $S^* \text{ transpose } S$ okay this should be greater than equal to zero

now [Vocalised Noise] this form what is the dimension of a star transpose let's look at the dimensions what is the dimension one row and two columns and a is two rows and one column so this must be two by two in this multiplication is to be compatible then this must two by two

it is indeed so because I is an identity matrix two by two S is a two by two matrix and $S^* \text{ transpose } S$ multiplied by S will also be a two by two matrix okay this is two by two this is two by two agree

such a form such a form let's say in general a star transpose a multiplied by any matrix a would be a scalar quantity this is a scalar quantity this is the power it is twice the power actually half multiplied by this is the power okay

so this is twice the power it is a scalar quantity and the variables you see obviously these are not variables this is a matrix of constants it contains zero one one zero and S one one S one two S two one S two two okay this is a matrix of constants the variables are here the pre multiplier and the post multiplier and therefore the variables will be involved here there will be no linear term

all terms would be quadratic alright you might have a one squared you might have a two squared you might have a one a two there cannot be a term like a one or a two is that is that clear

<a_side> ((no)) (00:40:40) <a_side>

no

you see a star transpose a shall not contain any linear term with regard to the variables alright it shall always come in product form and what does a contain a contains a one and a two so the only terms that is allowed here a one a two a one magnitude squared and a two magnitude squared because there is a complex conjugating such a form is called a quadratic form

quadratic form did you come across this term earlier no it is called a quadratic form we shall come [Vocalised Noise] across quadratic form once more in this class itself and that's why i'm tempted to introduce this at this moment

now [Vocalised Noise] it it is also it can also be proved from matrix theory that if a quadratic form is nonnegative if a quadratic form quadratic form is a scalar quantity if it is nonnegative okay the another name another name for this is positive positive definite okay

if it is greater then no this is called positive semi definite

<a_side> ((semi definite)) (00:42:05) <a_side>

positive semi definite okay

if this is equality sign is not there then it is called positive definite okay since it can be either greater than zero or equal to zero this form is also called it it is said the language is the quadratic form is positive semi definite okay

even if you forget the language it doesn't matter all that you have to remember is that if a quadratic form is greater than or equal to zero this is possible this is a theorem this is {poss} (00:42:40) it can be very easily proved but i will not go into the proof

if a quadratic form is greater than equal to zero then this implies that the determinant of the coefficient matrix here the coefficient matrix is a the determinant of the coefficient matrix is positive semi definite which means that

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$$\begin{aligned} & \# \underline{a}^T (\underline{I} - \underline{S}^* \underline{S}) \underline{a} \geq 0 \\ & \Rightarrow \det (\underline{I} - \underline{S}^* \underline{S}) \geq 0 \\ & \text{N is lossless} \\ & \det \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \right\} = 0 \end{aligned}$$

if this is so if a transpose T identity matrix minus s transpose T s a is greater than equal to zero this implies that the determinant of the coefficient matrix is greater than equal to zero okay

a quadratic form greater than equal to zero implies it can be proved that the determinant of this is greater than equal to zero alright now let's specialize the network let's specialize the network to a lossless network suppose n is lossless then the sign that shall apply here is equality sign the power absorbed by the network shall be zero

if the network itself is lossless it cannot dissipate any power it cannot absorb any power and therefore P capital P should be equal to zero which means that one zero zero one minus S [Vocalised Noise] one one star yes next term

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no it's a transpose

<a_side> ((S two one star)) (00:44:30) <a_side>

S two one star S one two star then S two two star is it okay multiplied by S [Vocalised Noise] one one S one two S two one S two two this shall be equal to zero

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pardon me

<a_side> (()) (00:44:53) <a_side>

no the determinant of this [Vocalised Noise] how do i show this[Vocalised Noise] the determinant of this shall be equal to zero

now how can the determinant be equal to zero this means that that two matrices have to be identical all terms should [Vocalised Noise] be equal to zero alright

(Refer Slide Time: 00:45:30 min)

The image shows a whiteboard with handwritten mathematical equations. The first equation is a determinant of a 2x2 matrix set equal to zero:

$$\det \begin{bmatrix} 1 - (S_{11}^* S_{11} + S_{21}^* S_{21}) & -(S_{11}^* S_{12} + S_{21}^* S_{22}) \\ -(S_{12}^* S_{11} + S_{22}^* S_{21}) & 1 - (S_{12}^* S_{12} + S_{22}^* S_{22}) \end{bmatrix} = 0$$

Below this, the terms are simplified to:

$$f_{11} f_{22} - f_{12} f_{12}^* = 0$$

which means that [Vocalised Noise] if i now multiply S one one star

<a_side> (()) (00:45:35) <a_side>

this is not (()) (00:45:36) pardon me

<a_side> (()) (00:45:40) <a_side>

hmm okay let's write down the total matrix okay total matrix is one minus S one one star S one one okay the total matrix S one one what is it next

<a_side> (()) (00:46:06) <a_side>

plus S two one star S two one correct this is the first term second term is zero minus okay which means minus S one one star S one two then plus S two one star S two two okay third term is minus [Vocalised Noise] yes

<a_side> (()) (00:46:42) <a_side>

$S_{12}^* S_{11} + S_{22}^* S_{21}$ is it okay yeah okay and the fourth term is $1 - S_{11} S_{22}$

<a_side> (()) (00:47:04) <a_side>

$S_{12}^* S_{11} + S_{22}^* S_{21}$ okay this determinant the determinant of this matrix is equal to zero okay

now what is the condition what is the condition can this two diagonals both of them can be equal to zero

<a_side> ((the product of both the diagonals are the same)) (00:47:33) <a_side>

product of both the diagonals is the same okay now what is this element and this element they are complex conjugates of each other alright and therefore what it means is that if I call this element as let's say f_{11} f_{12} f_{21} and f_{22} then $f_{11} f_{22} - f_{12} f_{21}$ [Vocalised Noise] should be equal to zero okay

what is the condition [Vocalised Noise] under which this would be valid that is the question

if this is zero then obviously this is also zero then this product should be equal to zero alright can this be non zero can this be non zero $f_{12} f_{21}$

<a_side> (()) (00:48:41) <a_side>

no

why not

<a_side> (()) (00:48:44) <a_side>

hmm

<a_side> (()) (00:48:46) <a_side>

if f_{12} is zero then f_{21} must be also be zero complex conjugate is zero is zero but the question is question is can we have f_{12} not equal to zero is that a possibility

<a_side> (()) (00:49:04) <a_side>

okay pardon me

<a_side> (()) (00:49:09) <a_side>

yeah

<a_side> (()) (00:49:19) <a_side>

what becomes real [Noise]

<a_side> (()) (00:49:25) <a_side>

you see this is a real quantity yeah that's where lies the you see this a real quantity so is this therefore the product of this two

<a_side> (00:49:38) <a_side>

is real

is real

whereas

<a_side> (()) (00:49:41) <a_side>

no no this product S one one star S one two [Vocalised Noise] so last S two one star S two

<a_side> (()) (00:49:54) <a_side>

that is also real correct correct

so it is a real quantity minus a real quantity equal to zero we will prove next time Thursday that that each of them has to be individually zero and i will give the logic next time