Circuit Theory
Prof. S.C. Dutta Roy
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecturer - 30
Scattering Matrix

This is thirtieth lecture. And we are going to discuss a new 2 port description; that means, the scattering matrix; scattering matrix description of a 2 port.

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This descriptions; scattering matrix description is not confined to 2 ports only; it can also be applied to multi ports. However, our description shall be in the context of 2 ports only.

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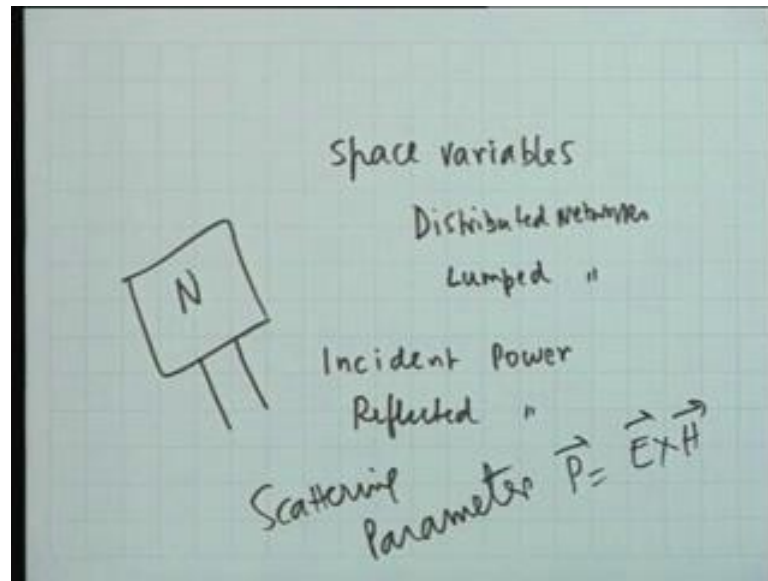
The scattering matrix description is particularly important in networks in which, the space variable is to be taken in to consideration. What are such networks called? Distributed networks as opposed to lumped networks. Therefore, scattering parameters are important descriptions in networks, which have to be considered in addition to the other the time variable the space variable also. In other words: in distributed networks.

Nevertheless they are equally applicable, they are quite general parameter. They are equally applicable to lumped networks also. And it is appropriate that, at this stage we know about scattering matrix parameters; particularly when you have a course on electromagnetic theory and its applications and learn about microwaves scattering matrix shall be the only description, either a field description or a scattering matrix description. And this scattering matrix description is in terms of incident power, is in terms of power instead of voltage and or current. It is in terms of incident power and reflected power at a particular port.

If, you take a network N and at a particular port, as you know at very high frequencies, it is it is impossible to define even define a voltage. You have to go for field descriptions that are, electric field and magnetic field. However power which is E cross H is as fundamental a quantity as the electric field or magnetic field. It is a combination of electric and magnetic fields. And at those frequencies the only description that, that is suitable and works is, the power description. And that power description brings in the parameters called scattering parameters.

Now, it is convenient to think in terms of incident and reflected power at high frequencies. You could also do that at low frequencies, nobody stops you. But, at low frequencies it is usually more convenient to describe the network in terms of voltages and currents. Voltage and current as the variables and therefore, Z parameters, Y parameters, H and transmission parameters, they describe the network quite well.

But, since scattering matrix parameters are universal, that is, they can be applied to low frequency as well as high frequency network. We shall deal with them in a very general manner. Most of our examples will be, with regard to lumped networks. But we would also like to look at 1 specific form of a distributed network namely; a transmission line. In fact, we shall do that in order to facilitate the definition of scattering matrix parameters. And you will understand the physical significance. I could do this mathematically that is, define the incident variable as this and the reflected variable as this and so on and so forth.

But, let us go a bit into the physical interpretation, that is, the link of scattering parameter with power. And this necessitates that we do when, we know a little bit about transmission line theory. I do not think you have been taught transmission line theory yet, or you are having it now. We shall there.

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Oh very good, then my job is simplified.

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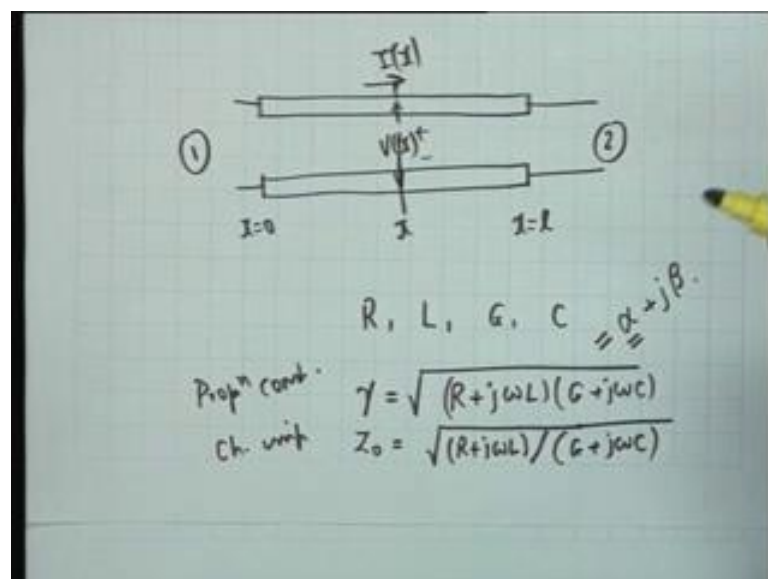
The transmission line; the telephone line for example, or the power line that goes overhead, typically it consists of 2 lines like this, they usually run parallel to each other, 2 lines like this. And any other transmission line can be thought, can be described by a parallel wire transmission. For example, a coaxial line is also a parallel wire, there is an inner core and there is an outer shield. Outer shield is at a constant distance with the inner core, also the length of the cable. And therefore, all most all transmission lines can be described in terms of a 2 wire line. And you know that, the space variable here is an important parameter.

If, we consider this as x equal to 0 and x equal to l, then at x that is shown here for example, an input step voltage does not appear at the output instantaneously, it appears after some delay and this is why, the space variable becomes important. You also know that a transmission line like this is described by 4 parameters R, L, G and C where: R and L are the series resistance and series inductance per unit length and G and C are respectively the conductance and the capacitance per unit length. These are shunt parameters whereas, these are series parameters. And you also know if you since, you have done it that, a transmission line is described in terms of 2, instead of the 4 primary parameters as they are called a primary constants.

You can have you can have a description in terms of just 2 derived parameters namely; the propagation constant gamma which is equal to, for sinusoidal excitation it is the product of R plus j omega L G plus j omega C the product of this and the characteristic impedance Z 0, this is the propagation constant and this is the characteristic impedance Z 0, which is equal to square root of R plus j omega L divided by G plus j omega C. It is the ratio of the 2, the impedance. It is impedance per unit length and the shunt admittance per unit length; it is the ratio of the 2 square root of that. Do the dimensions agree?

G is 1 by resistance. So, it is impedance squared, square root of impedance squared is impedance. What about the dimension of gamma?

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Dimension less.

You also know that, gamma is a complex quantity in general; it has a real part alpha and an imaginary part beta j beta where, the real part alpha is a positive quantity. The voltage

and current at any point of the line, let us say x distant x from the origin from port number 1, this is port number 2. The voltage and current at any point; if we denote this by V of x and the current is denoted by I of x, well this voltage and current in general, are not only functions of time, but also functions of space; space variable, we have considered only 1 dimensional propagation.

If it was space propagation for example, propagation of a disturbance created by an antenna in an otherwise still environment well, it travels in all the 3 directions and therefore, 3 variables would have been needed. Here it is a guided wave, guided along the transmission lines. So, the propagation is only along 1 direction, which we conveniently call the x direction.

If, the excitation is considered sinusoidal, then the quantities I and V are phasors. You can take either the peak value or the root mean squared value depends on your convenience, these are the phasors. And then time dependence is taken care of, by make it in to a phasor and only the space dependence x has to be taken care of. And you know that this is only a review. I am not going into the details.

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You know that the basic equations for transmission line behavior is dv dx equal to minus Z I where, Z is R plus j omega L that is the series impedance per unit length. And the other equation, the current equation is; di dx equal to minus Y times V where Y is equal to G plus j omega C. And a combination, these are 2 coupled equations: voltage current, current voltage, they are coupled equations. And if you eliminate 1 of them for example,

if you differentiate this and substitute for I, then you get a second order differential equation in the voltage d2 V dx 2 would be equal to gamma squared, gamma squared V where, gamma squared is simply the product Z Y and gamma; obviously, is the propagation constant.

Therefore, the solution is very easy because, it is a second order differential equation, without the first order differential coefficient. And the coefficient of V is a constant gamma squared. And the solution is V of x equal to some constant V i e to the minus gamma x plus some other constant V r e to the plus gamma x. It will have 2 independent solutions and gamma can have the value of plus minus, gamma has the value square root Z Y. And the auxiliary equation for this shall have the roots m equal to plus minus gamma and that is why this solution comes. You have done this solution earlier.

You know that this term the subscript i stand for the incident wave and subscript r stands for the reflected wave. Can anyone recall why this is called an incident wave?

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Direction of gamma.

You see the voltage as it reverses the line must decrease with distance because, it is a passive network. And the real part of gamma is the attenuation constant therefore, as x increases, the attenuation shall increase and therefore, this is the incident wave; a wave travelling in the direction of positive x. And this is the reflected wave that is, the wave travelling in the negative x direction. This is the incident wave and that is the reflected wave.

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And if you substitute this in the equation I equal to minus 1 over Z dV dx. If you substitute this solution, then you get the solution for I. And it can be written as V i divided Z 0 e to the minus gamma x minus V r by Z 0 e to the gamma x, that is, the incident current, this is the incident current and this is the reflected current. There is a sign change there is a negative sign here.

Now, these are our basic equations for a transmission line: V equal to V i e to the minus gamma x plus V r e to the gamma x. These are our basic transmission line equation from which now, we shall go in to the scattering matrix description. If there is a question at this point I will be happy to answer.

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That's right, if you are differentiating by V gamma comes here, but then this is also divided by Z.

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Gamma by Z is Z naught. That is correct.

Now, gamma by Z is not Z naught.

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One by Z naught that is how this 1 by Z naught comes.

I certify that these 2 equations are correct alright.

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Reflection at which point? At any point in the transmission line, the voltage component can be thought of as the superposition of 2 voltages; 1 travelling in the forward direction and the other travelling in the backward direction. And the 1 travelling in the reverse direction is called the reflected wave. The reflection can occur throughout the line, reflection occurs at every point in the line and it is the sum total of that, that you see V r e to the minus gamma e to the plus gamma x is the reflected wave which, is a sum total of reflections at all points, for which the coordinate is greater than x.

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Forward direction is consider as positive because, our coordinates where x equal to 0 and x is equal to L. Now, this is quite arbitrary, it depends on where you connect your source. Obviously, we have connected our source at port number 1. So, as we go away from the source, x is positive, this is considered as the origin.

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That means: if I connect the source here, then I will consider x equal to 0, as this point and the wave travelling in this direction from right to left will be considered as the incident wave

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That's right. That is correct.

Our direction will change; x equal to 0 and x equal to L. So, x increases in this direction it is a matter of convention that, we always take this source at the left, nothing to do with political inclinations. Source at the left and at the load at the right, this is our convention. You see in drawing networks, the left2 terminals, we call it port number 1, there is nothing sacred about exciting at port number 1, you can excite at any other port.

In fact, when you go to microwaves you will see the magic T for example, is a 3 port networks. There are networks which are 6 ports. Then there is nothing called input port, output port, you could excite at 3 ports and take the output at any of the other 3 port. There are ring couplers, in which it could be n number of ports n, can be greater than 6 also, there is nothing sacred.

As far as, 2 port is concerned we stick to the convention that, we excite at the left and take the output at the right which also, we have broken. You see in stating the reciprocity theorem for example, we interchanged the source and the load. So, there is nothing sacred.

We start from these 2 expressions. Let me write these expressions again.

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I V equals to V i e to the minus gamma x plus V r e to the gamma x. And I equal to V i divided by Z naught minus e to the minus gamma x minus V r by Z naught e to the gamma x. Now, I can write this second equation. I do some mathematical manipulations now. I can write this second equation as I Z 0 equals to V i e to the minus gamma x minus V r e to the gamma x.

If, I combine this equation with this equation 1 and 3, then I can find out expressions for V i e to the minus gamma x in terms of V and I Z 0. What I simply do is, V plus I Z 0 divided by 2. On the other hand, the reflected wave V r e to the gamma x can be written as: V minus I Z 0 divided by 2. That is very simple; simple manipulation.

The next thing that I do is I take 1 by square root Z 0. I take square root Z 0 common. Then what I get is half square root Z 0, out of this expression I take out. Then, I get and half. So, I get V divided by square root Z 0 plus I square root Z 0. And I write this as square root Z 0 half V by square root Z 0 minus I square root Z 0, there is a purpose in doing that. I, before I go in to why I did this manipulation, let me also point out to you

that, if the line if the transmission line is terminated in its characteristic impedance, then what happens? There is no reflected wave.

The reason is from here you see if V is equal to I Z 0 at X equal to l, what does termination mean? It means that at the end of the line you have a termination Z 0, this is V of l and this current is I of l. So, V r e to the power gamma l shall be equal to, V of l minus I of l Z 0 which is equal to 0 because, this says that V of l is equal to I of l Z 0. You see this point.

Therefore, V r e to the gamma l would be equal to 0. Now, e to the gamma l cannot be equal to 0, unless l is infinity.

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One is minus infinity. Therefore, this constant V r must be equal to 0 which means: that there is no reflected wave. And if there is no reflected wave, then these 2 terms shall be 0, these 2 terms shall be 0 and you see that the ratio of V to I at any point on the line, shall be exactly equal to Z 0. In other words, characteristic impedance terminated line at any point inputs an impedance of Z 0. The input impedance of a characteristic impedance terminated line is equal to Z 0 wherever, the impedance is measured, irrespective of where the impedance is measured.

Now, let us go back to this manipulation.

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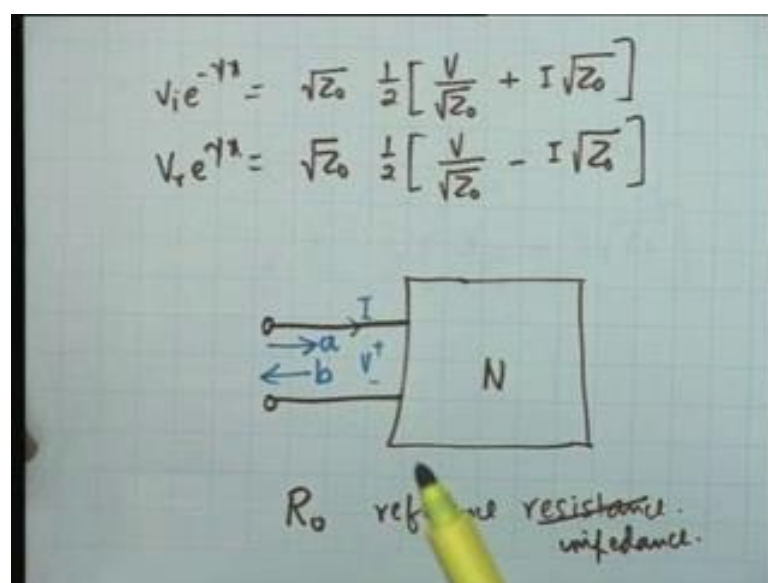
We have shown that V i e to the minus gamma x, let me write this again equal to Z 0 half V by square root Z 0 plus I square root Z 0. And V r e to the gamma x equal to square root Z 0 half V by square root Z 0 minus I square root Z 0. Now, these where done with reference to a transmission line, we found out expressions for the incident voltage wave and the reflected voltage wave.

Now, let us consider a general 2 port, forget about transmission line; a general 2 port N, general 1 port. The 1 port has only 2 terminals, where you can connect voltage source measure the current or connect a current source measure the voltage. Suppose, the voltage current description we do not want, we want a different kind of description in terms of power. Then what you do is; you define for every such 1 port, a constant R 0, a constant R 0, which is called a reference impedance, actually it is a resistance. Resistance is a special case of an impedance. So, it is a reference resistance. Let us put the term impedance; reference impedance, all we want is that this should be a 40 percent.

Let R 0 be arbitrary we choose, we define, we call R 0 as an arbitrary reference impedance. We will show lateral, how to choose R 0. But, given an N given a 1 port N, we defined N in arbitrary reference impedance R 0, and then we make the following definitions. We say.

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We are now confining to 1 port only, then while extending to 2 ports, we will have to define 2 such reference.

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No it is for the present it is arbitrary, we will see how to fix it later for the present it is arbitrary. Then, we define an incident parameter a and a reflected parameter b as follows. We define an incident parameter a and the reflected parameter b, not parameter a variable instead of voltage and current. Now, we will see how a and b are related to incident power and reflected power later, but at the present time we simply say it is a variable, we do not know how it is related to power, but we define like this.

We define a; incident parameter as half V by square root R 0 where, let V and I be the voltage and current at this port. Then, what I do is a we define simply as this term that is, if Z 0 is square root Z 0 is taken out, then whatever this term is this is what we define as a which means: that we define this as I square root R 0. You see, this is the incident you can see the relationship between the incident voltage in a transmission line and the incident variable of the 1 port. And b is called the reflected variable and it is defined as V by square root R 0 minus I square root R 0.

These are 2 quantities, 2 variables which are defined for the 1 port N in terms of conventional voltages and currents. Ultimately, we can do wave with voltage and current. Now, for being applicable to lumped networks in which, you can work in terms of voltage and current, there must be a relationship and this is the relationship that we have, that we take care of in the definition. And if you see this 2 expressions and the definitions of a and b, you see that a and b are very simply related to the incident voltage wave in a transmission line and the reflected voltage wave in a transmission line.

If, R 0 equal to Z 0 if the reference impedance is the same as the characteristic impedance, then in a transmission line a, in a transmission line a is simply equal to the incident voltage wave divided by square root of Z 0. Therefore, a is indeed connected to an incident variable. Similarly, b as you can see is V r the reflected wave the reflected voltage wave divided by square root Z 0.

So, a and b have a physical interpretation. We are not considering the dimensions at the movement. The dimensions; obviously, are not voltage because, they are voltage divided by a square root of an impedance. So, it would be volt per Ohm to the power half or volt Ohm to the minus half. We are not bothering about it now, we should relate ultimately a and b to power and then we shall see the peculiarity of this definition shall become obvious because, our main concern is to describe the network in terms of power; power as the variable.

But as far as a 1 port is concerned, these are the definitions of the incident and the reflected variables. We are not saying voltage, current or power.

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Pardon, I beg your pardon.

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I can choose R naught anything I like alright, then my V i e to the for a transmission line, it will not be as simple as this it would be a different kind of a relationship. But, why should 1 do that? Because after all, our purpose is to describe the multiport, describe the 2 port in terms of variables and parameters, so it is convenient.

Similarly, here also in the case of 1 port you will see how to choose R naught to make life simple. Instead of going in to complicated calculations, we can bring out things very easily by particular choice of the characteristics impedance and you will see how to do that. For a transmission line you are quite right, it is chosen as equal to the characteristic impedance at, in fact, in both ports.

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In transmission lines we do not concern ourselves yes, that is true because it simplifies things, it simplifies matters. And it also fits in physically, the interpretation of incident power, reflected power the pointing vector and so on and so forth. But let us go back to a 1 port. In order to understand the physical meaning of a and b a little better, let us consider power.

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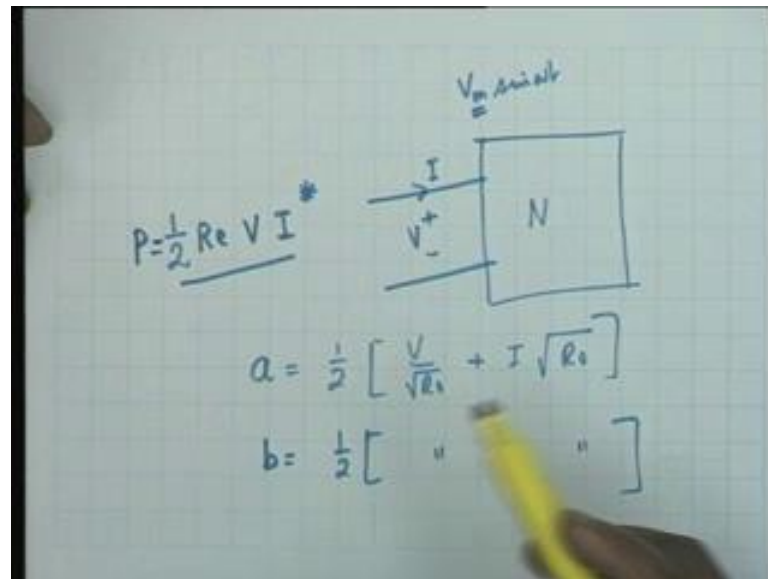
As you know, the power absorbed by a network N, when it is excited by a sinusoid V and the current is I, the power as you know is given by real part of VI star. Do you know this complex conjugate? Alright, Can you derive this? Well for historical reasons I will use the factor half which means that, I am considering V as the peak vector not the RMS vector. The phasor is considered with the peak value.

For example: if I have Vm sine omega t, then my voltage vector is Vm; the peak value the maximum value not the RMS value. And this is historical, you can do with half, then it is implied that V and I are RMS vectors. Now, the derivation of this; can there be, can you give me a very simple way of deriving this? I what I am interested in is, whether you know how to derive this, if you know we will skip it alright.

Now V and I, you know that a is equal to by definition a is equal to half V by R square root R naught plus I square root R naught. My intension now is to express P in terms of a and b, to get an interpretation of a and b in terms of incident and reflected power. So, this is half this minus this. Now, if I eliminate, now if I find V and I in terms of a and b.

Do you have a question?

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That's right. That is correct, that is why I took the factor half here.

So, you can easily see that a plus b times square root of R naught is simply equal to V is it not? That right. And similarly, a minus b this factor of half because twice into half becomes equal to 1, a minus b square root R naught becomes equal to I. So, the power absorbed by the network.

a minus b divided by R naught. Yes, thank you.

So, the power absorbed by the network is half real part of V, that is, a plus b multiplied by R naught and R naught will cancel square root R naught and therefore, multiplied by a star minus b star. And the real part of this is simply aa star minus bb star. The rest of it will be the imaginary part, aa star minus bb star which I can write as half mod a squared minus half mod b squared. And you can see that, the power absorbed by the network has been expressed in terms of 2 quantities: a positive quantity and a negative quantity.

Therefore, this can be interpreted as the incident power; that is the power that is tried to be fed to the network by the source. And this power half mod b squared is the reflected power or the power that is rejected by the network and sent back to the source. The total power absorbed by the network is the power that is fed in and the power that goes out. Rest of the power remains inside the network and is dissipated.

If, this was a lossless network, then mod b squared would have been equal to mod s squared because; no power would have been absorbed. But in a passive network, this

shall be greater than or equal to 0, because of passivity. If it is 0, then it is passive and lossless, if it is greater than 0 then it is passive and…

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Passive or dissipative passive and dissipative. And therefore, from this we get an interpretation of the incident variable a and the reflected variable b in terms of power, that is, aa star divided by 2 is the incident power, bb star divided by 2 is the reflected power alright. And this is how the 2 variables are related to physical quantity like power. First we related to voltage and current, but that as I said holds only if you can measure the voltage and current. In a microwave network, cannot have a, you cannot put a volt meter across a wave guide for example, and measure the voltage no it is not possible, it's a single metal and therefore, you have to measure the power.

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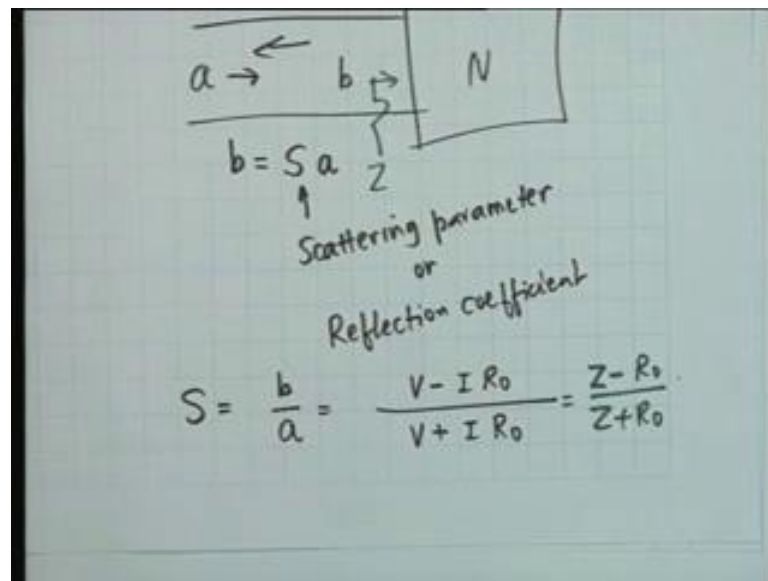
Now, let us go back to a and b. a is the incident variable, yes was there a question? a is the incident variable and b is the reflected variable. And 1 defines a relationship between b and a of a 1 port like this. The reflected variable is put as some constant S multiplied by the incident variable a. And S is called the scattering parameter of the 1 port N. In a 1 port, if we had the impedance description Z parameter description, all that you can define is the input impedance. In a 1 port, if you want to have a admittance description, all that you have is the input admittance.

Similarly, in a 1 port there are only 2 variables and therefore, you can express 1 in terms of the other. And S is a scattering parameter and is a constant for the 1 port. S is also

called for a 1 port; it is also called the reflection coefficient. And to see why it is called a reflection coefficient, you see that if, you look at the definitions of b and a, S is the ratio of b to a and if we cancel out the factors half, then we get this as V minus IR naught divided by V plus IR naught.

If we apply the definition and cancel out the factors half multiply by square root R naught, then this is what I get. And you can see that, if I divide both numerator and denominator by I, then I can write this as Z minus R naught divided by Z plus R naught where, Z is the input impedance, that is, it is the ratio or V by I. And as you know from transmission line theory; this is called the reflection coefficient or the miss match coefficient, that is, how much Z the input impedance is away from the reference impedance. So, this is also called the reflection coefficient.

You can see that if the reference impedance R naught is chosen equal to Z, then the reflection then the scattering parameter is 0. This has something to do it with the choice of R naught at a slightly later stage. But, let us look at a situation in which we have a real source and a real source is a voltage source with internal impedance in series or a current source with a internal admittance in parallel.

Let us take the voltage description of the real source.

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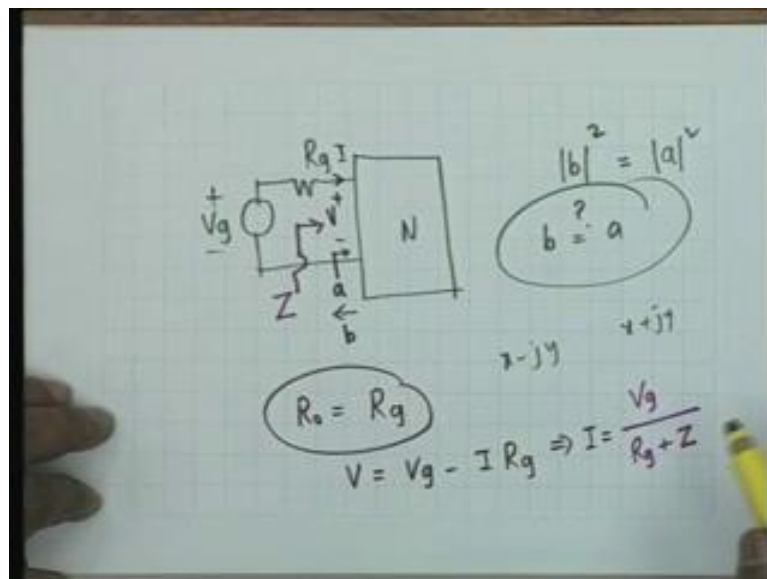
Vg and an R sub g.

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If it is lossless all that it means is, mod b squared equal to mod a squared, does that mean that b is equal to a? No, not necessarily. Magnitude squared may be equal, but the complex quantities may be quite unequal. For example, x minus jy and x plus jy they are the same mod, but the quantities are different. We will come to this when, when is this scattering parameter equal to 1? We will come to this a little later.

Let's have a network N. Let us have a network N which is driven by a real source Vg and Rg. And let the voltage V and the current I be defined like this. And at this point let us define an incident parameter a and a reflected parameter b at the input of the network. We have defined voltage, current, incident parameter, reflected parameter. Now, we also choose R 0 as equal to Rg. We choose the reference impedance with is sam as the input impedance, as the internal impedance of the source. This was arbitrary, so we can choose it. Let us choose it to be equal to Rg.

Then, let us see what is V? This is the choice, V is Vg minus IRj or I let us also suppose, the input impedance is equal to C, input impedance of the 1 port. We have defined VI. So, the ratio of V by I is Z. Then do not you see that I is simply equal to Vg by Rg plus Z. Vg by Rg plus Z this is perfectly alright. Now, let us look at what happens to a and b. The current here is simply voltage divided by Rg plus the impedance.

Let us see what happens to a and b.

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a is equal to 1 by 2 V by square root R 0 plus I square root R 0. Let us not impose R 0 at the present stage; we will see what happens when R 0 equal to Rg a little later. Let us

keep the identity over 0 for the present. I can write this as 1 by 2 square root R 0, if I take square root R 0 out, then I get V plus I times R 0. And V is Vg minus I Rg Rg multiplied by Vg divided by Rg plus Z. I substitute for I plus I times R 0. So, it would be Vg R 0 divided by Rg plus Z is that is this equation. What I did was; skipped a couple of steps, but let me introduce that this is 1 by 2 square root R 0 V plus IR 0. For V, I write Vg minus I times Rg plus I times R 0.

So, this I can now simplify to Vg, Vg I can take common for all the 3 divided by twice square root R 0 1 plus R 0 minus Rg divided by Rg plus Z. Now, what will happen if R 0 is equal to Rg? The second term obviously becomes 0 isn't that right? If, the reference impedance is chosen as equal to the internal impedance of a source; reference impedance so far was arbitrary. If this is chosen to be equal to this, then my a simply becomes Vg divided by twice square root R 0 not R 0 no longer R 0 Rg.

Now, let us look at the power P.

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As you know the power is half mod a squared minus mod b squared, which is equal to half a squared. If I take this out, what shall I get in the bracket? 1 minus mod S squared because, S is b by a alright mod S squared. You also know that S is equal to reflection coefficient, that is, Z minus Ro divided by Z plus R o and Ro Rg, Ro has already been chosen to be Rg.

Therefore, if Rg if Z equal to Rg, if the input impedance, if the network is so designed that, the input impedance is equal to the internal impedance of a source, then S is equal

to 0 identically. And therefore, the power simply becomes the incident power, there is no reflected power and all the power that is fed to the network is absorbed by the network. So, the power becomes equal to half mod a squared. And if you substitute the value of a, then you get Vg squared divided by 8 times Rg. Vg squared divided by a eight times Rg.

Now, if you recall the theorem of maximum power transfer, you know that maximum power is transferred to a network if the internal impedance of the source is the complex conjugate of the network impedance. You know this do not you? Here the internal impedance is resistive Rg. Therefore, if Z equal to Rg, then maximum power is transferred and therefore, this power we can call the maximum available power, that is, I am going very slow because this terms are new.

Maximum available power is the power; that is the maximum power that can be drawn from a source, generating an open circuit voltage Vg and having an internal impedance Rg. You cannot draw a power more than this.

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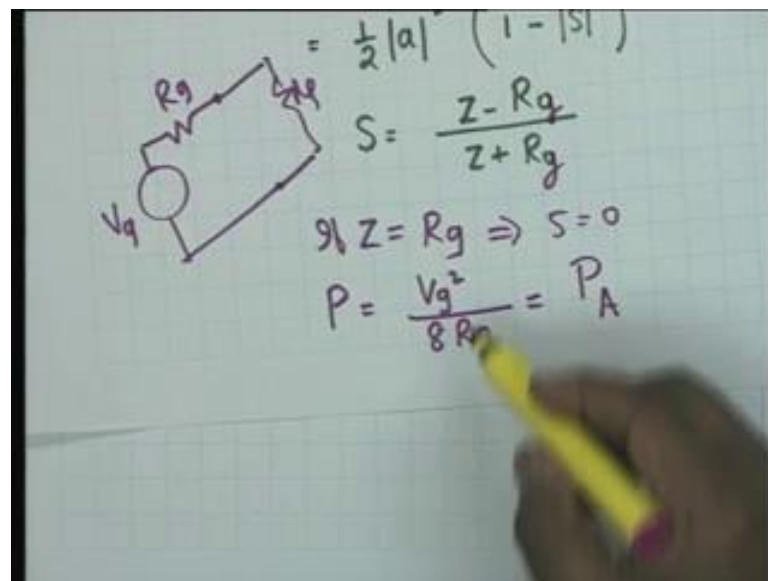
If you have a source which generates a peak voltage of V sub g peak phasor V sub g and a resist internal resistance Rg, you cannot draw from this source a power greater than this because, by the maximum power transfer theorem; maximum power is can be transferred from this source only to an impedance which is exactly to Rg. And if you terminate this in Rg, the power that is transferred to Rg is exactly equal to this. Therefore, this power is called the name for this is maximum available power.

Our final conclusion is that if you have a 1 port which is supplied from a voltage source Vg and internal impedance Rg, then maximum available power will be transferred to the load Va P A equal to Vg squared by 8 Rg when, the reference impedance R 0 is chosen to be equal to Rg. Not only that, the network is such that it is input impedance is equal to Z; Z no more transmission line, it's a simple 1 port which is equal to V by R.

So, given a 1 port; given a 1 port, 1 first identifies what is the internal impedance of the source and chooses that as the reference impedance. And, then if the 1 port is to be as effective as possible, you should design the 1 port so that, its input impedance is equal to the internal impedance of the source. And in the context, we go back to the definition as S is Z minus now Rg which is the same as R naught divided by Z plus R naught and you can write this as z minus 1 divided by z plus 1. I divide by, where z is equal to Z divided by R naught; obviously, z is a normalized input impedance of the network, normalized input impedance of the network and we decide to work is S normalized.
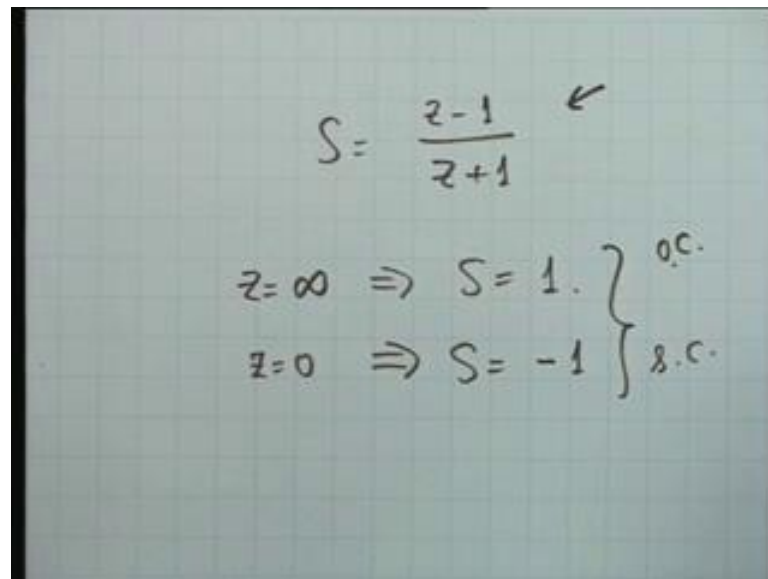
The scattering parameter is as usual, it does not change it is not normalized. What is normalized is, the input impedance of the network normalized with respect to the reference impedance. Now, follow me carefully. If we decide to work with z, if we decide to work with z the normalized input impedance then; obviously, our reference impedance is now 1 Ohm and this is the convention. You will see if you study microwaves later, that 1 always takes of 1 Ohm normalized impedance normalized 1

Ohm reference impedance for the scattering parameter, which simply means that all impedances are divided by the actual source impedance.

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Source impedance has to be the characteristic impedance if, it is to give maximum power to the transmission line.

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Now, so we would like to make a few comments about this relationship. z minus 1 divided b z plus 1, and then close the class. A few comments about this; there was a question about b equal to a b equal a then S equals to 1? Alright; that means, no power is absorbed. There is another way that S can be equal to 1. Suppose, the network the 1 port is an open circuit; that means, z equal to infinity, do not see that S is equal to 1? S is equal to 1.

Similarly, if the network itself is a short circuit, then S is equal to minus 1. These are 2 special values for open circuit and short circuit S equal to 1 for open circuit, S equal to minus 1 for a short circuit.

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Another characteristic or property of S comes into transparency if you look at this expression. Half a squared 1 minus mod S squared. This is was the expression for the power absorbed by the network. And you know that this is greater than or equal to 0 for a passive network. What can you say about mod s squared then?

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So, mod S squared must be less than or equal to 1, it can also be equal to 1. So, for a passive network, the scattering parameter, now it's a parameter this scattering parameter or the reflection coefficient, magnitude squared must be bounded to unity, bounded by unity, it cannot exceed unity for a reactive network. What is a reactive network? A reactive network is composed of only reactances; reactive network, reactance's means: inductances and capacitances which do not dissipate energy. In other words, a reactive network is also a lossless network. For a reactive network magnitude S squared shall be equal to 1.

The third property of S shall be proved next time, after defining a couple of important complex variable function. We require a bit of mathematics, but it is not much, there is nothing to be afraid of it. This is what we will do on Thursday.

Thank you.