Circuit Theory
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Lecture - 3
Network Equations: Initial and Final Conditions

In the third lecture, we shall talk of network equations and initial and final conditions. In this course, we will assume that you know KCL, KVL, ohm’s law of course and we also presume you know, Laplace transforms and Fourier transforms. Am I justified in assuming this? Okay.

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So there are, basically, there are two ways of writing network equations. Given a network N, you can solve it by writing two sets of, one of two sets of equations. First set is based on what is called a mesh basis network equations, network analysis can be based on mesh, identification of meshes and writing mesh equations or you can write node equations. You have written mesh and node equations in the previous course also. Here, we shall be a little more general, those initially in includes initial conditions and see how things differ. Basically, this will also be a little bit of review of what you have done earlier, but let me find out some of the basic foundations of mesh analysis and node analysis.

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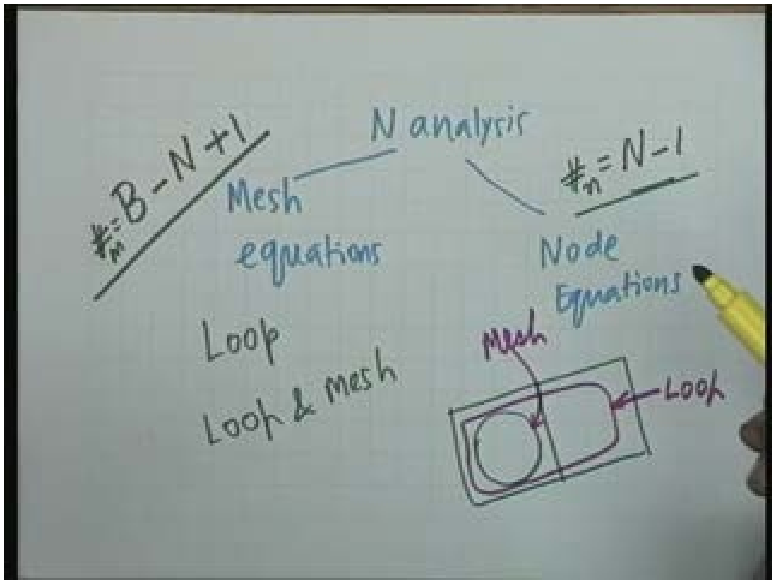

There is also a term loop equations, there is also a term loop, instead of mesh sometimes used, and you must be able to distinguish between a loop and a mesh. A mesh is the smallest size closed path which does not contain any other closed path inside this. A mesh is the smallest size closed path which does not contain any other closed path inside it. For example, if we have a network like this, I have simply drawn in terms of lines, each line, each part of the line may represent an element. This may be a resistance, this may be a source and so on and so forth. This is called a graph of the network where every branch is replaced by a line.

Now you see, if this is the network graph, then this qualifies for a mesh, but this does not qualify for a mesh. The colored one, the pink colored one is not a mesh, it is a loop. The pink colored one is a closed path. It contains another closed path, the green colored one, inside this. So this is not a mesh, this is a loop. On the other hand, this is a mesh. A mesh is a loop, but a loop is not necessarily a mesh. Is that okay? Both are closed paths, both are circuits. One can contain another circuit inside, the other cannot and therefore, all meshes are loops but all loops are not meshes.

I am sorry, my coat is thick, okay. All loops are not necessarily meshes, but all meshes are loops. Now, the number of equations that has to be written, that is, a very important factor and the number of mesh equations, the number of mesh equations is given by the number of branches B, the number of mesh equations is given by the number of branches B minus the number of nodes N plus 1. This is the number of mesh equations that have to be written. This is the set of, this is a set of independent mesh equations that have to be written in order to be able to solve the circuit.

On the other hand, in node equations, the number of node equations is simply equal to the number of nodes minus 1 that is it. Whereas the number of independent mesh equations involves the number of branches, the number of independent node equations does not involve number of branches. And in a given situation, for a given network, one of them will be advantageous as compared to the other. Naturally, if you can do with writing only 1 equation, why should you write 2? If the number of mesh equations required is 2, and the number of node equations required is 1, naturally, you shall prefer node basis analysis and this is what determines in a given situation, whether, you shall be using mesh analysis or node analysis.

There are many situations since the number is the same, then it depends on your personal prejudice, personal preference. There is nothing to choose between and this we shall illustrate by 2 examples, one of mesh analysis and the other of node analysis. The example that I take is 5 point 1 from the text book Kuo.

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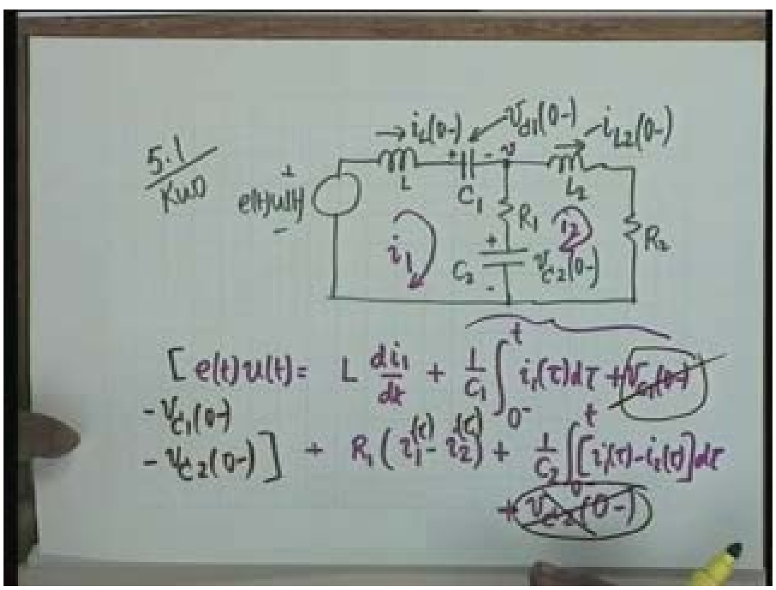

And the network is given like this; there is a voltage source e t into u t plus minus, then there is an inductor L which carries an initial current i L 0 minus. As you know, the initial conditions are now to be, instead of 0, it will be 0 minus, because we have to distinguish between 0 minus and 0 plus because of the possibility of occurrence of an impulse function or its derivative. The next element is a capacitor C 1 and the voltage across this is denoted by this polarity and the initial voltage is V C 1 0 minus, this is the initial voltage. Then a resistance R 1, a capacitance C 2 with this polarity of voltage and the initial voltage across this is V C 2 0 minus and then we have another inductor L 2 in which, the initial current is i L 2 of 0 minus and a resistance R sub 2. This is the network. This is a 2 mesh network. 2 meshes are required. 2 mesh equations shall be required and you can, you can identify, you can say the first mesh equation is i 1, we introduce a circulating current, hypothetical circulating current and the second mesh equation contains the current i 2, circulatory current i 2. How many loops can you form in this?

Students: 3.

Sir: 3, all right, 2 meshes and 1 big loop, 2 smaller loops and 1 big loop. Now if you discuss, if you describe this in terms of a graph, network graph, it depends on what nodes do you identify. Obviously, this network requires 2 mesh equations, 2 independent mesh equations. There is one more point that I want to mention about mesh equations is that, if you have written a set of mesh equations, then each element of the network must be at least in one mesh otherwise, you have not written it correctly. Is that clear? Each element of the network must, from part of at least one mesh, if not more.

For example, these two elements R 1 and C 2, they are in 2 meshes but L and C one are in mesh number one. Now you can also see that if you want to write node equations for this circuit, the only node equation that you can, you have to write only one. You see, this is the only voltage that is unknown V. If you can solve for this voltage, then you know the currents in all the branches, you know the voltages across each element and therefore, a node equation is to be preferred in this situation.

But let us write the mesh equations; let us see how to write the mesh equations. A certain discipline has to be followed in writing mesh equations and node equations. The discipline that I follow, you may or may not like it, you may follow your own discipline, but the discipline that I follow is, I identify the source in the mesh, the source, obviously, the only source is e t u t. All others, there are three other sources in this. One is for the inductor initial current, one is for the capacitor initial voltage and the third for the second capacitor initial voltage. These three as you know act as sources. The inductor current acts as a current source, the capacitor initial voltage acts as a voltage source. Similarly, here also there is a voltage source, but we shall take care of them while writing the equations.

We first find the driving energy source. The driving energy source e t u t, so we write e t u t is equal to, then you write the drops across the elements. The first one, the inductor, obviously, the drop is L d i 1 d t, the current across this is, current through this is, i 1 the initial current i L 0 minus does not specifically enter into this equation because it is a different cell coefficient, plus the voltage drop across C 1 shall be due to the current i 1 and due to the initial voltage, so it would be one by C 1 integral 0 minus to t, i 1 tau d tau plus V C 1 0 minus. This is the voltage drop across the capacitor C 1.

In writing this equation, you must be cautious, you must be careful about the polarities, that is, the flow of current i 1. Does it produce a voltage which is in agreement with a polarity of the initial voltage? If they are, then the sign is plus, if they are not, then you have to make one of them a negative sign. Then plus the voltage drop across R 1 shall be R 1 times i 1 minus i 2 because that is the current in this. The current is i 1 here, and the second mesh current goes up, so it is i 1 minus i 2 plus the drop across C 2 which for similar distances 1 by C 2 0 minus to t. Now the current that goes into C 2 is, for this polarity, is i 1 minus i 2 and therefore, it would be i 1 tau minus i 2 tau d tau plus, plus or minus? i 1 minus i 2 agrees with V C 2, so it would be V C 2 0 minus. This is the equation for mesh number one.

Now for solving such an equation, we shall write the second equation later, but for solving such an equation, the terms which are sources V C 1 0 is like it is an independent of time, it is a constant, it is, it acts as a source you transfer it to the left that is minus V C 1 0 minus. Then you cancel this and the term V C 2 0 minus, you also take it to the left hand side, so that on the right hand side remain only differential coefficients or integrals. All initial condition terms are transferred to the left and the effective source for this mesh would be the combination of these three terms, that is, the driving force, the driving source energy source and the two sources due to initial conditions. Yes?

Student: (…)

Sir: because this is a function of time. These are dependent variables, independent being tau ,we do not know them, so we keep all the unknown quantities on the right hand side, all known quantities on the left hand side. This is another way of looking at it because we have to, what we have to do is to solve for i 1 and i 2. Unfortunately, this is not an equation containing i 1 only. It also contains i 2 and therefore, we must write the second equation to be able to solve for the network and the second equation would be like this. There is no, there are no energy sources in this.

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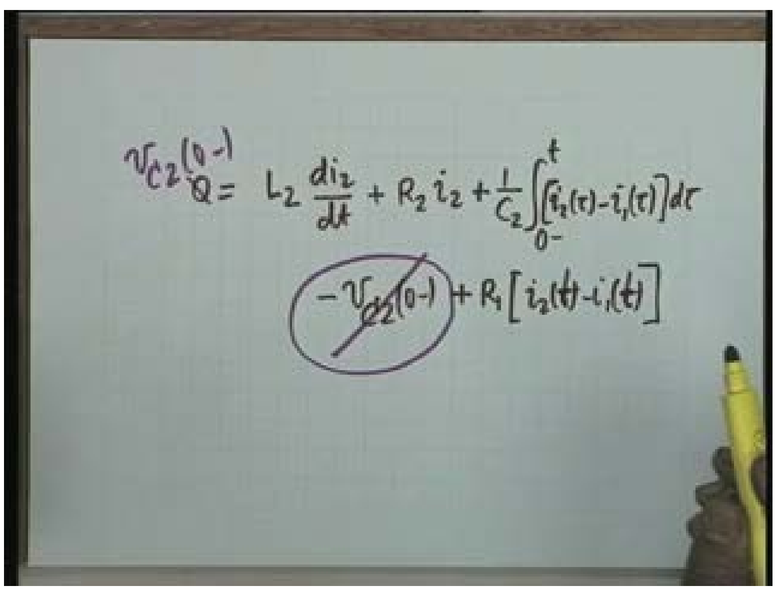

So I write simply 0 and I follow the direction of I 2. Let us start from here, L 2, then it becomes L 2 d i 2 d t, the initial condition does not matter, plus R 2 multiplied by i 2, this is the only current that flows in R 2. Then comes the question of C 2 is C two. If you write 1 plus 1 by C 2 integral 0 minus to t, i 2 t minus i 1 tau d tau, then the initial condition term, now, should come with a negative sign. So minus V C 2 0 minus, then plus R 1 multiplied by i 2 tau minus i 1 tau.

This is the set of mesh equations and you see, here also, you can transfer the time independent term or the initial condition term to the left hand side. Then you get V C 2 0 minus, this is a discipline that I follow and I find it very convenient, that is, on the right hand side you keep only the unknown quantities expressed in terms of differential coefficients or linear terms. i 2 is a linear term or integral terms, but the common link between them is that each quantity is time dependent. On the left hand side are the known quantities, known quantities can be time dependent or time independent, it does not matter. Time dependent example, is the driving force e t u t and what you have to do, is to solve for these two equations. You can solve either in the time domain or in the frequency domain. If it in the frequency domain, you have to take the Laplace transform of the two equations, then you know the differential equations or integral differential equations simply become algebraic equations and you can solve for them. We shall not, we shall not continue

Student: (…)

Sir: Okay, why was it i 2 minus i 1? Because we are going in this.

Student: (…)

Sir: Oh! Why not t? Is that the question?

Student: (…)

Sir: Oh! I have to write t, thank you very much. It would be i 2 t minus i 1 t. I made a mistake. tau is a dummy variable in the integral, it cannot be, it is, it is a function of time, thank you, we don't go ahead with the solution of this because we are illustrating only the writing of mesh equations and node equations. Now I must also point out, there is nothing secret about going clockwise you could also go anticlockwise.

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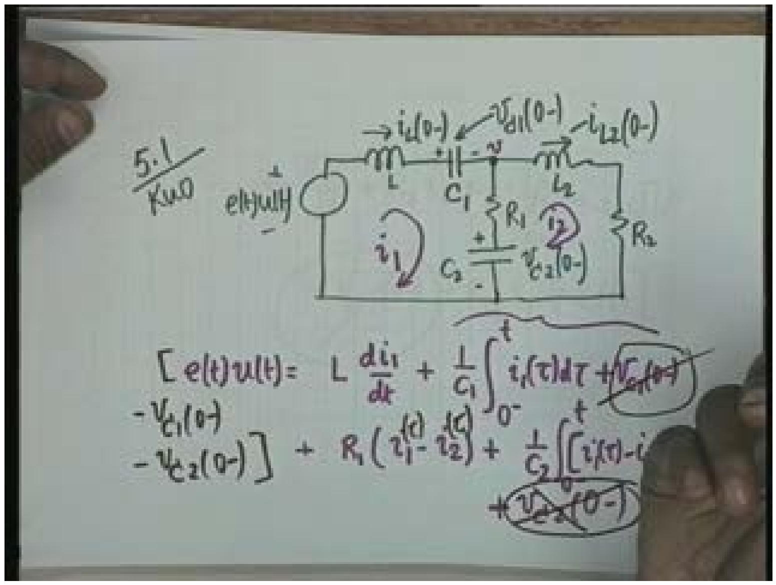

For example, in the choice of the mesh equations, we could go like this, we could have identified i 1 in the other direction, well, this is convenient because e t u t tries to send the current in this direction, in the direction of i 1, that is why we chose it. It is a, purely the matter of convenience, as long as you are careful about your credits and debits, positive and negative sign, you are perfectly stable with regard to your economic conditions. If you know how much, what is your positive side and what is your negative side. That is all that you have to know to write mesh equations. So you know you have to have Ohm’s law and KVL and a certain amount of discipline and a certain amount of caution with regard to signs of the initial conditions. Let us take an example of a node equation.

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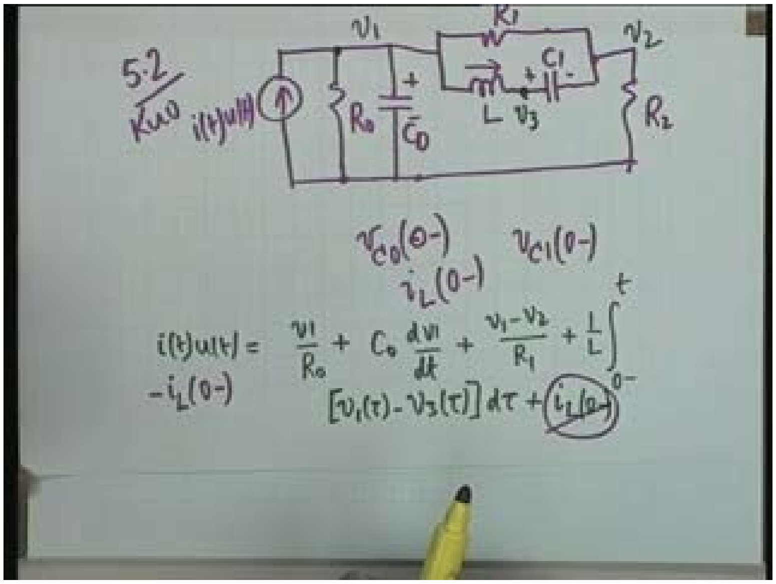

And this example is 5 point 2 in Kuo and this example is like this: we have a current source i t u t a unit step current, current source i t u t, then you have a resistance R 0 which possibly could be the internal resistance of the current source. It could be a non ideal source, R 0 could represent the internal resistance of the current source. Then you have a capacitor C 1 C 0 whose initial voltage is V C 0 0 minus, initial voltage is V C 0 0 minus with this polarity plus minus, then you have, to complicate matters, a resistance R 1 and an inductance L whose initial current is i L 0 minus. Then a capacitor C 1 whose initial voltage is V C1 0 minus, the polarities must be given, which polarity it was initially charged to? And then these two come together and we have a resistance R 2.

We have to write node equations for this. Obviously, the nodes whose voltages are not known are this node, call this V 1, this node, call this V 2 and in addition you require a node here, let us say V 3. These are the 3 unknown voltages that have to be determined, in order that you can determine the currents and voltages in all parts of the network, all parts of the network and you proceed in the same discipline, disciplinary manner as we have done earlier, that is, we write node equations for node 1, node 2 and node 3 by identifying the driving source. For node 1, the driving source is it u t so you write it u t is equal to

Student: Professor, what is the requirement of V 3?

Sir: Because there are 2 elements with 2 initial conditions. Unless you know this voltage you cannot write the equation for C 1, you cannot write the equation for L. You see, the current, so this would be L d V 1 minus V 3 d t. That is why we have to take the node V 3. If initial conditions are absent, if there is no initial condition, then you could simply take Laplace transforms circuit and then this could be treated as one branch. This is because of initial conditions that we have to identify the third node here.

So we are saying, the current arriving at the node 1 which is driving force i t u t, that must be equal to the current living R 0 which is V 1 by R 0 plus the current living C 0, this must be C 0 d v 1 d t. The initial condition does not show explicitly here, because it is a differential coefficient here, plus the current that lives R 1 which is, obviously, V 1 minus V 2 divided by R 1. Then the current that lives L, this is 1 over L integral 0 minus to t.

The voltage, that is, V 1 tau minus v 3 tau d tau, this is the current and then the initial current plus, is it plus or minus? Plus because it agrees with the current living so it would be i L 0 minus. This is the node equation for node number 1 and you can obey the same discipline, namely, we can transfer this term, the initial condition term to the left hand side and you can write minus i L of 0 minus. Then on the left hand side are only known quantities, on the right hand side only unknown quantities. Let us go ahead with the completion of this example. We next write the node equation for V 3. For V 3 is a there are only two currents to be taken care of, that is, the current through C 1 and the current through L.

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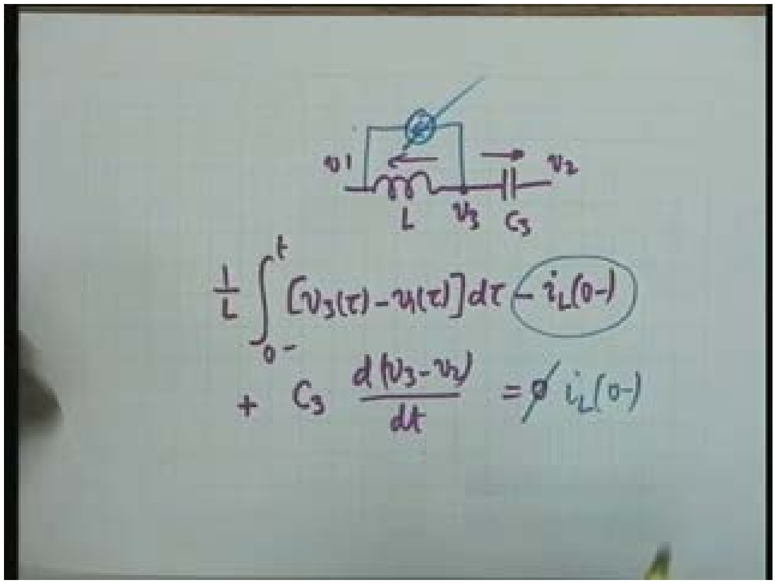

So, what we have is this situation L and C 3, this node voltage is V 2, this node voltage is V1 and this is V 3. So the node equation would be: 1 by L, the current going from V 3 to V 1, which way should we do? What I am going to do is that this, there is no driving source and therefore, this current, plus this current must be equal to 0. This is what I am going to write, so 1 by L integral 0 minus to t. Now notice the polarity V 3 tau minus V 1 tau. This is the current living V 3 d tau then minus i L of 0 minus plus, plus C 3, the capacitor then d V 3 minus V 2 d t, this should be equal to 0. This is the equation for node 3, V 3, is this clear, the polarity? Now, what you can do is, you can transfer this quantity to the right hand side and then you have the same discipline, that is, one side of the equation contains unknowns, the other side contains knowns.

Student: Sir, does not the initial flow of (…) current in the capacitor also?

Sir: That was at 0 minus. At 0 minus, the inductor might have had another path which was opened at t equal to 0. It is possible, it is possible to establish a current in the inductor. Say at t equal to 0 minus, perhaps, there was a current source here which has established a current i L 0 minus and then a t equal to 0. This is taken off so it is possible. Now the last equation, the last node, that is, V 2 node, this node has the following components.

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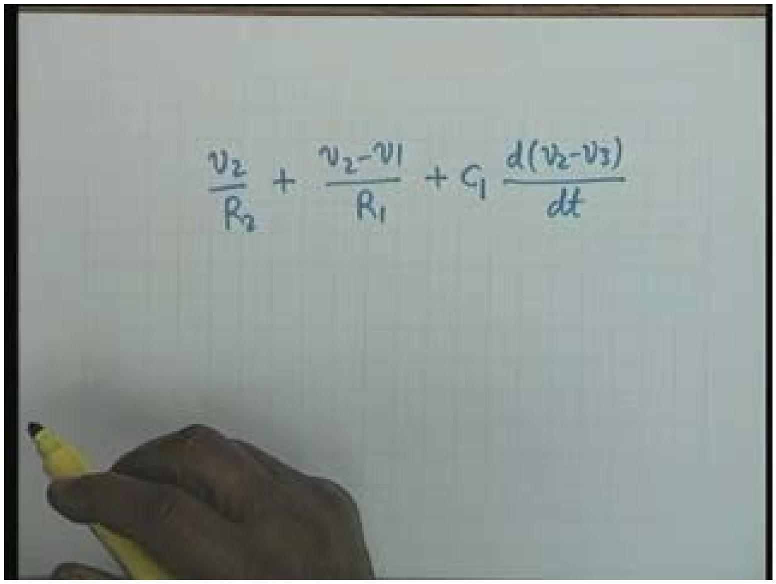

You see, V 2 by R 2 is the current leaving through R 2 plus V 2 minus. V 1 divided by R 1 is the current leaving through R 1 and plus C 1 d V 2 minus V 3 d t. These are the three currents. The current through R 2 is V 2 by R 2, the current through R 1 is V 2 minus V 1 by R 1 and the current through C 1, that will be C 1 d d t of V 2 minus V 3. So the sum of these three currents equal to 0 and there are no known quantities on this side and therefore, there is nothing to transfer. Also note that the initial condition on the capacitor does not arise anywhere and the reason is that the capacitor, we are writing equations in terms of currents and currents are differential coefficients of voltages as far as capacitors are concerned. So that takes care of an example of a node equation and we used an example from the text book Kuo.

Student: Excuse me sir, on this circuit, can we write mesh equations?

Sir: Of course we can write mesh equations.

Student: Sir, but will not it be the case that if we are taking the second part of the circuit, between C 0 and R2, take this in a particular mesh, then it would be, there would be one small circuit in that mesh.

Sir: No. What we will do is, if we want to do a mesh equation, what we will do is,

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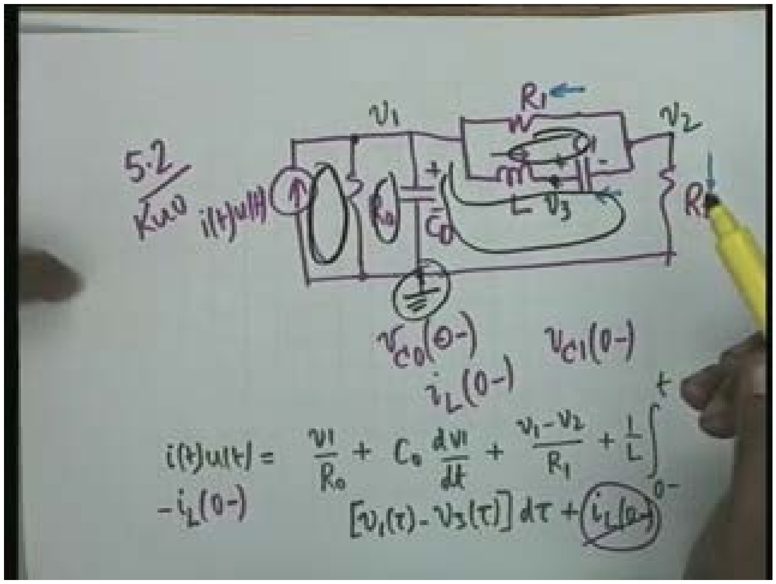

This is one mesh, let me use another colour, this will be, this will do. This is one mesh, this is one mesh, this is one mesh and this is one mesh. 4 mesh equations. We are doing, however, things are not that bad because this current i t u t is known. So the first mesh equation is known, second mesh equation is unknown, third is unknown, fourth is unknown. So you have to solve for 3 mesh currents and since only 2 node equations known. How many node equations did we write?

Students: 3.

Sir: 3, so there is nothing much to choose, they are identical. In the previous example

Student: Can you have a mesh passing through R 1?

Sir: Can you have mesh passing through R 1.

Student: (…)

Sir: No, we cannot, because then this will be inside that is not a mesh. This is C 0, R 1, R 2, this is a loop

Student: We can interchange the position of R1, L and C.

Sir: Agreed, that means you change, you interchange R 1 in because they are in parallel. Yes, we can do that but as drawn here, it is a plainer structure, depends on the way you look at it. If you interchange the two, yes.

Student: (…)

Sir: Pardon me, if you interchange these two, then through R 1 it would be a mesh.

Student: Otherwise it would be a loop.

Sir: Otherwise it would be a loop. Yeah, as we have written, yes.

Student: Sir, you said that there will be 2 known quantities, in case of mesh 1, there can be 1quantity. Sir, we can change R1 and C0 and we can get one more known quantity.

Sir: We can. I agreed, no. See, the choice of meshes is unique. Once you have chosen this, no question of interchange. No interchange, no. Once you have chosen the structure and the meshes, period. you cannot be in two minds, no. It might work in life but once, not in circuit theory.

Student: Sir, do we have the fourth node?

Sir: Where do you have the fourth node? Fourth node is here, the ground node which is taken as 0 potential. You see, whenever you say this voltage is V 1, we mean that we have a reference and this is why the number of independent node equations is equal to number of nodes minus 1. That minus 1 comes because of the reference

Student: What is the requirement of the second mesh? As we have already included the two components

Sir: What is the requirement of the second, this mesh. But is not the other way around. You see, every component must be in at least one mesh. It is not necessary that only one mesh should contain one component, no. the other way is not true, so a component may occur in 2, 3, 4 meshes.

Student: Then sir, 3 mesh will not solve the equation?

Sir: 3 meshes should solve because one of them is known. This i t u t is known and therefore, 3 independent equations. Agreed, that happens because this source is known not otherwise

Student: Sir, what is meant by branches in that equation you have written n minus 3 plus 1?

Sir: Oh, branches means, any 2 terminal element is a branch one. This is a branch, this is a branch, this is a branch, this consists of 2 branches connected together here.

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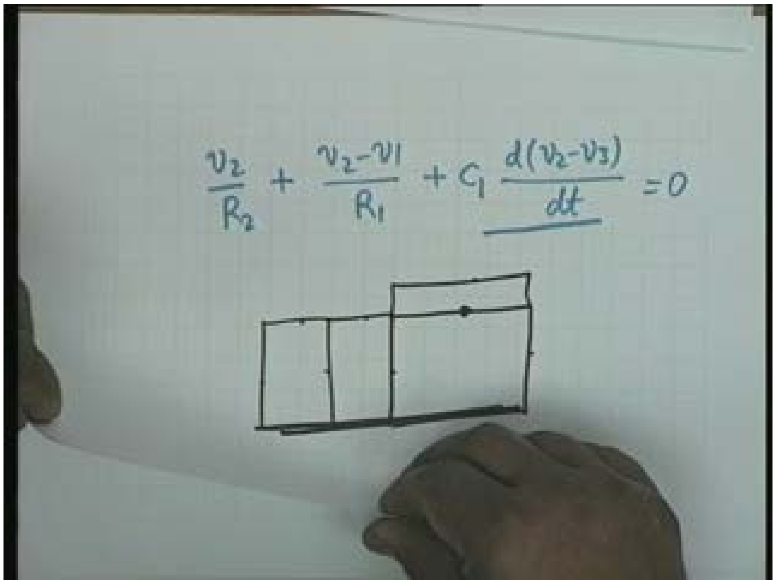

So, a branch is if we draw the skeleton of the network, for example, the skeleton of five point two would be like this. This is the graph of the network and the number of branch is 1, 2,3,4,5,6,7,8. This is not a branch because this is 0 potential, this is a short circuit.

Student: (…)

Sir: One at a time, yeah, two parallel components.

Student: Second one, they have 2 components.

Sir: You can consider this as one component is that okay

Student: I am asking that in that parallel connection, in the 2 part we had 2 components, so we will consider that as 2 branches or 1 branch?

Sir: It does not matter what you consider, depending on that, you have to write the required number of node equations. As you see, this series connection could be considered as one component but because of inductor and capacitor and because of initial conditions, we cannot take care of the initial condition by considering as one component and so therefore we consider this as 2 branches because this voltage is to be determined.

Student: So we have to add one more node also?

Sir: Yeah, that will be obvious in the context. For example, if we have two resistances in series then we would not have put a branch here

Student: Sir, but in the skeleton diagram you just made one branch.

I just made one branch; I made a mistake, two branches. You should know how to take care of teacher's mistakes and it is mutual, now it will be all right. Now once you have been able to write the equations, once you have been able to write the equations, the next task is to solve them.

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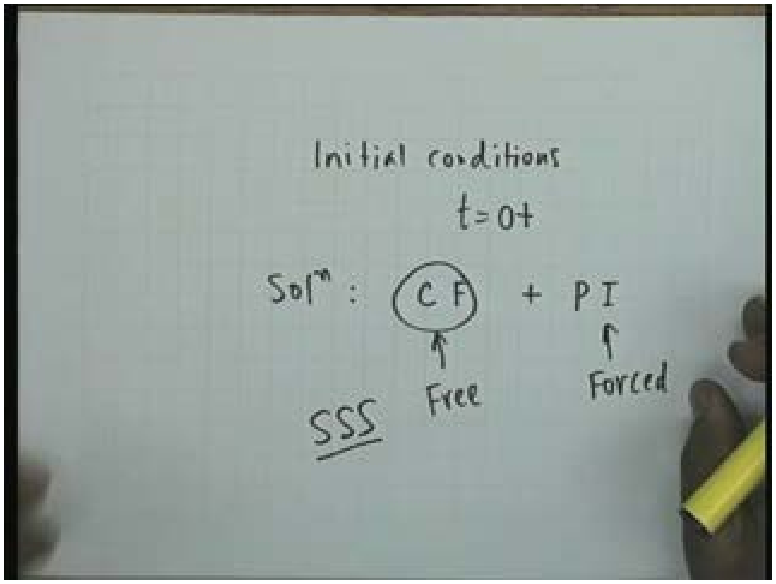

Now solution of an equation, differential equation, requires initial conditions and as I told you initial conditions are conditions at t equal to 0 plus, not at 0 minus. For solving a set of differential equations, the initial conditions have to be evaluated at t equal to 0 plus and this can be done in two ways, either from physical considerations or from the differential equation itself and we shall look at, how to find out initial conditions through a couple of examples again. But let me tell you that the solution is that of a voltage or a current always contains two parts; one is the complementary function C F and the other is the particular integral P I.

The complementary function, as you know, is called the free response or natural response. You were acquainted to these terms free and forced. It is either free response or natural response and the particular integral is the forced response, free and forced response. In addition, another term which is very often used is that, as time proceeds from 0 plus as t goes to infinity, the complementary function usually dies out.

The complementary function of the natural response which is the response due to the initial conditions of the network, usually die down exponentially and then we have left to it a steady state solution SSS. Steady state solution which is usually the particular integral but not necessarily so, the steady state solution is not necessarily the particular integral although this is what happens in most of the situations. Now let us look at the initial condition problem through a couple of examples.

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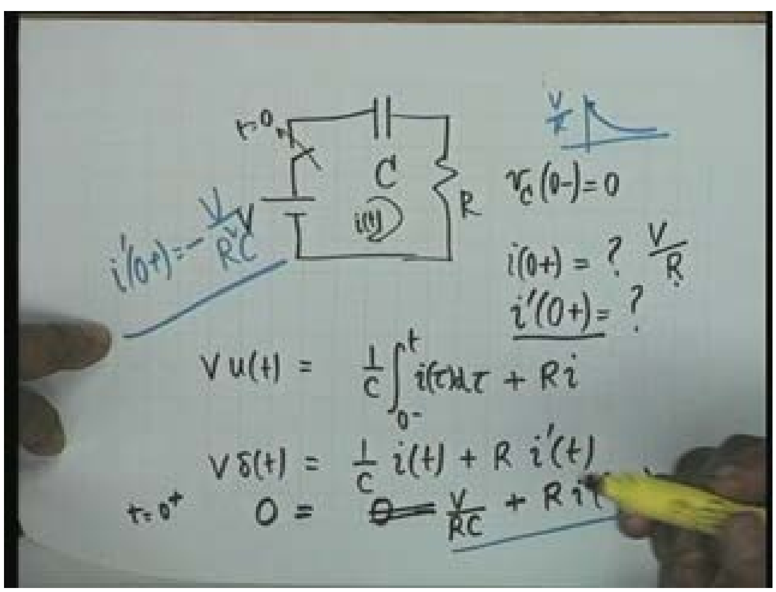

The first example that we take is that of a voltage source V, which is switched on at t equal to 0 to a capacitance resistance network C R, and it is given that V C 0 minus is equal to 0. We are required to find out if this is the current I, after the switch is on, that is, V u t is switched on in this circuit. What we have to find out is i of 0 plus, which is initial condition in addition one is required to find out, let us say, the statement of the problem is find i of 0 plus and i prime of 0 plus. Then there are two ways one can do it. One is from physical conditions, physically we argue that the capacitor voltage at t equal to 0 minus was 0 and the current does not have an impulse component and therefore, V c 0 plus must also be 0.

That is, a t equal to 0 plus, the capacitor acts as a short circuit, because it does not drop any voltage and therefore, i of 0 plus must be equal to V by R. i of 0 plus, at 0 plus V, C is a short circuit. Say a resistance R is connected and therefore, it must be V by R. This is from the physical condition. From the physical conditions however, we cannot find out i prime 0 plus. We have to write the differential equation. The differential equation is V u t equals to, now you write a single loop or mesh or whatever you call it, I write 1 by C integral 0 minus to t i tau d tau plus R i. This is my simple single loop equation. Now from here, from the differential equation, we could find out i of 0 plus, because 0 minus to 0 plus, this integral shall be equal to 0, i of t does not have an impulse component and therefore, i of 0 plus shall be equal to V divided by R. That is all and 0 plus, u t is equal to 1.

So we could have found this out from the physical, from the differential equation also. To find out i prime of 0 plus, that is the more interesting exercise. What we do is, we differentiate the equation. Then I get, this is equal to one by C, this would be simply i t plus R i prime t prime stands for differentiation with respect to time and in this equation, if I put t equal to 0 plus, what does the left hand side become? 0 at 0 plus delta t is 0. This becomes 0 because i of 0 plus is 0, so this is also 0. Now,

Student: Sir, 0 plus should be V by R.

Sir: V by R, so it should be V by R C, plus R i prime 0 plus, and therefore; i prime 0 plus becomes equal to minus V divided by R square C. This is the equation. Now, this was not needed for solving the equation. All you needed was i of 0 plus, but this is a matter of curiosity. What is this slope? The initial current is V by R but how does the current change? You see, you know that the current in the circuit starts from V by R and then goes down exponentially. This is why the slope is negative. The slope at the initial point at t equal to 0 is negative because it decreases and the value is V by R square by C.

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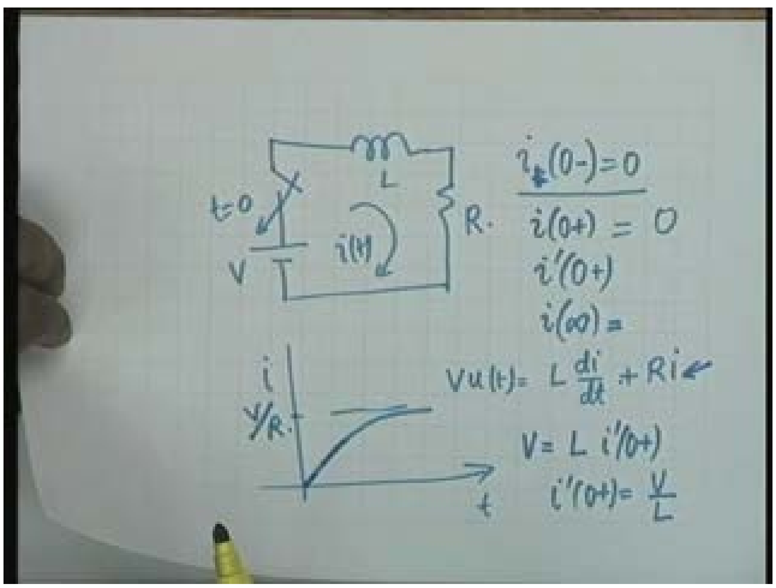

The next equation, next problem that we take is a similar problem but contains a capacitor. The same voltage source V i t contains an inductor L in series with resistance R, and it is given that i L of 0 minus. If this is i of t then i of 0 minus is equal to 0, that is, the inductor is initially uncharged and here also, you have to find out i of 0 plus let us say, i prime of 0 plus and in addition, let us say i of infinity, that is the steady state part. I forgot to mention, what would be the steady state solution here? 0, it is obvious, the current drops exponentially.

Now here my equation is, I can argue again from physical condition physical point of view, from physical considerations i of 0 minus is 0. The voltage source does not have an impulse therefore, i of 0 plus must also be 0. In other words, at t equal to 0 plus, the inductor acts as an open circuit, so the initial condition of the current is 0. Therefore, the current must start from here, current verses time, it must start from here. Then if I want to find out i prime of 0, this is 0 i prime of 0 plus, how does it rise?

Now can it fall? It may, if v polarity is negative. Is not that right? You must be very careful about polarities. If I see, if I write i in this direction and I reverse the polarity of v, then obviously, slope can be negative. But anyway, considering this situation, you have v times u of t is equal to L d i d t plus R I, this is the equation. This is a differential equation. There is no integration in this and you notice that if you put 0 plus here, then you simply get, if you put t equal to 0 plus you simply get L, V equal to L i prime 0 plus plus R, i of 0 plus which is 0 and therefore i prime of 0 plus would be equal to V by L. It is a positive quantity, it must rise like this and then i of infinity can again be found out from physical considerations or, or from the differential equation; say as time proceeds, i have infinity, the inductor acts as a short circuit and therefore, the current must rise to V by R. You can also look at it from the differential equation. At t equal to infinity d i d t i stabilizes. So d i d t is 0 and therefore, i must be equal to V by R. So the current rises like this, this is V by R. The point is, we have found out initial conditions from either physical considerations or from the differential equation. Yes?

Student: Can we find i of 0 plus from the equation?

Sir: From the equation? Can we find i of 0 plus from the equation? Yes, can someone answer this question? Can we find i of 0 plus from the equation here?

Student: Yes sir.

Sir: Tell me how at 0 plus, this is V okay this is R i 0 plus, what is d i d t 0 plus? We don’t know.

Student: (…)

Sir: That required i 0 plus. Well it is possible, but let us defer the discussion for a moment

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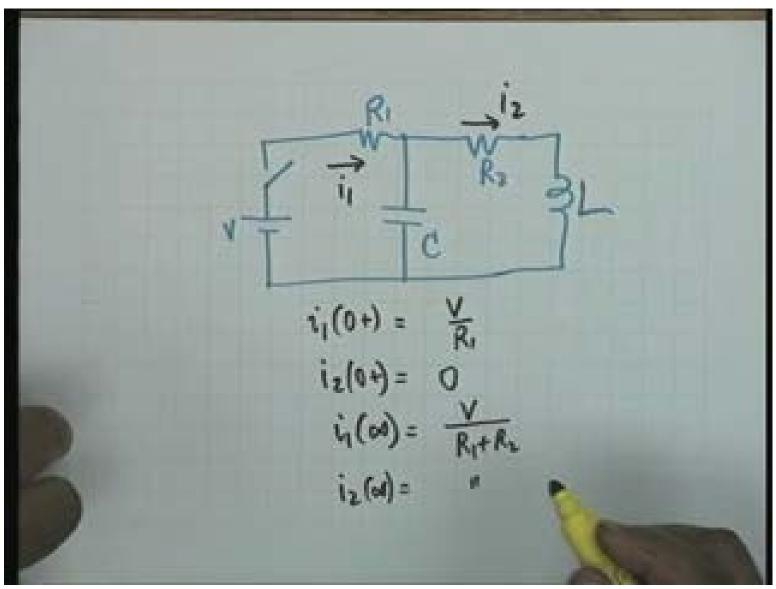

For an engineer, anything that works is a good solution. Either from physical consideration you find out which is the most convenient, well differential equation, it is convenient. It does not give directly. We have to go through some other means. Why go through some other means, if you can find out from physical conditions. We take a third example. In the third example, we have a capacitor as well as an inductor. We have a voltage source V, a resistance R 1, a capacitor C, a resistance R 2 and an inductor L and the currents are identified, not loop currents but branch currents. Let us say i 1 and i 2, obviously, this current would be i 1 minus i 2, the current through the capacitor.

Now we do not have to do anything else. We can simply appeal to the physical conditions and find out initial conditions. For example, i 1 0 plus, we of course assume that inductor and capacitor initially relaxed, that is, i 2 0 minus is 0 and the V c 0 minus is 0 we assume that. Then i 1 0 plus what would this be equal to? At t equal to 0 plus the capacitor is a short and therefore, i 1 0 plus must be V by R 1. What is i 2 0 plus

Student: 0.

Sir: 0, because if this is a short then no current passes and what is i 1 of infinity? when its, V by R 1 plus R 2. This is because the inductor behaves like a short circuit and the capacitor is open. So and what is i 2 of infinity? It is the same. Wonderful! We did not have to write the differential equation. We can do that if you so desire, but we did not have to write.

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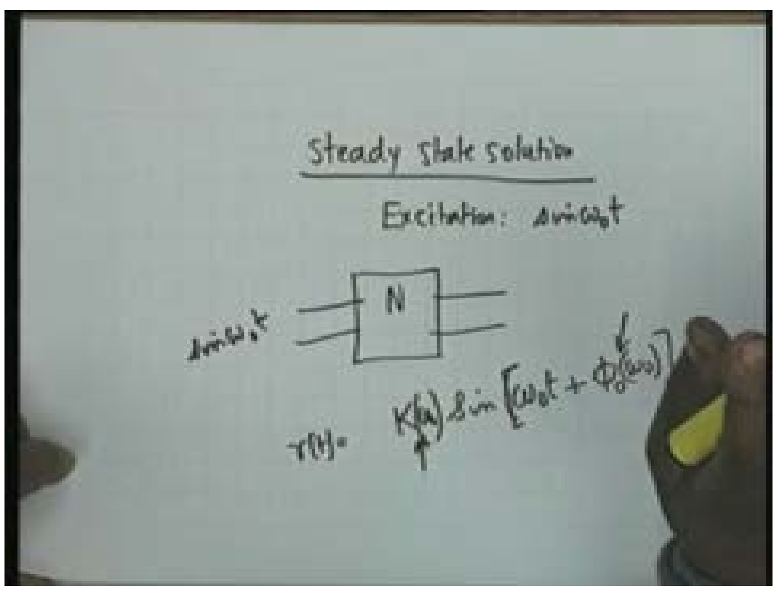

The next question is about steady state solution. We have already found out steady state solution for the previous 3 examples. Now there is one situation in which the finding the steady state solution in practice is an extremely important step and this is when the excitation is sinusoidal. Excitation is either sine of omega 0 t or cosine of omega 0 t. You know there is a preference for sine. In naming it, we could have called it co-sinusoidal, but co-sinusoidal, if we write cosine it is co-sinusoidal. But since it involves another two letters co, we go for an economic solution, sinusoidal. So by sinusoidal you mean sine, as well as cosine.

Now if to a network L L F P P network, the excitation is sinusoidal, let us say, sine omega 0 t. Then you know that the voltages in currents in the networks can be solved by means of phasers. The summing substance of phaser analysis is that if the excitation is sinusoidal, then for a linear network, the response must also be sinusoidal of the same frequency. The only difference would be that the response would be different in amplitude k, which will depend on the frequency omega and the response may not be in phase with excitation. So there may be a face difference V 0 of omega 0. The phase difference and the amplitude both shall depend on the frequency and you shall, yes?

Student: Professor, amplitude could be a product or, you know, it could be divided and may that sort of function or anyway.

Sir: Any function, a simply general function of omega 0. And to illustrate this, to illustrate this, we take a simple example.

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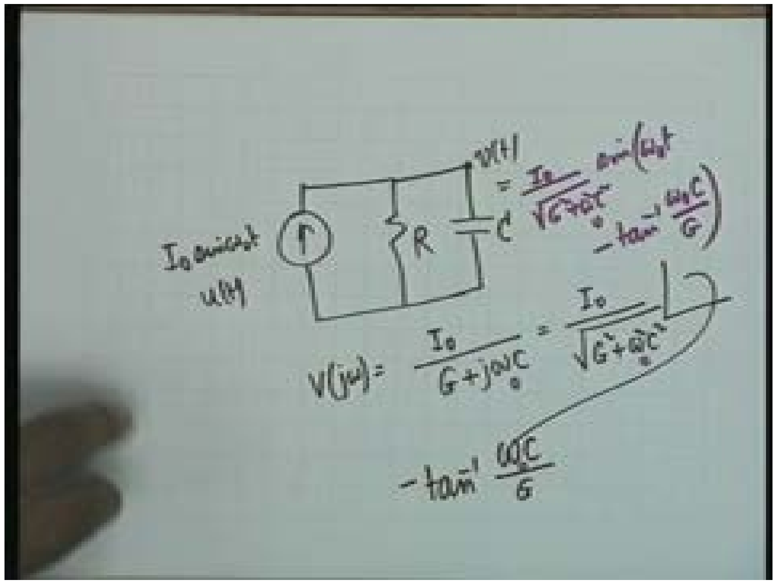

Let us say, we have an i 0 sine of omega 0 t, u t, of course, and a parallel combination of R and C and we are required to find out V t. Then we simply argue that capital V, the phaser, capital V g omega would be simply i 0. The phaser for this is i 0 multiplied by the impedance. The impedance is the reciprocal of the admittance and the admittance is G plus j omega C which i can write as i 0 divided by square root of G square plus omega square C square and the angle is minus the angle of the numerator minus the angle of the denominator. So minus tan inverse imaginary part divided by the real part, do you know this?

Students: Yes sir.

Sir: And therefore, V t, I can write by inspection without solving any differential equation. I can write as i 0 divided by square root of G square plus omega square C square. This is my k, this is my k and sine of omega 0 t minus, because the phaser is minus, tan inverse omega 0 omega 0 C divided by G. That eases our problem if the excitation is sinusoidal. We will meet after one hour.