Circuit Theory
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Lecture - 29
Problem Session 7: Two-port Networks (contd.)

This is lecture 29 and this is our problem solving session number 7. And we solve problems on 2 port parameters as a finale to 2 port parameters. 2 port parameters we have already discussed in theory and we end up this discussion with a few problems of as i said Non routine nature.

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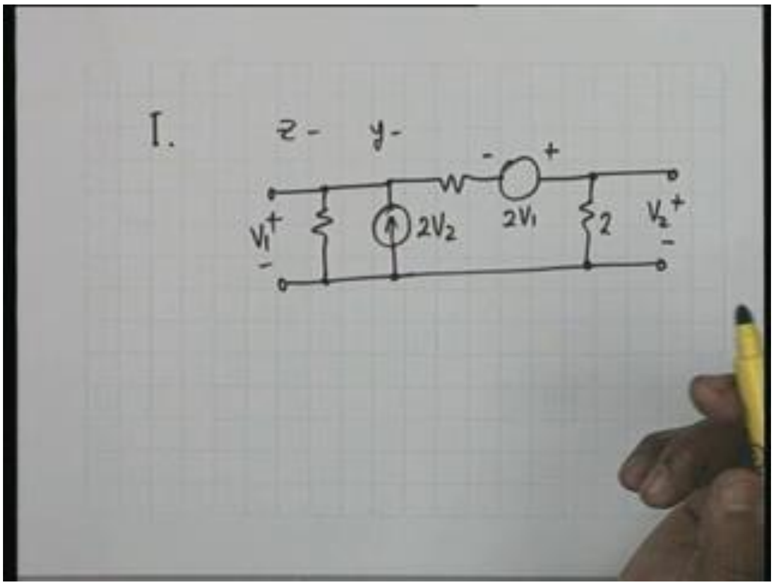

We start with problem number 1 and the problem is to determine the Z and y parameters of the network shown below, which as you see, is not a totally RNC network, it has controlled sources dependent, sources a current generator and a voltage generator with specified polarities. This is 2 ohm and this is V 2 this is V 1.Just to identify the ports. We have to find out the Z and y parameters of this network. 1 thing that, 1 should be careful about the occurrence of control sources is that if it is voltage for example, V1 is killed. For example, if we want find y2 2 then V1 this will be short circuited. If V2 is killed then this will also disappear and therefore, it offers simplification.

Another things that 1 should notice is in a similar manner, if V2 is short circuited if port 2 is short circuited. Then what will happen to this current generator.

It will be opened. V2 equal to 0 the current is 0. Current generator internal impedance is infinity and therefore, this will be opened. The other thing that one should be careful is, whether the network is reciprocal or not because the occurrence of controlled sources is has the potential of making the network Non-reciprocal and therefore, you should not only calculate Z1, 2 and say Z1, 2 equal to Z to 1. As you do with reciprocal networks or networks composed of bilateral elements controlled resources we cannot say, If controlled sources occur it is not necessary that the network is not reciprocal, but if controlled sources occur there is a possibility that the network may be Non-reciprocal. In other words, all the 4 parameters have to be computed not 3.

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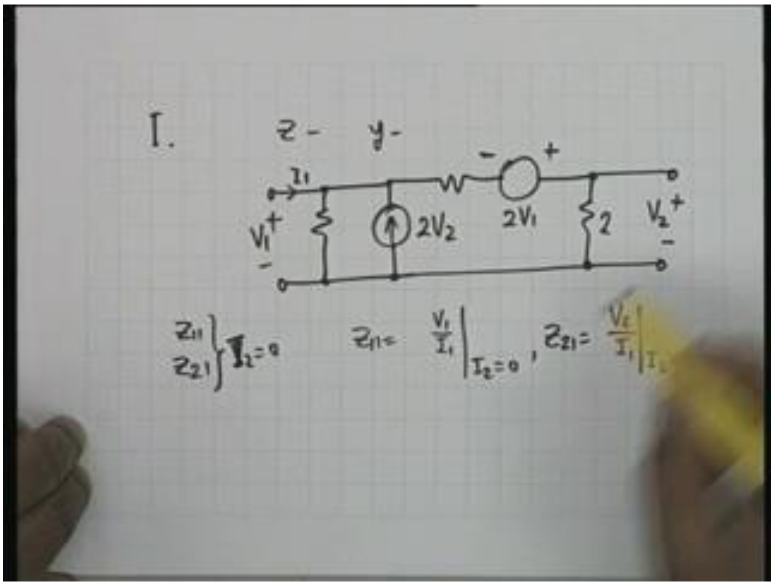

Now, let us see let us calculate Z1 1 and Z2 1 for both of them. The definition says V2 equal to 0 and Z1 1 is equal to V1 by I1.

I 2 is equal to 0

That is correct.

Thank you.

V1 by I1, I2 equal to 0 and Z2 1 is equal to V2 by I1 with I2 equal to 0. So, let us keep I2 equal to 0 and therefore, these terminals are left open and we have to find out this current I1 this current I1 and this voltage V2.Then we will be able find out both these parameters.

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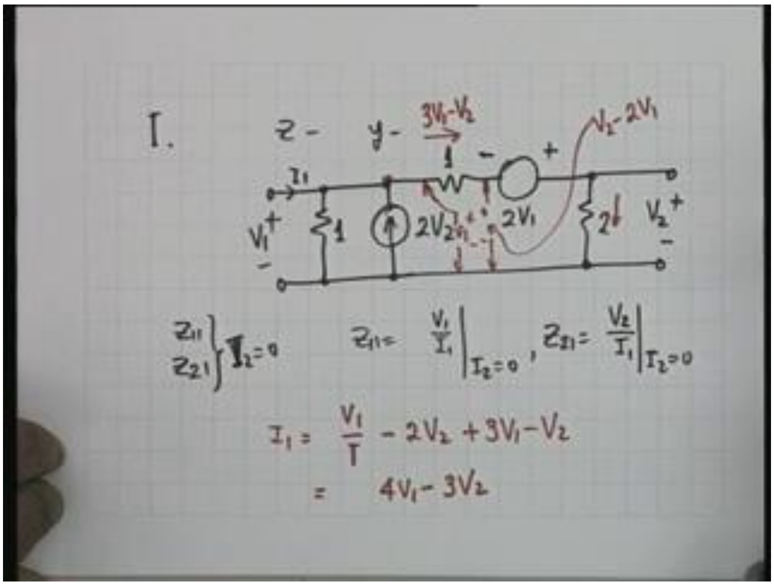

One thing that we notice is that if this voltage now, let me use a different colour. If this voltage is V1 then this voltage is also V1. Now, this resistance is 1 ohm that is given this resistance is 1 ohm and this resistance is also 1 ohm. I did not put the resistance values are here. Let’s go systematically. This voltage is V1 and what is this voltage, this is V2.

V 2 minus 2 V1.

This is V2 then there is drop to V1. So, this voltage is V2 minus 2 V1 with the same polarity plus minus; therefore, the current through 1 ohm this current shall be equal to this voltage minus this voltage divided by 1 agreed. So, V1 minus V2 plus 2 V1. In other words, it is 3V1 minus V2 agreed and that is solves the problem. Once you are able to identify this current, you see my equation shall be I1. If i write KCL at this point I1 I at this point. I one is equal to V1 by 1 that is this current. Current through this resistance then 2 V2 is coming. So, minus 2 V2 minus 2 V2 then 3 V1 minus V2 is going so 3V1 minus V 2.

If you simplify this it is simply 4V1 minus 3V2 agreed. This is equal to I1. We also notice, while this kind of a simple minded approach requires experience, requires practice while you also notice that this current 3 V1 minus V2 must be the same as this current that is V2 by 2 agreed. V2 by 2 it is the same current as this.

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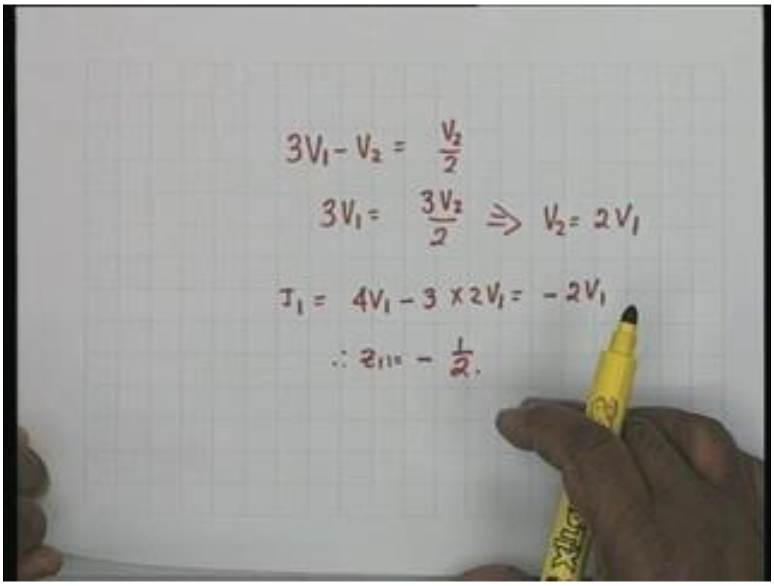

Therefore, the equation of our circuit is 3V1 minus V2 is equal to V2 by 2 or 3V1 equal to 3V2 by 2 which makes V2 equal to twice V1 agreed. V2 equal to twice V1, which means if I combine now, with the first equation I get I1 as equal to 4V1 minus 3 times V2 3 times 2V1 which is equal to minus 2V1. Is it have I made a mistake somewhere?

No.

Therefore, Z1 1 which is the ratio of V1 to I1 is minus half is it. I1 is minus 2V1 and therefore, Z1 1 is minus half.

It comes as negative that is a negative resistance is realized at port number 1. And happens because of controlled sources which mean, that the circuit is active wherever negative resistance occurs instead of dissipating energy it generates energy and therefore, the circuit is an active circuit.

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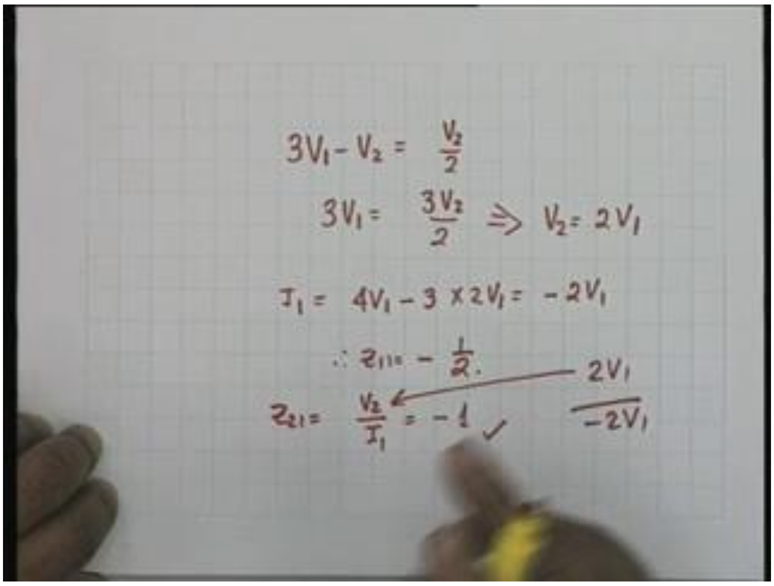

Number 2: I want to find out Z2 1 which is equal to V2 by I1.

No.

It is simply V2 by I1.

Yeah pardon me.

What did you say?

Minus one.

That’s right.

Because how do I find this V2 is 2V1 and I1 is minus 2V1.Everything expressed in terms of V1. So, Z2 1 is equal to minus 1. Now, this does not mean that Z1 2 should also equal be equal to minus one. You have to calculate that, in other words, there is no escape you have to take the network again and put I1 equal to 0 and the network then becomes.

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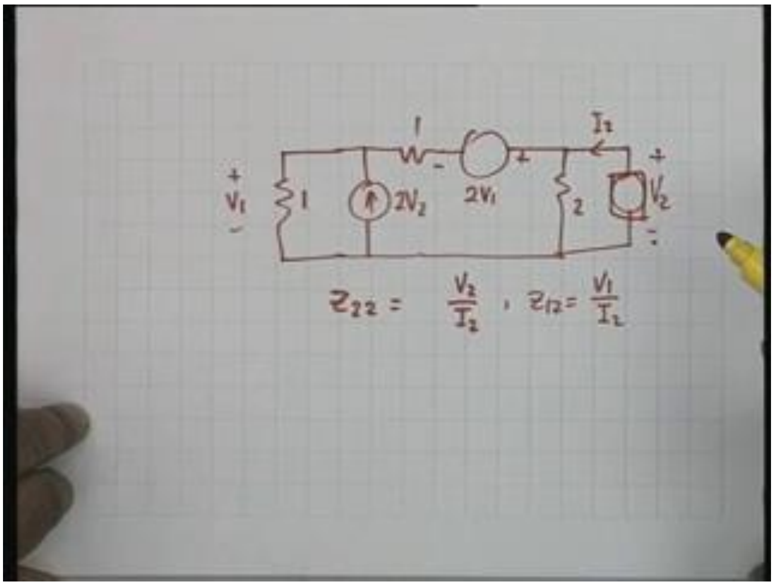

1 this is V1, I1 is 0. No entry of current from port 1, Port one is open circuited then you have a 2V2 and then 1 ohm. The source minus plus 2V1 2 then you have an I2. You have to calculate Z2 2 say you connect the voltage source. V2 and all you have to find out now is, I1 is 0; therefore, Z2 2 is equal to V2 by I2 and Z1, 2 is equal to.

Yeah.

V one divided by I2.

Uh it does not matter.

I am not shown, whether it is a voltage source or current source. It could be current source, it could be voltage source. All I, all that matters is this is V2, this voltage and the current drawn is I2 that is all. I have not drawn a well, i have a drawn a voltage source this is the symbol. If i have drawn a current source I would have shown a arrow, but it does not matter you simply call this as source. If necessary you call it a you indicate it by rectangle to indicate that, we do not know what this source.

We do not care.

Not not that we do not know.

We do not care.

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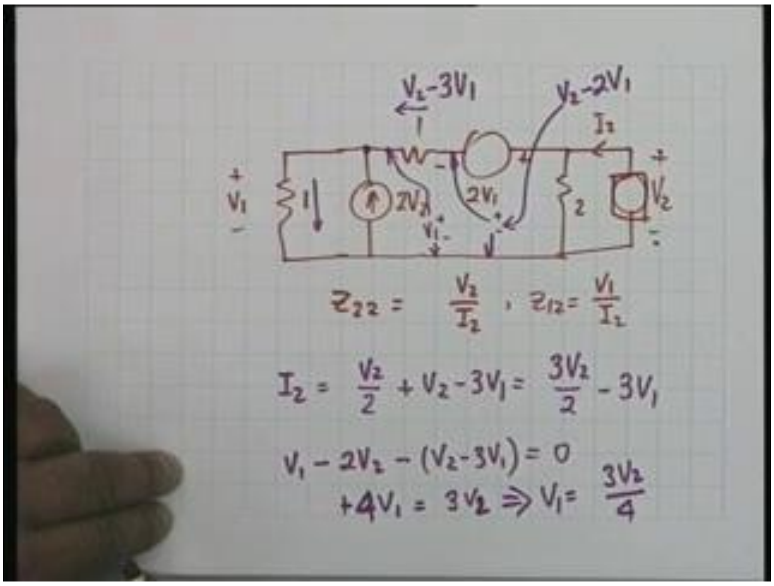

Now, to find I2, let us see what these voltages are. This voltage is V1 and this voltage as we find out earlier is V2 minus 2V1; therefore, this current let us take.

In this direction.

This current would be V2 minus 2V1 minus V1; that means, it will be V2 minus.

3V1 all right and then all that you have to do is to write the node equation at this node I2 would be equal to V2 by 2.

It is this current plus the current that goes via this branch which is V2 minus 3V1 that is; equal to 3V 2 by 2 minus 3V1 agreed. I2 equal to 3V2 by 2 minus 2 3V1 and then you find out the KCL at this node. The current through here, current through this branch is V1 by 1 then minus 2V2 because it goes it goes up and minus this current comes in minus V2 minus 3V1 this must be equal to 0.

Which says that Let us say

Minus 2V1 equal 3V2. So, no I have made a mistake.

This would be plus so 4V1.

4V1 equal to 3 V2.

Therefore, V1 equal to 3V2 divided by 4.

Don't allow me to make mistake.

V1 equal to 3V2 by 4 and therefore if I substitute this.

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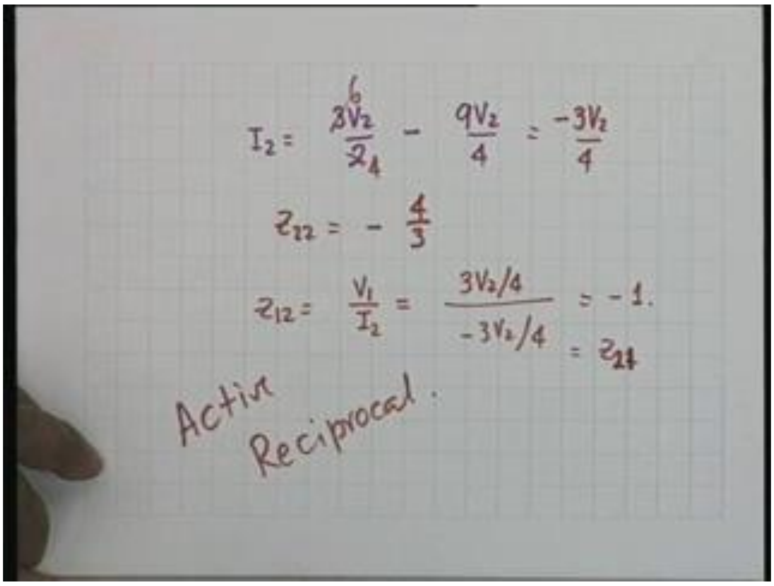

Then I get I 2 equal to 3V2 by 2 minus 3V1, that is 9V2 by 4. And I can write this as 6 V2 by 4; therefore, this is equal to

Minus 3V2 by 4 and therefore, Z2 2 is equal to minus 4 by 3. This is also negative. Finally Z1 2 which is equal to V1.

Yes.

V1 by I2, V1 is 3V2 by 4 and I2 is minus 3V2 by 4. So, this is equal is minus 1.

What was V1 2?

Also minus 1; therefore, even though Z2 1; therefore, even though controlled sources occur the circuit is reciprocal, but not passive. It is active and reciprocal. Is there any other way of working out this problem?

Pardon me.

What equations?

Mesh and loop equation while that we do not want to do.

Till we are first too

Because; that means, writing a set of simultaneous equation solving for the current sensor. If we do not want, if we can do it by inspection why not?

That is

Yes.

What we can do is.

You see after there are only 2 nodes.

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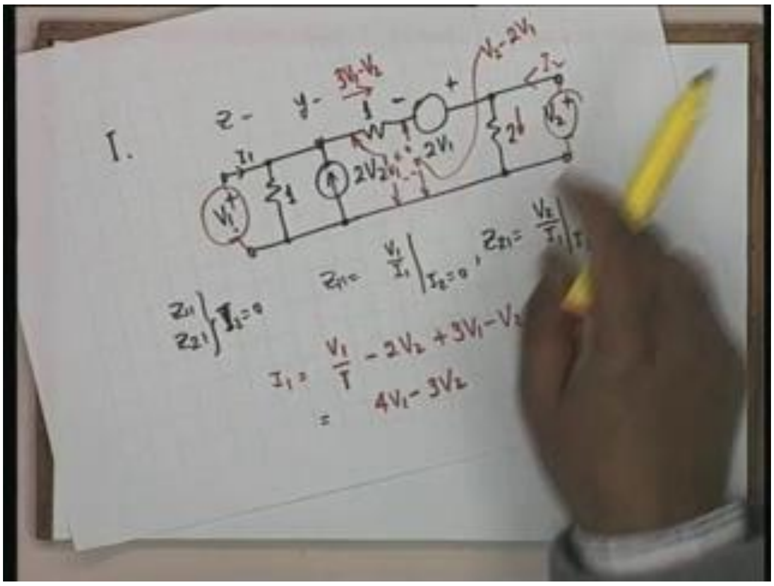

We could connect a voltage source here and write node equations here and here, i could write in terms of, I1 and I2 could be expressed in terms of V1 and V2. Then we get the Y parameters and then we could convert them to Z parameters or well no or that is what we will have to do. Otherwise, we will have to write V1 in terms of I1 and I2. Well that also can be done, you can apply Thevenin’s theorem here, make it into a single loop network. There are many ways that this can be done, but let us do in this simple minded manner going back to the roots. Remember going back to the roots, the chances of making a mistake is reduced considerably. And this problem is not too complicated that we cannot solve it by going back to the roots without any complication.

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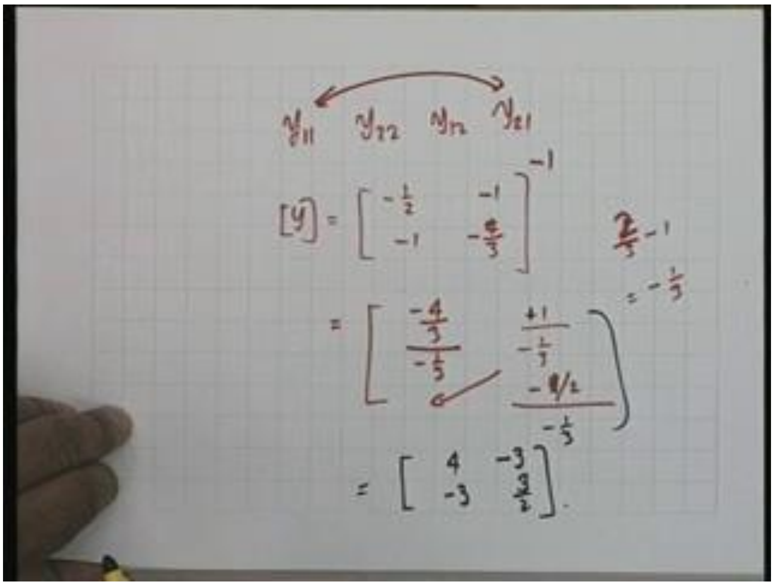

Let’s see y11, the y parameters y11, y22, y12 and y21. To find y11 and y21 this 2.

Pardon me

I can use the transformation also, that is 1 method. Well, the transformation means you will have to find the determinant of the metrics divide the corresponding, it is not too difficult. Let’s do that, then maybe we will go back to the roots and see. You see going back to the roots it becomes much simpler. The y parameters will become much simpler, but suppose we say what was our Z11 minus half and Z12 minus 1 minus 1 then

Minus.

4 by 3

Inverse of this.

So, this is equal to minus 4 by 3, this 1 divided by what is the determinant.

8 by 3 minus 1.

Is it right?

This multiplied by this

2 by 3 minus 1 which is equal to minus one-third agreed. So, this divides by minus one-third then minus 1 divided by minus one-third same comes here.

Here I get minus half

It should be 1 divided by how wonderful.

The negative sign goes off agreed. And this should be minus half divided by.

Minus one-third.

So, the Y parameters finally, become 4.

Minus 3 minus 3 and

3 by 2.

Let us see, if this agree if we make the calculations that is from the roots.

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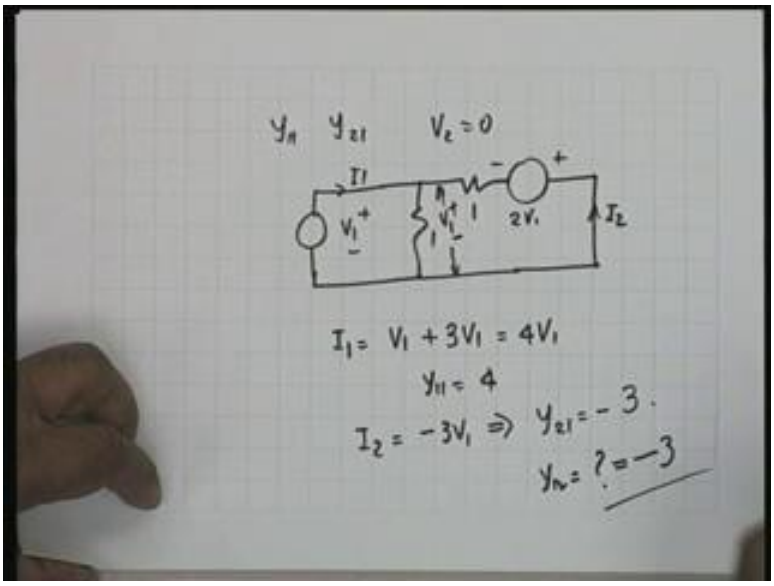

For y11 and y21, we have to make V2 equal to 0. And if we do that, the equation the circuit simply becomes this we have a source V1, I1 then we have this 1 ohm. 2V2 that currents generator vanishes, because V2 is made equal to 0. Current generator vanishes and therefore, I have minus plus 2V1 and a short circuit. The current through which is I2.

This becomes the circuit and what we have find out is I1 and I2 .Now, if you write the node equation at this point I1 is V1 by 1 and what is the voltage across, this is V1.

What is the voltage across this?

Is it minus 3,1 or plus 3,1?

Plus 3V1 and therefore, the current is 3V1.

You mustn’t make a mistake.

This is V1 minus plus and this is minus plus and therefore, the drop across this will be V1 minus minus 2V1 which is equal to 3V1. Suppose, the polarity was this and this was plus this was minus then i would simply V1 minus 2V1, but no it is V1 minus minus 2V1. So, 4V1 is equal to I1 which means is y1 will equal to 4. We have already found that and I2, I 2 is simply,

We have already found this current as 3V1. So, it is minus 3V1; therefore, y21 is equal to minus 3. That’s what we have found out. And in a similar manner we can find out y11 and Y12. Now, if we have found out V11, V12, V21 and V22. Do we still have to calculate y12?

No.

We argue that we have already shown that the network is reciprocal; therefore, y12 must be equal to minus 3. All that we have to calculate now is y11 and which is also simplicity itself .Because short circuiting the input terminal reduces the voltage source to 0. And there is only a current source all right and it becomes simpler to take care of this situation. We go to problem three problem 3.

We skip one of them and problem 3 says.

Yes

That is also removed.

That is correct.

Let me draw this network. This is interesting question.

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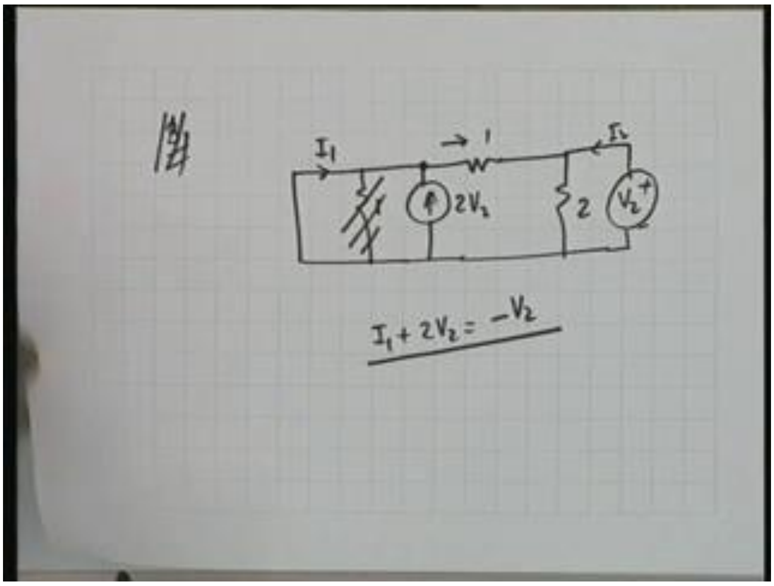

You see what happens, I am talking of the previous circuit V1 is made equal to 0.This is the current I1, it will be short circuit. 1 then you have 2V2 and then you have 1 ohm and 2 and V2.What his question was? I1 naturally no current shall flow through 1, but this current generator, it cannot be killed. If there is a V2 there shall be a current 2V2 and therefore, the current that enters here is, I1 plus 2V2 and that must be even the current that flows here, which is minus V2 by,

No

I1 plus 2V2 shall be equal to

Minus V2, because this point is the same as this point and therefore, this 1 ohm comes across V2.And then I2 shall be equal to this current plus this current.

That’s how one find so.

No.

Well we do not care where it flows. All we know is, at this juncture there is a current I1 coming. This 1 ohm becomes absolutely redundant 1 ohm can become a carrier current, but at this juncture 2V2 comes and I1 comes. We do not know why not? We do not know, whether the total current will flow through this or not.

No we do not know.

Okay.

Should not it flow through the

That is generated here.

It is a current generator.

But his question is should not I1 be equal to minus 2V2?

There is an additional current going here, these are not the only 2 currents, that is the third current and we have taken care of by writing

KVM at this particular KCM.

Yes.

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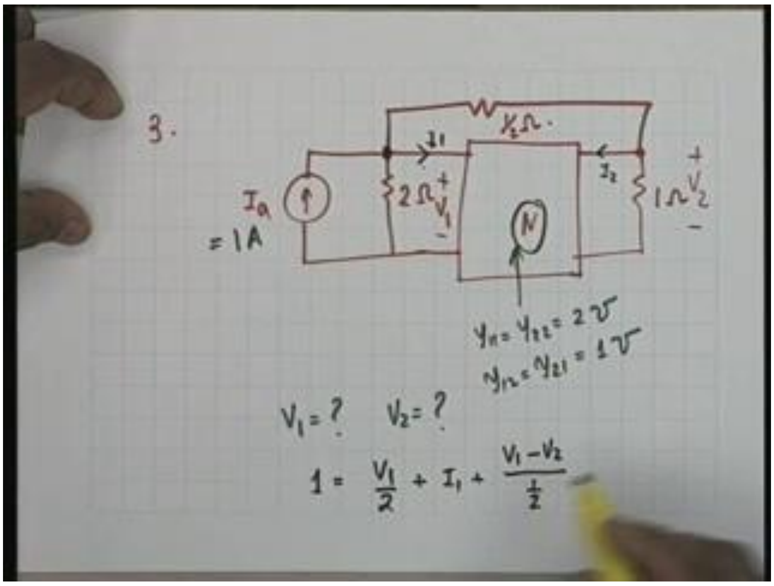

Question 3: Well, each question has to be done carefully, but there is a pot hole here which 1has to guard against. Question 3 is, we have a network in which 2 ohm’s comes in parallel with a current Ia and then we have a network N a 2 port N and a 1 ohm resistance, then there is a bridge half ohm. This is the composite network that is there is a 2 port which has which is augmented by 2 ohm’s, one ohm here and a bridge of half ohm and this is V2.This voltage is V2, this voltage is V1. What is given is that this parameter

the parameters of this network this i had given. This network is y11 equal to Y22 equal to 2 .y12 equal to y21 equal to 1 and I a is given as 1 ampere.

The question is to find V1 and V2.This is the question, you understood the question Now, can how do you solve it? Let's have suggestions

Can we replace this by a pi network?

No, we cannot because the other network, what you are trying to do is to use this as a parallel connection of 2 networks. Parallel connection half two mows 1,2 ohms 1 ohm, but it is not given that this 2 are connected together. So, you cannot make a parallel connection because this could be truly a four terminal network agreed. This is the temptation this is the pot hole. Fortunately in this particular case this also gives correct results and that is the, the point of the question. The question has been framed such that even if you consider this as a parallel connection of 2 true ports. You get the same result as you get by considering N is a truly four terminal network, but if you are happy with the answer. The examiner would be unhappy with the answer, because the method of calculation would be wrong. Now, therefore, what would be the correct method?

Now, go back to the roots. That is, what you do is the following; let's identify some currents and voltages and this is why i said you have to be very careful. I had done it both ways. The wrong way and the right way and they give the same result. Suppose, you have told that this is a short circuit, then there is absolutely no problem. You just find out the y parameters of this 5 pi network. And add to the y parameters of the network N and determine I1 and I2, but no this cannot be done here because this is not specific; therefore, we postulate currents I1 and I2.We postulate currents I1 and I2 and I write at this point.

The KCL will be 1would be equal to 1 ampere would be equal to V1 by 2 plus I1. V1 by 2 plus I1 plus V1 minus V2 divided by half. This is 1 of the equations and the other equation is at this node.

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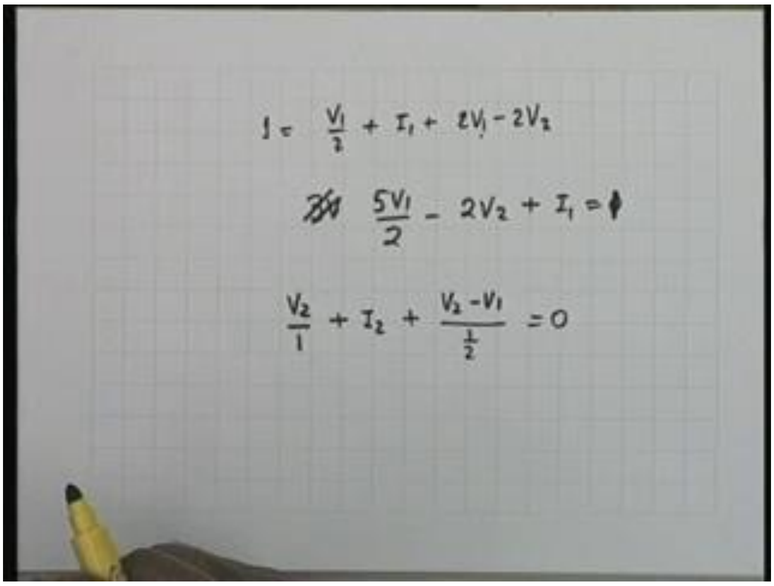

Therefore, this simplification this gives further we give 1 equal to V1 by 2 plus I1 plus 2V1 minus 2V2. Therefore, this is equal to 2V1 and V1 by 2. So, 3.

5V1 by 2 minus 2V2 plus I1 equals to 0.

Equal to 1.

The other equation is, if I look at the network V2 by 21 plus I2 plus V2 minus V1 divided by half this would be equal to 0.

Yeah

Yes we have.

Infact…

I am glad you discovered it. So, what is the way out?

You see writing this equation as assume and he has correctly caught me doing this mistake. You see my V2 reference is not necessarily same as V1 reference. And when I say the only problem is here V1 minus V2. If I do that then obviously, the references have to be the same agreed and therefore, this method going back to roots also does not work agreed. So, what is the way out? Shall i discard it at this point? Are you convinced that it should be discarded.

Because doing this simply means that we are assuming that, this 2 are connected together which is not correct? So, what is that we should do now.

What do we do now?

Well, that is called the convenient the true. If there is a problem. Now, we want t counter. We want to confirmed it what is the solution, can we go to find out the y11, y22 y12 and y21 this total network.

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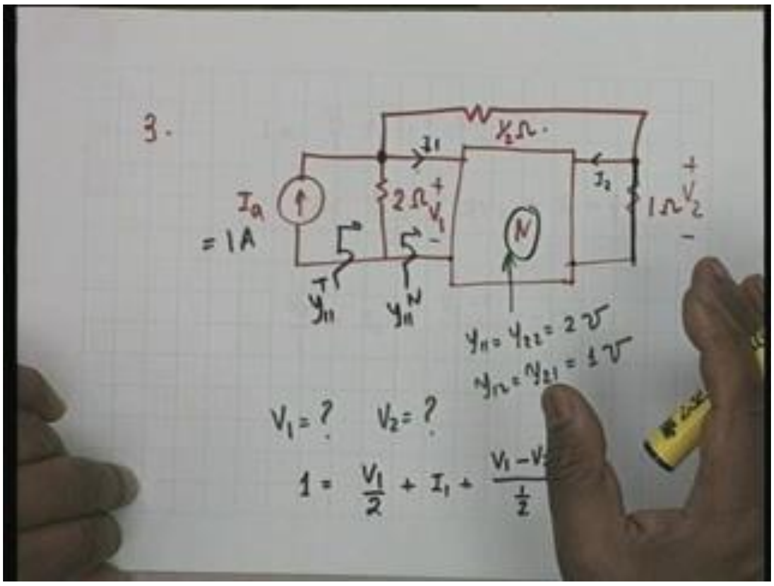

Can I do that.

How?

For example, if you want to find out y11. I have to short circuit this. If I short circuit this then what would be,

y11.

Same problem.

So, in other words, you see if I short circuit this let us do it. Short circuit this then my y11 total would be this admittance plus Y11 of the network N half mow plus y11 N plus I don't know how this half ohm.

Comes into this circuit because i do not know, what is connected between these2. And therefore, I have the same problem and I claimed that there is now way

One more variable how do you solve it?

The number of equations will be only 2.

All that you can connect is V1 I1, V2 I2 that is all.

Nothing else. And therefore, this problem is not solvable unless

These 2 points are connected. So, we now we can leave it all right as per suggestion we can go to the next problem. Next problem is question number 5. It is not that I am leaving the tough ones. No I am just taking the alternate problem.

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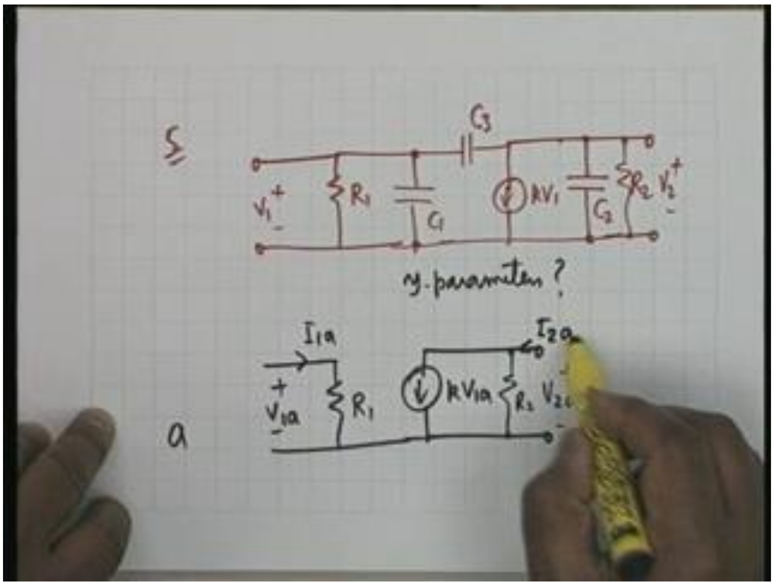

Determine the y parameter of the network question number 5 is determine the y parameters of this network. The network is this R1,V1 and then you have a C1.

C3 a controlled source KV1 and a capacitor C2.

If you notice carefully.

There is an R2 somewhere. Yes there is an R2 here.

If you notice carefully

There is a V2 here port number 2. If you notice carefully this is simply the equivalent circuit of a BJT or FET. Is it not? R1 is simply R phi. C1 is C phi. Are you acquainted at this terminology? C3 is C mu and C2 is C0 output capacitance. R two is R zero that is the collector dynamic resistance and this source kV1.K is simply,

GM and V1 is the voltage across R phi which of course, assumes that Rx is neglected. The base's spreading resistance is neglected. Now, to find the y parameters of this work, the easiest thing to do, easiest thing to do would be to view this as a parallel connection of 2 networks. And you see the how the parallel connection is chosen. One of the networks we choose fortunately there is a straight connection. It is a 3 terminal. So, 1 of the networks that we choose is R1. Let's call this V1 a, the a network and kV1 a that is it. No, let us include R2 also. This is V2 a. I1 a, I2 a this is my a network and the b network

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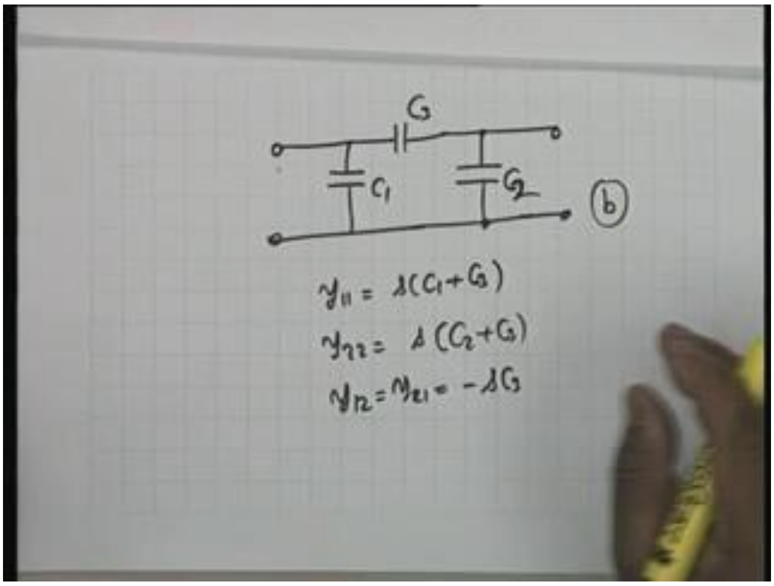

The b network is simply the capacitive network. That is; the b network is C1.

Yeah you have to calculate a parallel impedance. R1, C1 R2, C2. Yes we can do that. It can also be done that way. A little bit more calculation here; there is no calculation as you all see. This is a b network, for the b network you can see that y11 is sC1 plus C3. B network (( )) is s C1 plus C3, y 2 is equal s.

This is C2, sC2 plus C3 and y12, y21 is equal to minus sC3.This is the y parameter of this network no calculation is by inspection. What about the other network?

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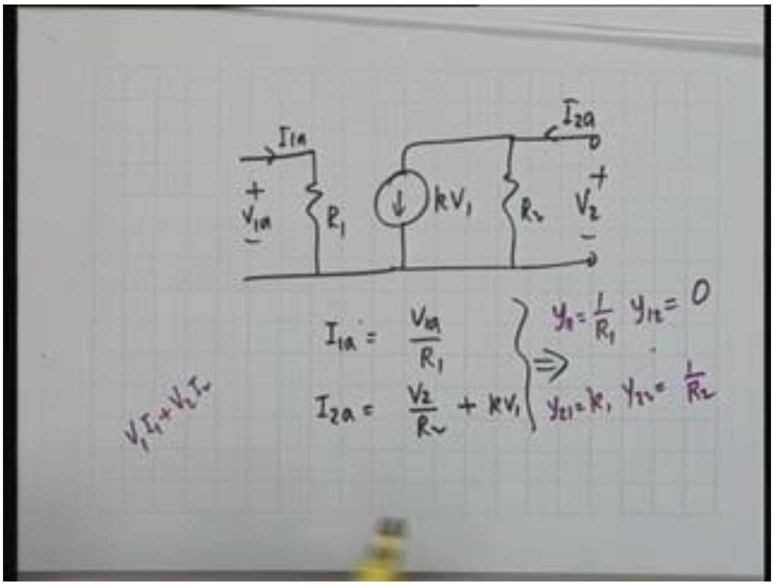

The other network is, R1 V1a, I1a kV1 R2. This is V2 and this is I2a and you can see, that all that I write is I1a equal to V1a divided by R1and I2a is equal to V2 divide by R2 plus kV1and the y parameters are obvious. What is y11?

1 by R1 y12 0 y2.

1 k and y22 equals to 1by R2. And because both of them are 3terminal you can simply, add the y parameters. I did not have to calculate the parallel combination of R1and C1 or R2 and C2. I need not to write I had to write 2node equation, but…

This is non reciprocal.

Yes.

BJT is non reciprocal.

Is it active BJT?

Yes it is active.

The activity; however, is not obvious yet. Activity will be clear if you write V1 I1 plus V2 I2.You can see that, it can be less than 0. It is possible to make it less than 0. So, this network is active and can a transistor be passive.

Can a transistor be passive.

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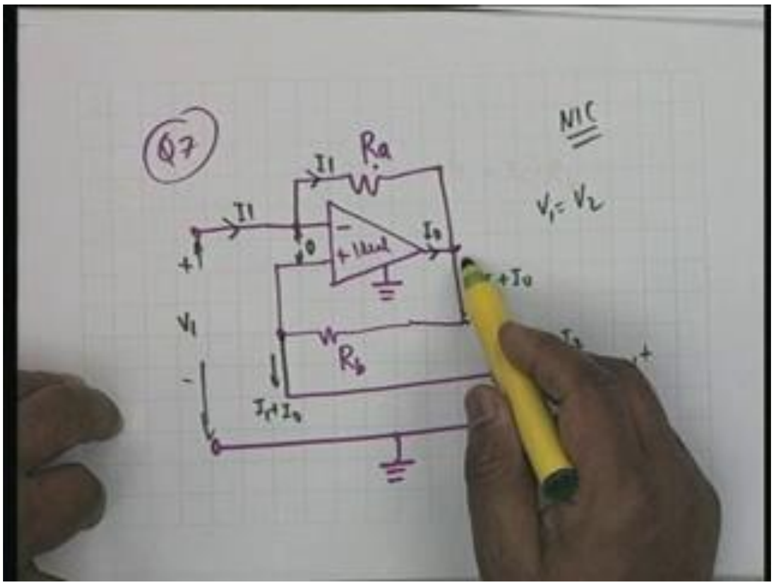

Last question of the day number 7 was there any problem in the fifth-one. Number 7 gives a an op amp circuit, a practical op amp circuit for an NIC negative impedance convertor and the circuit is like this. Let me draw this and the analysis will be extremely simple. This resistance is R sub b. It is not given in this figure, R sub b and this is connected of course, the op amp has a ground. This is the input port and from here you have a resistance Ra connected to the output. The second port is from here, to ground. The question is, this op-amp of course, we have to assume that this is ideal.

You have to show that this behaves as an NIC. The solution is extremely simple. What you have to do is assume a this is the port V1 and this current is I1. We have to show that if port number two if port number 2 is terminated in. Let’s say some impedance ZL we have to show that V1 I1 that is the input impedance is

It can be equal to minus ZL or it can be proportional to minus ZL. The proportionality constant can be anything. As long as the impedance is the, it is proportional to the negative of the terminating impedance it is an NIC. So, this voltage is V2 and this current is I2.

Proportionality constant someone has it is positive. It will be an NIC quite surprised. Proportionality constant can be negative. If the proportionality constant is negative for an NIC .

Then it is not an NIC. It is a positive impedance (( )) PIC. Now, let us see how to calculate this we invoked the idealness condition of an op-amp. One of the things that is obvious is V1 is equal to V2. Isn't that right because again idealness because the potential difference between this 2points is 0 virtual short not ground. Why did not you say ground? Because neither of this points are grounded. The ground is somewhere else.

This is not ground because this is you r port number 2. So, we do not say virtual ground be careful about this terminology. In this situation it is a virtual short because the ideal op-amp has infinite amount of gain. Once again, the ideal op-amp can just take a current and therefore, the current I1 must flow here agreed. The current I1 must flow here, when it comes here the op-amp gives out some current let us say I0 and therefore, this current would be I1 plus I0. And this current after coming here, nothing can go the op-amp and therefore, this must be equal to I1 plus I0 .

So, there are two things that is obvious, that is; I2 should be equal to minus I1 plus I0. This current and this current and the other thing that is obvious is that this drop. Drop across Ra should be the same as the drop across Rb, which means that I 1 Ra should be equal to I1 plus I naught into Rb.

What means?

Why this should be?

You see this 2points are virtual short and therefore, the drop across Ra should be equal to drop across Rb, but the drop across Rb is I1 plus I0 multiplied by Rb with this terminal positive.

So, there shall be…

A negative sign.

Why should I 1 flow through to because the op-amp is ideal? The input impedance in either port is infinity. If the impedance is infinity it cannot take a current. It cannot in the electronics language sync a current all right.

So, the current is diverted through Ra.

No, because there is no connection. Current has to have a connection, the potential of 2 points may be the same, but there can be no current through that because there is no physical connection.

Any other question?

No, I1 plus I0 is flowing like this and I2 flowing like this. They oppose each other and therefore, one must be the negative of the other.

Why are they approximately short? The op-amp ideal op-amp has a infinite gain. So, to produce a finite output voltage it requires 0 input voltage. 0 input voltage means voltage between this point and this point and therefore, they are short. They must be at the same what, Ideal minded infinite gain.

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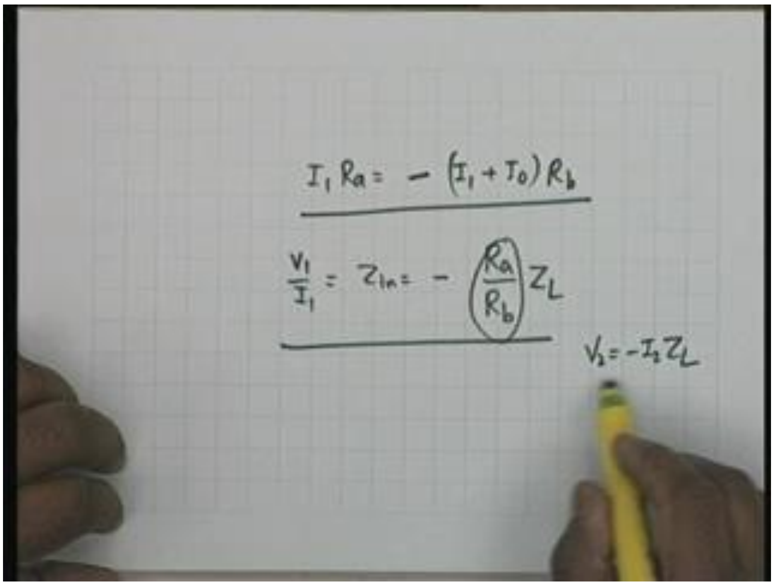

Now, any other, I1 Ra equal to minus I1 plus I0 Rv and therefore, V1 by I1 which is equal to Z input would be equal to minus Ra divided by Rb into ZL. The 2 simple manipulation of these 2 equations. I1 is equal to minus I1 I0 by Rv and so on.

What comes have to be plus?

No, where you must not forget that V2 is equal to minus I2 ZL. So, V1 by I1 is V2 I1 and V2 is minus I2 z L with minus I2 z L by I1 and we have already found out what I1 is. I0 does not come into the picture. It gets cancelled out and you noticed that if this 2 resistances are made equal. There was a negative impedance convertor with a conversion constant of 1. What is the dimension of the conversation constant?

Dimensionless you also noticed that by choosing Ra and Rb appropriately 1 can multiply impedance. Not only makes it negative, but then one can increase it or one can decrease. It the way 1 likes and this circuit has lot of uses in active filters and in integrated circuits.

Yeah, this is what it is.

How?

It is very simple, 2 equations V2 this is V1 V2 and V1 are equal and therefore, V1 by I1 is the same as.

Any other question, that is all for the day.