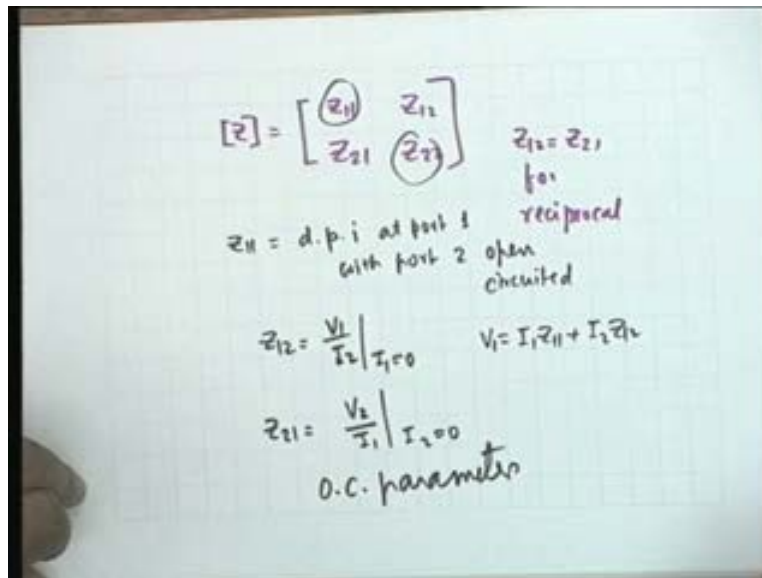


**Circuit Theory**  
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**Lecture - 21**  
**2 – Port Networks**  
**(Continued)**

This is the twenty first lecture and we, to continue our discussion on 2 port networks. As we did in the in the last lecture, the z parameters

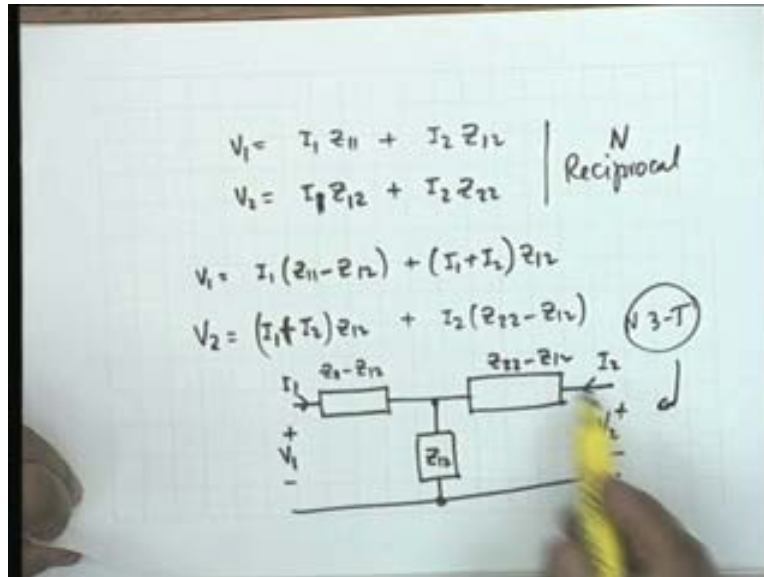
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The z parameters have 4, it is a 2 by 2 matrix, z 1 1, z 1 2, z 2 1 and z 2 2 and for reciprocal networks, z 1 2 is equal to z 2 1, for reciprocal networks and we have also seen how they are defined. z 1 1 and z 2 2 are driving point impedances at ports 1 and 2, with the other port open circuited. z 1 1 is driving point impedance at port 1 with port 2 opened, open circuited and z 1 2 is equal to, if I write the, yeah it is a transfer function. I 1 z 1 1 plus I 2 z 1 2 where z 1 2 is V 1 by I 2 with I 1 equal to 0. So it is a transfer function between port 2 and port 1 with port 1 opened and z 2 1 is V 2 by I 1 with I 2 equal to 0. Again, port 2 is open for this and therefore, all these 4 parameters are measured with 1 of the ports open circuited and therefore, these are called open circuit parameters sometimes called open circuit z parameters.

If  $z_{12}$  is equal to  $z_{21}$ , if this is true that is reciprocity holds and if the network is 3 terminal, then a particularly simple equivalent circuit can be derived like this.

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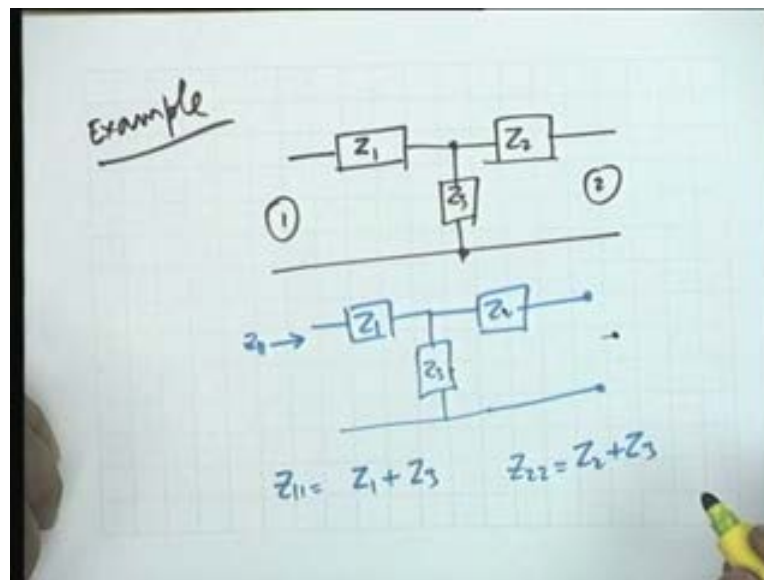


Let us take the equation defining equation  $V_1$  equal to  $I_1 z_{11}$  plus  $I_2 z_{12}$  and  $V_2$  equal to  $I_1 z_{12}$  plus  $I_2 z_{22}$ , we assume reciprocity therefore, it is  $I_1 z_{12}$  plus  $I_2 z_{22}$  this is reciprocity has been assumed  $N$  is reciprocal. You see, look at the first equation. I can write this as  $I_1 z_{11}$  minus  $I_2 z_{12}$ . I subtract  $I_1 z_{12}$  and add  $I_1 z_{12}$ , so I get  $I_1$  plus  $I_2$  multiplied by  $z_{12}$ , agreed. I have added, I have subtracted this  $I_1 z_{12}$ , I have added this  $I_1 z_{12}$ , no question?

And similarly in  $V_2$ , I can write  $I_1$  plus  $I_2 z_{12}$  and then I must subtract  $I_2 z_{12}$ . So I write  $I_2 z_{22}$  minus  $I_2 z_{12}$ , no harm done. But this step shows that if I have 3 impedances like this,  $z_{11}$  minus  $z_{12}$ , then  $z_{22}$  minus  $z_{12}$  and a third impedance here, which is exactly  $z_{12}$ , then this circuit, this network is a 3 port network. If  $N$  is 3 port, if  $N$  is 3 terminal, then this equivalent circuit would be a physical equivalent circuit also. Otherwise, if any 4 terminal, then this equivalent circuit is only mathematically equivalent to the equations, this is a representation of the mathematical equations.

This is called the z parameter, T equivalent circuit. It looks like a T, so it is called a T equivalent circuit. Of any 4 terminal network, the equivalent circuit has to be within inverted commas because it may not be physically equivalent. If the original network is 4 terminal, then it is not physically equivalent, it is only mathematically equivalent circuit. Nevertheless, this circuit, this equivalent circuit is of great help in finding out, in analyzing analyses and syntheses of networks. As an example, let us take a T network.

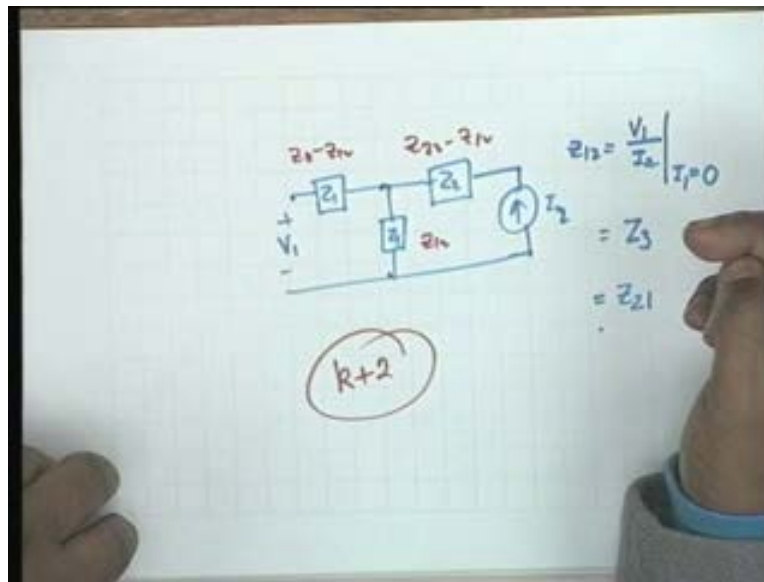
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Let say z A, let us take z 1 z 2 and z 3. This is port 1 and this is port 2. We are required to find out the z parameters of this circuit. One of the ways is go back to definition, whenever in doubt go back to the rules. Let us apply the definition, z 1 1, as you know, is the driving point impedance at port 1 with port 2 opened. If this is kept open, then obviously, z 1 1 is simply equal to z 1 plus z 3 agreed?

Similarly, you can see that z 2 2 would be simply equal to z 2 plus z 3. z 2 2 is impedance looking into this port, port 2 with port 1 opened and therefore, it is simply z 2 plus z 3, z 2 plus z 3, agreed. Let us look at the other 2 parameters.

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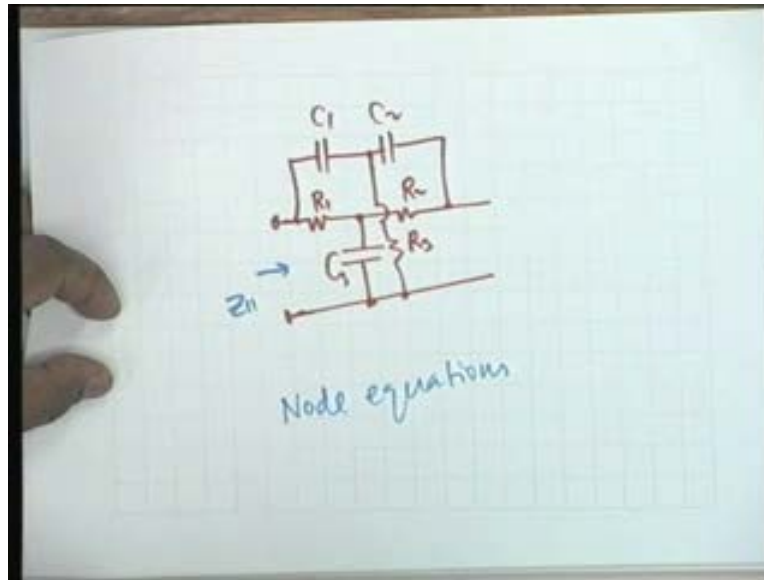


This is the network  $z_1, z_2, z_3$ . To find out  $z_{12}$ , you recall the definition of  $z_{12}$ , it is  $V_1$  divided by  $I_2$  with  $I_1$  equal to 0 and therefore, we keep this open and we find out  $V_1$ , with a current generator  $I_2$  here. Obviously, since this is open, there is no drop in  $z_1$ . So this  $V_1$  is the same as the drop across this and therefore, this is simply equal to  $z_3$ , simply  $z_3$ .  $V_1$  divided by  $I_2$ ,  $I_2 z_3$  is equal to  $V_1$ , so  $V_1$  by  $I_2$  is simply  $z_3$  and by the same token, if you interchange the current generator and the response, you will get this also as equal to  $z_{21}$ .

Now another way, this is by applying definition, another way would be to compare this network with the equivalent network. The equivalent, mathematical equivalence was  $z_{11} - z_{12}$ , this was  $z_{22} - z_{12}$  and this was  $z_{12}$ . So immediately you identify  $z_{12}$  as equal to  $z_3$  and therefore,  $z_{11}$  is  $z_1$  plus  $z_3$ ,  $z_{22}$  is  $z_2$  plus  $z_3$ . This is one of the usage of the equivalent circuit. It is a very simple case, so there is no problem.

But suppose, you have a more complicated network, in which, there are let us say  $K$  number of  $K$  plus 2 number of nodes. More complicated network and you cannot obtain this by inspection or by finding out the input impedance. For example, finding out input impedance is also a problem. Let me show you an example.

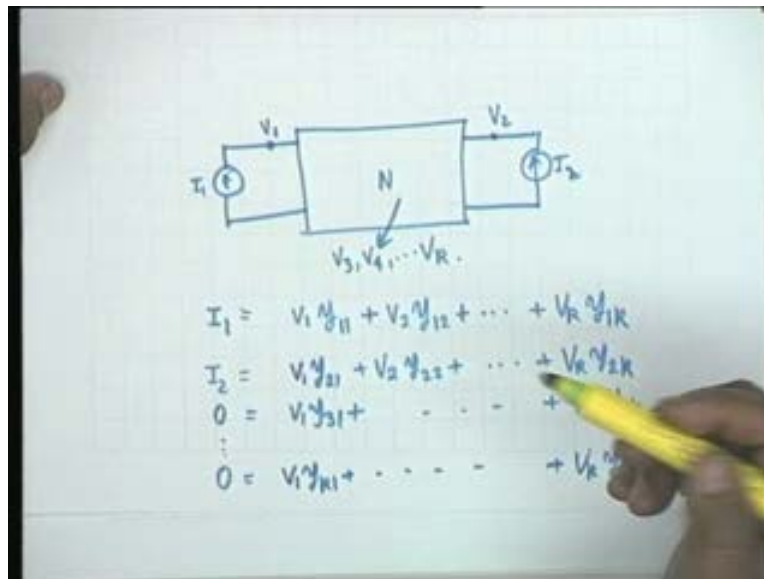
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Suppose you have a network like this. Suppose, you have a network like this, let us say  $R_1$ ,  $R_2$ ,  $R_3$ ,  $C_1$ ,  $C_2$ ,  $C_3$  and if you are required to find out the open circuit parameters, the  $z$  parameters. To find out  $z_{11}$ , you keep this open and find out the impedance looking here. Can you calculate this out by inspection? No way. No way, you know, this is not open, there is a path here and therefore, by inspection, it is impossible to find. What do you do, that is the question, how do you find  $z_{11}$ ?

The thing to do is to use node equations for analysis and let us do it systematically, node equations. For a complicated network in which one cannot find out the  $z$  parameters by inspection, one uses node equations and we look at the network like this. We have  $N$  and we have 2 ports which are available, so to the 2 ports, I connect 2 current generators  $I_1$  and  $I_2$ . These are, after all, the independent variables  $I_1$  and  $I_2$  and we identify this as one of the nodes. We call this  $V_1$ , this as 1 of the nodes, call this  $V_2$  and so on.

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Well,  $V_1$  is between this point and this point,  $V_2$  is between this point and this point and so on and, let us say, these are the external nodes. Let us say, there are other nodes inside the network, we call them  $V_3, V_4$  to let us say,  $V_k$ . There are  $k$  plus 2 nodes.

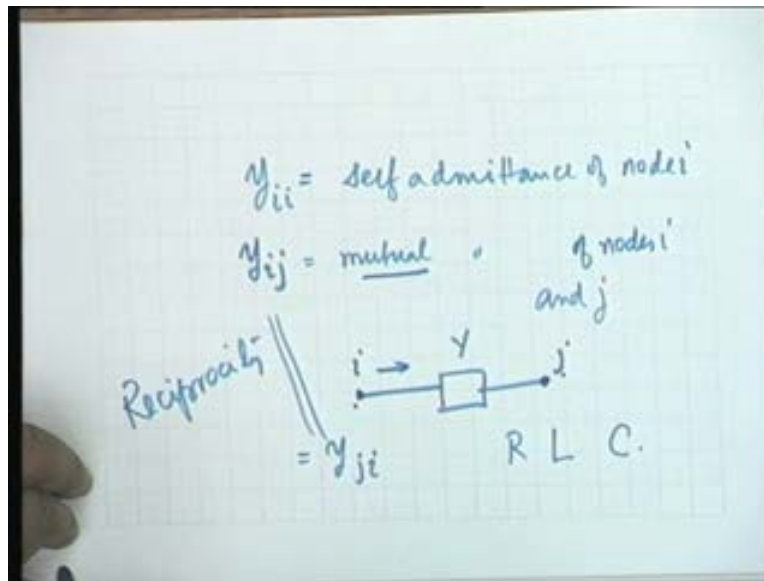
Student: (...)

Sir: No, there are 2 outside.  $V_3, V_4$ , no, internal nodes are these, internal node nodes are these. Now you can write node equations and the first 2 shall be  $I_1, I_2$ , they are generators. There are no other generators inside and therefore, 0 and so on. 0 equal to this. Now,  $I_1$  will be a function, in general, will be a function of all the node voltages and they shall be linearly related and therefore, I can write  $V_1$ , some  $Y_{11}$ , some admittance, because it is a current. Each term must be of the dimension of current, some admittance, I will explain these admittances later, plus  $V_2 Y_{12}$  plus etcetera plus  $V_k Y_{1k}$ . All these coefficients must be admittances.

Similarly, here I get  $V_1 Y_{21}$  plus  $V_2 Y_{22}$  plus etcetera plus  $V_k Y_{2k}$ . The third one, third node, there is no source there and I write 0 equal to  $V_1 Y_{31}$  plus etcetera plus  $V_k Y_{3k}$ , and the final node equation the  $k$ 'th node, shall be  $V_1 Y_{k1}$  plus etcetera plus  $V_k Y_{kk}$ . This is the general, we will take an example later to show how this is obtained, but in general, I can write

these equations. Now in these equations, the constants that occur have 2 varieties. The subscripts may be the same, subscripts may be different. If you notice, if I write the y matrix, the matrix of coefficients, then the diagonal elements are all of the same 2 subscripts, that is, of the form y I I. So in the off diagonal elements, off diagonal elements are all with different subscripts, I j type.

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Now  $y_{ii}$  is, if you recall how node equations are written, it is simply the sum of the admittances connected to that particular node. So it is called self admittance of node  $i$ . Now  $y_{ij}$ , that is, if  $i$  and  $j$  are different then  $y_{ij}$  represents the admittance connected between nodes  $i$  and  $j$ . So whenever you write the  $i$ 'th node equation,  $y_{ij}$  shall occur. The  $j$ 'th node equation also  $y_{ij}$  shall occur. So this is the mutual admittance because it occurs, it is shared by 2 nodes, it is a mutual admittance of nodes  $i$  and  $j$ .

Student: Sir, will we define the diagonal if  $k$  is even?

Sir: How do you define?

Student: The diagonal.

Sir: How do you define the diagonal? Any matrix shall be a diagonal, is not it right?  $y_{11}$ ,  $y_{22}$ ,  $y_{33}$ ,  $y_{44}$ . This is the diagonal, any matrix you have the diagonal.

Student: Sir, as you said that  $y_{ii}$  is the sum of admittance, connected to one node and  $y_{ij}$  is the admittance connected between the 2 nodes

Sir: 2 nodes  $i$  and  $j$ .

Student: Can you explain that a little bit sir?

Sir: Oh, we will, you see, whenever you write a node equation, let us say, between  $i$  and  $j$ , suppose, there is an element say, let us say, capital  $Y$ , then when we write the node equation for node  $i$ , you write  $V_i$  minus  $V_j$  multiplied by  $Y$ . When you write equation for  $j$ , you write  $V_j$  minus  $V_i$  multiplied by  $Y$ . So  $Y$  is common to the node equations, for node  $i$  as well as node  $j$  and this is why it is called mutual admittance. One thing that should be clear is that if this element, if the combined element or the combination of elements between nodes  $i$  and  $j$ , if they are bilateral elements, then obviously  $y_{ij}$  shall be the same as  $y_{ji}$ , is not that right?

If it is a diode, it will not be the case. If it is bilateral, then obviously  $y_{ij}$  shall be equal to  $y_{ji}$ , if the element consists of  $R$ ,  $L$  and  $C$  and therefore, one thing that one immediately recognizes is that if the network is reciprocal, then the half diagonal elements in corresponding positions will be identical, that is,  $y_{ij}$  shall be equal to  $y_{ji}$ . This also is a reflection of reciprocity. What we should now realize is that there are 2 kinds of elements in this matrix. One is the self admittance and other is the mutual admittance. The mutual admittances of nodes  $i$  and  $j$  have one and only one value, provided, the elements composing the admittance are bilateral.



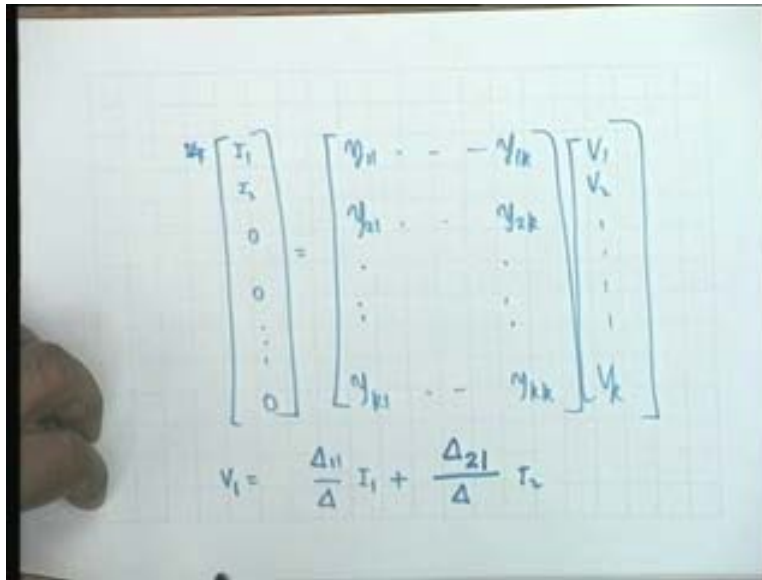
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$$y_{ij} = \frac{G_{ij} + sC_{ij} + \frac{1}{sL_{ij}}}{v_{i,j}}$$
$$y_{ij} = y_{ji}$$
$$\Delta_{ij} = \Delta_{ji}$$

In general, therefore, I can write  $y_{ij}$  as equal to  $G_{ij}$ , some conductance, plus some capacitance  $sC_{ij}$  plus  $1$  over  $sL_{ij}$ . In general, I can write it like this, if R L C elements compose this. Now if  $y_{ij}$ , well obviously, this is a general case, even if  $j$  equal to  $i$ , it is the same. All the admittances connected to a particular node shall be of this form. There will be a conductance, there will be a capacitance, there will be an inductance. So all  $ij$ ,  $i$  may be equal to  $j$ ,  $i$  may not be equal to  $j$ .

But the fact that  $y_{ij}$  equal to  $y_{ji}$  also shows that this matrix,  $y$  matrix, the co-factors with corresponding subscripts shall also be equal because the co-factors are obtained by deleting a particular row and a particular column and therefore,  $\Delta_{ij}$  shall be equal to  $\Delta_{ji}$  and you know how to find out  $\Delta_{ij}$ , you delete the  $i$ 'th row and the  $j$ 'th column and then determine the determinant and multiply by minus  $1$  to the power  $i + j$ . You must not miss that and the sign of the co-factor is extremely important. Now let us see let us see an example.

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$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & \dots & -y_{1k} \\ y_{21} & \dots & y_{2k} \\ \vdots & \ddots & \vdots \\ y_{k1} & \dots & y_{kk} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \end{bmatrix}$$
$$V_1 = \frac{\Delta_{11}}{\Delta} I_1 + \frac{\Delta_{21}}{\Delta} I_2$$

No, we have not finished the discussion. You see, what I wrote was that  $V_1$ , I am sorry,  $I_1 I_2 0 0 0$ , now I write the set of equations in matrix form. This is equal to the  $y$  matrix,  $y_{11}$  to  $y_{1k}$ ,  $y_{21}$   $y_{2k}$  and  $y_{k1}$ ,  $y_{kk}$ , I am not writing the intermediate values, multiplied by  $V_1 V_2 V_k$  and what we want to find out is the, our aim was to find out the  $z$  parameters, so you must express  $V_1$  and  $V_2$ . We are not interested in the other node voltages.  $V_1$  and  $V_2$  in terms of  $I_1$  and  $I_2$  and obviously, if you see,  $V_1$  is  $\Delta_{11}$  by  $\Delta$ , where  $\Delta$  is the admittance,  $\Delta$  is the determinant of this admittance matrix multiplied by  $I_1$  plus  $\Delta_{21}$ , what would be the subscript of this?

Student: (..)

Sir: No,  $I_1$  and this is  $I_2$ , I am finding of  $V_1$

Student:  $\Delta_{12}$ .

Sir:  $\Delta_{21}$ ? No, it is  $21$ ; second row and the first column.

Student: Are they identical?

Sir: They happen to be identical, yes they happen to be identical. But this procedure is general, whether they are identical or not, it does not matter.

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$$V_1 = \frac{\Delta_{11}}{\Delta} I_1 + \frac{\Delta_{12}}{\Delta} I_2$$

$$V_2 = \frac{\Delta_{21}}{\Delta} I_1 + \frac{\Delta_{22}}{\Delta} I_2$$

And so, what I get is  $V_1$  equal to  $\frac{\Delta_{11}}{\Delta} I_1$  plus  $\frac{\Delta_{12}}{\Delta} I_2$ . Similarly,  $V_2$  can be written as  $\frac{\Delta_{21}}{\Delta} I_1$  plus  $\frac{\Delta_{22}}{\Delta} I_2$  and you can see that this is  $z_{11}$ , this is  $z_{12}$ , this is  $z_{21}$  and this is  $z_{22}$ , this is the general procedure. If it is not possible to do it by inspection or by simplification, there are simplifications which one can effect, to be able to obtain the  $z$  parameters by inspection. But that we will do later.

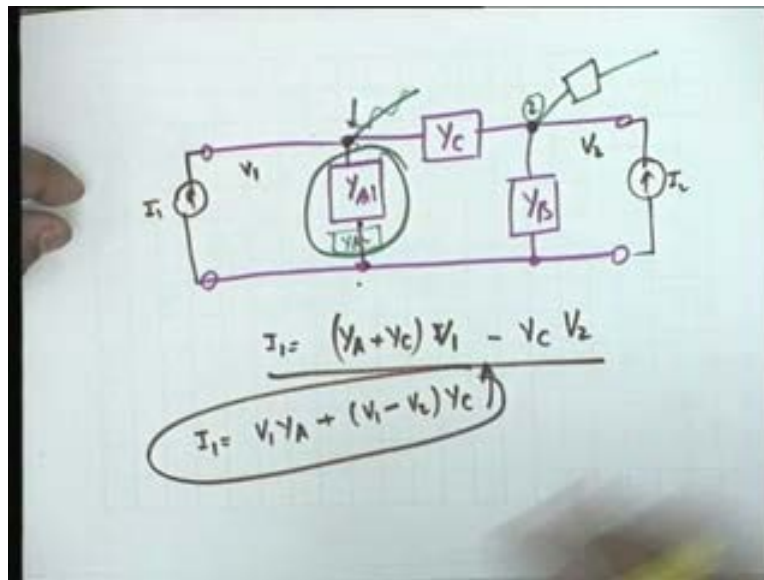
Student: Sir the example which you have just taken, I think the  $z_{11}$  and all the parameters could be determined by inspection.

Sir: Could be determined by inspection?

Student: It is just 2 Ts in parallel

Sir: But then, you have to simplify, then you have to make a conversion. It is not by inspection you have to convert the 2 T's into pi combine the 2 and so on. We will come to this, it is not inspection. Inspection means, you do not have to look at, you do not have to write a single line of equation. Inspection means, you just write down the results. It is not true for an example that I have taken. In fact, it is not true even for a very simple example. Let us take that example now, where we shall illustrate this procedure, by taking a simple pi network.

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Let us say, we have 3 admittances. YA YC and YB, this is my network and I am required to find out z parameters. You notice that I do require calculation. If we keep this open, here it is possible to write down the inspection, it is not that it is very difficult, but you can see that if we keep it open, then the input admittance will be YA plus YC YB divided by YC plus YB. And similarly, I can find out V 2 2 and so on, but it would be instructive to do it by the node analysis. Let us do that. We have a V 1, well we have a current generator I 1 here. We are doing it only to illustrate the generate procedure. There are many ways of finding the z parameters of this. And I 2 and what we are required to find out is V 1 and V 2.

Let us write the 2 node equation, there are only 2 nodes which, for which equations have to be written, this 1 and this 1 and you see, I 1 therefore, is equal to, what is the self admittance of this

node? Sum of all the admittances that is  $Y_A$  plus  $Y_C$ . So  $Y_A$  plus  $Y_C$  multiplied by  $V_1$ , then self mutual admittance is the admittance connected between nodes 1 and 2 and, that is,  $Y_C$  will come with negative sign, minus  $Y_C$  times  $V_2$ , is not that right?

The current entering here is the current going through this, plus the current going through this. And this current is  $V_1 Y_A$  and this current is  $V_1$  minus  $V_2 Y_C$ . That is how I get  $Y_A$  plus  $Y_C V_1$  minus  $Y_C V_2$ , is this point clear?

Student: No sir.

Sir: No? Let us write  $k c l$  at this node. What do I get?  $I_1$  equal to  $V_1 Y_A$  plus  $V_1$  minus  $V_2 Y_C$ , which is exactly the same as this equation.

Student: What is the general principle?

Sir: Oh, the general principle, if both of them are considered positive,  $V_1$  and  $V_2$ , then the mutual admittance will come with the negative sign, that is it.

Student: (..)

Sir: Yes, suppose,  $V_2$  was considered this is negative and this is positive, then things would have been definitely different.

Student: How do we take the self admittance?

Sir: Self admittance is simply the sum of all the admittances connected here. You see, this is  $Y_A$  this is  $Y_C$  and this a current generator. So the self admittance is 0 and therefore, this is a simple way of writing equation. Self admittance and mutual admittance, if there is any confusion go ahead and write  $k c l$ .

Student: Sir about but the nature, you are going to see the direct convention between the two?

Sir: That is correct.

Student: You cannot go around.

Sir: No, if you go around another node, then purpose is not served, because we are going to make the, investigate the connection between this node and all other nodes in the circuit and therefore, you must not go indirectly, you must go directly.

Student: Sir, then in some cases some nodes may not be connected to the other node.

Sir: Then the corresponding current will be absent. No current goes through that, corresponding current shall be absent, yes?

Student: Taking the self impedance,

Sir: Admittance.

Student: We should take the sum of the two admittances attached to it?

Sir: 2, 3, 4.

Student: and all the branches attached to the node?

Sir: All the branches attached to this node. Take the admittances of all the branches attached to this node, yes?

Student: And in this case the YC and YB will not come in series?

Sir: No, they are not connected in series. No, they are connected like this, there is no change in connection. It is only that the mathematical equation describing it is in terms of self and mutual

admittance; this is a matter of convenience and as I said if you are in doubt write  $k c l$ . It is always better to go to the rules.

Student: For instance, the self inductance in this case would be in voltage  $V_1$  and  $V_2$ . That is what gives us the second term  $Y C V_2$ , because it is connected between these two nodes.

Sir: This is mutual, anything connected.

Student: Sir but with respect to the first node.

Sir: With respect to the first node.

Student: The term that we are getting, that is in fact that is why you are taking up to the second node and we are letting it into the voltage  $V_2$  because it is connected between the  $V_1$  and  $V_2$ .

Sir: We had no alternative, this is dictated by  $k c l$ , as I said, I am writing  $k c l$  here, which is exactly the same as this. We have no choice, this is how it will come and what I am telling you is a short cut, that is, you do not need to write  $k c l$  every time, just write self admittance and mutual admittance and the convention is that all polarities, there is a reference, obviously reference is here, with respect to this all voltages are considered positive and therefore, mutual admittance will come as in a negative term.

Student: Sir, if there is further branching of  $Y C$ , suppose one more is **(...)**

Sir: Here?

Student: Sir, after  $Y C$ .

Sir: After  $Y C$ , there is one more.

Student: Then the self admittance at point 1 will be more?

Sir: Will not change, because there is a node 2 here. Self admittance is from here to the next node, from here to the next node, sum of all of this.

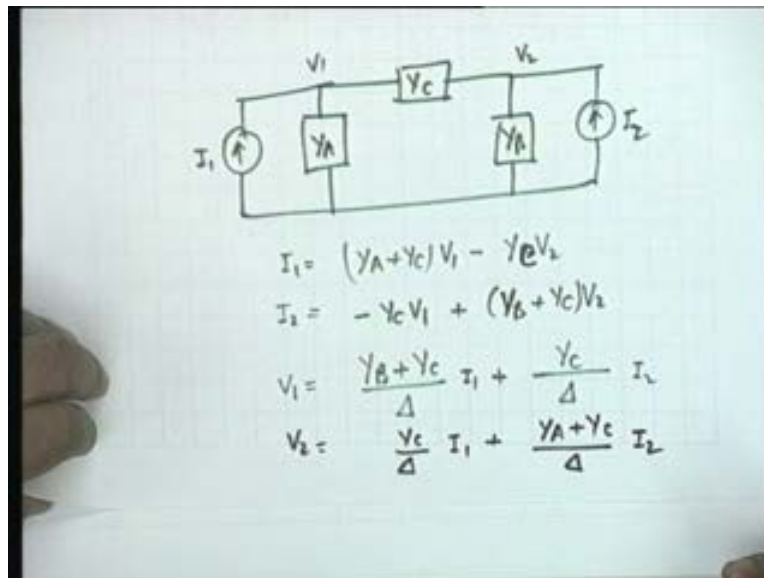
Student: Even then mutual will not change.

Sir: Even mutual will not change. Nothing will change because if you identify this is a node. But suppose, there is a question here, suppose, Y A consisted of 2 admittances like this, Y A 1 and Y A 2 and this is not recognized as a node, then you would have to take the total of this, agreed?

Student: Sir if between 1 and 2.

Sir: If between 1 and 2, there is 1 more, then you add them. If there is Y C 1 and then Y C 2 then Y C 1 plus Y C 2 shall figure in both self and mutual.

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Now we will come to further complications, when you come to this. This is Y C and this is Y B and this is I 1, I 2, this is V 2 and this is V 1 and our equation are I 1 equal to Y A plus Y C times V 1 minus Y B V 2 and the second equation, similarly, I 2 would be



Student: Y C sir, Y C.

Sir: Pardon me.

Student: Y C.

Sir: Y C that is correct, thank you. Y C V 2 the second one would be, mutual admittance image is the same.

Student: minus Y C.

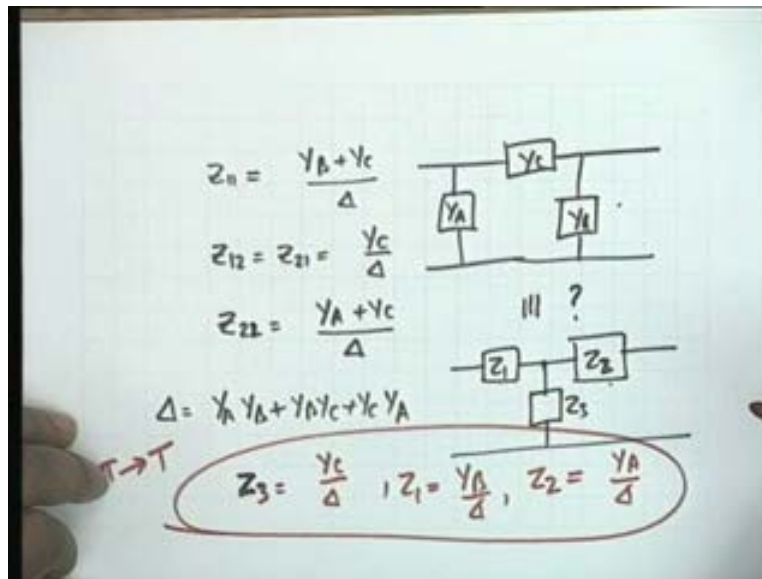
Sir: Minus Y C V 1 plus Y B plus Y C V 2 and what I am trying to do now is to find out V 1 and V 2. You see, V 1 is  $\frac{\Delta_{11}}{\Delta}$  by  $\Delta$ , what is  $\Delta_{11}$ ? First row and first column, so it is Y B plus Y C divided by  $\Delta$ , whatever  $\Delta$  is, multiplied by I 1 plus, again  $\Delta$  here I 2. This should be  $\Delta_{21}$  or  $\Delta_{12}$ , the sign here is negative and the sign of  $\Delta_{12}$  itself is negative. So it is simply Y C divided by  $\Delta$ , is that okay? I am, I have applied the same procedure. I have not written the matrix, I have done some, I have omitted some steps.

Similarly, you can write V 2 is equal to Y C by  $\Delta$  times I 1 plus, what should you have here?

Student: YA plus YC

Sir: YA plus YC divided by  $\Delta$  multiplied by I 2, so we immediately identify  $z_{11}$ ,  $z_{12}$ ,  $z_{21}$  and  $z_{22}$ , agreed? Let me write these results. This is obtained, not by instruction but by node analysis and this was to illustrate the general procedure that you will follow in the case.

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$z_{11}$  is  $Y_B$  plus  $Y_C$  divided by  $\Delta$ ,  $z_{12}$  equal to  $z_{21}$  is equal to  $Y_C$  divided by  $\Delta$  and  $z_{22}$ , I am sorry,  $z_{22}$  would be  $Y_A$  plus  $Y_C$  divided by  $\Delta$ . This is what is the network? Let me draw the network. Network is, we have a  $Y_A$ , we have  $Y_C$  and we have a  $Y_B$ . What is  $\Delta$ ? The determinant of the, determinant of the, it is  $Y_A$  plus  $Y_C$  multiplied by  $Y_B$  plus  $Y_C$  minus  $Y_C$  square. So it is simply  $Y_A Y_B$ , it a cyclic summation of products,  $Y_B Y_C$  plus  $Y_C Y_A$ .

Suppose, this network this 2 port network is equivalent to, with a question mark, we will find the condition, it is equivalent to the T network like this 2 and 3. Can you tell me if this is equivalent to this, can you tell me the values of the T network elements? Obviously,  $z_{12}$ ,  $z_{21}$  which is  $z_3$  and therefore,  $z_3$ , let me write in the different colours.  $z_3$  must be equal to  $Y_C$  by  $\Delta$ .  $z_3$  is  $Y_C$  by  $\Delta$  and this should be  $z_{11}$  minus

Student:  $z_{12}$ .

Sir:  $z_{12}$  and therefore, from this I get  $z_1$  equal to  $Y_B$  by  $\Delta$  and third element. No other alternative but to become  $Y_A$  by  $\Delta$  and you see that in the process we have derived the conversion formula for equivalents between a pi and a T. This is a pi to T conversion. This looks like a pi and this is a T. If these 2 networks are to be equivalent, then the T network parameters

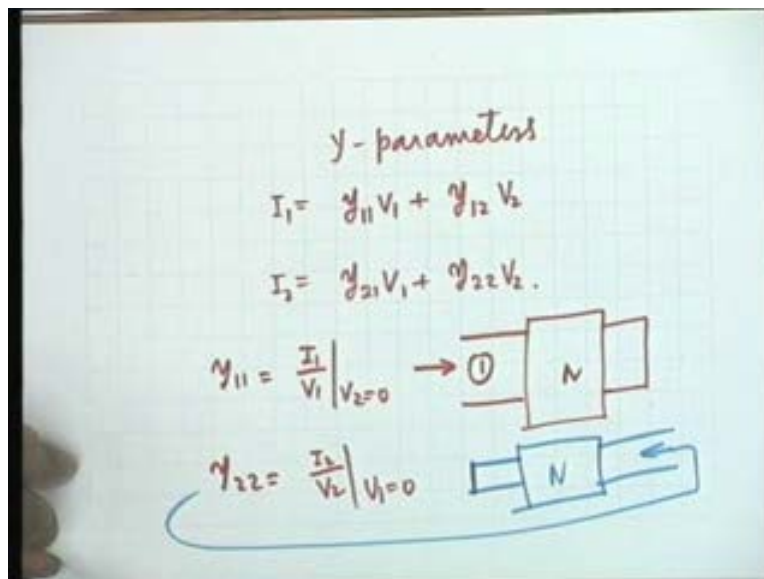
can be obtained in terms of the pi network parameters and these are the following. All that you need is to find out del and then elements independently Y C Y B and Y A.

The interesting thing to notice is that  $z_1$  is related to Y B,  $z_1$  is related to YB the opposite.  $z_2$  is related to YA. In that sense, there is symmetry, there is symmetry. Any question on this? We will have plenty of occasions to use these equivalents in future .

Student: (..)

Sir: The condition when it is true is that both of them must be 3 terminal, both of them must be 3 terminal, not otherwise. Suppose we had another element here, no, then no way, they are not equivalent. It is only when it is exactly a pi and it is a pi if this is a short, this is a T if this is a short, they must be equivalent. We next go to the Y parameters and in Y parameters, as a told you, the independent parameter, independent variables or  $V_1$  and  $V_2$   $V_2$  voltages and the depended variables at recurrence  $I_1$  and  $I_2$ .

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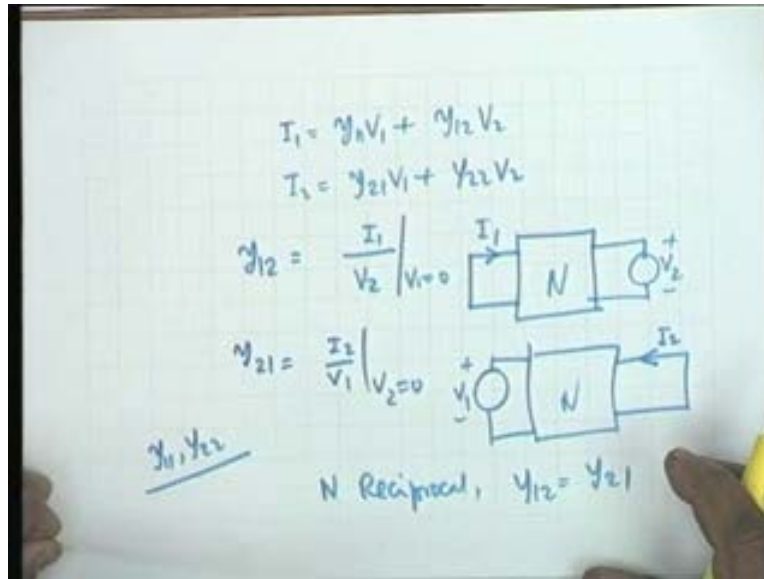


So you relate  $I_1$  to  $v_1$  and  $v_2$ , obviously, if it is a linear network, then it should be a liner combination of  $V_1$  and  $V_2$  and the coefficients are termed, I have to have the dimension of

admittance, they are termed  $y_{11}$  and  $y_{12}$ . Similarly, you say  $Y_{21}$ ,  $V_1$  plus  $Y_{22}$ ,  $V_2$  and if you look at the definition,  $y_{11}$  is equal to  $I_1$  by  $V_1$  with  $v_2$  equal to 0,  $v_2$  equal to 0 means short circuiting port 2, that is, you have the network, this is the driving point admittance at port 1  $I_1$  by  $V_1$  with port 2 short circuited.

In a similar manner, you can show that  $Y_{22}$  is  $I_2$  by  $V_2$  with  $V_1$  equal to 0. That means, it is the driving point admittance at port 2, with port 1 short circuited. This is where you measure the driving point admittance, with this short circuited.

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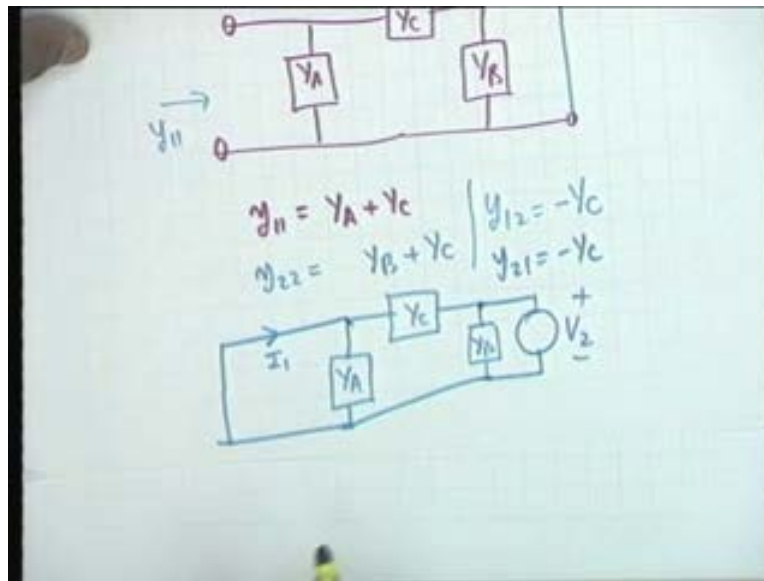


Then the other 2 parameters,  $y_{11} V_1$  plus  $y_{12} V_2$   $I_1$  equal to  $y_{21} V_1$  plus  $Y_{22} V_2$ . Let us say,  $Y_{12}$  this is equal to  $I_1$  divided by  $V_2$  with  $V_1$  equal to 0, which means that you measure the short circuit current at port 1,  $V_1$  equal to 0 means, let us see,  $V_1$  equal to 0 means you are short circuiting this. No voltage can be appear across the short circuit and you have to find  $I_1$ . Now 1 of common mistakes is that the direction of  $I_1$ , which direction should we take?

Student: (..)

Sir: It should go up, this is  $I_1$ . You must not change the directions of the current and you have connected the voltage source here,  $V_2$ . This ratio  $I_1$  by  $V_2$  shall be  $Y_{12}$  and similarly, for  $Y_{21}$ , it is  $I_2$  by  $V_1$  with  $V_2$  equal to 0, that is, you connected  $V_1$ , this is the network, short circuit this and measure this current, current coming up. If you notice,  $y_{11}$  and  $y_{22}$  are driving point parameters,  $y_{12}$  and  $y_{21}$  are transfer admittances. 2 of them are driving point and 2 are transfer. In the same manner that we discussed in the case of the  $z$  parameters, if the network is reciprocal, if  $N$  is reciprocal, then  $y_{12}$  and  $Y_{21}$  shall be the same. If  $N$  is reciprocal, then  $y_{12}$  shall be equal to  $y_{21}$ . Let us take an example, let us take this pi network.

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There is an admittance  $Y_C$  and there is an admittance  $Y_B$  here. If I apply the definition of  $y_{11}$ ,  $y_{11}$  is the driving point admittance with port 2 short circuited. If this is short circuited, then all that you get is  $Y_A$  and  $Y_C$  in parallel and therefore,  $y_{11}$  is  $Y_A$  plus  $Y_C$ . Is it clear? What is  $y_{12}$ ?  $y_{12}$  is this admittance with this short circuited. If you short circuit this, obviously,  $Y_B$  drops out of consideration, so  $Y_A$  comes in parallel to  $Y_C$  and this is  $y_{11}$ . In a similar manner, you can find out  $y_{22}$ , as equal to  $Y_B$  plus  $Y_C$ .

Let us find out  $y_{12}$  or  $y_{21}$ ,  $y_{12}$ , for  $y_{12}$ , you shall connect a voltage source as port 2,  $Y_C$  and  $Y_A$ , and what shall we do? Short circuit this and measure this current  $I_1$ . What do you

think the current would be? You see,  $Y_B$  is in parallel to  $V_2$ , so it becomes ineffective.  $V_2$  and this voltage is 0, this also becomes ineffective. So  $V_2$  by  $Y_C$  would be equal to  $I_1$ , no.

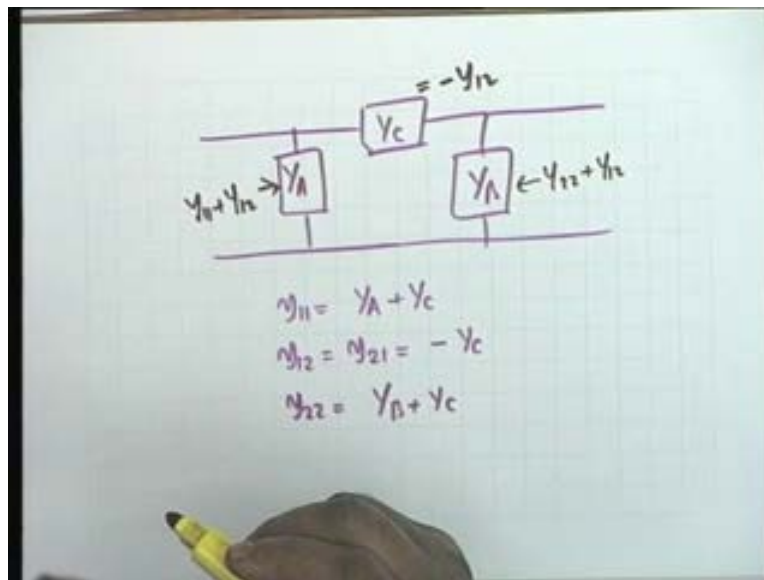
Student: Sir,  $V_2 Y_C$ .

Sir:  $V_2 Y_C$ , no even that is not correct.

Student: negative.

Sir: Negative of that and therefore,  $y_{12}$  shall be equal to minus  $Y_C$ , is that clear? Similarly,  $y_{21}$  is equal to minus  $Y_C$ . Let me write it down, which will lead me to a very interesting result.

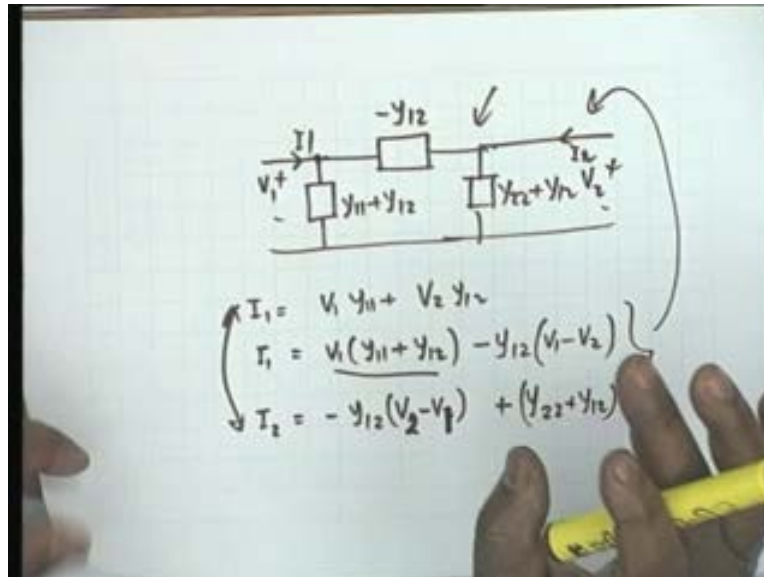
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I have  $Y_A$ ,  $Y_C$ ,  $Y_B$  and I have shown that  $y_{11}$  is equal to  $Y_A$  plus  $Y_C$ ,  $y_{12}$  is equal to  $y_{21}$  equal to minus  $Y_C$  and  $y_{22}$  is equal to  $Y_B$  plus  $Y_C$ . Now, if I want to replace this,  $Y_A$   $Y_C$   $Y_B$  by the 2 port parameters, let us see what happens. What is  $Y_C$ ?  $Y_C$  is minus  $y_{12}$ , what is  $Y_A$ ?  $Y_A$  is obviously, the sum of the 2, sum of the 2. So  $Y_{11}$  plus  $Y_{12}$  and  $Y_B$  is  $Y_{22}$  plus  $Y_{12}$ . Do not you see that we have obtained an equivalent circuit for a general network, at 3 terminal equivalent circuit. Again, either physical or mathematical, depending on whether it is 4

terminals or 3 terminal, as the case may be. Is the point clear? A general, now let me make a formal definition.

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No, but before that, what I derived is that a general 2 port can be represented, physically or mathematically by 3 admittances like this, minus  $y_{12}$  and this is  $y_{22}$ , plus  $y_{12}$ . This is what I have shown, by referring to a simple example. Now let me make a formal derivation. You see, my equations were  $I_1$  equal to  $V_1 y_{11}$  plus  $V_2 y_{12}$ . I can write this as  $V_1 y_{11}$  plus  $y_{12}$ . Then minus  $y_{12}$ ,  $V_1$  minus  $V_2$ , is that okay? This is  $I_1$  and  $I_2$ , I can write as minus  $y_{12}$ ,  $V_1$  minus  $V_2$  plus  $y_{22}$ , plus  $y_{12}$   $V_2$ , is that okay?

Student:  $v_2$  minus  $v_1$ .

Sir: No, this should be  $v_2$  minus  $v_1$  that is correct. Is it okay? because my other equation is  $I_2$  equal to  $y_{12} V_1$  plus  $y_{22} V_2$ . What I have done is that I have added  $y_{12} V_1$  to  $V_2$  and subtracted  $y_{12} V_1$  to  $V_2$ . Now do not you see that this is same as, these 2 equations are represented by the same as this,  $V_1$   $V_2$ , this is  $I_2$  and this is  $I_1$ . This, node equation, I am writing node equation.  $I_1$  equal to this node multiplied by self at node, the current show this plus current show this. The current show this is  $V_1 y_{11}$  plus  $y_{12}$  and the current through this is  $V$

1 minus  $V_2$  multiplied by minus  $Y_{12}$  and therefore, this is formally established as the pi equivalent circuit of a general 2 port. Equivalent circuit would be a physical equivalent circuit, if the general 2 port is 3 terminal, it is a mathematical equivalent circuit if the general port general 2 port is 4 terminal.

Student: Excuse me sir, we are getting  $y_{12}$

Sir:  $y$ ?

Student:  $y_{12}$ ,  $y_{12}$ .

Sir: That is right.

Student: What is the physical significance of negative impedance?

Sir: It may not be negative, you know,  $y_{12}$  itself will be negative. There is no physical significance of negative impedance is that it is negative. There are possibilities; it is possible to get negative resistance, negative inductance, negative capacitance, no problem. You get them, not with coils and parallel plates, we have to use active devices, yeah.

Student: Sir, just you are mentioning about this particular mathematical equivalents.

Sir: That is correct.

Student: Sir, so what kind of calculations can we make (..)

Sir: Oh, any calculation, if you are required to find out, let us say, this voltage or this voltage, no problem or the relationship between the voltages and current, no problem because whatever you do with those 2 equations, you can do with this circuit also. This is mathematical equivalence.

Student: Sir, then what is your problem with the physical thing? I mean what is lacking?



Sir: It may not be realized in that form, because that was a 4 terminal one and this is 3 terminals. Even if, all these elements are positive, it would not be physically equivalent of the previous circuit because that was a 4 terminal one this is a 3 terminal one.

Student: Sir but we can carry all the calculations.

Sir: You can carry all the calculations. Anything that you do with these 2 equations, you can do with this circuit also, because this circuit represents only base to equation, mathematical equivalence. Now, if it is not possible to calculate the y parameters by inspection, then what do you do? You do exactly the dual of what you did with z parameters.

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node eqns  
loop  
mesh eqns

$$\begin{matrix} Z_{ij} & Z_{ii} \\ = & R_{ij} + sL_{ij} + \frac{1}{sC_{ij}} \end{matrix}$$

In the case of z parameters you wrote node equations, here you shall write loop equations, not loop, mesh equations and in mesh equations, the coefficients would be coefficients of the dimension of impedance, that is, we shall have terms like  $Z_{ij}$  and  $Z_{ii}$  and let me introduce this definition, these terms that in loop, in mesh equations  $Z_{ii}$  is the self impedance of loop  $i$  and by self impedance, you simply mean, you go round the mesh and just add up the impedances.

This is the self impedance of mesh i and  $Z_{ij}$  is the mutual impedance between meshes i and j, that is, the impedance which is common to meshes, i and j and both of them, both of them are of the form in general  $R_{ij} + sL_{ij} + \frac{1}{sC_{ij}}$ .

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Handwritten matrix equation on a whiteboard:

$$\begin{bmatrix} V_1 \\ V_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & \dots & Z_{1k} \\ Z_{21} & \dots & Z_{2k} \\ \vdots & \ddots & \vdots \\ Z_{k1} & \dots & Z_{kk} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_k \end{bmatrix}$$

The matrix is labeled "mesh impedance matrix". To the left, it is noted that  $Z_{ij} = Z_{ji}$  and  $\Delta_{ij} = \Delta_{ji}$ .

So if you write mesh equations, the matrix that you shall get would be of the form  $V_1, V_2$ . These are the only 2 sources that you connect, all the rest are 0, would be equal to  $Z_{11}$ . Let me make capital  $Z_{11}$  to capital  $Z_{1k}$ ,  $Z_{21}$  to capital  $Z_{2k}$  and so on.  $Z_{k1}$  up to  $Z_{kk}$  multiplied by  $I_1 I_2$  to  $I_k$ , this will be the type of equations, set of equations that you get and whatever we have said about the y matrix, the admittance matrix, nodal admittance matrix also holds about the mesh impedance matrix. In other words, the self impedances occur along the diagonal and all of diagonal entries are mutual impedances, they are equal  $Z_{ij}$  equal to  $Z_{ji}$ . If the network consists of only bilateral elements, that is, if the network is reciprocal and therefore,  $\Delta_{ij}$  shall also be equal to  $\Delta_{ji}$ , where this  $\Delta$ , now refers to the determinant of the impedance matrix.

Student: (..)

Not necessarily, if you have a non-reciprocal, if you have a unilateral element, then no. It is not true because current cannot flow equally well in both direction.

Student: Sir but by taking  $\delta_{ij}$  we are just eliminating the  $i$ th and  $j$ th.

Sir: Yeah, but if its elements are not equal,  $Z_{21}$  is not equal to  $Z_{12}$ , then obviously, they will not be equal. Finally, what we do is we find out this currents  $I_1$  and  $I_2$ ,  $I_1$  would be  $\delta_{11}$  by  $\delta_{11}$  multiplied by  $V_1$ , plus  $\delta_{21}$  by  $\delta_{11}$  multiplied by  $V_2$ , from which you identify  $y_1$  and  $y_2$ . I think this is a good point to stop. We will continue this on the Tuesday, tomorrow, we shall be doing problem sets.