**Circuit Theory**
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**Lecture - 20**
**Concept of Delay and Introduction to 2-port Networks**

We are going to discuss a concept of delay and then begin our study of 2 port networks. 2 port networks will consume a bit of time because this is a very important topic in network theory, having applications in all other fields. The concept of delay is, can be introduced like this.

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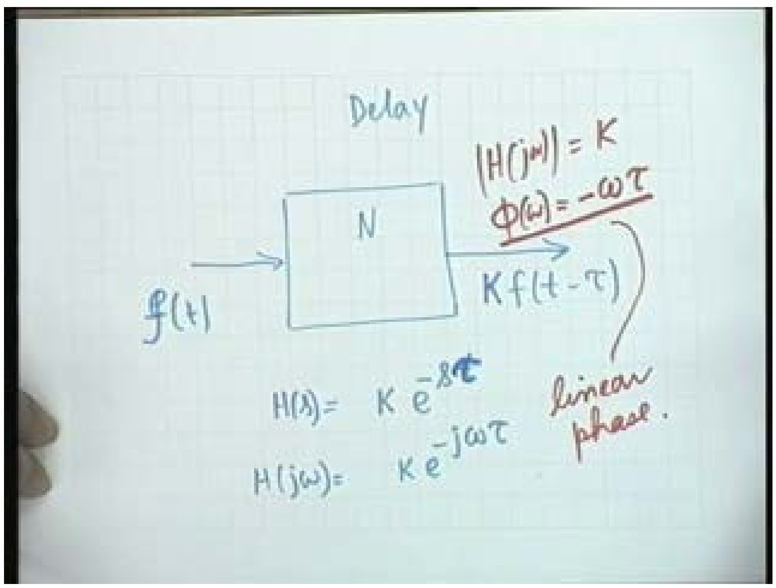


Suppose we have a network N and let us say, it is excited by, we are not showing a voltage or current, is excited by an input, let us say e of t, make it f of t where the f can be a voltage or it can also be a current and let the output be delayed by an amount tau. Let the network N be such, let the output is exactly of the same wave shape f, but it is delayed by tau. In addition, the network N may either amplify or attenuate the signal, so we put a constant K.

Now if you find the transfer function of this network H of s, Laplace of the output to Laplace of the input, it is simply e to the minus s t, s tau, e to the minus s tau, is that okay? Which means that the frequency response of this network is K e to the minus j omega tau, in other words, the magnitude  the magnitude of H is a constant magnitude of H of j omega is a constant and the phase phi omega is equal to minus omega tau.

Tau is the delay of the network and in the time domain description, you notice that there is no element, no parameter called frequency. It is simply a function of time. It is also a function of time.  Tau is a constant independent of any other parameter of the network. Now you notice that the phase under this condition is minus omega tau and therefore, phase is related linearly to frequency and therefore, this is a situation of linear phase. I could have phi omega equal to minus omega tau plus another constant, some are times phi, for example. Even then the phase is linear. It obeys a linear relationship with the frequency.

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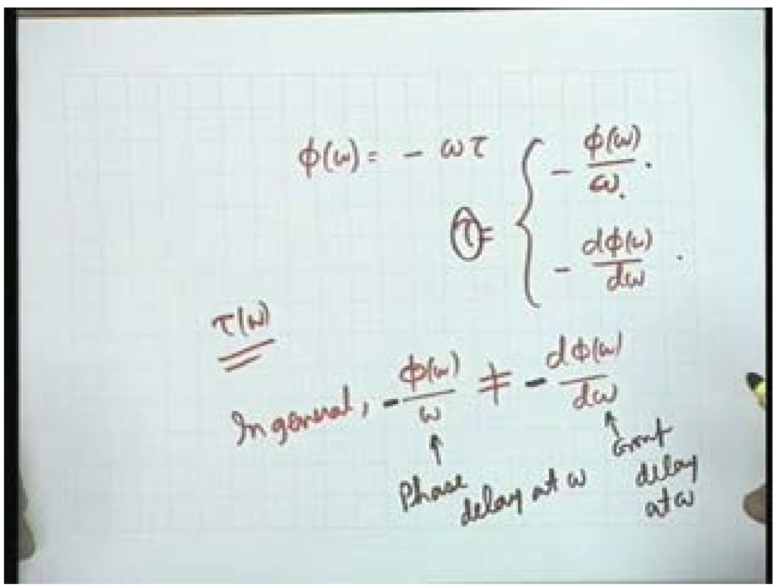


Now the impact or the importance of the linear phase is, shall be obvious in a few minutes. But look at this, phi omega equals to minus omega tau and tau, if phi omega is given, tau can be extracted in either of 2 ways. I can divide phi omega by omega, take a negative sign or I can differentiate phi omega with respect to omega and take the negative sign. Tau, in either case, tau is equal to, the same. However, this is a situation at frequency omega and when tau is a constant, there are circuits. There are networks in which tau is a function of frequency. The tau is a function of frequency and if tau is a function of frequency, tau of omega, then obviously, these 2 shall be different. These 2 are equal only if the phase is linearly related to omega with the constant equal to 0.

You see, if phi omega was minus omega tau plus another constant, then phi omega by omega would not be equal to d phi omega by d omega and therefore, this is the very special situation, in which, the phase is linearly related to frequency, with the phase at 0 frequency equal to 0, that is, the straight line passing through the origin. In general, this is not so. In general, phi omega by omega, in general, is not equal to d phi omega by d omega and, in general, this quantity with a negative sign is called the phase delay, at the frequency omega and this is called the group delay, at the frequency omega. You shall encounter these 2 terms, once again, in electromagnetic theory.

Student: Sir, why is there a negative sign?

Sir: Why a negative sign, because delay tau has to be positive. Tau has to be positive. You cannot advance a frequency, you do not,

Student: On the other side also.

Sir: Pardon me, on the other side also, yes, thank you. Oh, that was the mistake. So these are the general definitions of phase delay and group delay and the quantity that we shall be concerned, in network theory, would be group delay rather than phase delay. And therefore, we formally define the group delay or the delays, simply we shall call it delay and if we say delay, it shall mean group delay.

We define tau, the group delay or delay and, in general, it is a function of omega. It is defined as the negative gradient of phase minus d phi omega d omega. This is the definition of delay. Now the significance, why do we, well the physical significance, I have already told you that an ideal transmission system, which does not distortive wave form; an ideal transmission system. For example, a communications channel. The telephone line, for example, if someone speaks here, it should be heard on the other end of the line exactly as it is spoken and then the group delay. The delay is important.

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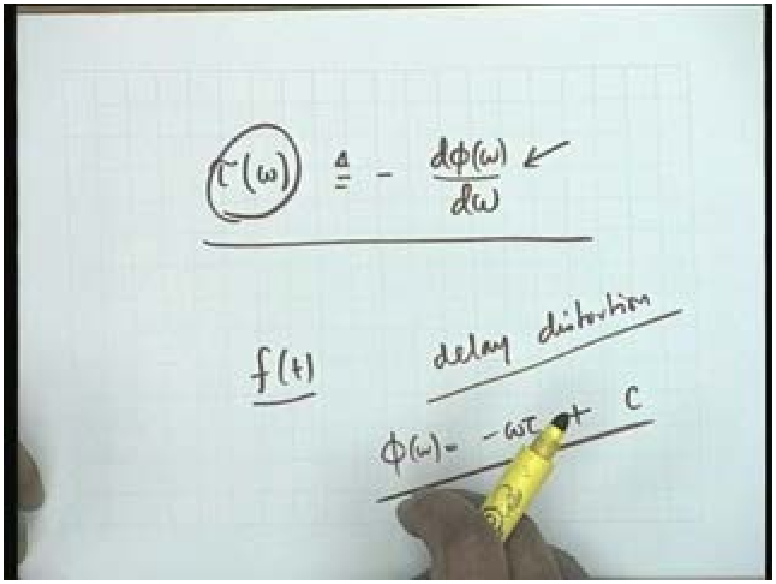


There is a delay. As soon as they speak, the speech does not reach the other end of the line. There is a delay. So delay is a necessary evil of any system. What we want is, let it be delayed, let it also be attenuated or amplified. If it is attenuated, we can put a repeater in the line somewhere. Repeater is an amplifier. A repeater to lift the signal, so that it can be harder at the other side, but what we do not want is, a distortion in delay. That is, we do not want tau to be a function of frequency.

If it does happen, then as you know, any function f of t, a speech signal, for example, consists of many frequencies. Right from 16 hertz to 16 kilo hertz and if these frequencies, these frequencies are together, at this sending end when I speak, these frequencies are together. Now if these frequencies arrived at different times at the receiving end, obviously, this speech would be distorted and this is called delay distortion and to avoid delay distortion, if there is to be no delay distortion, obviously, the group delay function or tau omega should be a constant and the condition for tau omega to be a constant is that, the channel should be linear phase.

You see, if the differential coefficient is taken, obviously, in linear phase, we can allow a constant, let us say c. We can allow a constant c and c is usually phi by 2. If at all there is a constant, it is equal to phi by 2. Is that point clear? So this is the importance of linear phase.

Student: What is it phi by 2?

Sir: Oh, that is because of a constant phase shaped phi by 2. That, we will see later. But the point that I want to illustrate to you is that the delay is a network parameter, a network performance parameter. It is not a parameter of the network, it is a performance characteristic of the network and ideally, we would need delay to be linearly related to frequency. Then there shall be no delay distortion.

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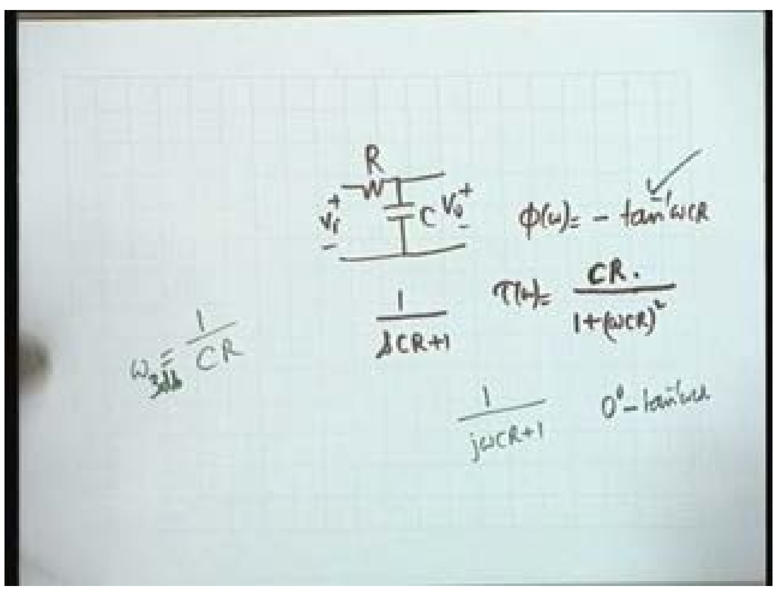


You can easily verify that any of these networks, simple networks, for example, R and C, the low past filter. What is the phase phi? Omega is equal to, well what is the transfer function of this? S C R plus 1. So if you put S is equal to j omega, this is the input and this is the output, so phi omega is minus tan inverse omega C R and therefore, tau omega is negative gradient of the phase. So it would be 1 by 1 plus omega C R whole square. Is that okay?

Now, the negative sign is taken care of, by the definition of tau and you notice, C R has the, if C R was not there, what would have happened? I f C R was not there, then the equation would have been dimensionally wrong because omega C R is dimensionless. It is adding to 1. You cannot add horses to donkeys. You can only add voltage to voltage, current to current, dimensionless quantity to a dimensionless quantity. This is perfectly all right, but tau omega has the dimension of time, so you must have C R, the time constant. Now it makes dimensionally correct. So and you notice.

Student: Sir, it will be phi by 2 minus?

Sir: Phi by 2 minus, why 1 by j omega C R plus 1, phase of the numerator is 0 minus, phase of the denominator is tan inverse omega C R, so this is correct. You notice that, at low frequencies, that is, omega C R much less than 1, the phase delay is indeed a constant, is not it right? And within the 3 db frequency, what is the 3 db frequency? Omega is equal to 1 by C R, this is the 3 db frequency, half power frequency and within the 3 db frequency, the delay remains to within a factor of 50 percent of its dc value.

The half delay bandwidth of this circuit is 1 by C R half delay, not 3 db. 3 db is the magnitude. It is not 1 by route 2, it is half. It is convenient to specify that way, because if omega equal to C R, you notice that tau omega is exactly half of the dc value, but even then there is delay distortion. Is not it right? Delay distortion will be absent only when omega C R is much less than 1, so that the delay remains a constant. Therefore, delay distortion is a fact of life and one tries to avoid it. Now let us see how delay is related to poles and zeros.

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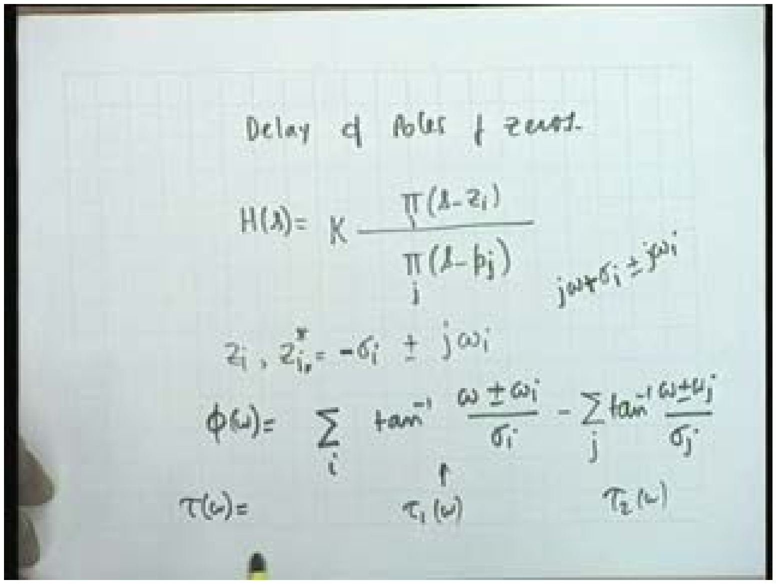


In general, as you know, H of s; the transfer function can be written as continued product as minus. Let us say z i over i divided by continue product s minus p j over j and there will be some constant K. z is and p js, they can be real, they can be complex. Real is a special case of complex, correct? Real pole or real 0 is a special case of a complex 0 or complex pole, in which the imaginary part is 0 and therefore, it suffices to consider complex poles and zeros. The real poles and zeros are special cases.

So let us say, z i equal to sigma i plus minus j omega i, then you notice that the angle, the phase of this shall be, phi omega, shall be, this is not z i, z i star, because we are considering real networks. If there is a complex pole, its conjugate must also occur, and phi omega you can easily see that it would be of the form of angle of the numerator, K does not contribute to phi, angle of the numerator, which will be tan inverse omega plus minus omega i divided by sigma i. There will be a plus term, there will also be a minus term over i.

We are taking pairs here. It will be implied that if there is a real pole, if there is a real 0, I beg your pardon, if there is a real 0, then there will be only 1 term that you can take care, minus summation over j tan inverse omega plus minus omega j divided by sigma j. This is the phase of the total system and tau omega you can now find out by taking minus d phi d omega.

Student: Sir, could you please explain why a constant omega here?

Sir: Why a constant here?

Student: (…)

Sir: How is it so? You see, j omega minus sigma i plus minus j omega i and therefore, I think I have, this would come. Let us put, this is minus, minus sigma then it will be j omega plus sigma i plus minus omega i. So the imaginary part is omega plus minus omega i. The real part is sigma i. That, obviously, sigma I is, what is the restriction at sigma i?

Student: It should be positive.

Sir: It should be?

Student: It should be positive.

Sir: No.

Student: It should be zero, sir.

Sir: Zeros are not restricted. It is, the restriction is on the poles. Whatever the case may be, this will contribute to a delay which will be negative. If the phase is positive, minus d phi d omega will be negative and this will contribute a delay tau to which will be positive and the overall delay must be positive. Overall delay must be positive, if the function will be such that the overall delay is positive. If the overall delay is negative, then obviously, the system is not realizable. Even if it is realizable, delay loses its meaning. Delay is a matter of definition, is not it? So delay losses its meaning. Anyway, let us look at one particular complex pole and see what happens.

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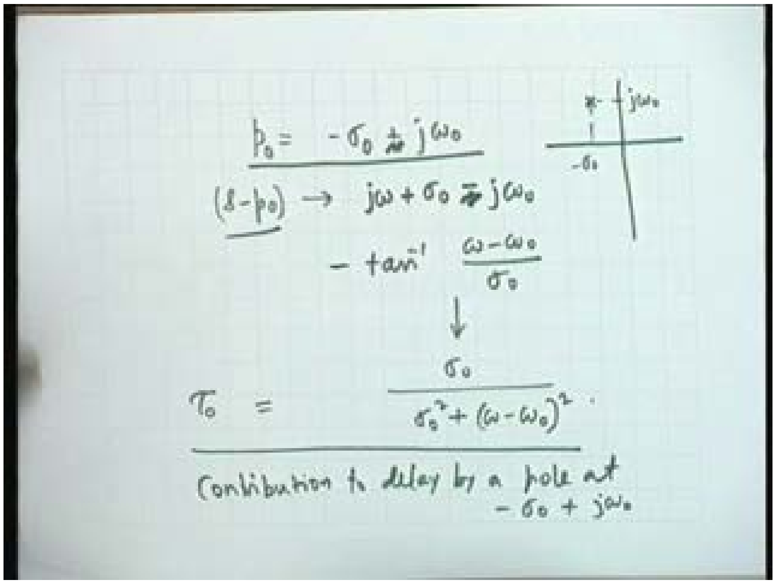


Let us look at pole due to, let us say, minus sigma 0 plus minus j omega 0. Let us say there is a pole p 0, which is this and therefore, the term s minus p 0, which with s equal to j omega becomes j omega minus j omega plus sigma 0 minus plus j omega 0. The pole factor would be s minus p 0. I am considering the particular pole. Let us take the plus sign. Let us take the plus sign here. One particular pole, its complex conjugate shall be can be treated same, in the same manner. If it is plus sign, then this will be minus sign. I am just taking one of the poles, here, where this is minus sigma 0 and this is j omega 0. Now due to this, the angle would be tan inverse omega minus omega 0 divided by sigma 0 and

Student: Sir, negative.

Sir: Negative. The angle contributed by this pole would be this and if I take minus d phi d omega, the expression for the delay, let us say, tau 0. The contribution to delay by the pole p 0, let us call it tau 0. Tau 0 would be sigma 0 divided by sigma 0 squared plus omega minus omega 0 whole square. This is the contribution to the delay by a single complex pole. Its conjugate, naturally, shall contribute to omega plus omega 0, agreed.

Student: Sir it should be sigma zero square in omega (…)

Sir: It should be sigma, no, it is not.

Student: Sir, why do you have negative sign there?

Sir: Why do you have negative sign, because this is a pole.

Student: But omega is zero sir and the contribution is positive.

Sir: When omega, the contribution is positive, for all values of omega, because omega minus omega 0 square cannot be negative and therefore, the contribution is always positive contribution due to a pole shall always be positive. Contribution to due to 0, will be negative, but the overall the sum of the 2 should be positive. Then delay has a meaning otherwise, it does not have a meaning. Now look at this, I emphasize, this is the contribution to delay. Let me write down, contribution to delay by a pole at minus sigma 0 plus j omega 0. You have to sum up all such terms, for all the poles. Let us, let me rewrite this expression.

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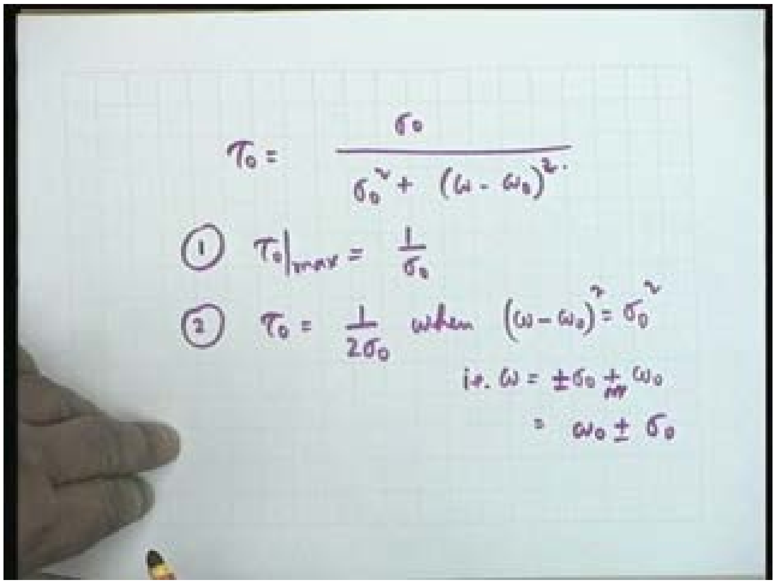

Tau 0 equal to sigma 0 divided by sigma 0 squared plus omega minus omega 0 squared. We notice several things that, number 1, the maximum value of tau 0, tau 0 maximum, obviously occurs, when the denominator is a minimum and the denominator is a minimum, the variable is omega, so the minimum value, this is 0, agreed and therefore, tau 0 maximum is 1 by omega 0. Second, the delay is equal to half of this maximum delay, that is, 1 over 2 sigma 0.

When omega, minus omega 0 squared is equal to sigma 0 squared, which means that omega, that is, omega is equal to sigma 0 plus minus omega 0 or is it the other way round? That is, omega 0 plus minus sigma 0. Now, obviously, these are the half delay points, exactly like half power points. The 2 frequencies omega 0 plus minus sigma 0 are the half delay points and if you go to the pole 0 sketch, omega 0 is somewhere here and this is sigma 0, minus sigma 0.

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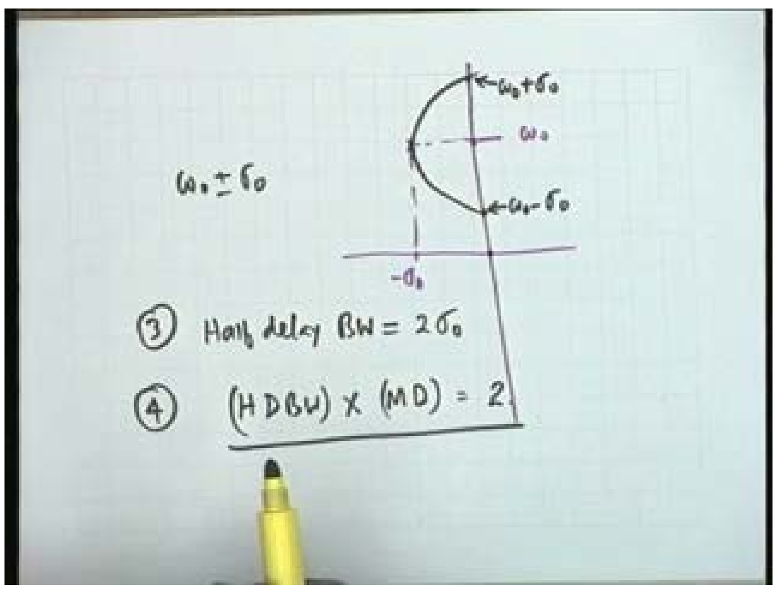


Now, if it omega 0 is the centre and sigma 0 as the radius, no, omega 0 is the centre and sigma 0 is the radius, correct. That is the distance, if you draw a circle. Next to that, if you draw a circle, where do they intersect? This will be omega 0 plus sigma 0 and this would be omega 0 minus sigma 0. So you can find, geometrically, the 2 half delay points as this, agreed. The 2 half delay points where omega 0 plus minus sigma 0 and you notice that if both of them have to exist, then omega 0 must be better than sigma 0, is that clear?

Due to a single pole, you see, otherwise it will go negative. That means, there are not two positive frequencies at which the delay is half of the maximum delay and the maximum delay occurs here at omega equal to 0 the dc value. You also notice that the half delay bandwidth is simply twice sigma 0. This is the third point to notice and the fourth point you notice is that half delay bandwidth multiplied by maximum delay is equal to 2, which is a kind of an uncertainty relationship, is not that right? That is, if you want a half delay large bandwidth, then your delay has to be small, agreed.

If you want a large delay, then it has to be a narrow bandwidth. The product is a constant and this kind of an uncertainty relationship you shall continue to get in electronics in communication and of course in physics, Heisenberg's uncertainty principle, about that momentum and position measurement. Now this reflects very well in network theory that they half delay bandwidth and the maximum delay, the product is a constant, you cannot increase more or you cannot decrease both. If 1 is increased, the other shall decrease. If the other shall, other increases the first one shall decrease.

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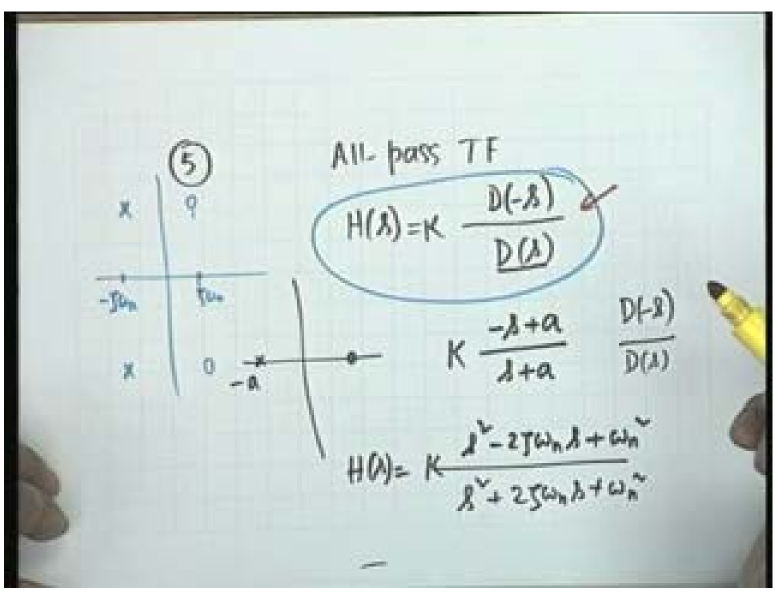

The final observation that we make at this point on delay is that, if you have an all pass transfer function. What is the characteristable of an all pass transfer function? If the denominator is D of s, the numerator would be?

Student: Twice of s

Sir: No. The numerator polynomial I have taken k out, so the polynomials, pardon me?

Student: D of s itself.

Sir: No. See if it D of s, pardon me?

Student: The numerator and denominator will cancel out and you will have a constant.

Sir: No, that it that should be for s equal to j omega magnitude should be the same. Magnitude of the numerator polynomial and the denominator polynomial should be the same, but I have already talked about all pass filters. Poles and zeros are mirror images of each other, so if D of s describe the poles, then D of minus s shall describe the zeros. Is not that clear? Let us have the first order. We have a pole here, let us say, at minus a. So my pole factor is s plus a. The numerator, that is, the 0 must be here which contributes to a factor of s minus a or minus s plus a and then you can adjust your constant. Adjust your constant, such that, it fits the given network and therefore, this is of the form, if the denominator polynomial is D of s, the numerator polynomial must be D of minus s.

It is the same story with regard to second order. Let us say, s squared plus twice zeta omega n s plus omega n squared. If this is the denominator and it is an all pass function, then simply change as to minus s. So this does not change but this changes, minus twice zeta omega n s plus omega n square. Then you adjust your k. This is the second order all pass filter and you notice that the poles and zeros shall be exactly mirror images. You go minus zeta omega n and you put the poles here, then because of the change in sign, all that changes is, it becomes plus zeta omega n and the zeros must be here.

You remember this, that the all pass function is described by a constant multiplied by 2 polynomials, multiplied by a rational function, one of which is the mirror image of the other. The polynomials are also called mirror image polynomial. The 2 polynomials are said to be relating through a mirror image which refers, obviously, to the routes of the polynomials. D of s routes at the poles, D of minus is routes are the zeros and they are in mirror image to, with respect to each other.

Now if that is so, what can you say about the angle of D of minus s and the angle of D of s?

Student: The same.

Sir: The same but it is change of sign.

Student: (…)

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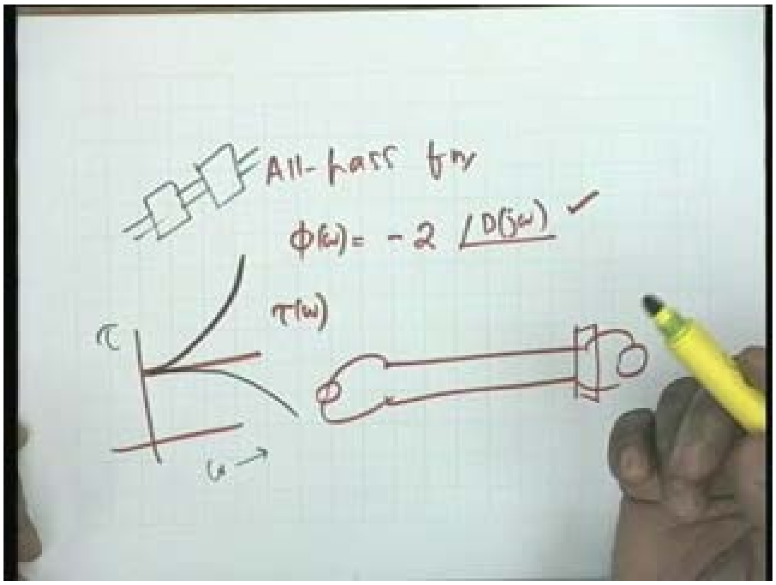


Sir: That is right and therefore, for an all pass network function, phi omega would be equal to minus twice angle of D j omega. Is that clear? Minus twice angle of D j omega. Angle of, minus angle of D j omega is contributed by the denominator and the same shall be contributed by the numerator. So if that is so, then for an all pass filter, finding out delay should be simpler because all you have to do is to find the angle of one of the polynomials multiplied by minus 2. That gives you the delay.

Now why am I discussing all pass in so much of details because that is the only use of all pass functions. All pass functions cannot discriminate in magnitude. It can only discriminate in frequency, in phase. So phi omega now, the tau omega that is given by an all pass filter, all pass function. It can be utilized to equalize channels, transmission channels, that is, you have a channel, let us say telephone line for example, and there is a source here that is a reception here and there is a delay distortion.

So what you can do is, if there is delay distortion then, before you connect the receiver, you can put an all pass network here if the delays, what I want is, my ideal is, delay should be a constant. Suppose the delay goes like this, then you can use an all pass filter whose delay goes like this, so that the sum of 2 delays is a constant. If you have 2 systems in cascade,

Student: Excuse me sir, this draft is between delay and frequency?

Sir: That is right, tau versus frequency. I want ideally to be a constant. If the channel goes like this, I can use an all pass function, whose delay goes like this, so that the sum of the 2 delays is a constant. Why am I taking this sum, because in cascaded system, the phases add the magnitudes?

Student: Multiplied.

Sir: Multiplied, if the phases add, the delay should also add and this is the use of all pass function. All pass functions are extremely important in coming (…) and the major use is delay equalization.

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Delay equalization, which means, that introducing an all pass function or all pass network to make the overall delay in the region of interest a constant or to make the system into a linear phase one. In that sense, we can also say that what you are equalizing is, it can also be called phase equalization system. Phase equalization means that phase is converted to a linear function of frequency, fine? Clear? Any question at this point?

Student: Sir this relation that you mentioned that you mentioned that uncertainty is a part of this, is it a design constraint at all, because it does not seem to be.

Sir: Is it a design constraint at all? You see, what I talked about was due to a single pole.

Student: Yeah.

Sir: Single pole. You have to combine it with its conjugate. You have to combine it to other poles. You have to combine into zeros and therefore, one cannot say whether the overall half delay bandwidth multiplied by overall delay is a constant or not, but for a individual pole, this is the case and you might design a network. You might design a second order or simply may be a first order network. For a first order network, this is obviously true. For a second order network also, it is true but it cannot be said whether it is true for the overall, but it is a kind of a universalization of a principle.

Student: Sir, that means if you are summing up all the phrases, you can say this.

Sir: You cannot say this, correct.

Student: And for,

Sir: Or you can say confidently, is about a single pole, yes.

Student: Yeah for that we will have to find (…) and again the half part?

Sir: Half delay, yes. Half delay bandwidth. Yes, any other question?

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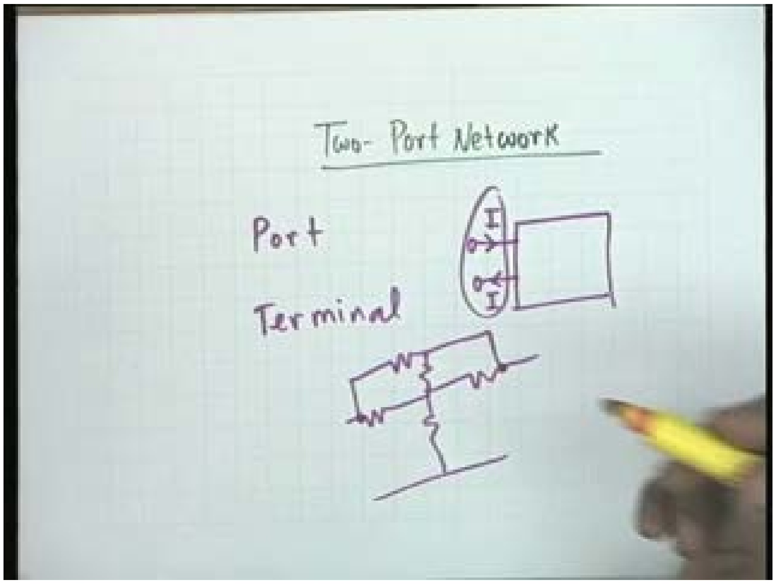


Sir: Well, we start our discussion on a 2 port networks, yes?

Student: Is t 0 the maximum value of?

Sir: Is t 0 the maximum? What is t 0? Not necessarily. Oh there are, I told you the equalization. You see, obviously, maximum will, is not here. It goes on, it could be that the delay shows a peak, that is also possible. In fact, the beat shows a peak. In the single pole case that we dealt, the delay was maximum when omega was equal to omega 0. So at omega 0 is a maximum, then it fall down. Any other?

We shall come back to delay later on in the course, when we talk about characterization of channels or networks in terms of its rise time and delay and we shall again meet the same uncertainty principle. This will come back a little later in the course after doing some of the more essential network descriptions and one of this description corresponds to concerns the 2 port networks. As I have already introduced to you, the terms port and terminal, would someone tell me what a port is?

Student: 2 terminals.

Sir: A port is a combination of 2 terminals that is all.

Student: Sir, from that we can input power or we can input (…).

Sir: Okay, so, suppose, a network is given, in which sources are already connected and your reception points are also known. The output points are also known. Now, how do you recognize the ports as a network is already given? This recording network, for example, that we have here; it is already given, it is already working a telephone network or a communication channel. It is given, how would you identify which 2 terminals form a port?

Student: The currents?

Sir: The currents, that is correct. A port is a pair of terminals. Unless you say that, it is not complete. It is a pair of terminals, such that the current entering into 1 terminal is exactly equal to the current leaving the other terminal. This is a port. In other words, this port can be terminated in a source or a load, does not matter, but for example, if you have a network like this, these 2 terminals may not form a port.

For forming a port, the essential condition is that the current entering one terminal must be equal the current leaving other terminal and if a network has 2 such ports, you call it 2 port. If it is only 1 port, then you call it a 1 port and if it is more than 2, we generally call it a multi port. We can use the terms 3 ports, 4 ports and so on. In fact there are, if you, if any of you go to microwave at your later stage in the carrier, you will encounter 6 port networks, like you encounter cars and buses on the road. 6 port network is a very common thing. Now what we are going to study is a 2 port network. 1 port is simple, all you can do is connect a current generator, measure the voltage or connect a voltage generator, measure the current.

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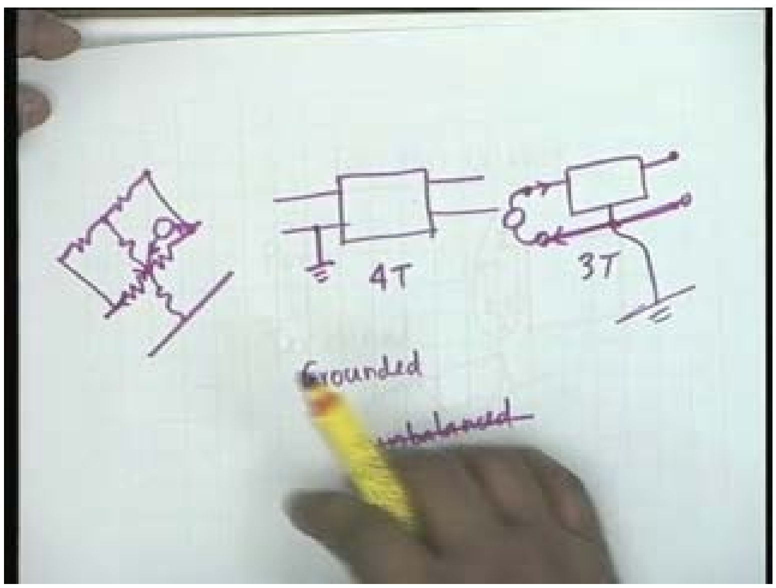


Let me review, let me recall also, some of the terms that we have introduced earlier with regard to a 2 port. A 2 port may consist of 4 terminals, in which, none of them are interconnected to any other. Say it is a true 4 terminal network when the potentials of each terminal is independently determined or you could have a 3 terminal network in which as indeed, there are 2 terminals at the top but there is a single terminal at the bottom. That is, the 2 ports have 1 common terminal. This is called a 3 terminal network.

Now this common terminal or any of these terminals can be connected to ground or may not be connected to ground. A network, in which any a 2 port network, in which any terminal is connected to ground, is called a grounded network. A grounded network, unlike grounded aero planes, it is nothing to do with that kind of a ground. It is simply setting a reference, let us say, grounded network. If a network, if a 2 port network has at least 1 terminal grounded, it is called an unbalanced network, unbalanced, because one of the terminals has been fixed at 0 potential. So the other terminals are unbalanced with respect to this.

Student: Sir, then what is this grounded, how is it different? So that means that 1 terminal in each port for the 2 port network should to be grounded, for it to be (…)

Sir: Unbalanced is it?

Student: No sir, for this gournded, does it mean that we should fix 1 terminal in each port to the ground? (…)

Sir: We will come back to this later. I understand your confusion. We will come back to this, we will not introduce this unbalanced terminal, we will do it later.

Student: Sir, in grounded, is 1 of the terminal is grounded?

Sir: 1 of the terminals is grounded.

Student: Sir, both the port are?

Sir: No, 1 of the terminals of the 4. Suppose this is grounded, then we called it a grounded. We will call it a grounded network. In general, these terms are popular terms and they have nothing to do with technical characterization. Technical characterization; 4 terminals, 3 terminals. This third terminal may either be connected to ground or may not be connected to ground, we do not care. It may be connected to ground or may not be connected to ground, but our voltage reference will measure the terminal, this terminal voltage with respect to this, to measure this terminal voltage with respect to this.

Student: Sir, in the 3 terminal case, the 2 output terminals may not form a port.

Sir: The output terminals have to form a port, otherwise.

Student: No sir, at the junction.

Sir: It is not a true port at this junction. No, we take terminals to the output. We take terminal to the input fairly and then you can connect a source here, the force this to be a terminal, the source has to become its current. We can do that, given any 2 terminals with the network disconnected or the network not energized, we can force any 2 terminals to become a port. Is that point clear?

Student: Yes sir.

Sir: No? Any 2 terminals to become a port, suppose, I have a complicated network like this and let us say, this terminal, this terminal and this terminal, they do not form a port. But I can connect a source then the current entering here must be the current leaving here. I can force 2 terminal is to become a port but that is not the issue. This not a question of forcing ports. A network has to be meaningful, it has to serve a particular path first, so it is designed and the ports are well identified ahead, in advance of the design. So that is

Student: Sir only when we require the source, (…)

Sir: Well, not necessarily, because there may be other shunt connections. There may be shunt, for example, let us see here. The current that is coming here is not necessarily going back here, is not it? So the question of identifying a port will be a trivial question. I am sorry, I took it in the other way round. Now the point, if the, we have to bother about now is given a 2 ports.

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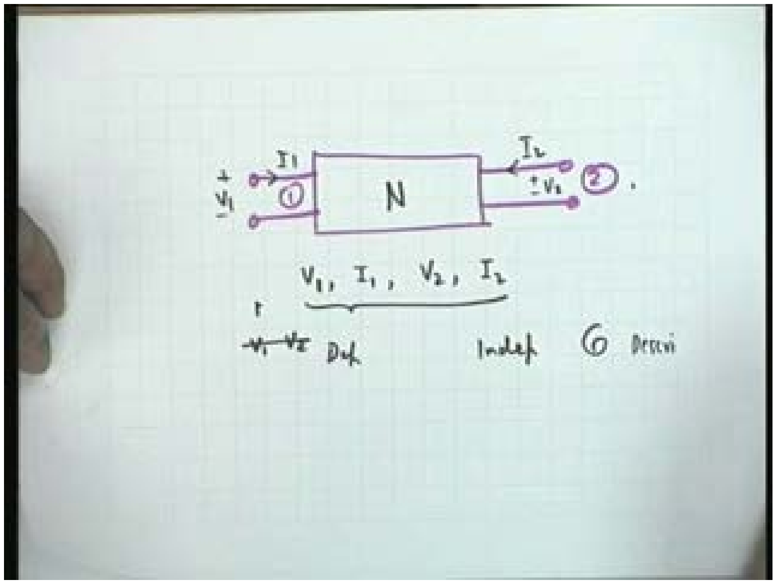


Will consider a general 2 ports, that is, a 4 terminal network, truly. We call this is port 1, we call this is port 2 and we identify 4 parameters, 2 voltages and 2 currents v 1, I 1 and v 2, I 2 and the convention is that currents go into the network exactly like the example of a 2 port that we have already dealt with transform. The current is going, this is the convention. So what we have is, and obviously, this is to be a port, then the current going out here must be I 1. The current going out here must be I 2. So the quantities that we have now, terminal quantities are 4 in number; v 1, I 1, v 2, I 2 and this is a black box. We may not know what is inside.

We want a terminal description or a port description or an external description of the system. What we can do is, we can consider any 2 of this as independent variable, then the other 2 shall be dependent variable because there are 2 ports. You can set 2 of this as independent, you can connect, for example, a voltage generator and a current generator here, then v 1 and I 2 shall be independent variables and I 1 and v 2 shall be dependent variables. Or you can connect both as voltage generator, you can connect both as current generator or a combination of voltage and current generator. Now, in how many ways can you do this, 2 quantities taken out of 4?

Student: 4 c 2.

Sir: So that is 4 c 2, which is exactly 6 therefore, there are 6 possible descriptions and these descriptions are if v 1 and v 2, these are dependent. Let me make a table dependent variables and independent variables and the description, let me go to the next page.

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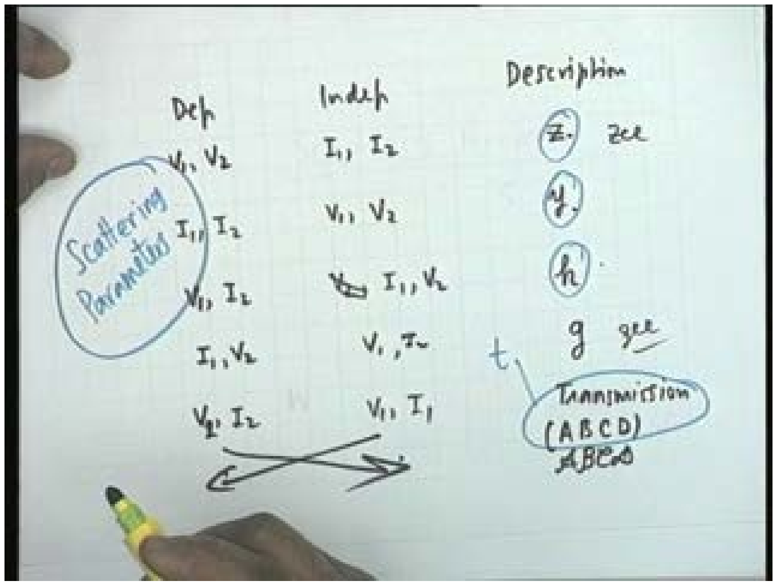


Dependent, independent and description, suppose v 1 and v 2 are dependent variables and independent variables are I 1 and I 2, then the description that we get are the z parameters, is so called impedance parameters or open circuit impedance parameters. As we shall see, if it is the other way round, that is, I 1, I 2 are the dependent variables and v 1, v 2 are the independent variables, then we get what are known as the y parameters, are the short circuit admittance parameters. We shall see the meanings of this open circuit and short circuit in a few moments.

On the other hand, if we have, let us say, v 1 and I 2 a combination v 1 and I 2 expressed in terms of v 2 and I 1, no, I would like to give the order same. So I will call this I 2, I 1 and v 2, I 1 and v 2. Then what we get are the hybrid parameters, h parameters. Hybrid parameters and why they are called hybrid shall be clear later. I am expressing a voltage and the current in terms of a current and the voltage and naturally, you see, the z parameters and y parameters, they should be inverses of each other.

Similarly, I can express I 1, v 2. I can bring I 1, v 2 here and v 1, I 2 here. The description will be the so called g parameter, small g. Please distinguish between g and z and this is z, g. Now another description is, I have

Student: Sir, that is v 2, I 1 and I 2, v1.

Sir: v 2, I, yeah, that is right. No, I do the same way. I 1, v 2 and v 1, I 2, there is a reason for putting into in this order, as we shall see later. We can have v 2, I 2 as dependent parameters and v 1, I 1 as independent parameters. Then this is called, this so called transmission parameters or also called A B C D parameters and if I repeat this, that this v 1, I 1, I bring here and v 2, I 2, I bring here. Then they are called inverse transmission parameters or they are sometimes denoted by script A B C D.

If I bring this here and this here, so these are the 6 possible cases and the cases that we shall discuss in details are, first of all, the z parameters, the y parameters, the hybrid parameters and the transmission parameters. The other 2 are not very much used all. The 4 of these are extremely useful. z and y are useful in almost all situations. These are parameters for all seasons. The h parameter, particularly, is useful in dealing with electronic circuits transistor circuits. The transmission parameters are useful in communication networks or communication systems, particularly when systems are to be cascaded, one after another. We will show that the A B C D parameters multiplied.

Well, in addition to these 6, for a 2 ports, a 2 port is a special case of a multiport in which the number of ports is equal to 2, for a multiport there is another description which is extremely useful at very high frequencies and these are called the scattering parameters. They are based on wave descriptions rather than voltage and current, and scattering parameters are extremely useful at microwaves.

We cannot do without them and this is an opportune moment to introduce scattering parameters to you also. Therefore, we shall discuss z parameters. This we shall call capital S parameters, scattering parameters or capital S. We shall introduce z y h A B C D and A B C D parameters for, are also sometimes called the small t parameters, transmission parameters small t. However, we shall prefer to use A B C D rather than small t because small t, t his slot, this symbol is reserved for a well respected physical variable namely time. We would not like to enter into that that kind of a controversy.

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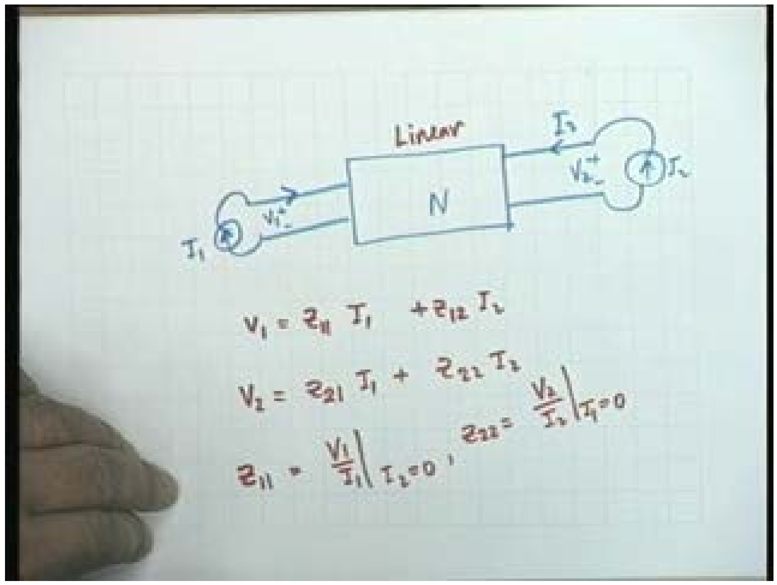


This scattering parameters, we shall describe for 2 port networks. They will be particularly useful for; number 1 transmission lines, which you shall study in electromagnetic theory and in microwaves, if you happen to take a course and a, let your course on microwave circuits well you cannot do without scattering parameters, so we shall discuss them also. To introduce the, first the z parameters, let me first write down the network description or the manner in which, in z parameters what are the independent variables the currents? I 1 and I 2. In other words, what we do is, connect a current generator here I 1, note the aero I 1 has to go here and let the voltage be v 1.

Connect a current generator here I 2, so that current enters here and the voltage is v 2. This is N. What we have to do is, to express v 1 and v 2. This are the only 2 other. These are the only 2 other parameters, only 2 other quantities, physical quantities that you can measure and therefore, v 1 and v 2, these are the dependent parameters. They have expressed in terms of I 1 and I 2 and because we are considering a network which is linear, the voltage v 1 shall be obtained by super position, that is, superposition of the effects due to 2 sources.

First take I 1, disconnect I 2. Find v 1, then take I 2, disconnect I 1, find v 1 and add the 2. In other words, v 1 shall be a linear combination of I 1 and I 2 and therefore, the coefficients, we can say k 1, k 2, f 1, f 2, whatever constants. But the useful nomenclature is z 1 1, z 1 2, these are the constants. Is that point clear? How we obtained this equation, a dependent variable v 1 is obtained by superposition of 2 effects. One due to I 1, the other due to I 2 and because the network is linear, they shall be superpositioned with constant multipliers and these multipliers we call z 1 1 and z 1 2.

Similarly, v 2 is obtained by taking z 2 1 I 1 and z 2 2 I 2. This is the, this is how the z parameters are defined and if you look at this, if you look at these parameters, one can immediately see that if you want to measure z 1 1, for example, this constant, all you have to do is to make I 2 0 and divide v 1 by I 1. So z 1 1 is the v 1 by I 1 with I 2 equal to 0. I 2 equal to 0 means the second port is open circuited, I 2 equal to 0 means no current, no current flow. Source is disconnected and therefore, no current flows because this is open circuited.

Similarly, you can see that z 2 2 is v 2 by I 2 under the condition that I 1 equal to 0. You also notice something interesting that z 1 1 is a driving point impedance, is not that right? It relates the voltage and current at the same port and therefore, it is a driving point impedance. Similarly, z 2 2 is also a driving point impedance.

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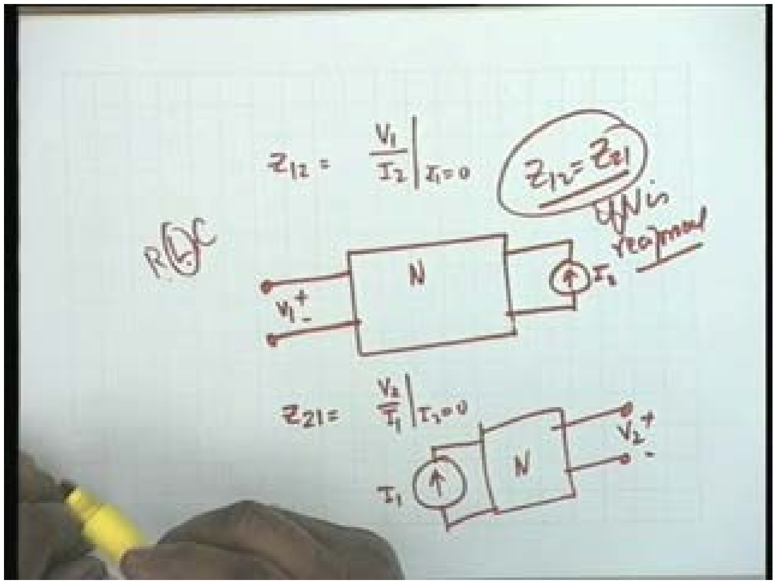


On the other hand, if you look at z 1 2, z 1 2 is v 1 by I 2 with I 1 equal to 0. Let us look at how this is done. There is a current source I 2 and this is kept open. I 1 is equal to 0, so the input port is open and you measure v 1. Obviously, z 1 2 is a transfer impedance, is not that right, because the effect and the cause are not at the same port. There are at different ports and therefore, this is a transfer function having the dimension of impedance. It is a ratio voltage to current. In a similar manner, z 2 1 is equal to v 2 by I 1 with I 2 equal to 0, which means that you excite by I 1, the network N and measure the voltage v 2.

If the network N is reciprocal, then the reciprocity theorem says that this and this are reciprocal situations and therefore, z 1 2 should be equal to z 2 1, if N is reciprocal. We are to start with considering only reciprocal network. We shall take a examples of nonreciprocal networks also later but we will see that if we consider R L C elements only, L includes mutual inductance, then the networks are reciprocal because each of these elements R L and C R are what is the term?

Student: (…)

Sir: No.

Student: Bilateral.

Sir: Bilateral. Each of this elements is bilateral because current flows equally well in both directions, therefore, the total network composed of bilateral elements has to be reciprocal and z 1 2 equal to z 2 1. In other words, we come to a very interesting phenomenon that, although in theory we required 4 constants, 2 of them are identical and therefore, this is a 3 parameter description of a 2 port. 3 parameter description holds only if the network is reciprocal. We will stop here and start again.