Circuit Theory

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Lecture - 18
Problem Session – 4: Network Functions and Analysis

This is the 18th lecture and we are on our problem session number 4. I have distributed two sets of problems. Problem set 4 and problem set 5, in the hope that we shall be able to do some problems from problem set 4 and also some from problem set 5. We start with 7.15. The first problem in problem set 4 and the problem is as follows.

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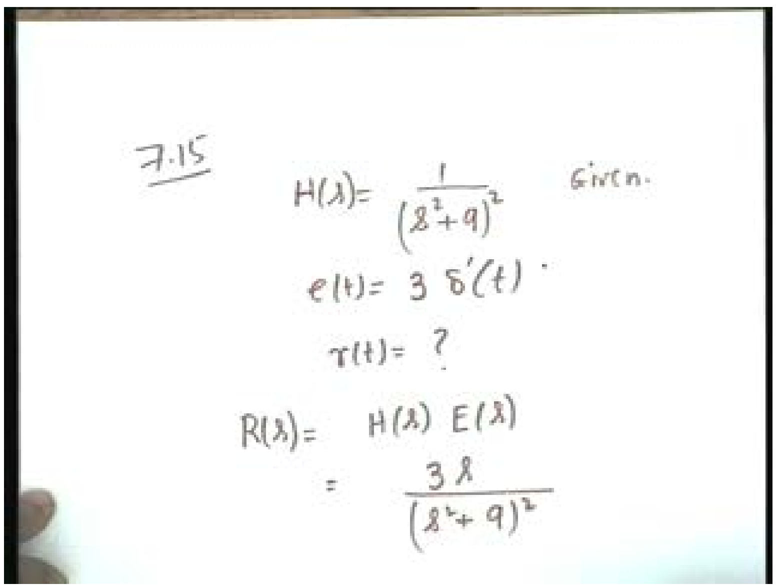

The system function is given as H of s equal to 1 by s squared plus 9 whole squared. This is given. If the excitation were e of t equal to 3 delta prime t, determine the response r of t. All that you have to do here, given the transfer function and the excitation, the response transform is equal to the transfer function multiplied by E of s and only thing you have to know is that the Laplace transfer of delta prime is simply s and therefore, you get 3 s divided by s squared plus 9 whole squared and the easiest thing to do is to view this as a product of two transforms and then take the convolution. This is not listed in a table and so what you do is,

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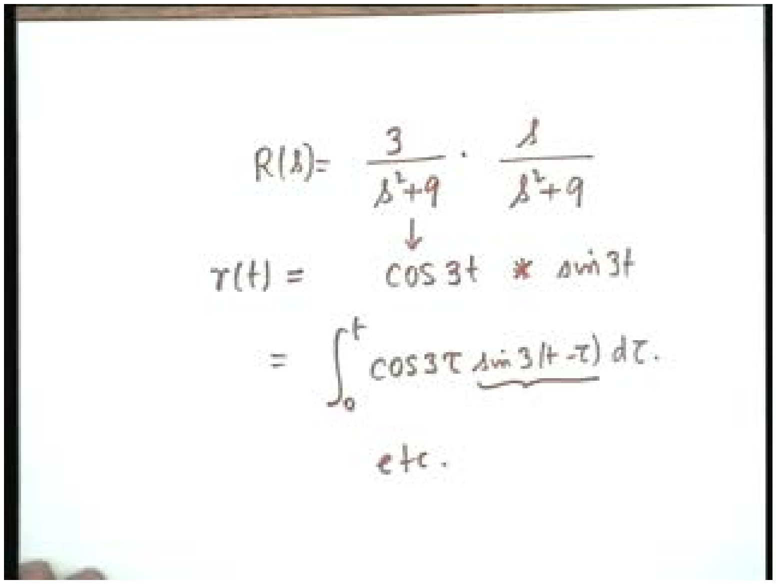

R of s, you write this as 3 divided by s squared plus 9, 3 divided by s squared plus 9 multiplied by s divided by s squared plus 9 and therefore, this is already listed in a table and therefore, r of t, the inverse of this is cosine. Is it cosine or sine? Cosine 3 t and the inverse of this is sine 3 t and therefore, you write r of t as equal integral 0 to t cosine 3 tau sine of 3 t minus tau d tau and then, you do the trigonometric simplifications. Sine of a minus b. You write this, multiply, take the t part out and so on and so forth. So I will not continue this.

Did I take the periodic convolution? No, I took the ordinary convolution. u t, of course is there but the upper limit is 0 to t. No, because this function sine 3 t, this is also okay. That is the reason. Both of them are multiplied by u t, thank you. It would have been incorrect if I had written like that and left it. The final function, of course, must be multiplied by u t. One second.

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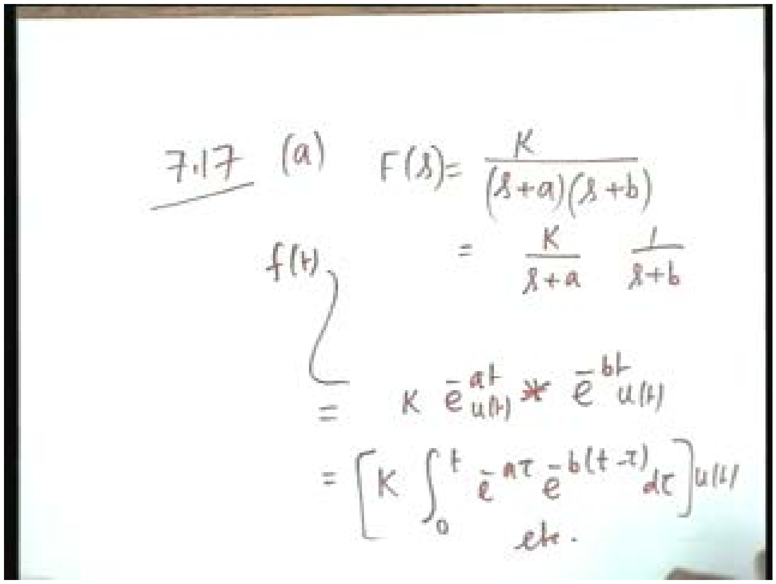

The next problem that I take is 7.17. I take only a typical problem. Let us say a. It says using the convolution integral, find the inverse transform of F of s equal to K divided by s plus a times s plus b. You have to find the inverse transform f of t by using the convolution integral. So what you do is, you write this as K by s plus a multiplied by s plus b and therefore, f of t is equal to K e to the minus at convolved with, once again u t and so this is K integral 0 to t e to the minus a tau e to the minus b t minus tau d tau and the whole thing shall be multiplied by u t and you can find out the response.

Let us go to tougher ones. The next one in my list is 7.19. The solutions to these problems, the ones that I leave out are available with Mr. Joshi, Wednesdays and Fridays, 4 onwards he would be available in his room. 7.19 is, using any method find v of t in this circuit where the e of t, there is a switch which closes at t equal to 0, there is a resistance 2 ohms and there is an inductance 100, this is v of t.

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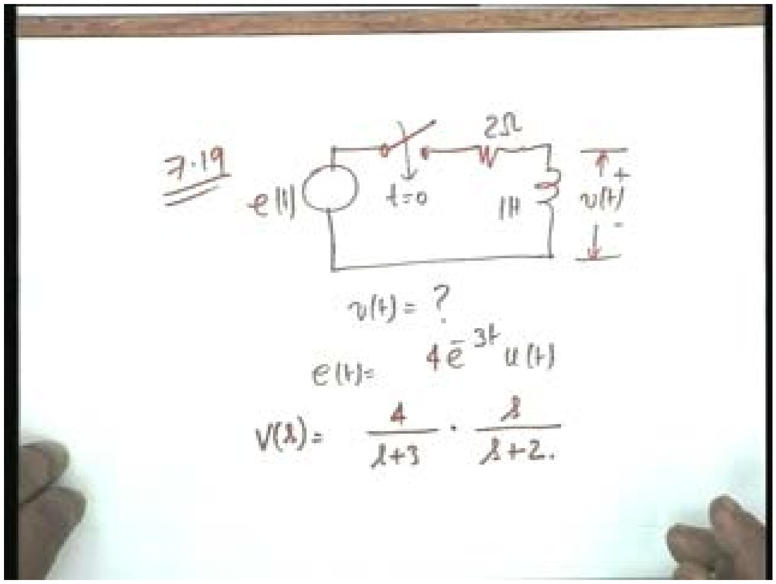

You are required to find out v of t if e of t is equal to 4 e to the minus 3 t u t. By Laplace transform, v of s is equal to e of s where this 4 divided by s plus 3 e of s multiplied by the potential division between 100 inductor and 2 ohm resistor. So it would be s divided by s plus 2, agreed? This can be written as, I have done this, for a change, I have solved it, completely.

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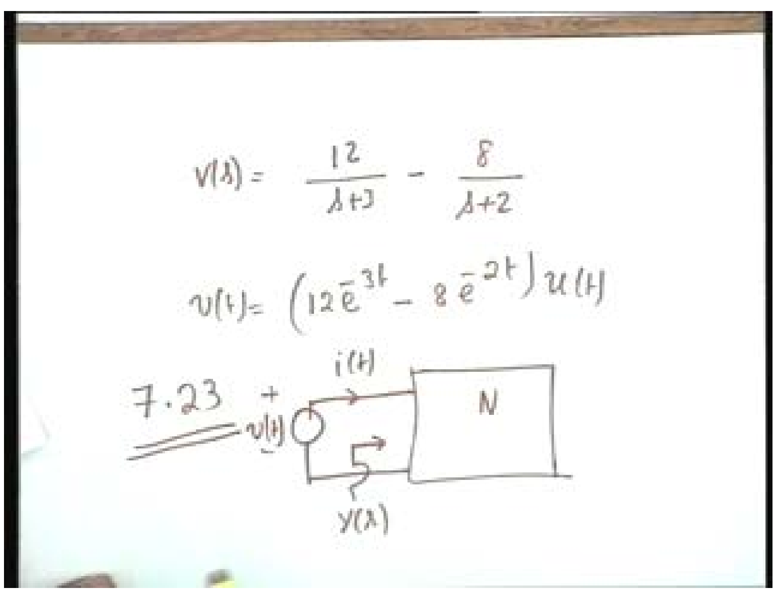

This can be written as 12 divided by s plus 3. I hope I am right, minus 8 divided by s plus 2, 12 times 2 is 24. Have I made a mistake? 4 s okay, 24 and 24 cancels, so 12 s minus 8 s is 4 s. I did by inspection and therefore, v of t is 12 e to the minus 3 t minus 8 e to the minus 2 t whole multiplied by u of t. Let us graduate then, any question? Let us graduate then, to a more interesting example. 7. 23. In between I have done 7.21 in the class as an example. That is, the poles are given. I have to find out the element values. I have done this.

So we go to 7.23.7.23 says there is a network N and there is a voltage source v of t plus minus and this current is, i of t. The admittance looking here is Y of s. It gives the following data. In this network, a unit step of voltage, that is, v of t equal to u t. A unit step of voltage is applied to the network and the resulting current, i of t is equal to 0 point 0 1 e to the minus t plus 0 point 0 2 amperes.

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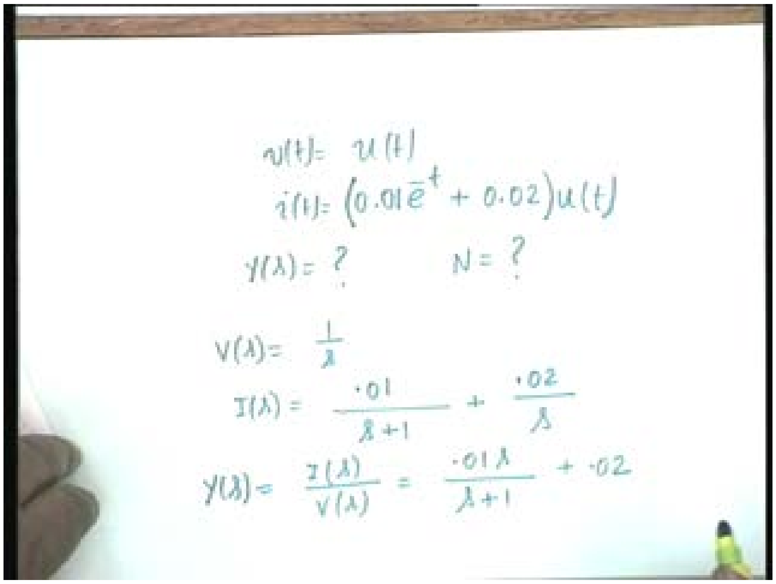

You are required to find out the admittance Y of s and a network that will yield this admittance function, so it is a synthesis problem. Given the excitation and the response, you are required to find out the input admittance. This is driving point admittance and you are required to find a network which realizes this particular admittance. Now if you take the Laplace transform, v of s is 1 over s and i of s is point 0 1 divided by, well, it is obvious that this should be multiplied by u t. It is not said so, but you, it is implied.

It is also implied that whatever the network N is because we have to work in the Laplace domain and find out admittance, input admittance, the network must be relaxed, initially. That is, the t will be 0 minus. The network contains no energy sources. These are implied; they do not have to be spelt out because if you recall the definition of network function is, transform of the output divided by transform of the input with the network initially relaxed.

So i of s would be point 0 1 divided by s plus 1 plus point 0 2 divided by s and therefore, Y of s, which is equal to i of s divided by v of s, will be given by point 0 1 s divided by s plus 1 plus point 0 2. This is my admittance and to find out a network, you are required to look at it carefully.

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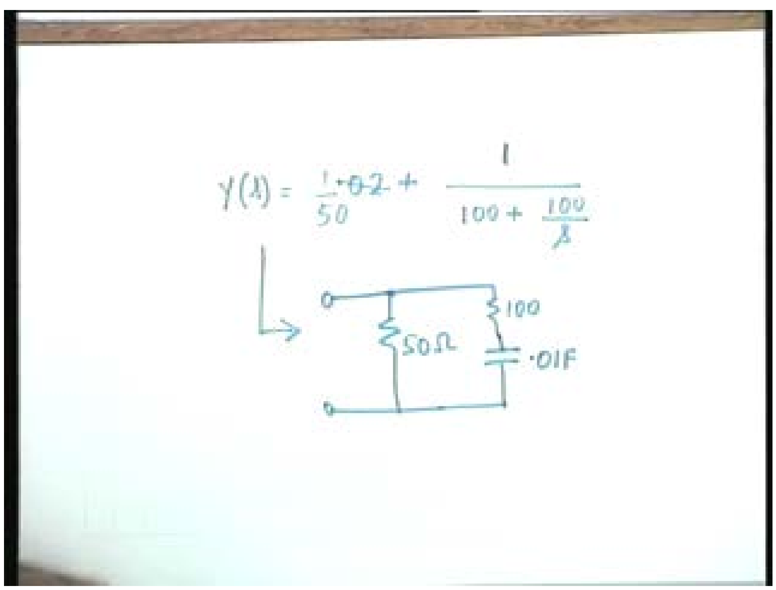

Y of s, admittance equal to point 0 2 plus, let us write the other term as 1 divided by, these are tricks of the trade and you will learn in due course, while working, while solving problems, divided by, this would be 100 plus 100 divided by s. Is that okay? Point 0 1 s, I divide both numerator and denominator. So I get 100 by s and 100.

Student: Better 1 by 50 plus

Sir: We will write 1 by 50, which means that this admittance is the parallel combination of this admittance and this admittance, so and this admittance is that of a 50 ohm resistor. Then this is admittance, so this must be an impedance. 100 plus 100 by s must be impedance. So it is 100 ohms.

Student: point 01

Sir: What?

Student: point 01 capacitance

Sir: Capacitor is point 0 1, that is, perfect. Then this is your first exercise in synthesis. We shall have many more. In fact, the next problem 7.24 is also a problem in synthesis, but we shall skip that. We shall leave that for you. Yes?

Student: Sir, why did not you check whether it is realizable or not?

Sir: Why did not you check whether it is realization or not? Can anyone answer this question?

Student: Because it is realized for real and rational

Sir: So real and rational does not mean necessarily realization.

Student: Poles are (..)

Sir: Yes, necessary but not sufficient. This is now, this answer to this question is the general philosophy of an engineer. Anything works. It is good enough. You see, by looking at the function, you can find out a network. Obviously, it must be realizable. If you could not find a network by obvious means, then the question of testing and all that comes. So be an engineer but do not lose your common sense, and the analytical capability, of course. We take problems set 5 now and the first problem, well, we skip the first one. First one is very simple. We take the second problem, that is, 8.2.

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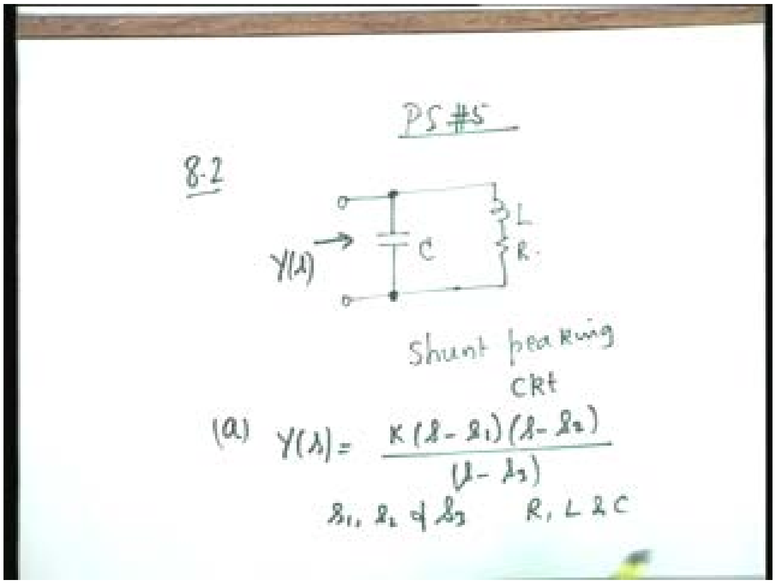

From PS 5, problem set 5, 8 point 2 says the circuit shown in the figure, this is the figure, c, then I have an L and then R. Obviously what we have done is, we have connected a capacitor across an inductance. Inductance inevitably has a resistance and that we have shown separately L and R. This circuit is a shunt peaking circuit, often used in video amplifiers. Shunt peaking circuit, you shall learn more about it in analog electronics, of course. Shunt peaking circuit used in video amplifiers.

The first question is, show that the admittance Y of s is of the form K s minus s 1 s minus s 2 divided by s minus s 3. Show that the admittance is of this form. This is the first question. Then express s 1, s 2 and s 3 in terms of R L and C, in terms of the element values. This would be straight forward, as we shall show. The second part of the question says that if s 1 is minus 10 plus j 10 to that 3, the 0 of the admittance is complex, 0 of the admittance is a pole of the impedance.

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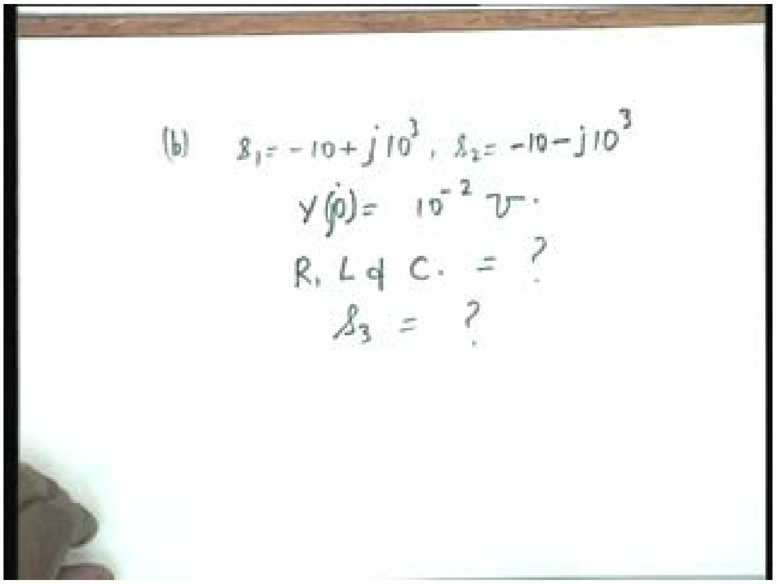

Is not it right? The impedance and admittance, driving point impedance and admittance are reciprocals of each other. So the pole of the impedance is complex, then s 2, one can write down without any further given data, it must be minus 10 minus j 10 to that 3. They must occur in complex conjugate pairs.

Third data that is given: Y of 0, Y of j 0. Y of j 0 and Y of 0 are the same 10 to the minus 2 moles. You have to find out R L and C and the numerical value of s 3, this is the question. I repeat, given this circuit C in parallel series connection of L and R, you have to first show that the admittance, driving point admittance is of this particular form. Express s 1, s 2 and s 3 in terms of R L and C, then a numerical part which says if s 1 s 2 are given like this and Y of 0 is given as 10 to the minus 2, find R L and C and s 3 in numerical terms. To solve the problem, we first solve the first part.

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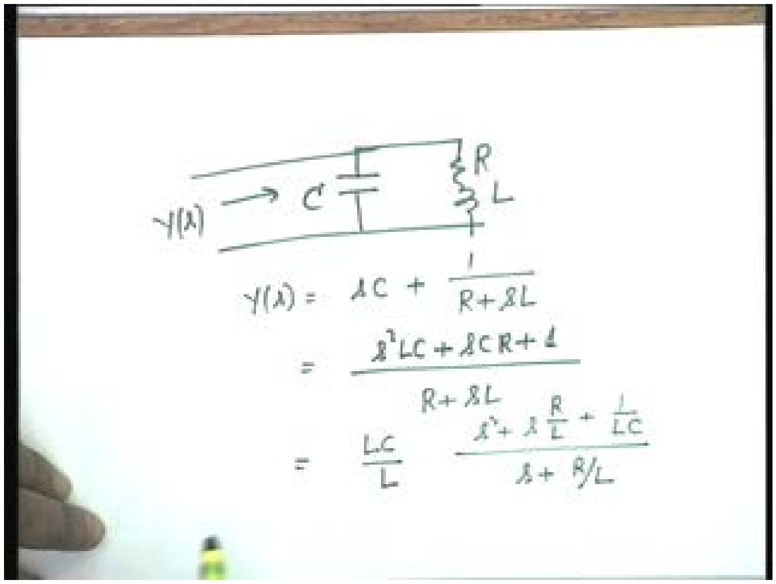

That is, we find out the admittance, C in parallel with R and L. This is my Y of s. So Y of s is equal to s c, the admittance of this plus 1 over R plus s L, which I can write as, we must be able to write this as K, some constant multiplied by a numerator polynomial with unity leading coefficient and a denominated polynomial, again with unity leading coefficient. So I get, let us write the term first, s squared L C plus s C R plus 1 which is equal to, now I make the numerator polynomial coefficient, leading coefficient unity.

So L C divided by L, from the denominator I get s square plus s R by L plus 1 over L C, then s plus R by L, which I can write as Y of s as equal to C multiplied by s plus R by L, then the numerator, I can write as s minus s 1 multiplied by s minus s 2, where s 1 and s 2 are the roots of the quadratic equations. So you get minus R by 2 L plus minus j. I take a j because it is given that the roots are complex square root of 1 by L C minus R squared by 4 L squared and therefore, I know what is s 1. I know s 2 and I know, of course, s 3 would be equal to minus R by L. The first part is solved, yes?

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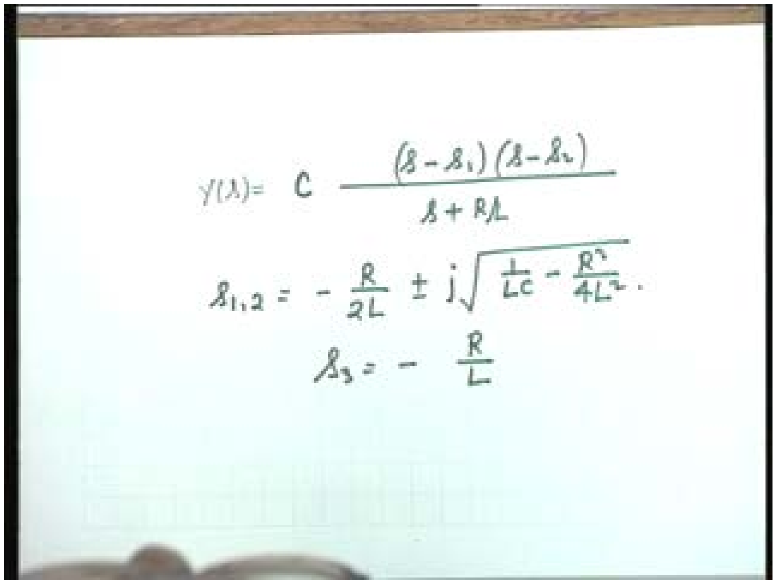

Student: You said that we are taking it as conjugate because it is given in the question, but initially, while discussing the question you told us that this s 1 is, this s 2 has to be conjugate of this. So

Sir: What is the question?

Student: Sir, now you are saying because it is given in the question, s 1 and s 2 are conjugates.

Sir: Yeah, that is it.

Student: initially to start with you said that if s 1 is this s 2 has to be a conjugate of this

Sir: Correct. Well, it is what both is. Ignore part b. Part a, this solution is correct, this solution is also correct. If 1 by L C is not greater than R square by 4 L squared, then the roots would be real. I have expressed it in this form because I have to solve part 2 which gives a complex conjugate pairs.

Student: In any way, roots can be real.

Sir: Roots can be real. This is not incorrect, this is correct. I have expressed it in this form, I could have expressed it plus minus square root of R square by 4 L squared minus 1 by L. That would have been perfectly valid. But because I have to solve the numerical part, I have expressed it in this form.

Number 2, second part of the question, if one of the roots is minus 10 plus j 10 to the 3, the other root has to be minus 10 minus j 10 to the 3, why? Why do they have to be conjugate? Because this circuit is real and therefore, its admittance is a real rational function and a real rational function shall always set roots in complex conjugate pairs. So the next part is Y of 0 is given as 10 to the minus 2 without any further calculation or without any look at the equation.

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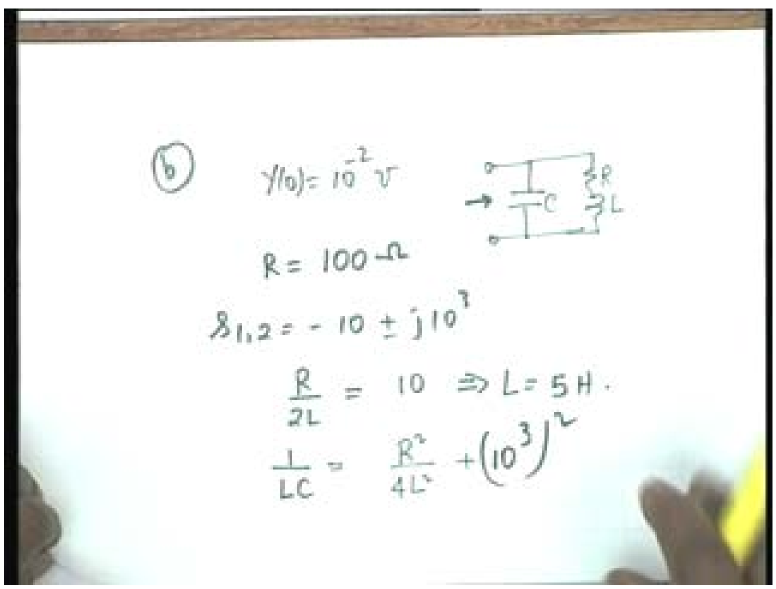

Is not it clear that a D C, at D C, what this network will see only this resistance and therefore, capital R must be equal to 100 ohms. Is that okay? Capital R must be equal to 100 ohms and since s 1 2 is equal to minus 10, this is part b, plus minus j 10 to the 3. R by 2 L, obviously, is equal to 10. The real part, which is minus R by 12, that must be equal to 10 and then therefore, L is equal to and capital R is 100, 500.

How do I find C? 1 by L C is equal to R square by 4 L square plus 10 to the 3, now.

Student: R squared by L C is equal to. Six.

Sir: This is 10 to the 6. That is all set. So you know R. You know L. Therefore, you can find out C and C my calculation gives it as approximately .2 micro Farad. All you have to do is to substitute in those relation values of L and R and therefore, s 3 is minus R by L and that should be equal to minus 20. The problem is completely solved.

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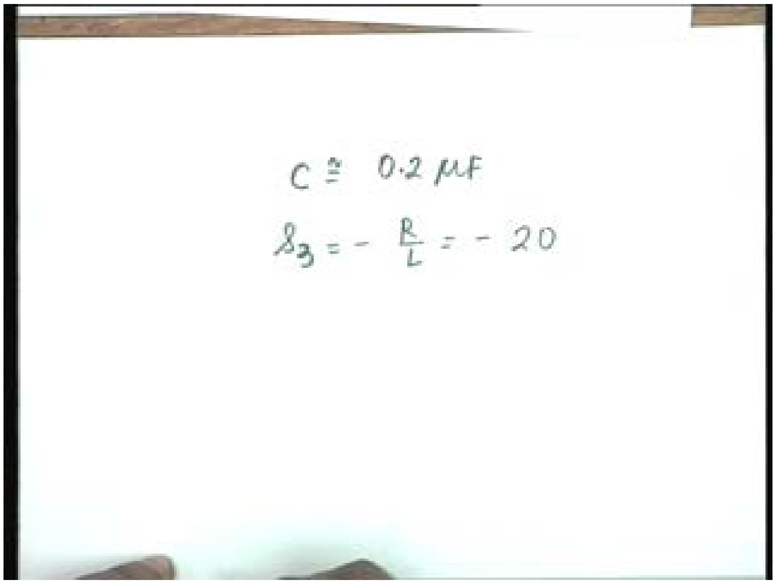

My next problem that I take is, I will skip 8. 3.8.5, 8 point 5 there are many problems, I will take only part h. Part h, the last one. That is, the problem is by means of the vector method sketch the amplitude and phase response amplitude and phase response.

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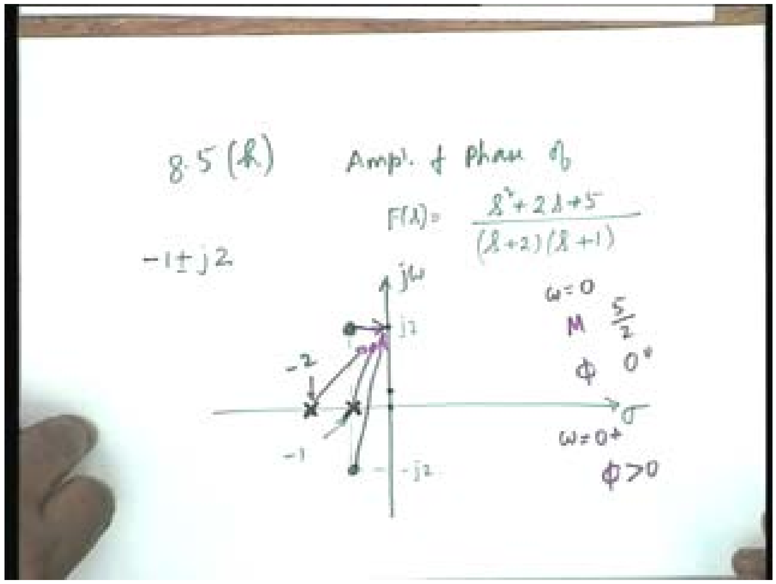

Vector method, no you cannot use mathematical method. That is algebraic simplifications and that is all. Amplitude and phase of f of s is given as s squared plus 2 s plus 5 divided by s plus 2 s plus 1. This is what you have to find out and therefore, the first thing you do is to locate the poles and zeros. Here also, I will solve part of it and leave the rest to you. Where are the zeros? Zeros are at

Student: Minus plus minus 2 j

Sir: Minus 1 plus minus j 2, so minus 1 plus minus j 2 this will be, I will take a point. This is 0. This is minus 1 and this is j 2, this is minus j 2. Then the poles are at minus 2, so somewhere here and minus 1. There is also a pole here. This is minus 2, should use some other. So all you have to do is to take points from here and go up representative points, find out the amplitudes and the phases.

Suppose, you take the point at the origin, omega equal to 0. Well, you do not have to measure the amplitudes because it is obvious from the equation, the magnitude would be

Student: 5 by 2

Sir: 5 by 2, the phase would be 0. Phi would be equal to 0 degree, because the angle of this is cancelled by the angle of this and the angle from here and here both are 0. They are same. What happens when omega equal to 0 plus? What about the angle? Capital N, I can find out now. I have to measure the various vectors but

Student: Sir we take only one convention for measuring the angle. Otherwise it will give you 2 pi or

Sir: Does not matter. I will confine between 0 and 2 phi, a principle value, as it is a square. Now what about 0 plus? Let us take a point here. Is the phase positive or negative? This is an important question. Obviously, what happens is, pardon me, is it positive or negative?

Student: Positive.

Sir: Positive, so phase is greater than 0. This is important to determine what happens when omega is 0 plus. Let us calculate the two, at least one representative point, let us say, of j 2. These are the vectors. Can you tell me what are the 0 vectors? This, obviously is 1. This one, this is 4 and this is 1. So under root 17, 4 plus 1, fine.

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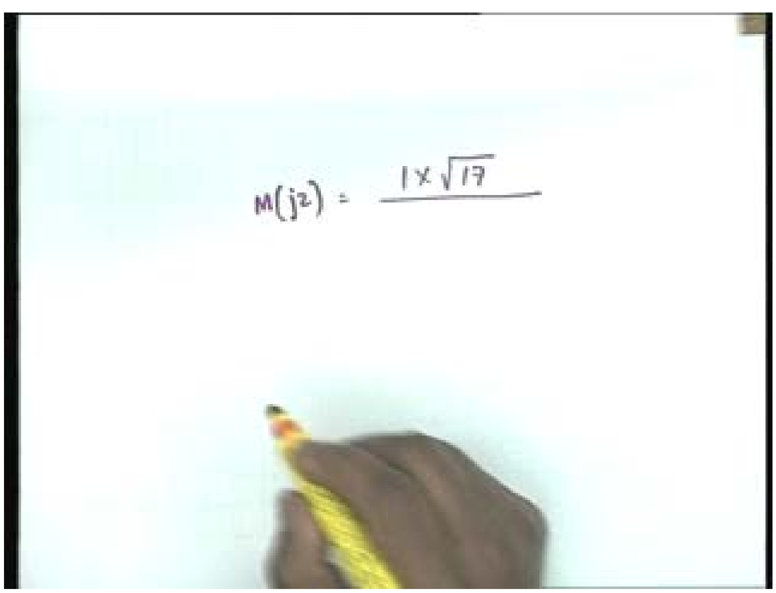

So the magnitude at j 2 would be 1 multiplied by under root 17 divided by

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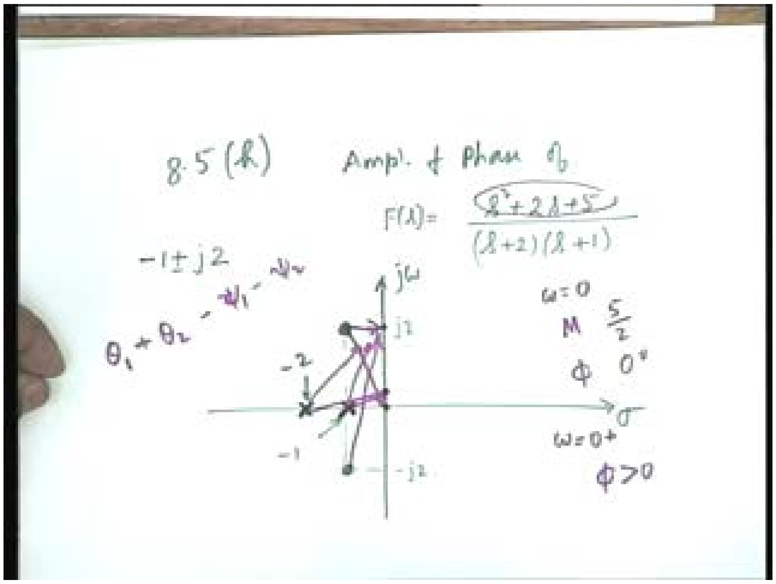

Student: Sir, can it be (..) ? How is it positive?

Sir: How is it positive? When the angle is here?

Student: Yeah

Sir: When the point is here, this angle you take, this as negative and this angle, this is positive. So they cancel.

Student: Sir but there are two conventions.

Sir: No, I am taking my principle value as I said.

Student: First, we are taking anti clockwise and otherwise, we are taking clock wise.

Sir: We said negative side. My total angle, I have to confine between 0 and 2 phi.

Student: This would confine the anything minus 5

Sir: We will do that from minus phi to phi. I can do it the other way also. It does not matter because it at in phase an ambiguity of 2 phi is of no concern, but an ambiguity of plus phi or minus phi is of great concern. So all I am doing is, I am taking this angle as negative and this angle as positive. If you are not satisfied, add 2 phi to it. That is all that.

Student: It changes the sign

Sir: Oh! That does not matter. 2 phi plus something is 2 phi plus theta is in the first quadrant, 2 phi minus theta is in the second quadrant, in the fourth quadrant. There is no ambiguity. There is an ambiguity at 2 phi, but 2 phi ambiguity does not make electrical engineers life miserable.

Student: That means, we cannot say that omega is equal to 0 plus 5 greater than 0 minus 5 greater than

Sir: No my convention here, you follow 1 convention that is all. You see, if you want to take the angle of this vector, as this angle, fine, you should continue to do that. It is not that for one particular frequency, you do this, for the other you take the negative angle, no.

Student: We should take one

Sir: Consistent. So if you take negative of this, then positive of this, these two are 0. The angle of this vector and this vector, this sum is either 2 phi or 0. Depending on the convention, I take it as 0. If it is 0, then when it is, when the omega is 0 plus, obviously this angle decreases. The negative decreases and this increases and therefore, the sum of these two is positive and this angle and this angle, they are of course positive and therefore, the total angle is positive.

Student: But it is added to the (..) coefficient

Sir: Absolutely wonderful.

Student: Is not we should write 2 pi?

Sir: Pardon me, no, wait a second. What they are saying is quite different. The angle of the numerator minus the angle of the denominator, so you have to add these two and subtract it from the angle of the numerator. So it would be the angle of 1 0 theta 1 plus the angle of the other 0 theta 2 minus, let us say psi 1 minus psi 2. The angle, so how do you resolve this issue? Actually, take a point and measure the angles and see if it is positive or negative, so one cannot say.

Student: It is (..)

Sir: By inspection, whether it is positive or negative, but it always comes as positive. I will verify. Wait, we will see one example. We will see at j 2.

Student: Sir, a particular case (..)

Sir: It is all right. Pardon me.

Student: A particular case make up another one.

Sir: Yeah, between 0 and j 2, I had verified. It always comes as positive; it is not obvious. You will have to argue it up.

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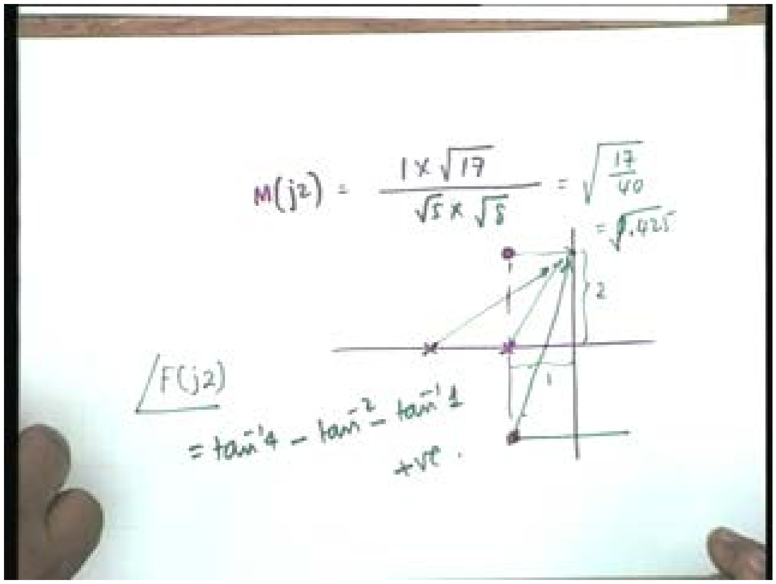

What about the other two vectors? Oh, my figure has become very complicated.

Student: Zeroes?

Sir: No, these are zeros. How can I make such mistakes, minus 1 and minus 2. Let me use another. I want to find out this vector, this vector, this one and this one. This distance is 1 and this distance is 2. So 1 times root 17, that is, perfectly root 17 is this length. What about this length, root 5 and this length?

Student: Root 8.

Sir: Root 8. So that is it. Square root of 17 by 40 and this comes as point. I did not calculate this square root of .425. You better calculate this one. But what is interesting is, angle of this, angle at j 2. This angle is 0, this angle tan inverse 4, tan inverse 4 minus tan inverse 2 minus tan inverse 1. It will be negative. I found this to be a positive. If it is not, correct me. Now as you go ahead, you will have to take a few representative points.

No, do not waste time on calculation. Now you do this later. I have told you that my solutions, you have to take with pinch of salt. Do not forget that you had, I want you to work yourself and verify and sometimes, I make intentional mistakes. That also, I have told you, I did and you pointed out once, well, in many more occasions and the more occasions there are, the happier I am, at least my intentional mistakes.

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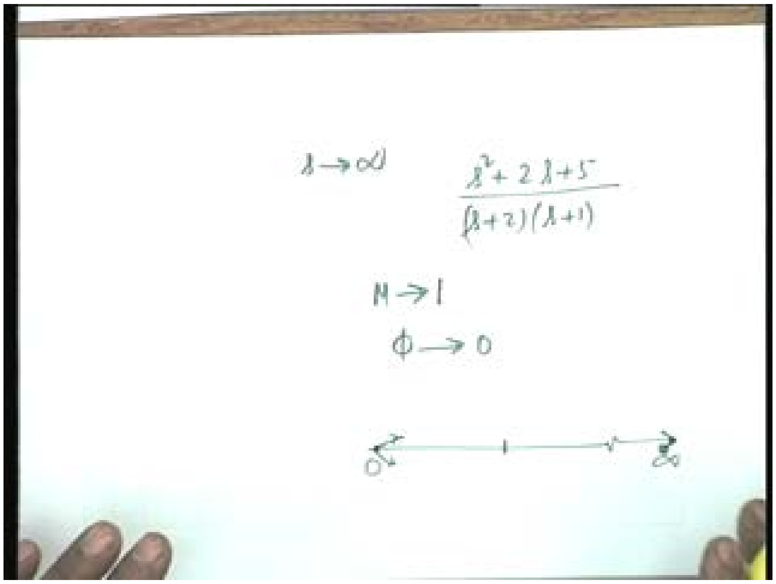

What happens when s goes to infinity? Now for that, I do not have to look at, I can look at the pole 0 diagram. I can also look at the transfer function. At infinity, obviously, the magnitude function becomes 1 and the phase, the angle goes to 0 magnitude goes to 1. The question that I am leaving you with is the phase always positive is it? It starts from 0, it goes back to 0 in the excursion. Is there any other point, any other frequency at which the phase can be 0?

That is a good question, because if there is a frequency, at least the phase is also 0. See, at 0, it is 0. At infinity, it is 0. So it can either come like this or it can come like this. It can either go like this or go like this. Fine, but in between, does it really monotonic? Is it always negative? Is there a point where there is a crossing. There could be all these points. I leave you with 3 question marks.

Positive or negative, is there a 0 crossing are not and so on. The next problem that we take is 8.8 or else keep 8. 9.

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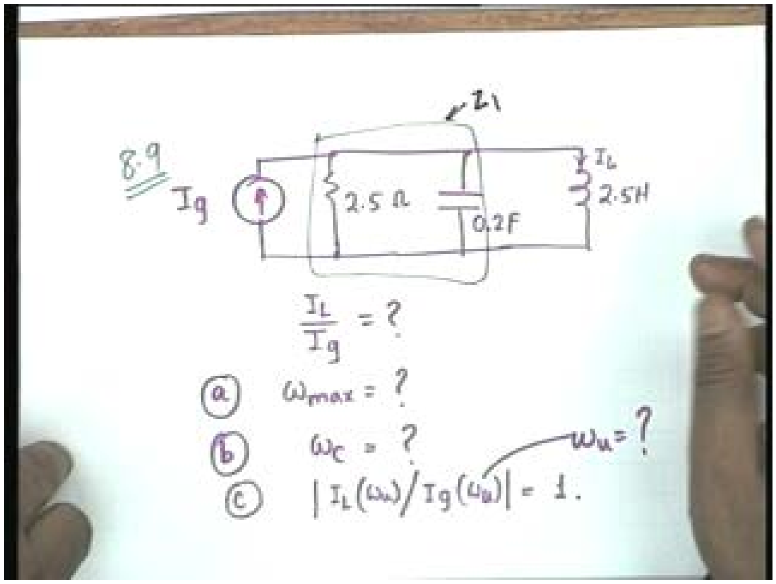

8. 9 for the circuit shown, a circuit is this. We have a current generator to bring variety into experience. We have a current generator in parallel with 2 point 5 ohms. Then, a point 2 Farad capacitor and an inductor of value, 2 point 5 Henry. This current is the load current. The transfer function here is again dimension less, but is a current ratio. It is a ratio of I L to I g. You are first required to find out I L to I g, I L by I g, that is, the transfer function and then find, the question is then, find the point omega max where the amplitude is a maximum.

Point omega max, the half power point omega C and the point omega u where I L omega u is to I g omega u is equal to 1 and the constraint is, use only geometric construction. You are required to find out omega u. In received part, you are required to find out omega u and you have to use only geometric construction. The problem, although looks a bit complicated, is simplicity itself. The solution, how we solve this to find I L, we will consider this as one impedance, Z 1, let us say. Then we use current deficient between g 1 and 2 point 5 in the inductor. That is how we find I L by inspection. No loop analysis, no node analysis, nothing and the result is

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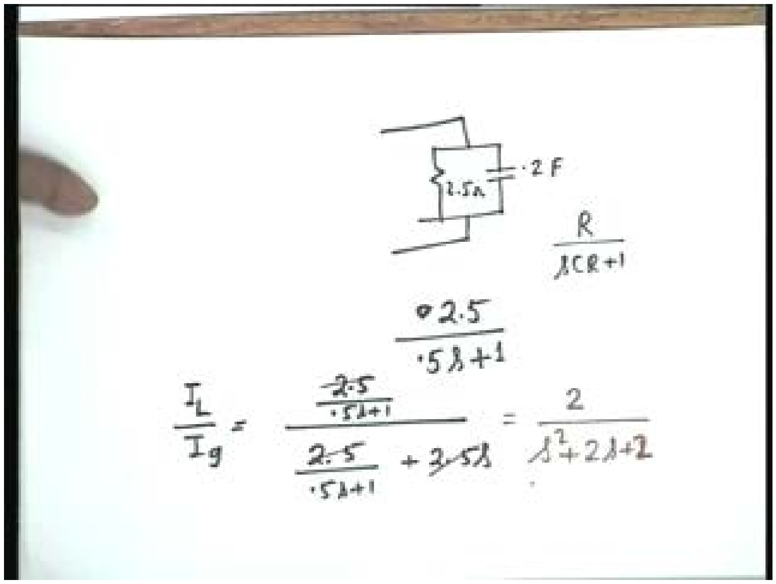

If I have a point 2 Farad capacitor and a 2 point ohm resistor, the impedance of this I have told you is R divided by s C R plus 1. So it will be point 2 divided by .2 into .2 5, is .5 right?

Student: 2.5

Sir: 2 point 5 multiplied by point 2 is point 5.

Student: The numerator is 2 point 5

Sir: Thank you. Numerator is 2 point 5, 2 point 5 by point 5 is plus 1. This is the impedance. This is the impedance of this and therefore, I L by I g equal to this impedance point 5 s plus 1 divided by the sum of the 2 impedances. That is, 2 point 5 divided by point 5 s plus 1, plus 2 point 5, agreed? The current through this would be this impedance, multiply this impedance, divided by the sum of these two. This is the current ratio.

Pardon me. Is it okay? This is Z 2, then I L by I g equal to Z 1 divided by Z 1 plus Z 2. That is it, elementary.

Student: That has to s 7 s 1 s is equal to, that is.

Sir: Where? Here?

Student: Is upper there, inductance.

Sir: Oh yeah, I have made a mistake and I could have come back anyway but good that we do not do this now. Now I can cancel 2 point 5. So my function is simplified to the following if you allow. Why not?

Student: 2 by s squared plus 2 s plus 1

Sir: 2 by s square plus 2 s plus 1. That is it.

Student: 2, plus 2.

Sir: Oh, 2.

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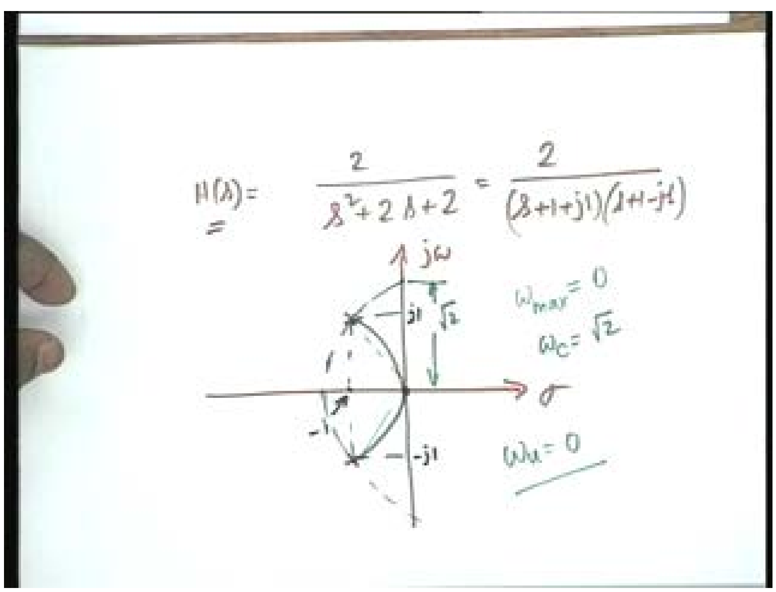

So my transfer function is now, I am in this. This s squared plus 2 s plus 2 and it represents a single tuned. It is similar to a single, this is a single tuned circuit 1 L, 1 C, 1 R, but a different kind of combination. It does not matter. I can write this is as 2 by s plus 1 plus j 1 s plus 1 minus j 1. Now no more analytics, I have to use only geometric construction. After I found out H of s, only geometric construction and let us look at the construction.

1 and 1 and therefore, the poles are here. This is j 1 minus j 1 and this is minus 1. Second part of the problem is to find the point omega max. Obviously, if I draw the peaking circle it touches here because this as the centre, 1 is the radius, it will touch here. Therefore, omega max is equal to 0. Therefore, it is truly a low pass filter. Maximum occurs there and there shall be only one upper half cut-off and that is obtained by taking this as centre and this as radius, you draw a circle. This is the half power circle.

Now, what would this be? Root 2 and therefore, omega C is equal to root 2 radius. What else is required to be found out? Where the amplitude is maximum and then point omega u, where this transfer function magnitude is 1. How do I do that? Oh, look at this. s equal to 0. At this point, I do not have to do K by sine psi and all that. So omega u, obviously, is equal to 0.

Sir: Pardon me.

Student: This is mathematical.

Sir: Yes, this is mathematical. If the magnitude is to be 1, then I go back to this K by sine psi or something.

Student: k sine psi.

Sir: But this is the only point where the angle is phi by 2 and sine of pi by 2 is 1 K is 1 here.

Student: Earlier you said this is a dual to the single tuned circuit.

Sir: This is dual to the single tuned circuit. This is parallel tuned circuit, but actual dual, it is not the dual, actual dual would have been C parallel L parallel R. This is the actual dual.

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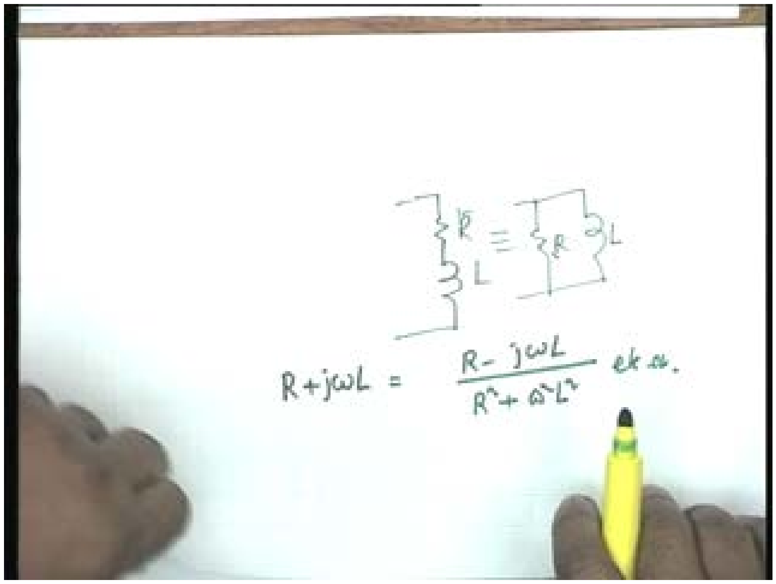

However, as we shall see later or even we can see it even now R and L, a series combination can be converted to a parallel combination, where the resistance and inductance both shall be frequency dependent. For example, what I do is R plus j omega L. I write this as R minus j omega L divided by R squared plus omega squared L squared etcetera. You can do the rest. So it can be considered as dual, provided we take the frequency dependent resistance and inductance R 1 and L 1.

Student: But it is in the configuration, they all are 3 and (..)

Sir: No.

Student: Yes sir, yes sir.

Sir: Oh, I am sorry, I beg your pardon. I was confusing with the other problem, other problem in which R 1 and L 1 are same. Yeah, this is truly a parallel. I beg your pardon. The next problem that I work is, I have 7 minutes. I will skip 10, 13, and 14. Let us do 15 that would be my last problem. 8 point 15.

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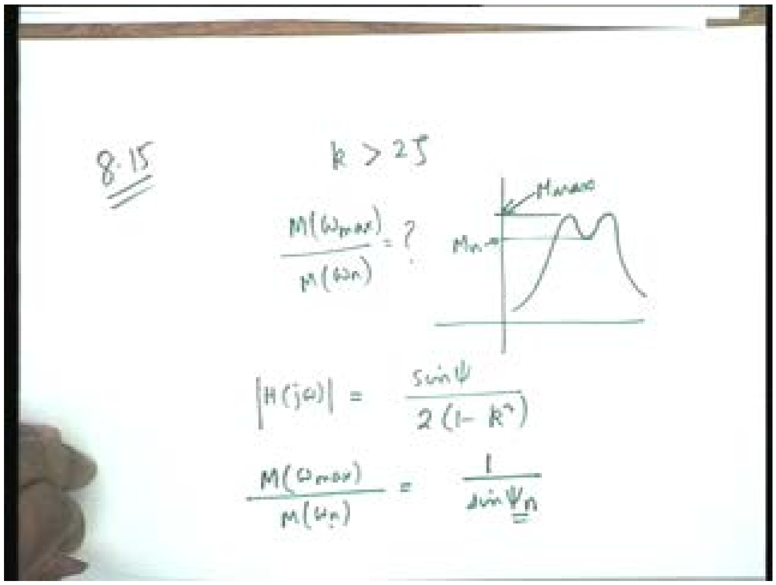

This will utilize the result that you did in the morning. That is, the double tuned circuit. For the over coupled case of a double tuned circuit, that is, K is greater than 2 zeta derive an expression for the peak to value ratio. That is M omega max divided by M omega n. You recall that for the over coupled case, the response is like this. So this is M max and this is M n, let us say. I have to find out the ratio of these two. Now if you recall, we had put H of j omega. This is what has to be found out. We had H of j omega in this form with all the usual approximations and all that, in the form that this is equal to sine psi, pardon me.

Student: M is only the magnitude

Sir: M is the magnitude. Sine psi divided by 2 1 minus k square, you recall this?

Student: Yes.

Sir: Therefore, the maximum value is 1 by 2 1 minus k squared and therefore, a M omega max to M omega n would be equal to, both will have 2 1 minus k squared and omega max occurs in psi is phi by 2. So it is only 1 by sine of psi and n, let us say. Psi n is the angle between the two vectors when omega is equal to omega n. So let us look at, is this point clear? In the ratio, maximum occurs in psi is phi by 2 and let us say, at omega n, the angle is psi n, angle between the 2 vectors. Then it is simply 1 by psi n. Now let us look at psi n. This will prove to be interesting.

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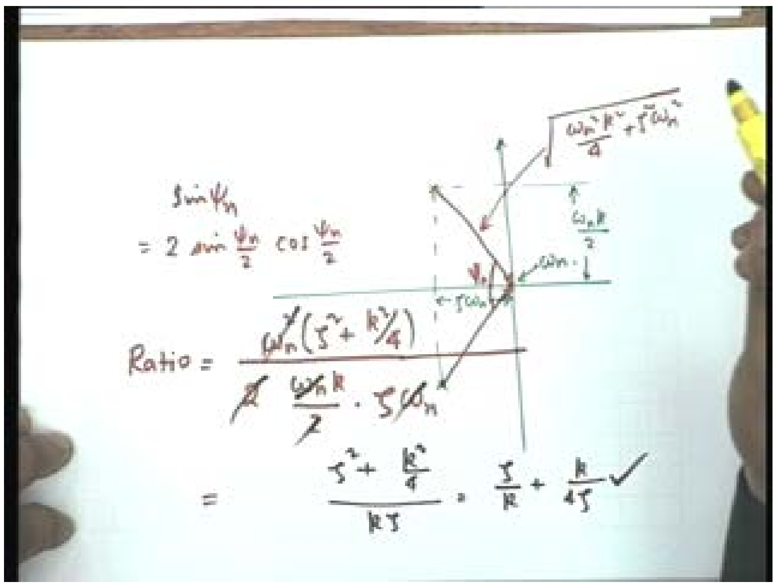

This is omega n and my poles are here. I am drawing a exaggerated figure. These are my poles. What I have to find out is, let me, how much is this? Omega n k divided by 2 and this is zeta omega n. So what I have to find out is, this is 1 vector, this is the other vector, I have to find out psi n. I can write psi sine of psi n as equal to twice sine psi n by 2 cosine of psi n by 2, because it is psi n by 2 which I can find out easily. Now to find out sine psi n by 2, this zeta omega n and this is omega n k by 2.

So the hypotenuse is square root of omega n squared k squared by 4 plus zeta square omega n squared. Is that okay? This is okay. This is the hypotenuse. So my ratio, the required ratio is equal to 1 over 2. Sine of psi n is perpendicular. That is, omega n k by 2 perpendicular divided by hypotenuse. That is this much. I can take it in the numerator. Cosine psi n will also give me this. So what I have is omega n squared zeta squared plus k squared by 4. What will be the numerator of cosine psi n by 2? Zeta omega n. Is this okay?

So this is equal to, I cancel out, I do not want omega n square. This and this cancel 2 and 2 cancels, so I divide by k zeta. I think I have made a mistake.

Student: Sir, it is the inverse.

Sir: What is inverse?

Student: Ratio. Sine psi is inverse.

Sir: Sine psi is okay, cosine sin also okay, but where have I made mistake?

Student: k squared by 4.

Sir: What is k square by 4?

Student: It is alright.

Sir: So I get zeta squared plus k squared by 4 divided by k zeta. So this becomes zeta by k plus k by 4 zeta. That is correct. Now, can this ratio be equal to 1? That is, the maximum and the minimum values are the same. Is it possible?

Student: I think it can be critical damping.

Sir: Critical damping, what is critical damping? Zeta equal to?

Student: 2 k

Sir: Zeta equal to 2 k.

Student: k is equal to 2 zeta. I think k is equal to 2 zeta.

Sir: I am sorry here. What you had is x plus x by 4. When is it maximum?

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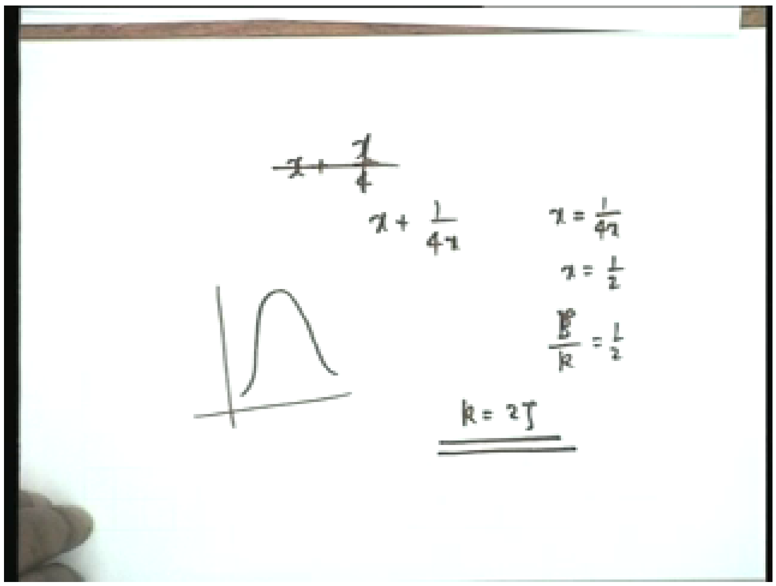

Sir: No, I am sorry, x, x plus 1 over 4 x. This is maximum when x is equal to 1 over 4 x. So x is equal to half. Do not you know when this is maximum?

Student: k is equal to zeta (..)

Sir: So x, x is k zeta by k, so zeta by k equal to half and therefore, k equal to 2 zeta. That is the critically coupled case. Does this stand to reason? Yes, of course, in critical coupled case, there is one maximum and that is the coincidence of three maxima. The two maxima that were occurring in the side, they gradually come together and the dip also goes up and the three coincide. So this is the case for critical coupling and it is under this condition that you get the maximum bandwidth without splitting of the peaks. So this is indeed, this indeed stands to reason. I think we will stop at that. Thank you.